Machine Learning 2 **Due: April 26, 2016**

Homework 3

Instructor: Ke Tran Email: m.k.tran@uva.nl

Student: Andrea Jemmett UvA ID: 11162929
Collaborators: N/A Email: andreajemmett@gmail.com

Problem 1. Consider the inference problem of evaluating $p(\mathbf{x}_n|\mathbf{x}_N)$ for the graph shown in Figure 1, for all nodes $n \in \{1, \ldots, N-1\}$. Show that the message passing algorithm can be used to solve this efficiently, and discuss which messages are modified and in what way.

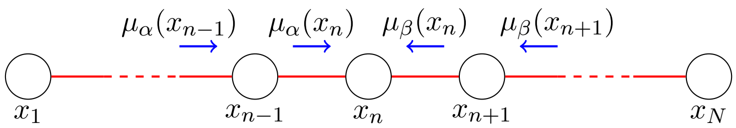

Figure 1: Chain of nodes model

Problem 2. Apply the sum-product algorithm to the chain of nodes model in Figure 1 and show that the results of message passing algorithm are recovered as a special case, that is

$$\begin{aligned}
p(x_n) &= \frac{1}{Z}\mu_\alpha(x_n)\mu_\beta(x_n) \\
\mu_\alpha(x_n) &= \sum_{x_{n-1}} \psi_{n-1,n}(x_{n-1}, x_n)\mu_\alpha(x_{n-1}) \\
\mu_\beta(x_n) &= \sum_{x_{n+1}} \psi_{n+1,n}(x_{n+1}, x_n)\mu_\beta(x_{n+1})
\end{aligned}$$

where $\psi_{i,i+1}(x_i, x_{i+1})$ is a potential function defined over clique $\{x_i, x_{i+1}\}$.

Problem 3. Run sum-product algorithm on the graph in Figure 2 with node x_3 designed as the root. Using the computed messages given in *p.409 Bishop*.

1. Show that the correct marginals are obtained for x_1 and x_3.
2. Show that the sum-product algorithm on this graph gives the correct joint distribution for x_1, x_2.

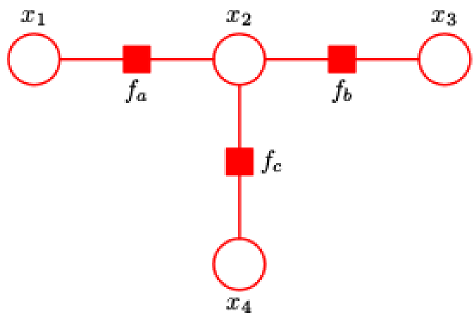

Figure 2: A simple factor graph

Problem 4. Show that the marginal distribution for the variables $\mathbf{x}_s$ in a factor $f_s(\mathbf{x}_s)$ in a tree-structured factor graph, after running the sum-product message passing algorithm, can be written as

$$p(\mathbf{x}_s) = f_s(\mathbf{x}_s) \prod_{i \in \mathrm{ne}(f_s)} \mu_{x_i \to f_s(x_i)}$$

where $\mathrm{ne}(f_s)$ denotes the set of variable nodes that are neighbors of the factor node f_s