UNIVERSITY OF AMSTERDAM

Multi-Agent Systems

False-Name Bidding and Economic Efficiency in Combinatorial Auctions

Colleen Alkalay-Houlihan and Adrian Vetta

Combinatorial Auctions

In combinatorial auctions, bidders express a valuation for every possible subset of the entire set of goods.

Definition

A combinatorial auction $\mathcal{A}$ is a tuple $\langle G, I, v \rangle$ where:

- $G = \{x_1, x_2, \ldots, x_m\}$ is a set of goods
- $I = [n] = \{1, 2, \ldots, n\}$ a collection of bidders
- v a valuation function such that for each set of goods $S \subseteq G$, bidder $i \in I$ has a non-negative value $v_i(S)$.

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A feasible allocation of the goods to these bids is a collection of pairwise-disjoint sets, i.e., an assignment $\mathcal{T} = \{T_1, T_2, \ldots, T_n\}$ such that:

- $T_i \subseteq G \; \forall i \in I$
- $T_i \cap T_j \; \forall i \neq j$

In the standard sealed-bid auction, each bidder i submits a bid vector $\mathbf{b}_i$ consisting of a bid $b_i(S)$ for each package S.

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Vickrey–Clarke–Groves Mechanism

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The social welfare of a feasible allocation $\mathcal{T}$ is $\omega(\mathcal{T}) = \sum_{i \in I} v_i(T_i)$.

Clearly, this objective is achievable only if the bidders bid truthfully; that is, they declare $\mathbf{b}_i = \mathbf{v}_i$.

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The Vickrey–Clarke–Groves Mechanism is a Groves mechanism $(\mathcal{X}, p)$, such that

$$\mathcal{X}(v) \in \arg\max_{x} \sum_{i} v_i(x)$$

$$p_i(v) = \max_{x} \sum_{j \neq i} v_j(x) - \sum_{j \neq i} v_j(\mathcal{X}(v))$$

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False-Name-Proof Mechanisms

Bidder	License 1	License 2	License 1 & 2
Dodgers	\$1 bn	\$1 bn	\$9 bn
Horizon	\$4 bn	\$4 bn	\$4 bn

Table: Auction of two broadband licenses

Under the VCG mechanism, Dodgers is assigned both licenses and pays \$4 billion.

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Table: Auction of two broadband licenses with false-names

The VCG mechanism will now allocate License 1 to Horizon-1 and License 2 to Horizon-2, while Dodgers receives no license at all. Horizon-1 and Horizon-2 both pay \$1 billion.

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Research Question

Truthful bidding is only a means to an end. The auctioneer desires truthful bidding as it should allow it optimise its objective – in this case economic efficiency.

Thus, if the incentives provided by a false-name-proof mechanism to ensure truthfulness themselves negatively impact this objective, then that mechanism will have little appeal to the auctioneer

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Is it possible to design a mechanism that will achieve high economic efficiency even if the bidders can manipulate the mechanism by making false-name bids?

The answer is yes and we quantify the extent to which the VCG mechanism has this property.

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Submodularity

We say that a collection of goods are substitutes if the demand for one is non-decreasing in the price of the others.

Goods are complements if the demand for one is non-increasing in the price of the others.

Example

- Substitute: Two different car models
- Complementary: Cars and gasoline

Definition

A set function $f : 2^X \to \mathbb{R}$ is submodular if and only if for all $A \subseteq B \subseteq X$ and all $x \in X \setminus B$: $f(A \cup \{x\}) - f(A) \geq f(B \cup \{x\}) - f(B)$.

✓ Goods are substitutes for bidder $i \in I$ if and only if bidder i's indirect utility function is submodular.

Example

Diminishing returns

$$f(\$0 \cup \$1) - f(0) \geq f(\$1M \cup \$1) - f(\$1M)$$

Results

Theorem

Given a combinatorial auction where each bidder has α near-submodular valuation, any Nash equilibrium S for the VCG mechanism obtained when one bidder makes false-name-bids has welfare

$$\omega(S) \geq \frac{1}{1+\alpha} \cdot \mathrm{OPT}$$

Conclusions

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Setting an auction with a particular substitutability level, allows the auctioneer to impose restrictions on the degree of submodularity ofn the bidding functions. This in turn controls the economic efficiency of the VCG mechanism.

Auctioneer $\rightarrow$ Substitutability $\rightarrow$ Submodularity $\rightarrow$ Efficiency

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