L^2-Norm

The ℓ^2-norm (also written "l^2-norm") $|\mathbf{x}|$ is a vector norm defined for a complex vector

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad (1)$$

by

$$|\mathbf{x}| = \sqrt{\sum_{k=1}^{n} |x_k|^2}, \qquad (2)$$

where $|x_k|$ on the right denotes the complex modulus. The ℓ^2-norm is the vector norm that is commonly encountered in vector algebra and vector operations (such as the dot product), where it is commonly denoted $|\mathbf{x}|$. However, if desired, a more explicit (but more cumbersome) notation $|\mathbf{x}|_2$ can be used to emphasize the distinction between the vector norm $|\mathbf{x}|$ and complex modulus $|z|$ together with the fact that the ℓ^2-norm is just one of several possible types of norms.

For real vectors, the absolute value sign indicating that a complex modulus is being taken on the right of equation (2) may be dropped. So, for example, the l^2-norm of the vector $\mathbf{x} = (x_1, x_2, x_3)$ is given by

$$|\mathbf{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2}. \qquad (3)$$

The ℓ^2-norm is also known as the Euclidean norm. However, this terminology is not recommended since it may cause confusion with the Frobenius norm (a matrix norm) is also sometimes called the Euclidean norm. The l^2-norm of a vector is implemented in the Wolfram Language as Norm[m, 2], or more simply as Norm[m].

The "L^2-norm" (denoted with an uppercase L) is reserved for application with a function $\phi(x)$

SEE ALSO

Angle Bracket, Complete Set of Functions, L1-Norm, L2-Space, L-infty-Norm, Parallelogram Law, Vector Norm

EXPLORE WITH WOLFRAM|ALPHA

WolframAlpha

2,5 torus knot

More things to try: 2,5 torus knot | expand (x^2 + 1)(x^2 - 1)(x+1)^3 | LT e^t sin 2t

REFERENCES

Gradshteyn, I. S. and Ryzhik, I. M. *Tables of Integrals, Series, and Products, 6th ed.* San Diego, CA: Academic Press, pp. 1114-1125, 2000.

Horn, R. A. and Johnson, C. R. "Norms for Vectors and Matrices." Ch. 5 in *Matrix Analysis.* Cambridge, England: Cambridge University Press, 1990.

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SUBJECT CLASSIFICATIONS

Calculus and Analysis › Norms ›

History and Terminology › Notation ›