EEN020 Computer Vision
Assignment 2
Calibration and DLT

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Calibrated vs. Uncalibrated Reconstruction

Theoretical Exercise 1

Let $\mathbf{X}$ the estimated 3D points, $\mathbf{x}$ the image projection points and $[\mathcal{R} \quad \mathbf{t}]$ a camera matrix of an uncalibrated camera. Then, we have:

$$\lambda[\mathcal{R} \quad \mathbf{t}]\mathbf{X} = \mathbf{x}, \quad \text{for a scalar } \lambda \in \mathbb{R}$$

But, if we apply a projective transformation $\mathcal{T}$ on the 3D points and the inverse transformation on the camera matrix, we can write:

$$\lambda[\mathcal{R} \quad \mathbf{t}]\mathcal{T}^{-1}\,\mathcal{T}\mathbf{X} = \mathbf{x}$$

We conclude that the identical image projection points can be retrieved for any projective transformation when estimating motion $\big([\mathcal{R} \quad \mathbf{t}]\mathcal{T}^{-1}\big)$ and structure $\big(\mathcal{T}\mathbf{X}\big)$ simultaneously with uncalibrated cameras.

Computer Exercise 1

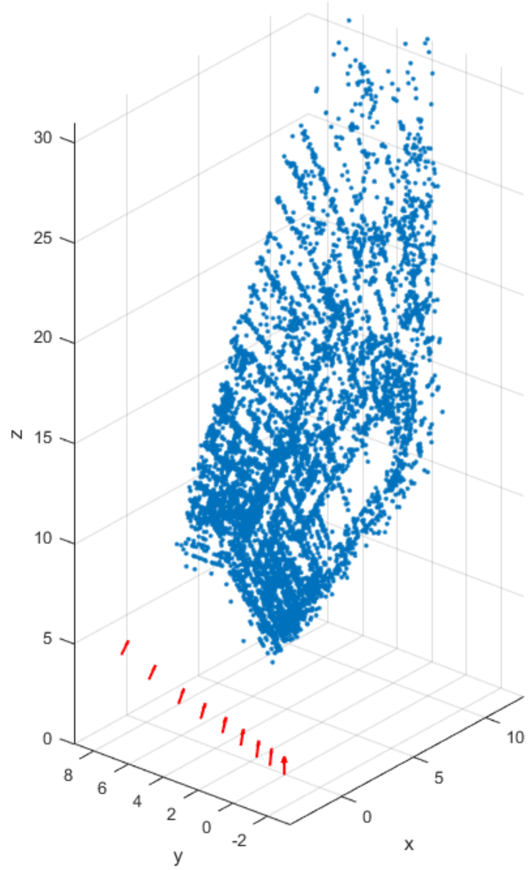

Figure 1: The uncalibrated reconstruction and the principal axes of the cameras.

By observing Figure 1, the uncalibrated projective reconstruction points X do not preserve the physical properties of the building. All lengths and the wall angle are distorted. What is preserved is only straight lines.

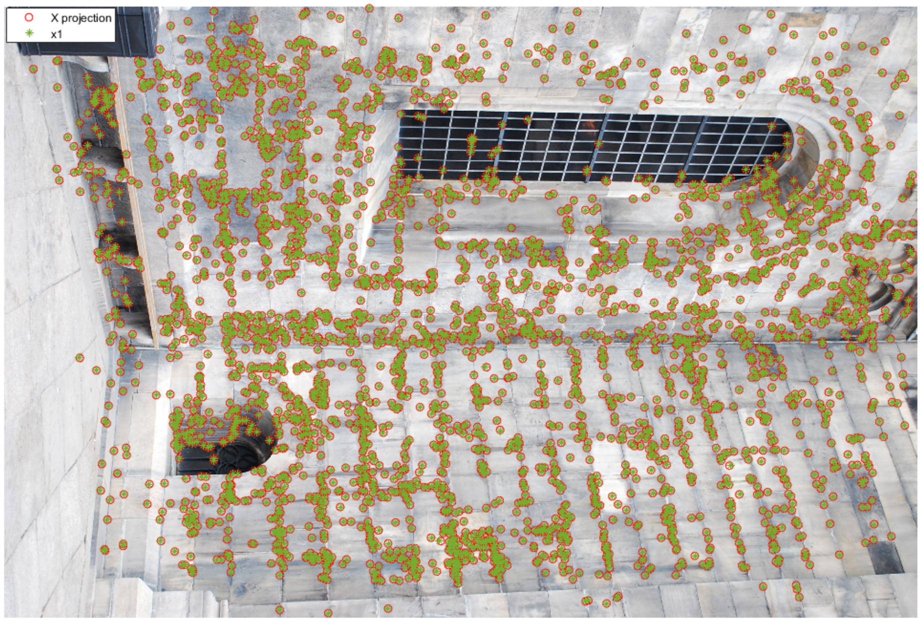

Figure 2: The projected reconstructed points X and the homogeneous coordinates x1 on image 1.

In Figure 2, the projected X points and the homogeneous coordinates x_1 overlap everywhere.

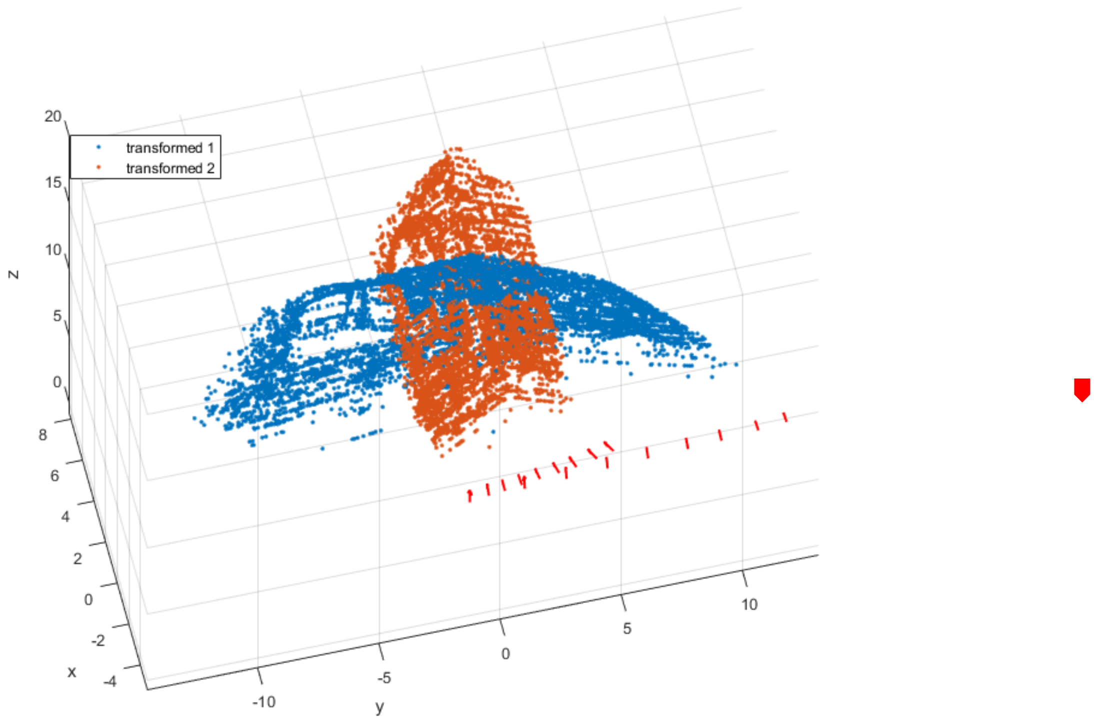

Figure 3: The transformed X points and the principal axes of the transformed cameras.

In Figure 3, the transformed points and principal vectors are the results of two different projective transformations operating on X and the cameras. The second transformation looks more 'reasonable' than the first. Specifically, the characteristics of the image appear to be in proportion.

Finally, in Figure 4, the points x_1 and the two transformed point sets appear to coincide, wherever they exist.

Figure 4: The the homogeneous coordinates x1 and the projected transformed X points on image 1.

Theoretical Exercise 2

Calibration matrices are formed in a way such, that when they operate on $[\mathcal{R} \quad \mathbf{t}]\mathbf{X}$, the result is a similarity transformation (or an affine transformation, if $\gamma \neq 1$ or/and $s \neq 0$). The major difference between projective transformations like in *Computer Exercise 1* and similarity or affine transformations is the degrees of freedom, which are connected to the preserved properties. The ambiguity from projective transformations is due to the lack of preservation of angles and parallel lines, properties that similarity transformations preserve. However, similarity transformations do not preserve distances. Moreover, in the same manner a projection transformation can be applied as in *Theoretical Exercise 1*, rotation or translation can be applied on calibrated camera matrices and the respective 3D points. Therefore, there is still ambiguity for calibrated cameras.

Camera Calibration

Theoretical Exercise 3

For a square matrix K, if $det(K) \neq 0$, then $\exists K^{-1}$ such, that $K^{-1} = \frac{adj(K)}{det(K)}$.

$$det(K) = f \begin{vmatrix} f & y_0 \\ 0 & 1 \end{vmatrix} = f^2 \neq 0$$

$$adj(K) = \Big(cof(K)\Big)^T = \begin{pmatrix} \begin{vmatrix} f & y_0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & y_0 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 0 & f \\ 0 & 0 \end{vmatrix} \\ -\begin{vmatrix} 0 & x_0 \\ 0 & y_0 \end{vmatrix} & \begin{vmatrix} f & x_0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} f & 0 \\ 0 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & x_0 \\ f & y_0 \end{vmatrix} & -\begin{vmatrix} f & x_0 \\ 0 & y_0 \end{vmatrix} & \begin{vmatrix} f & 0 \\ 0 & f \end{vmatrix} \end{pmatrix}^T =$$

$$= \begin{pmatrix} f & 0 & 0 \\ 0 & f & 0 \\ -fx_0 & -fy_0 & f^2 \end{pmatrix}^T = \begin{pmatrix} f & 0 & -fx_0 \\ 0 & f & -fy_0 \\ 0 & 0 & f^2 \end{pmatrix}$$

$$K^{-1} = \frac{1}{f^2} \begin{pmatrix} f & 0 & -fx_0 \\ 0 & f & -fy_0 \\ 0 & 0 & f^2 \end{pmatrix} = \begin{pmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{pmatrix}$$

$$A\,B = \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/f & 0 & -x_0/f \\ 0 & 1/f & -y_0/f \\ 0 & 0 & 1 \end{pmatrix} = K^{-1}$$

Therefore, K^{-1} can be factorised as shown above. The geometrical interpretation can be given when these matrices operate on unnormalized points in homogeneous coordinates, (x, y) Firstly, matrix B translates them by (x_0, y_0). Then, matrix A normalizes the translated points by the focal length f. Analytically:

$$B\,x = \begin{pmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x - x_0 \\ y - y_0 \\ 1 \end{bmatrix}$$

$$A\,(B\,x) = \begin{pmatrix} 1/f & 0 & 0 \\ 0 & 1/f & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{x-x_0}{f} \\ \frac{y-y_0}{f} \\ 1 \end{bmatrix}$$

Therefore, a normalized point in homogeneous coordinates is given by:

$$\mathbf{x}' = K^{-1}\,\mathbf{x} = A\,B\,\mathbf{x} \Longleftrightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{x-x_0}{f} \\ \frac{y-y_0}{f} \end{bmatrix}$$

The operation $K^{-1}\,\mathbf{x}$ provides the points on the image plain with independence of the camera's intrinsic characteristics.
If the unnormalized point is the principal point, meaning $(x, y) = (x_0, y_0)$, the corresponding normalized point is $(0, 0)$:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \frac{x_0-x_0}{f} \\ \frac{y_0-y_0}{f} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

An unnormalized point with distance f from the principal point satisfies the expression:

$$\sqrt{(x - x_0)^2 + (y - y_0)^2} = f$$

With the following implications we show that, since $f > 0$:

$$\begin{aligned} \sqrt{(x - x_0)^2 + (y - y_0)^2} &= f \Longleftrightarrow \\ (x - x_0)^2 + (y - y_0)^2 &= f^2 \Longleftrightarrow \\ \frac{(x - x_0)^2}{f^2} + \frac{(y - y_0)^2}{f^2} &= 1 \Longleftrightarrow \\ {x'}^2 + {y'}^2 &= 1 \Longleftrightarrow \\ \sqrt{{x'}^2 + {y'}^2} &= 1 \end{aligned}$$

The distance of the normalized points (x', y') from the normalized principal point $(0, 0)$ becomes 1.

The camera with calibration matrix $K = \begin{pmatrix} 400 & 0 & 400 \\ 0 & 400 & 300 \\ 0 & 0 & 1 \end{pmatrix}$ has focal length $f = 400$ and principal point $(400, 300)$. Therefore, the normalized points of $\mathbf{v} = (0, 300, 1)$ and $\mathbf{u} = (800, 300, 1)$ are given by:

$$\mathbf{v}' = \begin{bmatrix} \frac{0-400}{400} \\ \\ \frac{300-300}{400} \\ \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad \mathbf{u}' = \begin{bmatrix} \frac{800-400}{400} \\ \\ \frac{300-300}{400} \\ \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

The angle θ between the viewing rays projecting to these points is given by:

$$\cos\theta = \frac{\mathbf{v}' \cdot \mathbf{u}'}{|\mathbf{v}'||\mathbf{u}'|} = \frac{-1 \cdot 1 + 0 \cdot 0 + 1 \cdot 1}{\sqrt{2} \cdot \sqrt{2}} = 0$$

$$\theta = \arccos(0) = \frac{\pi}{2}$$

Let the calibration matrix K and the rotation matrix R of a camera be:

$$K = \begin{pmatrix} \gamma f & s & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} \mathbf{R_1}^T \\ \mathbf{R_2}^T \\ \mathbf{R_3}^T \end{pmatrix}$$

The center of the calibrated camera is in the nullspace of the camera matrix. However, the same center is also in the nullspace of the uncalibrated camera matrix:

$$\begin{aligned} K[R|\mathbf{t}]\mathbf{C} = 0 &\Leftrightarrow \\ K^{-1}K[R|\mathbf{t}]\mathbf{C} = K^{-1}0 &\Leftrightarrow \\ [R|\mathbf{t}]\mathbf{C} = 0 & \end{aligned}$$

The principal axis of the uncalibrated camera is given by $\mathbf{R_3}^T$ and the principal axis of the calibrated camera is given by

$$\mathbf{K_3}^T R = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} R = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \mathbf{R_1}^T \\ \mathbf{R_2}^T \\ \mathbf{R_3}^T \end{pmatrix} = \mathbf{R_3}^T$$

Consequently, the camera center and the principal axis are the same for the calibrated and the uncalibrated camera matrix.

RQ Factorization and Computation of K

Theoretical Exercise 4

$$K\,R = \begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} \begin{pmatrix} \mathbf{R_1}^T \\ \mathbf{R_2}^T \\ \mathbf{R_3}^T \end{pmatrix} = \begin{pmatrix} a\mathbf{R_1}^T + b\mathbf{R_2}^T + c\mathbf{R_3}^T \\ 0\mathbf{R_1}^T + d\mathbf{R_2}^T + e\mathbf{R_3}^T \\ 0\mathbf{R_1}^T + 0\mathbf{R_2}^T + f\mathbf{R_3}^T \end{pmatrix} = \begin{pmatrix} a\mathbf{R_1}^T + \mathbf{R_2}^T + c\mathbf{R_3}^T \\ \mathbf{R_2}^T + e\mathbf{R_3}^T \\ f\mathbf{R_3}^T \end{pmatrix}$$

The corresponding KR matrix for the given matrix P is :

$$KR = \begin{pmatrix} 2400\sqrt{2} & 0 & 800\sqrt{2} \\ 700\sqrt{2} & 2400 & -700\sqrt{2} \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} \mathbf{A_1}^T \\ \mathbf{A_2}^T \\ \mathbf{A_3}^T \end{pmatrix}$$

We will demand that a, b, c, d, e, f are positive, $\mathbf{R_i}^T\mathbf{R_i} = 1$ and $\mathbf{R_i}^T\mathbf{R_j} = 0$ for $i \neq j$. Starting from the last row, we have:

$$f\mathbf{R_3} = \mathbf{A_3} \iff \begin{bmatrix} f\,R_{31} \\ f\,R_{32} \\ f\,R_{33} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix} \iff \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix} = \begin{bmatrix} \frac{1}{f\sqrt{2}} \\ 0 \\ -\frac{1}{f\sqrt{2}} \end{bmatrix}$$

$$\begin{aligned} ||\mathbf{R_3}^T|| = 1 \iff R_{31}^2 + R_{32}^2 + R_{33}^2 = 1 \iff \\ \frac{1}{2f^2} + 0 + \frac{1}{2f^2} = 1 \iff f^2 = 1 \iff \\ f = 1 \end{aligned}$$

$$\mathbf{R_3}^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Continuing to the middle row, we have:

$$\begin{aligned} d\mathbf{R_2}^T + e\mathbf{R_3}^T = \mathbf{A_2}^T \iff \\ d\mathbf{R_2}^T\mathbf{R_3} + e\mathbf{R_3}^T\mathbf{R_3} = \mathbf{A_2}^T\mathbf{R_3} \iff \\ e = \mathbf{A_2}^T\mathbf{R_3} = \begin{bmatrix} 700\sqrt{2} & 2400 & -700\sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} \iff \\ e = 1400 \end{aligned}$$

$$
\begin{aligned}
d\mathbf{R_2}^T + e\mathbf{R_3}^T = \mathbf{A_2}^T &\iff \\
d\mathbf{R_2}^T = \mathbf{A_2}^T - e\mathbf{R_3}^T &\iff \\
\begin{bmatrix} d\,R_{21} \\ d\,R_{22} \\ d\,R_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 2800 \\ 0 \end{bmatrix} &\iff \begin{bmatrix} R_{21} \\ R_{22} \\ R_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 2800/d \\ 0 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
||\mathbf{R_2}^T|| = 1 \iff R_{21}^2 + R_{22}^2 + R_{23}^2 = 1 &\iff \\
0 + (\frac{2800}{d})^2 + 0 = 1 &\iff \\
d = 2800
\end{aligned}
$$

$$
\mathbf{R_2}^T = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}
$$

Finishing with the top row, we have:

$$
\begin{aligned}
a\mathbf{R_1}^T + b\mathbf{R_2}^T + c\mathbf{R_3}^T = \mathbf{A_2}^T &\iff \\
a\mathbf{R_1}^T\mathbf{R_3} + b\mathbf{R_2}^T\mathbf{R_3} + c\mathbf{R_3}^T\mathbf{R_3} = \mathbf{A_2}^T\mathbf{R_3} &\iff \\
c = \mathbf{A_2}^T\mathbf{R_3} = \begin{bmatrix} 2400\sqrt{2} & 0 & 800\sqrt{2} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix} &\iff \\
c = 1600
\end{aligned}
$$

$$
\begin{aligned}
a\mathbf{R_1}^T + b\mathbf{R_2}^T + c\mathbf{R_3}^T = \mathbf{A_2}^T &\iff \\
a\mathbf{R_1}^T\mathbf{R_2} + b\mathbf{R_2}^T\mathbf{R_2} + c\mathbf{R_3}^T\mathbf{R_2} = \mathbf{A_2}^T\mathbf{R_2} &\iff \\
b = \mathbf{A_2}^T\mathbf{R_2} = \begin{bmatrix} 2400\sqrt{2} & 0 & 800\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} &\iff \\
b = 0
\end{aligned}
$$

$$a\mathbf{R_1}^T + b\mathbf{R_2}^T + c\mathbf{R_3}^T = \mathbf{A_2}^T \iff$$
$$a\mathbf{R_1}^T = \mathbf{A_2}^T - b\mathbf{R_2}^T - c\mathbf{R_3}^T \iff$$
$$\begin{bmatrix} a\,R_{11} \\ a\,R_{12} \\ a\,R_{13} \end{bmatrix} = \begin{bmatrix} 1600\sqrt{2} \\ 0 \\ 0 \end{bmatrix} \iff \begin{bmatrix} R_{11} \\ R_{12} \\ R_{13} \end{bmatrix} = \begin{bmatrix} 1600\sqrt{2}/a \\ 0 \\ 0 \end{bmatrix}$$

$$||\mathbf{R_1}^T|| = 1 \iff R_{11}^2 + R_2^2 + R_3^2 = 1 \iff$$
$$(\frac{1600\sqrt{2}}{a})^2 + 0 + 0 = 1 \iff$$
$$a = 1600\sqrt{2}$$

Finally, the matrices K and R are formed below:

$$K = \begin{pmatrix} 1600\sqrt{2} & 0 & 1600 \\ 0 & 2800 & 1400 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix}$$

- focal length: $f = K_{22} = 2800$
- skew: $s = K_{12} = 0$
- aspect ratio: $\gamma = \frac{K_{11}}{f} = \frac{2\sqrt{2}}{7}$
- principal axis: $(x_0, y_0) = (K_{13}, K_{23}) = (1600, 1400)$

Direct Linear Transformation DLT

Theoretical Exercise 5

$$\min_{||\mathbf{v}||\in\mathbb{R}^n} ||M\mathbf{v}||^2$$

For every squared quantity in $\mathbb{R}^n$ it is always true that $||\mathbf{x}||^2 \geq 0$. In this particular case:

$$||M\mathbf{v}||^2 \geq 0\,, \quad ||\mathbf{v}|| \in \mathbb{R}^n$$

Consequently, the above linear least squares system always has the minimum value 0.

$$\min_{||\mathbf{v}||^2=1} ||M\mathbf{v}||^2$$

For every orthogonal matrix Q applies that $Q^TQ = QQ^T = I$ and $Q^T = Q^{-1}$. If M has a singular value decomposition $M = U\Sigma V^T$, with U, V orthogonal then:

$$\begin{aligned} ||M\mathbf{v}||^2 &= (M\mathbf{v})^T(M\mathbf{v}) = \mathbf{v}^TM^TM\mathbf{v} = \mathbf{v}^T(U\Sigma V^T)^T(U\Sigma V^T)\mathbf{v} = \\ &= \mathbf{v}^T(\Sigma V^T)^TU^TU\Sigma V^T\mathbf{v} = \mathbf{v}^T(\Sigma V^T)^T\Sigma V^T\mathbf{v} = (\Sigma V^T\mathbf{v})^T\Sigma V^T\mathbf{v} = \\ &= ||\Sigma V^T\mathbf{v}||^2 \end{aligned}$$

$$||V^T\mathbf{v}||^2 = (V^T\mathbf{v})^TV^T\mathbf{v} = \mathbf{v}^TVV^T\mathbf{v} = \mathbf{v}^T\mathbf{v} = ||\mathbf{v}||^2$$

Then, if $||\mathbf{v}|| = 1$, then $||V^T\mathbf{v}|| = 1$.

$$\min_{||\tilde{\mathbf{v}}||^2=1} ||\Sigma\tilde{\mathbf{v}}||^2$$

This problem gives the same minimal value with the above problem, since:

$$||V^T\mathbf{v}^*|| = ||\tilde{\mathbf{v}}^*|| \quad \text{for } ||\mathbf{v}^*|| = ||\tilde{\mathbf{v}}^*|| \ = 1$$

Therefore, for the diagonal matrix Σ:

$$||\Sigma\tilde{\mathbf{v}}^*||^2 = ||M\mathbf{v}^*||^2$$

There are always at least two solutions to these problems, because $||\mathbf{v}|| = ||-\mathbf{v}||$.

Finally, we form the Lagrangian of the problem

$$\min_{||\mathbf{v}||^2=1} ||M\mathbf{v}||^2$$

$$\mathcal{L}(\mathbf{v}, \lambda) = ||M\mathbf{v}||^2 - \lambda(||\mathbf{v}||^2 - 1)$$

$$\begin{aligned} \nabla\mathcal{L}(\mathbf{v}^*, \lambda) = \mathbf{0} &\iff \\ 2M^T M\mathbf{v}^* - 2\lambda\mathbf{v}^* = \mathbf{0} &\iff \\ M^T M\mathbf{v}^* = \lambda\mathbf{v} & \end{aligned}$$

Considering the above eigenvalue equation, the minimization problem and the facts below:

- $||M\mathbf{v}^*||^2 = ||\Sigma V^T \mathbf{v}^*||^2 = \lambda||\mathbf{v}^*||^2$
- $||\mathbf{v}^*|| = 1$
- $\Sigma \in \mathbb{R}^{m\times n}$ is diagonal with diagonal elements in decreasing order
- $rank(M) = rank(\Sigma)$
- The non-zero singular values of M, found on the diagonal entries of Σ, are the square roots of the non-zero eigenvalues of $M^T M$.

the solution to the eigenvalue problem is the eigenvector corresponding to the smallest eigenvalue. This eigenvector is given by the last row of V^T. If $rank(\Sigma) < n$, M does not have full rank and $M\mathbf{v}^* = \mathbf{0}$ has an explicit solution.

Theoretical Exercise 6

$$\begin{aligned} \mathbf{x} \sim \mathcal{P}\mathbf{X} &\Longrightarrow \\ \mathcal{N}\mathbf{x} \sim \mathcal{N}\mathcal{P}\mathbf{X} &\Longrightarrow \\ \tilde{\mathbf{x}} \sim \mathcal{N}\mathcal{P}\mathbf{X} & \end{aligned}$$

For $\mathcal{N}\mathcal{P} = \tilde{\mathcal{P}}$ and $\mathcal{N}$ invertible, we have $\mathcal{P} = \mathcal{N}^{-1}\tilde{\mathcal{P}}$.

Computer Exercise 2 (including optional)

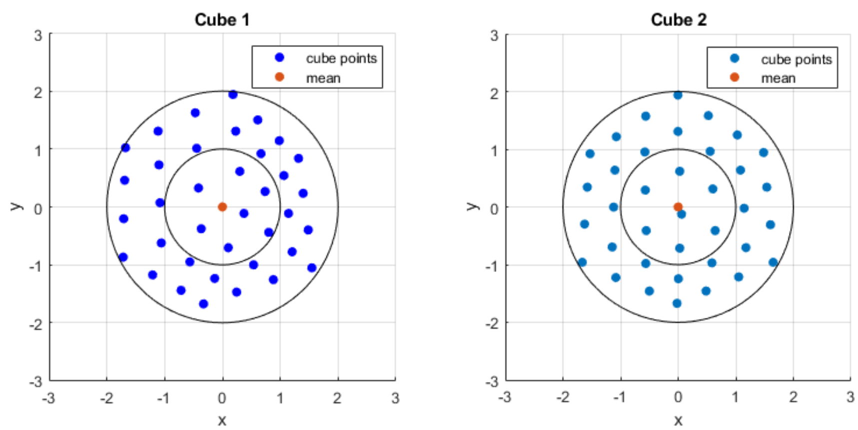

Figure 5: Normalized points for the two images. The standard deviations are the circles with the smaller radius.

The means of the normalized points is of the order 10^{-14}, so they are close to zero and the standard deviations are equal to 1 for both axes. This is visualized in Figure 5.

After performing DLT, we can conclude that the minimum eigenvalue for each image is close to zero, as presented in Table 1. Moreover, the value of the least squares is equal to the respective eigenvalue, proving that $||\mathbf{v}|| = 1$.

DLT	Image 1	Image 2
λ_{min}	$2.26\,10^{-4}$	$1.48\,10^{-4}$
$\|\|M\mathbf{v}\|\|$	$2.26\,10^{-4}$	$1.48\,10^{-4}$

Table 1: Values of the minimum eigenvalues λ_{min} and the corresponding values of $||M\mathbf{v}||$.

By observing Figure 6, the projected points and the lines fit the cubes. Moreover, the projected points overlap with the measured image points.

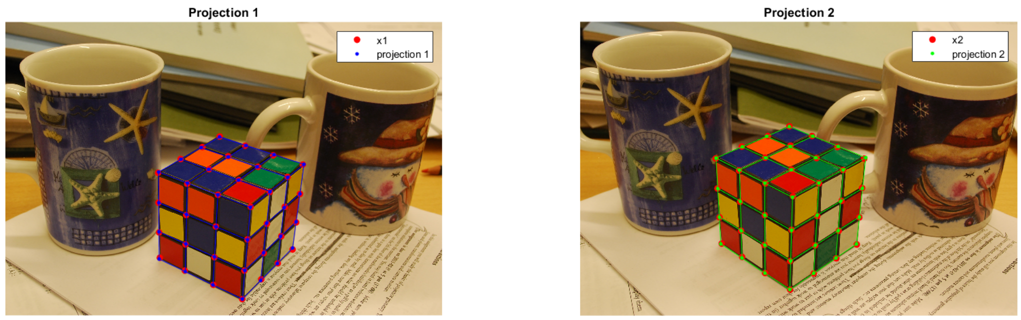

Figure 6: Projected normalized model points with unnormalized camera matrices on the two images.

The 3D shape formed by the lines in Figure 7 and the principal axes of the cameras look accurate.

The calibration matrix of the first camera normalized by its last element K_{33} is:

$$K_1 = \begin{pmatrix} 2448.1 & -18.0 & 960.1 \\ 0 & 2446.4 & 675.7 \\ 0 & 0 & 1 \end{pmatrix}$$

Why is there no ambiguity?
Any real square matrix $P_{3\times 3}$ can be decomposed as $P = KR$, where K is upper triangular and R is orthogonal. If P is invertible and the diagonal elements are positive, then the factorization is unique. In this instance, the diagonal elements of both camera matrices are positive and their determinants are not zero, making them invertible. Therefore, their inner parameters are unique.

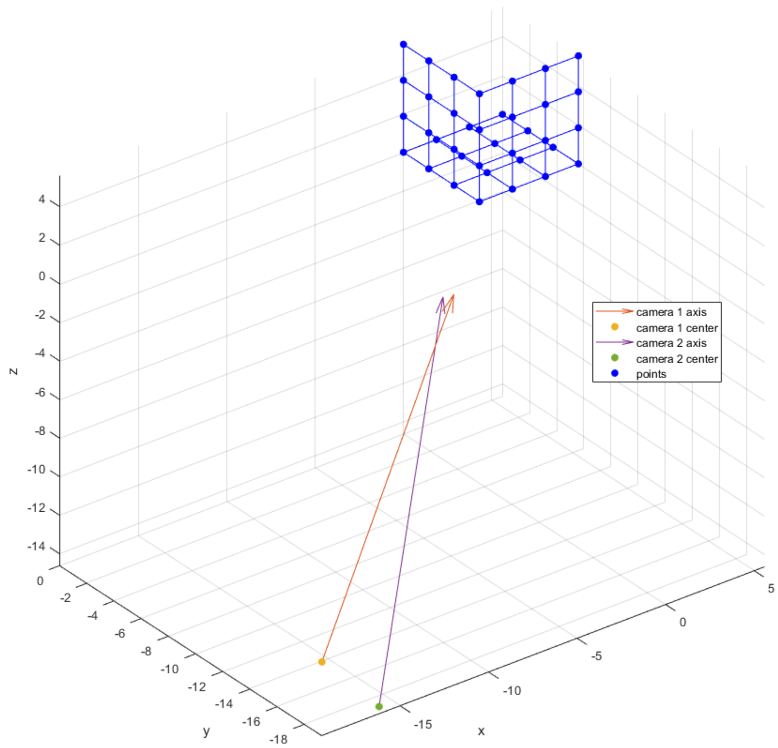

Figure 7: 3D plot of the camera centers, the principal axes and the model points X.

The values of e_{RMS} in Table 2 show that the distance between the initial measured points and the normalized projected points is shorter than the unnormalized ones. Also, the error of the full points is lower than that of the smaller sample.

e_{RMS}	whole	sample
normalized	3.62	4.54
unnormalized	4.90	7.06

Table 2: Values of e_{RMS}.

Computer Exercise 4

For clearer images, please see the MATLAB figures in the submitted directory:

$$/Assignment\_2/compEx4/compEx4\_Plots$$

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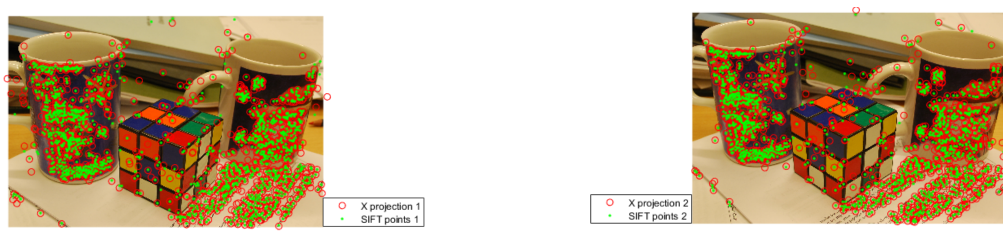

Figure 8: Triangulated points X with camera matrices $K[R|\mathbf{t}]$ and SIFT points x.

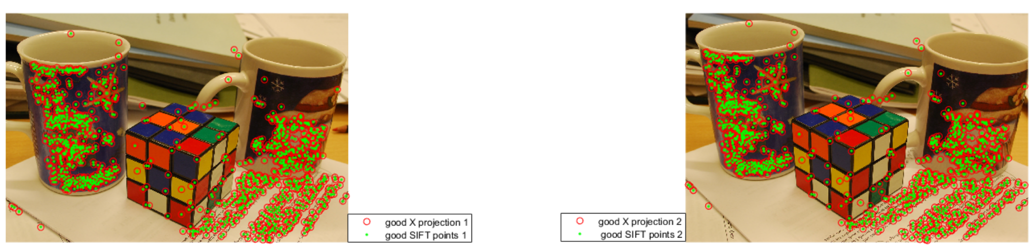

Figure 9: Good points X triangulated with camera matrices $K[R|\mathbf{t}]$ and good points x.

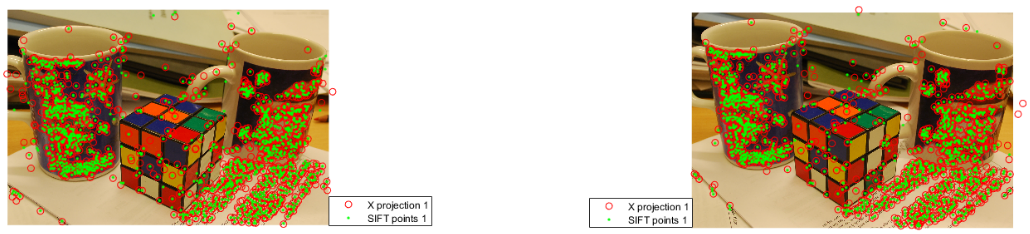

Figure 10: Triangulated points X with camera matrices $[R|\mathbf{t}]$ and SIFT points x.

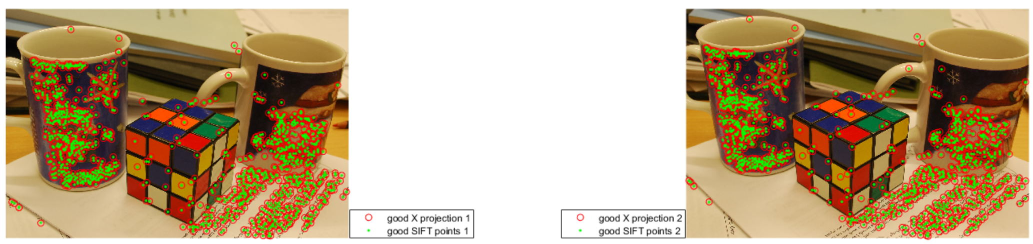

Figure 11: Good points X triangulated with camera matrices $[R|\mathbf{t}]$ and good points x.

After observing the 3D plots of the points in Figures 12 and 13 below, the dominant objects (cups and paper) are relatively distinguishable.

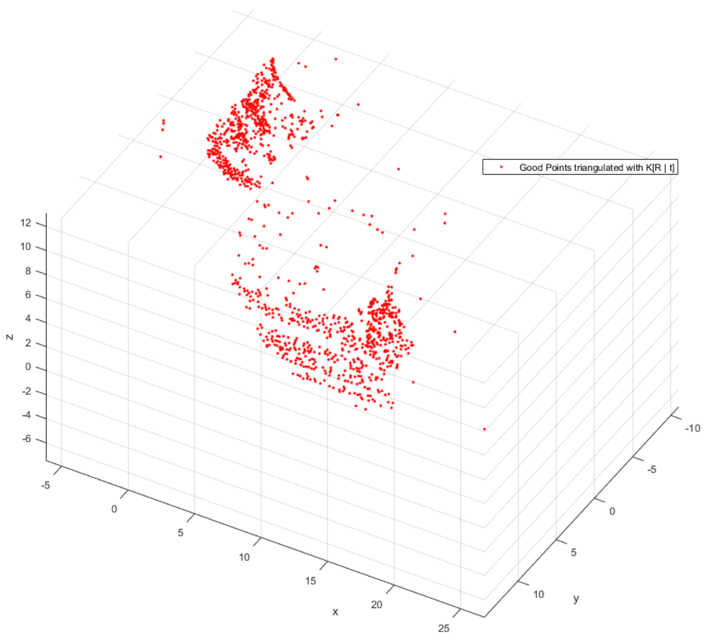

Figure 12: 3D good triangulated points with camera matrices $K[R|\mathbf{t}]$

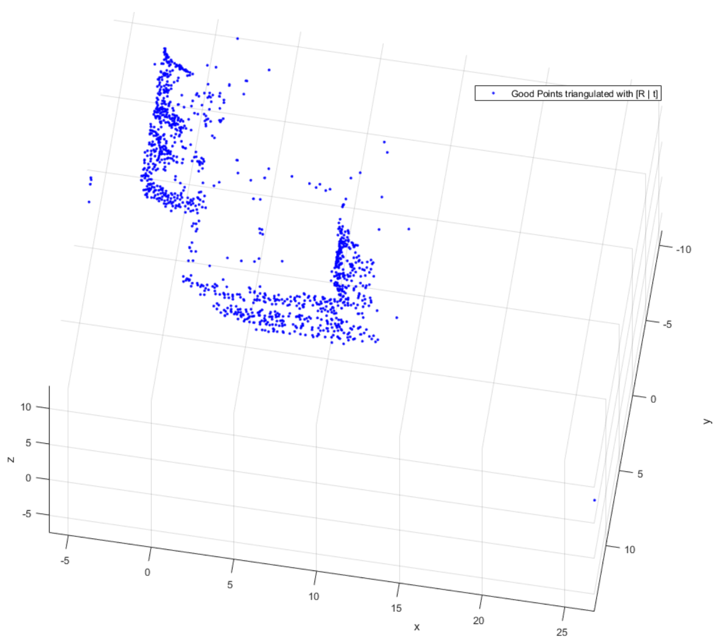

Figure 13: 3D good triangulated points with camera matrices $[R|\mathbf{t}]$