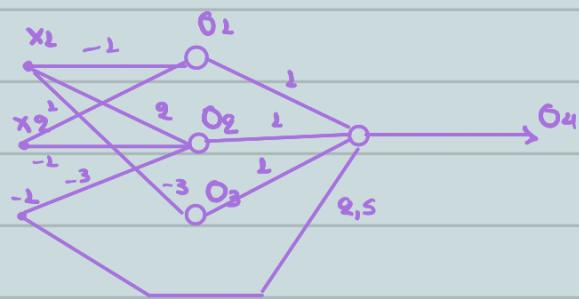
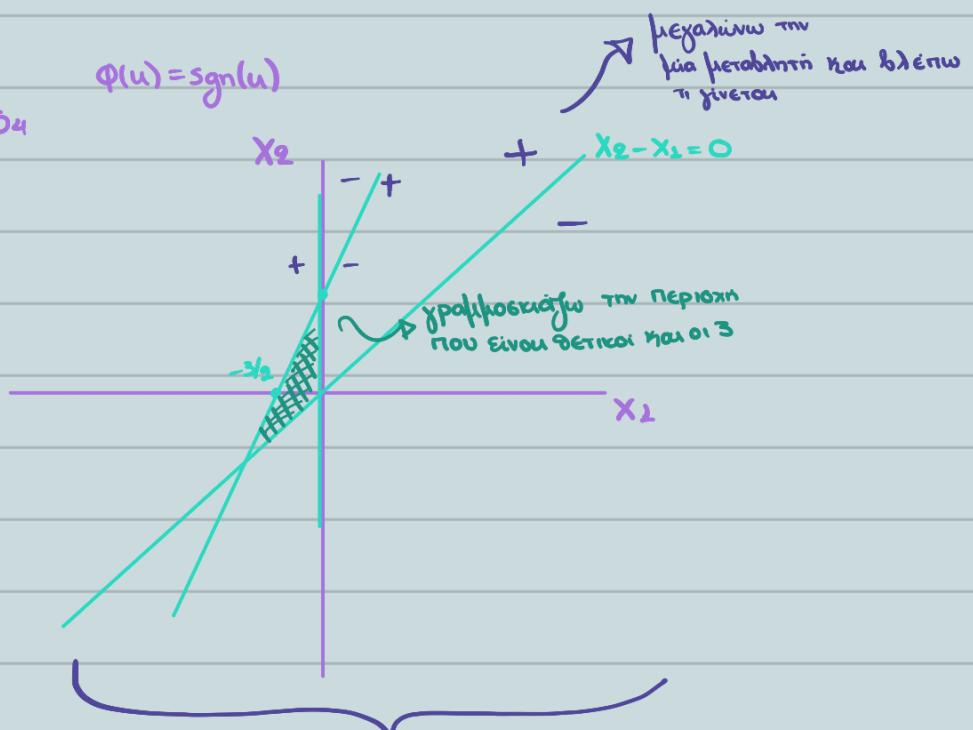


Aσκηση 1



$$\phi(u) = \text{sgn}(u)$$



$$O_1 = \text{sgn}(x_2 - x_1)$$

$$O_2 = \text{sgn}(2x_2 - x_1 + 3)$$

$$O_3 = \text{sgn}(-3x_1)$$

$$O_4 = \text{sgn}(O_1 + O_2 + O_3 - 9,5)$$

$$O_1: x_2 - x_1 = 0 \Rightarrow x_2 = x_1$$

$$O_2: 2x_2 - x_1 + 3 = 0$$

$$O_3: -3x_1 = 0$$

Aσκηση 2

$$\underline{W}^{k+1} = \underline{W}^k + \frac{c}{2} (\underline{d}^k - \underline{O}^k) \underline{y}^k$$

$$\underline{W}^{k+1} = \underline{W}^k + c \cdot \frac{\underline{e}^k}{2} \frac{\underline{y}^k}{\|\underline{y}^k\|^2}$$

Ζω συγάλλημα μειώνεται κατά παράγοντα 1-1, $0 < c_k < 1$.

$$\begin{aligned} \underline{d}^{k+1} &= \underline{d}^k \\ \underline{e}^{k+1} &= \underline{d}^{k+1} - \underline{W}^{k+1} \underline{y}^{k+1} = \underline{d}^k - \underline{W}^k + c \cdot \frac{\underline{e}^k}{2} \frac{\underline{y}^k}{\|\underline{y}^k\|^2} \end{aligned}$$

$$\underline{e}^k = \underline{d}^k - \underline{W}^k \underline{y}^k$$

$$\underline{y}^{k+1} = \underline{y}^k$$

$$= \underline{d}^k - \underline{W}^k \underline{y}^k - c \frac{\underline{e}^k}{2} \frac{\underline{y}^k \underline{y}^k}{\|\underline{y}^k\|^2}$$

$$\|\underline{y}^k\|^2 = \underline{y}^k \cdot \underline{y}^k$$

$$= \underline{e}^k - c \frac{\underline{e}^k}{2} \frac{1}{\|\underline{y}^k\|^2} \Rightarrow \frac{\underline{e}^{k+1}}{\underline{e}^k} = 1 - \frac{c}{2}$$

Ζω update που γρνούμονοιστε συγκανει.

Avgnon 3

$$\epsilon(\underline{w}) = \frac{1}{2} [(w_2 - w_1)^2 + (1 - w_2)^2]$$

$$\nabla \epsilon(\underline{w}) = \begin{bmatrix} \frac{\partial \epsilon(\underline{w})}{\partial w_2} \\ \frac{\partial \epsilon(\underline{w})}{\partial w_1} \end{bmatrix} = \begin{bmatrix} -2(w_2 - w_1) + (-2)(1 - w_2) \\ w_2 - w_1 \end{bmatrix} = \begin{bmatrix} 2w_1 - w_2 - 1 \\ w_2 - w_1 \end{bmatrix}$$

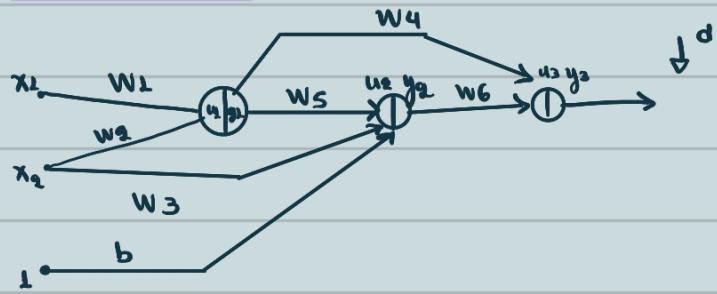
$$\nabla \epsilon(\underline{w}) = 0 \Rightarrow \begin{cases} 2w_1 - w_2 - 1 = 0 \\ w_2 - w_1 = 0 \end{cases} \Rightarrow \begin{cases} w_2 = 1 \\ w_1 = w_2 \end{cases} \Rightarrow \underline{w}^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\nabla^2 \epsilon(\underline{w}) = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \quad \forall \underline{x} \in \mathbb{R}^{N \times 1}$$

$$A \in \mathbb{R}^{N \times N}$$

$$\underline{x}^T A \underline{x} > 0$$

Άσκηση 4



$$\phi(x) = \frac{1}{1 + e^{-x}}$$

- a) Έκφρασης για τα y_1, y_2, y_3
b) $\delta_1, \delta_2, \delta_3$

$$e = d - y_3$$

γ) Διχέσεις ανανέωσης των δαπών
(παραβιέτρων)

$$\delta) W_1 = W_2 = W_3 = W_4 = W_5 = \pm, W_6 = -\pm, b = -\frac{\pm}{2}$$

$$(x_1, x_2) = (0, 0), e = \frac{1}{4}, \delta_1 = \frac{1}{64}, \text{ για δαπών } W_{ij}$$

$$y_1 = \phi(u_1), u_1 = w_1 x_1 + w_2 x_2, y_3 = \phi(u_3), u_3 = w_6 y_2 + w_4 y_1$$

$$y_2 = \phi(u_2), u_2 = w_5 y_1 + w_3 x_2 + b$$

$$\delta_3 = e \cdot \phi'(u_3)$$

$$\delta_3 = w_6 \delta_3 \phi'(u_2)$$

$$\delta_1 = (w_5 \delta_2 + w_4 \delta_3) \phi'(u_1)$$

$$\Delta w_1 = -n \delta_1 \cdot x_1$$

$$\Delta w_2 = -n \delta_1 \cdot x_2$$

$$\Delta w_3 = -n \delta_2 \cdot x_2$$

$$\Delta w_5 = -n \delta_2 y_1$$

$$\Delta w_6 = -n \delta_3 y_2$$

$$\Delta w_4 = -n \delta_3 y_1$$

$$\Delta b = -n \delta_2 \cdot 1$$

$$u_1 = w_1 \cdot 0 + w_2 \cdot 0 = 0$$

$$y_1 = \phi(0) = \frac{1}{1 + e^0} = \frac{1}{2}$$

$$u_2 = \pm \cdot \frac{1}{2} + \pm \cdot 0 + \left(-\frac{\pm}{2}\right) = 0, y_2 = \phi(0) = \frac{1}{2}$$

$$u_3 = -\pm \cdot \frac{1}{2} + \pm \cdot \frac{1}{2} = 0, y_3 = \phi(0) = \frac{1}{2}$$

$$\phi'(x) = \phi(x)(1 - \phi(x))$$

$$\delta_3 = \frac{1}{4} \cdot y_3(1 - y_3) = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

$$\delta_2 = -\pm \cdot \frac{1}{16} y_2(1 - y_2) = -\frac{\pm}{64}$$

$$\delta_1 = \left(\pm \cdot \frac{-\pm}{64} + \pm \cdot \frac{1}{16}\right) y_1(1 - y_1) = \left(-\frac{\pm^2}{64} + \frac{\pm}{16}\right) \frac{1}{2} = \frac{1}{64}$$

$$= \frac{1}{64} \Rightarrow \left(\frac{-\pm^2}{64} + \frac{\pm}{16}\right) \frac{1}{4} = \frac{1}{64} \Rightarrow$$