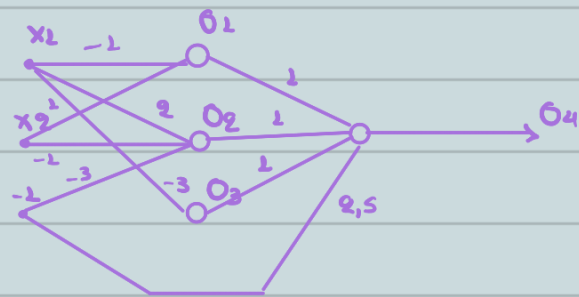
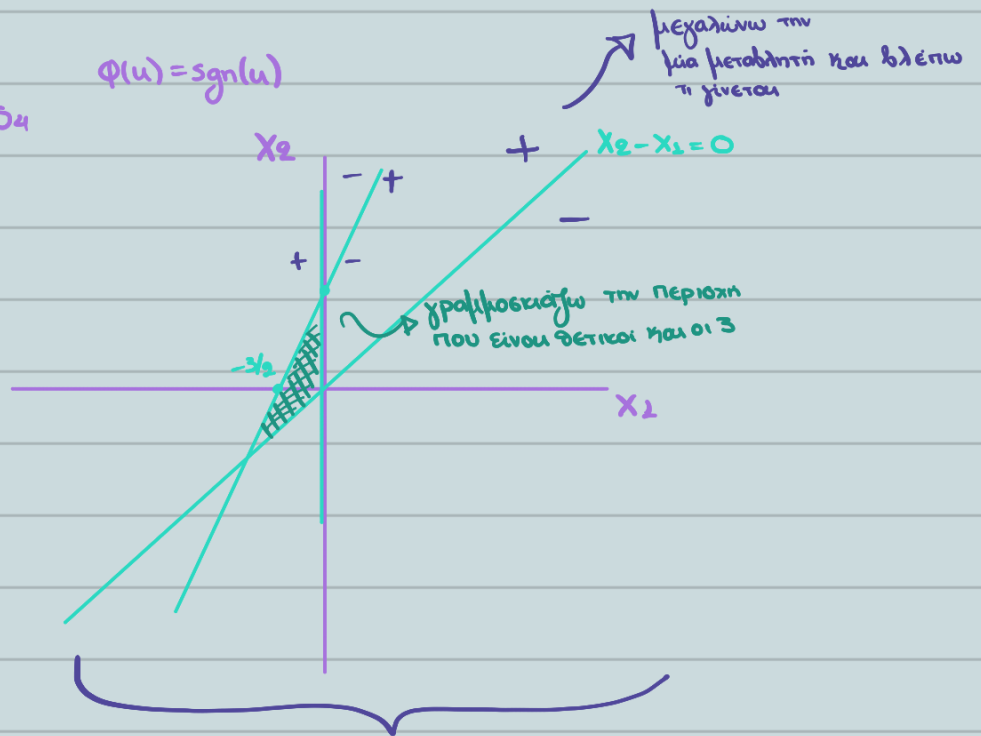


## Άσκηση 1



$$\phi(u) = \text{sgn}(u)$$



$$O_1 = \text{sgn}(x_2 - x_1)$$

$$O_2 = \text{sgn}(2x_1 - x_2 + 3)$$

$$O_3 = \text{sgn}(-3x_1)$$

$$O_4 = \text{sgn}(O_1 + O_2 + O_3 - 2.5)$$

$$O_1: x_2 - x_1 = 0 \Rightarrow x_2 = x_1$$

$$O_2: 2x_1 - x_2 + 3 = 0$$

$$O_3: -3x_1 = 0$$

Classifier

## Άσκηση 2

$$\underline{w}^{k+1} = \underline{w}^k + \frac{c}{2} (d^k - o^k) \underline{y}^k$$

$$\underline{w}^{k+1} = \underline{w}^k + c \cdot \frac{e^k}{2} \frac{\underline{y}^k}{\|\underline{y}^k\|^2}$$

$$e^k = d^k - \underline{w}^k \underline{y}^k$$

$$\underline{y}^{k+1} = \underline{y}^k$$

Το βυάθμα μειώνεται κατά παράγοντα 1-1,  $0 < c_k < 1$ .

$$d^{k+1} = d^k$$

$$e^{k+1} = d^{k+1} - \underbrace{\underline{w}^{k+1}}^{\text{transpose}} \underline{y}^{k+1} = d^k - \underline{w}^k + c \cdot \frac{e^k}{2} \frac{\underline{y}^k}{\|\underline{y}^k\|^2} \cdot \underline{y}^k =$$

$$= d^k - \underline{w}^k \underline{y}^k - c \frac{e^k}{2} \frac{\underline{y}^k \underline{y}^k}{\|\underline{y}^k\|^2}$$

$$\|\underline{y}^k\|^2 = \underline{y}^k \cdot \underline{y}^k = e^k - c \cdot \frac{e^k}{2} \cdot 1 \Rightarrow \frac{e^{k+1}}{e^k} = 1 - \frac{c}{2}$$

Το update που χρησιμοποιούμε συρραίνει.

### Aufgaben 3

$$\epsilon(\underline{w}) = \frac{1}{2} [(w_2 - w_1)^2 + (1 - w_1)^2]$$

$$\nabla \epsilon(\underline{w}) = \begin{bmatrix} \frac{\partial \epsilon(\underline{w})}{\partial w_1} \\ \frac{\partial \epsilon(\underline{w})}{\partial w_2} \end{bmatrix} = \begin{bmatrix} -2(w_2 - w_1) + (-2)(1 - w_1) \\ w_2 - w_1 \end{bmatrix} = \begin{bmatrix} 2w_1 - w_2 - 1 \\ w_2 - w_1 \end{bmatrix}$$

$$\nabla \epsilon(\underline{w}) = 0 \Rightarrow \begin{cases} 2w_1 - w_2 - 1 = 0 \\ w_2 - w_1 = 0 \end{cases} \Rightarrow \begin{cases} w_1 = 1 \\ w_2 = w_1 \end{cases} \Rightarrow \underline{w}^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

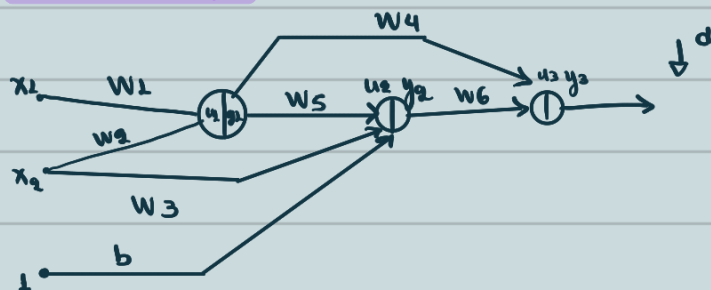
$$\nabla^2 \epsilon(\underline{w}) = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\forall x \in \mathbb{R}^{N \times 1}$$

$$A \in \mathbb{R}^{N \times N}$$

$$\underline{x}^T A \underline{x} > 0$$

## Άσκηση 4



$$\phi(x) = \frac{1}{1 + e^{-x}}$$

α) Εργασίες για τα  $y_1, y_2, y_3$

β)  $\delta_1, \delta_2, \delta_3$

$$e = d - y_3$$

γ) Σχέσεις ανανέωσης των βαρών (παραμέτρων)

$$\delta) w_1 = w_2 = w_3 = w_4 = w_5 = 2, w_6 = -2, b = -\frac{2}{2}$$

$$(x_1, x_2) = (0, 0), e = \frac{1}{4}, \delta_1 = \frac{1}{64}, \text{ζήκη βαρών } w_i;$$

$$y_1 = \phi(u_1), u_1 = w_1 x_1 + w_2 x_2, y_3 = \phi(u_3), u_3 = w_6 y_2 + w_4 y_1$$

$$y_2 = \phi(u_2), u_2 = w_5 y_1 + w_3 x_2 + b$$

$$\delta_3 = e \cdot \phi'(u_3)$$

$$\Delta w_1 = -n \delta_1 \cdot x_1$$

$$\Delta w_5 = -n \delta_2 y_1$$

$$\Delta w_4 = -n \delta_3 y_1$$

$$\delta_2 = w_6 \delta_3 \phi'(u_2)$$

$$\Delta w_2 = -n \delta_1 \cdot x_2$$

$$\Delta w_6 = -n \delta_3 y_2$$

$$\delta_1 = (w_5 \delta_2 + w_4 \delta_3) \phi'(u_1)$$

$$\Delta w_3 = -n \delta_2 \cdot x_2$$

$$\Delta b = -n \delta_2 \cdot 1$$

$$u_1 = w_1 \cdot 0 + w_2 \cdot 0 = 0$$

$$y_1 = \phi(0) = \frac{1}{1 + e^0} = \frac{1}{2}$$

$$u_2 = 2 \cdot \frac{1}{2} + 2 \cdot 0 + \left(-\frac{2}{2}\right) = 0, y_2 = \phi(0) = \frac{1}{2}$$

$$u_3 = -2 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2} = 0, y_3 = \phi(0) = \frac{1}{2}$$

$$\phi'(x) = \phi(x)(1 - \phi(x))$$

$$\delta_3 = \frac{1}{4} \cdot y_3(1 - y_3) = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$$

$$\delta_2 = -2 \cdot \frac{1}{16} y_2(1 - y_2) = -\frac{2}{64}$$

$$\delta_1 = \left(2 \cdot \frac{-2}{64} + 2 \cdot \frac{1}{16}\right) y_1(1 - y_1) = \left(\frac{-2^2}{64} + \frac{2}{16}\right) \frac{1}{4}$$

$$= \frac{1}{64} \Rightarrow \left(\frac{-2^2}{64} + \frac{2}{16}\right) \frac{1}{4} = \frac{1}{64} \Rightarrow$$