PHYS512 A5

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1

For the initial parameter guess of [60,0.02,0.1,0.05,2.00e-9,1.0], $\chi^2 = 15267.94$ with 2501 degrees of freedom. Using the more accurate guess of [69, 0.022, 0.12, 0.06, 2.1e-9, 0.95], $\chi^2 = 3272.20$. These values would not be considered an acceptable fit. We would expect a good fit to yield a χ^2 value closer to the number of degrees of freedom (2501). We can also compare the plots generated by these parameters. Clearly, the plot generated using the second array of parameters is more accurate.

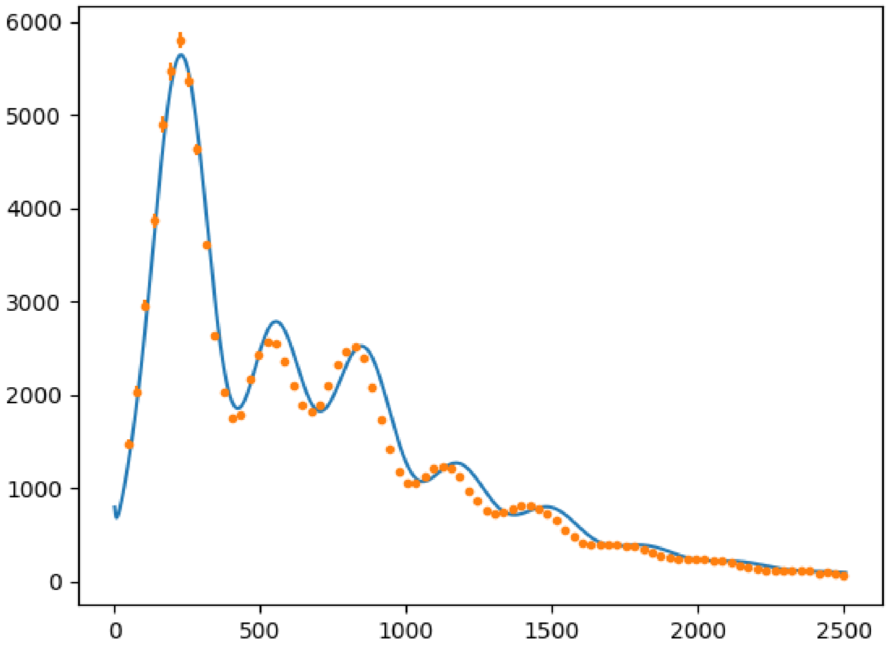

Figure 1: Plot of the initial best-fit guess versus the model

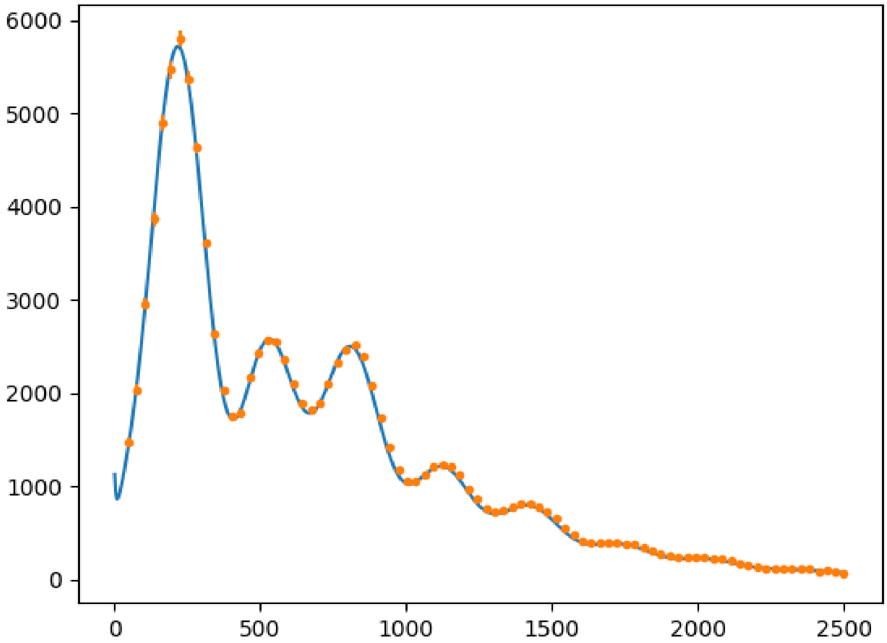

Figure 2: Plot of the refined best-fit guess versus the model

2

Newton's method was used to find the best-fit parameters for this problem. The function "newtons" can be found in the utils.py file. It closely follows the example in the class git repository. Newton's method yielded the following parameter value.

- $H_0 = 69.00 \pm 1.821 \times 10^{-24}$
- $\Omega_b h^2 = 0.0220 \pm 9.683 \times 10^{-21}$
- $\Omega_c h^2 = 0.1200 \pm 5.152 \times 10^{-21}$
- $\tau = 0.0600 \pm 2.498 \times 10^{-21}$
- $A_s = 2.1178 \times 10^{-19} \pm 2.498 \times 10^{-21}$
- $n_s = 0.9500 \pm 5.983 \times 10^{-13}$

These values are identical to the guess values with the exception of A_s. This being the case, we achieve $\chi^2 = 3059.16$ which is reasonable and a plot closely fitting the model. The errors do seem rather small. It is possible that I should have used the provided data to compute the error rather than the standard error as used in assignment 4.

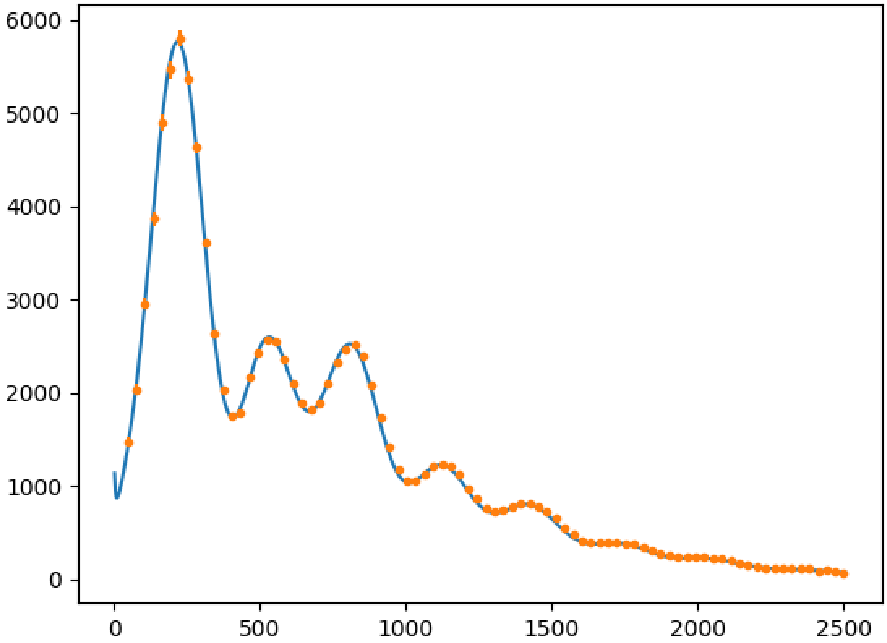

Figure 3: Best-fit plot for question 2.

3

The MCMC chain converged after about two hours using 10000 iterations. We can see the plot of the Fourier transform of the Baryon density chain. The left half of the Fourier transform is flat which indicative of convergence. Looking at the MCMC plot we notice that we indeed slightly oscillate

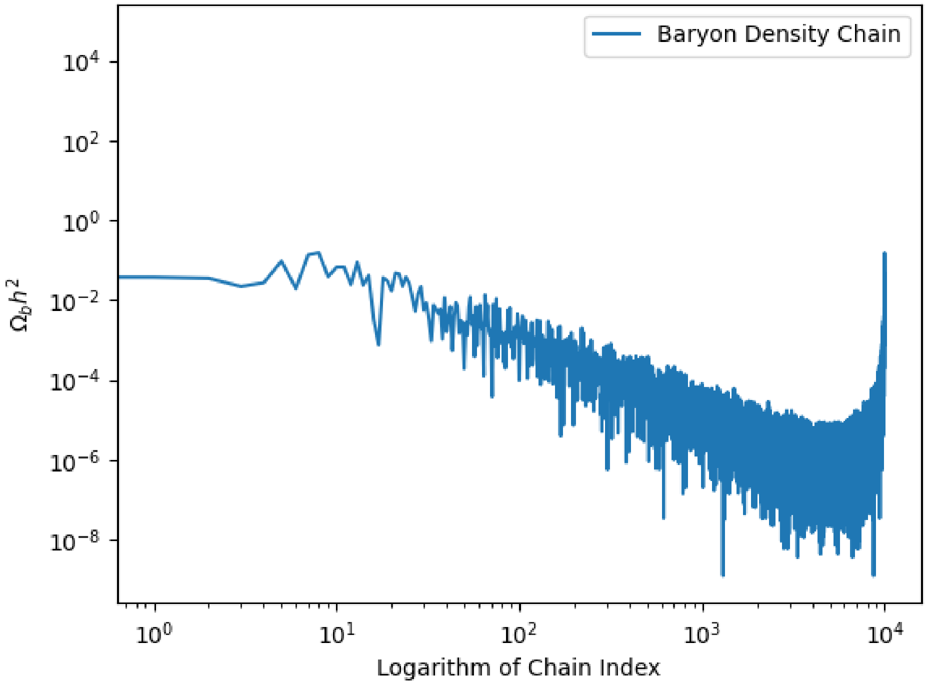

Figure 4: Squared amplitude of the Fourier transform of the Baryon density chain

around a value of 0.02327. Similar behaviour is seen with the other parameters. The curvature matrix was used for generating the MCMC step size as suggested in the assignment handout. The curvature matrix was multiplied by a random Gaussian vector with the same length as the parameter vector. The difference of χ^2 values from the current step and previous step were used to determine whether the step is accepted.

The parameters generated from the MCMC are listed below.

- $H_0 = 69.00 \pm 8.606 \times 10^{-8}$
- $\Omega_b h^2 = 0.02346 \pm 1.801 \times 10^{-4}$
- $\Omega_c h^2 = 0.1202 \pm 8.929 \times 10^{-5}$
- $\tau = 0.06000 \pm 7.785 \times 10^{-8}$
- $A_s = 2.151 \times 10^{-9} \pm 5.349 \times 10^{-12}$
- $n_s = 0.9501 \pm 2.425 \times 10^{-5}$

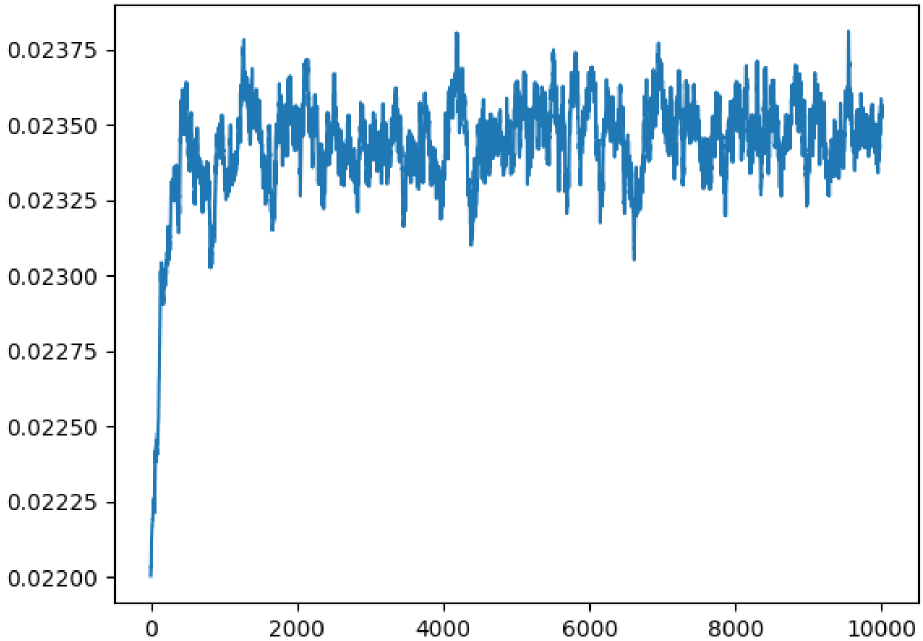

Figure 5: Baryon density chain versus number of iterations

To calculate Ω_Λ, we use the following equation:

$$\Omega_\Lambda + \Omega_b + \Omega_c = 1 \tag{1}$$

We can re-arrange this, and substitute the values we have calculated ($\Omega_b h^2$ and $\Omega_c h^2$):

$$\Omega_\Lambda = 1 - \left(\frac{\Omega_b h^2}{h^2} - \frac{\Omega_c h^2}{h^2}\right) \tag{2}$$

Calculating the error can be done by noting that $\sigma_{\Omega bc} \approx |\Omega_{b/c}|$. Expanding this, we get:

$$|\Omega_{b/c}| = \sqrt{\left(\frac{\sigma_h}{h}\right)^2 + \left(\frac{\sigma_{b/ch^2}}{\Omega_{b/ch^2}}\right)^2} \tag{3}$$

Furthermore, since $h = H_0/100$, the error will follow $\sigma_h = \sigma_{H0}/100$. It follows then that:

$$\sigma_{\Omega_\Lambda} = \sqrt{\Omega_b^2 + \Omega_c^2} \tag{4}$$

By plugging our values into these formulas, we get:

$$\Omega_\Lambda = 0.698 \pm 0.257 \tag{5}$$

4

We include the optical constraint by weighting the chain from question 3 summing a sample from random normal distribution and a Gaussian distribution with variance equal to the error in tau, and a mean of tau. This new probability is re-normalized resulting in a list of ”weights”, which is then used to scale our output chain from question 3. We remove the first 115 entries of the list since they have χ^2 values greater than 2800. We then feed these new importance parameters into our MCMC function and calculate the following values:

- $H_0 = 69.00 \pm 2.278 \times 10^{-6}$
- $\Omega_b h^2 = 0.02300 \pm 1.528 \times 10^{-4}$
- $\Omega_c h^2 = 0.1177 \pm 6.300 \times 10^{-4}$
- $\tau = 0.06000 \pm 3.579 \times 10^{-6}$
- $A_s = 2.134 \times 10^{-9} \pm 4.963 \times 10^{-12}$
- $n_s = 0.9527 \pm 1.358 \times 10^{-3}$

We note that this leads to an excellent chi squared value of $\chi^2 = 2642.36$. We observe a similar graph for the Fourier transform and thus make the same conclusion; the MCMC converges.

We also note that the fit is exceptional as we can see below.

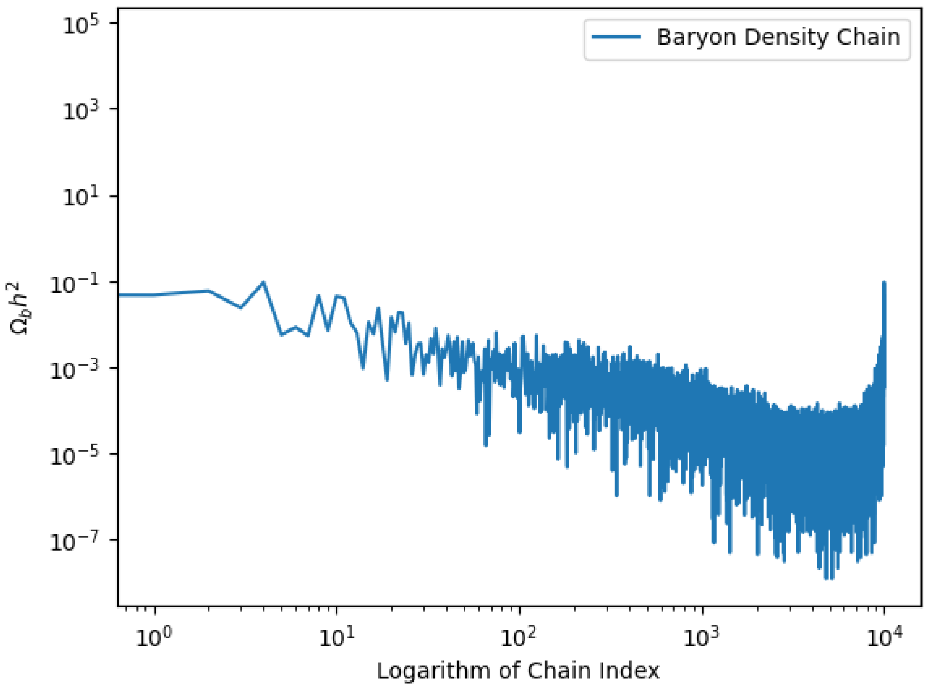

Figure 6: Fourier transform of the Baryon density chain versus number of iterations (Question 4)

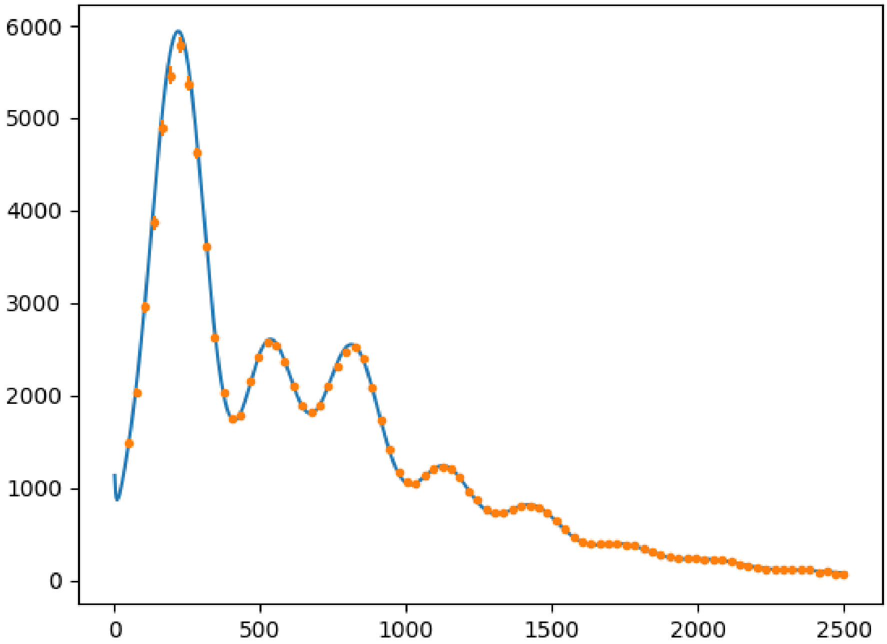

Figure 7: Baryon density chain versus number of iterations (Question 4)