PHYS512 A6

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1

From choosing the values of $a = 4$ and $b = -2$ through trial-and-error, I was able to show that the points given in the provided text file group together on about 30 planes.

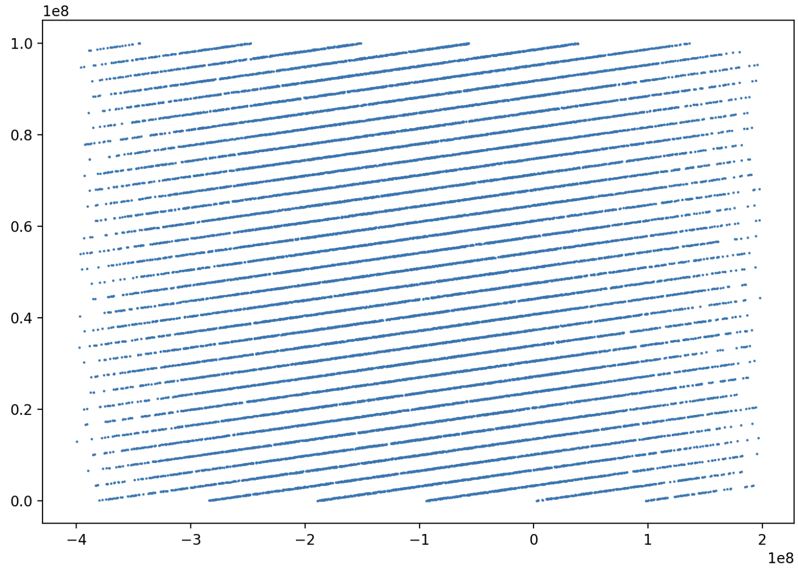

Figure 1: Plot of the planes formed by plotting $(ax + by, z)$

Trial and error did not result in a similar result for the python random library. A scatter plot for the same values of a and b can be seen below. Unfortunately, I was unable to get the c library work.

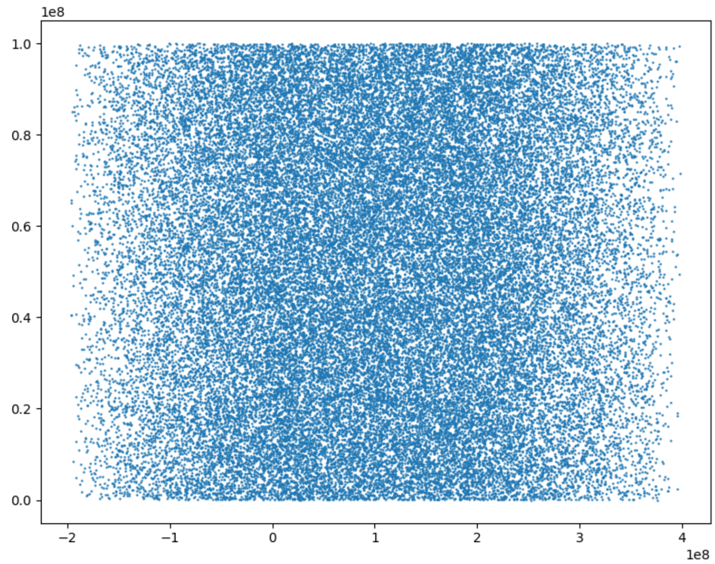

Figure 2: Plot of the planes formed by plotting $(ax + by, z)$ for points generated using the Python "random" library

2

For us to be able to use a function for the rejection method, it must be lie "close" to the distribution we are trying to calculate a value for, and it must lie above the distribution. A Desmos plot of a Lorentzian, Gaussian and a Power Law function show that the Lorentzian will be the best function to use for the rejection method. The Gaussian is disqualified since it intersects with the exponential, and while the power law envelopes the exponential distribution, the Lorentzian envelops the exponential better.

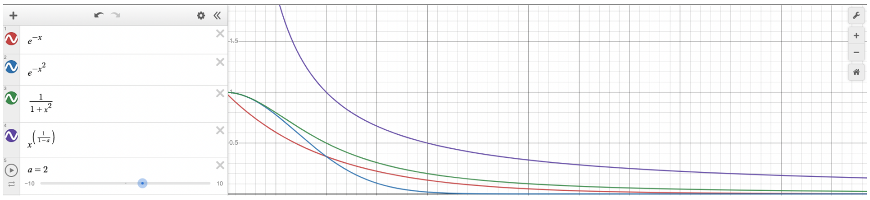

Figure 3: Plot of the different deviates in comparison to the exponential distribution

Similarly to what was done in class, we use $y = \tan^{-1}(x/2)$. Thus $x = 2\tan(y)$ and $\frac{dx}{dy} = \sec^2(y)$. We multiply tan to renormalize the function since we are only concerned with positive values of x. Therefore, we get $P'(y) = P(x)\frac{dx}{dy} = \frac{\exp(-2\tan(y))}{2\cos^2(y)}$, plotted between 0 and $\pi/2$. The acceptance condition will be that the Lorentzian divided by the exponential at some value x is greater x, for all x. Written mathematically,

$$\frac{L(x)}{\exp(-x)} < x, \forall x \tag{1}$$

The resulting plot can be seen below.

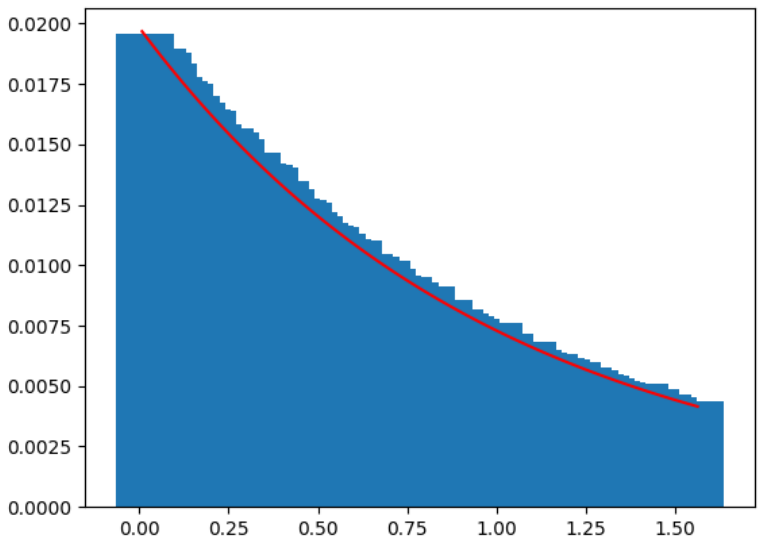

Figure 4: Plot of the exponential distribution (in red) and the sampled points

We quickly conclude that these samples closely follow the exponential distribution.

3

In question 3, I obtained a result that looked slightly off, given that the output did not appear to be normalized. If u goes from 0 to 1, v goes from 0 to 1. I cannot comment on the efficiency of this generator. Below you can see the generated plot.

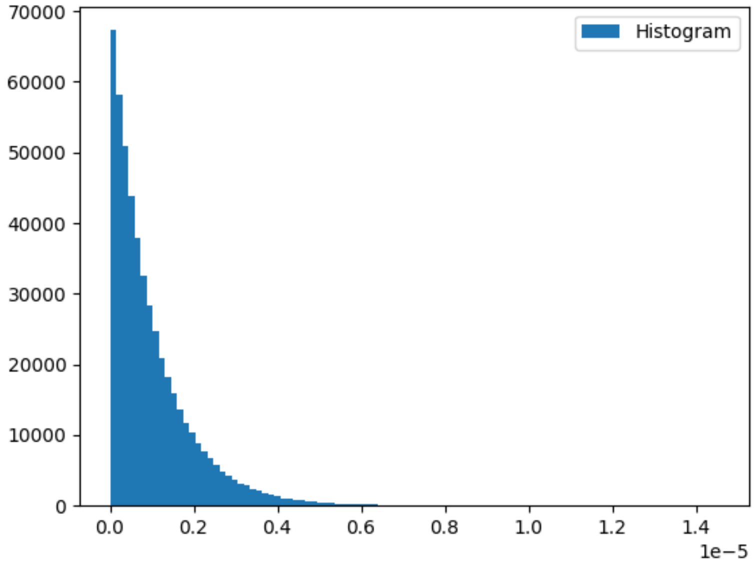

Figure 5: ratio of uniform method plot