PHYS512 A3

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1

The "rk4_stepd" integration method uses 11 function calls per step, whereas the classic "rk4_step" method uses only 4 function calls. The error is similar for both methods as can be found in the plot below.

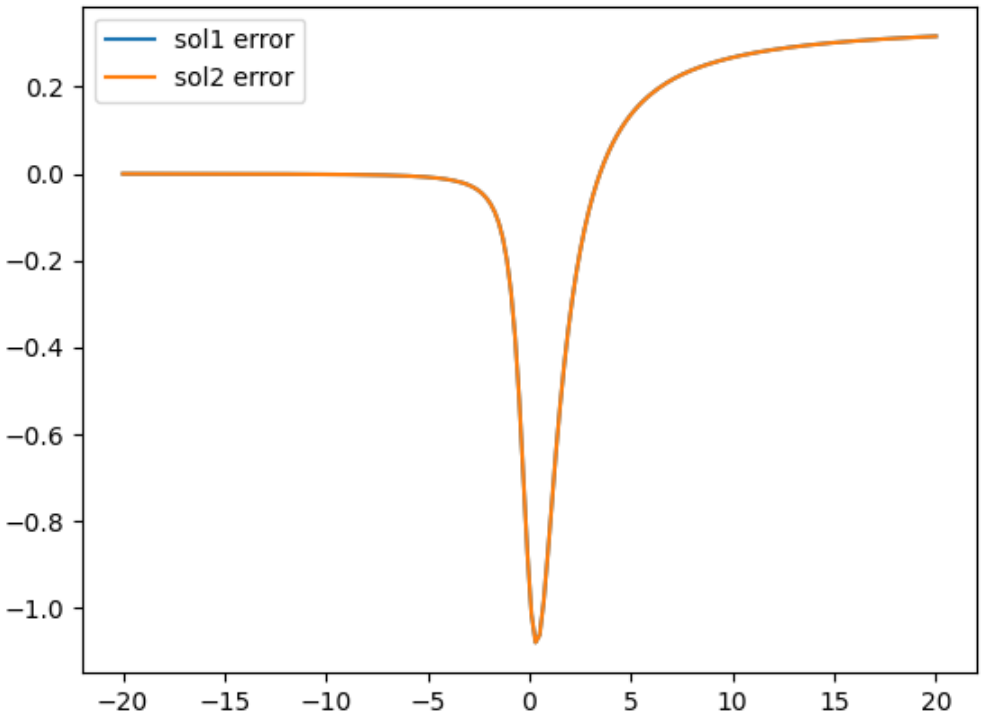

Figure 1: Plot of the errors of the "rk4_step" (blue) and "rk4_stepd" (orange) integration methods.

2

2.1

The initial solution to this problem used the classic "solve_ivp" method from the scipy library, however the code took too long to run. Once the method was changed to Radau, the code ran far quicker and produced a plot.

2.2

Below is the plot of the ratio of Lead (Pb206) to Uranium (U238) on a log scale from 1 microsecond to 10^{20} seconds. This plot intuitively makes sense, given that the graph approaches 0.5 as time

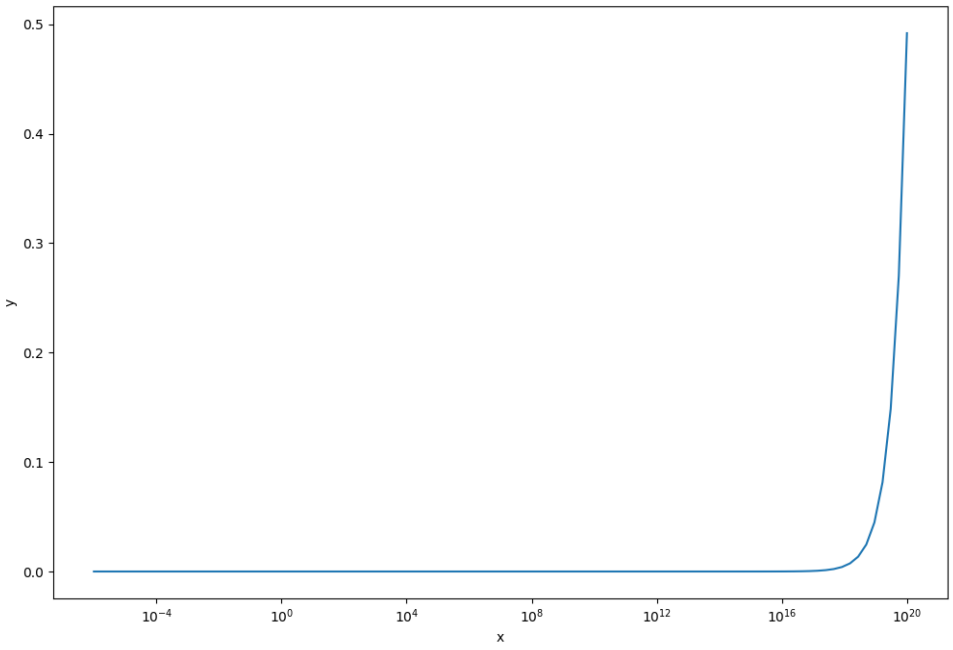

Figure 2: Plot of the ratio of Lead (Pb206) and Uranium (U238) versus time

approaches the half-life of Uranium.

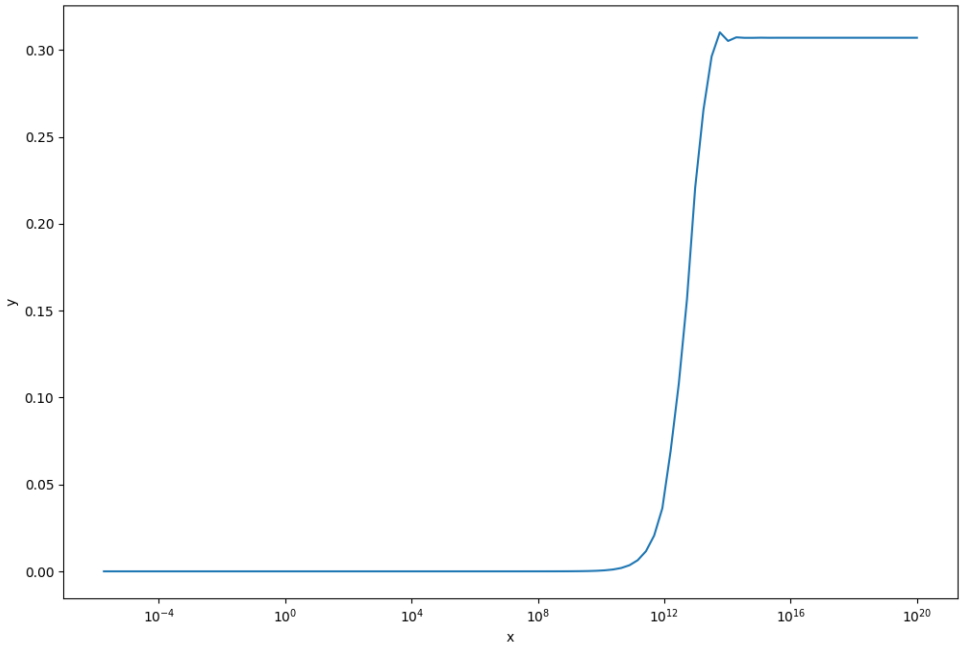

Figure 3: Plot of the ratio of Thorium (Th230) and Uranium (U234) versus time

3

3.1

We can pseudo-linearize the problem by first fully expanding the right-hand side and moving z_0 over, and then group terms together, where $A = -2ax_0$, $B = -2ay_0$, $C = a$ and $D = z_0 + a(x_0^2 + y_0^2)$

$$\begin{aligned} z &= z_0 + ax^2 - 2ax_0x + ax_0^2 + ay^2 - 2ay_0y + ay^2 \\ &= Ax + By + C(x^2 + y^2) + D \end{aligned} \tag{1}$$

3.2

The best fit parameters can be obtained from solving the above system of equations after getting the fit for A, B, C and D. From the fit, we see that $a = 0.001667$, $x0 = -1.360$, $y0 = 58.22$ and $z0 = -1512.88$.

3.3

Please see the code attached in this submission fr the steps followed to calculate the focal length and the error in the focal length. The focal length was found to be $f = 1499.66$ and $f_{error} = 0.214$