PHYS512 A4

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1.1

Newton's method with 100 iterations was used to solve this problem. The initial guess for the Lorentzian parameters are $a = 1.5$, $t_0 = 0.000192$, and $\omega = 0.00001$. The final best-fit parameters are $a = 1.4214$, $t_0 = 1.9200 \times 10^{-4}$, and $\omega = 1.7956 \times 10^{-5}$. Below is a plot of a Lorentzian plotted with the raw data.

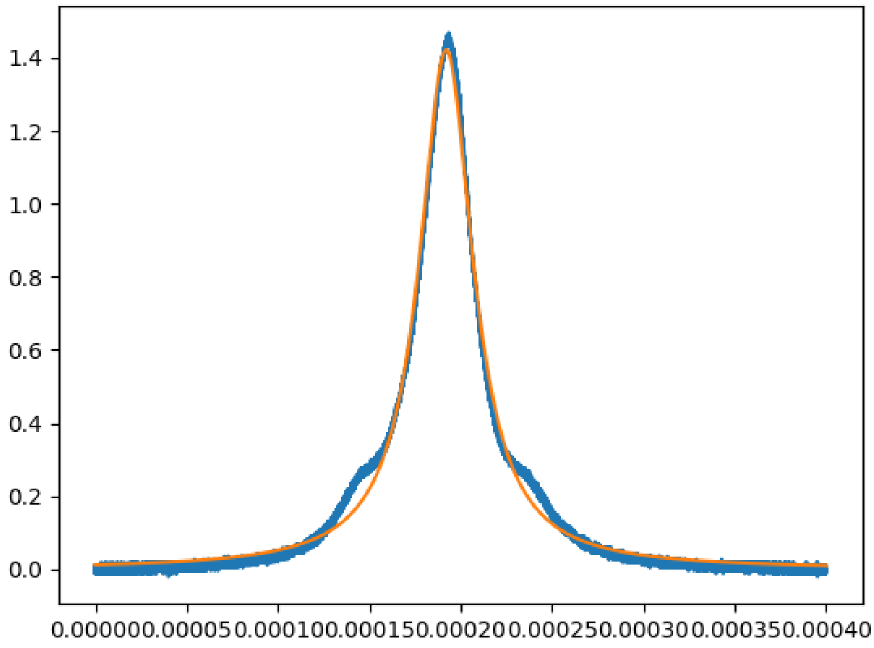

Figure 1: Best-fit Lorentzian and raw data vs. time

1.2

The errors in the parameters are as follows. $a = 1.755 \times 10^{-5}$ $t_0 = 5.823 \times 10^{-15}$ and $\omega = 3.139 \times 10^{-10}$

1.3

The best-fit parameters for the Lorentzian using the numerical gradient are $a = 1.4228$, $t_0 = 1.924 \times 10^{-4}$, and $\omega = 1.7924 \times 10^{-5}$. When comparing the numerical gradient best-fit values to the analytic gradient we note that values are accurate to two decimal places, and are thus statistically similar. The plot of the numerical best-fit Lorentzian is included below.

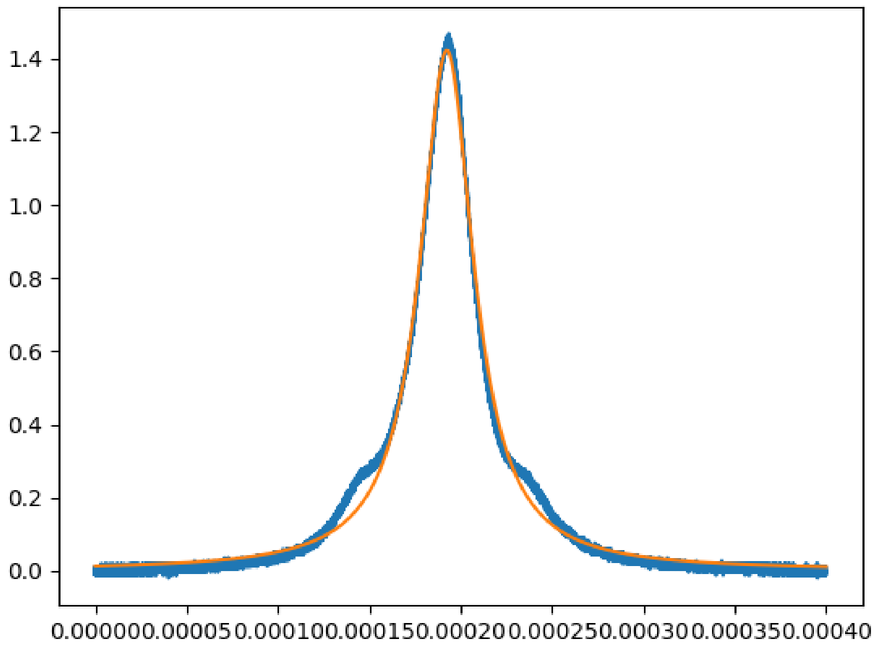

Figure 2: Best-fit numerical Lorentzian and raw data vs. time

1.4

To model the data as the sum of the three Lorentzians we use a similar approach for calculating the numerical gradient for the single Lorentzian. We will note that the two additional Lorentzians will contribute to the left and right of the main peak. Thus, their amplitude will be much smaller. Given that small dimples to the left and right of the peak occur at a y-value of about 0.3, I chose the guess values of $b = 0.3$ and $c = 0.35$. The time shift term was guessed to be $dt = 0.00005$, re-using the guesses made for a, t_0, and ω from previous questions.

The errors on these parameters were computed using the gradient and covariance matrix (see code). The errors computed are as follows. $a = 1.9045 \times 10^{-5}$, $b = 1.8164 \times 10^{-5}$, $c = 1.7786 \times 10^{-5}$, $t_0 = 2.2548 \times 10^{-10}$, $t_1 = 2.718 \times 10^{-9}$, and $\omega = 4.0381 \times 10^{-10}$.

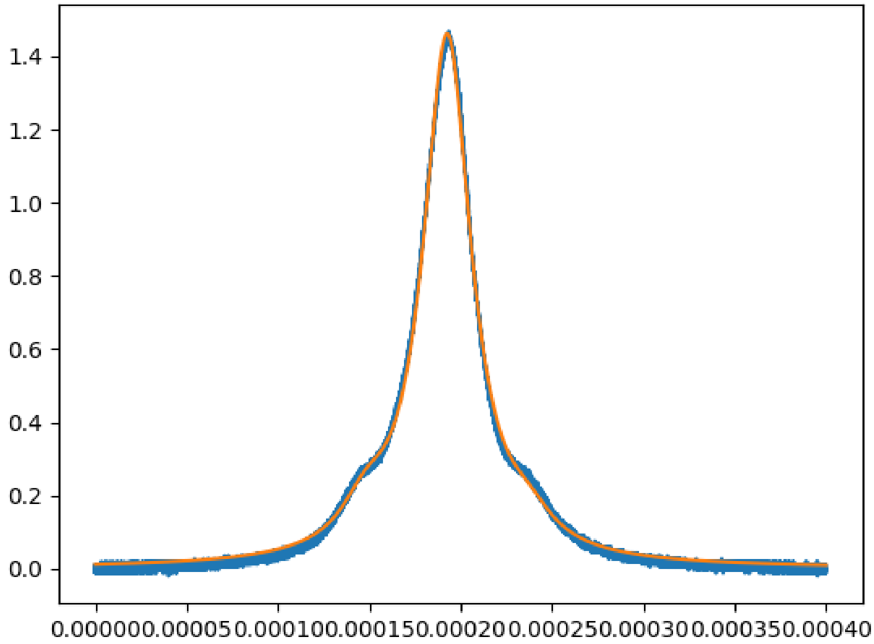

Figure 3: Best-fit numerical Lorentzian sum and raw data vs. time

1.5

From inspecting figure 4, we can see that the noise varies considerably close to $t \approx 0.0002$ seconds. This leads me to believe that the model could use further refinement, namely improving the values of a, b and c, since the noise behaves less predictably in the region most affected by these values.

1.6

The resulting χ^2 values differ by only 10^{-5}, far less than 1. This is reasonable given that we are only slightly perturbing our p values.

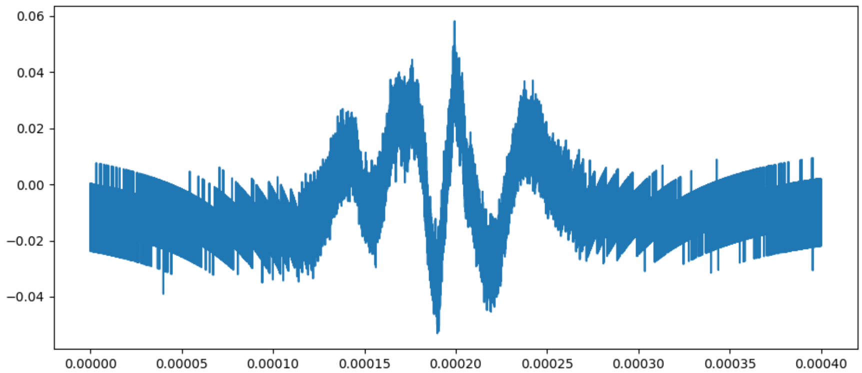

Figure 4: Best-fit numerical Lorentzian sum and raw data vs. time

1.7

For some reason, my mcmc simulation failed to yield convergence despite me using the code from the lecture slides. This will be improved for the next assignment.

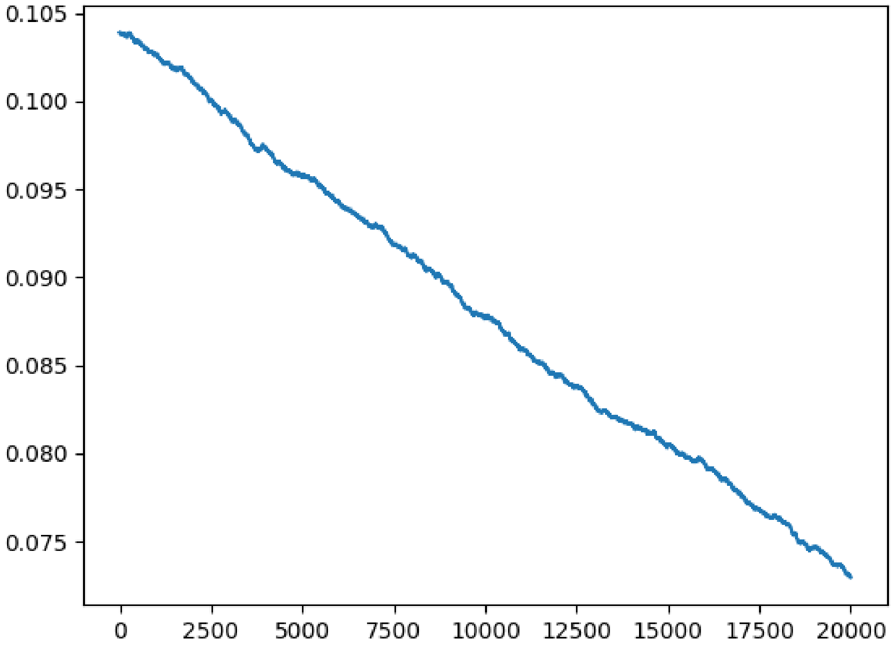

Figure 5: Markov Chain Monte Carlo Simulation output for the b parameter

1.8

We can compute the width $\mu = \frac{dt}{\omega} \times 9$ GHz of the cavity resonance by using our best-fit parameters $dt = 4.4567 \times 10^{-5} \pm 2.718 \times 10^{-9}$ and $\omega = 1.6065 \times 10^{-5} \pm 4.0381 \times 10^{-10}$. Therefore,

$\mu = 24.97 \pm 0.21$ GHz