PHYS512 Final Project

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December 2022

1 List of Abbreviations

- b: Radius of Coil [m]
- C_1: Capacitance of C_1 Capacitor [F]
- C_2: Capacitance of Tuning Capacitor [F]
- δ_p: Collision-less Skin Depth [m]
- e: Charge of an Electron [C]
- $e_c(T_e)$: Collisional energy lost per ion produced [eV]
- ϵ: Eccentricity [unitless]
- l: Length of the Cylindrical Plasma Channel [m]
- m: Atomic Mass [kg]
- μ_0: Vacuum Permeability [H/m]
- N: Number of Turns in the Coil
- n_s: Plasma Number Density [$1/m^3$]
- $P(t)$: Power Absorbed by Plasma [W]
- R: Radius of Plasma Channel [m]
- R_s: Resistance of Plasma [Ω]
- R_T: Source Resistance [Ω]
- σ_{eff}: Effective Conductivity [1/(Ωs)]
- T_e: Electron Temperature [eV]
- V_s: Sheath Voltage Drop [V]
- V_T: Power Supply Voltage [V]
- ν_{eff}: Effective Heating [1/s]
- ν_{iz}: Ionization Rate [1/s]
- ν_{loss}: Loss Rate [1/s]
- $W_e(n_s, T_e)$: Plasma Energy [J]
- ω: Angular Frequency [rad/s]
- X_1: C_1 Reactance [Ω]
- X_s: Plasma Reactance [Ω]

2 Background

2.1 Basic Thruster Design

The basic thruster design consists of a (non-conductive) cylinder with radius R and length l. A coil with N winds is wrapped around the cylinder, the radius of this coil being b. An AC voltage is applied across the terminals of the coil to ionize a gas within the tube. A figure of this can be seen below.

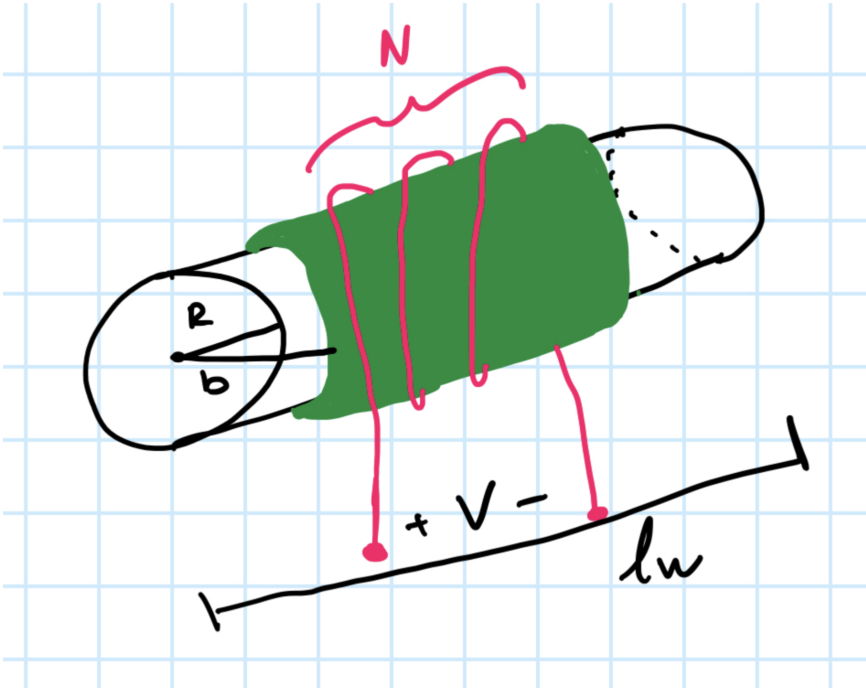

Figure 1: Sketch of a cylinder (black), surrounded by a dielectric (green), wrapped with a coil (red)
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2.1.1 Inductive Plasma Discharge

An impedance-matched thruster coil can be modeled as the circuit seen in Figure 2.

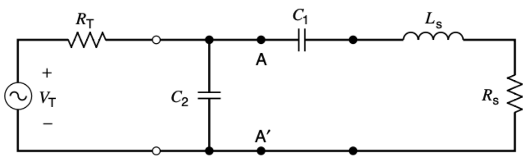

Figure 2: Equivalent impedance matching circuit for inductive discharge [1]
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The values of V_T (thruster input voltage), R_T (power supply resistance) and C_1 are fixed with values of $V_T = 600$V, $R_T = 50\Omega$, and $C_1 = 90$ nF respectively, L_s (plasma inductance) and R_s (plasma resistance) are dependent on the state of the plasma, and C_2 is the tuning capacitor. A quick derivation from Lieberman [1] yields an expression for the tuning capacitor which can be seen in equation 1. Below is a list of the terms in the expression.

- The impedance of C_1: $X_1 = \frac{1}{j\omega C_1}$
- The impedance of L_s: $X_s = j\omega L_s$
- The coil inductance: $L_s = \frac{\mu_0 \pi R^2 N^2}{l}\left(\frac{b^2}{R^2} - 1\right)$
- The plasma resistance: $R_s = \frac{N^2 \pi R}{\sigma_{eff} l \delta_p}$
- The effective conductivity: $\sigma_{eff} = \frac{e^2 n_s}{m \nu_{eff}}$
- The skin depth: $\delta_p = \sqrt{\frac{m}{e^2 \mu_0 n_s}}$

$$C_2 = \frac{1}{\omega}\frac{R_s^2 + (X_1 + X_s)^2}{(X_1 + X_s)} \tag{1}$$

The equation above was achieved by simple AC circuit analysis. In a nutshell, the above expression gives us the required capacitance for maximum power transfer into the plasma. The main takeaway here is the dependence of R_s, the plasma resistance, on n_s, the plasma number density. We see that the tuning capacitance C_2, is dependent upon R_s, so to obtain the value of this capacitor, we first need to obtain the value of n_s. We also note that the tuning capacitor is dependent on frequency. Given that the thruster will operate at a fixed frequency between 1MHz and 5 MHz (this is a design constraint given by the owner of the project, and it's derivation is outside of the scope of this report), we will choose an arbitrary frequency of $f = 2.5$ MHz. Furthermore, the value of $C_1 = 90$ nF, again, chosen arbitrarily.

2.1.2 Pulsed Discharges

The thruster is designed to be either fully on or off. We can think about this as the circuit radiating the plasma with a constant power value when on, and a power of zero when off. Thus, the input power can be modelled by a step function.

To calculate the pulsed plasma density, we solve a system of ordinary differential equations.

$$\frac{dn_s}{dt} = n_s\left[\nu_{iz}(T_e) - \nu_{loss}(T_e)\right] \tag{2}$$

$$\frac{dT_e}{dt} = T_e\left[\frac{P(t)}{W_e(n_s, T_e)} - \frac{2}{3}\left(\frac{e_c(T_e)}{T_e} + 1\right)\nu_{iz}(T_e) - \frac{2}{3}\left(\frac{V_s(T_e)}{T_e} + \frac{3}{2}\right)\nu_{loss}(T_e)\right] \tag{3}$$

While these equations may seem involved, the first is simply a conservation of particles, where the change in plasma number density is equal to the plasma number density multiplied by the difference in ions generated and ions lost to de-ionization.

The second equation is a conservation of energy for the particles, where the change in electron temperature is proportional to the electron temperature multiplied by a fixed plasma energy, the energy created by generating ions, and the energy lost from deionization. These equations are coupled, and thus need to be solved together. There are many terms in here that can be described with further granularity, but for your sake and for mine, I will leave that up to the reader. These can be found in the **gas.py** and **thruster.py** files, or one can skim through Chapter 10 of Lieberman. Again, the granularity of these equations is outside the scope of this report, and it is far more important that we focus on the ODE solution and the calculation of standard error.

The final remark for this section, is that to improve how "realistic" these results are, I injected Gaussian noise into the power supply function $P(t)$. Noise was added to each power value, with an amplitude of 100, normally distributed with zero mean and a variance of 1.

3 Results

The beauty of this project is that the plasma can be simulated for three different gas species: Ar, N_2, and Xe. The plasma number density n_s, the electron Temperature T_e, and the capacitance C_2 was simulated for all three of these species, and compared on the plot. Additionally, the standard error was computed for the capacitance of these three species, which will be listed below. The length of the pulse was taken to be 0.125 ms. The reason for this choice is that after the steady-state behaviour of both electron temperature, plasma number density and tuning capacitance is constant, and this steady-state is reached quite quickly, as one will see with the graphs. Furthermore, the ODE solver takes non-linear time to run, and thus decreasing the time over which we integrate saves computational effort while yielding the same results.

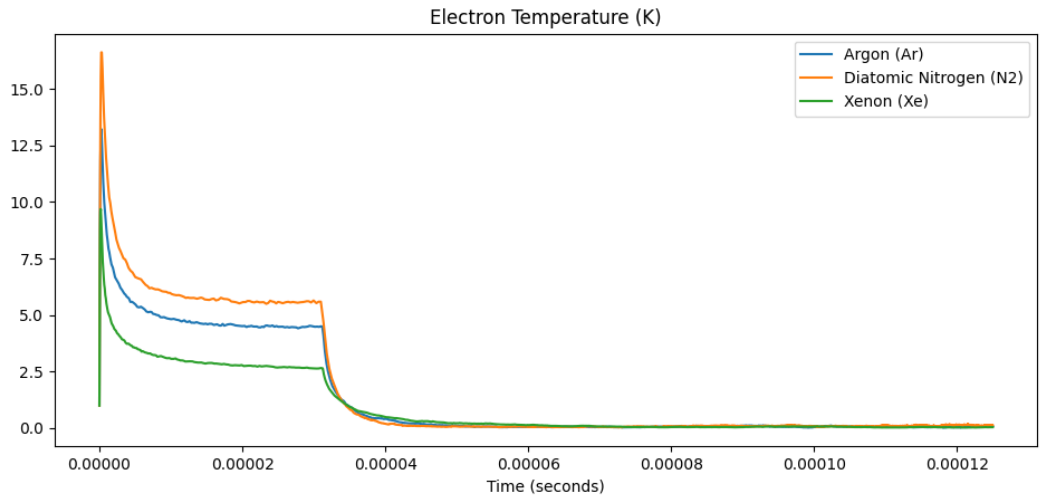

Figure 3: Electron temperature versus time.

The results seen in the electron temperature graph agree with those obtained in Lieberman Chapter 10 [1]. We can also see the effect of noise in the result. Below we see the plasma number density for each of the species in Figure 4.

We see that the data here approaches a flat slope, as stated previously. We further notice that for each species, the plasma number density remains within an order of magnitude of one another. We note that the noise is less prominent in the plasma number density, as the power term is only

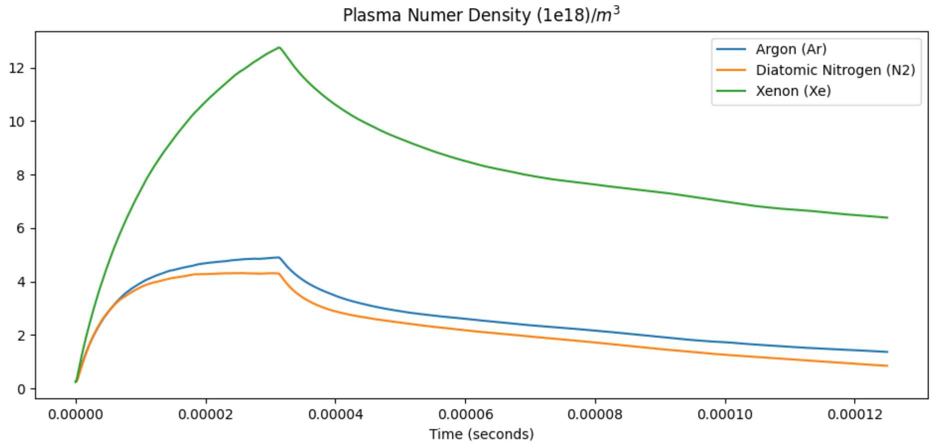

Figure 4: Electron temperature versus time.

included explicitly in the electron temperature equation. Finally, we list the capacitance of each plasma species in Figure 5.

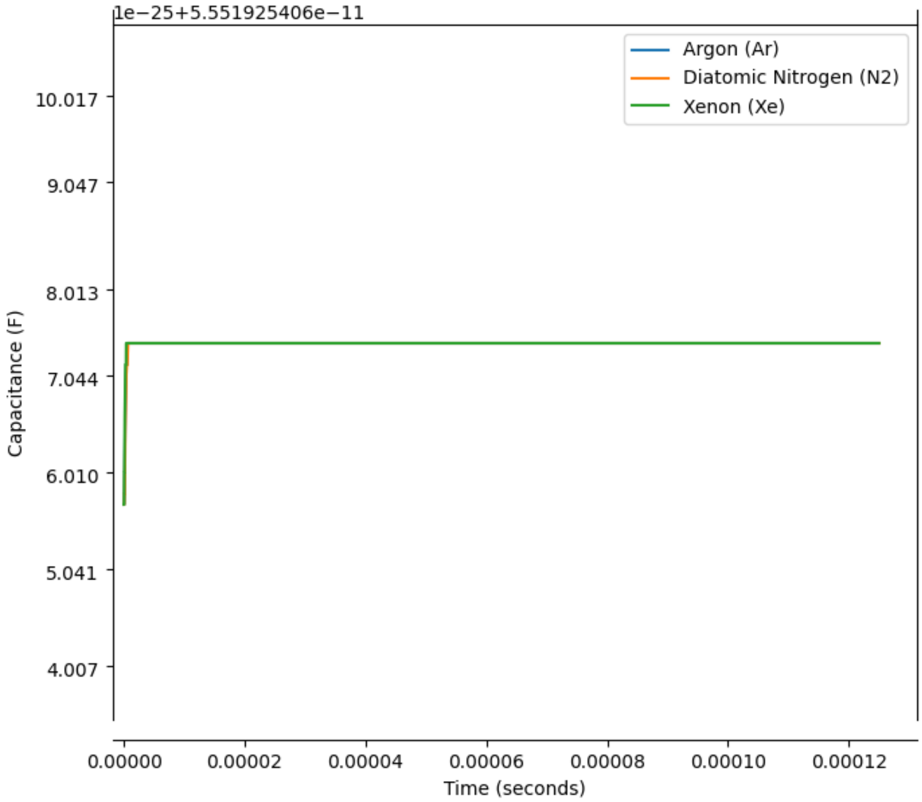

Figure 5: Tuning Capacitance versus time.

We see here that the plasma number density variations have litter effect on the capacitance value after the initial ramp-up seen in Figure 4, between $0 < t < 0.00003$ s. I computed the standard error for these capacitors by taking the standard deviation of each and dividing it by the square-root of the number of samples to obtain the tolerance. These values are listed in the table below.

- $C_{Ar} = 5.5519 \times 10^{-11} + / -1.4884 \times 10^{-29}$ F
- $C_{N_2} = 5.5519 \times 10^{-11} + / -1.2822 \times 10^{-29}$ F
- $C_{Xe} = 5.5519 \times 10^{-11} + / -7.6330 \times 10^{-29}$ F

As we can seen graphically, the values for these capacitors are incredibly similar. This can be further verified by noting that the plasma number densities are within an order of magnitude of each other, and the R_s term does not necessarily dominate in equation 1. We do however note, that diatomic nitrogen is the least sensitive. This is wonderful news, given that in practice, the upper atmosphere (the environment in which this thruster would be operational), is predominantly oxygen and nitrogen. The code for this assignment can be found in the included directory. Please contact me with any questions you may have.

4 References

1. M. A. Lieberman and A. J. Lichtenberg, Principles of plasma discharges and materials processing. Hoboken, NJ: Wiley-Interscience, 2005.