

Assignment 2

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GATE EC PROBLEM 30

If E denotes the expectation, the variance of a random variable X is given by ?

- (A) $E[X^2] - E^2[X]$
- (B) $E[X^2]$
- (C) $E[X^2] + E^2[X]$
- (D) $E^2[X]$

SOLUTION

The expectation of a random variable X is given by

$$E[X] = \sum_{\text{all } x} x \Pr(x)$$

The expectation of a random variable is also known as mean of random variable and denoted as μ

The expectation of a function $g(X)$ is given by

$$E[g(X)] = \sum_{\text{all } x} g(x) \Pr(x)$$

The variance of a random variable X is given as

$$\text{Var}(X) = E[(X - \mu)^2]$$

We know that

$$E[(X - \mu)] = \sum_{\text{all } x} (x - \mu)^2 \Pr(x) \quad (1)$$

$$E[(X - \mu)] = \sum_{\text{all } x} (x^2 - 2\mu x + \mu^2) \Pr(x) \quad (2)$$

$$= \sum_{\text{all } x} x^2 \Pr(x) - 2\mu \sum_{\text{all } x} x \Pr(x) + \mu^2 \sum_{\text{all } x} \Pr(x) \quad (3)$$

$$= E[X^2] - 2\mu \cdot \mu + \mu^2 (1) \quad (4)$$

$$= E[X^2] - \mu^2 \quad (5)$$

$$= E[X^2] - E^2[X] \quad (6)$$

\therefore

Hence option A is the correct answer

THE PYTHON CODES

We run a simulation using binomial distribution with random probabilities and random number of values the random variable can take. We then plot variance calculated using the above formula on the X-axis while on the Y-axis we use variance from the inbuilt function inside the scipy library. If the formula is correct then the plotted points should be around the $x=y$ line. The graph is shown below.

