

Assignment-1

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PROBLEM 1.7

It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

SOLUTION 1.7

The repeated selections of articles in a random sample space are Bernoulli trials. Let X denote the number of times of selecting defective articles in a random sample space of 12 articles.

Clearly X , has a binomial distribution with $n=12$ and $p=10\%=\frac{1}{10}=0.1$ and $q=1-p=1-\frac{1}{10}=\frac{9}{10}$

$$\therefore X = B(n, p)$$

The probability mass function(p.m.f) for a general binomial distribution is given by

$$\Pr(X = k) = \binom{n}{k} \times p^k \times q^{n-k}$$

In this case $n=12$, $p=\frac{1}{10}$ and $q=\frac{9}{10}$, So

$$\Pr(X = k) = \binom{12}{k} \times \left(\frac{1}{10}\right)^k \times \left(\frac{9}{10}\right)^{12-k}$$

where $\Pr(X = k)$ is the probability of getting k number of defective articles and $k \in \{0, 1, 2, \dots, 12\}$

In this case we want $\Pr(X = 9)$

$$\Rightarrow \Pr(X = 9) = \binom{12}{9} \times \left(\frac{1}{10}\right)^9 \times \left(\frac{9}{10}\right)^3 \quad (1)$$

$$= \frac{12!}{9!3!} \times \frac{1}{10^9} \times \frac{9^3}{10^3} \quad (2)$$

$$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} \times \frac{729}{10^{12}} \quad (3)$$

$$= \frac{16038}{10^{11}} \quad (4)$$

So, the probability of there being 9 defective articles in a random sample space of 12 articles is $\Pr(X = 9) = 1.6038 \times 10^{-7}$

PYTHON CODE

Since the probability of producing 9 defective articles is very very low we need to run a large number of simulations. But a large number of simulations takes a large amount of time which is inefficient so we need to strike a balance between time and the precision to the original value. So a simulation of 4×10^8 would likely yield a better results in a better time

The codes can be downloaded from

Vasishtadepu/AI5002/tree/main/Assignment1/Codes