

Assignment 2

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GATE EC PROBLEM 30

If E denotes the expectation, the variance of a random variable X is given by ?

- (A) $E[X^2] - E^2[X]$
- (B) $E[X^2]$
- (C) $E[X^2] + E^2[X]$
- (D) $E^2[X]$

SOLUTION

Before we start the proof we need to know 3 properties of expectation

$$E[f(x) + g(x)] = E[f(x)] + E[g(x)] \quad (1)$$

If k is a constant value then

$$E[k \cdot g(x)] = k \cdot E[g(x)] \quad (2)$$

$$E[k] = k \quad (3)$$

Now variance of random X is given by

$$Var(X) = E[(X - \mu)^2] \quad \text{where } \mu = E[X]$$

$$Var(X) = E[X^2 - 2\mu \cdot X + \mu^2]$$

$$= E[X^2] - E[2\mu \cdot X] + E[\mu^2] \quad \text{from (1)}$$

$$= E[X^2] - 2\mu \cdot E[X] + \mu^2 \quad \text{from (2) and (3)}$$

$$= E[X^2] - 2\mu^2 + \mu^2 \quad (\because E[X] = \mu)$$

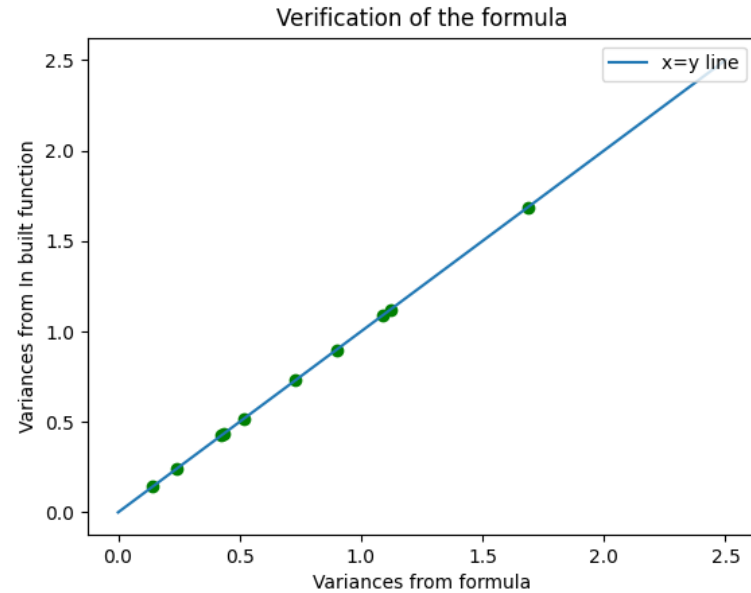
$$= E[X^2] - \mu^2$$

$$= E[X^2] - E^2[X] \quad (\because \mu = E[X])$$

Hence option (A) is correct.

GRAPH USING PYTHON

We consider a binomial distribution with random variable X and assign randomly the values it can take and probability is also random. We calculate two variances one using the formula $Var(X) = E[X^2] - E^2[X]$ and other using the inbuilt function in `scipy.stats`. We plot the graph between two and compare them with the line $x = y$. With the green points representing the variance points.



As we can see from the above graph all the points lie on the line $x = y$ so the formula is correct