Assignment 3

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GATE 2019(IN) Q.No 27(Eng section)

The function $p(x)$ is given by $p(x) = \frac{A}{x^\mu}$ where A and μ are constants with $\mu > 1$ and $1 \leq x < \infty$ and $p(x) = 0$ for $-\infty < x < 1$. For $p(x)$ to be a probability distribution function, the value of A should be equal to?

1) $\mu - 1$
2) $\mu + 1$
3) $\dfrac{1}{\mu - 1}$
4) $\dfrac{1}{\mu + 1}$

Solution

Given function $p(x)$ is given by

$$p(x) = \begin{cases} 0 & \text{if } \quad -\infty < x < 1 \\ \dfrac{A}{x^\mu} & \text{if } \quad 1 \leq x < \infty \end{cases}$$

For any function $g(x)$ to be a probability distribution function the following conditions has to be to be satisfied.

(1) $\quad g(x) \geq 0$

(2) $\quad \int\limits_{-\infty}^{\infty} g(x)\, dx = 1$

Since $p(x)$ = $\dfrac{A}{x^\mu}$ for $x \geq 1$ we need $A > 0$ for the 1st condition to be satisfied.

Now we calculate the definite integral for the second condition

$$\int\limits_{-\infty}^{\infty} p(x)\, dx = \int\limits_{-\infty}^{1} p(x)\, dx + \int\limits_{1}^{\infty} p(x)\, dx \tag{1}$$

$$= 0 + \int\limits_{1}^{\infty} \frac{A}{x^\mu} dx \tag{2}$$

$$= \left[\frac{-A}{(\mu - 1)\, x^{\mu-1}}\right]_1^\infty (\mu > 1) \tag{3}$$

$$= 0 + \frac{A}{\mu - 1} \quad (\because \mu > 1) \tag{4}$$

$$\int\limits_{-\infty}^{\infty} p(x) = \frac{A}{\mu - 1} \tag{5}$$

From (2) the above equality should also be equal to 1.

$$\frac{A}{\mu - 1} = 1 \tag{6}$$

$$\implies A = \mu - 1$$

Hence option (A) is the correct answer.

Graphs using python

Using matplotlib in python we plot graphs of pdf by taking various values of μ the resulting graph looks something like this