Assignment 2

ADEPU VASISHT

GATE EC PROBLEM 30

If E denotes the expectation, the variance of a random variable X is given by ?
(A) $E[X^2] - E^2[X]$
(B) $E[X^2]$
(C) $E[X^2] + E^2[X]$
(D) $E^2[X]$

SOLUTION

The expectation of a random variable X is given by

$$E[X] = \sum_{all\ x} x \Pr(x)$$

The expectation of a random variable is also known as mean of random variable and denoted as μ

The expectation of a function $g(X)$ is given by

$$E[g(X)] \sum_{all\ x} g(x) \Pr(x)$$

The variance of a random variable X is given as

$$Var(X) = E[(X - \mu)^2]$$

We know that

$$E[(X - \mu)] = \sum_{all\ x} (x - \mu)^2 \Pr(x) \tag{1}$$

$$E[(X - \mu)] = \sum_{all\ x} \left(x^2 - 2\mu x + \mu^2\right) \Pr(x) \tag{2}$$

$$= \sum_{all\ x} x^2 \Pr(x) - 2\mu \sum_{all\ x} x \Pr(x) \tag{3}$$

$$+ \mu^2 \sum_{all\ x} \Pr(x)$$

$$= E[X^2] - 2\mu.\mu + \mu^2 (1) \tag{4}$$

$$= E[X^2] - \mu^2 \tag{5}$$

$$= E[X^2] - E^2[X] \tag{6}$$

$$\therefore$$

Hence option A is the correct answer

THE PYTHON CODES

We run a simulation using binomial distribution with random probabilities and random number of values the random variable can take. We then plot variance calculated using the above formula on the X-axis while on the Y-axis we use variance from the inbuilt function inside the scipy library. If the formula is correct then the plotted points should be around the x=y line. The graph is shown below.
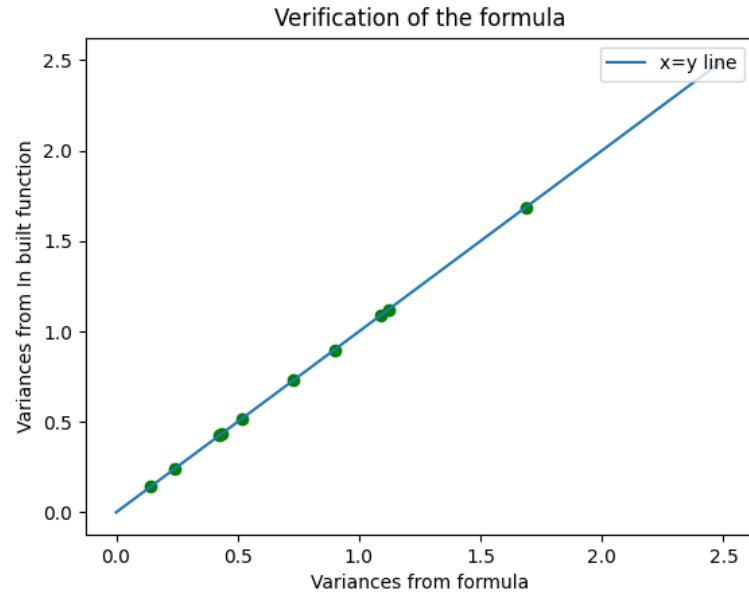
As we can see from the above graph the points are close to the $x = y$ line hence the formula is correct.

The codes can be found in the link below

Vasishtadepu/AI5002/tree/main/assignment2/Codes