Assignment 2

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The function $p(x)$ is given by $p(x) = \frac{A}{x^{\mu}}$ where A and μ are constants with $\mu > 1$ and $1 \leq x < \infty$ and $p(x) = 0$ for $-\infty < x < 1$. For $p(x)$ to be a probability distribution function, the value of A should be equal to?

(A) $\mu - 1$

(B) $\mu + 1$

(C) $\dfrac{1}{\mu - 1}$

(D) $\dfrac{1}{\mu + 1}$

Solution

Given function $p(x)$ is given by

$$p(x) = \begin{cases} 0 & \text{if} \quad -\infty < x < 1 \\ \dfrac{A}{x^{\mu}} & \text{if} \quad 1 \leq x < \infty \end{cases}$$

For any function $g(x)$ to be a probability distribution function the following conditions has to be to be satisfied.

(1) $g(x) \geq 0$

(2) $\int\limits_{-\infty}^{\infty} g(x)\,dx = 1$

Since $p(x) = \dfrac{A}{x^{\mu}}$ for $x \geq 1$ we need $A > 0$ for the 1^{st} condition to be satisfied.

Now we calculate the definite integral for the second condition

$$\int\limits_{-\infty}^{\infty} p(x)\,dx = \int\limits_{-\infty}^{1} p(x)\,dx + \int\limits_{1}^{\infty} p(x)\,dx$$

$$= 0 + \int\limits_{1}^{\infty} \frac{A}{x^{\mu}} dx \quad (\because p(x) = 0 \text{ in } (-\infty, 1))$$

$$= \left[\frac{-A}{(\mu - 1)\,x^{\mu - 1}}\right]_1^{\infty} \quad (\because \mu > 1)$$

$$= 0 + \frac{A}{\mu - 1} \quad (\because \mu > 1)$$

$$\int\limits_{-\infty}^{\infty} p(x) = \frac{A}{\mu - 1}$$

From (2) the above equality should also be equal to 1.

$$\frac{A}{\mu - 1} = 1$$

$$\implies A = \mu - 1$$