Assignment-1

Adepu Vasisht

PROBLEM 1.7

It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

SOLUTION 1.7

The repeated selections of articles in a random sample space are Bernoulli trials. Let X denote the number of times of selecting defective articles in a random sample space of 12 articles.

Clearly X, has a binomial distribution with n=12 and p=10%=$\frac{1}{10}$=0.1 and q=1-p=1-$\frac{1}{10}$=$\frac{9}{10}$

$$\therefore X = B(n,p)$$

The probability mass function(p.m.f) for a general binomial distribution is given by

$$\Pr(X = x) = \binom{n}{x} \times p^x \times q^{n-x}$$

In this case n=12 , p=$\frac{1}{10}$ and q=$\frac{9}{10}$, So

$$\Pr(x) = \binom{12}{x} \times (\frac{1}{10})^x \times (\frac{9}{10})^{12-x}$$

where $\Pr(x)$ is the probability of getting x number of defective articles and $x \in \{0, 1, 2.......12\}$

In this case we want $\Pr(9)$

$$\Pr(9) = \binom{12}{9} \times (\frac{1}{10})^9 \times (\frac{9}{10})^3$$

$$= \frac{12!}{9!3!} \times \frac{1}{10^9} \times \frac{9^3}{10^3}$$

$$= \frac{12 \times 11 \times 10}{3 \times 2 \times 1} \times \frac{729}{10^{12}}$$

$$= \frac{16038}{10^{11}}$$

So, the probability of there being 9 defective articles in a random sample space of 12 articles is $\Pr(9) = 1.6038 \times 10^{-7}$

PYTHON CODE

Since the probability of producing 9 defective articles is very very low we need to run a large simulation. But a large simulation takes a large amount of time which is inefficient so we need to strike a balance between time and the precision to the original value. So a simulation of 4×10^8 would likely yield a better results in a better time

The codes can be downloaded from

Vasishtadepu/AI5002/tree/main/Assignment1/Codes