Assignment 2

ADEPU VASISHT

GATE EC Problem 30

If E denotes the expectation, the variance of a random variable X is given by ?

(A) $E[X^2] - E^2[X]$
(B) $E[X^2]$
(C) $E[X^2] + E^2[X]$
(D) $E^2[X]$

Solution

The expectation of a random variable X is given by

$$E[X] = \sum_{all\ x} x \Pr(x) = \mu \tag{1}$$

The variance of the random variable X is given by

$$Var(X) = \sum_{all\ x} (x-\mu)^2 \Pr(x) \tag{2}$$

We know that sum of all the probabilities is 1 i.e

$$\sum_{all\ x} \Pr(x) = 1 \tag{3}$$

We expand the variance equation (2) from above

$$Var(X) = \sum_{all\ x} (x-\mu)^2 \Pr(x)$$

$$= \sum_{all\ x} \left(x^2 - 2x\mu + \mu^2\right) \Pr(x)$$

$$= \sum_{all\ x} x^2 \Pr(x) - 2\mu \sum_{all\ x} \Pr(x) + \mu^2 \sum_{all\ x} \Pr(x)$$

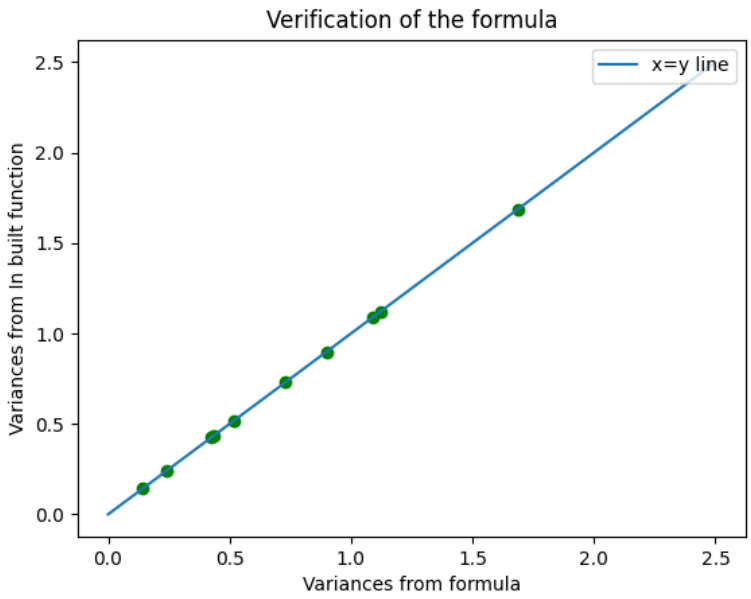
$$= E[X^2] - 2\mu \cdot \mu + \mu^2 \, (1) \, (\because (1)\, and\, (3))$$

$$= E[X^2] - E^2[X]$$

Hence option A is the correct answer

Graph using Python

We consider a binomial distribution with random variable X and assign randomly the values it can take and probability is also random. We calculate two variances one using the formula $Var(X) = E[X^2] - E^2[X]$ and other using the inbuilt function in scipy.stats. We plot the graph between two and compare them with the line $x = y$. With the green points representing the variance points.



As we can see from the above graph all the points lie on the line $x = y$ so the formula is correct