CSE 15: Discrete Mathematics
Homework 3

Spring 2020

Introduction

This assignment is about propositional logic and its applications. There will be logical puzzles (system specification), logical equivalences, and the very important concept of logical consequence. As always, you will also be practicing your LaTeX skills.

Exercises

Create a new LaTeX document and type out the solutions to the following exercises. Your document should include an appropriate title, your name, as well as a date. Please number your solutions appropriately. Upload your `.tex` and your `.pdf` files under the relevant CatCourses assignment.

Knights and Knaves

Recall the Knights and Knaves puzzle we saw in class. In that fictional world there are two types of people. Knights - who always tell the truth, and Knaves - who always lie. It is impossible to distinguish them by appearance, but only by the truth of their statements.

1. One day a traveller was wandering around the island of Knights and Knaves, when he encountered two local inhabitants, P and Q. The traveller asked: "Is any of you a knave?". P replied: "At least one of us is a knave".

   Can you find out what P and Q are? If so, what are they? If not, explain why not, and what other information you would need to know.

2. Later on, the traveller met two other locals, A and B. He asked whether either of them is a knight. A replied: "If B is a knave, then I am a knave too".

   What are A and B?

Logical Identities

Simplify the following propositions. Show all steps of your solutions.

1. $\neg(p \to (q \to p))$
2. $\neg((p \wedge q) \to (q \vee p))$

Logical Equivalences

Determine whether or not the following pairs of propositions are equivalent. Show all steps.

1. $p \to (q \to r)$ and $(p \wedge q) \to r$
2. $p \to (q \to r)$ and $(p \to q) \to r$

Logical Consequence

Determine if the following inferences are valid. Explain why, or why not.

1.
    Jimmy is smart
    Smart people are rich
    ―――――――――――――
    Jimmy is rich

2.
    Islands are surrounded by water
    Puerto Rico is surrounded by water
    ―――――――――――――――――――
    Puerto Rico is an island