Definition 0.0.1. Consider the natural numbers $\mathbb{N} = \mathbb{N}_0$, with $+, \cdot$ defined as usual. Then $(\mathbb{N}, +, 0)$ is a commutative monoid, as is $(\mathbb{N}, \cdot, 1)$, and $+$ and $\cdot$ satisfy distributivity just as in a ring, i.e. with negatives for addition.

Consider the natural numbers $\mathbb{N}_{>0}$, with $+, \cdot$ defined as usual. Then $(\mathbb{N}, +)$ is a commutative semigroup, $(\mathbb{N}, \cdot, 1)$ is a commutative monoid, and $+$ and $\cdot$ satisfy distributivity just as in a ring.

Consider the natural numbers $\mathbb{N}_{>k}$ for any $k \in \mathbb{N}$ which is at least 1. Then $(\mathbb{N}, +)$ is a commutative semigroup, as is $(\mathbb{N}, \cdot, 1)$, and $+$ and $\cdot$ satisfy distributivity just as in a ring.

Example 0.0.2. In particular we note that for a ring like structure the absence of invertibles allows for maps to not fix units and zeros but still be compatible with addition and multiplication. As an example consider that chain of inclusions $\ldots \supset \mathbb{N}_{>k} \supset \ldots \supset \mathbb{N}_{>0} \supset \mathbb{N}_0$.