Chapter 1

2024-04-21 – Cyclotomic Polynomials

1.1 Explicit construction

For the cyclotomic polynomials there is always a coefficient which is exactly $1 \in \mathbb{Z}$ and maps to the relevant $1 \in R$ for any commutative unital zero-divisor-free factorial ring over which we consider the cyclotomic polynomial. Hence recall the famous Eisenstein's criterion to look up in your favourite algebra reference, with a minute simplification assuming there is a coefficient $a_i = 1$.

Proposition 1.1.1. *Let A be a factorial ring with quotient field K: $A \hookrightarrow Quot(A) = K$. Let $f = \sum_i a_i X^i \in A[X]$ be a polynomial with coefficients in A of degree N such that $a_N = 1$ and $a_0 = \pm 1$.*

If there exists $p \in A$ which is indecomposable, satisfying $p | a_i$ for $i = 1, \ldots, N-1$, then f is indecomposable over $K[X]$ as well as $A[X]$.

Proof. $\square$