Chapter 1

2024-04-20 – The basic Hopf fibre bundles

1.1 Explicit construction

Definition 1.1.1. For $k = \mathbb{R}^n$ assume a multiplication $\mu\colon k \otimes_{\mathbb{R}} k \to k$ making $k^\times \supset \mathbb{S}(k) \cong \mathbb{S}^{n-1}$ into an H-space. Then we can construct a fibre bundle as follow: Consider the unit sphere in $\mathbb{S}(k \times k)$, which can be identified as $\mathbb{S}(\mathbb{R}^{2n}) = \mathbb{S}^{2n-1}$, and the one-point compactification of $\mathbb{S}^k = \mathbb{S}^{dim_{\mathbb{R}} k} = \mathbb{S}^n$.

Notice that for $\mathbb{S}(k^2)$ the components can be written as pairs of elements of k with norms each less than or equal to one, and norm-squares summing to one. Thus consider the map:

$$\begin{aligned} m_k\colon \quad \mathbb{S}^{2n-1} = \mathbb{S}(k \times k) \to & \quad \mathbb{S}^k \\ a, b \mapsto & \quad a \cdot b^{-1}. \end{aligned}$$

Proposition 1.1.2. *This is a fibre bundle for $k = \mathbb{R}, \mathbb{C}, \mathbb{H}$.*