Computer Assigment

A set of n lamps illuminates an area that we divide into m regions (pixels). We let l_i denotes the lighting level in region i, so the $m-$vector l gives the illumination levels across all regions. We let p_i denote the power at which lamp i operates, so the $n-$vector p gives the set of lamp powers. The vector of illumination levels is a linear function of the lamp powers, so we have $l = Ap$ for some $m \times n$ matrix A. The jth column of A gives the illumination pattern for lamp j, i.e., the illumination when lamp j has power 1 and all other lamps are off. We will assume that A has linearly independent columns (and therefore is tall). The ith row of A gives the sensitivity of pixel i to the n lamp powers. Your mission is to find lamp powers that result in a desired illumination pattern l^{des}, such as $l^{\text{des}} = \mathbf{1}$, ($\mathbf{1}$ the vector with all ones) which is uniform illumination with value across the area. In other words, we seek p so that $Ap \approx l^{\text{des}}$. We can use least squares to find $\hat{p}$ that minimizes the sum square deviation from the desired illumination, $\|Ap - l^{\text{des}}\|_2^2$. As an example use $n = 10$ lamps, the area being an 25×25 grid with $m = 625$ pixels, each $1m^2$. The (x, y) positions of lamps and their height above floor are

$$\{(4.1, 20.4, 4), (14.1, 21.3, 3.5), (22.6, 17.1, 6), (5.5, 12.3, 4), (12.2, 9.7, 4)\}$$

$$\{(15.3, 13.8, 6), (21.3, 10.5, 5.5), (3.9, 3.3, 5), (13.1, 4.3, 5), (20.3, 4.2, 4.5)\}$$

The illumination decays with an inverse square law, so A_{ij} is proportional to d_{ij}^{-2}, where d_{ij} is the (3-D) distance between the center of the pixel and the lamp position. The matrix A is scaled so that when all lamps have power one, the average illumination level is one. The desired illumination pattern is $\mathbf{1}$, i.e., uniform with value 1.

1. Using colormap, create two graphs to show the illumination of the two patterns: The first with all lamps set to 1 and the other that minimize the sum square deviation with a desired uniform illumination. Which are the Root Mean Squared (RMS) errors in both cases?

2. Create the histogram of patch illumination values for all lamp powers one, and for lamp powers found by LS. Explain the results.

3. The above choice of points for the lamps was made in such a way to ensure positivity in the corresponding power of the lamps. Now you have to add an extra constraint. The total energy consumption of the lamps will be equal 10 and non of them could be negative. Find the new power distribution of the lamps in order to ensure the least square (under constraints) optimality.

4. Challenge. Try to find, new points for the lamps to beat the RMS error of the first question. The lamps can be in any height between 4 and 6 meters and of course inside the area. Again, the total energy of the lamps should be 10 and non of them should be negative. Run different -random- choices of positions until you find a better solution. For this choice of points, create present the colormap picture of the illumination as well as the histogram of the intensities of the pixels of the area.

Remarks

- The library optimize of SciPy could be useful.
- Below an example of colormap and histogram