2010 American Control Conference
Marriott Waterfront, Baltimore, MD, USA
June 30-July 02, 2010

Precision Coordination and Motion Control of Multiple Systems via Iterative Learning Control

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Abstract— **In this paper, we focus on improving the trajectory tracking and formation coordination performance of multiple systems through the use of iterative learning control. A Norm Optimal framework is used to design optimal learning filters based on varying design objectives. The general norm optimal framework is reformatted to enable separate weighting on individual system trajectory tracking, coupled system trajectory tracking, and coordinated system formation or shape tracking. A general approach for designing a norm optimal learning controller for this coupled system is included. The novel structure of the weighting matrices used in this approach enables one to focus on individual design objectives (e.g. trajectory tracking, formation tracking) and formation approaches (e.g. leader reference, formation center, and neighbor reference tracking) that affect the overall performance of the coupled systems within the same framework. The capabilities of the proposed controller are validated through simulation results.**

I. INTRODUCTION

Multiple system control is comprised of a multitude of independent systems, which are coupled together through a common desired outcome. As the drive to enhance efficiency and positioning from the macro to the nano-scale increases, the ability to improve system coordination and precision motion control becomes more critical. In this paper we present a method for improving precision coordination and motion control of multi-input multi-output (MIMO) systems that execute the same task repetitively. Examples of multiple systems coupled through a common outcome which perform the same task repetitively can be found in manufacturing applications [1]–[3], surveying [4], [5], and agricultural applications such as crop dusting and spraying, [6].

A general approach for controlling the positioning of multiple systems is to implement individual controllers on each independent agent. However, one of the unique advantages of working with MIMO systems in which the desired output is coupled in the form of a particular formation is the ability to focus on the coordination of these agents as a group objective, in addition to the individual objective of trajectory tracking. The formation-keeping approach has been inspired by observations from nature, such as flocking or schooling. Majority of the work in this area has focused on developing optimal and robust feedback controllers in the time domain [7]–[10]. More recent approaches have included the use of Iterative Learning Control [11], [12]. However, most methods which enable one to focus on tracking and coordination of a combined MIMO system require different control design strategies depending on the formation and the objectives of the combined system. Previous results in [13] demonstrated performance improvements obtained through the use of a controller which coupled individual systems through the error signals. This coupled approach makes the control input, and thereby the system output, dependent on the performance of the other systems. The work presented in this paper seeks to extend the idea of coupling multiple systems through a common desired output and formation.

The primary objective of the paper is to present a novel control methodology for precision coordination and motion control of multiple systems which perform the same task repetitively. More specifically, this paper will focus on 1) the development of a generalized structure for an iterative learning controller which incorporates a coupled approach to improving the performance of multiple systems, and 2) a description of a design methodology for generating this coupled iterative learning controller. The remainder of the paper is as follows. Section II presents the class of systems considered in this paper. A description of a few techniques for coupling multiple systems is provided in Section III. Sections IV and V introduce the novel norm optimal framework and design methodology which will be used to develop the learning controllers. Simulation results for learning controllers designed to satisfy varying design objectives will be given in Section VI. Section VII provides concluding remarks and future directions.

II. CLASS OF SYSTEMS

In this paper we consider linear, causal, discrete-time MIMO systems, P, given as

$$P \triangleq \begin{cases} x_j(k+1) = A(k)x_j(k) + B(k)u_j(k) \\ \delta y_j(k) = C(k)x_j(k) + D(k)u_j(k), \end{cases} \tag{1}$$

$$y_j(k) = \delta y_j(k) + y_o(k) + d_j(k) \tag{2}$$

where $k = 0\ldots, N-1$ is the discrete time index, $j = 0\ldots$ is the iteration index, $u_j(k) \in \mathbb{R}^{q_i}$ is the control, $y_j(k) \in \mathbb{R}^{q_o}$ is the output, $y_o(k) \in \mathbb{R}^{q_o}$ is iteration-invariant, $d_j(k) \in \mathbb{R}^{q_o}$ corresponds to stochastic external disturbances, $x_j(k) \in \mathbb{R}^n$ are system states, and $(A(k), B(k), C(k), D(k))$ are appropriately sized iteration-invariant real-valued matrices. It is assumed that $x_j(0) = x_o$ for all j, and that $y_o(k)$ can be used to capture iteration-invariant initial-condition responses, feedback control, and external disturbances. As illustrated by the matrices, $(A(k), B(k), C(k), D(k))$, P is defined as time-varying over a single profile, but iteration-invariant from trial-to-trial. In the lifted-domain [14], [15], the discrete-time behavior of the system is represented by its

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convolution matrix $\mathbf{P}$ using impulse response data $H_{i,j}(k)$.

$$\mathbf{P} = \begin{bmatrix} H_{0,0} & & 0 \\ \vdots & \ddots & \\ H_{N-1,0} & \cdots & H_{N-1,N-1} \end{bmatrix}. \quad (3)$$

For simplicity, the analysis and results presented in this paper are for a linear time-invariant (LTI) system. For MIMO LTI systems, (A, B, C, D) are time-invariant, real-valued matrices and $H_{i,j}$ contains the impulse response from each of the q_i inputs to each of the q_o outputs.

The nominal plant $\mathbf{P}$ assumes perfect system modeling. However, many system models contain some form of model uncertainty. To address this in the controller design, assume that the true system $\mathbf{P}_t$ corresponds to the nominal model $\mathbf{P}$ plus an uncertainty Δ_P: $\mathbf{P}_t = \mathbf{P}(I + \Delta_P)$, with the multiplicative uncertainty $\Delta_P = \mathbf{W}\Delta$ and $||\Delta||_{i2} \leq 1$.

During trial j, system $\mathbf{P}_t$ maps the input signal u_j to the measured output signal y_j, i.e., $\mathbf{y}_j = \mathbf{P}_t\mathbf{u}_j + \mathbf{d}_j$, with $\mathbf{u}_j$, $\mathbf{y}_j$, and $\mathbf{d}_j$ of the form defined for $\mathbf{u}_j$ in (4), respectively.

$$\mathbf{u}_j = \begin{bmatrix} u_j^T(0) & u_j^T(1) & \cdots & u_j^T(N-1) \end{bmatrix}^T \quad (4)$$
$$\text{with} \quad u_j^T(k) = \begin{bmatrix} u_j^1(k) & \cdots & u_j^{q_i}(k) \end{bmatrix}.$$

For this work we adopt a widely used norm optimal ILC update law [15], [16]

$$\mathbf{u}_{j+1} = \mathbf{L_u}\mathbf{u}_j + \mathbf{L_e}\mathbf{e}_j \quad (5)$$

with $\mathbf{e}_j = \mathbf{y}_r - \mathbf{y}_j$, where $\mathbf{y}_r$ is the reference signal and is assumed iteration invariant. In (5), $\mathbf{L_u}$ and $\mathbf{L_e}$ are solutions to a quadratic optimization problem detailed shortly in Section IV. These lifted matrices are generally non-causal, time-invariant filters on the control and error signals, respectively. In this paper, a new format for these filters is introduced in order to enable a norm optimal learning controller to address specific performance and robustness challenges of controlling multiple systems. The coupling of multiple systems in the form of the output performance is presented in the following section.

III. COUPLING OF MULTIPLE SYSTEMS

A. Contour Error

When coupling multiple independent systems or agents, one may couple these agents through the desired coordinated output of the combined MIMO system. For agents consisting of two or more individual axes, an additional error component known as the contour error can be identified. Contour errors for a general class of multi-axis agents can be defined with respect to the individual error signals, $e_1 ... e_{q_o}$, and trajectory dependent gains known as coupling gains [17], [18], $c_1(\theta, k) ... c_{q_o}(\theta, k)$, where k is the time interval from $k = 0, 1, \dots, N-1$, θ is the instantaneous angle of the reference trajectory with respect to the horizontal axis of the coordinate system, and $1, 2 ... q_o$ are the individual uncoupled axes. Mathematically, for two axes this can be shown as [17]

$$\varepsilon(k) = c_1(\theta, k) \cdot e_1(k) + c_2(\theta, k) \cdot e_2(k). \quad (6)$$

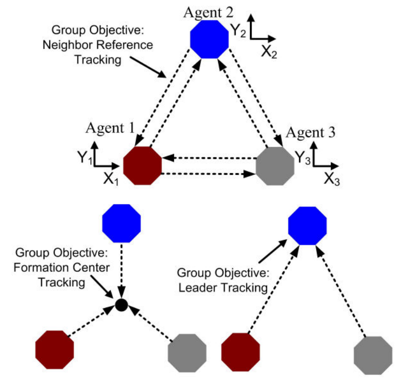

Fig. 1. Formation tracking techniques described in terms of a group objective. Note that in all three cases, the individual systems maintain their own objectives independently from the group objective.

Linearized coupling gains for this 2 degree-of-freedom (DOF) example have the format given in (7). Note that the use of trajectory-dependent coupling gains leads to a time-varying controller.

$$c_1(\theta, k) = -\sin\theta(k); c_2(\theta, k) = \cos\theta(k). \quad (7)$$

B. Formation Control

Contour tracking describes a technique for coupling the output trajectory from multiple axes for a given agent. An approach for coupling the individual agents, known as formation control, ensures shape or formation-keeping among multiple agents. In this technique, there are two distinct design objectives; maintain the formation of multiple agents, and minimize individual axis tracking errors. There are three general techniques for determining a coupled system's formation position: unit-centered referenced, leader referenced, and neighbor referenced [7].

In the unit-centered approach, the formation center is calculated by averaging the xyz positions of all the agents. Each agent determines and maintains its own formation position relative to that center. In leader reference control, each agent determines its formation position with respect to the lead agent. Lastly, with neighbor reference control, each agent maintains a position relative to neighboring agents. These techniques can be shown graphically as in Fig. 1.

Previous work in [19] introduced a norm optimal ILC design which reformats the general norm optimal framework to enable the controller to focus on improving the trajectory tracking performance and robustness of a multi-axis system. The objective of the work presented in this paper is to extend the norm optimal design strategies to include individual and group objectives in an effort to improve trajectory tracking performance and group formation coordination through the use of modified weighting matrices in the norm optimal framework. The generalized structure for the modified framework is given in the following section.

IV. NORM OPTIMAL FRAMEWORK

The norm optimal learning controller in (5) results from a quadratic optimization problem, [20], [21]. In this problem, we want to minimize an objective $\mathcal{J}$, with $\mathcal{J}$ corresponding to the sum of weighted norms as shown in (8),

$$\mathcal{J} = \mathbf{e}_{j+1}^T \mathbf{Q} \mathbf{e}_{j+1} + \mathbf{u}_{j+1}^T \mathbf{S} \mathbf{u}_{j+1} + (\mathbf{u}_{j+1} - \mathbf{u}_j)^T \mathbf{R} (\mathbf{u}_{j+1} - \mathbf{u}_j). \tag{8}$$

where $(\mathbf{Q}, \mathbf{R}, \mathbf{S})$ are symmetric, positive definite matrices of appropriate dimension and $\mathbf{P}^T \mathbf{Q} \mathbf{P} + \mathbf{S} + \mathbf{R} > \mathbf{0}$. The generalized form of these matrices is $(\mathbf{Q}, \mathbf{R}, \mathbf{S}) \equiv (q\mathbf{I}, s\mathbf{I}, r\mathbf{I})$ with q, s, r real-valued positive scalars. Applying the substitution $\mathbf{e}_{j+1} = \mathbf{e}_j - \mathbf{P}(\mathbf{u}_{j+1} - \mathbf{u}_j)$, differentiating $\mathcal{J}$ with respect to $\mathbf{u}_{j+1}$, setting the result to zero, and rearranging the solution, yields the general N.O. controller,

$$\begin{aligned} \mathbf{u}_{j+1} &= \mathbf{L_u} \mathbf{u}_j + \mathbf{L_e} \mathbf{e}_j \\ \mathbf{L_u} &= (\mathbf{P}^T \mathbf{Q} \mathbf{P} + \mathbf{S} + \mathbf{R})^{-1} (\mathbf{P}^T \mathbf{Q} \mathbf{P} + \mathbf{R}) \\ \mathbf{L_e} &= (\mathbf{P}^T \mathbf{Q} \mathbf{P} + \mathbf{S} + \mathbf{R})^{-1} \mathbf{P}^T \mathbf{Q}. \end{aligned} \tag{9}$$

An essential part of the design process involves determining the weighting matrices $(\mathbf{Q}, \mathbf{S}, \mathbf{R})$. In this section, the format of the weighting matrices is modified to enable one to focus on individual objectives, a group objective, or some combination of the two.

A. Individual Agent Control Design

As the previous section stated, the weighting matrices are generally of the form $(\mathbf{Q}, \mathbf{R}, \mathbf{S}) \equiv (q\mathbf{I}, r\mathbf{I}, s\mathbf{I})$. While this approach works well for time-invariant unmodelled dynamics and external disturbances, previous work in [19] demonstrated performance and robustness improvements from implementing time-varying weighting matrices aimed at addressing time and position dependent disturbances, dynamics, and tracking errors. Applying the design framework described in [19], a norm optimal $\mathbf{Q}$ weighting matrix for independent multi-axis agents can be defined as,

$$\mathbf{Q}_{1,2,\dots,p} = [\mathbf{\Gamma 1}_Q + \mathbf{\Gamma 2}_Q \cdot \mathbf{C}_Q^T \mathbf{C}_Q] \tag{10}$$

where $1, 2, \dots, p$ identifies the individual multi-axis agent within the combined MIMO system. Note that norm optimal $\mathbf{S}$ and $\mathbf{R}$ matrices for independent agents would be of the same form as $\mathbf{Q_{1\dots p}}$.

In (10), the $\mathbf{C}_Q$ matrix corresponds to the coupling matrix used to define contour error with respect to the individual axis errors as a function of the reference trajectory. The matrices $\mathbf{\Gamma 1}_Q$ and $\mathbf{\Gamma 2}_Q$ refer to the amount of weighting applied to the coupled or individual signals, respectively. These matrices are of the form provided in (11), where the inner block diagonal matrices are shown for a 2 DOF system and the gains are defined as $\gamma 1(k) = \gamma(k)$ and $\gamma 2(k) = 1 - \gamma(k)$, respectively.

$$\mathbf{\Gamma j} = \begin{bmatrix} \begin{bmatrix} \gamma_j(1) & 0 \\ 0 & \gamma_j(1) \end{bmatrix} & & 0 \\ & \ddots & \\ 0 & & \begin{bmatrix} \gamma_j(N) & 0 \\ 0 & \gamma_j(N) \end{bmatrix} \end{bmatrix}_{,j=1,2} \tag{11}$$

The gain $\gamma(k)$ is used to determine the weighting applied to the individual and coupled signals, respectively. From (11), $(\gamma(k) = 1)$ refers to all of the weighting being applied to the individual signals, while $(\gamma(k) = 0)$ results in only the coupled signals being weighted.

B. Coupled Agent Control Design

After all the agent weighting matrices have been determined, the individual trajectory tracking objective, in terms of a coupled versus individual agent approach, needs to be addressed. Using a similar form to that presented in IV-A, the norm optimal weighting matrix for coupled versus individual system error tracking can be defined as,

$$\bar{\mathbf{Q}} = [\mathbf{B1}_Q \cdot \mathbf{Q} + \mathbf{B2}_Q \cdot \mathbf{K}_Q^T \mathbf{Q}_{FC} \mathbf{K}_Q] \tag{12}$$

Note that $\bar{\mathbf{S}}$ and $\bar{\mathbf{R}}$ are matrices of the same form as $\bar{\mathbf{Q}}$. In (12), $\mathbf{Q}$ is a diagonal matrix containing the individual multi-axis agent weighting matrices $(\mathbf{Q}_1, \mathbf{Q}_2, \dots \mathbf{Q}_p)$ along the diagonal, while $\mathbf{Q}_{FC}$ is a nominal weighting matrix of the form provided in (10) designed for formation center tracking. Matrices $\mathbf{B1}$ and $\mathbf{B2}$ are used to turn off/on the weighting on the individual and coupled agent tracking, respectively. Setting $(\mathbf{B1} = \mathbf{I}, \mathbf{B2} = \mathbf{0})$ results in individual agent tracking, whereas coupled agent tracking is achieved by setting $(\mathbf{B1} = \mathbf{0}, \mathbf{B2} = \mathbf{I})$. $\mathbf{K}_Q$ is a non-square matrix containing the time-invariant gains used to define the formation center with respect to the individual agents within the combined MIMO system and is of the form shown in (13), where each agent is assumed to have 2 DOF.

$$\mathbf{K} = \begin{bmatrix} \begin{bmatrix} \kappa_1(1) & & 0 \\ & \ddots & \\ 0 & & \kappa_1(N) \end{bmatrix} \cdots \begin{bmatrix} \kappa_p(1) & & 0 \\ & \ddots & \\ 0 & & \kappa_p(N) \end{bmatrix} \end{bmatrix} \tag{13}$$

Recall that $1, 2, \dots p$ identifies the individual agents. The diagonal elements are given as,

$$\kappa_{(\cdot)}(k) = \begin{bmatrix} \kappa x(k) & 0 \\ 0 & \kappa y(k) \end{bmatrix} \tag{14}$$

C. Formation Control Design

The final element in the modified norm optimal framework is an additional component in the design of the weighting on the error signals enabling formation or shape tracking. As discussed previously, there exist applications in which the ability to maintain a specific formation (or group objective) may outweigh the individual tracking objective. For these systems, it is important to have the ability to vary the weighting on the individual versus group objective within the controller design. A weighting matrix capable of independent objective weighting is given in (15).

$$\hat{\mathbf{Q}} = \mathbf{\Sigma}_Q \cdot [\mathbf{X1}_Q \cdot \bar{\mathbf{Q}} + \mathbf{X2}_Q \cdot \mathbf{F}^T \mathbf{F}] \tag{15}$$

In (15), $\mathbf{F}$ contains the gains defining the formation errors with respect to the individual agent tracking errors. This results in a non-symmetric matrix in which only certain combinations of the individual tracking errors are weighted to ensure a desired formation or shape. These gains are time-invariant for MIMO systems in which the same reference trajectory is applied to each individual agent.

Similar to the matrices $\mathbf{B1}$ and $\mathbf{B2}$, $\mathbf{X1}$ and $\mathbf{X2}$ are directly related to each other through their diagonal elements $(\chi_1(1), \ldots \chi_p(N))$ and $(1-\chi_1(1), \ldots 1-\chi_p(N))$, which are used to determine the ratio of weighting on the individual objective versus the group objective. Generally $\chi_{(\cdot)}(k)$ has a value other than 0 or 1 in order to weight a combination of individual and group objectives.

The diagonal matrix $\mathbf{\Sigma}_Q$ (16) is used to determine the overall gain on the error signals with respect to the control and change in control signals. Similarly, $\mathbf{\Sigma}_S$ and $\mathbf{\Sigma}_R$ in $\hat{\mathbf{S}} = \mathbf{\Sigma}_S \cdot \bar{\mathbf{S}}$ and $\hat{\mathbf{R}} = \mathbf{\Sigma}_R \cdot \bar{\mathbf{R}}$ describe the overall gain on the control signals and change in control signals, respectively.

$$\mathbf{\Sigma}_Q = \begin{bmatrix} \begin{bmatrix} \sigma_{Q1}(1) & 0 \\ 0 & \sigma_{Qp}(1) \end{bmatrix} & & 0 \\ & \ddots & \\ 0 & & \begin{bmatrix} \sigma_{Q1}(N) & 0 \\ 0 & \sigma_{Qp}(N) \end{bmatrix} \end{bmatrix} \tag{16}$$

The gains $(\sigma_{Q(\cdot)}(k), \sigma_{S(\cdot)}(k), \sigma_{R(\cdot)}(k))$ are similar to the gains (q, s, r) from the original form of the norm optimal weighting matrices in (8) in that they weight the different components of the cost function. Therefore, the tuning rules presented in [19] can be used to design the gains $(\sigma_{Q(\cdot)}(k), \sigma_{S(\cdot)}(k), \sigma_{R(\cdot)}(k))$, respectively.

Each of the design elements in IV-A - IV-C offers a means of weighting different aspects of the error signals, control signals and change in control signals. A multi-step design methodology providing details on the individual terms in the weighting matrices is provided in the next section. In order to validate the capabilities of the modified weighting matrix design, Section VI presents results obtained from simulating modified norm optimal learning controllers with generic models of stable second order systems.

V. DESIGN AND IMPLEMENTATION

Subsection III-B introduced three common approaches for controlling trajectory and formation tracking of multiple agents. The general approach to formation control includes decoupling the individual agent tracking objective from the group objective. In this manner, an optimal solution for accomplishing each objective can be obtained. This approach enables the group to maintain a set shape or formation, while also enabling each agent to accomplish its desired task independently. Applying this decoupled approach, a four step design methodology for generating optimal learning controllers can be determined (Fig. 2).

A. Design Methodology

Following the decoupled approach, the first step involves generating individual optimal weighting matrices $(\mathbf{Q}_{(\cdot)}, \mathbf{S}_{(\cdot)}, \mathbf{R}_{(\cdot)})$ of the form given in (10) for each multi-axis

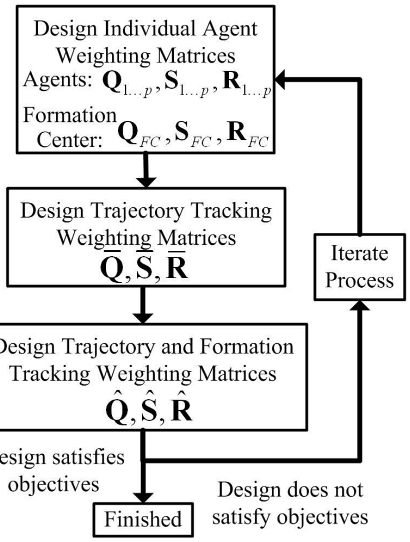

Fig. 2. *Design methodology used to determine the weighting matrices for the optimal learning controllers.*

agent within the combined MIMO system. While the framework is general enough to allow separate $(\mathbf{Q}_{(\cdot)}, \mathbf{S}_{(\cdot)}, \mathbf{R}_{(\cdot)})$ designs for each multi-axis agent, many combined MIMO systems are comprised of multiple *identical* agents. In this case, a single design may be applied to each individual agent. Along with the individual agent designs, optimal $(\mathbf{Q}_{FC}, \mathbf{S}_{FC}, \mathbf{R}_{FC})$ weighting matrices for the formation center should be determined at this time. Nominal weighting matrices, valid for the formation center as well as the individual agents, will often suffice.

Having designed the weighting matrices for each agent, the individual weighting matrices are combined to form $(\mathbf{Q}, \mathbf{S}, \mathbf{R})$, while the formation center weighting matrices are used to generate the coupled agent approach. The coupling gains $\kappa x(k), \kappa y(k)$ for a 2 DOF agent are calculated as the ratio of the horizontal and vertical components from the agent to the formation center position. For example, the coupling gains for a three agent system are identically given as $\kappa x(k) = \kappa y(k) = 1/3$, for all $k = 1, 2, \ldots N$. Finally, diagonal matrices $\mathbf{B1}$ and $\mathbf{B2}$ need to be determined based on the desired tracking objective. The following protocol can be used to switch between different trajectory tracking methods: for individual agent tracking set $(\mathbf{B1} = \mathbf{I}, \mathbf{B2} = \mathbf{0})$, for formation center tracking set $(\mathbf{B1} = \mathbf{0}, \mathbf{B2} = \mathbf{I})$, and for leader reference tracking set $b_1(k) = 1, b_{(2,\ldots p)}(k) = 0$ for all k in $\mathbf{B1}$ and $\mathbf{B2} = \mathbf{0}$. Note, neighbor reference trajectory tracking is a form of leader reference tracking in which there are multiple *leaders*.

The final step in the design methodology focuses on formation tracking. In (15), $\mathbf{X1}$ and $\mathbf{X2}$ weight the individual objective (trajectory tracking) versus the group objective (formation tracking), respectively. Prior to determining these weighting matrices, one must define the desired formation or shape of the combined system. Figure 3 illustrates an

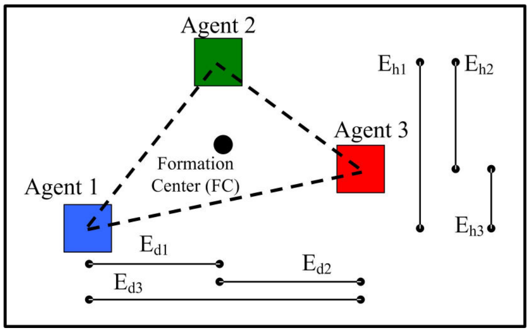

Fig. 3. *Example formation for a 3 agent coupled MIMO system*

example formation for a three agent MIMO system. The three individual agents are coupled through the desired formation defined in terms of the formation error signals $\mathbf{E}_{d1}, \mathbf{E}_{d2}, \mathbf{E}_{d3}, \mathbf{E}_{h1}, \mathbf{E}_{h2}, \mathbf{E}_{h3}$.

Redefining the formation error signals in terms of the individual axis errors for the independent agents, the formation matrix $\mathbf{F}$ for the system illustrated in 3 is given below.

$$\begin{aligned}\begin{bmatrix}\mathbf{E}_{d1}\\ \mathbf{E}_{d2}\\ \mathbf{E}_{d3}\\ \mathbf{E}_{h1}\\ \mathbf{E}_{h2}\\ \mathbf{E}_{h3}\end{bmatrix} &= \begin{bmatrix}\mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0}\\ -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{I}\\ \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\end{bmatrix}\begin{bmatrix}\mathbf{E}_{x1}\\ \mathbf{E}_{y1}\\ \mathbf{E}_{x2}\\ \mathbf{E}_{y2}\\ \mathbf{E}_{x3}\\ \mathbf{E}_{y3}\end{bmatrix} \\ \mathbf{E}_{d,h} &= \mathbf{F}\cdot\mathbf{E}_{(1,2,3)}\end{aligned} \tag{17}$$

Lastly, the overall gains on the error, control, and change in control signals ($\sigma_{Q(\cdot)}(k), \sigma_{S(\cdot)}(k), \sigma_{R(\cdot)}(k)$) are designed based on the tuning guidelines provided in [19]. Simulation results are used to evaluate the performance of the optimal learning controllers. If the coupled system performance does not satisfy the desired objectives, the design process can be iterated. The next section presents simulation results for a three agent system.

VI. SIMULATION RESULTS

Using the methodology and MIMO system presented in V, norm optimal learning controllers for three different design scenarios were constructed. These scenarios were selected to evaluate the system performance while focusing on the individual trajectory tracking objective, the group formation objective, and a combination of the two using formation center trajectory tracking. The simulation set-up includes the following assumptions: dynamically similar agents enabling nominal $(\mathbf{Q}, \mathbf{S}, \mathbf{R})$ weighting matrices, multiplicative model uncertainty applied to each agent independently, identical raster reference trajectory applied to each agent, and a non-repetitive external disturbance applied only to agent 1.

Using these assumptions, optimal weighting matrices were designed. In order to simplify the design and evaluation, the same $\hat{\mathbf{S}}$ and $\hat{\mathbf{R}}$ matrices were used for all three cases. The first scenario, which focused on individual agent trajectory tracking, applied zero weighting to formation tracking. The gain selection for this case included $\mathbf{\Gamma 1}_{Q,S,R} = \mathbf{I}, \mathbf{B1}_{Q,S,R} = \mathbf{I}$,

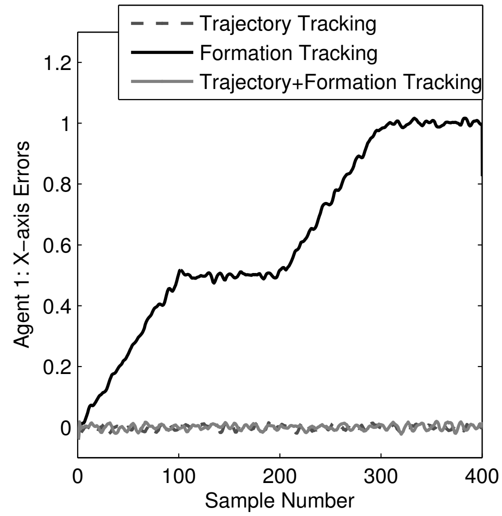

Fig. 4. *Agent 1 x-axis tracking errors for the three tracking cases.*

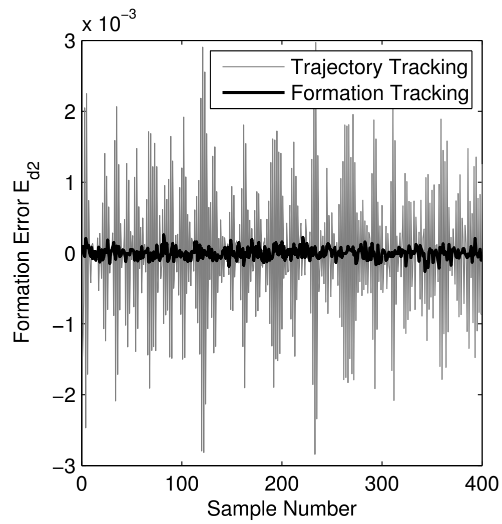

Fig. 5. *Formation error* $\mathbf{E}_{d2}$ *for trajectory versus formation tracking.*

and $\mathbf{X1}_{Q,S,R} = \mathbf{I}$. The second scenario focused on the group formation objective by setting the individual objective gain $\mathbf{X1}_Q = \mathbf{0}$, thereby negating the design of $\bar{\mathbf{Q}}$. The third case required the calculation of both the $\mathbf{K}$ and $\mathbf{F}$ coupling matrices. As previously described, the formation center coupling matrix for a three agent system sets $\kappa_i(k) = 1/3 \cdot \mathbf{I}(2)$ for all $k = 1, 2, \ldots N$ and $i = 1, 2, 3$, while $\mathbf{F}$ is given in (17). For equal weighting on the individual and group objectives set $\chi_i(k) = 1/2$ for all $k = 1, 2, \ldots N$ and $i = 1, 2, 3$. The overall gains for all three scenarios were selected as $(\sigma_{Qi}(k) = 1, \sigma_{Si}(k) = .005, \sigma_{Ri}(k) = .001)$ for all $k = 1, 2, \ldots N$ and $i = 1, 2, 3$, respectively.

Figures 4 and 5 indicate the performance trade-offs between individual trajectory tracking and formation tracking. A controller designed to optimize trajectory tracking minimizes the individual axis errors, while indirectly reducing the formation errors. Controllers designed to achieve a group objective result in the lowest formation errors, but very poor trajectory tracking. The coupling of the individual axis errors into the formation errors enables the formation error to tend to zero (Fig. 5), while the individual axis errors remain large (Fig. 4). Weighting the tracking objectives equally and applying a formation center trajectory tracking approach, the MIMO system is able to minimize trajectory and formation tracking errors simultaneously.

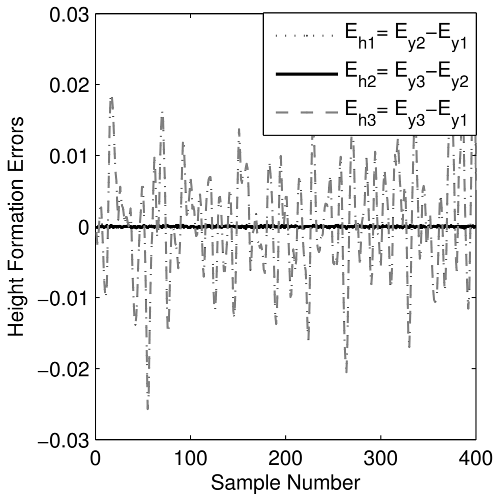

Fig. 6. *Height formation errors for the formation tracking case.* $\mathbf{E}_{h2}$ *is not derived from Agent 1 and therefore converges to much smaller values.*

In addition to formation and trajectory tracking, it is important to determine the affect of external disturbances on the combined system. This affect can be seen in Fig. 6, in which the three height formation error signals for the formation tracking case are given. The addition of a non-repeating external disturbance signal to agent 1 degrades the formation tracking performance for the combined system as shown in Fig. 6. These results indicate that formation error signals defined with respect to the individual axis errors of agent 1 will exhibit performance limitations resulting from the applied disturbance signals. The formation error signal $\mathbf{E}_{h2}$ does not demonstrate the same limitations since it is not derived from the error signals of agent 1, thereby allowing the signal to converge to much smaller values.

These results validate the design flexibility of the norm optimal framework presented in IV. The ability to vary the design objectives, as well as the formation method, through the selection of different gains within a single framework is unique to this approach.

VII. CONCLUSION

This paper discusses precision coordination and motion control of multiple systems that perform the same task repetitively. Using norm optimal weighting matrices, we combine individual trajectory tracking and group formation tracking for varying formation approaches into a single framework. The flexibility of this framework enables the construction of diverse learning controllers for a variety of applications.

After introducing the techniques for coupling multiple systems, a detailed description of the framework and design methodology was provided. Using the methodology presented in the paper, norm optimal learning controllers for three distinct tracking objective and formation approaches were designed for comparison in simulation. Simulation results demonstrated the variation in the system performance due to different design objectives or formation approaches. Future work will focus on implementing the optimal learning controllers on an experimental testbed.

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