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Simulation of Gamma Ray Burst afterglows using PLUTO code

by

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"The cosmos is full of surprises, and sometimes they come with gamma rays and a sense of wonder."

— **DR. SARAH CARTER**

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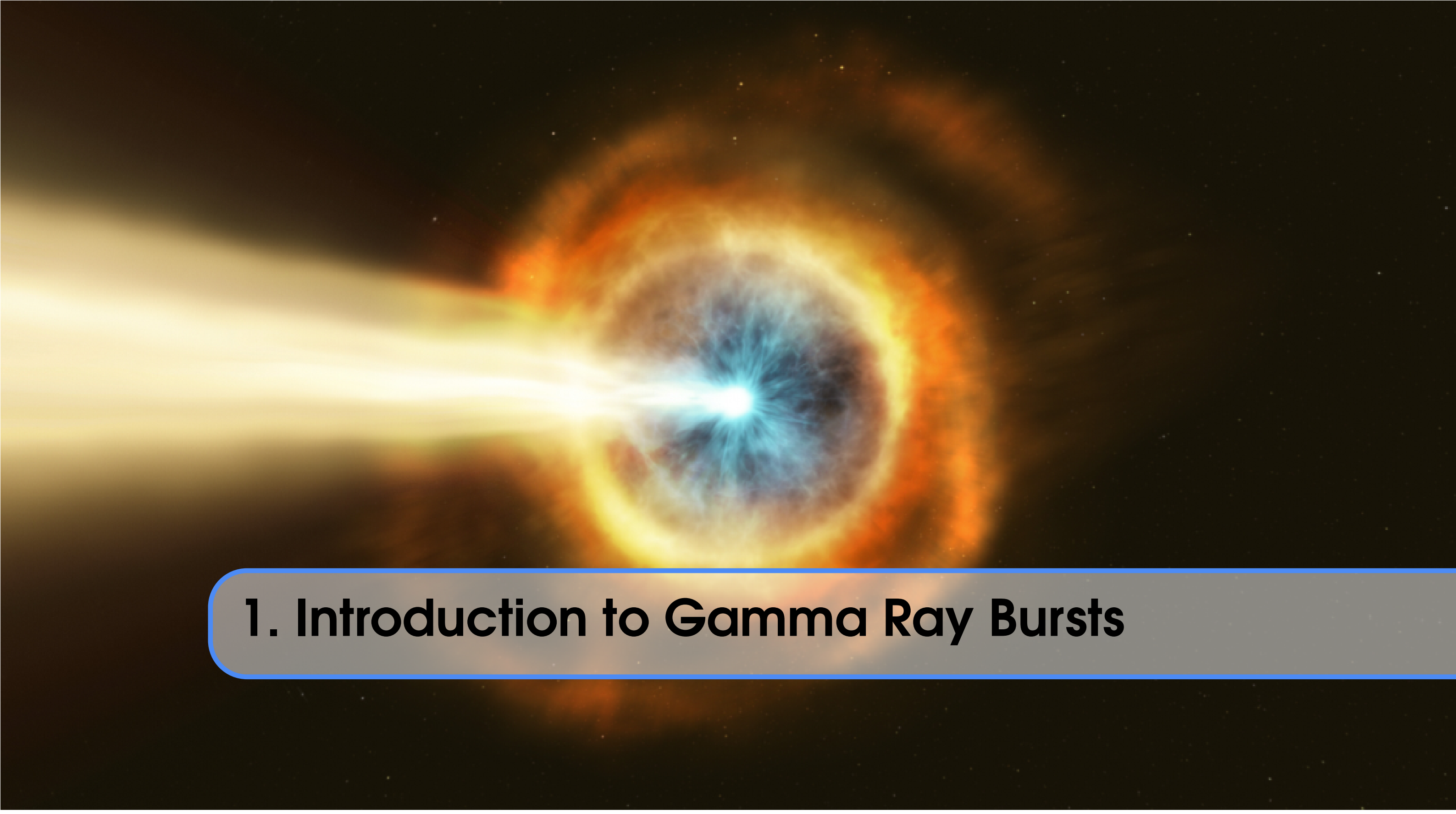


1. Introduction to Gamma Ray Bursts

1.1 The accidental discovery

On a cold night in January 1967, a U.S. satellite named Vela 3, built to detect Soviet nuclear tests, found something surprising: a burst of high-energy gamma rays from deep space. Instead of finding secret activities on Earth, it discovered a cosmic explosion.

Dr. Sarah Carter, a young astrophysicist, noticed the strange data. At first, the military kept the discovery secret because they didn't want people to think their satellite was faulty.

Over the years, scientists debated the cause of these gamma-ray bursts (GRBs). Some thought they were from exploding stars, others thought black holes. Sarah kept searching for answers with her tools and curiosity.

By the 1990s, new technology helped. The Compton Gamma Ray Observatory (CGRO) provided more data, and Sarah and her team realized GRBs were happening all over the universe, like cosmic fireworks. These bursts were powerful and far away, making the universe seem like it was having a huge, long-distance party.

In 1997, the BeppoSAX satellite from Italy and the Netherlands found the exact location of a GRB. Ground-based telescopes saw its afterglow, and it turned out the burst was billions of light-years away, incredibly old.

More discoveries followed. Some GRBs came from massive stars collapsing into black holes, while others were from neutron stars merging, creating gravitational waves detectable by LIGO.

Today, GRBs are a key part of understanding the universe, showing the violent and dynamic processes that shape it. Dr. Carter, now a famous astrophysicist, often smiles when she thinks about those early days. **"Who would have thought a satellite meant to find Soviet nukes would discover the universe's biggest light show?"** she says.

1.2 What is a Gamma-Ray Burst?

Let's start from the basics! As we know about the spectrum of electromagnetic waves ranging from radio and microwaves to x-ray and gamma rays with small window of optical wavelengths in the middle. Gamma-rays are of high frequency radiations, and that's why are very energetic ones. The energy is so high that we can observe it from billions of light years far distance.

Now what is the Gamma-Ray Burst (GRB)? So GRBs are one of the most energetic events that occur in the universe. They are all over the sky as you can see in 1.1.

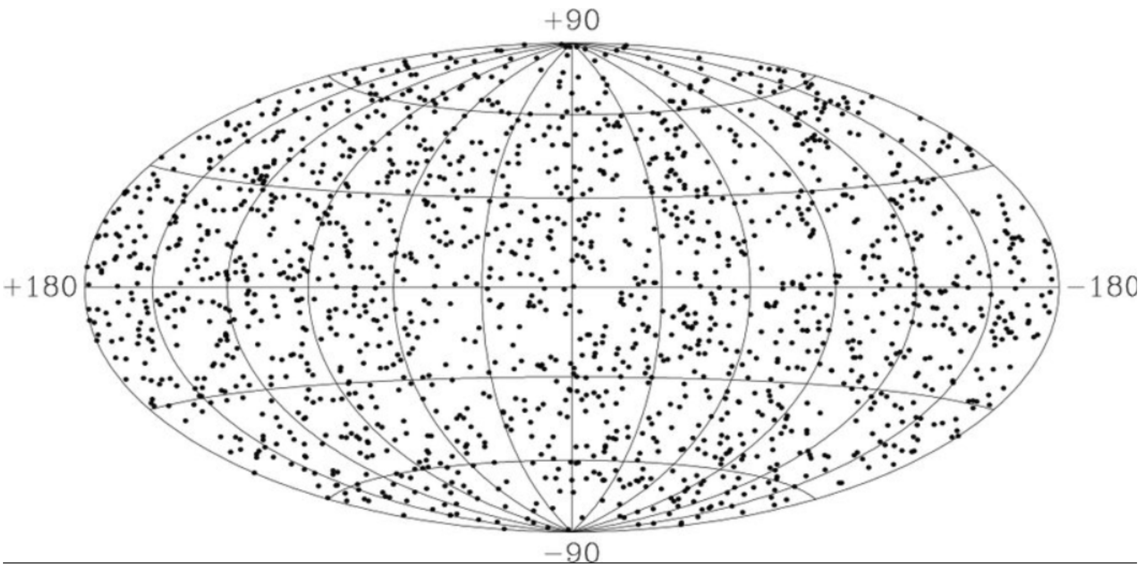


Figure 1.1: These are around 2700 GRBs on the sky detected by Burst And Transient Source Experiment (BATSE).

The above GRBs[1] are not the only one that are in the universe. The actual number can be much much bigger than "2700". We are not able to detect all of them because they release their energy in very sharp conical jets which are, in most of the cases, away from us. We can only detect those which are directly targeted towards us.

These GRBs are classified into two categories depending on their time-period of detection-:

1. **Short GRBs :** These types of GRBs have very small detection time-period, i.e; of the order of ms to some seconds.
2. **Long GRBs :** Long GRBs are those which have time period of the order of hundreds of seconds.[2]

Other than their time duration difference, they also differ in their respective spectra based on the hardness ratio. BATSE collected energy photons in 4 different channels as shown below The Hardness ratio is defined as

$$HR = \frac{Ch-3 \text{ photon counts}}{Ch-2 \text{ photon counts}} \tag{1.1}$$

[1]High Energy Astrophysics, Stephan Rosswog and Marcus Briiggen, 2nd ed.

[2]The brightest GRB ever detected is nicknamed as **B.O.A.T.** (Brightest Of All Time), designated as GRB-221009A.

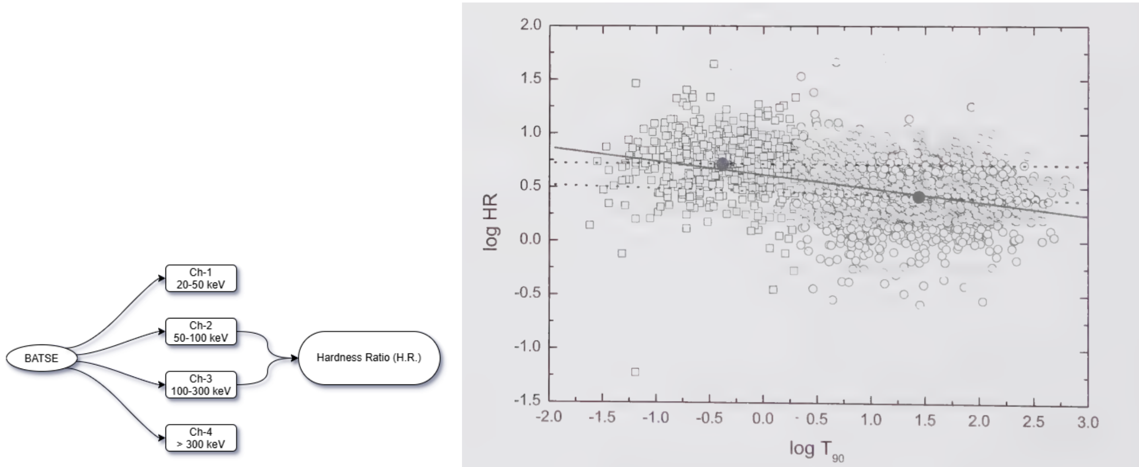


Figure 1.2: (a) shows the channel distribution of BATSE according to the respective photon energies emitted during GRBs, and (b) shows the distributions of short GRBs (□) and long GRBs (◦) with their hardness ratio and time period. Here T_{90} means the observation time of a GRB with its 5% to 95% energy output.

In the upcoming sections, we will first look at the model of GRBs that give rise to the collapsar and merger theories of gamma ray bursts. We will then explore how short gamma-ray bursts (GRBs) result from the mergers of compact binaries, such as neutron star-neutron star (NS-NS) or neutron star-black hole (NS-BH) systems. Meanwhile, long GRBs are caused by the collapse of the core of massive stars.

1.3 Fireball model

1.3.1 Source distribution

The distance scale to the bursting sources could be either spherically distributed in a restricted volume or could have a constant source number density in the nearby vicinity of the observer. The BATSE found that GRB sources are isotropic and inhomogeneous. In simple words, the number density is nearly constant out to some large distance, which then drops very rapidly.

And what about their distance from us? In 1997, BEPPOSAX found out that atleast long GRBs are coming from very far distance outside out galaxy.

1.3.2 Energy and collimated outflow

if we assume that the GRB's central engine is emitting the energy isotropically in all directions, then

$$E_{iso} = 4\pi d^2 f \tag{1.2}$$

where f is the time integrated flux, also known as *fluence*.

But if we assume the source to outflow its energy into two cones (with solid angle $\Delta\Omega$), then that's what we observe from them, i.e;

$$E_{true} = E_{iso}\left(\frac{\Delta\Omega}{4\pi}\right) \tag{1.3}$$

Here, the quantity $(4\pi/\Delta\Omega)$ is termed as **Beaming Factor**. Radiation with smaller $\Delta\Omega$ will be strongly beamed.

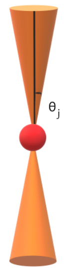


Figure 1.3: Collimated outflow with two cones of opening jet angle θ_j

The achromatic breaks of GRBs agrees with this concept. As the shock moves relativistically (with lorentz factor γ), observer will see the radiations with an opening angle of $1/\gamma$ ($<\theta_j$). With moving forward, the jet will interact with interstellar medium (ISM) which will decrease its γ and increase $1/\gamma$. Just when $1/\gamma > \theta_j$, observer will start to see the cone and realise that it was not the part of a sphere, and then light-curve will drops very rapidly.

Such steepening of light-curve, which is independent of the wavelength is called **Achromatic Break**.

1.3.3 GRB fireball model

As GRBs central engines are very far away from us, yet they are very energetic, so for any model to fit the observations of GRBs, it should rapidly release a large amount of energy. As discussed earlier, the engine is isotropic and inhomogeneous, the flow is optically thick initially, and becomes thin as it passes through the matter to cool down. This energy then fractionally dissipated at very large distance to produce ultra-relativistic electrons that emit the gamma rays by synchrotron radiations and inverse-compton effect. The rest energy is transferred further into the ambient medium, where the blast waves interact with the interstellar medium to produces electromagnetic emission, called afterglow.

The fireball model needs to have some important properties. It should have a large η (= E/m_0c^2) for ultra-relativistic outflow production, and to produce time variability as observed in the bursts. There are 5 main stages in the fireball model:

1. **Energy deposition by central engine:** The central engine contains a very large amount of energy inside it in a very small (around 100 km) region. This fireball consists of a thick plasma of photons, $e^- - e^+$ pairs and baryons.
2. **Expansion:** For a large η value, the fireball accelerates, i.e; γ increases linearly with r, and then it reached the asymptotic value;

$$\gamma(r) = \gamma_{asym} \approx \eta \tag{1.4}$$

   and temperature & number density change accordingly.
3. **Preburst (photon escape):** In Thomson scattering, the optical thickness is given as

$$\tau = n_e \sigma_T D \tag{1.5}$$

   So, in the expansion phase, the number density falls, which implies that the optical thickness (τ) decreases. Just when $\tau < 1$, the photons escape from the surface leads to the preburst with thermal spectrum.
4. **GRB production - Internal Shocks:** During the expansion, the fireball gives maximum energy to the kinetic energy of baryons. If the density profile is not homogeneous, but have some regions of higher lorentz factor than others,

the faster one will catch the slower moving regions. They then produce the gamma rays by the phenomena of *synchrotron radiation*.

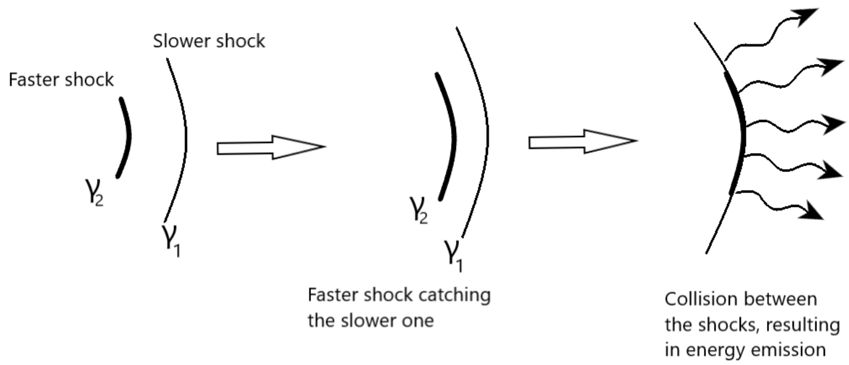


Figure 1.4: Due to uneven density, the faster part of outflow will catch the slower one, and collide to emit energy in a wide range from Radio waves to Gamma rays.

Numerical calculations shows that they occur very far away from the central engine. Here, the density is very low so that the produced gamma rays escape freely into the interstellar medium.

5. **Afterglow - External Shocks:** The remaining outflow material interacts with the medium and dissipates its kinetic energy. Energy flow starts with gamma rays, tend to lower energies as the outflow decelerates.
The energy spectrum of the afterglows depends on the nature of the interstellar medium, and distributed in different power-laws.

$$\rho_{ext} \propto r^{-k} \tag{1.6}$$

where $k = 0$ corresponds to the uniform medium, while $k = 2$ means stratified medium. [3]

1.4 Central Engine models

1.4.1 Collapsars

Observations showed that many long GRBs occur with the supernovae explosion. Their output is electromagnetic energy which is very similar to the collimated outflow energy of the gamma-ray burst. The difference is the outflow of the core-collapsed supernova is non-relativistic, while that of GRB is relativistic.

This model involves the Wolf-Rayet star[4] with He-core. When its core collapses, it becomes a black hole(BH) with matter accreted around it. As it is easy to fall material relativistically near the axis than in the accretion plane (because of centrifugal barrier resistance), that energy deposition then launches a jet along

[3] Sari, R., Piran, T., & Narayan, R. 1998, ApJ, 497, L17.

[4] Wolf-Rayet stars are very massive stars with no H-envelope outside and very strong stellar winds.

the axis of the black hole. This energy deposition can happen either by $\nu_i - \bar{\nu}_i$ annihilation or any process related to the spin of BH.

Now the Gamma ray production occurs due to the time-dependent accretion from the disk. There will be a shear flow at the interface of jet and stellar mantle.[5] There will be some velocity fluctuations, that will make the material to gain different Lorentz factors, which will then produce internal shocks. Eventually, the flow will become optically thin and gamma rays will produce via synchrotron and inverse compton effect.

1.4.2 Compact Binary Mergers

Neutron star mergers are the model for the production of short GRBs. Observations have shown that short GRBs most of the time coincide with gravitational wave from the merger. These mergers involve the collision of two neutron stars.

When two neutron stars in a binary system spiral towards each other due to gravitational wave emission, at last they merge. This merger results in the formation of a massive neutron star or a black hole, surrounded by a hot, dense accretion disk composed of neutron-rich material. The collision and formation of a black hole lead to the creation of powerful jets. The formation of jet involves **Neutrino-Antineutrino annihilation** and **Magnetic processes**. These jets are highly collimated and relativistic, moving close to the speed of light. The energy required to launch these jets comes from the rapid rotation of the resulting massive neutron star or black hole and the intense gravitational and magnetic fields.

As the jets burst through the ISM, they interact with it, leading to the formation of shocks (or internal shocks). These shocks then produce gamma-rays through processes such as **Synchrotron Emission** [6], **Inverse Compton Scattering** [7].

1.5 Conclusion

With a brief recap, we learnt in this chapter that how we discovered the Gamma-Ray Bursts, and their basic terminology. Then in next section, we learnt about fireball model, and modified it based on calculated observations. In that section, we gained knowledge of internal and external shocks which lead to afterglows. In the last section, we found that there can be two types of central engine models, with collapsar model used for long GRBs, while merger model used for short GRBs.

Now in the next chapter, we will build the base of fluid dynamics, which we will then implement in PLUTO Code for the simulation of afterglows.

[5]This is known as Kelvin-Helmholtz Flow. When two different fluids interact at their interface, they slip past or flow around one another centrifugally.

[6]Electrons accelerated in the magnetic fields emit gamma rays via synchrotron radiation.

[7]High-energy electrons can transfer energy to lower-energy photons, boosting them to gamma-ray energies.

2. Fluid Dynamics

2.1 Introduction

Fluid dynamics is the branch of physics that studies the behavior of fluids (liquids and gases) in motion. Understanding how fluids move and interact with their surroundings is crucial in many areas of science and engineering. In astrophysics, fluid dynamics plays a vital role in explaining various phenomena, from the formation of stars and galaxies to the powerful explosions observed in the universe.

When we talk about fluids in astrophysics, we often refer to gases and plasma states of matter that dominate the universe. They are influenced by various forces, like gravity and pressure.

One of the most fascinating and energetic events in the universe is the gamma-ray burst (GRB). These bursts are brief but extremely intense emissions of gamma rays, which are the most energetic form of light. GRBs are believed to be associated with catastrophic events like the collapse of massive stars or the merging of neutron stars. To understand how GRBs are produced and propagate, we must delve into the principles of fluid dynamics.

In this chapter, we will explore the fundamental concepts of fluid dynamics and how they apply to astrophysical phenomena. We will examine how the movement and interaction of gases and plasma contribute to the formation and evolution of GRBs.

2.2 Fluids

2.2.1 Fundamental equations of fluid dynamics

Fluid is regarded as a continuous medium, with very great number of molecules in a small volume element. The mathematical description of state of a fluid can be understood from the equation of continuity;

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{2.1}$$

where ρ and $\mathbf{v}$ are the density and velocity of the fluid, respectively and can be a function of space and time ($\rho(x,y,z,t)$, $\mathbf{v}(x,y,z,t)$).

From here, we can find a vector $\mathbf{j} = \rho\mathbf{v}$, known as the mass flux density, with direction along the flow of fluid, and magnitude equal to the mass of fluid in unit time and through a unit perpendicular area. This vector is very important. We will see its use in jump conditions of shocks in sec. 2.4.1.

The other important equation that governs fluid dynamics is **Euler Equation**. If p is the pressure acting on the surface of the bounding volume, then by equating forces, we can get

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{\rho}\nabla\mathbf{p} \tag{2.2}$$

If the fluid is ideal, there will be no heat exchange, means no entropy change, then

$$\begin{aligned} \frac{ds}{dt} &= 0 \\ \frac{\partial s}{\partial t} + (\mathbf{v}\cdot\nabla)s &= 0 \end{aligned} \tag{2.3}$$

With further modification, this equation will turn out to be the **Adiabatic Equation**.

$$\frac{\partial(\rho s)}{\partial t} + \nabla\cdot(\rho s\mathbf{v}) = 0 \tag{2.4}$$

We can determine the fluid state by 5 quantities -: 3 components of velocity, pressure and density. For the case of ideal fluid, we can find them using Continuity, Euler and Adiabatic equation.

2.2.2 Bernoulli's Equation

Now for adiabatic process, s = constant, $dw = Tds + Vdp = \frac{dp}{\rho}$, where w is enthalpy. While talking about the steady flow of the fluid, the equation will become

$$\frac{\partial}{\partial l}\left(\frac{1}{2}v^2 + w\right) = 0 \tag{2.5}$$

where l is the tangential direction of the fluid flow. In gravitational field, this will turn out as

$$\frac{1}{2}v^2 + w + gz = constant \tag{2.6}$$

This is known as **Bernoulli's Equation**.

2.2.3 Energy and Momentum Flux

The energy and momentum flux are two things that are very important and will be used when we will approach the dynamics relativistically, which is the ultimate goal of this chapter.

Taking a small volume element of fluid, then the energy of unit volume is

$$\text{Energy Density} = \frac{1}{2}\rho v^2 + \rho\varepsilon \tag{2.7}$$

with 1st term is the contribution of kinetic energy, and 2nd term is the internal energy. With the help of 2.1, 2.2, 2.3 and the fact that $w = \varepsilon + \frac{p}{\rho}$, we can arrive to the expression

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + \rho\varepsilon\right) = -\nabla\cdot\left[\rho\mathbf{v}\left(\frac{1}{2}\rho v^2 + w\right)\right] \tag{2.8}$$

Here this $\rho\mathbf{v}\left(\frac{1}{2}v^2 + w\right)$ is known as **Energy flux density** vector, and is the amount of energy carried by unit mass of fluid during its motion.

Now to find momentum flux density, we will use similar arguments, but as momentum is a tensor quantity, we will do calculations in tensor notation. The momentum of a unit volume is $\rho\mathbf{v}$. We can find its changing rate.

$$\frac{\partial}{\partial t}(\rho v_i) = \rho\frac{\partial v_i}{\partial t} + v_i\frac{\partial\rho}{\partial t} \tag{2.9}$$

For 1st term, we can use 2.1 $\left(\frac{\partial\rho}{\partial t} = -\frac{\partial}{\partial x_k}(\rho v_k)\right)$, and for 2nd term we can use 2.2 $\left(\frac{\partial v_i}{\partial t} = -v_k\frac{\partial v_i}{\partial x_k} - \frac{1}{\rho}\frac{\partial p}{\partial x_k}\right)$. So 2.9 will become

$$\frac{\partial}{\partial t}(\rho v_i) = -\frac{\partial\Pi_{ik}}{\partial x_k} \tag{2.10}$$

Here Π_{ik} is known as **Momentum flux density** tensor, and can be expressed as

$$\Pi_{ik} = p\delta_{ik} + \rho v_i v_k \tag{2.11}$$

If $\mathbf{n}$ is the outward normal to the surface of unit volume, then

$$\Pi_{ik}n_k = p\mathbf{n} + \rho\mathbf{v}(\mathbf{v}\cdot\mathbf{n}) \tag{2.12}$$

and we can define Π_{ik} now as the i-th component of momentum flowing in unit time through unit area normal to x_k-axis.[1]

2.3 Sound Waves

Let's connect Gamma Ray Burst with sound waves. Although they are very different kind of things; sound waves are matter waves travel through the medium by compressing it; GRBs propagate through interstellar medium and produce shock waves. So with analogy of sound waves, we can understand the underlying physics of GRBs.

There is a small amplitude oscillatory motion in sound waves, so the velocity $\mathbf{v}$ will be very small. So we can neglect the term $(\mathbf{v}\cdot\nabla)\mathbf{v}$ in equation 2.2. Also let p_0 & ρ_0 be the constant equilibrium pressure & density and p' & ρ' be their variations in sound wave. Then,

$$\begin{aligned} p &= p_0 + p' \\ \rho &= \rho_0 + \rho' \end{aligned} \tag{2.13}$$

The continuity equation and Euler equation will become

$$\frac{\partial\rho'}{\partial t} + \rho_0(\nabla\cdot\mathbf{v}) = 0 \tag{2.14}$$

[1] If $\mathbf{n}$ is along the fluid flow, then momentum transfer will be only the longitudinal component = $p + \rho v^2$. It $\mathbf{n}$ is normal to the fluid flow, then there will be only transverse component = p.

$$\frac{\partial \mathbf{v}}{\partial t} + \left(\frac{1}{\rho_0}\right) \nabla p' = 0 \tag{2.15}$$

Now sound wave in ideal gas is adiabatic, so the pressure and the density of the fluid will be interrelated;i.e, small change in ρ will change p also as follows:

$$p' = \left(\frac{\partial p}{\partial \rho}\right)_s \rho' \tag{2.16}$$

So, equation 2.14 will become

$$\frac{\partial p'}{\partial t} + \rho_0 \left(\frac{\partial p}{\partial \rho}\right)_s \nabla \cdot \mathbf{v} = 0 \tag{2.17}$$

Equations 2.15 and 2.17 give the complete description of the sound waves. For mathematical simplicity for further quantities to derived, it is convenient to take $\mathbf{v} = \nabla \phi$. This gives us then the well known wave equation:

$$\frac{\partial^2 \phi}{\partial t^2} - c^2 \nabla^2 \phi = 0 \tag{2.18}$$

with $c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$, the speed of sound in a given medium. The speed of fluid can be written as

$$v = \frac{p'}{\rho c} = \frac{c\rho'}{\rho} \tag{2.19}$$

From here, we can find the energy density of sound wave which is $\frac{1}{2}\rho v^2 + \rho \varepsilon$. Putting $\rho = \rho_0 + \rho'$ and $\varepsilon = \varepsilon_0 + \varepsilon'$, and then integrating on the whole volume, total energy we get will be

$$\int_V \left(\frac{1}{2}\rho_0 v^2 + \frac{1}{2}\frac{c^2 \rho'^2}{\rho_0}\right) dV = \int_V E\, dV \tag{2.20}$$

From equation 2.8, energy flux density is $\rho \mathbf{v}\left(\frac{1}{2}v^2 + w\right) \approx \rho w \mathbf{v}$, as $\mathbf{v}$ is very small. Using $w = w_0 + w'$, and $w' = \left(\frac{\partial w}{\partial p}\right)_s p' = \frac{p'}{\rho}$:

$$\text{Total Energy Flux} = \oint (\rho w_0 \mathbf{v} + p'\mathbf{v}) \cdot d\mathbf{f} = \oint p'\mathbf{v} \cdot d\mathbf{f} \tag{2.21}$$

where first term corresponds to the total change in the mass of fluid, which will be zero. The second term is known as the *sound energy flux density*. From here, another important relation can be find out,

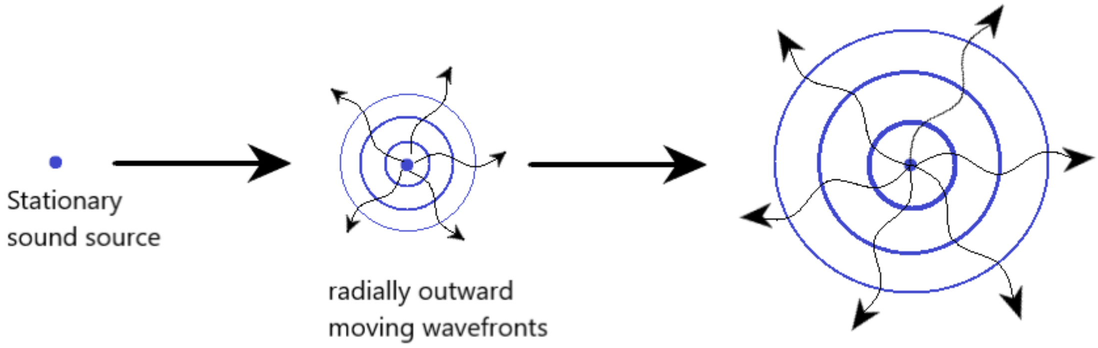
$$\frac{\partial E}{\partial t} + \nabla \cdot (p'\mathbf{v}) = 0 \tag{2.22}$$

2.4 Shock Waves

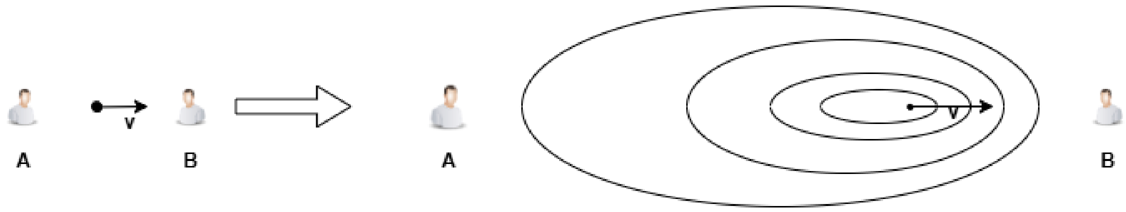
Speed of light is a barrier to each and every material thing in this universe. Not any single thing can travel faster than the speed of light. The whole theory of relativity is based on this fact. But that's for light, not for SOUND!

Sound wave is a matter wave, that means in a denser medium, the sound wave travels with higher velocity. In Earth's atmosphere, its velocity is ~340 m/s.

This velocity is achievable. Let's see what happens when we approach this limit, and cross it.



(a)



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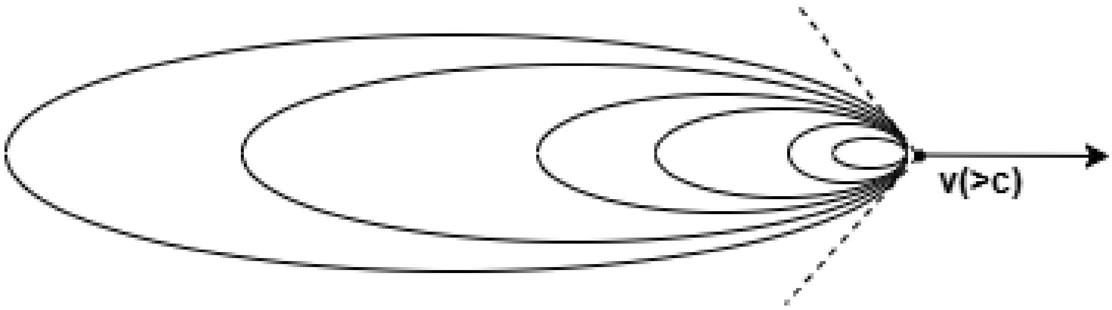

(c)

Figure 2.1: (a) When the source is stationary, the wavefronts move spherically outward; (b) When the source is moving with some velocity, the observer A will hear the low frequency sound, while the observer B will hear the high frequency sound; (c) When the source of sound is moving with velocity higher than the speed of sound, the wavefronts will get congested and take a conical shape.

Let there is a stationary source of sound. When it starts emitting sound, and if the environment is homogeneous (in this case, air), the wavefront of sound waves will spread homogeneously in all directions, as shown in figure 2.1a.

Now let's say the source is moving with some velocity $\mathbf{v}$, towards observer B, and away from observer A. As the source is moving, it is emitting the sound waves, so the wavefront near the observer B will be compressed, while that near observer A will be rarefied. Because of the relative motion of source and waves, the wavefront will be elliptical (or ellipsoidal), instead of circular (or spherical). This is known as the Doppler Shift in Sound.

What if the source is moving as fast as the speed of sound in the medium? The wavefront near the observer B will be compressed so much that it will make a cone of compressed air just at the back of the source. If $\mathbf{c}$ is the speed of sound in the medium, then we can define **Mach Number** as:

$$\mathbf{M} = \frac{v}{c} \tag{2.23}$$

If $\mathbf{M} < 1$ then the source or the moving body is called as *Subsonic*; and if $\mathbf{M} > 1$ then the source is known as *Supersonic*. The image in the starting of this chapter is showing the phenomena of sonic boom.

If a gas in steady motion with velocity $\mathbf{v}$ experiences a perturbation [2], the effect of the perturbation will propagate with the same velocity $\mathbf{v}$ carried along the gas flow. Also it will propagate relative to the gas with c in any direction $\mathbf{n}$. So the total velocity of perturbation will be $\mathbf{v} + c\mathbf{n}$.This can be shown as below:

Here, α is known as **Mach angle**, and $sin\alpha = \frac{c}{v}$. In subsonic flow, the effect will be seen in both upstream as well as downstream, while in supersonic flow, the effect will only be seen in downstream. Also, in supersonic flow, the disturbance staring from O doesn't affect gas outside the cone.

2.4.1 Surface of Discontinuous: Jump Conditions

As we can see in figure-2.1c, because of very high velocity of source, the forward wavefronts are compressed, and created a conical shape, which is the surface of discontinuity. The concerned quantities like density, pressure and velocities etc. vary discontinuously across this surface. Now on this surface, some boundary conditions should be satisfied, known as **Jump Conditions**. Let's discuss about them.

The mass coming from one side should be equal to the mass leaving the other side. The mass flux can be written as ρv_x per unit area. So,

$$\begin{aligned} \rho_1 v_{x_1} &= \rho_2 v_{x_2} \\ [\rho v_x] &= 0 \end{aligned} \tag{2.24}$$

With same argument, we can say that energy flux is also continuous. So, from 2.8,

$$\begin{aligned} \rho_1 v_{x_1}\left(\frac{1}{2}v_1^2 + w_1\right) &= \rho_2 v_{x_2}\left(\frac{1}{2}v_2^2 + w_2\right) \\ \left[\rho v_x\left(\frac{1}{2}v^2 + w\right)\right] &= 0 \end{aligned} \tag{2.25}$$

One more continuity can be found from the momentum flux. The force exerted from both the sides should be equal. From 2.12 we can write

$$\begin{aligned} p_1 + \rho_1 v_{x_1}^2 &= p_2 + \rho_2 v_{x_2}^2 \\ [p + \rho v_x^2] &= 0 \\ [\rho v_x v_y] &= 0 \\ [\rho v_x v_z] &= 0 \end{aligned} \tag{2.26}$$

From here, there can be two cases of surface of discontinuity:

[2] A slightly change in pressure p, density ρ, or velocity $\mathbf{v}$ is termed as perturbation here.

1. **Mass Flux = 0:** so $\rho_1 v_{x1} = \rho_2 v_{x2}$, which means $v_{x1} = v_{x2}$. So the normal component of the velocities will be continuous, and from 2.26, we found out that pressure is also continuous.
2. **Mass Flux $\neq$ 0:** so normal velocity components and the pressure will be discontinuous, but from 2.26 the tangential components will be continuous. The following conditions must be hold in this type of discontinuous surface:

$$\begin{aligned} [\rho v_x] &= 0 \\ \left[\frac{1}{2}v_x^2 + w\right] &= 0 \\ [p + \rho v_x^2] &= 0 \end{aligned} \tag{2.27}$$

This kind of discontinuity is known as **Shock**.

2.5 Relativistic Fluid Dynamics

To derive the fluid dynamical equations in relativistic regime, we should know the energy-momentum 4-tensor.

$$T = \begin{bmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix} \tag{2.28}$$

where T_{00} is defined as Energy density, T_{0i} or T_{i0} are momentum components, and T_{ij} is momentum flux density tensor. Consider a volume element in its local frame of reference[3], then the momentum density in this frame will be zero, while the T_{00} will be the proper internal energy density. So,

$$T = \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{bmatrix} \tag{2.29}$$

Using gravitational metric tensor, $g^{ik} = diag(-1, 1, 1, 1)$ and $w = e + p$, we can compress this whole tensor into

$$T^{ik} = wu^i u^k + pg^{ik} \tag{2.30}$$

In the observer reference frame, this equation can be converted into

$$\begin{aligned} T^{\alpha\beta} &= \frac{wv_\alpha v_\beta}{c^2(1 - v^2/c^2)} + p\delta_{\alpha\beta} \\ T^{0\alpha} &= \frac{wv_\alpha}{c(1 - v^2/c^2)} \\ T^{00} &= \frac{w}{(1 - v^2/c^2)} - p = \frac{e + pv^2/c^2}{(1 - v^2/c^2)} \end{aligned} \tag{2.31}$$

[3]Reference frame in which $u^0 = 1$ and $u^\alpha = 0$ is known as Local reference frame. Here, u^i is fluid 4-velocity.

Let's compare it with non-relativistic case. In relativistic case, the internal energy contains the rest mass energy mnc^2 also. But in non-relativistic case, the fluid element is in motion, so the change will be from mn to $\rho\sqrt{1-\frac{v^2}{c^2}} \approx \rho - \rho\frac{v^2}{2c^2}$. So,

$$\begin{aligned} T^{00} &= \rho c^2 + \rho\varepsilon + \frac{1}{2}\rho v^2 \\ T^{\alpha\beta} &= \rho v_\alpha v_\beta + p\delta_{\alpha\beta} \end{aligned} \tag{2.32}$$

which is same as what we have found in 2.11.

2.5.1 Shock waves in relativistic flow

Shock waves in relativistic fluid dynamics are the fundamental part of the high-energy astrophysical phenomena. Blandford and McKee in 1976 gave the solution of this problem which we will refer here[4].

Let there is a surface of discontinuity with side-1 (unshocked gas) having parameters e_1, p_1, w_1, ρ_1, n_1, γ_1 (Lorentz factor) and $\hat{\gamma}_1$ (ratio of specific heats), and side-2 (shocked gas) having similar parameters. Let Γ be the lorentz factor of shock itself, then jump conditions can be written as:

$$\begin{aligned} \frac{e_2}{n_2} &= \gamma_2\frac{w_1}{n_1} \\ \frac{n_2}{n_1} &= \frac{\hat{\gamma}_2\gamma_2+1}{\hat{\gamma}_2-1} \\ \Gamma^2 &= \frac{(\gamma_2+1)[\hat{\gamma}_2(\gamma_2-1)+1]^2}{\hat{\gamma}_2(2-\hat{\gamma}_2)(\gamma_2-1)+2} \end{aligned} \tag{2.33}$$

At the shock front, the jump conditions are termed as **Rankine-Hugoniot Conditions**:

$$\begin{aligned} [\rho u^i] &= 0 \\ [T^{0i}] &= 0 \\ [T^{ij}] &= 0 \end{aligned} \tag{2.34}$$

describing conservation laws of mass, momentum and energy respectively. Now, Blandford-McKee solution provides the self-similar description of a relativistic blast wave, particularly for highly relativistic case ($\Gamma >> 1$).

Let's us see some direct results provided by them. The position of shock front at time t is:

$$R_{sh}(t) = t\left[1-\frac{1}{2\Gamma^2}\right] \tag{2.35}$$

The self similar variable introduced here:

$$\chi = [1+2\Gamma^2](1-r/t) \tag{2.36}$$

The number density n_2 and pressure p_2 behind the shock front will follow the following profiles:

$$\begin{aligned} \rho_2 &= 2n_1\Gamma^2\chi^{-7/4} \\ p_2 &= \frac{2}{3}w_1\Gamma^2\chi^{-17/12} \end{aligned} \tag{2.37}$$

[4] Blandford, R. D., & McKee, C. F. 1976, ApJ, 19, 1130.

And the total energy E of the blast wave will be

$$E = \frac{8\pi w_1 t^3 \Gamma^2}{17} \tag{2.38}$$

So, these are some results which Blandford and McKee found out. They are very good solutions especially in context of GRBs. In the next chapter, we will numerically simulate this phenomena and will explore their complex nature.

3. Pluto Code: Simulations

3.1 Introduction

PLUTO is a freely-distributed software for the numerical solution of mixed hyperbolic/parabolic systems of partial differential equations (conservation laws) targeting high Mach number flows in astrophysical plasma dynamics. The code is designed with a modular and flexible structure whereby different numerical algorithms can be separately combined to solve systems of conservation laws using the finite volume or finite difference approach based on Godunov-type schemes.

PLUTO is a highly portable software and can run from a single workstation up to several thousands processors using the Message Passing Interface (MPI) to achieve highly scalable parallel performance.[1]

At present, Pluto can simulate for following types of physics in 1d, 2d and 3d systems using Cartesian, Cylindrical, Polar and Spherical coordinates:

1. Classical Hydrodynamics (HD)
2. Magnetic Hydrodynamics (MHD)
3. Relativistic Hydrodynamics (RHD)
4. Relativistic Magnetic Hydrodynamics (RMHD)
5. Resistive Relativistic Magnetic Hydrodynamics (ResRMHD)
6. Radiative Relativistic Magnetic Hydrodynamics (RadRMHD)

There are many types of solvers , time-stepping methods and reconstructions also. Depending on the problems, we use different things.

3.2 Familiarising with PLUTO

Our main focus will be on RHD. Our ultimate goal is to simulate the jet break and the afterglow of GRB with given initial conditions. Before this, to familiarise with

[1] From the official site of Pluto Code - https://plutocode.ph.unito.it/

Pluto, let us mimic the blast wave in 1D with the example of Sedov wave.

3.2.1 Sedov Wave & Shock tube Wave

Sedov wave, also known as Sedov-Taylor blast wave, refers to a blast wave generated by a very strong explosion. They are specific self-similar solution to the blast wave problem. They form the strong shocks due to the instantaneous release of a large amount of energy from a small region of high density and high pressure to the region of low density and low pressure.

On the other hand, Shock tube wave is a simple wave generated in a 1D tube with partition somewhere between the length of the tube, and different density, pressure and velocities of the gas on both the sides.

3.2.2 Problem Setup

Setup of this problem is like this: We have a 1D container with normalised length of 1. The container is divided into two parts: One part is having high density and high pressure gas, while the other part is having same gas with low density and low pressure. Let the partition is at 0.1. Figure 3.1 is showing the setup. We will use cartesian geometry so that we can compare it with shock-tube wave. Now we have to find how the density, pressure and velocity will evolve with time.

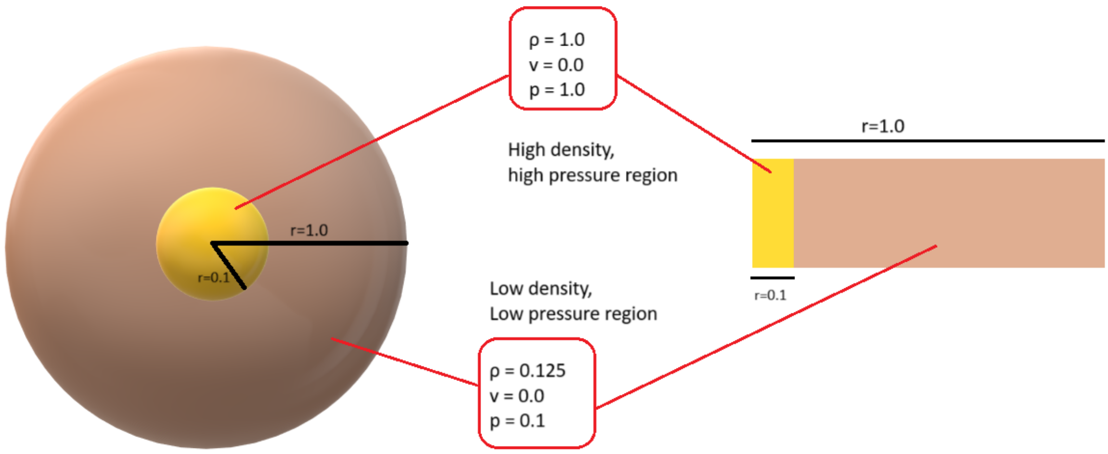

Figure 3.1: Sedov blast wave problem setup in cartesian geometry.

3.2.3 Results

Changing these initial values in the *pluto.ini* and *init.c* files. Let t_{stop} = 1.0, and tab = 0.25. So simulation will run for 1.0 normalised time with time step of 0.25 time units. So 5 tabulated data files will be generated with the following arrangement.

X	Y	Density	V_x	V_y	V_z	Pressure
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...
...	...	...	...	...	...	...

Let's plot Density, Velocity and Pressure with distance at t=0.25.

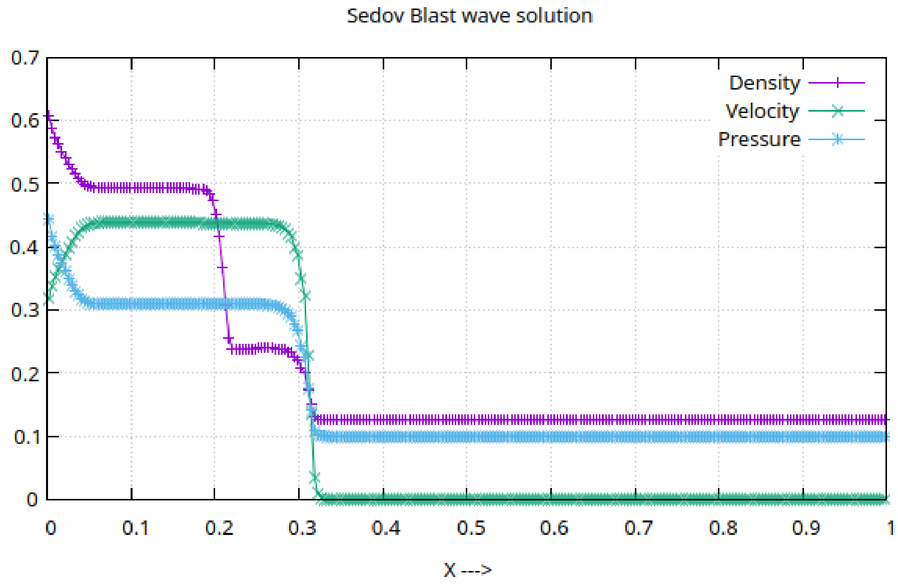

Figure 3.2: Sedov wave solution at t=0.25 time units.

There are different number of steps in density and pressure plots. As this is the classical riemann problem, in general, the solution can be studied in 6 regions[2]:

- Region 1: initial left state that has not been yet influenced by rarefaction or shock waves
- Region 2: wave traveling to the left (may be rarefaction or shock)
- Region 3: region between the wave moving to the left and the contact discontinuity
- Region 4: region between the contact discontinuity and the wave moving to the right
- Region 5: wave traveling to the right (may be rarefaction or shock)
- Region 6: initial right state that has not been yet influenced by rarefaction or shock waves

The computed results are perfectly matching with the solution of 1D classical riemann problem. Now in the next section, we will look into the simulation part of the density and pressure evolution in afterglows. We will also see the velocity changes during that period.

3.3 Afterglow Simulations

Now we are near our ultimate goal of simulation. We are going to simulate the relativistic jet propagation. We will take the uniform medium ($\rho \propto r^{-k}$, here $k = 0$) with constant pressure. A conical beam with a small aperture is injected with the speed β from the lower radial boundary. The beam pressure and the ambient pressure is matched here, so jet is pressure-matched, and the density is joined with sharp transition at the border of the nozzle.

We will try to simulate two cases, one is ($\gamma \approx 50$) and other is ($\gamma \approx 20$).

[2]F. D. Lora-Clavijo and J. P. Cruz-Perez and F. S. Guzman and J. A. Gonzalez, 2013

3.3.1 Case-1: $\gamma = 50$

Here, we will take jet velocity to be very very high, i.e., $\beta = 0.9998$ implying $\gamma \approx 50$. Let's see what changes we will have to make in *init.c, pluto.ini* and *definitions.h* files.

- **definitions.h->** RHD, Dimensions = 2, Geometry = Spherical, Equation of State = TAUB. Let's define 4 parameters: GAMMA (jet's lorentz fator), RHO_IN (density inside the boundary), RHO_OUT (density outside the boundary), PRESS_IN (beam pressure). We will give their values in *pluto.ini*.
- **pluto.ini->** (X1,X2) = (r, θ), r is from 1.0 to 1000.0, and θ is from 0 to $\pi/2$. Let's take $t_{stop} = 10000.0$ with time step of 100.0. Now for boundaries,
  $r = 0$: user defined condition (will be described in init.c),
  $r = end$: outflowing boundary
  $\theta = 0$: axisymmetric[3]
  $\theta = \pi/2$: eqtsymmetric[4]
  The conditions are defined at θ-boundaries to make the system spherically symmetric. Now, instead of making tabulated files, we will make **vtk**-files, which we will use in **VisIt** software to visualize the evolutions. As we have taken $t_{stop} = 10000.0$, let's make 100 vtk data files. In the end, let GAMMA=50, RHO_IN=1.0, RHO_OUT=100.0 and PRESS_IN=0.01.
- **init.c->** We will define one function named "Profile" which will join the jet values with ambient ones at the nozzle border. For our purpose let this be sharp transition. We can extract the initial values of density, pressure and velocities from *pluto.ini* file.
  Now, we want to study shocks and jet propagation in GRBs, so we will give different density, velocity and pressure to some range. Here, let $\rho = 2 \times \rho_{out}$, $v_{x1} = 0.1$ and $p = 10^{-4} \times p_{in}$ between $r = 50$ and $r = 100$.

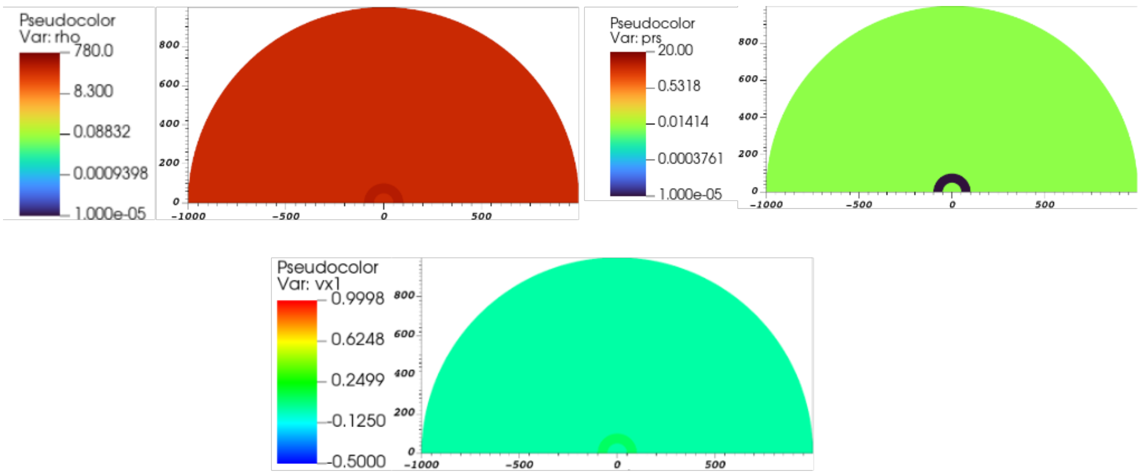

Figure 3.3: Initial values of density, pressure and velocity defined in "init.c" file.

As said earlier, at $r = 0$, user will define the boundary, so let's do this. Let the jet will start after 10 time units. Also, at $\theta = 5^\circ$, there is no outflow, so we are giving reflecting boundary here. At last, the jet values and ejecta values can be taken from the *pluto.ini* file.

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3 The ambient conditions will not vary around the azimuthal axis.
4 The ambient properties will be mirrored across the equitorial plane $z = 0$.

After giving all the required inputs, these are the results we get:

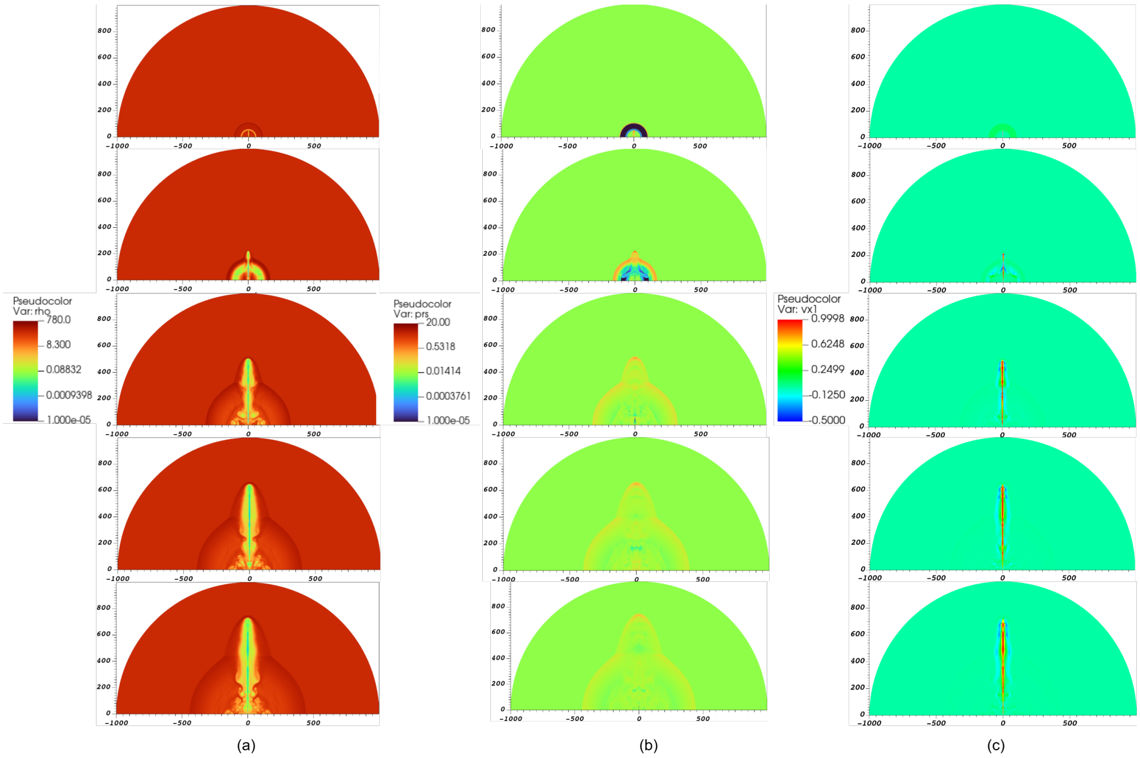


Figure 3.4: The evolutions of (a) density, (b) pressure and (c) radial velocity are shown here. The 5 snapshots in each evolution corresponds to the time t=100.0, t=1000.0, t=5000.0, t=8000.0, t=10000.0

As discussed in section 1.4, some shear flow will be developed at the interface of jet and ejecta resulting in Kelvin-Helmholtz flow (Vorticity, spinning of the fluid), that you can see at $t = 5000.0$. Also, flow have different lorentz factors, which collide with each other and produce internal shocks (the wave like structure is forming near the axis).

One important thing to notice is that we can see a cocoon like structure here surrounding the jet. While jet breaking, the cocoon produces a much slower and less collimated outflow. This can be the reason of X-ray flashes observations.

3.3.2 Case-2 : $\gamma = 20$

In this case, I have changed just two things: first is I have setup the jet ejection time to be 20 units, while I gave 20 as lorentz factor of beam. The initial conditions are shown in figure-3.5. After starting simulation, final results are shown in figure-3.6.

Here, on comparing with the results of $\gamma = 50$ case, we can see that the jet is not that much stronger for the present case. The reasons are two: the jet ejection time we have taken is 20 units, instead of 10 units; and we have taken slower velocity of jet here.

One more thing we can see is that the internal shocks are more prominent in former case (the blue color corresponds to negative velocity, showing the reverse shocks).

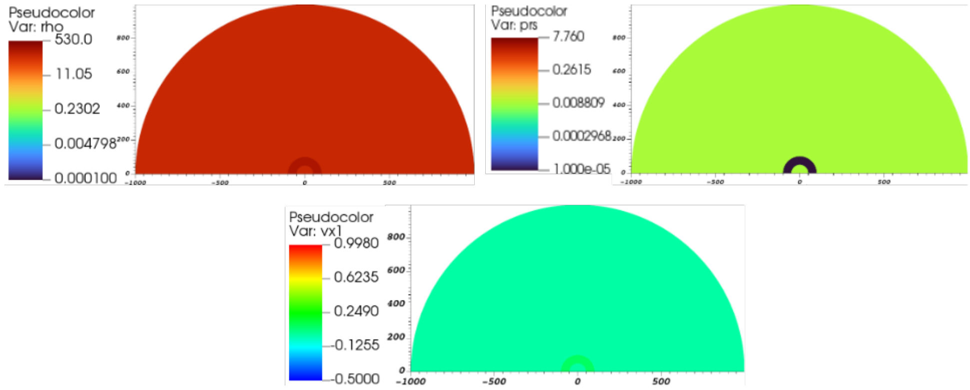
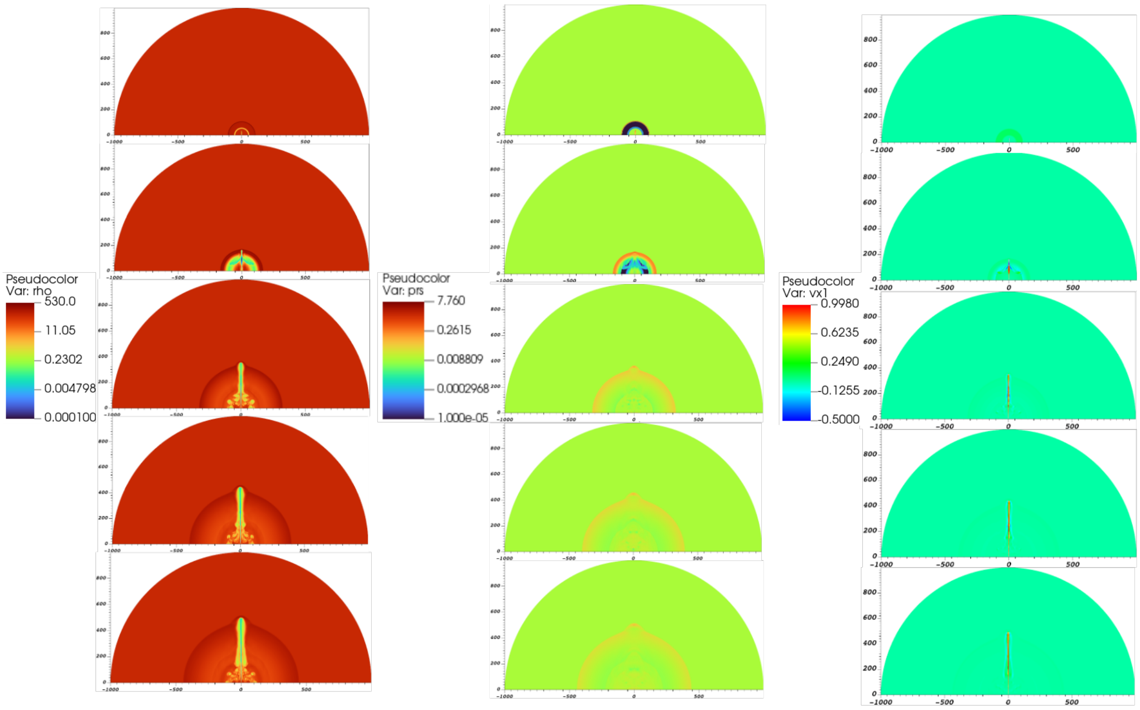


Figure 3.5: Initial distributions of density, pressure and velocity.



(a) (b) (c)

Figure 3.6: The evolutions of (a) density, (b) pressure and (c) radial velocity are shown here. The 5 snapshots in each evolution corresponds to the time t=100.0, t=1000.0, t=5000.0, t=8000.0, t=10000.0

So, we have simulated the afterglow effects on the density, pressure and velocity for GRBs. Let's recap what we have done totally in the following chapter.

4. Summary

In this internship, my aim was to simulate the afterglows' density, pressure, and velocity evolution over time in two dimensions. To achieve this, I began by gaining a basic understanding of gamma-ray bursts (GRBs), their unintentional discovery, and their classifications(1.2). I then moved on to understanding the modeling of GRBs(1.3), which provided insights into the collapsar and merger models of the central engines(1.4).

For the simulation part, I needed to understand the dynamics of GRBs, which I learned from studying fluid dynamics and its relativistic version. Initially, I learned about fluids and the governing Bernoulli and Euler equations(2.2). Next, I delved into shock waves, their production, and the jump conditions at their surfaces of discontinuity(2.4). Finally, I explored relativistic fluid dynamics and used the Blandford-McKee solutions for blast waves(2.5).

In the last chapter, I introduced the PLUTO Code(3.1), which I used to generate the simulation data. To become familiar with PLUTO, I compared the solutions of Sedov blast waves with those provided by existing references(3.2). I then proceeded to address the actual problem: simulating the GRB afterglows(3.3).

Now, the question is, if they are very very far away from us, they will not cause any harm to the Earth and the people. So why are we studying all this? Despite GRBs not posing any direct threat to Earth, their study is so much valuable. Through their study we can understand the high energy processes that we can't do on Earth. GRBs are the events occurred at the time of death of massive stars, which can tell us about the stellar evolution. One important knowledge GRBs can give us is the knowledge of the early universe, cosmic evolution and the universe expansion. Also, GRB events are associated with the gravitational waves, that is also a major and important field of astronomy.

So studying GRBs enriches our understanding of the cosmos and accelerates scientific progress.

5. References

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