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2024-03-13

Contest (1)

template.cpp101 lines

```
// #pragma comment(linker, "/stack:200000000")
// #pragma GCC optimize("Ofast")
// #pragma GCC optimize("O3,unroll-loops")
// #pragma GCC target("sse,sse2,sse3,ssse3,sse4")
// #pragma GCC target("avx2,bmi,bmi2,popcnt,lzcnt")

// #define _GLIBCXX_DEBUG
// #define _GLIBCXX_DEBUG_PEDANTIC

#include <cassert>
#include <iomanip>
#include <iostream>
#include <vector>
#include <algorithm>
#include <map>
#include <set>
#include <functional>
#include <array>
#include <numeric>
#include <queue>
#include <deque>
#include <cmath>
#include <climits>

using namespace std;

const int MOD = 998244353;
const long double PI = 3.141592653589793;
using ll = long long;
const ll INF = 1e18;

// #define int ll

// —————> sashko123's defines:

#define intn int //Vasya sorry :(
#define p_b push_back
#define fi first
#define se second
#define pii std::pair<int, int>
#define oo LLONG_MAX
#define big INT_MAX
#define elif else if

int input()
{
    int x;
    cin>>x;
    return x;
}

// —————> end of sashko123's defines (thank you Vasya <3)

template<typename T>
using graph = vector<vector<T>>;

template<typename T>
istream& operator>>(istream& in, vector<T>& a) {
    for (auto& i: a) {
        in >> i;
    }
    return in;
}

ll fast_pow(ll a, ll b, ll mod) {
```

```
    if (b == 0)
        return 1;
    if (b % 2) {
        return (1ll * a * fast_pow(a, b - 1, mod)) % mod;
    }
    ll k = fast_pow(a, b / 2, mod);
    return (1ll * k * k) % mod;
}

ll fast_pow(ll a, ll b) {
    if (b == 0)
        return 1;
    if (b % 2) {
        return (1ll * a * fast_pow(a, b - 1));
    }
    ll k = fast_pow(a, b / 2);
    return (1ll * k * k);
}

void solve() {

}

int32_t main(int32_t argc, const char * argv[]) {
    cin.tie(0);
    cout.tie(0);
    ios_base::sync_with_stdio(0);
    // insert code here...
    int tt= 1;
    // std::cin >> tt;
    while (tt--) {
        solve();
    }
    return 0;
}

fast-input.h35 lines

double readNumber() {
    const int BSIZE = 4096;
    static char buffer[BSIZE];
    static char* bptr = buffer + BSIZE;
    auto getChar = []() {
        if (bptr == buffer + BSIZE) {
            memset(buffer, 0, BSIZE);
            cin.read(buffer, BSIZE);
            bptr = buffer;
        }
        return *bptr++;
    };
    char c = getChar();
    while (c && (c < '0' || c > '9') && c != '-')
        c = getChar();
    bool minus = false;
    if (c == '-') minus = true, c = getChar();
    double res = 0;

    while (c >= '0' && c <= '9') {
        res = res * 10 + c - '0';
        c = getChar();
    }

    if (c == '.') {
        c = getChar();
        double cur = 0.1;
        while (c >= '0' && c <= '9') {
            res = res + (c - '0') * cur;
            c = getChar();
        }
    }
}
```

```
        cur /= 10.0;
    }
}

return minus ? -res : res;
}

.bashrc3 lines

alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++20 \
-fsanitize=undefined,address'
xmodmap -e 'clear lock' -e 'keycode 66=less greater' #caps =<

.vimrc6 lines

set cin aw ai is ts=4 sw=4 tm=50 nu noeb bg=dark ru cul
sy on | im jk <esc> | im kj <esc> | no ; :
" Select region and then type :Hash to hash your selection.
" Useful for verifying that there aren't mistypes.
ca Hash w !cpp -dD -P -fpreprocessed \
tr -d '[:space:]' \
\ md5sum \
cut -c-6

hash.sh3 lines

# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum | cut -c-6

Algebra (2)

xor-basis.h
Description: Xor basis, all elements in the main set can be constructed
using xor operation and elements in the basis
Time: insert per element - O(log(A_max))

<vector>hash46 lines

template<typename T = int, int max_bit = 31>
struct xor_basis
{
    std::vector<T> basis; // basis[i] -> element with smallest
        set bit equal to i
    int sz; // Current size of the basis

    xor_basis() {
        basis.assign(max_bit);
    }

    bool insert(T val) {
        for (int i = 0; i < max_bit; i++) {
            if (((val >> i)&1) == 0)
                continue;
            if (!basis[i]) {
                basis[i] = val;
                sz++;
                return true;
            }
            val ^= basis[i];
        }
        return false;
    }

    bool contains(T val) {
        for (int i = 0; i < max_bit; i++) {
            if (((val >> i)&1) == 0)
                continue;
            if (!basis[i]) {
                return false;
            }
            val ^= basis[i];
        }
    }
}
```

```
        return true;
    }

    T max_element() { // not-sure
        T val = 0;
        for (int i = max_bit - 1; i >= 0; i--) {
            if (basis[i] && !((val>>i)&1)) {
                val ^= basis[i];
            }
        }
        return val;
    }
};
```

fft.h

Description: FFT implementation

Time: $(n + m) \cdot \log(n + m)$

`<vector>`, `<complex>` hash56 lines

```
using cd = std::complex<double>;

void fft(std::vector<cd> &a, bool invert) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;

        if (i < j)
            swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <= 1) {
        double ang = 2 * PI / len * (invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            cd w(1);
            for (int j = 0; j < len / 2; j++) {
                cd u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }

    if (invert) {
        for (cd &x : a)
            x /= n;
    }
}
```

```
template<typename T>
std::vector<T> multiply(const std::vector<T>& a, const std::
    vector<T>& b) {
    std::vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end
        ());
    int n = 1;
    while (n < a.size() + b.size())
        n <= 1;
    fa.resize(n);
    fb.resize(n);

    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);
```

```
    std::vector<T> result(n);
    for (int i = 0; i < n; i++)
        result[i] = round(fa[i].real());
    result.resize(a.size() + b.size() - 1);
    return result;
}
```

nnt.h

Description: NNT implementation by modulo 998244353

Time: $(n + m) \cdot \log(n + m)$

`<vector>` hash68 lines

```
const int root = 31; // primitive root of module
const int root_1 = fast_pow(root, MOD - 2, MOD); // (primitive
    root) ^ -1
const int root_pw = 1 << 23; // max power of 2 in MOD - 1

inline int inverse(int n, int mod) {
    return fast_pow(n, mod - 2, mod);
}

void nnt(std::vector<int> &a, bool invert, int mod) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;

        if (i < j)
            std::swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <= 1) {
        int wlen = invert ? root_1 : root;
        for (int i = len; i < root_pw; i <= 1)
            wlen = (int)(1LL * wlen * wlen % mod);

        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; j++) {
                int u = a[i+j], v = (int)(1LL * a[i+j+len/2] *
                    w % mod);
                a[i+j] = u + v < mod ? u + v : u + v - mod;
                a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod
                    ;
                w = (1ll * w * wlen % mod);
            }
        }

        if (invert) {
            int n_1 = inverse(n, mod);
            for (int& x : a)
                x = (1ll * x * n_1 % mod);
        }
    }

    std::vector<int> multiply(const std::vector<int>& a, const std
        ::vector<int>& b, int mod) {
        int n = a.size(), m = b.size();
        int k = n + m - 1;
        int l = 1;
        while (l < k)
            l <= 1;

        std::vector<int> A(l), B(l);
        for (int i = 0; i < n; i++) {
```

```
        A[i] = a[i];
    }
    for (int i = 0; i < m; i++) {
        B[i] = b[i];
    }

    nnt(A, false, mod);
    nnt(B, false, mod);
    for (int i = 0; i < B.size(); i++) {
        A[i] = (1ll * A[i] * B[i]) % mod;
    }
    nnt(A, true, mod);
    A.resize(k);
    return A;
}
```

Numeric (3)

pollard-rho.h

Description: Finds divider of N.

Time: $\mathcal{O}\left(N^{1/4} \cdot \log(N)\right)$

`<numeric>` hash20 lines

```
long long mult(long long a, long long b, long long mod) {
    return (__int128)a * b % mod;
}

long long f(long long x, long long c, long long mod) {
    return (mult(x, x, mod) + c) % mod;
}

long long rho(long long n, long long x0=2, long long c=1) {
    long long x = x0;
    long long y = x0;
    long long g = 1;
    while (g == 1) {
        x = f(x, c, n);
        y = f(y, c, n);
        y = f(y, c, n);
        g = std::gcd(abs(x - y), n);
    }
    return g;
}
```

primitive-root.h

Description: Primitive root of n

hash54 lines

```
constexpr long long safe_mod(long long x, long long m) {
    x %= m;
    if (x < 0) x += m;
    return x;
}

long long pow_mod(long long x, long long n, int m) {
    if (m == 1) return 0;
    unsigned int _m = (unsigned int)(m);
    unsigned long long r = 1;
    unsigned long long y = safe_mod(x, m);
    while (n) {
        if (n & 1) r = (r * y) % _m;
        y = (y * y) % _m;
        n >>= 1;
    }
    return r;
}

int primitive_root(int m) {
    if (m == 2) return 1;
```

```
if (m == 167772161) return 3;
if (m == 469762049) return 3;
if (m == 754974721) return 11;
if (m == 998244353) return 3;
int divs[20] = {};
divs[0] = 2;
int cnt = 1;
int x = (m - 1) / 2;
while (x % 2 == 0) x /= 2;
for (int i = 3; (long long)(i)*i <= x; i += 2) {
    if (x % i == 0) {
        divs[cnt++] = i;
        while (x % i == 0) {
            x /= i;
        }
    }
}
if (x > 1) {
    divs[cnt++] = x;
}
for (int g = 2;; g++) {
    bool ok = true;
    for (int i = 0; i < cnt; i++) {
        if (pow_mod(g, (m - 1) / divs[i], m) == 1) {
            ok = false;
            break;
        }
    }
    if (ok) return g;
}

template <int m> constexpr int primitive_root = primitive_root(m);
```

Data structures (4)

```
fenwick-tree.h
Description: Fenwick Tree, update(+=) at element, sum at segment. R is excluded.
Time: update - O(log N), get - O(log N)
<cassert>, <vector> hash32 lines
template <class T> struct fenwick_tree {
public:
    fenwick_tree() : _n(0) {}
    fenwick_tree(int n) : _n(n), data(n) {}

    void add(int p, T x) {
        assert(0 <= p && p < _n);
        p++;
        while (p <= _n) {
            data[p - 1] += T(x);
            p += p & -p;
        }
    }

    T sum(int l, int r) {
        assert(0 <= l && l <= r && r <= _n);
        return sum(r) - sum(l);
    }

private:
    int _n;
    std::vector<T> data;

    T sum(int r) {
        T s = 0;
        while (r > 0) {
```

```
        s += data[r - 1];
        r -= r & -r;
    }
    return s;
}
};

fenwick-tree-2d.h
Description: 2d fenwick tree, update(+=) at element, sum at 2d segment. R is excluded.
Time: update - O(log N * log M), get - O(log N * log M) hash53 lines
template <class T> struct fenwick_tree_2d{
    struct fenwick_tree {
        int n;
        unordered_map<int, T> data;
        fenwick_tree(): n(0) {};
        fenwick_tree(int n): n(n) {};

        void add(int p, T x) {
            assert(0 <= p && p < n);
            p++;
            while (p <= n) {
                data[p - 1] += T(x);
                p += p & -p;
            }
        }

        T pref_sum(int r){
            T s = 0;
            while (r > 0) {
                s += data[r - 1];
                r -= r & -r;
            }
            return s;
        }
    };

    int n, m;
    std::vector<fenwick_tree> data;

    fenwick_tree_2d(int n,int m): n(n), m(m), data(n, fenwick_tree(m)) {};

    void add(int x, int y, T val) {
        assert(0 <= x && x < n);
        x++;
        while (x <= n) {
            data[x - 1].add(y, val);
            x += x & -x;
        }
    }

    T pref_sum(int xr, int yr){
        T s = 0;
        while (xr > 0) {
            s += data[xr - 1].pref_sum(yr);
            xr -= xr & -xr;
        }
        return s;
    }

    T sum(int x1, int y1, int xr, int yr) {
        return pref_sum(xr, yr) - pref_sum(xr, y1) - pref_sum(x1, yr) + pref_sum(x1, y1);
    }
};
```

```
segment-tree.h
Description: Segment tree, update(+=) at element, sum at segment. R is excluded.
Time: update - O(log N), get - O(log N)
<array> hash38 lines
template<typename T, int N = (1 << 18)> struct segment_tree {
    std::array<T, 2 * N> tree;
    segment_tree() {
        tree.fill(T());
    }

    void update(int pos, T val) {
        pos += N;
        tree[pos] += val;
        pos >>= 1;
        while (pos > 0) {
            tree[pos] = tree[pos << 1] + tree[(pos << 1) | 1];
            pos >>= 1;
        }
    }

    T get_sum(int l, int r) {
        l += N;
        r += N;

        T ans = T();
        while (l < r) {
            if (l & 1) {
                ans += tree[l++];
            }
            if (r & 1) {
                ans += tree[--r];
            }
            l >>= 1;
            r >>= 1;
        }
        return ans;
    }

    T get(int pos) {
        return tree[N + pos];
    }
};

lazy-segment-tree.h
Description: Segment tree, update(+=) at segment, sum at segment. R is excluded.
Time: update - O(log N), get - O(log N)
<array>, <vector> hash82 lines
template<typename T> struct lazy_segment_tree {
    struct node{
        T sum = 0;
        T promise = 0;
    };

    int n;
    std::vector<node> tree;

    lazy_segment_tree(int n_, const vector<T>& init): n(n_) {
        tree.assign(4 * n, node{});
        build(1, 0, n, init);
    }

    void build(int v, int l, int r, const vector<T>& init) {
        if (l + 1 == r) {
            tree[v].sum = init[l];
            return;
        }
    }
```

```
    if (l >= r)
        return;
    int mid = (l + r) / 2;
    build(2 * v, l, mid, init);
    build(2 * v + 1, mid, r, init);

    tree[v].sum = (tree[2 * v].sum + tree[2 * v + 1].sum);
}

void update(int l, int r, T value) {
    update(1, 0, n, l, r, value);
}

T get(int l, int r) {
    return get(1, 0, n, l, r);
}

T get(int pos) {
    return get(1, 0, n, pos, pos + 1);
}

void push(int v, int l, int r) {
    if (tree[v].promise == 0)
        return;

    tree[v].sum += tree[v].promise * (r - l);
    if (l + 1 < r) {
        tree[2 * v].promise += tree[v].promise;
        tree[2 * v + 1].promise += tree[v].promise;
    }
    tree[v].promise = 0;
}

void update(int v, int tl, int tr, int l, int r, T value) {
    push(v, tl, tr);
    if (l >= tr || tl >= r)
        return;
    if (l <= tl && tr <= r) {
        tree[v].promise += value;
        push(v, tl, tr);
        return;
    }
    int mid = (tl + tr) / 2;
    update(2 * v, tl, mid, l, r, value);
    update(2 * v + 1, mid, tr, l, r, value);

    tree[v].sum = tree[2 * v].sum + tree[2 * v + 1].sum;
}

T get(int v, int tl, int tr, int l, int r) {
    push(v, tl, tr);
    if (l >= tr || tl >= r)
        return 0;
    if (l <= tl && tr <= r) {
        return tree[v].sum;
    }
    int mid = (tl + tr) / 2;
    auto left = get(2 * v, tl, mid, l, r);
    auto right = get(2 * v + 1, mid, tr, l, r);

    return left + right;
}
};
```

persistent-segment-tree.h
Description: Persistent segment tree, update(+=) at segment, sum at segment. R is excluded.
Time: update - $\mathcal{O}(\log N)$, get - $\mathcal{O}(\log N)$
 <array> hash83 lines

```
template<typename T> struct persistent_segment_tree {
    struct node {
        T sum;
        int left, right;

        node(T val = T()): sum(val), left(-1), right(-1) {}
        node(int left_, int right_): sum(T()), left(left_), right(right_) {}
    };
    std::vector<node> tree;

    node create_node(T val = 0) {
        return node(val);
    }
    node create_node(int left, int right) {
        auto v = node(left, right);
        v.sum = (left != -1 ? tree[left].sum : 0) + (right != -1 ? tree[right].sum : 0);

        return v;
    }
    int LAST = 0;
    template<typename... params>
    int new_node(params... args) {
        tree[++LAST] = create_node(args...);
        return LAST;
    }
    int n;
    persistent_segment_tree(int n_, int sz_, const vector<T>& a, int& first_root): n(n_) {
        tree.resize(sz_);
        first_root = build(0, n, a);
    }

    int build(int l, int r, const vector<T>& a) {
        if (l + 1 == r) {
            return new_node(a[l]);
        }
        int mid = (r + 1) / 2;
        int left = build(l, mid, a);
        int right = build(mid, r, a);
        return new_node(left, right);
    }

    int update(int root, int pos, T val) {
        return update(root, 0, n, pos, val);
    }

    T get(int root, int l, int r) {
        return get(root, 0, n, l, r);
    }

    T get(int root, int pos) {
        return get(root, 0, n, pos, pos + 1);
    }

    int update(int v, int l, int r, int pos, T val) {
        if (l + 1 == r) {
            return new_node(val + tree[v].sum);
        }

        int mid = (l + r) / 2;
        if (pos < mid) {
            return new_node(
                update(tree[v].left, l, mid, pos, val),
                tree[v].right
            );
        } else {
```

```
            return new_node(
                tree[v].left,
                update(tree[v].right, mid, r, pos, val)
            );
        }
    }

    T get(int v, int tl, int tr, int l, int r) {
        if (tr <= l || r <= tl)
            return 0;
        if (l <= tl && tr <= r) {
            return tree[v].sum;
        }

        int mid = (tl + tr) / 2;
        return get(tree[v].left, tl, mid, l, r) + get(tree[v].right, mid, tr, l, r);
    }
};
```

li-chao-tree.h
Description: Li-Chao tree, online convex hull for maximizing $f(x) = k * x + b$, for minimization use $(-k) * x + (-b)$
Time: add - $\mathcal{O}(\log N)$, get - $\mathcal{O}(\log N)$
 hash74 lines

```
template<typename T> struct li_chao_tree {
    const T MX = 1e9 + 1;
    struct line {
        T k = 0;
        T b = -INF;

        T f(T x) const {
            return k * x + b;
        }
    };
    struct node {
        line ln;
        node* left = nullptr;
        node* right = nullptr;
    };

    node* new_node() {
        const int N = 100000;
        static node* block;
        static int count = N;

        if (count == N) {
            block = new node[N];
            count = 0;
        }
        return (block + count++);
    };

    node* root = new_node();

    T get(T x) {
        return get(root, -MX, MX, x);
    }

    void add(line ln) {
        add(root, -MX, MX, ln);
    }

    T get(node*& v, T l, T r, T x) {
        if (!v || l > r) {
            return -INF;
        }
        T ans = v->ln.f(x);
        if (r == l) {
            return ans;
        }
```

```
    }
    T mid = (r + 1) / 2;
    if (x <= mid) {
        return max(ans, get(v->left, l, mid, x));
    } else {
        return max(ans, get(v->right, mid + 1, r, x));
    }
}

void add(node*& v, T l, T r, line ln) {
    if (l > r)
        return;
    if (!v) {
        v = new_node();
    }
    T m = (r + 1) / 2;
    bool left = v->ln.f(l) < ln.f(l);
    bool md = v->ln.f(m) < ln.f(m);
    if (md)
        swap(v->ln, ln);
    if (l == r) {
        return;
    }
    if (left != md) {
        add(v->left, l, m, ln);
    } else {
        add(v->right, m + 1, r, ln);
    }
}

};
```

rope.h
Description: Rope data structure
Time: all get queries $\mathcal{O}(\log N)$

```
<ext/rope> hash11 lines
using namespace __gnu_cxx;
```

```
rope<int> v;
for(int i = 1; i <= n; ++i)
    v.push_back(i);
rope<int> cur = v.substr(l, r - l + 1); // start, length
v.erase(l, r - l + 1); // start, length
v.insert(v.mutable_begin(), cur);

for(rope<int>::iterator it = v.mutable_begin(); it != v.mutable_end(); ++it)
    cout << *it << " ";
```

ordered-set.h
Description: A red-black tree with the ability to get an element by index (find_by_order) and index of a specific element (order_of_key)
Time: get - $\mathcal{O}(\log N)$, segment tree is 2 times faster
`<ext/pb_ds/assoc.container.hpp>`, `<ext/pb_ds/tree.policy.hpp>` hash4 lines
using namespace __gnu_pbds;

```
template<typename T>
using ordered_set = tree<T,null_type,less<T>,rb_tree_tag,
    tree_order_statistics_node_update>;
```

sparse-table.h
Description: Min sparse table.
Time: build - $\mathcal{O}(N * \log N)$, get - $\mathcal{O}(1)$
`<cassert>`, `<array>`, `<vector>` hash29 lines
template<typename T>
struct sparse_table {
 static const int K = 20;
 std::array<std::vector<T>, K> ar;

```
    std::vector<int> lg;  
    int n;  
  
    sparse_table(int n, const vector<T>& a): n(n) {  
        lg.resize(n + 1);  
        lg[1] = 0;  
        for (int i = 2; i <= n; i++)  
            lg[i] = lg[i >> 1] + 1;  
  
        ar[0] = a;  
  
        for (int k = 0; k + 1 < K; k++) {  
            ar[k + 1].resize(n);  
            for (int i = 0; i + (1 << k) < n; i++) {  
                ar[k + 1][i] = min(ar[k][i], ar[k][i + (1 << k) ]);  
            }  
        }  
  
        T get(int l, int r) {  
            assert(0 <= l < n && l < r && r <= n);  
            int power = lg[r - l];  
            return min(ar[power][l], ar[power][r - (1 << power)]);  
        }  
    };
```

xor-trie.h
Description: Binary t for integer numbers. Get finds maximum xor of two numbers.
Time: add - $\mathcal{O}(\log A)$, get - $\mathcal{O}(\log A)$

```
<cassert>, <array> hash35 lines
struct xor_trie_node {
    int cnt = 0;
    std::array<xor_trie_node*, 2> mp = {nullptr, nullptr};

    void add(int mask, int k = 30) {
        cnt++;
        if (k == -1)
            return;
        int bit = (mask>>k)&1;

        if (!mp[bit])
            mp[bit] = new xor_trie_node();
        mp[bit]->add(mask, k - 1);
    }

    void remove(int mask, int k = 30) {
        cnt--;
        if (k == -1)
            return;
        int bit = (mask>>k)&1;

        assert(mp[bit] && mp[bit]->cnt > 0);
        mp[bit]->remove(mask, k - 1);
    }

    int get(int mask, int k = 30) {
        if (k == -1)
            return 0;
        int bit = (mask>>k)&1;
        int cur= bit;
        if (mp[!bit] && mp[!bit]->cnt)
            cur = !bit;
        return ((cur^bit) << k) | mp[cur]->get(mask, k - 1);
    }
};
```

Graphs (5)

articulation-points.h

Description: Finds all articulation points.
Time: $\mathcal{O}(N + M)$

`<set>`, `<vector>`, `<functional>` hash35 lines

template<typename T>
using graph = std::vector<std::vector<T>>;

std::set<int> find_articulation_points(int n, graph<int> g) {
 std::vector<int> used(n), tin(n), fup(n);
 int T = 0;
 std::set<int> nodes;
 std::function<void(int, int)> dfs = [&](int v, int p = -1)
 {
 used[v] = true;
 tin[v] = fup[v] = T++;
 int cnt = 0;
 for (auto to : g[v]) {
 if (to == p)
 continue;
 if (used[to]) {
 fup[v] = std::min(fup[v], tin[to]);
 } else {
 dfs(to, v);
 fup[v] = std::min(fup[v], fup[to]);
 if (fup[to] >= tin[v] && p != -1) {
 nodes.insert(v);
 }
 cnt++;
 }
 }

 if (cnt > 1 && p == -1)
 nodes.insert(v);
 };
 for (int i = 0; i < n; i++) {
 if (!used[i])
 dfs(i, -1);
 }
 return nodes;
}

bridges.h

Description: Finds all bridges in the undirected graph.
Time: $\mathcal{O}(N + M)$

`<vector>`, `<functional>` hash30 lines

template<typename T>
using graph = std::vector<std::vector<T>>;

std::vector<std::pair<int, int>> find_bridges(int n, graph<int> g) {
 std::vector<int> used(n), tin(n), fup(n);
 int T = 0;
 std::vector<std::pair<int, int>> edges;
 std::function<void(int, int)> dfs = [&](int v, int p = -1)
 {
 used[v] = true;
 tin[v] = fup[v] = T++;
 for (auto to : g[v]) {
 if (to == p)
 continue;
 if (used[to]) {
 fup[v] = std::min(fup[v], tin[to]);
 } else {
 dfs(to, v);
 fup[v] = std::min(fup[v], fup[to]);
 if (fup[to] == tin[to]) {

```

        edges.push_back({v, to});
    }
}

};
for (int i = 0; i < n; i++) {
    if (!used[i])
        dfs(i, -1);
}
return edges;
}

```

dsu.h

Description: Disjoint Set Union**Time:** $\mathcal{O}(n)$ time (amortized) by sizes<algorithm>, <cassert>, <vector> hash59 lines

```

struct dsu {
public:
    dsu() : _n(0) {}
    dsu(int n) : _n(n), parent_or_size(n, -1) {}

    int merge(int a, int b) {
        assert(0 <= a && a < _n);
        assert(0 <= b && b < _n);
        int x = leader(a), y = leader(b);
        if (x == y) return x;
        if (-parent_or_size[x] < -parent_or_size[y]) std::swap(
            x, y);
        parent_or_size[x] += parent_or_size[y];
        parent_or_size[y] = x;
        return x;
    }

    bool same(int a, int b) {
        assert(0 <= a && a < _n);
        assert(0 <= b && b < _n);
        return leader(a) == leader(b);
    }

    int leader(int a) {
        assert(0 <= a && a < _n);
        if (parent_or_size[a] < 0) return a;
        return parent_or_size[a] = leader(parent_or_size[a]);
    }

    int size(int a) {
        assert(0 <= a && a < _n);
        return -parent_or_size[leader(a)];
    }

    std::vector<std::vector<int>> groups() {
        std::vector<int> leader_buf(_n), group_size(_n);
        for (int i = 0; i < _n; i++) {
            leader_buf[i] = leader(i);
            group_size[leader_buf[i]]++;
        }
        std::vector<std::vector<int>> result(_n);
        for (int i = 0; i < _n; i++) {
            result[i].reserve(group_size[i]);
        }
        for (int i = 0; i < _n; i++) {
            result[leader_buf[i]].push_back(i);
        }
        result.erase(
            std::remove_if(result.begin(), result.end(),
                [&](const std::vector<int>& v) {
                    return v.empty();
                }),
            result.end());
        return result;
    }
}

```

```

    }

private:
    int _n;
    // root node: -1 * component size
    // otherwise: parent
    std::vector<int> parent_or_size;
};

```

dynamic-connectivity-problem.h

Description: Disjoint Set Union with roolbacks**Time:** $\mathcal{O}(q * \log(q) * \log(n))$, q - number of queries, n - number of vertices<array>, <vector> hash110 lines

```

struct Query {
    int v, u;
    bool united;
    Query(int _v, int _u) : v(_v), u(_u) {}
};

struct DSU {
    struct DSU_save {
        int v, rang_v;
        int u, rang_u;
    };
    int n;
    std::vector<int> pred, rang;
    std::vector<DSU_save> saves;

    DSU(int n_) : n(n_) {
        pred.resize(n);
        rang.resize(n);
        for (int i = 0; i < n; i++) {
            pred[i] = i;
            rang[i] = 0;
        }
    }

    int get(int v) {
        if (pred[v] == v) {
            return v;
        }
        return get(pred[v]);
    }

    bool merge(int u, int v) {
        u = get(u);
        v = get(v);
        if (u == v)
            return false;
        if (rang[u] < rang[v])
            std::swap(u, v);
        saves.push_back({v, rang[v], u, rang[u]});
        pred[v] = u;

        if (rang[u] == rang[v]) {
            rang[u]++;
        }
        return true;
    }

    void rollback() {
        if (saves.empty())
            return;
        auto [v, rang_v, u, rang_u] = saves.back();
        rang[v] = rang_v;
        rang[u] = rang_u;

        pred[v] = v;
    }
}

```

```

    pred[u] = u;

    saves.pop_back();
}

};

struct dynamic_connectivity_problem {
    std::vector<std::vector<Query>> tree;
    int q, n;
    DSU dsu;
    dynamic_connectivity_problem(int q_, int n_) : q(q_), n(n_),
        dsu(DSU(n_)) {
        tree.resize(4 * q);
    }

    void add(Query a, int l, int r) {
        add(l, 0, q, l, r, a);
    }

    void add(int v, int tl, int tr, int l, int r, const Query& a) {
        if (l <= tl && tr <= r) {
            tree[v].push_back(a);
            return;
        }
        if (l >= tr || tl >= r)
            return;
        int mid = (tr + tl) / 2;
        add(2 * v, tl, mid, l, r, a);
        add(2 * v + 1, mid, tr, l, r, a);
    }

    void dfs(int v, int l, int r) {
        if (l >= r)
            return;

        for (auto& q: tree[v]) {
            q.united = dsu.merge(q.u, q.v);
        }
        if (l + 1 == r) {
            int x = dsu.get(0);
            // do something
        } else {
            int mid = (r + l) / 2;
            dfs(2 * v, l, mid);
            dfs(2 * v + 1, mid, r);
        }

        for (auto& q: tree[v]) {
            if (q.united)
                dsu.rollback();
            q.united = false;
        }
    }
};

```

kuhn-matching.h

Description: Fast pair matching algorithm. To find the minimum vertex cover start dfs from each vertice in the left that is not in maximum matching, from the left side chose unvisited vertices and from right chose visited.**Time:** $\mathcal{O}(n * (n + m))$ <vector>, <utility> hash44 lines

```

using namespace std;

template<typename T>
using graph = vector<vector<T>>>;

bool dfs(int v, graph<int>& g, vector<int>& mt, vector<int>&
    rev_mt, vector<int>& used) {

```

```
    if (used[v] == 1)
        return false;
    used[v] = 1;
    for (auto to : g[v]) {
        if (mt[to] == -1) {
            rev_mt[v] = to;
            mt[to] = v;
            return true;
        }
    }
    for (auto to : g[v]) {
        if (dfs(mt[to], g, mt, rev_mt, used)) {
            rev_mt[v] = to;
            mt[to] = v;
            return true;
        }
    }
    return false;
}

pair<int, vector<int>> pair_matching(int n, int m, graph<int> g)
{
    vector<int> mt(m, -1), used, rev_mt(n, -1);
    int cnt = 0;
    for (int it = 0; ; it++) {
        bool found = false;
        used.assign(n, 0);
        for (int i = 0; i < n; i++) {
            if (rev_mt[i] == -1 && dfs(i, g, mt, rev_mt, used))
            {
                cnt++;
                found = true;
            }
        }
        if (!found) {
            break;
        }
    }
    return {cnt, mt};
}
```

lca.h
Description: Finds lowest common ancestor of two vertices using sparce table
Time: $\mathcal{O}(n * \log(n))$ - build, $\mathcal{O}(1)$ - get

```
struct LCA {
    vector<pair<int, int>> traversal;
    vector<int> pos;
    graph<int> g;
    int n;
    sparse_table<pair<int, int>> st;

    LCA(int n_, graph<int> g_, int root = 0): n(n_), g(g_) {
        pos.resize(n);
        dfs(root, -1, 0);
        st = sparse_table<pair<int, int>>(traversal.size(),
            traversal);
    }

    void dfs(int v, int pred, int depth) {
        pos[v] = traversal.size();
        traversal.push_back({depth, v});
        for (auto to : g[v]) {
            if (to != pred) {
                dfs(to, v, depth + 1);
                traversal.push_back({depth, v});
            }
        }
    }
}
```

```
    }

    int get(int a, int b) {
        if (a == b)
            return a;
        return st.get(min(pos[a], pos[b]), max(pos[a], pos[b]) + 1)
            .second;
    }
};
```

two-sat.h
Description: 2-sat implementation
Time: $\mathcal{O}(n)$

```
<vector> hash74 lines

template<typename T>
using graph = std::vector<std::vector<T>>>;

struct two_sat {
    graph<int> g, rev;
    std::vector<int> used, order, comp, ans;
    int n;

    two_sat(int _n): n(_n) {
        g.assign(2 * n, {});
        rev.assign(2 * n, {});
    }

    void add_edge(int u, int v) {
        g[u].push_back(v);
        rev[v].push_back(u);
    }

    void add_clause_or(int a, bool val_a, int b, bool val_b) {
        add_edge(a + val_a * n, b + !val_b * n);
        add_edge(b + val_b * n, a + !val_a * n);
    }

    void add_clause_xor(int a, bool val_a, int b, bool val_b) {
        add_clause_or(a, val_a, b, val_b);
        add_clause_or(a, !val_a, b, !val_b);
    }

    void add_clause_and(int a, bool val_a, int b, bool val_b) {
        add_clause_xor(a, !val_a, b, val_b);
    }

    void top_sort(int v) {
        used[v] = 1;
        for (auto to : g[v]) {
            if (!used[to])
                top_sort(to);
        }
        order.push_back(v);
    }

    void compress(int v, int id) {
        comp[v] = id;
        for (auto to : rev[v]) {
            if (comp[to] == -1)
                compress(to, id);
        }
    }

    bool satisfiable() {
        order.clear();
        used.assign(2 * n, 0);
        comp.assign(2 * n, -1);
        ans.assign(n, 0);
    }
}
```

```
    for (int i = 0; i < 2 * n; i++) {
        if (!used[i])
            top_sort(i);
    }
    reverse(order.begin(), order.end());
    int id = 0;
    for (auto v : order) {
        if (comp[v] == -1)
            compress(v, id++);
    }

    for (int i = 0; i < n; i++) {
        if (comp[i] == comp[i + n])
            return false;
        ans[i] = (comp[i + n] < comp[i]);
    }
    return true;
}

};
```

Strings (6)

z-function.h
Description: Z-functions, z[i] equal to the length of largest common prefix of string s and suffix of s starting at i.
Time: $\mathcal{O}(N)$, N - size of string s

```
vector<int> z_function(const string& s) {
    int n = s.size();
    vector<int> z(n);
    int l = 0, r = 0;
    for(int i = 1; i < n; i++) {
        if(i < r) {
            z[i] = min(r - i, z[i - l]);
        }
        while(i + z[i] < n && s[z[i]] == s[i + z[i]]) {
            z[i]++;
        }
        if(i + z[i] > r) {
            l = i;
            r = i + z[i];
        }
    }
    return z;
}
```

Geometry (7)

point.h
Description: geometry formulas

```
<complex>, <iostream> hash124 lines

class point : public std::complex<long double> {
    using ld = long double;
    static constexpr long double PI = 3.141592653589793;

public:
    point() : std::complex<long double>() {}

    point(ld x, ld y) : std::complex<long double>(x, y) {}

    point(std::complex<long double> obj) : std::complex<long double>(obj) {}

    ld x() {
        return this->real();
    }
}
```



```

ld y() {
    return this->imag();
}

ld x() const {
    return this->real();
}

ld y() const {
    return this->imag();
}

// a_x * b_x + a_y * b_y
static ld dot_product(const point& a, const point& b) {
    return (conj(a) * b).real();
}

// a_x * b_y - a_y * b_x
static ld cross_product(const point& a, const point& b) {
    return (conj(a) * b).imag();
}

static ld squared_distance(const point& a, const point& b) {
    return norm(a - b);
}

static ld distance(const point& a, const point& b) {
    return abs(a - b);
}

// angle_of_elevation of line (a, b) to oX
static ld angle_of_elevation(const point& a, const point& b)
{
    return arg(b - a);
}

// k from y = k * x + b
static ld slope_of_line(const point& a, const point& b) {
    return tan(arg(b - a));
}

static point from_polar(ld r, ld theta) {
    return std::polar(r, theta);
}

static point rotate_above_pivot(const point& a, const ld
    theta, const point& pivot = point(0, 0)){
    return (a - pivot) * std::polar<ld>(1.0, theta) + pivot;
}

point& rotate(const ld theta, const point& pivot = point(0,
    0)) {
    *this = point::rotate_above_pivot(*this, theta, pivot);
    return *this;
}

// angle of ABC
static ld angle(const point& a, const point& b, const point&
    c) {
    return abs(remainder(arg(a-b) - arg(c-b), 2.0 * PI));
}

static point project_on_vector(const point& p, const point& v
    ) {
    return v * dot_product(p, v) / norm(v);
}

```

```

static point project_on_line(const point& p, const point& a,
    const point& b) {
    return a + (b - a) * dot_product(p - a, b - a) / norm(b - a
    );
}

static point reflect_accros(const point& p, const point& a,
    const point& b) {
    return a + conj((p - a) / (b - a)) * (b - a);
}

// intersection of lines (a, b) and (p, q). if parallel
// returns {false, ...} else {true, intersection}.
friend std::pair<bool, point> intersection_of_lines(const
    point& a, const point& b, const point& p, const point& q
    ) {
    ld c1 = cross_product(p - a, b - a), c2 = cross_product(q -
        a, b - a);
    if (c1 == c2) {
        return {false, {}};
    }
    return {true, (c1 * q - c2 * p) / (c1 - c2)}; // undefined
    if parallel
}

// returns a, b, c from a * x + b * y + c == 0 by two points
friend std::tuple<ld, ld, ld> get_line(const point& p, const
    point& q) {
    ld a = (p.y() - q.y());
    ld b = -(p.x() - q.x());
    ld c = p.y() * (p.x() - q.x()) - p.x() * (p.y() - q.y());
    return {a, b, c};
}

friend ld distance_from_point_to_line(const point& p, const
    point& a, const point& b) {
    point q = project_on_line(p, a, b);

    return point::distance(p, q);
}

friend ld distance_from_point_to_segment(const point& p,
    const point& a, const point& b) {
    point q = project_on_line(p, a, b);
    if (std::min(a.x(), b.x()) <= q.x() && q.x() <= std::max(a.
        x(), b.x()))
        return point::distance(p, q);
    else
        return std::min(distance(p, a), distance(p, b));
}

friend std::istream& operator>> (std::istream& in, point& p)
{
    ld x, y;
    in >> x >> y;
    p = point(x, y);
    return in;
}
};

```

faces-of-planar-graph.h

Description: Finds faces of planar graph. Rreturns a vector of vertices for each face, outer face goes first. Inner faces are returned in counter-clockwise orders and the outer face is returned in clockwise order.

Time: $\mathcal{O}(n \log(n))$

```

<vector> hash89 lines
template<typename T>
using graph = vector<vector<T>>>;

```

```

struct Point {
    int64_t x, y;

    Point(int64_t x_, int64_t y_): x(x_), y(y_) {}

    Point operator - (const Point & p) const {
        return Point(x - p.x, y - p.y);
    }

    int64_t cross (const Point & p) const {
        return x * p.y - y * p.x;
    }

    int64_t cross (const Point & p, const Point & q) const {
        return (p - *this).cross(q - *this);
    }

    int half () const {
        return int(y < 0 || (y == 0 && x < 0));
    }
};

std::vector<std::vector<size_t>> find_faces(std::vector<Point>
    vertices, graph<int> adj) {
    size_t n = vertices.size();
    std::vector<std::vector<char>> used(n);
    for (size_t i = 0; i < n; i++) {
        used[i].resize(adj[i].size());
        used[i].assign(adj[i].size(), 0);
        auto compare = [&](size_t l, size_t r) {
            Point pl = vertices[l] - vertices[i];
            Point pr = vertices[r] - vertices[i];
            if (pl.half() != pr.half())
                return pl.half() < pr.half();
            return pl.cross(pr) > 0;
        };
        std::sort(adj[i].begin(), adj[i].end(), compare);
    }
    std::vector<std::vector<size_t>> faces;
    for (size_t i = 0; i < n; i++) {
        for (size_t edge_id = 0; edge_id < adj[i].size();
            edge_id++) {
            if (used[i][edge_id]) {
                continue;
            }
            std::vector<size_t> face;
            size_t v = i;
            size_t e = edge_id;
            while (!used[v][e]) {
                used[v][e] = true;
                face.push_back(v);
                size_t u = adj[v][e];
                size_t e1 = std::lower_bound(adj[u].begin(),
                    adj[u].end(), v, [&](size_t l, size_t r) {
                        Point pl = vertices[l] - vertices[u];
                        Point pr = vertices[r] - vertices[u];
                        if (pl.half() != pr.half())
                            return pl.half() < pr.half();
                        return pl.cross(pr) > 0;
                    }) - adj[u].begin() + 1;
                if (e1 == adj[u].size()) {
                    e1 = 0;
                }
                v = u;
                e = e1;
            }
            std::reverse(face.begin(), face.end());
            int sign = 0;
            for (size_t j = 0; j < face.size(); j++) {

```

```
        size_t j1 = (j + 1) % face.size();
        size_t j2 = (j + 2) % face.size();
        int64_t val = vertices[face[j]].cross(vertices[
            face[j1]], vertices[face[j2]]);
        if (val > 0) {
            sign = 1;
            break;
        } else if (val < 0) {
            sign = -1;
            break;
        }
    }
    if (sign <= 0) {
        faces.insert(faces.begin(), face);
    } else {
        faces.emplace_back(face);
    }
}
return faces;
}
```

Techniques (A)

techniques.txt	159 lines
Recursion	
Divide and conquer	
Finding interesting points in N log N	
Algorithm analysis	
Master theorem	
Amortized time complexity	
Greedy algorithm	
Scheduling	
Max contiguous subvector sum	
Invariants	
Huffman encoding	
Graph theory	
Dynamic graphs (extra book-keeping)	
Breadth first search	
Depth first search	
* Normal trees / DFS trees	
Dijkstra's algorithm	
MST: Prim's algorithm	
Bellman-Ford	
Konig's theorem and vertex cover	
Min-cost max flow	
Lovasz toggle	
Matrix tree theorem	
Maximal matching, general graphs	
Hopcroft-Karp	
Hall's marriage theorem	
Graphical sequences	
Floyd-Warshall	
Euler cycles	
Flow networks	
* Augmenting paths	
* Edmonds-Karp	
Bipartite matching	
Min. path cover	
Topological sorting	
Strongly connected components	
2-SAT	
Cut vertices, cut-edges and biconnected components	
Edge coloring	
* Trees	
Vertex coloring	
* Bipartite graphs (=> trees)	
* 3^n (special case of set cover)	
Diameter and centroid	
K'th shortest path	
Shortest cycle	
Dynamic programming	
Knapsack	
Coin change	
Longest common subsequence	
Longest increasing subsequence	
Number of paths in a dag	
Shortest path in a dag	
Dynprog over intervals	
Dynprog over subsets	
Dynprog over probabilities	
Dynprog over trees	
3^n set cover	
Divide and conquer	
Knuth optimization	
Convex hull optimizations	
RMQ (sparse table a.k.a 2^k-jumps)	
Bitonic cycle	
Log partitioning (loop over most restricted)	
Combinatorics	

Computation of binomial coefficients
Pigeon-hole principle
Inclusion/exclusion
Catalan number
Pick's theorem
Number theory
Integer parts
Divisibility
Euclidean algorithm
Modular arithmetic
* Modular multiplication
* Modular inverses
* Modular exponentiation by squaring
Chinese remainder theorem
Fermat's little theorem
Euler's theorem
Phi function
Frobenius number
Quadratic reciprocity
Pollard-Rho
Miller-Rabin
Hensel lifting
Vieta root jumping
Game theory
Combinatorial games
Game trees
Mini-max
Nim
Games on graphs
Games on graphs with loops
Grundy numbers
Bipartite games without repetition
General games without repetition
Alpha-beta pruning
Probability theory
Optimization
Binary search
Ternary search
Unimodality and convex functions
Binary search on derivative
Numerical methods
Numeric integration
Newton's method
Root-finding with binary/ternary search
Golden section search
Matrices
Gaussian elimination
Exponentiation by squaring
Sorting
Radix sort
Geometry
Coordinates and vectors
* Cross product
* Scalar product
Convex hull
Polygon cut
Closest pair
Coordinate-compression
Quadtrees
KD-trees
All segment-segment intersection
Sweeping
Discretization (convert to events and sweep)
Angle sweeping
Line sweeping
Discrete second derivatives
Strings
Longest common substring
Palindrome subsequences

Knuth-Morris-Pratt
Tries
Rolling polynomial hashes
Suffix array
Suffix tree
Aho-Corasick
Manacher's algorithm
Letter position lists
Combinatorial search
Meet in the middle
Brute-force with pruning
Best-first (A*)
Bidirectional search
Iterative deepening DFS / A*
Data structures
LCA (2^k-jumps in trees in general)
Pull/push-technique on trees
Heavy-light decomposition
Centroid decomposition
Lazy propagation
Self-balancing trees
Convex hull trick (wcipeg.com/wiki/Convex_hull_trick)
Monotone queues / monotone stacks / sliding queues
Sliding queue using 2 stacks
Persistent segment tree