MA 180 - Discrete Mathematics

Exercise Set 01 (21/01/2025)

Q1 Determine whether each of the following sentences is a statement:

I. In 2003 George W. Bush was the president of the United States.

II. $x + 3$ is a positive integer.

III. Fifteen is an even number.

IV. If Jennifer is late for the party, then her cousin Zachary will be quite angry.

V. What time is it?

VI. As of June 30, 2003, Christine Marie Evert had won the French Open a record seven times.

Q2 Identify the primitive statements in Q1.

Q3 Let p, q be primitive statements for which the implication $p \to q$ is false. Determine the truth values for each of the following:

I. $p \wedge q$

II. $\neg p \vee q$

III. $q \to p$

IV. $\neg q \to \neg p$

Q4 By developing a series of logical equivalences prove the followings:

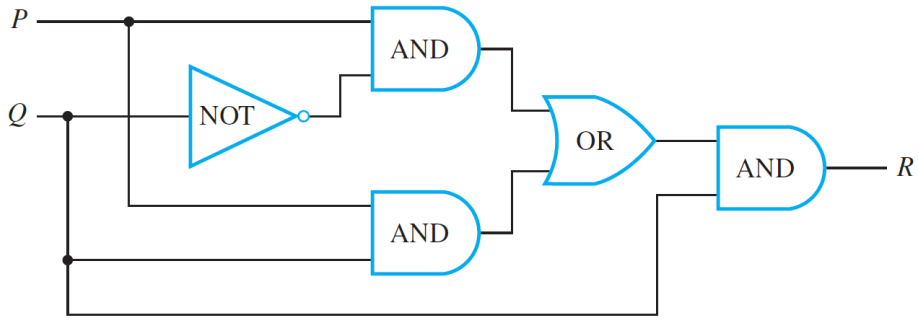
I. $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

II. $(p \wedge q) \to (p \vee q) \equiv T$

III. $\neg(\neg p \wedge q) \wedge (p \vee q) \equiv p$

IV. $[(p \vee q) \wedge (p \vee \neg q)] \vee q \equiv p \vee q$

Q5 Examine the two combinatorial circuits depicted in the figure. Show that the two logic circuit functions are identical when the output expressions are equal for the same inputs P and Q.



(a)

(b)

Q6 Let p, q, r, s denote the following statements:

- p: I finish writing my computer program before lunch.
- q: I shall play tennis in the afternoon.
- r: The sun is shining.
- s: The humidity is low.

Write the following in symbolic form:

I. If the sun is shining, I shall play tennis this afternoon.

II. Finishing the writing of my computer program before lunch is necessary for my playing tennis this afternoon.

III. Low humidity and sunshine are sufficient for me to play tennis this afternoon.

Q7 Determine the truth value of each of the following implications:

I. If $3 + 4 = 12$, then $3 + 2 = 6$.

II. If $3 + 3 = 6$, then $3 + 4 = 9$.

III. If Thomas Jefferson was the third president of the United States, then $2 + 3 = 5$.

Q8 Construct a truth table for each of the following compound statements, where p, q, r denote primitive statements:

I. $\neg(p \vee \neg q) \rightarrow \neg p$

II. $p \rightarrow (q \rightarrow r)$

III. $(p \to q) \to r$

IV. $(p \to q) \to (q \to p)$

V. $[p \wedge (p \to q)] \to q$

VI. $(p \wedge q) \to p$

VII. $q \leftrightarrow (\neg p \vee \neg q)$

VIII. $[(p \to q) \wedge (q \to r)] \to (p \to r)$

Q9 Which of the compound statements in **Q8** are tautologies?

Q10 Verify that $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology.

Q11 In the following program segment i, j, m and n are integer variables. The values of m and n are supplied by the user earlier in the execution of the total program.

```
for i := 1 to m do
  for j := 1 to n do
    if i ≠ j then
      print i + j
```

How many times is the **print** statement in the segment executed when

I. $m = 10, n = 10$

II. $m = 20, n = 20$

III. $m = 10, n = 20$

IV. $m = 20, n = 10$