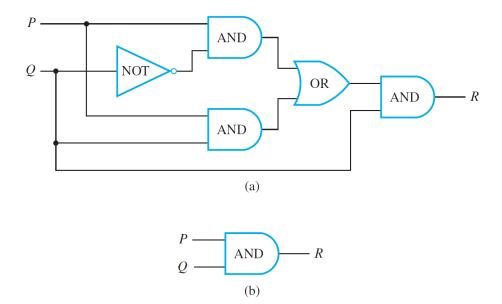
MA 180 - Discrete Mathematics

Exercise Set 01 (21/01/2025)

- Q1 Determine whether each of the following sentences is a statement:
 - I. In 2003 George W. Bush was the president of the United States.
- II. x + 3 is a positive integer.
- III. Fifteen is an even number.
- IV. If Jennifer is late for the party, then her cousin Zachary will be quite angry.
 - V. What time is it?
- VI. As of June 30, 2003, Christine Marie Evert had won the French Open a record seven times.
- Q2 Identify the primitive statements in Q1.
- Q3 Let p, q be primitive statements for which the implication $p \to q$ is false. Determine the truth values for each of the following:
 - I. $p \wedge q$
- II. $\neg p \lor q$
- III. $q \rightarrow p$
- IV. $\neg q \rightarrow \neg p$
- Q4 By developing a series of logical equivalences prove the followings:
 - I. $\neg (p \lor (\neg p \land q)) \equiv \neg p \land \neg q$
- II. $(p \land q) \to (p \lor q) \equiv T$
- III. $\neg(\neg p \land q) \land (p \lor q) \equiv p$
- IV. $[(p \lor q) \land (p \lor \neg q)] \lor q \equiv p \lor q$
- Q5 Examine the two combinatorial circuits depicted in the figure. Show that the two logic circuit functions are identical when the output expressions are equal for the same inputs P and Q.



Q6 Let p, q, r, s denote the following statements:

- p: I finish writing my computer program before lunch.
- q: I shall play tennis in the afternoon.
- r: The sun is shining.
- s: The humidity is low.

Write the following in symbolic form:

- I. If the sun is shining, I shall play tennis this afternoon.
- II. Finishing the writing of my computer program before lunch is necessary for my playing tennis this afternoon.
- III. Low humidity and sunshine are sufficient for me to play tennis this afternoon.

Q7 Determine the truth value of each of the following implications:

- I. If 3 + 4 = 12, then 3 + 2 = 6.
- II. If 3 + 3 = 6, then 3 + 4 = 9.
- III. If Thomas Jefferson was the third president of the United States, then 2 + 3 = 5.

Q8 Construct a truth table for each of the following compound statements, where p, q, r denote primitive statements:

I.
$$\neg (p \lor \neg q) \to \neg p$$

II.
$$p \to (q \to r)$$

III.
$$(p \to q) \to r$$

IV.
$$(p \rightarrow q) \rightarrow (q \rightarrow p)$$

$$V. [p \land (p \to q)] \to q$$

VI.
$$(p \land q) \rightarrow p$$

VII.
$$q \leftrightarrow (\neg p \lor \neg q)$$

VIII.
$$[(p \to q) \land (q \to r)] \to (p \to r)$$

Q9 Which of the compound statements in Q8 are tautologies?

Q10 Verify that $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ is a tautology.

Q11 In the following program segment i, j, m and n are integer variables. The values of m and n are supplied by the user earlier in the execution of the total program.

for
$$i := 1$$
 to m do
for $j := 1$ to n do
if $i \neq j$ then
print $i + j$

How many times is the **print** statement in the segment executed when

I.
$$m = 10, n = 10$$

II.
$$m = 20, n = 20$$

III.
$$m = 10, n = 20$$

IV.
$$m = 20, n = 10$$