

### Problems

7.1

Vapor pressure equations (Table 7.2):

$$n\text{-Hexane: } P_A^{\circ} = 15.9155 - \frac{2738.42}{\theta + 226.2} ; \theta \text{ in } ^\circ\text{C}; \text{boiling point} = 68.8^\circ\text{C}$$

$$n\text{-Octane: } P_B^{\circ} = 15.9635 - \frac{3128.75}{\theta + 209.35} ; P_A^{\circ} \text{ and } P_B^{\circ} \text{ in mm Hg.}$$

Boiling pt = 125.7 °C

(a) Construction of  $T-x-y$  and  $x-y^*$  diagrams.

Assume ideal solution behavior. The bubble point remains within  $T_A$  and  $T_B$  ( $T_A \leq T \leq T_B$ ). Select a set of values of  $\theta$  (or  $\theta$ ) calculate  $P_A^{\circ}$  and  $P_B^{\circ}$  and then calculate  $x$  and  $y^*$ .

Sample calculation:  $\theta = 90^\circ\text{C} \Rightarrow P_A^{\circ} = 1415, P_B^{\circ} = 252 \text{ mm Hg}$

$$\text{Total pressure} = 760 \text{ mm Hg} = P_A^{\circ}x_A + P_B^{\circ}(1-x_A) = 1415x_A + 252(1-x_A)$$

$$\Rightarrow x_A = 0.437; x_B = 1-x_A = 0.563.$$

$$y_A = \frac{P_A^{\circ}x_A}{P} = \frac{(1415)(0.437)}{760} = 0.813; y_B = 0.187$$

Calculations are repeated for a few other temperatures.

Calculated results and plots (with tie lines)

$\theta$	75 °C	80	85	90	95	100	105	110	115	120
$x$	0.794	0.656	0.538	0.437	0.350	0.274	0.207	0.148	0.0953	0.048
$y^*$	0.961	0.921	0.872	0.814	0.745	0.664	0.57	0.460	0.334	0.189

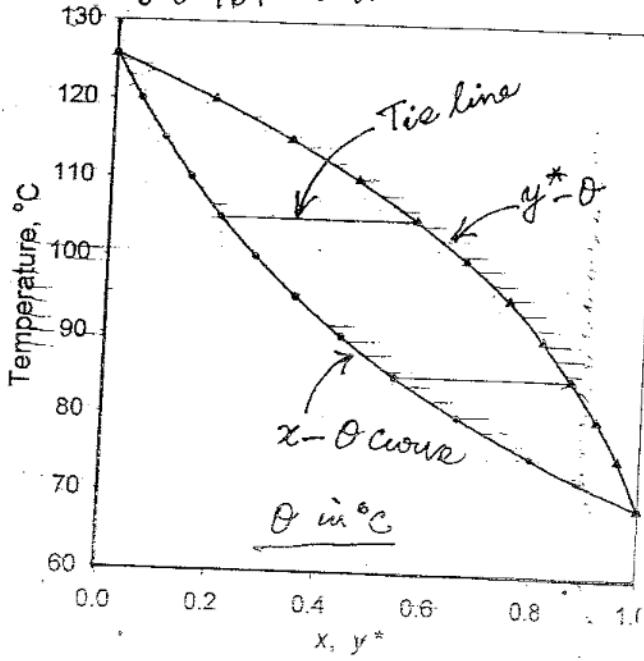


Fig Prob 7.1:  $T-x-y^*$  plots

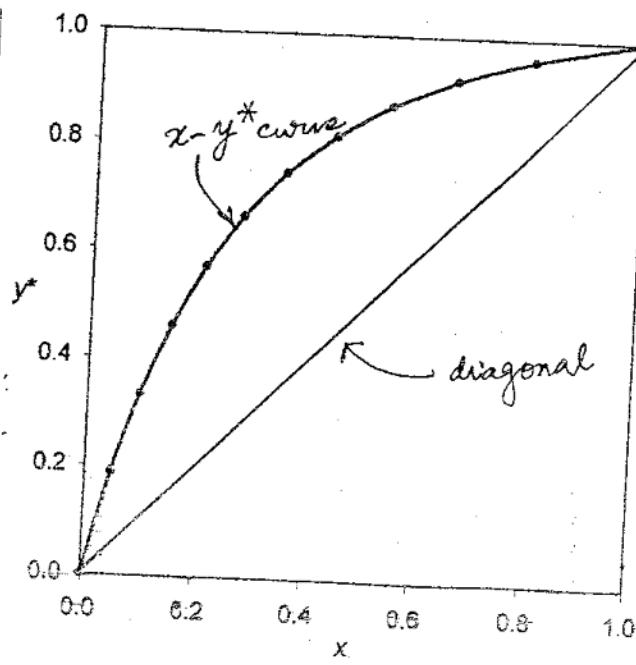


Fig Prob 7.1:  $x-y^*$  plot

(78)

(b) A liquid mixture containing 10 mole% n-hexane:

First calculate the bubble point [by trial or by interpolation, of the data calculated in part (a)]

Try  $\theta = 115.5^\circ\text{C}$ .  $P_A^{\infty} = 2638$  and  $P_B^{\infty} = 554 \text{ mm Hg}$

$$\text{Total pressure, } P = x_A P_A^{\infty} + (1-x_A) P_B^{\infty} = (0.1)(2638) + (0.9)(554) \\ = 762 \text{ mm Hg} \approx 1 \text{ atm.}$$

$$\text{Relative volatility, } \alpha_{AB} = \frac{P_A^{\infty}}{P_B^{\infty}} = 2638/554 = \underline{4.76}.$$

For a liquid containing 90 mole% n-hexane, try

$\theta = 71.7^\circ\text{C}$  as the bubble point  $\Rightarrow P_A^{\infty} = 831.4$ ;  $P_B^{\infty} = \underline{128 \text{ mm Hg}}$

$$P = (831.4)(0.9) + (128)(0.1) = \underline{761 \text{ mm Hg}}, \text{ acceptable.}$$

$$\alpha_{AB} = 831.4/128 = \underline{6.5}.$$

The relative volatility varies  $\Rightarrow$  the solution is not totally ideal.

(c) Initial vapor composition of a liquid ( $x_A=0.2$ ) at its bubble point can be calculated following the procedure given above.

(i) Take  $P = 1 \text{ atm} = 760 \text{ mm Hg}$ .

Trial temperature,  $\theta = 105.5^\circ\text{C} \Rightarrow P_A^{\infty} = 2121$ ,  $P_B^{\infty} = 420.7 \text{ mm}$

$$P = (2121)(0.2) + (420.7)(0.8) = 761 \text{ mm Hg} \text{ (acceptable)}$$

$$y_A = \frac{(2121)(0.2)}{761} = \underline{0.442}.$$

(ii) Take  $P = 2 \text{ atm} = 1520 \text{ mm Hg}$ .

Trial bubble point,  $\theta = 132.5^\circ\text{C} \Rightarrow P_A^{\infty} = 3943$ ,  $P_B^{\infty} = 920 \text{ mm Hg}$

$$P = (3943)(0.2) + (920)(0.8) = 1524 \text{ mm Hg} \approx \underline{2 \text{ atm}}$$

$$y_A = \frac{(3943)(0.2)}{1524} = \underline{0.517}$$

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(d) Consider a condensate of composition  $x_A = 0.517$   
 [the same as that of the vapor in d(ii)].

Trial bubble point,  $\theta = 111.7^\circ\text{C}$ .  $P_A^{20} = 2468$ ,  $P_B^{20} = 509 \text{ mm Hg}$

$$\text{Total pressure} = (2468)(0.517) + (509)(0.483) = 1522 \text{ mm Hg} \approx 2 \text{ atm.}$$

Composition of the initial vapor,

$$y_A^* = \frac{(2468)(0.517)}{1522} = 0.838 \quad (\text{See Note 2 below})$$

Note 1. If we consider a liquid of composition  $x_A$  and temperature  $\theta^\circ\text{C}$ , the total pressure exerted is

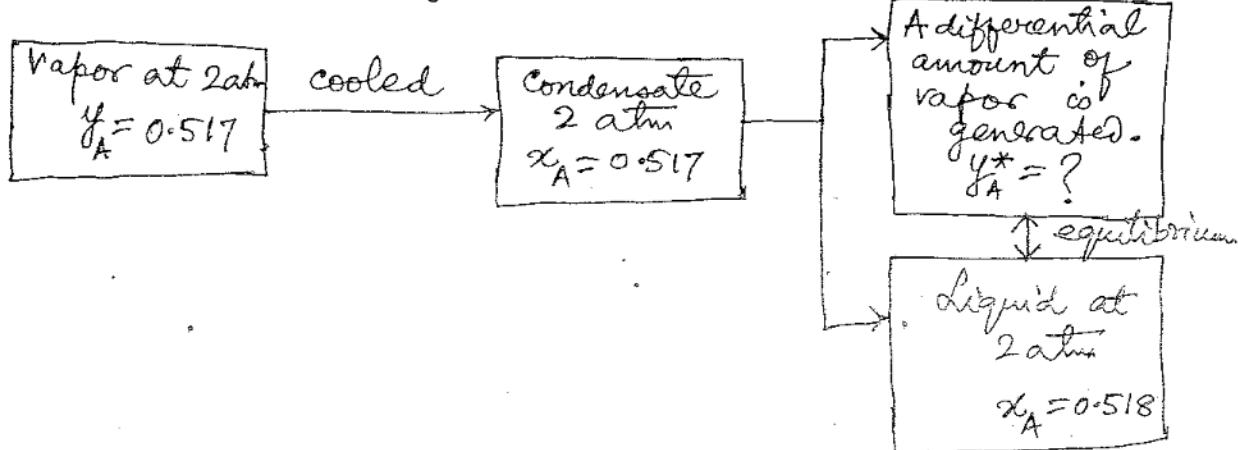
$$\begin{aligned} P &= x_A P_A^{20} + (1-x_A) P_B^{20} \\ &= x_A \cdot \exp \left[ 15.9155 - \frac{2738.42}{\theta + 226.2} \right] + (1-x_A) \exp \left[ 15.9635 - \frac{3128.75}{\theta + 209.85} \right] \end{aligned}$$

The above equation has three unknowns —  $P$ ,  $x_A$  and  $\theta$ .

If the temperature  $\theta$  and either  $P$  or  $x_A$  are given, the solution of the above equation for the unknown is straightforward. However, if  $P$  and  $x_A$  are given, the equation can be solved for  $\theta$  by trial or by using any numerical method such as Newton-Raphson method.

Note 2:

The condensation and vaporization processes in part (d) may be schematically represented as follows.



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- (7.2) Follow the procedures given in Section 7.1.6.

Calculation of the bubble point:

Average molecular weight of the mixture,

$$M_{av} = M_1 x_1 + M_2 x_2 + M_3 x_3 = [ \text{methanol-1; ethanol-2; } n\text{-propanol-3}]$$

$$(32)(0.3) + (46)(0.3) + (60)(0.4) = 47.4$$

This is closer to the molecular weight of ethanol. Select ethanol as the 'reference compound'. Boiling point of ethanol at 1 atm =  $78.3^\circ\text{C}$  = trial bubble point.

Calculate the relative volatilities of the other two compounds at the trial temperature.

$$\text{Vapor pressures at } 78.3^\circ\text{C} : P_1^{\infty} = 1260.2 ; P_2^{\infty} = 760 ; P_3^{\infty} = 349.3 \text{ mm Hg}$$

$$\alpha_{12} = 1260.2 / 760 = 1.658 ; \alpha_{32} = 0.46.$$

$$\text{Eq. (7.14b)} \rightarrow K_r \sum_i \alpha_{i2} x_i = K_r [(1.658)(0.3) + (1)(0.3) + (0.46)(0.4)] = 1.$$

$$\Rightarrow K_r \cdot (0.9814) = 1 \Rightarrow K_r = 1.019.$$

$$\text{Now put } K_r = P_2^{\infty} / P \Rightarrow P_2^{\infty} = (1.019)(760) \text{ mm Hg} = 774.4 \text{ mm Hg}$$

Boiling point of the reference compound (ethanol) at 774.4 mm Hg pressure =  $78.8^\circ\text{C}$ , which is the revised estimate of the bubble point of the mixture.

$$\text{At } 78.8^\circ\text{C}, P_1^{\infty} = 1283.2, P_3^{\infty} = 357 \text{ mm Hg}.$$

$$K_1 = P_1^{\infty} / P = 1283.2 / 760 = 1.657 ; K_2 = 1.019 ; K_3 = 0.47$$

$$\sum K_i x_i = (1.657)(0.3) + (1.019)(0.3) + (0.47)(0.4) = 0.9908.$$

This is a bit too low. A third cycle of calculation is needed.

$$\text{Take } K_r = K_2 / 0.9908 = 1.019 / 0.9908 = 1.0285.$$

$$P_2^{\infty} = (1.0285)(760) = 781.6 \text{ mm Hg} ; \text{ corresponding temperature } = \underline{79^\circ\text{C}}$$

$$P_1^{\infty} = 1292 ; P_3^{\infty} = 360 \text{ mm Hg. } K_1 = 1292 / 760 = 1.7 ; K_3 = 0.474.$$

$$\sum K_i x_i = (1.7)(0.3) + (1.0285)(0.3) + (0.474)(0.4) = 1.008.$$

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This value is pretty close to 1. However, a still better estimate of the bubble point of the mixture may be obtained by interpolation.

$$\text{Bubble point} = 79 - \frac{1.008 - 1}{1.008 - 0.9908} \cdot (79 - 78.8) = \underline{\underline{78.9^\circ\text{C}}}$$

Dew point of a vapor of composition  $y_1 = 0.3$ ,  $y_2 = 0.3$  and  $y_3 = 0.4$  at 1.5 atm total pressure.

Ethanol is chosen as the reference compound since its molecular weight is nearest to the average molecular weight.

Boiling point of ethanol at 1.5 atm (1140 mm Hg) total pressure = 86.95°C which is taken as the trial 'dew point'.

At  $86.95^\circ\text{C}$ ,  $P_1^o = 1702$  and  $P_3^o = 503.9$  mm Hg.

$$\alpha_{12} = 1702/1140 = 1.493; \alpha_{32} = 503.9/1140 = 0.442.$$

$$\text{Eq. (7.15b)} \rightarrow \frac{1}{K_2} \left( \frac{0.3}{1.493} + \frac{0.3}{1.0} + \frac{0.4}{0.442} \right) = 1 \Rightarrow K_2 = 1.406 = \frac{P_2^o}{P}$$

$$P_2^o = (1.406)(1140) = \underline{\underline{1602.8 \text{ mm Hg}}} \text{ Corresponding temperature} = 98.5^\circ\text{C}.$$

This is the second approximation to the dew point.

$$\text{At } 98.5^\circ\text{C}, P_1^o = 2479.3, P_3^o = 796 \text{ mm Hg.}$$

Calculate the K-values at  $98.5^\circ\text{C}$ .

$$K_1 = 2479.3/1140 = 2.175; K_2 = 1602.8/1140 = 1.405; K_3 = 0.698.$$

$$\sum_i (y_i/K_i) = 0.3/2.175 + 0.3/1.405 + 0.4/0.698 = \underline{\underline{0.9245}}.$$

This is much smaller than 1. Try next cycle of calculation.

$$\alpha_{12} = 2479.3/1602.8 = 1.547; \alpha_{32} = 796/1602.8 = 0.4969.$$

$$K_2 = K_2 = \sum_i \frac{y_i}{\alpha_{i2}} = \frac{0.3}{1.547} + \frac{0.3}{1.0} + \frac{0.4}{0.4969} = \underline{\underline{1.3}}.$$

$$K_2 = P_2^o/P \Rightarrow P_2^o = (1.3)(1140) = 1482 \text{ mm Hg} \Rightarrow \underline{\underline{\text{B.P.} = 94.2^\circ\text{C}}}.$$

Take this as the next revised estimate of the dew point.

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At  $94.2^\circ\text{C}$ ,  $P_1^v = 2162$  and  $P_3^v = 674 \text{ mm Hg}$ .

$$K_1 = 2162/1140 = 1.896; K_2 = 1482/1140 = 1.3; K_3 = 674/1140 = 0.591.$$

$$\sum_i \frac{y_i}{K_i} = \frac{0.3}{1.896} + \frac{0.3}{1.3} + \frac{0.4}{0.591} = 1.066 \rightarrow \text{not acceptable.}$$

Let us do another cycle of calculation:

$$\alpha_{12} = 2162/1482 = 1.46; \alpha_{32} = 674/1482 = 0.455$$

$$K_2 = \frac{0.3}{1.46} + \frac{0.3}{1} + \frac{0.4}{0.455} = 1.3846.$$

$$P_2^v = (1.3846)(1140) = 1578.4; \text{ corresponding boiling point} = 96^\circ\text{C}$$

$$\text{At } 96^\circ\text{C}, P_1^v = 2290; P_3^v = 723 \text{ mm Hg.}$$

$$K_1 = 2290/1140 = 2.009; K_2 = 1578.4/1140 = 1.3846; K_3 = \frac{723}{1140} = 0.634.$$

$$\sum_i \frac{y_i}{K_i} = \frac{0.3}{2.009} + \frac{0.3}{1.3846} + \frac{0.4}{0.634} = 0.9969 \approx 1$$

This value is reasonable. Required dewpoint = 96°C.

(7.4) Use Eq(7.30) to determine the distillate composition for different fractional vaporization. Given:  $\underline{\alpha=2.2}; \underline{z_F=0.4}$

$$-\frac{W}{D} = \frac{x_D - z_F}{x_W - z_F} \Rightarrow \frac{W}{D}(0.4 - x_W) + 0.4 = x_D = \frac{2.2x_W}{1 + 1.2x_W}$$

Solve the above equation for different values of W/D

Fraction vaporized, $\frac{D}{W+D}$	$\frac{W}{D}$	$x_W$	$x_D$
0.1	9.0	0.3806	0.575
0.2	4.0	0.361	0.5547
0.4	1.5	0.3243	0.5136
0.6	0.6667	0.29	0.4733
0.8	0.25	0.259	0.435
0.9	0.2454	0.2454	0.47

The lowest bottom composition occurs when only a differential amount of liquid remains — i.e.  $x_D \approx 0.4$ . Corresponding  $x_W = 0.232 \Rightarrow$  the bottom composition cannot be 10 mol% ethanol.

(7.5) Two-stage flash distillation

Stage 1: 60 mole% of the feed is vaporized  $\Rightarrow \frac{W_1}{D_1} = \frac{0.4}{0.6} = \frac{2}{3}$ .

$$Eq(7.30) \rightarrow \frac{2}{3}(0.4 - x_{w_1}) + 0.4 = \frac{2 \cdot 2 x_{w_1}}{1 + 1.2 x_{w_1}} \quad z_{F_1} = 0.4 \\ \alpha = 2.2$$

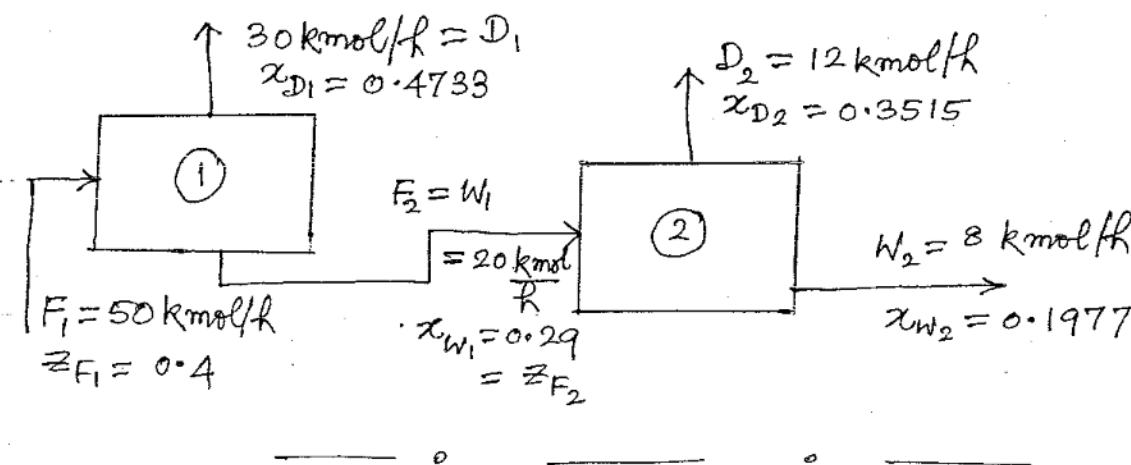
$$\Rightarrow x_{w_1} = 0.29 = \text{liquid composition}; \quad x_{D_1} = 0.4733.$$

Feed,  $F_1 = 50 \text{ kmol/h}$ ; distillate,  $D_1 = (50)(0.6) = 30 \text{ kmol/h}$   
 $W_1 = 20 \text{ kmol/h}$ .

Stage 2: 60 mole% of the bottom liquid ( $z_{F_2} = x_{w_1} = 0.29$ ) is vaporized in the second flash drum.

$$\frac{2}{3}(0.29 - x_{w_2}) + 0.29 = \frac{2 \cdot 2 x_{w_2}}{1 + 1.2 x_{w_2}} \Rightarrow x_{w_2} = 0.1977; \quad x_{D_2} = 0.3515.$$

$$D_2 = (20)(0.6) = 12 \text{ kmol/h}; \quad W_2 = 8 \text{ kmol/h}.$$



(7.6) Since the flowrates and concentrations are given in kg and mass%, respectively, it is convenient to use the equilibrium data after conversion to mass fractions,  $w_L$  and  $w_V$ , of acetone.

Molecular weight: acetone = 58; phenol = 94.

Sample calculations:

$$x = 0.01 \Rightarrow w_L = \frac{(0.01)(58)}{(0.01)(58) + (0.99)(94)} = 0.0062$$

$$y = 0.67 \Rightarrow w_V = \frac{(0.67)(58)}{(0.67)(58) + (0.33)(94)} = 0.556$$

## Calculated equilibrium data

(7-14)

$x$	0.01	0.04	0.1	0.2	0.3	0.4	0.5
$y$	0.67	0.776	0.852	0.91	0.94	0.962	0.98
$w_L$	0.0062	0.025	0.064	0.1336	0.209	0.291	0.3816
$w_V$	0.556	0.681	0.78	0.862	0.906	0.94	0.968

The equilibrium data are plotted. Graphical solution  
to be done

### First Stage flash

$D_1$  = mass of distillate (kg);  $w_1$  = mass of bottom product;

$z_{F_1}$  = feed concentration (mass fraction)

$w_{D_1}$ ,  $w_{w_1}$  = distillate and bottom concentration, mass fraction.

$F_1 = 3000 \text{ kg/h}$ , 50% flashed in stage 1

$$\Rightarrow D_1 = 1500 \text{ kg/h}; w_1 = 1500 \text{ kg/h}; z_{F_1} = 0.4 \text{ mass fraction.}$$

$$-\frac{w_1}{D_1} = \frac{w_{D_1} - z_{F_1}}{w_{w_1} - z_{F_1}} \Rightarrow -\frac{1500}{1500} = \frac{w_{D_1} - 0.4}{w_{w_1} - 0.4} = -1$$

$w_{D_1}$  and  $w_{w_1}$  are in equilibrium — to be determined graphically.

Locate the point  $(z_F, z_F) \rightarrow (0.4, 0.4)$  on the  $w_L-w_V$  diagram. Draw a line of slope  $-w_1/D_1 = -1$ , which is the first stage operating line that should meet the equilibrium line at  $(w_{w_1}, w_{D_1}) \rightarrow (0.0502, 0.75)$ .

### Second Stage flash

$F'_2 = w_1 = 1500 \text{ kg/h}$ ;  $z_{F_2} = w_{w_1} = 0.0502$ .

Locate the point  $(z_{F_2}, z_{F_2}) \rightarrow (0.0502, 0.0502)$ . The bottom product from this stage must not contain more than 1% acetone, i.e.  $w_{w_2} = 0.01$ .

Locate the point on the equilibrium line where

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$w_{w_2} = 0.01$ . Corresponding equilibrium vapor composition,  
 $w_{D_2} = 0.593$  (from the diagram).

The fraction vaporized can be determined algebraically or graphically.

$$-\frac{W_2}{D_2} = \frac{0.593 - 0.0502}{0.01 - 0.0502}$$

$$\text{and } W_2 + D_2 = 1500$$

$$\Rightarrow W_2 = 1396.6 \text{ kg/h}; \\ D_2 = 103.4 \text{ kg/h}$$

Fraction of the second stage feed to be vaporized

$$= \frac{D_2}{W_2 + D_2} = \frac{103.4}{1500} = 6.9\%$$

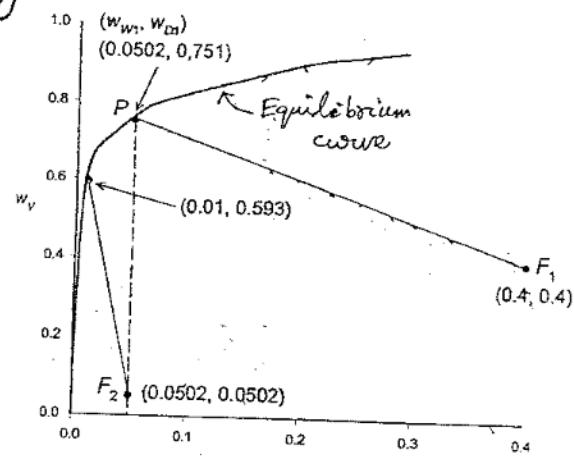


Figure Prob 7.6

(7.7) Feed,  $F = 1 \text{ kmol/h}$ ;  $Z_F = 0.5$ ;  $D = 0.5 \text{ kmol/h}$ ;  $W = 0.5 \frac{\text{kmol}}{\text{h}}$ .

Material balance:  $F \cdot Z_F = D x_D + W x_W$  ] solve to get

$$\text{Equilibrium relation: } \frac{x_D(1-x_D)}{x_W/(1-x_W)} = \alpha = 3.0 \quad x_D = 0.634 \\ x_W = 0.366$$

Products: Distillate,  $D = 0.5$ ,  $x_D = 0.634$ , vapor at  $80^\circ\text{C}$ .

Bottom product,  $W = 0.5$ ,  $x_W = 0.366$ , liquid at  $80^\circ\text{C}$ .

Feed temperature =  $40^\circ\text{C}$ ; ideal solution  $\Rightarrow \Delta H_s = 0$

Take the reference temperature  $40^\circ\text{C}$ . (Heat of solution)

Enthalpy of the feed,  $H_F = 0$

Enthalpy of the vapor product (use Eq 7.19) at  $80^\circ\text{C}$

$$H_v = (0.634)[(80-40)(20) + 1250] + (1-0.634)[(80-40)(30) + 1700] \\ = \underline{2361.1 \text{ kcal/kmol}}$$

$$\text{Liquid} \rightarrow H_L = (0.366)(80-40)(20) + (1-0.366)(80-40)(30) \\ = \underline{1053.6 \text{ kcal/kmol}}$$

$$\text{Heat to be supplied, } Q = 0.5 H_v + 0.5 H_L - H_F$$

$$= (0.5)(2361.1) + (0.5)(1053.6) - 0 = \underline{1707.3 \text{ kcal/h}}$$

(7-16)

(7.8) Given:  $H_L = 9,000 + 1,000x$ ;  $H_v = 35,000 + 8,000y$  kJ/kmol

(a) Heat of condensation of saturated pure vapor of A

$$\text{Put } x=1, y=1 \Rightarrow (\Delta H_v)_A = H_v - H_L$$

$$= [35,000] + 8,000(1) - [9,000] + 1,000(1) = \underline{\underline{33,000 \text{ kJ/kmol}}}$$

For pure vapor of B ( $x=0, y=0$ ),  $(\Delta H_v)_B = \underline{\underline{26,000 \text{ kJ/kmol}}}$

(b) Consider a saturated liquid mixture with  $x=0.3$ .

$$\text{Enthalpy of the saturated liquid} = 9,000 + (1,000)(0.3) = 9,300 \text{ kJ/kmol}$$

Actual enthalpy = 8,500 kJ/kmol  $\Rightarrow$  the liquid is Subcooled.

Now consider a saturated vapor with  $y=0.5$ .

$$\text{Enthalpy} = 35,000 + (8,000)(0.5) = 39,000 \text{ kJ/kmol.}$$

$$\text{Actual enthalpy of the vapor} = 40,000 \text{ kJ/kmol}$$

$\Rightarrow$  The vapor is Superheated.

(7.9) Use Eq(7.34)  $\rightarrow y_{D_i} = \frac{z_{F_i}(1+W/D)}{(1+W/K_i D)}$

$$\text{Given: } z_{F_1} = 0.38, z_{F_2} = 0.225, z_{F_3} = 0.395, W/D = 1.$$

$$y_{D_1} = \frac{(0.38)(1+1)}{1+(1/42)} = \underline{\underline{0.446}}. \text{ Similarly, } y_{D_2} = \underline{\underline{0.208}}; y_{D_3} = \underline{\underline{0.346}}.$$

$$\text{Liquid composition: } x_{W_1} = y_{D_1}/K_1 = 0.446/1.42 = \underline{\underline{0.314}}.$$

$$x_{W_1} = 0.208/0.86 = 0.242; x_{W_3} = \underline{\underline{0.444}}.$$

- (7.10) Bubble point of a homogeneous mixture of A and B at 1.5 atm total pressure (assume ideal solution):

$$\text{Total pressure, } P = x_A P_A^{\circ} + x_B P_B^{\circ} = 0.5 (P_A^{\circ} + P_B^{\circ}) \\ = 0.5 (1.75 P_B^{\circ} + P_B^{\circ}) = 1.375 P_B^{\circ} = 1.5 \text{ bar} = \frac{(1.5)(760)}{1.013} = 1125.4 \text{ mm Hg}$$

$$\Rightarrow P_B^{\circ} = 1125.4 / 1.375 = 818.45 \text{ mm Hg.}$$

$$\text{Given: } \ln P_B^{\circ} (\text{mm Hg}) = 14.243 - 2570 / (\theta, {}^{\circ}\text{C} + 232.5)$$

For  $P_B^{\circ} = 1125.4 \text{ mm Hg}$ ,  $\theta = 108.8 {}^{\circ}\text{C}$  = required bubble point.

Bubble point of a heterogeneous mixture of the above solution and water at 1.5 bar total pressure:

$$P = 1125.4 \text{ mm Hg} = P_w^{\circ} + 1.375 P_B^{\circ}; \ln P_w^{\circ} = 18.5882 - \frac{3984.92}{\theta {}^{\circ}\text{C} + 233.43}$$

Solving the above equation by trial,  $\theta = 95.5 {}^{\circ}\text{C}$

- (7.11) (a) Use Eq. (7.51b)  $\rightarrow \ln \frac{Fx_F}{Wx_W} = \alpha \ln \frac{F(1-x_F)}{W(1-x_W)}$ .

75 mole% of the benzene leaves in the vapor product

$$\Rightarrow D \cdot x_D = (0.75) Fx_F \Rightarrow Wx_W = (0.25) \cdot Fx_F \Rightarrow \frac{Fx_F}{Wx_W} = 4.0$$

$$\ln 4 = 2.51 \ln \left[ \frac{F}{W} \cdot \frac{(1-x_F)}{(1-x_W)} \right] = 2.51 \ln \left[ \frac{4x_W}{x_F} \cdot \frac{(1-x_F)}{(1-x_W)} \right] \text{ Since } \alpha = 2.51$$

Put  $x_F = 0.65$  to get  $x_W = 0.447$ .

$$\frac{F}{W} = 4 \cdot \frac{x_W}{x_F} = \frac{(4)(0.447)}{0.65} = 2.751. \quad F = 10 \text{ kmol}$$

$$\Rightarrow W = 3.635 \text{ kmol}; \quad D = 6.365 = \text{moles of distillate produced.}$$

- (b) The stillpot contains 35 mole% benzene  $\Rightarrow x_W = 0.35$ .

Put the values of the different quantities in Eq (7.51a).

$$\ln \frac{F}{W} = \frac{1}{\alpha-1} \ln \frac{x_F(1-x_W)}{x_W(1-x_F)} + \ln \frac{1-x_W}{1-x_F}$$

$$\Rightarrow \ln \frac{10}{W} = \frac{1.0}{2.51-1} \ln \frac{0.65(1-0.35)}{0.35(1-0.65)} + \ln \frac{1-0.35}{1-0.65} \Rightarrow W = 2.359 \text{ kmol}$$

$$D = 7.641 \text{ kmol}$$

Prob 7.11 contd.

7.18

(c) 50 mole% of the feed is vaporized  $\Rightarrow F/W = 2.0$ .

Put  $F = 10$ ,  $W = 5$ ,  $x_F = 0.65$  and  $\alpha = 2.51$  in Eq(7.51a)  
to get  $x_w = 0.516$ , and  $D = 5 \text{ kmol}$ .

(d) Again use Eq(7.51b)  $\rightarrow \ln \frac{F x_F}{W x_w} = \alpha \ln \frac{F(1-x_F)}{W(1-x_w)}$

The distillate has 75 mole% benzene.

Material balance  $\rightarrow W x_w = F x_F - D x_D = (10)(0.65) - 0.75D$

$$\Rightarrow \ln \frac{(10)(0.65)}{6.5 - 0.75D} = 2.51 \ln \frac{10(1-0.65)}{W - (6.5 - 0.75D)} = 2.51 \ln \frac{3.5}{(10-D) - (6.5)}$$

Solve to get  $D = 7.29 \text{ kmol}$ ;  $W = 2.71 \text{ kmol}$ ;  $x_w = 0.381$ .

7.12

$F = 70 \text{ kmol/h}$ ;  $z_F = 0.85$ .

(a) 70 mole% of the vapor is condensed.

$$W = (70)(0.7) = 49 \text{ kmol/h}; D = 21 \text{ kmol/h}$$

Use Eq (7.30) to calculate  $x_D$  and  $x_w$  which are at equilibrium.

$$-\frac{W}{D} = \frac{x_D - z_F}{z_F - x_w} \Rightarrow \frac{x_D - 0.85}{0.85 - x_w} = -\frac{49}{21} = -2.33$$

Draw the equilibrium line. Locate the point  $(z_F, z_F) - E(0.85, 0.85)$  on the  $x-y$  plane and draw a line of slope -2.33 to meet the equilibrium line at the point P. The coordinate of P is  $(0.815, 0.93)$  from the plot  $\Rightarrow x_w = 0.815$ ;  $x_D = 0.93$ . Composition of the uncondensed vapor,  $x_D = 0.93$ .

(b) The uncondensed vapor leaving the partial condenser has 91.5 mole% methanol  $\Rightarrow x_D = 0.915$ .

(7.12) contd...

7.17

Locate the point  $P'$  for  $x_0 = 0.915$ . Join  $FP'$ . Slope of the line  $FP'$  is  $-0.929 = -W/D$ .

$$W+D = 70, \text{ and } W = 0.929 D.$$

$$\text{Solving } W = 36.3 \text{ kmol/h and } D = 33.7 \text{ kmol/h.}$$

$$\text{Condensate rate} = 36.3 \text{ kmol/h}$$

Note Equilibrium data from Example 7.15 have been used.

$x$	0.7	0.8	0.9	0.95	1.0
$y$	0.87	0.915	0.958	0.979	1.0

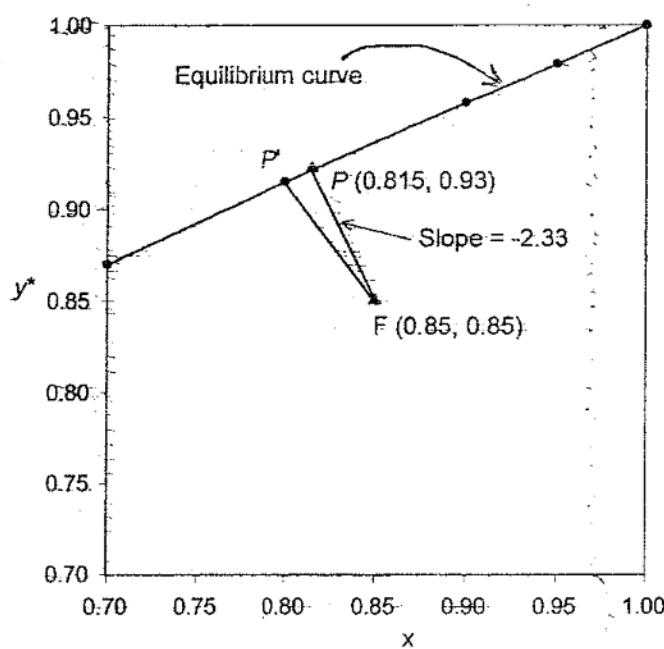
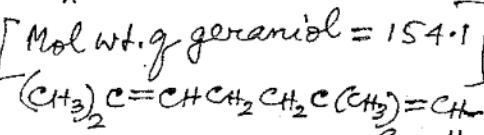


Fig: Prob 7.12.

(7.13) Use Eq. (7.42)  $\rightarrow m_A = m_B \cdot \frac{EP_A^v}{P - EP_A^v}$

$$m_A = (2 \text{ kg})(0.95)/154.1 = 0.01233 \quad [\text{Mol wt. g geraniol} = 154.1]$$

$$P_A^v = 8.65 \text{ mm Hg} = 0.0115 \text{ bar}$$



$$\text{Vaporization efficiency, } E = 0.9. \quad P - EP_A^v = P_B^v = 1.34 \text{ bar}$$

$$\text{Moles of steam required, } m_B = (0.01233) \cdot \frac{1.34}{(0.90)(0.0115)} = 1.596 \text{ kmol}$$

$$\text{Steam rate} = 50 \text{ kg/h} \Rightarrow \text{time required} = \frac{28.73}{50} \text{ h} = 0.5746 \text{ h} = 34.5 \text{ min}$$

$$= \frac{28.73}{50} \text{ h} = \underline{\underline{34.5 \text{ min}}}$$

(7.14) Initial moles of the essential oil =  $(20)(0.4)/170 = 0.04706$  7.20  
 Moles of impurity (mol wt = 280),  $m_c = (20)(0.6)/280 = 0.04255 \text{ kmol}$   
 50% of the oil is recovered; moles oil left =  $0.04706/2 = 0.02353$

Vapor pressure of water at  $108^\circ\text{C}$  (see Prob 7.13),  $= m_{Af}$

$$\underline{P_B^v = 1.34 \text{ bar}}; \underline{P_A^v = 6.5 \text{ mm Hg} = 8.66 \times 10^{-3} \text{ bar}}$$

Vaporization efficiency,  $E = 0.85$ .

$$P = EP_A^v + P_B^v = 1.347 \text{ bar.}$$

Use Eq. (7.45) to calculate the distillation time.

$$\begin{aligned} \dot{m}_B t &= \left( \frac{P}{EP_A^v} - 1 \right) (m_{Ai} - m_{Af}) + \frac{P m_c}{EP_A^v} \cdot \ln \frac{m_{Ai}}{m_{Af}} \\ &= \left[ \frac{1.347}{(0.85)(8.66 \times 10^{-3})} - 1 \right] [0.04706 - 0.02353] \\ &\quad + \frac{(1.347)(0.04255)}{(0.85)(8.66 \times 10^{-3})} \ln 2 = 9.68 \text{ kmol} = 174.2 \text{ kg} \end{aligned}$$

If the steam rate is 50 kg/h, time =  $\frac{174.2}{50} = 3.5 \text{ h}$

Rate of distillation when 50% of the oil is vaporized:

Use Eq. (7.44).

$$-\frac{dm_A}{dt} = \dot{m}_B \cdot \frac{Ex_A P_A^v}{P - Ex_A P_A^v}; \quad x_A = \frac{0.02353}{0.02353 + 0.04255} = 0.356$$

$$\dot{m}_B = 50 \text{ kg/h} = 2.778 \text{ kmol/h} \quad (\text{for } \dot{m}_B = 50 \text{ kg/h mentioned in the question})$$

$$\begin{aligned} -\frac{dm_A}{dt} &= (2.778) \cdot \frac{(0.85)(0.356)(8.66 \times 10^{-3})}{1.34} \\ &= 5.432 \times 10^{-3} \text{ kmol/h} = (5.432 \times 10^{-3})(170) \\ &= 0.923 \text{ kg/h} \end{aligned}$$

7.15

7.21

Equilibrium relation:  $\alpha = 2.8 \Rightarrow y^* = \frac{2.8x}{1+1.8x} \quad \text{--- (i)}$

$x$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$y^*$	0	0.237	0.412	0.545	0.651	0.737	0.808	0.867	0.918	0.962	1.0

Given:  $z_F = 0.45$ ; the feed is 50% vapor  $\Rightarrow q = 0.5$ ;  $x_D = 0.96$ ,

Slope of the feed line  $= \frac{q}{q-1} = \frac{0.5}{0.5-1} = -1. \quad x_w = 0.95$

Equation of the feed line:

It passes through  $(z_F, z_F) \rightarrow (0.45, 0.45)$  and has a slope  $= -1$ .

Equation  $\rightarrow y = -x + c \Rightarrow 0.45 = -0.45 + c \Rightarrow c = 0.9$

Feed line  $\rightarrow y = 0.9 - x \quad \text{--- (ii)}$

Determination of the minimum reflux ratio:

It can be done graphically or algebraically since the equilibrium data are given in the form of an equation.

Algebraic approach: The intersection of the feed line and the equilibrium curve is obtained by solving Eqs. (i) and (ii).

$$y = \frac{2.8x}{1+1.8x} = 0.9 - x \Rightarrow x = 0.325, y = 0.574.$$

The operating line for minimum reflux should pass through  $(0.325, 0.574)$  and  $(x_D, x_D) \rightarrow (0.96, 0.95)$ .

$$\text{Equation} \rightarrow \frac{y - 0.574}{0.95 - 0.574} = \frac{x - 0.325}{0.96 - 0.325} \Rightarrow y = 0.608x + 0.376$$

$$\text{Slope of this line is } 0.608 = R_m/R_m + 1 \Rightarrow R_m = 1.551$$

The minimum reflux ratio is  $R_m = 1.551$

Determination of the number of real trays for a reflux ratio,  $R = 1.5 R_m = (1.5)(1.551) = 2.326$ .

7.15 contd...

(for the rectifying section)

(7.22)

Actual operating line,  $y = \frac{R}{R+1}x + \frac{x_D}{R+1}$ , Eq. (7.73)

$$\Rightarrow y = \frac{2.326}{1+2.326} \cdot x + \frac{0.96}{1+2.326} = 0.7x + 0.289.$$

Draw the equilibrium line. Locate the point  $(x_D, x_D) \rightarrow D(0.96, 0.96)$  on the diagonal. Draw the rectifying section operating line through  $(0.96, 0.96)$  with an intercept of 0.289 on the Y-axis; draw the feed line. Locate the point  $W(x_w, x_w) \rightarrow (0.95, 0.95)$  and draw the stripping section operating line through  $W$  and the intersection of the feed line and the rectifying section operating line.

Draw steps between the equilibrium line and the operating lines. Number of ideal stages = 10

Feed plate  $\rightarrow$  6th

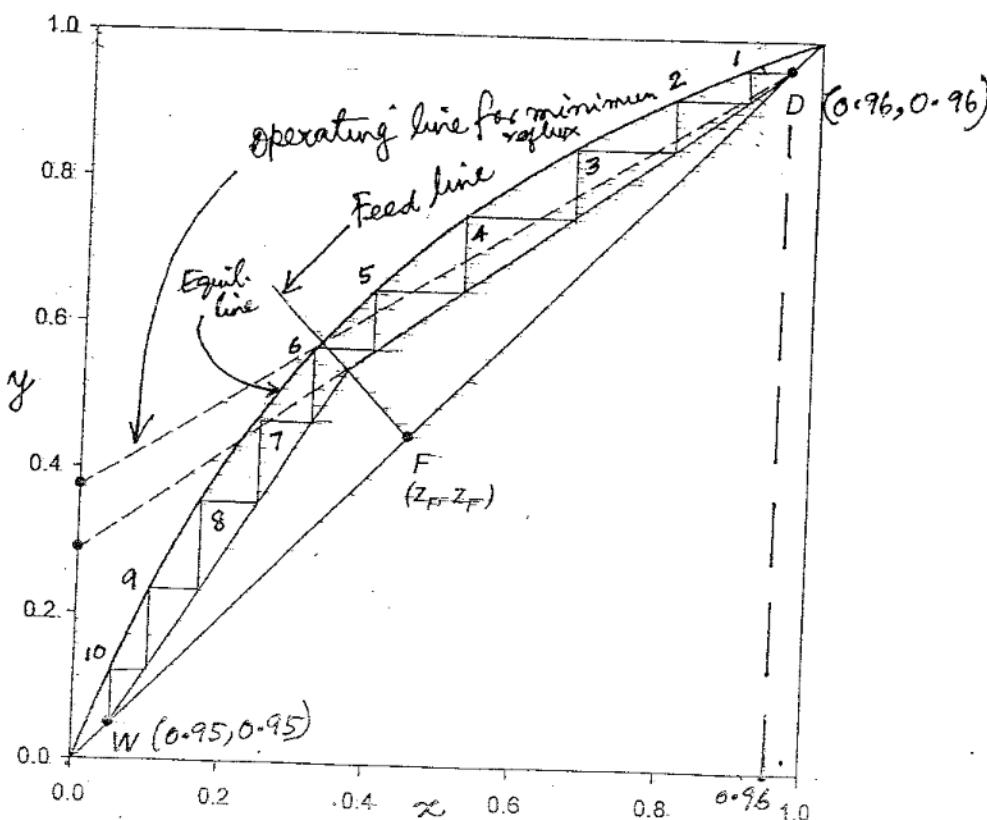


Fig. Prob 7.15. Construction of stages.

(7.16) The rate of product withdrawal ( $D$  and  $W$ ) will be necessary in the calculations. These can be obtained from material balance.

$$F = 40 \text{ kmol/h}; z_F = 0.55; x_0 = 0.975.$$

2% of the dimethylamine leaves at the bottom

$$\Rightarrow Wx_w = (0.02) \cdot Fz_F \Rightarrow Wx_w = (0.02)(40)(0.55) = 0.44$$

Dimethylamine balance:  $Fz_F = Dx_D + Wx_w$ .

$$\Rightarrow (40)(0.55) = D(0.975) + 0.44 \Rightarrow D = 22.1 \text{ kmol/h}$$

$$x_w = \frac{Wx_w}{W} = \frac{0.44}{17.9} = \underline{\underline{0.0246}} \quad W = 17.9 \text{ kmol/h}$$

### (a) Minimum reflux ratio

Locate the points  $F(z_F, z_F) \rightarrow (0.55, 0.55)$ ;  $D(x_D, x_D) \rightarrow (0.975, 0.975)$  on the  $x-y$  plane; draw the equilibrium curve. Since the feed is a saturated liquid, the feed line is a vertical line through  $F$ . The feed line intersects the equilibrium curve at the point  $P$ . Join  $DP$  and extend it to meet the  $y$ -axis at  $E_1$ .  $E_1 \rightarrow (0, 0.5)$ .

$$\Rightarrow \frac{x_D}{R_m + 1} = 0.5 \Rightarrow \frac{0.975}{R_m + 1} = 0.5 \Rightarrow R_m = 0.95 \\ \text{minimum reflux.}$$

### (b) Actual reflux ratio; $R = 1.4 R_m = (1.4)(0.95) = 1.33$ .

$$\frac{x_D}{R+1} = \frac{0.975}{1.33+1} = 0.418. \text{ Locate the point } E_2 \rightarrow (0, 0.418)$$

Join  $DE_2$  that meets the feed line at  $P'$ . Locate  $W(x_w, x_w) \rightarrow (0.0246, 0.0246)$  and join  $P'W$ .

$DP'$  is the rectifying section operating line and  $P'W$  is the stripping section operating line. Conduct stages between the equilibrium curve and the operating lines.

(7.24)

Prob (7.16) contd...

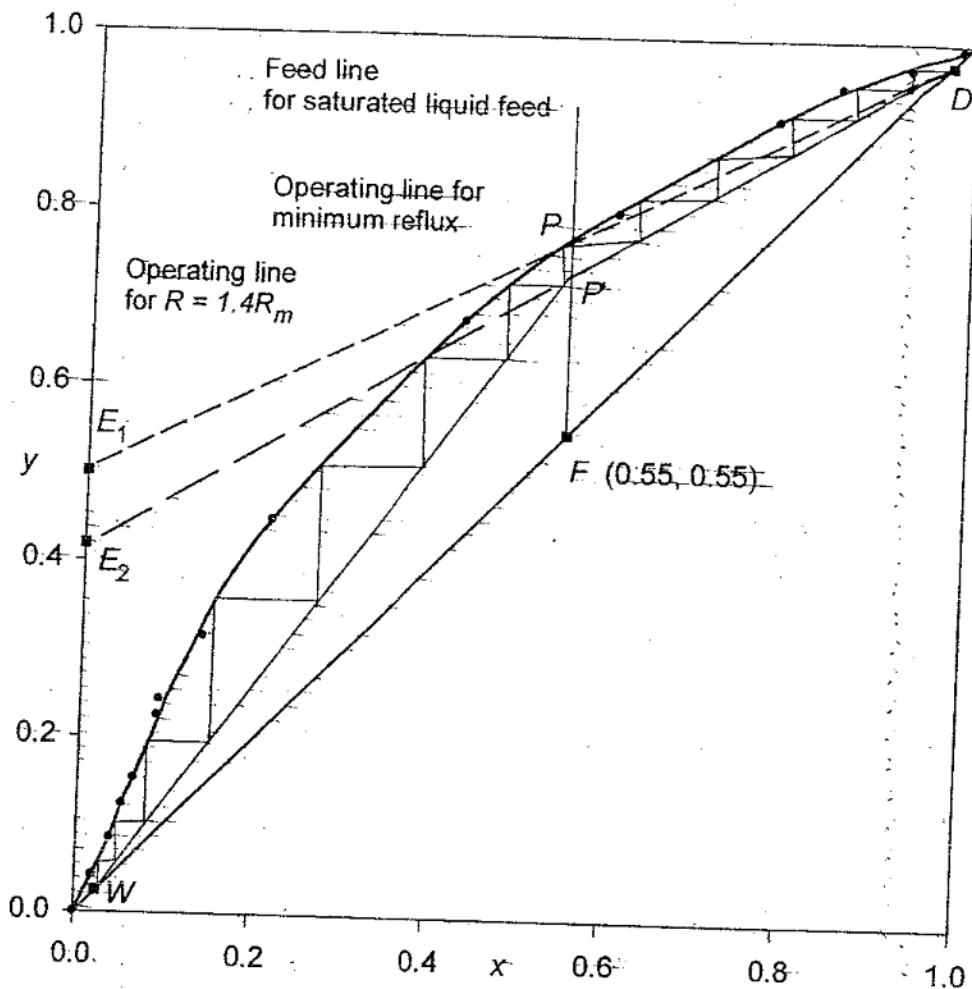


Fig. Prob 7.16: Graphical construction of stages.

The number of theoretical stages = 13.5; feed plate  $\rightarrow$  6

(c) Slopes of the operating lines

$$\text{Rectifying section: Slope} = \frac{R}{R+1} = \frac{1.33}{2.33} = \underline{\underline{0.571}}.$$

$$\text{Stripping section: Slope} = \bar{L}/\bar{V}.$$

$\bar{V} = V = D(1+R)$ , since the feed is a saturated liquid.

$$\bar{V} = (22.1)(1+1.33) = \underline{\underline{51.5}} \text{ kmol/h}$$

$$\bar{L} = L + F = R \cdot D + F = (1.33)(22.1) + 40 = \underline{\underline{69.4}} \text{ kmol/h}$$

$$\text{Slope} = \bar{L}/\bar{V} = 69.4/51.5 = \underline{\underline{1.347}}.$$

(d) Subcooled reflux [Prob(7.16) contd...]

Product rate =  $D$ ; external reflux ratio =  $R$ ; vapor flow rate to the condenser =  $V'$ ; vapor flow rate in the rectifying section =  $V$ . Vapor condensed at the top tray because of the cold reflux =  $V - V'$ .

$$\Rightarrow V - V' = RD/20 \Rightarrow V = V' + RD/20 = D(R+1) + RD/20$$

Liquid rate in the rectifying section,

$$L = RD + RD/20$$

Slope of the rectifying operating line,

$$\frac{L}{V} = \frac{RD + RD/20}{D(R+1) + RD/20} = \frac{(1+0.05)R}{(1+0.05) \cdot R + 1} = \frac{(1.05)(1.33)}{(1.05)(1.33)+1} = 0.5827.$$

Draw the rectifying section operating line through the point  $D$  with a slope 0.5827. The line meets the feed line at  $P$ . Join  $WP$ . Draw steps.

Number of theoretical stages = 12.8

Note

The number of plates in this case is less since the internal reflux ratio is higher.

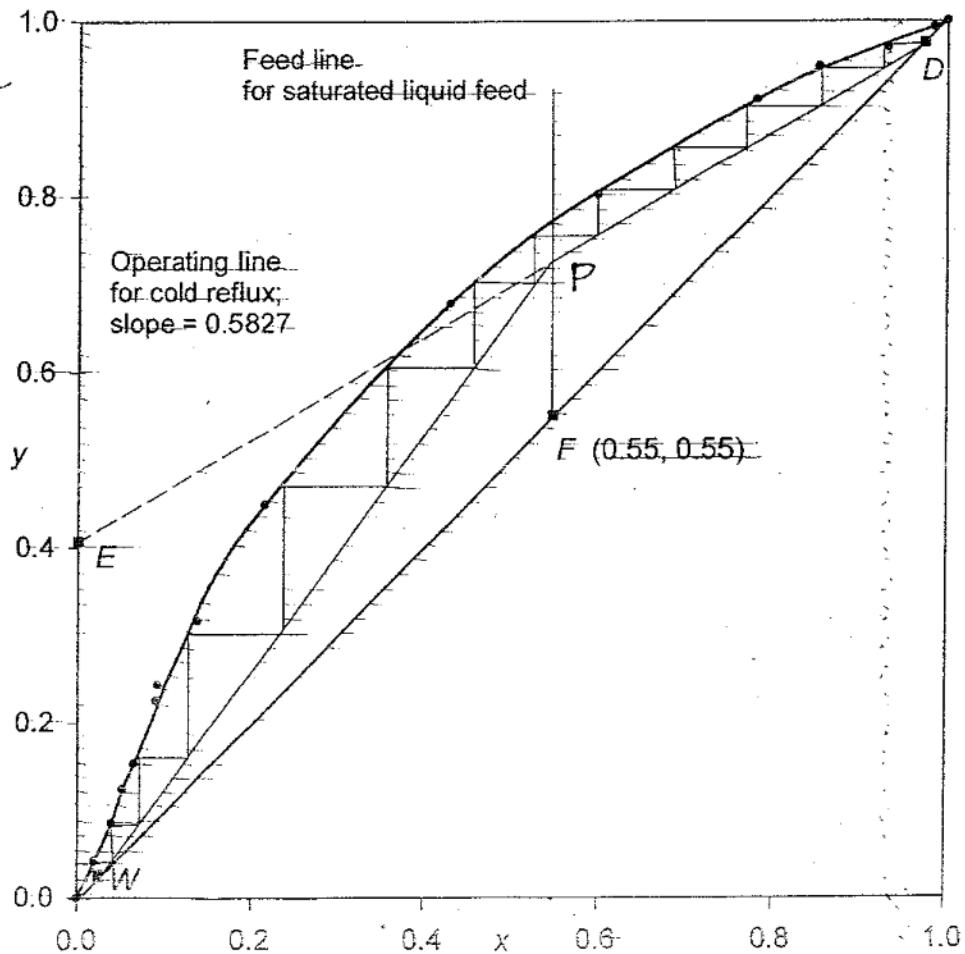
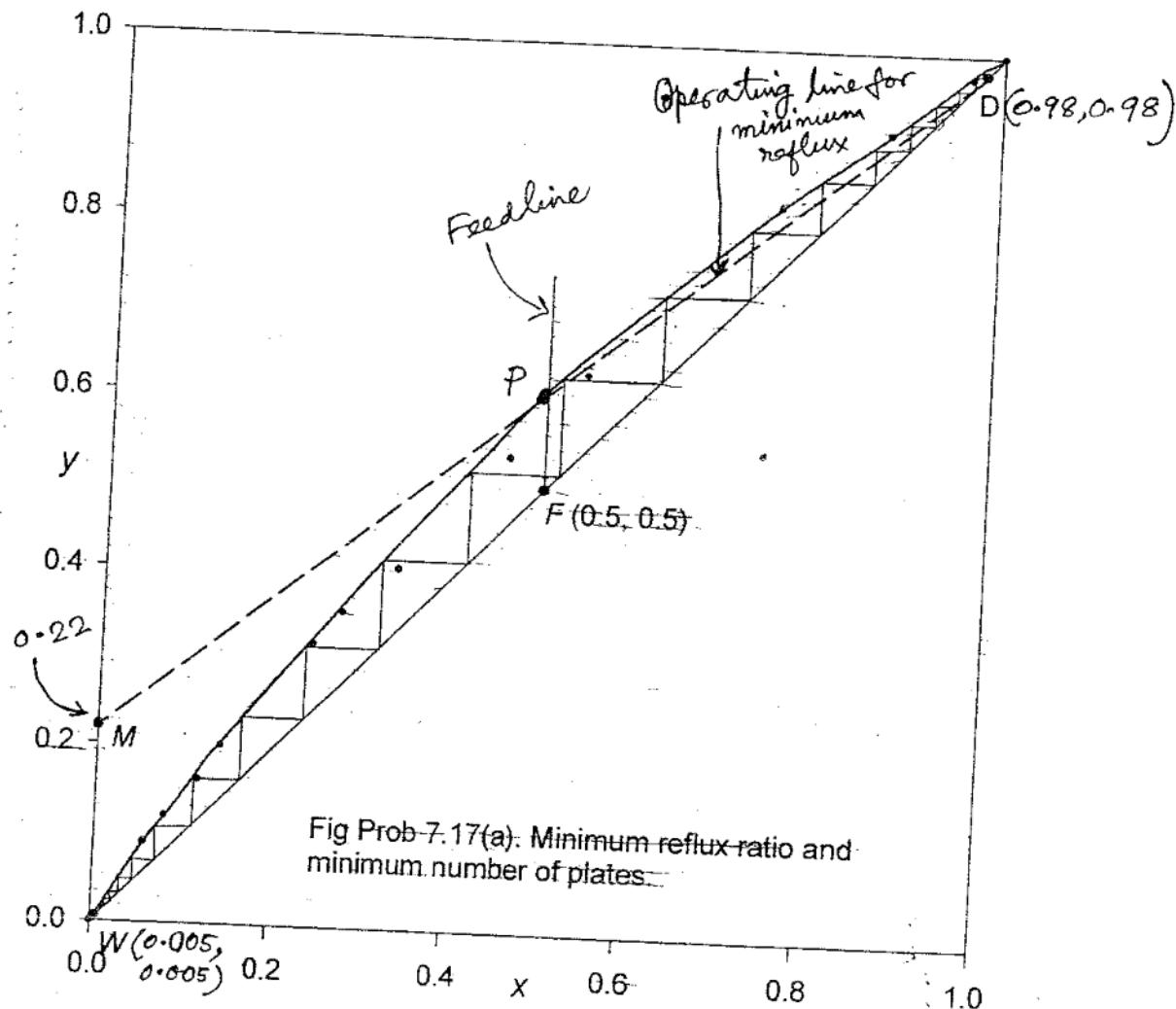


Fig. Prob. 7.16(b). Cold reflux - graphical construction of stages.

7.17  $F = 120 \text{ kmol/h}$ ;  $x_D = 0.98$ ;  $x_w = 0.005$ ;  $Z_f = 0.5$ .

(a) The minimum reflux ratio: Since the feed is a saturated liquid, the feed line is vertical. Draw the smoothed equilibrium curve (note that there is a considerable scatter in the data), locate the points  $D(0.98, 0.98)$ ,  $W(0.005, 0.005)$  and  $F(0.5, 0.5)$ . Draw the (vertical) feed line through  $F$  that meets the equil.



curve at  $P$  which is the pinch point. Join  $DP$  and extend it to meet the  $y$ -axis at  $M(0, 0.22)$ .

Then  $\frac{x_D}{R_m + 1} = 0.22 \Rightarrow \frac{0.98}{R_m + 1} = 0.22 \Rightarrow R_m = 3.45$

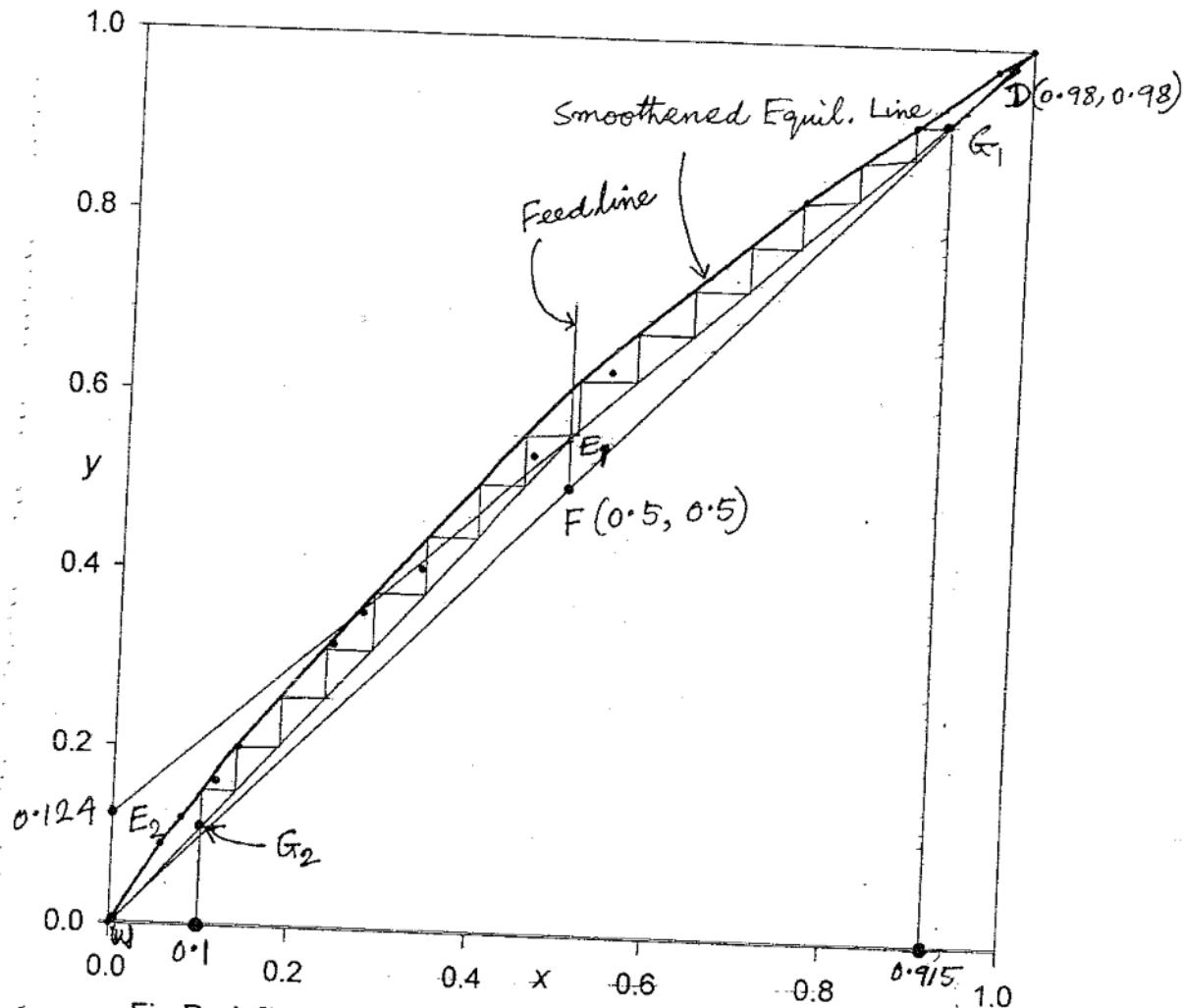
(b) The minimum number of trays (for total reflux) is obtained by drawing steps between the equilibrium curve and the diagonal. Number of trays = 30

[see Fig Prob 7.17(a)]

7.17 contd...

(c) Number of ideal trays for  $R = 2 R_m = (2)(3.45) = 6.9$ :

Intercept of the rectifying section operating line on the  $y$ -axis  $= \frac{x_0}{R+1} = \frac{0.98}{6.9+1} = 0.124$ . Locate the points  $D(0.98, 0.98)$ ,  $W(0.005, 0.005)$ ,  $F(0.5, 0.5)$ . Draw the feed

Fig Prob 7.17(b). Number of theoretical plates for  $R = 2R_m$ .

line through  $F$  and join  $D$  and  $E_2(0, 0.124)$ .  $DE_2$  meets the feed line at  $E_1$ . Join  $E_1, W$ .

The number of ideal trays is to be determined by drawing steps between the equilibrium curve and the operating lines.

It is to be noted that the operating lines and the equilibrium curves are very close in both high and low concentration regions making construction of steps difficult. One approach to tackle this problem

(7.17) contd...

(7.28)

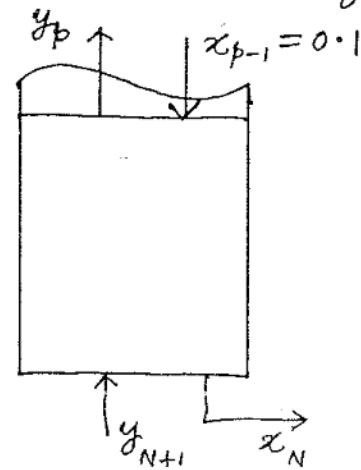
is to draw the high and the low concentration regions of the diagram, fig. Prob 7.17(6), on an enlarged scale and then to construct the stages. The alternative approach is to draw steps in the region where the equilibrium curve and the operating lines are not too close. The number of stages in the remaining regions is determined using the Kremser equation assuming that the equilibrium line is linear in the high and the low concentration regions. We follow the second approach here.

Start drawing steps from  $x = 0.1$  (in the stripping section) to  $x = 0.915$  (in the rectifying section).

Now consider the region  $x = 0.005$  to  $x = 0.1$ . The slope of the equilibrium line,  $m = 1.5$  ( $y = 1.5x$  for small  $x$ ).

Recall Eq. (4.41). Replace the mole ratio units by mole fraction units (since the total vapor and liquid rates remain constant).

$$N = \frac{\log \left[ \frac{(y_{N+1} - \alpha x_0)}{y_1 - \alpha x_0} \right] \left( 1 - \frac{1}{A} \right) + \frac{1}{A}}{\log A} \quad (i)$$



Material balance:  $D + W = F = 120$

$$Dx_D + Wx_W = Fz_F \Rightarrow D(0.98) + W(0.005) = (120)(0.5)$$

Solving  $D = 60.9$ ,  $W = 59.1$ .

### Rectifying section

Vapor flowrate,  $V = (R+1)D = (6.9+1)(60.9) = 481.1 \text{ kmol/h}$

Liquid rate,  $L = RD = (6.9)(60.9) = 420.2 \text{ kmol/h}$

Stripping section (saturated liquid feed,  $F = 120 \text{ kmol/h}$ )

Vapor rate  $\bar{V} = V = 481.1$ ; liquid rate,  $\bar{L} = L + F = 440.2 \text{ kmol/h}$

Slope of the stripping section operating line  $= \bar{L}/\bar{V} = 1.1228$

It passes through the point  $W(0.005, 0.005)$ .

$$\Rightarrow \text{Equation: } y = 1.1228x + 6.14 \times 10^{-4} \quad (ii)$$

(7-17) contd ...

(7-29)

Refer to the sketch of the lower part of the column.

Put  $x_N = 0.005 \Rightarrow y_{N+1} = 0.005$  (Point W)

$x_{p-1} = 0.1, y_p = 0.1117$  [from the operating line eqn. (ii)]

Over the region  $0 \leq x \leq 0.1$ , the equilibrium line can be approximated by  $y = 1.5x$

$$\Rightarrow m = 1.5 \text{ and } \bar{A} = \frac{\bar{L}}{m\bar{V}} = \frac{1.1228}{1.5} = 0.7487 ; d = m = 1.5$$

$$\text{Put } x_{p-1} = x_0 = 0.1 ; y_p = y_1 = (1.1228)(0.1) + 6.14 \times 10^{-4} = 0.1129$$

$$\text{Eq (i)} \rightarrow N = \frac{\log \left[ \frac{0.005 - (1.5)(0.1)}{0.1129 - (1.5)(0.1)} \cdot \left( 1 - \frac{1}{0.7487} \right) + \frac{1}{0.7487} \right]}{\log (0.7487)}$$

$$\Rightarrow \text{Number of plates (for } 0.005 \leq x \leq 0.1) = 13.$$

Number of ideal plates for  $0.915 \leq x \leq 0.98$  in the rectifying section:

The equilibrium line can be approximated as

$$y = m'x + c' = 0.7x + 0.3 \quad (\text{iii})$$

$$\Rightarrow y_m = 0.7x_m + 0.3$$

$$\text{The operating line eq is } y = \frac{R}{R+1}x + 0.124 = 0.8734x + 0.124$$

$$\Rightarrow y_{n+1} = 0.8734x_n + 0.124$$

Material balance over a section of the column

$$V(y_{n+1} - y_1) = L(x_n - x_0) = L \left( \frac{y_n - c'}{m'} - x_0 \right)$$

$$\Rightarrow y_{n+1} = \bar{A}y_n + \left[ y_1 - \frac{L}{V} \left( \frac{c'}{m'} + x_0 \right) \right] ; \bar{A} = \frac{L}{m'V}$$

Solution of the above equation (see Section 4.6.3)

$$y_n = \frac{y_0 - y_1}{1 - \bar{A}} \cdot (\bar{A})^n + \frac{1}{(1 - \bar{A})} \left[ y_1 - \frac{L}{V} \left( \frac{c'}{m'} + x_0 \right) \right]. \quad (\text{iv})$$

7.17 contd...

7.30

To calculate the number of ideal plates, put  $n=N$ ,

$$\bar{A} = \frac{L}{m'V} = \frac{0.8734}{0.7} = \underline{1.2477}; \quad x_N = 0.915; \quad y_N = 0.7x_N + 0.3 \\ = 0.9405; \\ x_0 = 0.98 = y_1; \quad y_0 = 0.7x_0 + C' = (0.7)(0.98) + 0.03 = \underline{0.986}.$$

$$\text{Eq. (iv)} \rightarrow 0.9405 = \frac{0.986 - 0.98}{1 - 1.2477} (1.2477)^N \\ + \frac{1}{1 - 1.2477} \cdot \left[ 0.98 - 0.8734 \left( \frac{0.3}{0.7} + 0.98 \right) \right]$$

Number of ideal stages =  $N = 19$

Number of ideal stages for  $0.1 < x < 0.915$  is obtained by graphical construction (see Fig Prob 7.17b) = 15

Total number of ideal plates =  $19 + 15 + 13 = \underline{47}$

Feed plate  $\rightarrow (15+8)\text{th}$ , i.e. 23rd plate from the top.

(d) Condenser vapor load,  $V = (R+1)D = \underline{481.1 \text{ kmol/h}}$   
[see the calculations under (c)]

7.18

(7.31)

Feed:  $x_F = 0.3$ , Saturated vapor. The feed line is a horizontal through  $(0.3, 0.3) \Rightarrow y = 0.3$  — (i)

Equilibrium line ( $\alpha = 4.12$ )  $\rightarrow y = \frac{4.12x}{1 + 3.12x}$  (ii)

Intersection of the feed line and the equilibrium line is obtained by solving (i) and (ii)  $\rightarrow x = 0.0942, y = 0.3$

The operating line for minimum reflux passes through D  $\rightarrow (x_D, x_0) \rightarrow (0.98, 0.98)$  and  $(0.0942, 0.3)$ .

$$\text{Slope} = \frac{0.98 - 0.3}{0.98 - 0.0942} = 0.768 = \frac{R_m}{R_m + 1}$$

Minimum reflux ratio,  $R_m = 3.31$

Calculation of the boilup ratio (for minimum reflux):

Material balance:  $F = D + W; F(0.3) = D(0.98) + W(0.01)$   
 $\Rightarrow D = 0.299F; W = 0.701F$ .

Since the feed is a saturated vapor,

$$\bar{V} = V - F = (R_m + 1)D - F = (3.31 + 1)(0.299F) - F = 0.2887F$$

$$\text{Boilup ratio, } R_v = \frac{\bar{V}}{W} = \frac{0.2887F}{0.701F} = 0.412$$

7.19

(a) The purity of the products (both top and bottom products) may be improved by increasing the reflux ratio.

(b) The action will lead to reduction of reflux ratio and less product purity.

(c) Increased steam pressure  $\rightarrow$  increased temperature driving force in the reboiler  $\rightarrow$  higher reboiler heat duty  $\rightarrow$  higher vapor rate  $\rightarrow$  higher reflux ratio  $\Rightarrow$  larger condenser heat duty and improved product quality.

(7.32)

7.20

Given:  $z_F = 0.6$ ;  $x_D = 0.96$ ;  $x_6 = 0.55$ ;  $y_6 = 0.72$ ;  $y_7 = 0.63$ .  
 Rectifying section operating line  $\rightarrow y_{n+1} = ax_n + b$

Passes through  $(x_D, x_D) \rightarrow (0.96, 0.96)$ . Also  $y_7 = 0.63$  and  
 $0.96 = a(0.96) + b$ , and  $0.63 = a(0.55) + b$   $x_6 = 0.55$ .

Solve to get  $a = 0.805$ , and  $b = 0.187$ .

Rectifying section operating line :  $y_{n+1} = 0.805 x_n + 0.187$   
 Slope  $= \frac{R}{R+1} = 0.805 \Rightarrow R = 4.134$

Relative volatility,  $\alpha = \frac{y_6/(1-y_6)}{x_6/(1-x_6)} = \frac{0.72/0.28}{0.55/0.45} = 2.104$

Material balance:  $F(0.6) = D(0.96) + W \cdot x_W$ ;  $F = D + W$

97% of A is recovered in the distillate  $\Rightarrow (0.97)(F)(0.6) = D(0.96)$

$$0.6 = \frac{D}{F}(0.96) + \left(\frac{F-D}{F}\right) \cdot x_W \quad \Rightarrow \frac{D}{F} = 0.606$$

$$\Rightarrow 0.6 = (0.606)(0.96) + (1-0.606)x_W$$

$$\Rightarrow x_W = 0.0463.$$

Number of ideal plates: Can be obtained by using the Smoker's equation or by graphical construction. The feed line is horizontal. The graphical construction is shown below.

$$\alpha = 2.104 \Rightarrow \begin{array}{c|cccccc|c} x & 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 \\ \hline y & 0 & 0.345 & 0.584 & 0.76 & 0.894 & 1.0 \end{array}$$

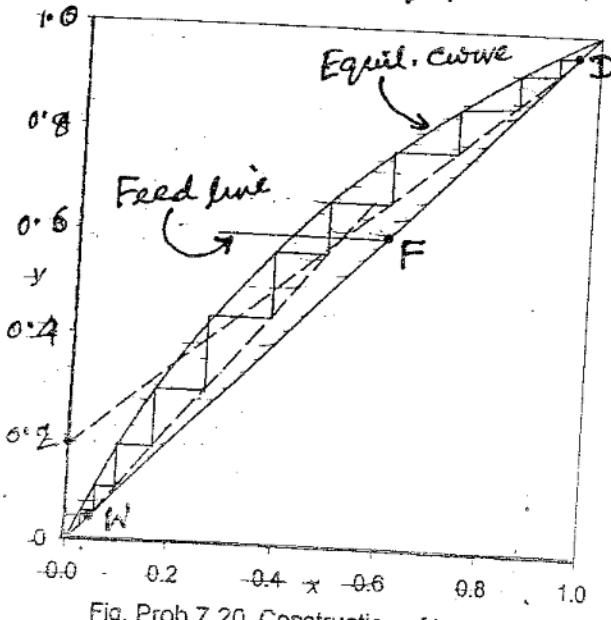


Fig. Prob 7.20. Construction of trays.

Total number of ideal stages = 10.3  
 Feed is to be supplied to the sixth tray from the top.

(7-34)

7.22

Given:  $x_D = 0.98$ ;  $R = 3.0$ ;  $x_5 = 0.54$ ;  $\alpha = 2.51$ .

$$\text{Rectifying section op. line} \rightarrow y_{n+1} = \frac{3}{3+1}x_n + \frac{0.98}{3+1} = \underline{\underline{0.75x_n + 0.245}}$$

(a) Put  $x_n = x_5 = 0.54$  to get  $y_6 = (0.75)(0.54) + 0.245 = \underline{\underline{0.65}}$

The concentration  $=$  concentration of vapor entering the 5-th tray.  
of the vapor leaving the 5th tray  $\rightarrow \frac{y_5/(1-y_5)}{x_5/(1-x_5)} = 2.51 \Rightarrow y_5 = \underline{\underline{0.747}}$

Hints (b) Feed  $\rightarrow$  Saturated liquid  $\Rightarrow q = 1$ ; feed line  $x = \underline{\underline{0.25}}$

Point of intersection of the feed line and the rectifying section operating line ( $y = 0.75x + 0.245$ )

is  $x = \underline{\underline{0.25}}, y = \underline{\underline{0.4325}}$

At the fifth tray,  $(x_5, y_5) \rightarrow (0.55, 0.747)$

$\Rightarrow$  The fifth tray belongs to the rectifying section.

(c) Enrichment of the vapor across the 4-th tray:

$$y_5 = 0.75x_4 + 0.245; y_5 = 0.747 \Rightarrow x_4 = \underline{\underline{0.67}}$$

$$\frac{y_4/(1-y_4)}{x_4/(1-x_4)} = \alpha = 2.51 \Rightarrow y_4 = \underline{\underline{0.836}}$$

$$\text{Enrichment across the 4th tray} = y_4 - y_5 = 0.836 - 0.747 \\ = \underline{\underline{0.089}}$$

(d) 97% of A in the is recovered in the distillate.

$$\Rightarrow (0.97) \cdot F Z_F = D x_0 \Rightarrow (0.97) \cdot F \cdot (0.25) = D (0.98) \\ \Rightarrow F = 4.041 D = 4.041 (F-W)$$

$$W = 3.04 D; D = \underline{\underline{0.2475 F}}$$

Since the feed is a saturated liquid,  $\bar{V} = V = (R+1)D = 4D$

$$\text{Boilup ratio; } R_v = \frac{\bar{V}}{W} = \frac{4D}{3.04D} = \underline{\underline{1.316}}$$

## CHAPTER 8: LIQUID-LIQUID EXTRACTION

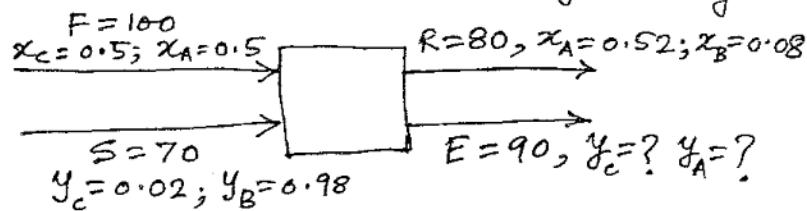
(8.1)

Short and MCQ's

(8.1) (i) 1.0 ; at the plait point, the compositions of the two phases are the same  $\Rightarrow$  selectivity is unity.

(8.2)

From total material balance,

$$E = 100 + 70 - 80 = 90 \text{ kg}$$


Solute (c) balance :  $(100)(0.5) + (70)(0.02) = (80)(0.4) + (90)(y_c)_E$

Carrier (A) balance :  $(100)(0.5) + (70)(0) = (80)(0.52) + (90)(y_A)_E$

Selectivity,  $\beta = \frac{(y_c)_E / (y_A)_E}{(x_c)_R / (x_A)_R} = \frac{(0.215) / 0.0933}{(0.40) / (0.52)} = 3$

(8.3) (iii) 85% ; (8.4) Solvent 3 ; (8.5) (ii) ; (8.6) (i)

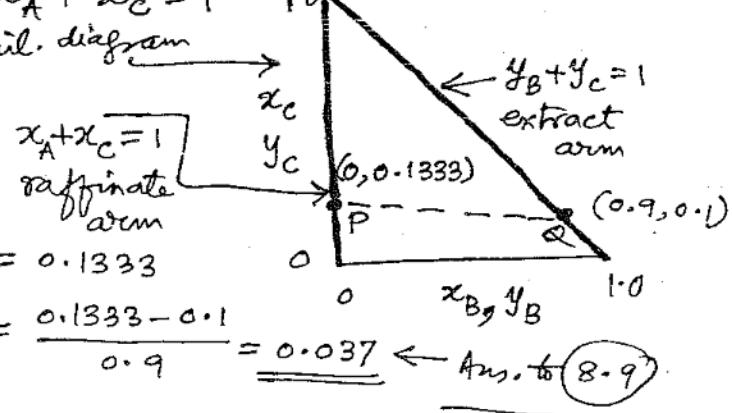
(8.7)  $\frac{R_N}{\Delta m} = \frac{S_m \Delta m}{R_N S_m} = \infty \Rightarrow \Delta m = 0 \Rightarrow R_N = S_m \text{ and } F = E_1$

(8.8) A and B are immiscible  $\Rightarrow y_A = 0$  and  $x_B = 0$ .

$\Rightarrow y_B + y_c = 1$ , and  $x_A + x_c = 1$

At the point Q on the extract arm,  $y_c = 0.1$   $y_B = 0.9$

Equil. diagram



At the point P on the raffinate arm,  $x_c = y_c / 0.75 = 0.1333$

Slope of the tie-line, PQ =  $\frac{0.1333 - 0.1}{0.9} = 0.037$

(8.10) (i) increased.

(8.11) Consider the equil. diagram, fig Ex 8.2

Locate F at  $(x_c)_F = 0.2$ , S and E at  $(y_c)_E = 0.1$  on the extract arm. Join FS. The tie-line through E intersects FS at M.

From the graph,  $\frac{F}{S} = \frac{MS}{FM} = 1.05 \Rightarrow S = \frac{F}{1.05} = \frac{1}{1.05} = 0.95 \text{ kg}$

(8.12) No, since the mutual solubility will be more.

(8.2)

(8.13) By total material balance,  $R = 500 + 400 - 550 = \underline{350} \text{ kg}$   
 Solute balance:  $(500)(0.4) + (400)(0) = (500)(0.33) + (350)(x_c)$

$$\text{Distribution coefficient, } K = \frac{x_c}{x_c} = \frac{0.33}{0.053} = \underline{6.23}$$

(8.14) Use Table 8.2

Propionic acid or phosphoric acid  $\rightarrow$  belong to Solute class 2  
 a solvent of class (column) 1 or class 7 or 8 is suitable

Pyridine (Solute class 7 matches it better) — use a solvent of  
 BTX (Solute class 11) — no solvent class shows negative  
 interaction; select one with zero interaction

o-Nitrophenol (Solute class 10, intramolecular hydrogen bonding)  
 use a solvent from class 4 or 1.

(8.15) (ii) Type II ; (8.16) (ii) 5% ; (8.17) No; intersecting tie-lines mean multiplicity of equilibrium concentrations.

(8.18) (i) Enthalpy ; (8.19) (ii) ; (8.20)

$$(8.21) (i) ; (8.24) (i) ; (8.25) (iii) \bar{a} = \frac{64}{d_{32}} = \frac{(6)(1-0.7)}{500 \times 10^{-6}} = \underline{\underline{3600 \frac{m^2}{m^3}}}$$

(8.26) (ii) 3-15 cm.

(8.27) (ii) 0.4 m

(8.29) (iii) Pod extractor

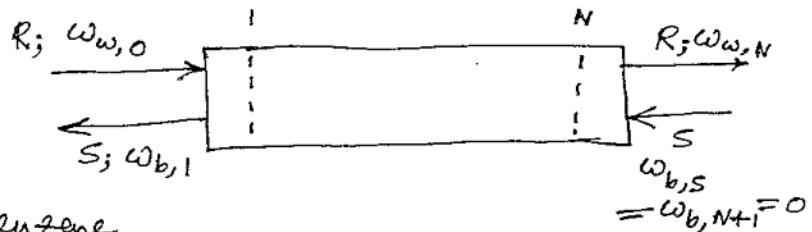
**PROBLEMS**

**8.1**

Since the solute concentrations are low, the flowrates of the carrier and the solvent phases (miscible) may be taken to be constant.

**8.3**

$$R = \text{Raffinate (water)} \\ \text{phase flow rate} \\ = 2000 \text{ kg/h}$$



$$w_{w,0} = \text{mass fraction of benzene} \\ \text{in the feed } = 0.02$$

$$S = \text{Solvent flowrate} = ? \quad w_b = \text{mass fraction solute in} \\ \text{the solvent phase.}$$

$$98\% \text{ of the solute is removed} \Rightarrow w_{w,N} = (0.02)(0.02) = 0.0004$$

For the minimum solvent rate, the pinch point occurs at the feed end of the cascade  $\Rightarrow w_{w,0}$  and  $w_{b,1}$  are at equilibrium.  $\Rightarrow w_{w,0}/w_{b,1} = 1.707 \Rightarrow w_{b,1} = w_{w,0}/1.707 = \frac{0.02}{1.707} = 0.0117$

(a) Solute material balance for the minimum solvent rate,  $S_m$ :

$$R(w_{w,0} - w_{w,N}) = S_m (w_{b,1} - w_{b,N+1})$$

$$\Rightarrow 2000(0.02 - 0.0004) = S_m (0.0117 - 0) \Rightarrow S_m = 3350 \text{ kg/h.}$$

(b) Actual solvent rate  $= 1.3 S_m = (1.3)(3350) = 4355 \text{ kg/h}$

Since the flowrates are 'constant' and the equilibrium relation is linear, the number of theoretical stages may be determined by using the Kremser equation. With the above notation, Eq. (4.44) reduces to the form

$$N = \frac{\log \left[ \left( \frac{w_{w,0} - K \cdot w_{b,N+1}}{w_{w,N} - K \cdot w_{b,N+1}} \right) (1 - \bar{E}) + \bar{E} \right]}{\log (1/\bar{E})} \quad (i)$$

$$\bar{E} = K \cdot R/S = (1.707)(2000)/(4355) = 0.784; \quad K = 1.707; \quad w_{b,N+1} = 0$$

$$N = \frac{\log \left[ \left( \frac{0.02 - 0}{0.0004 - 0} \right) (1 - 0.784) + 0.784 \right]}{\log (1/0.784)} = 10 \text{ ideal stages}$$

(c) Given,  $N = 4$ ; the same solvent rate  $\Rightarrow \bar{E} = 0.784; w_{w,N} = ?$

(8.1) Contd...

From Eq (i) above,

$$A = \frac{\log \left[ \left( \frac{w_{w,0} - 0}{w_{w,N} - 0} \right) (1 - 0.784) + 0.784 \right]}{\log (1/0.784)} \Rightarrow w_{w,0}/w_{w,N} = 8.625$$

$$\Rightarrow w_{w,N} = 0.02/8.625 = 0.00232$$

$$\Rightarrow \text{Fractional solute removal} = (0.02 - 0.00232)/0.02, \underline{i.e. 88.4\%}$$

(d)  $N = 5$ ;  $w_{w,0} = 0.02$ ;  $w_{w,N} = 0.0004$ ;  $S = ?$ 

$$5 = \frac{\log \left[ \left( \frac{0.02 - 0}{0.0004 - 0} \right) (1 - E) + E \right]}{\log (1/E)} \Rightarrow E = 0.528$$

$$\Rightarrow K \cdot R/S = 0.528 \Rightarrow S = K \cdot R / 0.528 = \frac{(1.707)(2000)}{0.528} = \underline{\underline{6466 \text{ kg/h}}}$$

(e) Use the result given in Prob 4.30.

$$A = \frac{\log (0.02 / 0.0004)}{\log (1 + S/KR)} \Rightarrow R = 2000, K = 1.707$$

$$\Rightarrow S = 5665 \text{ kg/h} \Rightarrow \text{Total Solvent reqd.} = (4)(5665) = \underline{\underline{22,660 \text{ kg/h}}}$$

\* \* \* \* \*

(8.2)

Given:  $F = 500 \text{ kg}$ ;  $(x_c)_F = 0.5$ ;  $(x_B)_F = 0$ ;  $F/S = 1.1$ 

$$S = 500/1.1 = 454.5 \text{ kg}; (y_B)_S = 0.98; (y_c)_S = 0.02$$

$$\text{At the 'mixing point', } (x_c)_M = \frac{(500)(0.5) + (454.5)(0.02)}{500 + 454.5} = \underline{\underline{0.271}}$$

The equil. data are plotted on the rectangular coordinate.

Locate the points  $F$  [ $(x_B)_F = 0, (x_c)_F = 0.5$ ];  $S$  [ $(x_B)_S = 0.98, (x_c)_S = 0.02$ ]; and the mixing point  $M$  on the line  $FS$  at  $(x_c)_M = 0.271$ .The tie line through the point  $M$  is drawn taking help of the  $x_c$ - $y_c$  curve drawn alongside the main figure.From the terminal points of the tie line,  $ER$ ,

$$(x_c)_R = 0.18; (y_c)_E = 0.32$$

Solute material balance:  $F(x_c)_F + S(y_c)_S = E(y_c)_E + R(x_c)_R$ 

$$\Rightarrow (500)(0.5) + S(0.02) = E(0.32) + R(0.18) \quad (i)$$

(8.4)

8.2 Contd...

8.5

$$\text{Total material balance: } 500 + 454.5 = E + R \quad (i)$$

$$\text{Solving (i) and (ii), } E = 623.5 \text{ kg; } R = 331 \text{ kg} \quad (ii)$$

Acetone removed from the aqueous phase

$$= (500)(0.5) - (331)(0.18) = 190.4 \text{ kg; i.e., } \frac{190.4}{500} \Rightarrow 76.2\%$$

\* \* \* \*

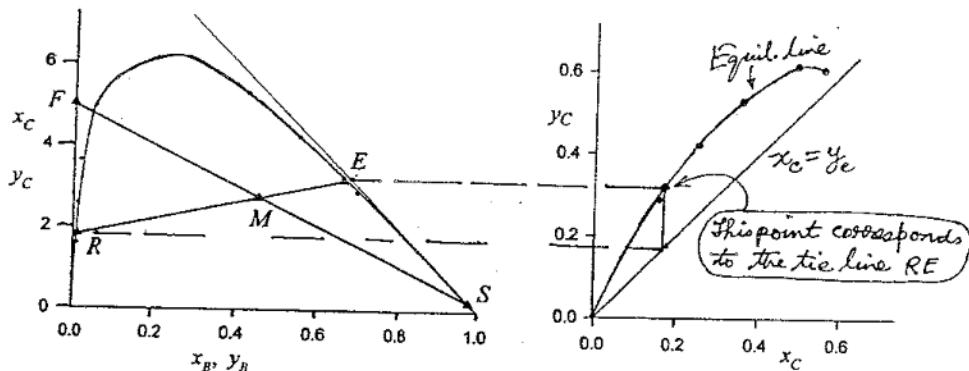


Fig. Prob. 8.2. Single stage extraction.

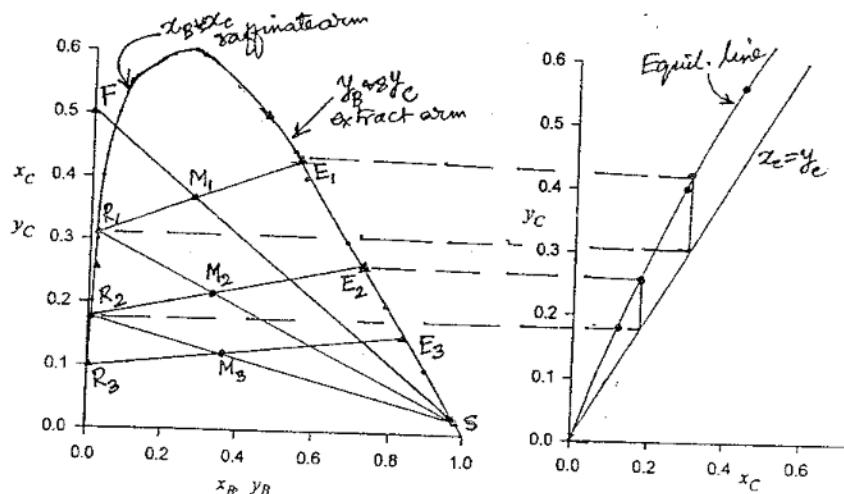


Fig. Prob 8.3. Three stage crosscurrent cascade.  
crosscurrent

\* \* \* \*

8.3 Refer to fig. 8.8 for three stages. Equal amount of solvent is added to each stage  $\Rightarrow S_1 = S_2 = S_3 = S$ .

Stage 1 :  $F = 500 \text{ kg/h}$ ;  $(x_c)_F = 0.5$ ;  $S = 5 = 200 \text{ kg/h}$ ;  $(x_c)_S = 0.02$

At the mixing point,  $M_1$ ,  $(x_c)_{M_1} = \frac{(500)(0.5) + (200)(0.02)}{700} = 0.363$

Draw the LLE curve and locate the points  $F(0, 0.5)$  and  $S(0.98, 0.02)$  on it; join  $FS$  and locate  $(x_c)_{M_1} = 0.363$  on the line  $FS$ ; draw the tie line  $R_1 E_1$  through the point  $M_1$ .

From the figure,  $(x_c)_{R_1} = 0.315$ ,  $(y_c)_{E_1} = 0.42$

Material balance :  $F(x_c)_F + S(0.02) = E_1(0.42) + R_1(0.315)$

$S = 200$ ,  $F = 500$ ,  $(x_c)_F = 0.5 \Rightarrow E_1 = \underline{319 \text{ kg/h}}$  and  $F + S = E_1 + R_1$ ,  $R_1 = \underline{381 \text{ kg/h}}$

Stage 2

$R_1 = 381$ ;  $S_2 = S = 200$ ;  $(x_c)_{R_1} = 0.315$ ;  $(y_c)_S = 0.02$

Mixing point,  $(x_c)_{M_2} = \frac{(381)(0.315) + (200)(0.02)}{381 + 200} = 0.2135$ .

Draw the line  $R_1 S$  and locate the point  $(x_c)_{M_2}$  on it; draw the tie line  $R_2 E_2$  through  $M_2$ .

From the figure,  $(x_c)_{R_2} = 0.18$ ;  $(y_c)_{E_2} = 0.26$

By a solute balance,  $R_2 = \underline{338.2}$ ,  $E_2 = \underline{242.8}$

Stage 3

For this stage,  $(x_c)_{M_3} = 0.1205$  which is located

on  $R_2 S$ . The tie line through  $(x_c)_{M_3}$  is  $R_3 E_3$

$(x_c)_{R_3} = 0.102$ ;  $(y_c)_{E_3} = 0.15$ ; by material balance,  $R_3 = \underline{330.2}$

Total solute balance over the cascade:

$$E_3(y_c)_{E_3} + E_2(y_c)_{E_2} + E_1(y_c)_{E_1} = F(x_c)_F + 3S(y_c)_S - R_3(x_c)_{R_3}$$

which is approximately satisfied. The discrepancy arises out of errors in reading values from the plots.

Acetone removed from the feed =  $F(x_c)_F - R_3(x_c)_{R_3} = \underline{216.3 \text{ kg/h}}$

Fractional removal =  $\frac{216.3}{250} = \underline{0.865}$

\*

\*

\*

\*

(8.7)

**8.4** The procedure is similar to that used for Prob. 8.3. The outline of the solution is given. Refer to fig. 8.8.

Stage 1:  $F = 1000 \text{ kg/h}$ ;  $(x_c)_F = 0.5$ ;  $S = S_1 = 250 \text{ kg/h}$ ;  $(x_c)_{S_1} = 0$

The LLF diagram is prepared on the rectangular coordinate.

At the mixing point of Stage 1,  $(x_c)_{M_1} = \frac{(1000)(0.5) + (250)(0)}{1250} = 0.40$

The points F and  $S_1$  are located and  $(x_c)_{M_1}$  is located on the line  $FS_1$ . The tie line through  $M_1$  is drawn having terminals  $R_1$  and  $E_1$ ;  $(x_c)_{R_1} = 0.358$ ;  $(y_c)_{E_1} = 0.47$ .

Solving the total and solute material balance equations,

$$E_1 = 468.7; R_1 = 781.3$$

Similarly we can proceed for Stages 2, 3 and 4 in which the given amounts of solvent are added.

Results:  $(x_c)_{R_2} = 0.22$ ,  $(y_c)_{E_2} = 0.306$ ,  $R_2 = 603.5$ ,  $E_2 = 477.8$

For Stage 3,  $(x_c)_{R_3} = 0.11$ ,  $(y_c)_{E_3} = 0.17$ ,  $R_3 = 489.5$ ,  $E_3 = 464$

For Stage 4,  $(x_c)_{R_4} = 0.05$ ,  $(y_c)_{E_4} = 0.075$ ,  $R_4 = 516$ ,  $E_4 = 373.5$

Mixing points:  $(x_c)_{M_2} = 0.2586$ ;  $(x_c)_{M_3} = 0.1392$ ;  $(x_c)_{M_4} = 0.0605$

See Fig. Prob 8.4. Total amount of solute recorded

$$= \sum_i E_i (y_c)_{E_i} = (468.7)(0.47) + (477.8)(0.306) \\ + (464)(0.17) + (373.5)(0.075) = 473.4 \text{ kg}$$

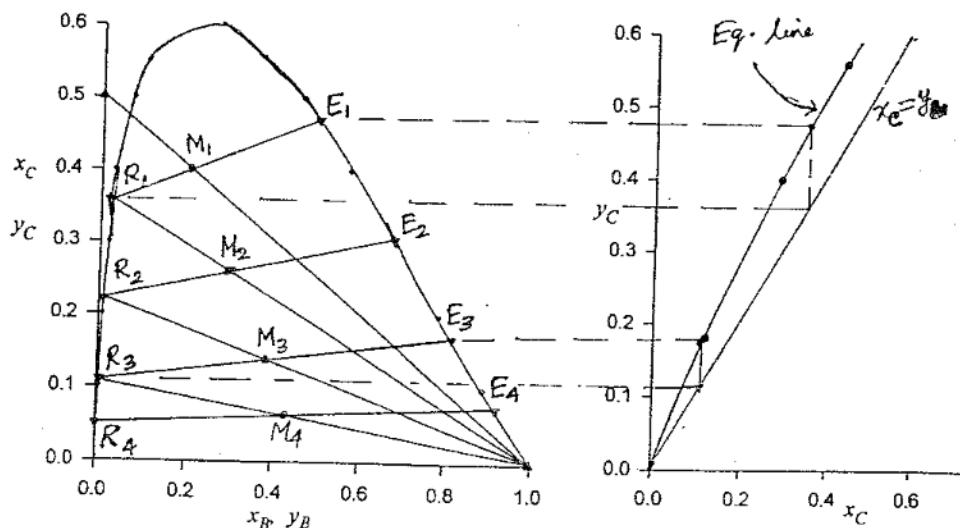


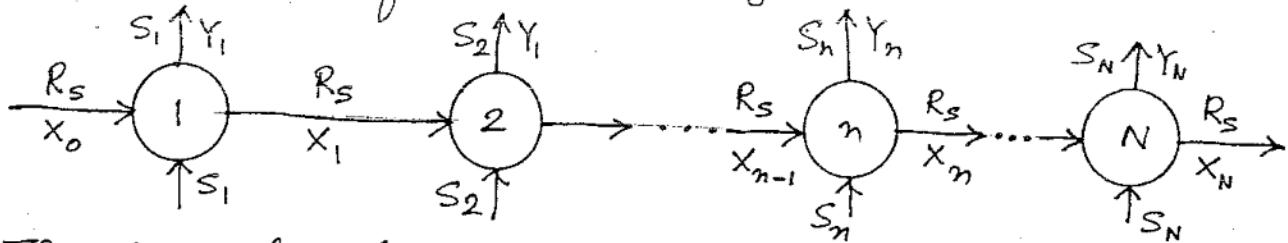
Fig. Prob 8.4. Four stage current cascade, quantities of solvent added to the stages are different.



8.5

A sketch of the cascade is given below

8.8



The phase flow rates,  $R_s$  and  $S_i$ , are on solute-free basis; the concentrations are in the "mass ratio" unit.

Overall mass balance:  $R_s(x_0 - x_N) = S_1 Y_1 + S_2 Y_2 + \dots + S_N Y_N = \sum_{n=1}^N S_n Y_n$  (i)

Total amount of solvent  $\sum_{n=1}^N S_n$  used = amount of solute recovered.

Profit,  $\bar{P}$  = price of extracted solute - cost of processing

$$\Rightarrow \bar{P} = c_1 \cdot R_s(x_0 - x_N) - c_2 \sum_{n=1}^N S_n \quad \dots \quad (\text{ii})$$

For the  $n$ -th stage,  $R_s(x_{n-1} - x_n) = S_n Y_n$

$$\Rightarrow S_n = \frac{R_s(x_{n-1} - x_n)}{Y_n} = \frac{R_s(x_{n-1} - x_n)}{\psi(x_n)} = \frac{R_s(x_{n-1} - x_n)}{\alpha x_n} \quad (\text{iii})$$

From Eqs. (i), (ii) and (iii), for  $N=4$ ,

$$\bar{P} = c_1 \cdot R_s(x_0 - x_4) - \frac{c_2 R_s}{\alpha} \sum_{n=1}^4 \left( \frac{x_{n-1} - x_n}{x_n} \right)$$

$$\Rightarrow \bar{P} = \frac{\bar{P}}{c_1 \cdot R_s} = (x_0 - x_4) - \gamma \sum_{n=1}^4 \left( \frac{x_{n-1} - 1}{x_n} \right); \quad \gamma = \frac{c_2}{c_1 \alpha}$$

$$\Rightarrow \bar{P} = x_0 - x_4 + 4\gamma - \gamma \left( \frac{x_0}{x_1} + \frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_4} \right) \quad (\text{iv})$$

To maximize  $\bar{P}$  (i.e.  $\bar{P}$ ), put  $\frac{\partial \bar{P}}{\partial x_1} = \frac{\partial \bar{P}}{\partial x_2} = \frac{\partial \bar{P}}{\partial x_3} = \frac{\partial \bar{P}}{\partial x_4} = 0$

Differentiating  $\bar{P}$ ,  $\frac{\partial \bar{P}}{\partial x_1} = \frac{\gamma x_0}{x_1^2} - \frac{\gamma}{x_2} = 0 \Rightarrow x_1^2 = x_0 x_2 \quad (\text{v})$

$$\frac{\partial \bar{P}}{\partial x_2} = \frac{\gamma x_1}{x_2^2} - \frac{\gamma}{x_3} = 0 \Rightarrow x_2^2 = x_1 x_3 \quad (\text{vi})$$

Similarly, from  $\frac{\partial \bar{P}}{\partial x_3} = 0$ ,  $x_3^2 = x_2 x_4 \quad (\text{vii})$  & from  $\frac{\partial \bar{P}}{\partial x_4} = 0$ ,  $x_4^2 = x_3 \quad (\text{viii})$

From Eqs (v) to (viii),  $x_1 = (\gamma x_0)^{1/5}$ ;  $x_2 = (\gamma^2 x_0^{3/5})^{1/5}$ ;  $x_3 = (\gamma^3 x_0^{2/5})^{1/5}$ ;  $x_4 = (\gamma^4 x_0)^{1/5}$

It can be shown that the "sufficient condition" for maximum are

Solvent rate to Stage 1 [Eq.(iii)],  $S_1 = \frac{R_s}{\alpha} \left( \frac{x_0}{x_1} - 1 \right)$  also satisfied.

and so on.

8.9

8.6  $F = 2000 \text{ kg/h}; (x_c)_F = 0.35; S = 2970 \text{ kg/h}; (y_c)_S = 0 \text{ (pure solvent)}$   
 raffinate concentration,  $(x_c)_{R_N} = 0.1$

LLE curve is drawn using rectangular coordinates; locate the points F, S and  $R_N$ . At the mixing point (M),  $x_M = \frac{(2000)(0.35)}{4970} = 0.41$   
 Locate M on the line FS, join  $R_N M$  and extend to meet the 'extract arm' of the LLE curve at  $E_1$ . Join and extend  $FE_1$  and  $R_N S$  to get the difference point,  $\Delta$ . Draw a set of lines from  $\Delta$  to intersect the LLE curve at a set of points

[Fig Prob 8.6(a)]

$x_c$	0.1	0.12	0.15	0.18	0.21	0.25	0.28	0.32	0.34
$y_c$	0.0	0.1	0.2	0.04	0.06	0.082	0.10	0.14	0.16

The equilibrium curve is drawn as  $x_c$  vs  $y_c$  on Fig Prob 8.6(b) and the operating line is drawn by joining the above points. Stps are drawn between the eq. and op. lines to find out the number of ideal stages,  $N = 5.9$ .

To obtain the minimum liquid rate, it is seen that  $\Delta_m$  can be obtained by drawing the tie line ( $FE_{1,m}$ ) through F. Join  $R_N$  and  $E_{1,m}$ ; the line intersects FS at  $M_m$ . From the figure 8.6(b),  $FM_m = 5.6$  and  $M_m S = 5.55$   
 By the Lever Arm Rule,  $F/S_m = M_m S_m / FM_m \Rightarrow \frac{2000}{S_m} = \frac{5.55}{5.6} = 0.9911$   
 $\Rightarrow S_m = \underline{2018 \text{ kg/h}} = \text{minimum solvent rate}$

The actual solvent rate (S) is  $2970 / 2018 = 1.47$  times the minimum.

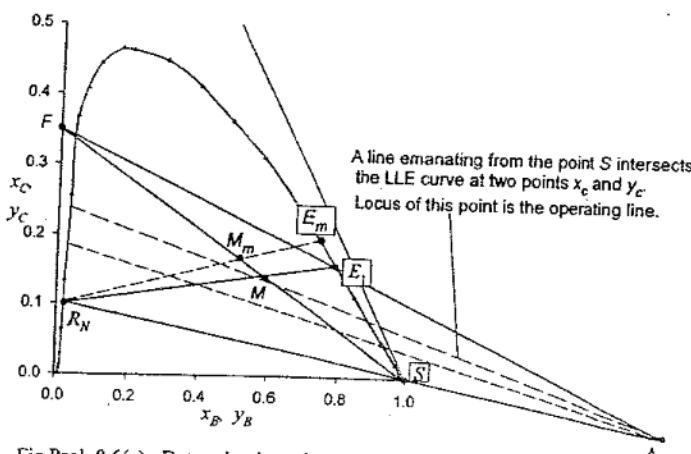
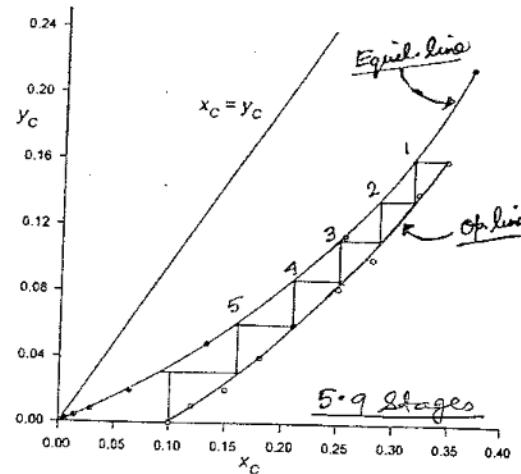


Fig Prob 8.6(a) : Determination of (i) points on the operating line  
 (ii) the points  $E_m$  and  $M_m$  for the minimum liquid rate.



MT Fig Prob 8.6(b) - determine the number of ideal stages.



(8.7)

$$F = 1000 \text{ kg/hr}; (x_c)_F = 0.35; (x_c)_{R_N} = 0.03; S = 1300 \text{ kg/hr}$$

(8.10)

$$\text{At the mixing point (M), } (x_c)_M = \frac{(1000)(0.35)}{2300} = 0.152 \quad (y_c)_S = 0$$

Locate the points F, S, R<sub>N</sub> on the LLE plot. Locate M on the line FS, join R<sub>N</sub>M to meet the Eq. curve at E<sub>1</sub>. Join and extend FE<sub>1</sub> and R<sub>N</sub>S to meet at Δ. Determine the points on the operating line by drawing a set of lines from Δ. Find out the number of ideal stages by step construction on the x<sub>c</sub>-y<sub>c</sub> plane. Number of ideal stages = 4

To determine the minimum solvent rate, draw the tie line through F that meets the extract arm of the LLE curve at (E<sub>1</sub>)<sub>m</sub>. Join R<sub>N</sub>E<sub>1,m</sub> that intersects the line FS at M<sub>m</sub>. Using the Lever Arm Rule,

$$F/S_m = \frac{M_m S_m}{F S_m} = \frac{5.8}{3} = 1.93 \Rightarrow S_m = \frac{1000}{1.93} = 517 \text{ kg/hr}$$

The construction is shown in fig Prob 8.7.

$$\text{Op. line} \rightarrow \begin{array}{c|c|c|c|c|c|c|c} x_c & 0.03 & 0.09 & 0.145 & 0.19 & 0.24 & 0.28 & 0.32 \\ \hline y_c & 0 & 0.03 & 0.06 & 0.08 & 0.12 & 0.15 & 0.19 \end{array}$$

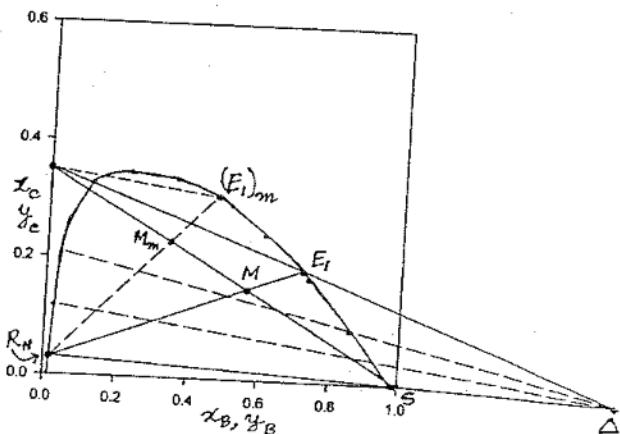
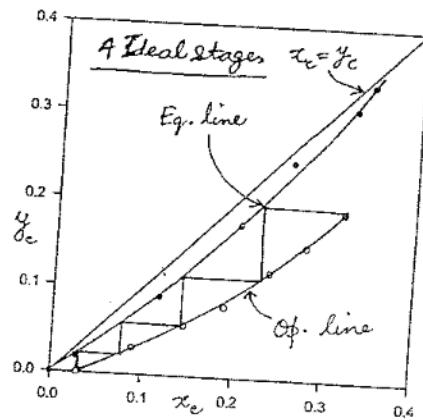


Fig Prob 8.7. Determination of the number of ideal stages.



8.8

$$F = 1000 \text{ kg/h}; (x_c)_F = 0.5; (x_c)_{R_N} = 0.02; (y_c)_S = 0$$

(8.11)

Draw the LLE curve and locate the points F, R<sub>N</sub> and S. Draw the tie line through F, F E<sub>1,m</sub>. Extend the lines F E<sub>1,m</sub> and R<sub>N</sub>S to get the point Δ<sub>m</sub>. The minimum liquid rate can be found from the location of Δ<sub>m</sub>, S<sub>m</sub> = 647.5 kg/h.

$$\text{Actual liquid rate} = 1.3 S_m = (1.3)(647.5) = \underline{\underline{842}} \text{ kg/h.}$$

Obtain the mixing point (M) on FS where  $(x_c)_M = \frac{500}{1842} = 0.271$

Join and extend R<sub>N</sub>M to get E<sub>1</sub> on the LLE line. Get the point Δ, the intersection of R<sub>N</sub>S and FE<sub>1</sub> extended. Obtain the points on the operating line by drawing a set of lines through Δ. Points on the op. line are

$x_c$	0.02	0.08	0.12	0.19	0.25	0.305	0.36	0.415	0.45	0.49
$y_c$	0.0	0.04	0.065	0.113	0.158	0.20	0.242	0.28	0.33	0.365

The number of ideal stages by step construction = 3.4

See the figures below, Fig Prob 8.8.

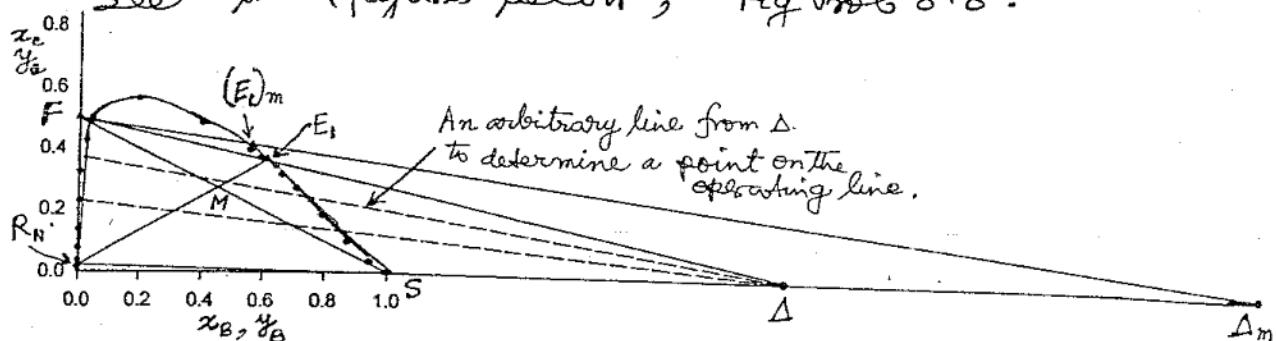
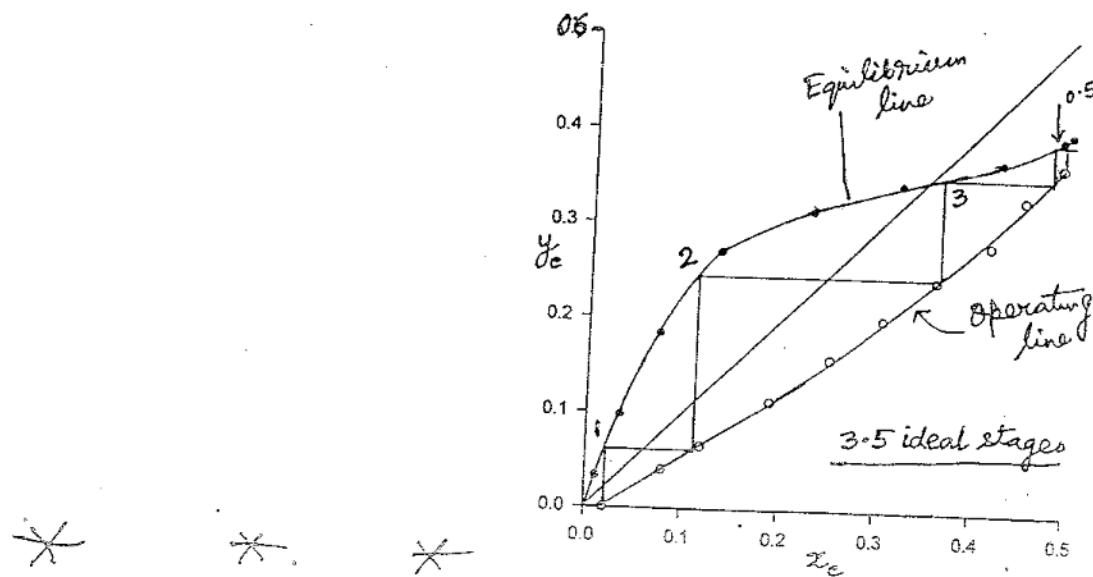


Fig. Prob 8.8(a): Solvent rate and points on the operating line



(8.9)

The data points under raffinate are to be converted to  $X-Z$  and those under extract to  $Y-Z$ . A sample calculation is shown for the first row of the data table.

(8.12)

$$X = \frac{x_c}{x_A + x_c} = \frac{0.095}{0.697 + 0.095} = 0.12; Z = \frac{x_B}{x_A + x_c} = \frac{0.208}{0.697 + 0.095} = 0.263$$

$$Y = \frac{y_c}{y_A + y_c} = \frac{0.005}{0.111 + 0.005} = 0.0431; Z = \frac{y_B}{y_A + y_c} = \frac{0.884}{0.111 + 0.005} = 7.62$$

Transformed data for preparation of the Maloney-Schubert plot:

$X$	0.12	0.17	0.252	0.314	0.376	0.59
$Z$	0.263	0.266	0.284	0.309	0.353	1.0

$Y$	0.043	0.132	0.252	0.33	0.392	0.59
$Z$	7.62	6.752	5.62	4.78	4.15	1.0

The data are plotted in Fig Prob 8.9. The lower part represents the raffinate phase and the upper part, the extract phase; P is the plait point. The equilibrium diagram on solvent free basis ( $x-y$ ) is plotted below  $X-Z$  and  $Y-Z$  curves. Fig Prob 8.9 shows the Jancke plot (the upper figure).

(b) At the mixing point ( $M'$ )

$$X_{M'} = \frac{(500)(0.3)}{500} = 0.3; Z = \frac{500}{500} = 1.0 \quad ; \quad F = 500 \text{ kg/h}, (x_c)_F = 0.30 \quad (z_A)_F = 0.70 \quad S = 500$$

The point  $M'$  is located on the Maloney-Schubert plot and the tie-line through  $M'$  is drawn ( $P'Q'$ ).

At  $P'$ ,  $X = 0.298$  and  $Z = 0.29$  for the raffinate phase

At  $Q'$ ,  $Y = 0.316$  and  $Z = 5.0$  " " extract phase.

Let the flowrates (solvent free basis) be extract:  $E'$ , raffinate:  $R'$ .

Total material balance:  $E' + R' = F + S = F + S$

Solvent balance:

$$= 500 + 0$$

$$E'Z + R'Z = FZ_F + SZ_S = 500 \text{ kg solvent}$$

$$\Rightarrow E'(5.0) + (500 - E')(0.29) = 500 \Rightarrow E' = 75.4, R' = 424.6$$

(8.9) contd...

Total phase flowrates :

$$E = E'(1+z) = (75.4)(1+5) = \underline{452.4} ; R = R'(1+z) = (424.6)(1.29)$$

$$= \underline{\underline{547.7}} \text{ kg/h}$$

$$\text{Mass of ethylene glycol extracted} = E'Y = (75.4)(0.316)$$

$$\text{Mass of glycol in the raffinate} = \underline{\underline{23.5}} \text{ kg}$$

$$( \text{Total solute} = 126.5 + 23.5 = 150 \text{ kg/h} ) = R'X = (424.6)(0.298)$$

$$\% \text{ Solute recovered} = \frac{23.5}{150} \times 100 = \underline{\underline{15.67\%}}$$

(8.13)

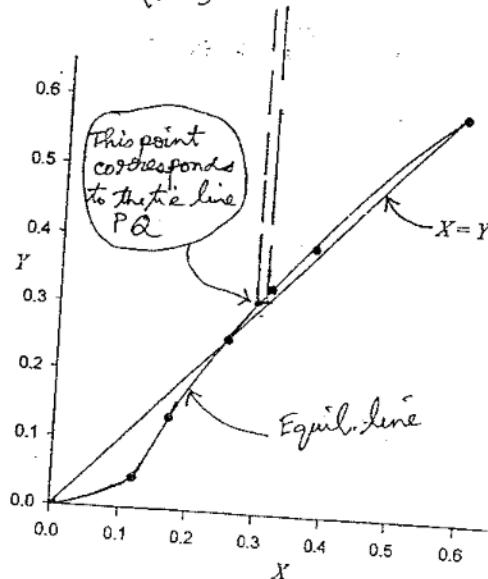
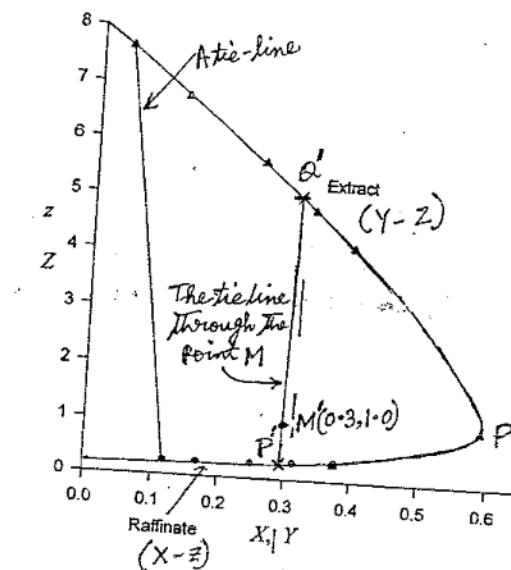


Fig Prob 8.9

(8.14)

(8.10) The final concentration ( $x_2$ ) of the solute in the dispersed phase (see Example 8.7) if 99% approach to equilibrium is achieved,

$$x_2 = x_{eq} + (x_1 - x_{eq})(1 - 0.99)$$

$$= 0.01504 + (0.3 - 0.01504)(0.01) = 0.01519$$

Calculate the required time from Eq. (8.32)

$$\ln \left[ \frac{0.03 - \frac{(3.77)(0.03)(1200)/(4500)}{1 + (3.77)(1200)/(4500)}}{0.01504 - \frac{(3.77)(0.03)(1200)/4500}{1 + (3.77)(1200)/(4500)}} \right] = \frac{(1.940)(7.352)(1010)(1.577 \times 10^{-5})}{1200} \cdot t$$

$$\Rightarrow 4.6025 = (0.1893)(2.0053) \cdot t$$

$$\Rightarrow t = 12 \text{ second}$$

## CHAPTER 9: SOLID-LIQUID EXTRACTION

9.1

Short and MCQ's

- (9.1) (i) and (iii); (9.2) (ii), (iv) and (vi); (9.3) (i) ethanol,  
 (ii) water, (iii) ammonium cyanide, (iv) ammoniacal solution,  
 (v) hexane, (vi) water; (9.4) (i) solid-phase diffusion,  
 (ii) liquid-phase diffusion, (iii) liquid-phase diffusion;  
 (9.5) (i); (9.6) (i); (9.7) (i)

- (9.8) NaOH concentrations in the overflow and in the liquid retained in the underflow are equal.

$$\text{Total Solute} = 0.15 \text{ kg}; \text{total solvent (water)} = 1.35 \text{ kg}; \\ \text{fractional solute concentration} = \frac{0.15}{1.35 + 0.15} = 0.1 = \text{solute}$$

$$\text{Mass of solution retained in the underflow} = (0.3 \text{ kg/kg inert})(0.5 \text{ kg inert}) = 0.15 \text{ kg}$$

$$\text{Mass of overflow} = 1.5 - 0.15 = 1.35 \text{ kg}$$

$$\text{Mass of NaOH extracted in the overflow} = (1.35)(0.1) = 0.135 \text{ kg}$$

$$\text{Fractional recovery} = \frac{0.135}{0.15} \Rightarrow 90\% .$$

- (9.9) (i); (9.10) (ii); (9.11) (ii), since the solution of higher density tends to move downward; (9.13) (iii);

- (9.14) (iii); (9.15) (i) Bollman, moving basket, moving belt, (ii) moving belt, (iv) moving belt, (v) Rotocel.

- (9.16) (i), (iii), (v) and (vii).

- (9.19) Overflow: Solution conc. =  $Y_c$ , solid free basis.  
 1 kg solvent means  $\frac{1}{(1-Y_c)}$  kg solution.

$$Z_V = 0.1 Y_c \text{ kg solid per kg solvent} \Rightarrow \frac{0.1 Y_c}{1/(1-Y_c)}$$

i.e.

$$\underline{\text{Underflow}} \quad Z_V = \frac{0.1 Y_c (1-Y_c)}{1/(1-Y_c)} \text{ kg solid per kg solution}$$

$$\Rightarrow Z_L = \frac{0.35 - 0.4 X_c^2}{1 - (0.35 - 0.4 X_c^2)} \text{ mass fraction solid in the sludge}$$

$$\text{The curves can now be drawn.}$$

PROBLEMS

(9-2)

- 9.1  $Y_c, Z_v$  values for the overflow, and  $X_c, Z_L$  values for the underflow can be calculated from the given data. Take the data set given in the last row of the table, for example. For 1 kg solution, mass of oil,  $m_c = 0.5 \text{ kg}$ , mass of solvent,  $m_B = 0.5 \text{ kg}$ , mass of solid in the overflow,  $m_A = 0.0$ ; mass of solids in the underflow,  $m'_A = 0.46 \text{ kg}$

$$Y_c = \frac{m_c}{m_B + m_c} = \frac{0.5}{0.5 + 0.5} = 1; Z_v = \frac{m_A}{m_B + m_c} = \frac{0.029}{0.5 + 0.5} = 0.029$$

$$X_c = \frac{m_c}{m_B + m_c} = \frac{0.5}{0.5 + 0.5} = 1; Z_L = \frac{m'_A}{m_B + m_c} = \frac{0.46}{0.5 + 0.5} = 0.46$$

Calculated data for all the data sets are given below.

$Y_c$	0	0.05	0.20	0.25	0.30	0.35	0.4	0.45	0.50
$Z_v$	0	0.002	0.005	0.007	0.01	0.013	0.017	0.022	0.029

$X_c$	0	0.05	0.20	0.25	0.30	0.35	0.4	0.45	0.50
$Z_L$	0.67	0.66	0.64	0.625	0.6	0.58	0.55	0.51	0.46

For preparing a right triangular diagram, we have to convert the given data to  $(Y_c, Y_B)$  for overflow and  $(X_c, X_B)$  for underflow. Again consider the last row of the given data table.

$$Y_c = \frac{m_c}{m_A + m_B + m_c} = \frac{0.5}{0.5 + 0.5 + 0.029} = 0.486; Y_B = \frac{0.5}{1.029} = 0.486$$

$$X_c = \frac{m_c}{m'_A + m_B + m_c} = \frac{0.5}{1.46} = 0.342; X_B = \frac{0.5}{1.029} = 0.342$$

Calculated data for all the sets:

$Y_c$	0	0.05	0.199	0.2482	0.297	0.3446	0.3933	0.4403	0.486
$Y_B$	1.0	0.95	0.796	0.745	0.693	0.6416	0.59	0.538	0.486

$X_c$	0	0.0301	0.122	0.1538	0.1875	0.2215	0.258	0.298	0.342
$X_B$	0.6	0.5723	0.488	0.461	0.4375	0.411	0.387	0.387	0.342

(9.1) contd...

(9.3)

The calculated data are plotted in Fig Prob 9.1(a) in the rectangular coordinates —  $x_B$  against  $x_C$  for the underflow, and  $y_B$  against  $y_C$  for the overflow. Three tie lines are shown.

The calculated data are also plotted as the Ponchon-Savarit diagram —  $Z_L$  against  $x_C$  for the underflow, and  $Z_V$  against  $y_C$  for the overflow. The tie lines are vertical.

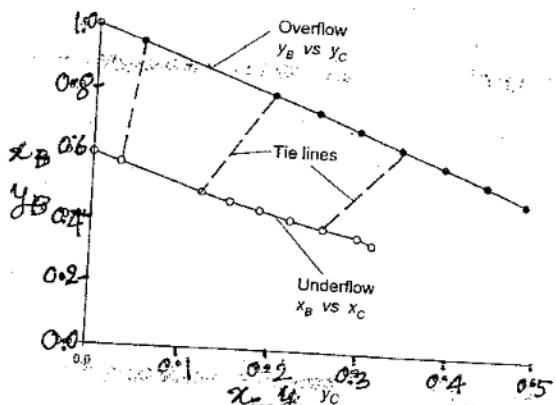


Fig. Prob. 9.1(a): Plot of underflow and overflow concentrations.

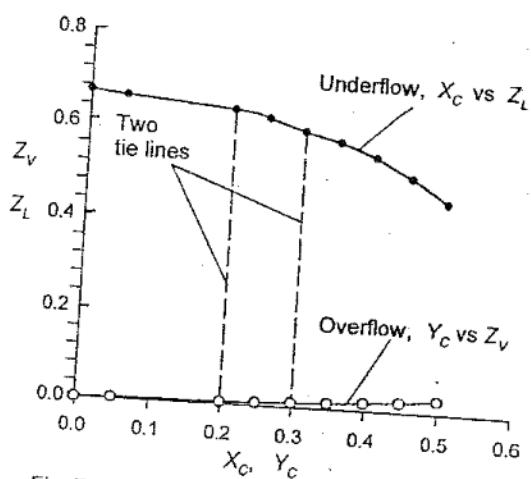


Fig. Prob. 9.1(b): Ponchon-Savarit plots of overflow and underflow concentrations.

(9.2)

\* \* \* \* \*

The overflow and underflow curves of fig. Prob 9.1 will be used.

Single stage contact of 1000 kg feed with 1800 kg solvent

(a) Solution using  $x_B-x_C$  and  $y_B-y_C$  plots, Fig Prob 9.2(a)

$$F = 1000 \text{ kg}; (x_C)_F = 0.25; S = 1800; (y_C) = 0$$

$$\text{At the mixing point } M, (x_C)_M = \frac{(0.25)(1000)}{1000 + 1800} = 0.0893$$

The points F and S are located and joined. The mixing point is located on FS and the tie line  $^{(IV)}$  through M is drawn to meet the overflow line at V and the underflow

(9.2) contd...

line at L. From the figure,  $(x_c)_L = 0.07$ ,  $(y_c)_v = 0.13$

By material balance,  $L + V = F + S = 2800$

$$\text{and } (L)(0.07) + V(0.13) = (1000)(0.25) \Rightarrow L = 1900 \text{ kg (underflow)}$$

$V = 900 \text{ kg (overflow)}$ .

$$\text{Fractional oil removal} = \frac{(900)(0.13)}{(1000)(0.25)} = 0.47$$

Solution using  $x_c$ - $Z_L$  and  $y_c$ - $Z_v$  plots, Ponchon-Savarit type diagram, Fig Prob 9.2(b)

$$F' = 250 \text{ kg and } S' = 1800 \text{ kg (solid-free basis)}$$

At the mixing point ( $M'$ ),  $Z_{M'} = \frac{750 \text{ kg solid}}{(250+1800) \text{ kg lig}} = 0.366$

$$\text{Also, } (x_c)_{F'} = \frac{250}{250} = 1, (Z)_{F'} = \frac{750 \text{ kg solid}}{250 \text{ kg lig}} = 3.0$$

$$(y_c)_{S'} = 0, (Z)_{S'} = 0 \text{ (since the feed solvent is solid-free)}$$

Locate  $F'$  and  $S'$  on the graph, join  $F'$ 's' and locate  $M'$  at  $(Z)_{M'} = 0.366$  on the line  $F'S'$ . Draw the tie-line (it is a vertical line in this case) through  $M$  that meets the overflow curve at  $V'$  and the underflow curve at  $L'$ .

$$(x_c)_{L'} = 0.12 \Rightarrow (y_c)_{V'}, (Z)_{L'} = 0.655$$

$$\text{Total material balance (solid free)} : L' + V' = F' + S' = 250 + 1800$$

$$\text{Solid balance} : (L')(0.655) + (V')(\sim 0.0) = 750 \Rightarrow L' = 1145 \text{ kg}$$

$$\text{Total underflow} = (1145)(1+0.655) = 1895 \text{ kg}$$

$$\text{" overflow} = 905 \text{ kg, Fractional removal of oil} = 45\%$$

Two-stage crosscurrent contact, 1500 kg solvent in each stage

This part of the problem is solved using the  $x_c$ - $x_B$  and  $y_c$ - $y_B$  plots, see Fig. Prob 9.2(a).

$$\text{Stage 1} : F = 1000 \text{ kg; } (x_c)_F = 0.25; S = 1500; (x_c)_S = 0$$

$$\text{Mixing point } M_1 \rightarrow (x_c)_{M_1} = \frac{250}{2500} = 0.1$$

Locate  $F$  and  $S$  and locate  $M_1$  on  $FS$ . The tie-line

(9.4)

9.5

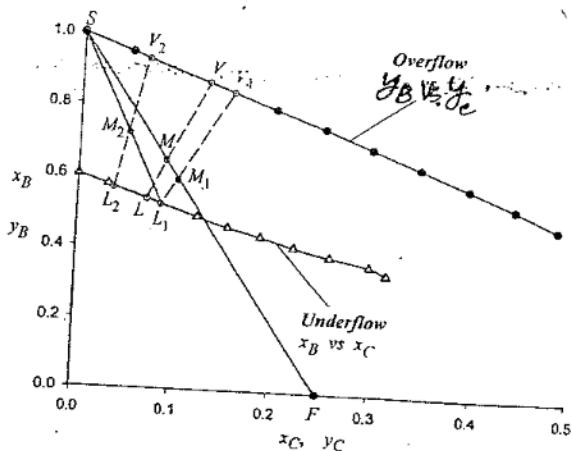


Fig. Prob. 9.2(a): Cross-current leaching, right triangular coordinates

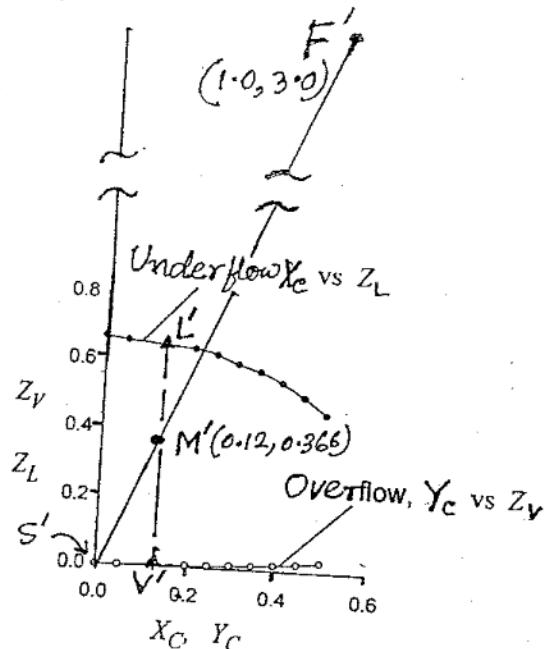


Fig. Prob. 9.2(b): Ponchon-Savarit plot for single-stage leaching.

through  $M_1$  is  $L_1, V_1$ .  $(x_c)_{L_1} = \underline{0.085}$ ,  $(y_c)_{V_1} = \underline{0.155}$   
By material balance,  $L_1 = \underline{1964 \text{ kg}}$ ,  $V_1 = \underline{536 \text{ kg}}$

Stage 2 Underflow from Stage 1:  $L_1 = 1964$ ,  $(x_c)_{L_1} = \underline{0.085}$   
Solvent,  $S = 1500 \text{ kg}$ ; at the mixing point ( $M_2$ ),  $(x_c)_{M_2} = \underline{0.0482}$   
Locate  $M_2$  on the line  $SL_1$ , and draw the tie line  $L_2V_2$  through  $M_2$ . At  $L_2$ ,  $(x_c)_{L_2} = \underline{0.0355}$ ;  $(y_c)_{V_2} = \underline{0.067}$   
Material balance equations:  $L_2 + V_2 = L_1 + S = \underline{3464}$   
Solute balance:  $L_2(0.0355) + V_2(0.067) = (1964)(0.085)$   
 $\Rightarrow L_2 = \underline{1996}$ ;  $V_2 = \underline{1468 \text{ kg}}$

Total oil recovered in stages 1 and 2  
=  $(V_1)(y_c)_{V_1} + (V_2)(y_c)_{V_2} = (536)(0.155) + (1468)(0.067)$   
=  $181.4 \text{ kg}$

Fractional recovery =  $\frac{181.4}{250} = \underline{72.6\%}$



(9.3) The points on the underflow and overflow curves (Ponchon-Savart) can be obtained from the given data.

$$Y_C \rightarrow 0.0 \quad 0.05 \quad 0.10 \quad 0.15 \quad 0.20$$

$$Z_V \rightarrow 0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0$$

$$X_C \rightarrow 0.0 \quad 0.05 \quad 0.10 \quad 0.15 \quad 0.20$$

$$Z_L \rightarrow \frac{1}{1.39} = 0.72 \quad \frac{1}{1.72} = 0.581 \quad 0.49 \quad 0.37 \quad 0.26$$

The clear solution (overflow; it has no solid in it) and the solution retained in sludge (underflow) have the same concentration of wash water rate:

$$\text{Feed, } F = 1000 \text{ kg having } 44.3\% \text{ solids} \Rightarrow F' = 1000 - 44.3$$

$$Z_{F'} = \frac{44.3}{557} = 0.795 ; (X_C)_{F'} = \frac{33.5}{33.5 + 22.2} = 0.601$$

Extract leaving the cascade,  $V'_1$ ;  $(X_C)_{V'_1} = 0.15$  (given).

Sludge leaving the cascade =  $L'_N$ ; mass of water in underflow =  $m_w$  kg.

$$(X_C)_{L'_N} = \frac{16.75}{m_w + 16.75} \rightarrow (Z)_{L'_N} = \frac{44.3}{m_w + 16.75} = 0.654$$

The point  $(X_C)_{L'_N}, (Z)_{L'_N}$  lies on the underflow curve.

By trial,  $m_w = 660$  kg (so that the point  $L'_N$  actually lies on the underflow curve)

$\Rightarrow (X_C)_{L'_N} = 0.0247, (Z)_{L'_N} = 0.654$

$$S' + (1000)(0.222) = 660 + (V'_1)(0.85)$$

$$\text{Alkali balance : } 335 - 16.75 = (V'_1)(0.15)$$

$$\Rightarrow V'_1 = \underline{\underline{2122 \text{ kg}}} ; S' = \underline{\underline{2241 \text{ kg}}} = \text{rate of supply of wash water.}$$

Determination of number of ideal stages (Fig Prob 9.3):

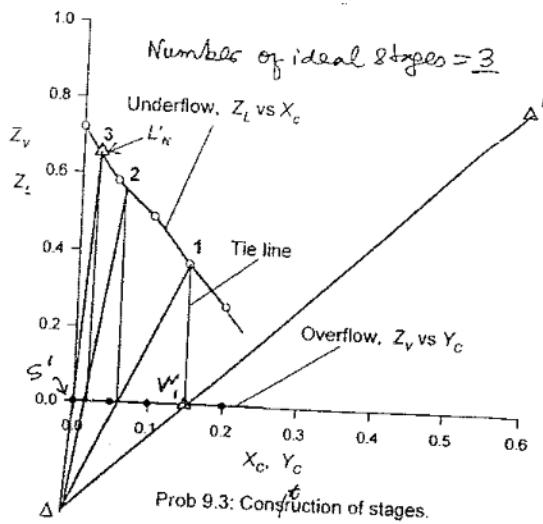
Locate the points  $F'(0.601, 0.795)$ ,  $S'(0, 0)$ ,  $V'_1(0.15, 0)$  and  $L'_N(0.0247, 0.654)$  on the Ponchon-Savart diagram.

Join and extend  $F'V'_1$  and  $L'_NS'$  to meet at A.

Draw stages as usual. Number of ideal stages = 3.

(9.3) Contd...

(9.7)



(9.4) Data points for  $X_C(Z)_L$  and  $Y_C(Z)_V$ , curves are derived from the given data.

$Y_C \rightarrow$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$(Z)_V \rightarrow$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$X_C \rightarrow$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$(Z)_L \rightarrow$	2.303	1.826	1.94	1.89	1.818	1.752	1.68	1.612

Solid feed;  $F' = 250$  kg (solid free basis)

$$(Z)_F = \frac{750}{250} = 3.0; (X_C) = \frac{250}{250} = 1.0$$

$$S' = 2100 \text{ (solid-free)}, (Y_C)_{S'} = 0.005$$

Underflow leaving the battery:

$$(X_C)_{L_N} = \frac{0.015}{0.015 + 0.315} \text{ (extrapolated)}$$

$$= 0.045$$

The points  $F'$ ,  $L'_N$  and  $S'$  are located.

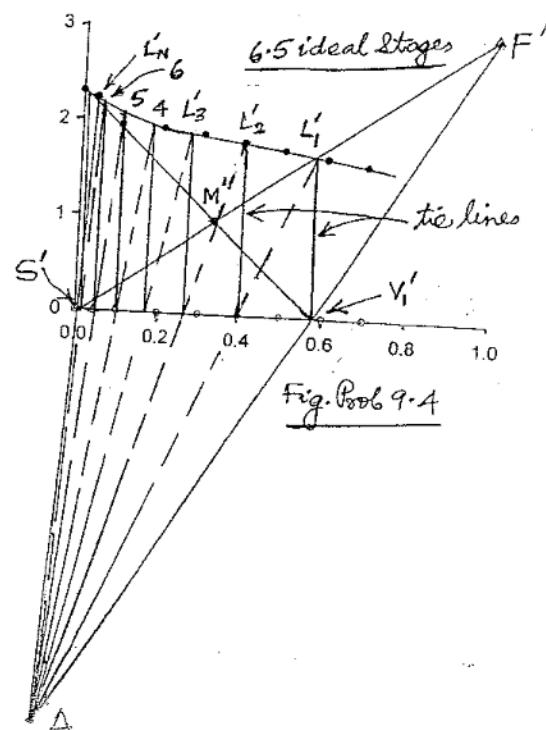
$F'S'$  is joined and  $M'$  is located on it at  $(Z)_{M'} = \frac{750}{250+2100} = 0.32$

Join  $L'_N M'$  and extend to meet the overflow curve at  $V'_1$ .

Join and extend  $L'_N S'$  and  $F' V'_1$  to meet at  $\Delta$ . Construct ideal stages as usual.

Number of ideal stages = 6.5

At  $V'_1$ ;  $(Y_C)_{V'_1} = 0.56 = \text{mass fraction}$   
of oil in the extract



(9.8)

- (9.5) The overflow is solid free, the underflow is constant. Eq (9.17) can be used directly;  $x_0, x_1, x_N$  and  $\alpha$  are to be calculated first. Ref. to Fig. 9.13b for notations.
- $F = 1000 \text{ kg/h}$ ; 15% solute, 2% water, 83% inert solid.
- Mass of inert =  $830 \text{ kg}$ ; mass of water entering with feed =  $20 \text{ kg/h}$ , mass of solute entering =  $150 \text{ kg}$ ; mass of solute leaving the cascade with the underflow ( $0.005 \text{ tons}$  of solids) =  $(830)(0.005) = 4.15$ .
- Mass of solution leaving in the underflow =  $(830)(0.4) = 332 \text{ kg/h}$

Concentration of solution leaving the cascade,  $\frac{4.15}{332} = 0.0125 = x_1$

To calculate  $V'_0$ , we have to make a solute balance in the cascade,  $y_N = 0.15 = x_N$ .

$$4.15 + [m_w - (332 - 4.15)] \frac{0.15}{0.85} = 150 \Rightarrow m_w = 1154.3 \text{ kg/h}$$

(See Example 9.5)

Wash water entering,  $V'_0 = m_w - \text{water entering with feed}$

$$\gamma = \frac{V'_0 / L'}{1134.3 / 332} = \frac{1154.3 - 20}{1134.3} = \frac{3.416}{\underline{\underline{1134.3}}} \text{ kg/h}$$

$$\text{From Eq (9.19), } N = \frac{\log \left[ 1 + (3.416 - 1) \left( \frac{0.15 - 0}{0.0125 - 0} \right) \right]}{\log (3.416)} = 2.8$$

\* \* \* \* \*

(9.6) Constant underflow ( $L'_1 = L'_2 = \dots = L'$ ); no solids in the overflow,

$$\text{From Eq (9.14), } y_{n-1} = \frac{L'}{V'_0} \cdot x_n + y_0 - \frac{L'}{V'_0} x_1 \quad (i)$$

$$\text{Morphree efficiency, } E_M = \frac{x_n - x_{n-1}}{x_n - x_{n-1}^*} \Rightarrow x_{n-1}^* = x_n - \frac{x_n - x_{n-1}}{E_M} \quad (ii)$$

$$\text{Also } y_{n-1} = \alpha x_{n-1}^* \quad (iii)$$

From Eqs. (i), (ii) and (iii),

$$y_{n-1} = \frac{L'}{V'_0} x_n + y_0 - \frac{L'}{V'_0} x_1 = \alpha x_{n-1}^* = \left[ x_n - \frac{x_n - x_{n-1}}{E_M} \right] \cdot \alpha$$

(9.6) contd... On Simplification,

$$X_n - \alpha X_{n-1} = [bX_1 - (E_M Y_0 / \alpha)]a ; a = \frac{1}{1 + E_M (\frac{L'}{\alpha V'_0} - 1)} ; b = \frac{L' E_M}{\alpha V'_0}$$

Solving the difference equation:

$$X_n = K a^n + \frac{[bX_1 - (E_M Y_0 / \alpha)]a}{1-a}$$

$$\text{Put } n=0 \text{ to get, } K = X_0 - \frac{bX_1 - (E_M Y_0 / \alpha)a}{1-a}$$

Put  $n=N$  to get the number of "real stages".

$$N = \frac{\log \left( \frac{X_N - \gamma}{X_0 - \gamma} \right)}{\log a} ; \gamma = \frac{L' X_1 - \alpha X_0 V'_0}{L' - \alpha V'_0}$$

(9.7) Let  $f_n = \frac{\text{kg solution retained}}{\text{kg inert}} \quad * \quad * \quad * \quad *$  at the  $n$ -th stage

Assume  $\frac{1}{f_n} = A + BX_n ; L'_n = W \cdot f_n$ ,  $W$  = constant solid flowrate in the underflow.

Put the above relation in Eq. (9.14) and use  $X_{n-1} = \gamma_{n-1}$

$$V'_0 \left( \frac{A + BX_n}{W} + 1 - \frac{A + BX_n}{A + BX_1} \right) X_{n-1} = X_n + V'_0 \left( \frac{A + BX_n}{W} \right) Y_0$$

On Simplification (see Chen, 1964),  $- \frac{A + BX_n}{A + BX_1} \cdot X_1$

$$X_n X_{n-1} + \alpha X_n + b X_{n-1} + c = 0 \quad (\text{this is non-linear Riccati difference equation})$$

$$a = -\frac{A + BY_0 \gamma}{B(r-1)} ; b = \frac{A\gamma + BX_1}{B(r-1)} ; c = \frac{A(X_1 - \gamma Y_0)}{B(r-1)} ; \gamma = \frac{V'_0}{L'}$$

Solution for the number of ideal stages ( $N$ ) may be

$$\frac{X_N + b + \beta_2}{X_N + b + \beta_1} = \left( \frac{X_0 + b + \beta_2}{X_0 + b + \beta_1} \right) \left( \frac{\beta_2}{\beta_1} \right)^N \quad \text{obtained from}$$

$\beta_1$  and  $\beta_2$  are the unequal real roots

$$\beta = \frac{(a-b) \pm \sqrt{(a+b)^2 - 4c}}{2}$$

Other cases of the roots are discussed in Chen (1964).

(9.8) The problem is similar to Example 9.5 and Prob. 9.5. Take help of those to solve it. Calculated values of the important quantities are given.

$$F = 2 \text{ tons/h} = 2000 \text{ kg/h}; \text{inerts} = 1560 \text{ kg/h}; \text{water} = 40 \text{ kg}$$

$$L' = (1560)(0.5) = 780 \text{ kg/h}; \text{solute leaving with the final sludge} = 12 \text{ kg/h}$$

$$X_1 = \frac{12}{780} = 0.0154; X_0 = Y_0 = 0.$$

$$m_w \text{ calculated by solute balance} = 2966.7; \text{wash water rate}, V'_0 = m_w - 40 = 2926.7 \text{ kg/h}; r = V'_0/L' = \underline{3.752}$$

From Eq (9.17), no. of ideal stages,  $N = \underline{2.5}$

\* \* \* \*

(9.9)  $Y_c - Z_v$  and  $X_c - Z_L$  values are calculated from the given data.

Example of calculation (see the first row)

$$Y_c = \frac{0.046}{0.046 + 0.952} \approx 0.046; Z_v = \frac{0.002}{0.046 + 0.952} \approx 0.002$$

$$X_c = \frac{0.026}{0.026 + 0.542} = 0.04577; Z_L = \frac{0.432}{0.026 + 0.542} = 0.7606$$

Calculated values:

$Y_c \rightarrow$	0.046	0.032	0.021	0.011	0.006	0.002
$Z_v \rightarrow$	0.002	0.001	0.0	0.0	0.0	0.0
$X_c \rightarrow$	0.04577	0.0326	0.0215	0.01096	0.006	0.002
$Z_L \rightarrow$	0.7606	0.715	0.663	0.6611	0.6584	0.6556

The overflow and underflow curves are drawn in Fig Prob 9.9

The tie-lines are vertical for practical purpose.

$$F = 400 \text{ kg slurry/h}; F' = (400)(0.49 + 0.029) = 207.6 \text{ kg (solid-free).}$$

$$\text{Wash water} = 500 \text{ kg/h}; (X_c)_{F'} = \frac{(400)(0.029)}{207.6} = 0.0559$$

Mixing point,  $M'$ .

$$(X_c)_{M'} = \frac{(400)(0.029)}{207.6 + 500} = 0.0164$$

$$(Z)_{M'} = \frac{(400)(0.48)}{207.6 + 500} = 0.272$$

The points  $F'$ ,  $M'$  and  $S'(0,0)$  on the graph.

(9.9) Contd...

(9.11)

The underflow composition  $\rightarrow$  protein concentration  
 $(6.2\%) = \underline{0.002}$

The water content in the underflow for a protein concentration of 0.002 mass fraction can be obtained from the given data by interpolation. Use the last two rows of data on underflow composition.

Water content (if protein conc. = 0.002)

$$= 0.5994 + (0.6028 - 0.5994) \left( \frac{0.0036 - 0.002}{0.0036 - 0.0012} \right) = \underline{\underline{0.6017}}$$

For the sludge leaving the last stage,

$$(X_C)_N = \frac{0.002}{0.6017 + 0.002} = \underline{\underline{0.00331}}$$

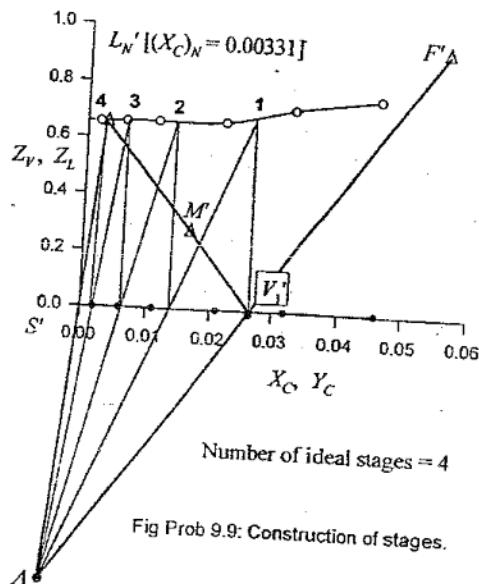
The point  $L'_N$  is located on the underflow curve for  $(X_C) = 0.00331$ .

Join  $L'_N M'$  and extend to meet the overflow curve at  $V'_1$ .

Join and extend  $F'V'_1$  and  $L'_N S'$  to meet at the difference point  $\Delta$ .

Draw stages.

Number of ideal stages  
 $= \underline{\underline{4}}$



- (15) Since the makeup water is supplied at a temperature  $T_L < T_w$ , the steady state water temperature would be a little lower than  $T_w$ . The air temperature and humidity would follow the adiabatic saturation line on the psychrometric chart.
- (33) As the 'approach' increases, the driving force becomes larger and the required packed height decreases. If the 'cooling range' increases, the driving force becomes less and the required packed height increases. If there is a wide seasonal variation of the wet-bulb temperature, the packed height should be calculated on the basis of the largest anticipated  $T_w$ . However, when the actual wet-bulb temperature drops, the fan r.p.m. can be regulated to put less air into the tower so that the desired cooling may be achieved with a lesser consumption of electrical energy.
- (38) (iii); (39) (ii); (40) Sketch (i) shows the correct orientation. For (ii) and (iii) water splash will occur, more for (iii). Further, orientation (ii) will allow more light into the tower, promoting algal growth.
- (41) Since air enters into the tower laterally, the cumulative air rate increases along the height. The V-shaped passage allows gradually increasing flow area up the packing in order to

① (ii) ; ② (iii), this is the humidity of saturated air at  $30^{\circ}\text{C}$  and 1.3 atm - can be obtained from the chart or using the vapor pressure equation of water ; ③ (i), it can be calculated as  $c_{PL} \cdot \Delta T / 20$  ; ④ (ii) ; ⑤ (iii), it can be calculated from the vapor pressure eqn.

$$Y'_s = \frac{P^v}{P - P^v} \times \frac{18.02}{28.97} \Rightarrow 1.6007 \frac{dY'_s}{dT} = \left[ \frac{1}{P - P^v} + \frac{P^v}{(P - P^v)^2} \right] \frac{dP^v}{dT}$$

$\frac{1}{P^v} \frac{dP^v}{dT} = \frac{3984.92}{(T - 39.724)^2}$ . Then put the values of  $P^v$  and  $T (= 298\text{K})$ .

⑥ (i) ; ⑦ (ii) ; ⑧ (ii) ; ⑨ (iii) ; ⑩ (iii), see below

$$\frac{h_L}{k_{y_1}} = \frac{H'_c - H'}{T_L - T_{L_i}} \quad [\text{Eq.(10.28)}] ; \quad T_{L_i} = 39^{\circ}\text{C}, \quad P^v = 0.0692 \text{ bar},$$

$$Y'_s = 0.0456 \text{ kg/kg dry air}, \quad H'_c = 156.5 \text{ at } 39^{\circ}\text{C} \quad [\text{Eq.(10.8)}],$$

$$H' = 135 \text{ kJ/kg dry air (given)}, \quad T_L = 42^{\circ}\text{C} \Rightarrow h_L/k_{y_1} = 7.17 \frac{\text{kJ}}{\text{kg°C}}$$

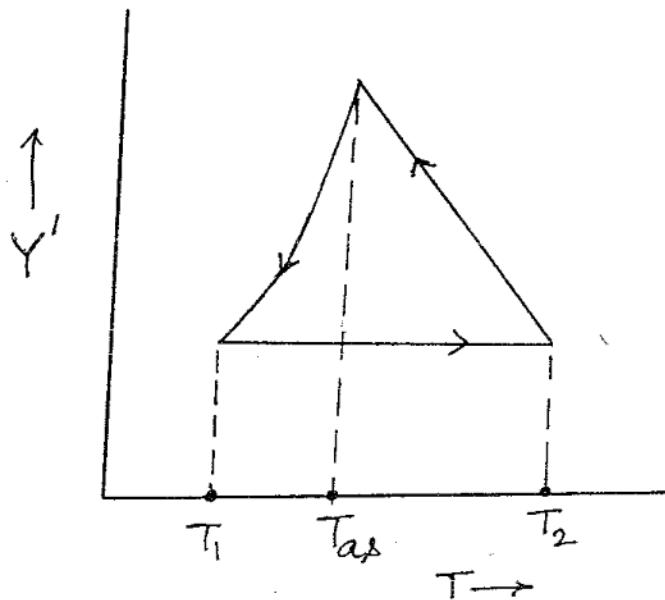
- ⑪ None of these quantities changes ; ⑫ (iii) ; ⑬ (ii) ;  
 ⑭ (ii) ; ⑮ given at the end ; ⑯ (i) ; ⑰ (ii) ; ⑱ (iii) ;  
 ⑲ (i) and (ii) ; ⑳ (i) ; ㉑ (iii) ; ㉒ (ii) ; ㉓ (iii),  
 since the fans are supported on an elevated structure ;  
 ㉔ (i) ; ㉖ No ; ㉗ items (ii), (iv) and (vi) will improve  
 the capacity ; ㉘ (i) ; ㉙ power plants, metallurgical  
 industries, rolling mills, refineries, etc. ; ㉚ (i) ;  
 ㉛ (iii) ; ㉞ (ii) and (iii).

- (15) Since the makeup water is supplied at a temperature  $T_L < T_w$ , the steady state water temperature would be a little lower than  $T_w$ . The air temperature and humidity would follow the adiabatic saturation line on the psychrometric chart.
- (33) As the 'approach' increases, the driving force becomes larger and the required packed height decreases. If the 'cooling range' increases, the driving force becomes less and the required packed height increases.
- If there is a wide seasonal variation of the wet-bulb temperature, the packed height should be calculated on the basis of the largest anticipated  $T_w$ . However, when the actual wet-bulb temperature drops, the fan r.p.m. can be regulated to put less air into the tower so that the desired cooling may be achieved with a lesser consumption of electrical energy.
- (38) (iii); (39) (ii); (40) Sketch (i) shows the correct orientation. For (ii) and (iii) water splash will occur, more for (iii). Further, orientation (ii) will allow more light into the tower, promoting algal growth.
- (41) Since air enters into the tower laterally, the cumulative air rate increases along the height. The V-shaped passage allows gradually increasing flow area up the packing in order to

10.1(b)

accommodate the increasing air rate. Since the horizontal length of the flow path of air through the packing is less than the vertical flow path in a counterflow tower, the pressure drop is less in a cross-flow tower.

(43)



(44) See the last para of Section 10.5.2.

(45) See Section 10.5.4.

## PROBLEMS

(10.2)

(Minor differences from the answers given here are likely owing to 'error' in reading data from the humidity chart.)

(10.1) (i)  $T_G = 33^\circ\text{C}$ ,  $T_w = 23^\circ\text{C}$ . Humidity =  $0.014 \text{ kg/kg dry air}$  (from the chart)

(ii) Enthalpy,  $H' = (2500)(0.014) + [1.005 + (1.88)(0.014)](33)$   
 $= 69 \text{ kJ/kg dry air}$  [Eq.(10.8)]

(iii) Dew point,  $T_d = 16^\circ\text{C}$  (follow the horizontal line through  $Y' = 0.014$  to the saturation curve and read the temperature)

(iv) Humid volume,  $v_H = \left(\frac{1}{28.97} + \frac{0.014}{18.02}\right)(22.4)\left(\frac{33+273}{273}\right)$   
 $= 0.886 \text{ m}^3/\text{kg dry air}$  [Eq.(10.6)].

(v) Humid heat,  $c_H = 1.005 + 1.88 Y' = 1.005 + (1.88)(0.014)$   
 $= 1.031 \text{ kJ/(kg dry air)}(^\circ\text{C})$ ; Eq.(10.7).

(b)  $T_w$  of the air sample if it is heated to  $50^\circ\text{C}$ :

Follow the adiabatic saturation line through the point  $T_G = 50^\circ\text{C}$  and  $Y' = 0.014$  to reach the saturation curve at  $T_w = 27^\circ\text{C}$ .

(c) First check if some water vapor condenses out when the air is cooled from  $33^\circ\text{C}$  to  $15^\circ\text{C}$ .

Vapor pressure of water at  $15^\circ\text{C}$ ,  $P^w = 0.0168 \text{ bar}$

Saturation humidity =  $\frac{0.0168}{1.013 - 0.0168} \cdot \frac{18.02}{28.97} = 0.0105$

(this can be obtained from the chart also)

The saturation humidity at  $15^\circ\text{C}$  is less than  $0.014 \text{ kg/kg}$

$\Rightarrow$  condensation of water occurs,  $(0.014 - 0.0105) = 0.0035 \text{ kg/kg}$

Heat rejected on cooling of air from  $33^\circ\text{C}$  to  $15^\circ\text{C}$

= Sensible heat + latent heat

=  $(c_H)(\Delta T) + (\Delta Y')(R) = (1.03)(33-15) + (0.0035)(2500)$

=  $27.3 \text{ kJ per kg dry air.}$

(10.1) contd..

(d) First calculate the partial pressure of water vapor in air at  $33^{\circ}\text{C}$  and  $\gamma' = 0.014$

$$p_A = \frac{(0.014/18.02)}{(0.014/18.02) + (1/28.97)} \cdot (1.013) \text{ bar} = 0.0223 \text{ bar}$$

As the total pressure is doubled, the partial pressure of the moisture in it is also doubled to  $p_A = 0.0446 \text{ bar}$

Vapor pressure of water at  $50^{\circ}\text{C} = 0.1221 \text{ bar}$

$$\text{Relative humidity} = \frac{0.0446}{0.1221} \times 100 = 36.5\%$$

Dew point of the air at  $50^{\circ}\text{C}$  and 2 atm: It cannot be obtained from the chart since the chart is for air at 1 atm total pressure.

Obtain the temperature at which  $P^v = 0.0446 \text{ bar}$

$$\ln(0.0446) = 11.96481 - \frac{3984.923}{T-39.724} \Rightarrow T = 304 \text{ K}$$

$$= 31^{\circ}\text{C} = \text{dew point of the air}$$

\* \* \* \*

(10.2) Inlet air,  $T_G = 34^{\circ}\text{C}$ ,  $T_w = 23^{\circ}\text{C} \Rightarrow \gamma' = 0.0116/16$

$$\begin{aligned} \text{Enthalpy} &= (0.01)(2500) + [1.005 + (1.88)(0.01)](34) \\ &= 61 \text{ kJ/kg dry air.} \end{aligned}$$

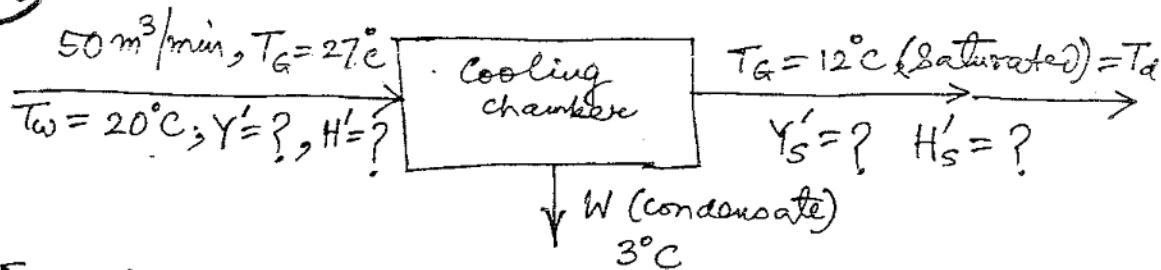
Since the air is contacted with water at the wet bulb temp. ( $23^{\circ}\text{C}$ ), humidification occurs along the adiabatic saturation line  $\Rightarrow$  the enthalpy of air remains constant irrespective of its humidity and temperature.

Exit moisture  $\rightarrow 95\%$ , Saturated  $\Rightarrow \gamma' = 0.0151 \text{ lb/lb}$ , temp is pretty close to  $23^{\circ}\text{C}$ .

The relative humidity can be calculated following Prob 10.1.

\* \* \* \*

10.3



Feed air:  $T_G = 27^\circ\text{C}, T_w = 20^\circ\text{C} \Rightarrow Y' = 0.0137 \text{ kg/kg dry air}$ .

Humid volume [Eq.(10.6)] =  $0.868 \text{ m}^3/\text{kg dry air}$ ;

$$\text{mass of dry air} = \frac{50}{0.868} = 57.58 \text{ kg dry air.}$$

The exit gas is saturated at  $12^\circ\text{C} \Rightarrow Y'_s = 0.01 \text{ kg/kg}$

The rate of water condensation =  $(57.58)(0.0137 - 0.01)$ ,  
 $W = 0.213 \text{ kg/min}$

Enthalpy of the feed gas, Eq.(10.8),  $H' = (2500)(0.0137)$

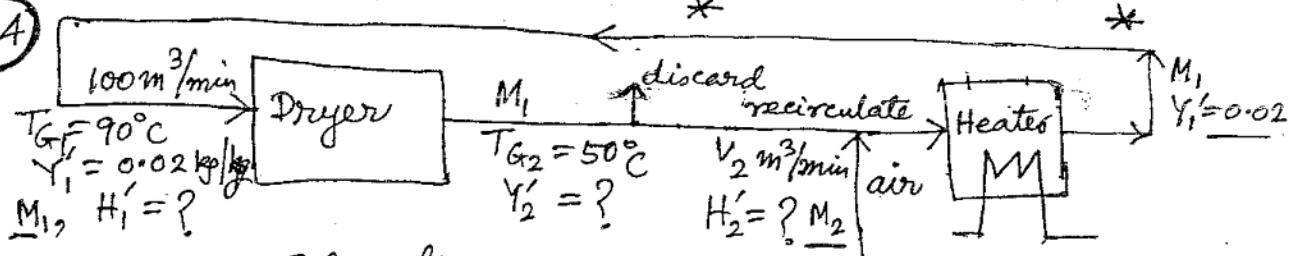
$$+ [1.005 + (1.88)(0.0137)](27) = 62 \text{ kJ/kg}$$

Enthalpy of the exit air ( $12^\circ\text{C}, Y'_s = 0.01$ ),  $H'_s = 37.3 \text{ kJ/kg}$

Enthalpy of condensate ( $3^\circ\text{C}$ ) =  $(4.187)(3) = 12.6 \text{ kJ/kg}$

$$\begin{aligned} \text{Heat load} &= (62 - 37.3)(57.58) - (12.6)(0.213 \text{ kg/min}) \\ &= 1419.5 \text{ kJ/min} \end{aligned}$$

10.4



Schematic of air flow

First calculate/obtain the temperature, humidity and enthalpy of individual gas streams.

(i) Hot gas to the dryer,  $T_G = 90^\circ\text{C}, Y'_1 = 0.02 \text{ kg/kg}$

(10.4) Contd.. Humid heat, Eq (10.6),  $v_H = \left( \frac{1}{28.97} + \frac{0.02}{18.02} \right) (22.4) \cdot \frac{273+90}{273}$  (10.5)

Air feed to the dryer =  $100 \text{ m}^3/\text{h}$ ,  $= 1.0904 \text{ m}^3/\text{kg dry air}$   
 $\Rightarrow$  Mass of air fed to the dryer,

$$= \frac{100}{1.0904} \text{ kg} = 91.71 \text{ kg/h (dry basis)} = M_1$$

Moisture in with the hot air =  $(91.71)(Y'_1) = (91.71)(0.02) = 1.834 \text{ kg/min}$

Air leaves the dryer at  $50^\circ\text{C}$ , adiabatic 'humidification' of air occurs,  $Y_2 = 0.0362 \text{ kg/kg dry air}$  (obtained by following the adiabatic saturation line)

Enthalpy of the feed air,  $H'_1 = (2500)(0.02) + [1.005 + (1.88)(0.02)](90)$   
 $= 143.8 \text{ kJ/kg dry air}$

Recirculated air: Enthalpy,  $H'_2 = 143.8 \text{ kJ/kg}$ , the same mass,  $M_2 = ?$  as the hot feed air to the dryer.

Fresh air Temp,  $T_{G3} = 27^\circ\text{C}$ ;  $T_w = 20^\circ\text{C} \Rightarrow Y'_3 = 0.0137 \text{ kg/kg}$   
 Enthalpy,  $H'_3 = (2500)(0.0137) + [1.005 + (1.88)(0.0137)](27) = 62 \text{ kJ/kg}$   
 $M_3 = ?$

Now the mass of recycled air ( $M_2$ ) and that of the fresh air ( $M_3$ ) can be calculated by mass and moisture balance.

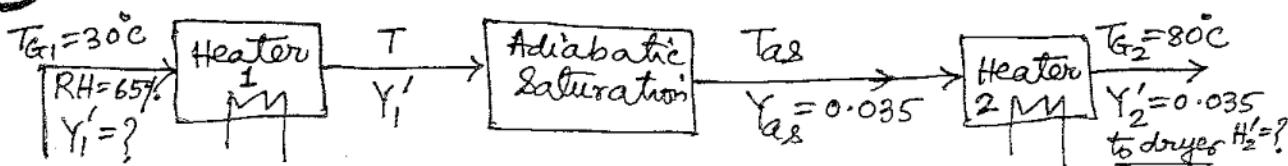
$$M_1 = M_2 + M_3 \Rightarrow M_2 + M_3 = 91.71$$

$$M_1 Y_1 = M_2 Y'_2 + M_3 Y'_3 \Rightarrow (91.71)(0.02) = M_2(0.0362) + M_3(0.0137)$$

$$\text{Solving the above two equations, } M_2 = 25.69 \text{ kg} / \quad M_3 = 66.02 \text{ kg} /$$

Heat requirement in the heater =  $M_1 H'_1 - M_2 H'_2 - M_3 H'_3$   
 $\Rightarrow Q = (143.8)(91.71 - 25.69) - (66.02)(62) = 5400 \text{ kJ/min}$

(10.5)



Humidity of the feed air:

Vapor pressure of water at  $30^\circ\text{C} = 0.042 \text{ bar}$ ; partial pressure of moisture in the air ( $\text{RH} = 65\%$ ) =  $(0.042)(0.65) = 0.0273 \text{ bar}$

(10.5) Contd..  $Y'_1 = \frac{0.0273}{1.013 - 0.0273} \cdot \frac{18.02}{28.97} = 0.01723 \text{ kg/kg dry air}$  (10.6)

The temperature  $T$  (see the sketch) is not known, but on adiabatic saturation, its humidity becomes  $Y'_{as} = 0.035$ . Start from  $Y'_{as} = 0.035$  on the saturation humidity line, move along the adiabatic saturation and reach the point where the humidity is  $Y'_1 = 0.01723$ . The corresponding temperature is  $T = 78^\circ\text{C}$ . Also the adiabatic saturation temperature (for  $Y'_{as} = 0.035$ ) is  $T_{as} = 34^\circ\text{C} \rightarrow$  Ans. to parts (a) and (b).

Now air at  $34^\circ\text{C}$  and  $Y' = 0.035$  is heated to  $80^\circ\text{C}$  in Reator 2. Its enthalpy is

$$H'_2 = (2500)(0.035) + [1.005 + (1.88)(0.035)](80) = 173.2 \text{ kJ/kg}$$

Psychrometric condition of air fed to the dryer :

$$T = 80^\circ\text{C}, Y' = 0.035, H' = 173.2 \text{ kJ/kg}$$

The total heat load can be found to be  $5.837 \times 10^5 \text{ kJ/h}$

\*

\*

\*

\*

(10.6)

Moist warm air :  $T_G = 70^\circ\text{C}, T_w = 60^\circ\text{C} \Rightarrow H'_i = 0.1525 \text{ kg/kg}$

The dehumidified air is essentially at equilibrium with water at  $26^\circ\text{C} \Rightarrow$  its temp =  $26^\circ\text{C}$ , saturated

$\Rightarrow$  its humidity =  $H'_o = 0.0282 \text{ kg/kg}$

Moisture removed =  $0.1525 - 0.0282 = 0.1243 \text{ kg/kg dry air}$

\*

\*

\*

\*

(10.7)

Inlet air at the bottom of the cooling tower,

$$T_{G_1} = 32^\circ\text{C}, T_{w_1} = 22^\circ\text{C}, Y'_1 = 0.013 \text{ kg/kg}, H'_1 = 65.5 \text{ kJ/kg}$$

Water temperature at the bottom,  $T_{L_1} = 29^\circ\text{C}$ .

Draw the saturation humidity line and locate the lower terminal point of the operating line, Q (29, 65.5) on the  $H-T_L, T_G$  plane.

Calculation of the minimum air rate : Draw a tangent

(10.7) Contd..

(10.7)

to the 'equilibrium curve' (i.e. the saturation humidity curve) from the point Q [See Fig Prob 10.7(a)]. The slope of the operating line is

$$\frac{L \cdot C_{OL}}{(G_s)_{min}} = \frac{9.37 \text{ kJ}}{\text{kg}^\circ\text{C}} \cdot \text{Given: } L = \frac{2,25,000 \text{ gfm}}{100 \text{ m}^2} = \frac{8505 \text{ kg/m}^2}{\text{kg/m}^2}$$

$$(G_s)_{min} = \frac{(8505)(4.187)}{9.37} = \frac{3800 \text{ kg/m}^2}{\text{kg/m}^2} \quad C_{OL} = 4.187 \text{ kJ/kg}^\circ\text{C}; 1 \text{ gal} = 3.78 \text{ liter} \approx 3.78 \text{ kg}$$

Actual air rate,  $G_s = \frac{6000}{1+Y_1} = \frac{6000}{1.013} = \frac{5923 \text{ kg/m}^2}{\text{kg/m}^2}$

Actual slope of the operating line =  $\frac{(8505)(4.187)}{5923} = 6.01$

(The actual rate of air supply is  $\frac{5923}{3800}$ , i.e. 1.56 times the minimum)

Draw the actual operating line of slope 6.01 through the point Q. Since warm water enters at  $42^\circ\text{C}$ , the upper terminal of the operating line corresponds to  $T_L = 42^\circ\text{C}$ . Locate P at the intersection of the operating line and the vertical line through  $T_L = 42^\circ\text{C}$ . Since the 'overall volumetric coefficient' is given, the enthalpy driving force at any point is  $H'^* - H'$ ; Fig 10.7(a). The driving force line is vertical. (Note that at P,  $H' = 147$ )

Draw a set of <sup>such</sup> vertical lines <sup>('tie-lines')</sup> and read the values of  $H'^* - H'$  for a set of values of  $H'$ . Such a set of values is given below.

$$H' = 65.5 \quad 83 \quad 102 \quad 115 \quad 127.5 \quad 147$$

$$H'^* = 95.5 \quad 111.5 \quad 129 \quad 129 \quad 159 \quad 186$$

$$\frac{1}{H'^* - H'} = 0.0333 \quad 0.0351 \quad 0.037 \quad 0.037 \quad 0.0317 \quad 0.0256$$

$\frac{1}{H'^* - H'}$  is plotted against  $H'$  and area under the curve for  $H' = 65.5$  to 147 is determined, fig Prob 10.7(b)

$$\text{Area} = N_{to G} = 2.83$$

[The data points to draw the 'equilibrium curve' can be easily calculated from the vapor pressure equation for water;

$T_g, ^\circ\text{C} = 20$	$25$	$27$	$32$	$35$	$37$	$40$	$42$	$45$
$H'^* = 57.4$	$76.3$	$85$	$110.6$	$129$	$143$	$183.5$	$183.5$	$213.7$

$\frac{\text{kJ}}{\text{kg dry air}}$

(10.8)

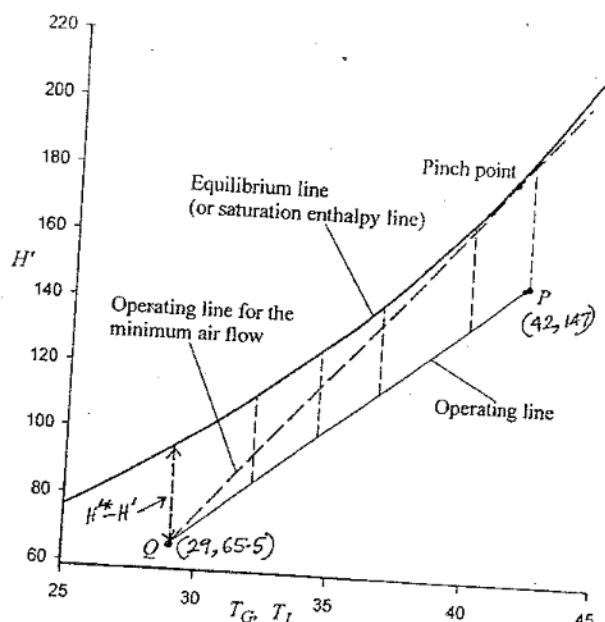
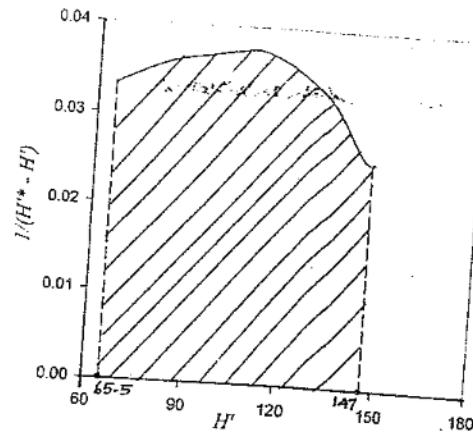


Fig. Prob. 10.7(a)

Figure Prob 10.7(b). Graphical integration for  $N_{IG}$ 

(10.8) The problem is similar to the previous one.

Inlet air:  $T_{G_1} = 30^\circ\text{C}$ ,  $T_{W_1} = 25^\circ\text{C}$ ,  $\gamma' = 0.019$ ,  $H' = 78.7 \text{ kJ/kg}$

Cooled water leaves at  $T_{W_1} = 30^\circ\text{C}$ . Locate  $\& (30, 78.7)$ .

For the minimum air rate, slope of the operating line touching the equilibrium line  $= \frac{8.78}{8.78}$ ,

$$L = 5500 \text{ kg/m}^2 \cdot \text{h}. (G_s)_{\min} = \frac{(5500)(4.187)}{8.78} = 2623 \text{ kg/m}^2 \cdot \text{h}$$

Actual air rate,  $G_s = (1.25)(2623) = 3279 \text{ kg/m}^2 \cdot \text{h}$

Corresponding slope of the actual operating line  $= \frac{(5500)(4.187)}{3279} = 7.02$

Draw the operating line of this slope through the point  $\&$  and locate the point  $P$  on it corresponding to  $T_{L_2} = 45^\circ\text{C}$  (Fig Prob 10.8a).

Draw a set of vertical lines between the operating

(10.8) contd...

(10.9)

line and the equilibrium of length  $H^* - H'$ .  
A set of such values are given below.

$H'$	78.7	90	102.3	114	135	150	175	186
$H^* - H'$	18.75	17.25	15	15	15.75	17.25	21.3	31.05
$\frac{1}{H^* - H'}$	0.0533	0.058	0.0667	0.0667	0.0635	0.058	0.047	0.0317

$\frac{1}{H^* - H'}$  values are plotted against  $H'$  and graphical integration is done as usual (Fig Prob 10.8b); area under the curve =  $N_{tOG} = 6.14$

$$H_{tOG} = \frac{\dot{V}_S}{K_y \bar{a}} = \frac{3279}{2500} = 1.312 \text{ m}$$

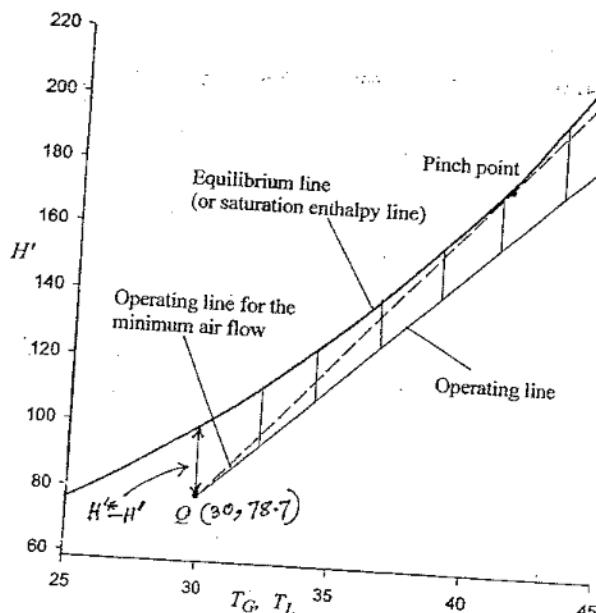


Fig. Prob. 10.8(a)

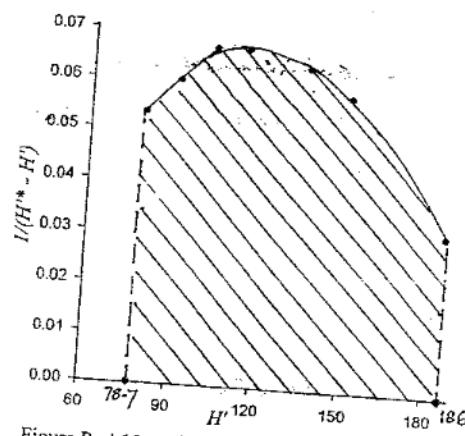


Figure Prob 10.8(b). Graphical integration for  $N_{tOG}$

10-9

Inlet air:  $T_{G_1} = 85^\circ\text{C}$ ,  $T_{w_1} = 46^\circ\text{C} \Rightarrow \gamma_1' = 0.0687 \text{ kg/kg dry air}$   
Enthalpy,  $H_1' = 224 \text{ kJ/kg dry air}$

10-10

Outlet air: wet bulb temp =  $31^\circ\text{C} \Rightarrow \gamma_2' = 0.0272$ ;  $H_2' = 100.7$   
(The outlet air has a wet bulb temp =  $31^\circ\text{C}$   
= its adiabatic saturation temp.)

Inlet water temp,  $T_{L_2} = 26^\circ\text{C}$ . Locate the point P ( $26^\circ\text{C}$ ,  
 $100.7 \text{ kJ/kg}$  on the H-T plane)  
The upper terminal point of the operating line (that  
corresponds to the bottom of the tower in case of dehumidification) is not known. But at the bottom enthalpy  
of the feed moist air is  $H_1' = 224 \text{ kJ/kg}$ .

Now we have to calculate the minimum water rate.  
Since the equil. line is convex downward, the pinch point  
is obtained by drawing a horizontal line  $H' = 224$  to  
meet the equilibrium line (see Fig Prob 10.9) at Q.

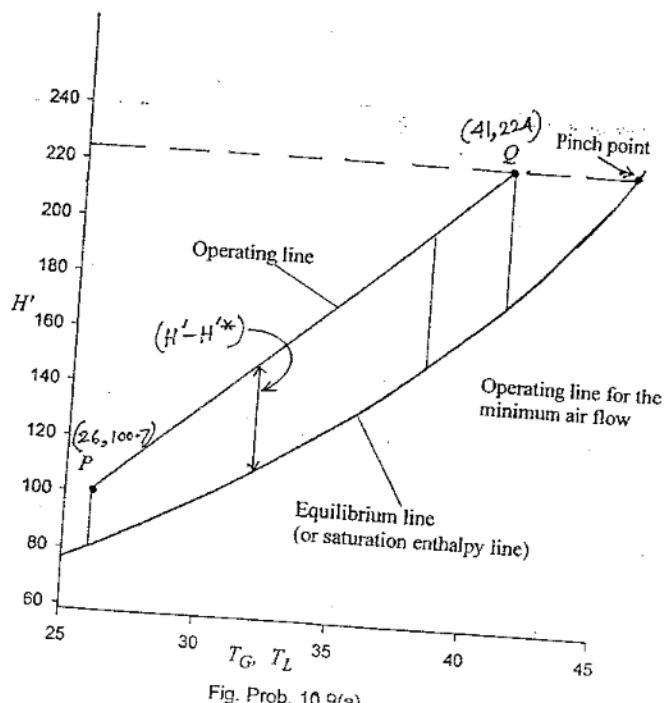
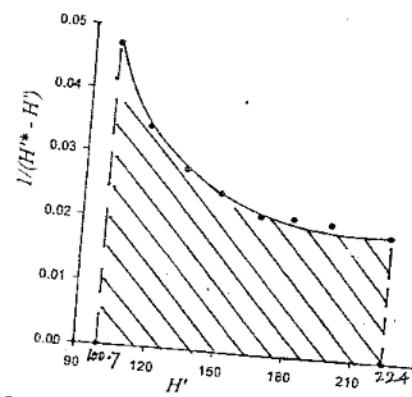


Fig. Prob. 10.9(a)

Fig. Prob 10.9(b). Graphical integration for  $N_{OG}$

Join  $PQ'$  (this is the operating line for the minimum water rate). At  $Q'$ , the water temp is  $T_{L1,\max} = 45.5^\circ\text{C}$ . The minimum water rate can be calculated from the following enthalpy balance.

$$\begin{aligned} G_s(H'_1 - H'_2) &= L_{s,\min}(T_{L1,\max} - T_{l2}) \cdot c_{wL} \\ \Rightarrow 4680(224 - 100.7) &= L_{s,\min}(45.5 - 26)(4.187) \\ \Rightarrow L_{s,\min} &= 7450 \text{ kg/m}^2\text{h} \end{aligned}$$

$$\begin{aligned} G_s &= \frac{G_1}{1 + Y_1} \\ &= \frac{5000}{1 + 0.687} \\ &= \underline{\underline{4680 \text{ kgdry air/m}^2\text{h}}} \end{aligned}$$

The minimum water rate can be determined from the slope of the line  $PQ'$  as well.

$$\begin{aligned} \text{Actual liquid rate} &= (1.25)L_{s,\min} = (1.25)(7450) \\ &= \underline{\underline{9330 \text{ kg/m}^2\text{s}}} \end{aligned}$$

Actual outlet water temp is given by

$$4680(224 - 100.7) = 9330(T_{L1} - 26)(4.187) \Rightarrow T_{L1} = 41^\circ\text{C}$$

Ans to part(ii)

$$\begin{aligned} \text{(ii) Water condensed per hour} &\quad \text{Corresponding operating line: } PQ \\ &= \text{moisture removed from air} = G_s(Y'_1 - Y'_2) \\ &= 4680(0.0687 - 0.0272) = 194 \text{ kg/m}^2\text{h} \\ &\quad (\text{As the water rate changes by about } 2\%) \end{aligned}$$

$$\text{(iii) The packed height : } N_{tOG} = \int_{H'_2}^{H'_1} \frac{dH'}{(H' - H'^*)}$$

As before, draw a set of vertical lines ('tie-lines') between the equilibrium line and the operating line. Obtain the values of  $\frac{1}{(H' - H'^*)}$  for different values of  $H'$ .

$$\begin{aligned} H' &= 100.7 \quad 116.2 \quad 133.5 \quad 149 \quad 167 \quad 181.5 \quad 198 \quad 224 \\ \frac{1}{H' - H'^*} &= 0.047 \quad 0.0342 \quad 0.0278 \quad 0.0244 \quad 0.0213 \quad 0.0213 \quad 0.0208 \quad 0.0194 \end{aligned}$$

$$\text{Value of integral obtained graphically} = 3.23 = N_{tOG}$$

$$\frac{H_{tOG}}{K_y, \bar{a}} = \frac{G_s}{K_y, \bar{a}} = \frac{4680}{2300} = \underline{\underline{2.035 \text{ m}}}$$

$$\begin{aligned} \text{Height of the packed section} &= (2.035)(3.23) \\ &= \underline{\underline{6.57 \text{ m}}} \end{aligned}$$

\* \* \* \*

10-12

(10-10) Vapor pressure of water at  $27^\circ\text{C} = 0.0352 \text{ bar}$   
 Saturation humidity at this temperature =  $0.0224 \text{ kg/kg}$   
 $= Y'_s$  of the feed air.

Humidity of the inlet air ( $T_G = 80^\circ\text{C}$ ,  $T_w = 27^\circ\text{C}$ ) =  $0.001 \text{ kg/kg}$   
 Outlet air is 95% saturated air leaving  
 the tower =  $(0.95)(0.0224) = 0.0213 \text{ kg/kg} = Y'_2$

Since the water is at the adiabatic saturation  
 temperature of the air, use Eq(10.38) to  
 calculate  $N_{t0G} \Rightarrow N_{t0G} = \ln \frac{Y'_s - Y'_1}{Y'_s - Y'_2}$   
 $= \ln \frac{0.0224 - 0.001}{0.0224 - 0.0213} = 3$

Rate of supply of water required  
 $\equiv$  Rate of vaporization loss  
 $= G_s(Y'_2 - Y'_1) = (4000)(0.0213 - 0.001) = 88.4 \text{ kg/h}$   
 \* \* \* for  $1 \text{ m}^2$  of tower cross-section.

(10-11) Range of cooling =  $45 - 28 = 17^\circ\text{C} = 30.6^\circ\text{F}$   
 Evaporation loss\* (Eq 10.40),  $E = (10,000)(30.6)(0.0008) = 245 \text{ gph}$   
 Drift and leakages,  $D = 0.1\%$  of the feed water =  $10 \text{ gph}$   
 Ratio of TDS in the water and in the makeup

$$r = \frac{G_2}{C_1} = \frac{600}{250} = 2.4$$

$$\text{Blowdown rate, } B = \frac{[245 - (10)(2.4 - 1)]}{(2.4 - 1)} = 165 \text{ gph.}$$

$$\text{Make up water} = B + D + E = 165 + 10 + 245 = 420 \text{ gph}$$

\* If we assume that the latent heat of vaporization comes solely from the sensible heat of water, evaporation rate =  $(10,000)(3.78) \text{ kg/h} \cdot (17)(4.187)/2500$

=  $1076 \text{ kg/h} = 285 \text{ gph}$  which is a little more than that calculated above. This is reasonable.

(10.12) The figure here shows the path followed by the drying air.

We shall calculate — <sup>The 'heat duty' of dryer → enthalpies and humidities of air entering and leaving the dryer → air flow → Temp.  $T_1$  → LMTD in the air heater → area of the heater.</sup>

Heat duty of the dryer : Heat required for evaporation of 300 kg/h water =  $(300)(2370) = 7.11 \times 10^5 \text{ kJ/h}$ ; add 15% to take care of sensible heat supply to the solid and heat losses  $\Rightarrow Q = (7.11 \times 10^5)(1.15) = 8.176 \times 10^5 \text{ kJ/h}$ .

Feed air :  $T_G = 25^\circ\text{C}$ , R.H. = 70%. Vapor pressure of water at  $25^\circ\text{C} = 0.0313 \text{ bar} \Rightarrow Y = \frac{(0.0313)(0.7)}{1.013 - (0.0313)(0.7)} = \frac{18}{28.97} = 0.01373 = Y_1$

Exhaust air :  $T_G = 55^\circ\text{C}$ ,  $T_w = 35^\circ\text{C} \Rightarrow Y_2 = \frac{0.0297}{0.0297 - 0.01373} = Y_2$

Rate of supply of drying air =  $\frac{\text{moisture removed/h}}{Y_2 - Y_1} = \frac{300 \text{ kg/h}}{0.0297 - 0.01373} = G_s$   
Total air rate,  $G = (18785)(1 + 0.01373) = 19043 \text{ kg/h} \leftarrow G_s = 18785 \text{ kg/h, dry basis}$   
Enthalpy of exhaust air  $= H_2 = (2500)(0.0297) + [1.005 + (0.0297) \times 132.6 \text{ kJ/kg dry air}] (55)$   
see Eq (10.8),  $H_2 = (2500)(0.0297) + [1.005 + (0.0297) \times$

Heat duty,  $Q = G_s(H'_1 - H'_2) \Rightarrow 8.176 \times 10^5 = (18785)(H'_1 - 132.6)$

$$\Rightarrow H'_1 = 176.1 \text{ kJ/kg dry air.}$$

Corresponding temperature is given by

$$176.1 = (2500)(0.01373) + [1.005 + (1.88)(0.01373)](T_1 - 0) \Rightarrow T_1 = 137.5^\circ\text{C}$$

Latent heat of steam at 5 atm, gauge =  $2090 \text{ kJ/kg}$ , temp,  $T_{st} = 159^\circ\text{C}$

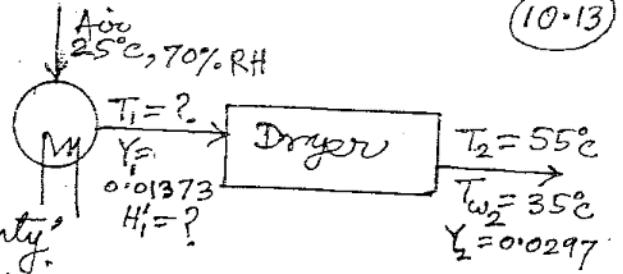
Air heater : temperature driving force,  $(\Delta T)_{in} = 159 - 25 = 134^\circ\text{C}$

$$(\Delta T)_{out} = 159 - 137.5 = 21.5^\circ\text{C}$$

$$\Rightarrow (\Delta T)_m = \frac{61.5^\circ\text{C}}{h = 32 \text{ W/m}^2\text{ }^\circ\text{C}}, \text{ given} \quad \text{Area of the heater} = \frac{Q}{h \cdot (\Delta T)_m} = \frac{8.176 \times 10^5 \times 1000}{(32)(61.5)(3600)} = 115.5 \text{ m}^2$$

$$\text{Rate of steam condensation} \Rightarrow A = 115.5 \text{ m}^2$$

$$= \frac{8.176 \times 10^5 \text{ kJ/h}}{2090 \text{ kJ/kg}} = 391 \text{ kg/h.}$$



(11.1)

- ① (ii), since sand is totally nonhygroscopic and all the moisture is "free"; ② (ii) and (iv); ③ (a) ~~>~~ (iii), compared to a 5 cm thick bed, a 10 cm bed of the same material will have a higher critical moisture; ④ (b)  $\rightarrow$  (i), the transport resistance to water diffusion is less if the particle size is larger, (c)  $\rightarrow$  (i), see Fig. 11.7(a) for these answers; ⑤ (iii)  $\rightarrow$   $28^\circ\text{C}$ , which is the wet bulb temperature of the drying air; ⑥ See p. 11.4;

⑦  $k_c / k_{Y_1} C_H = 1$ , for

$$Y' = 0.018, C_H = 1040 \text{ J/kg}^\circ\text{C}, k_{Y_1} = 0.144 \text{ kg/m}^2\text{s}(\Delta Y') = 520 \frac{\text{kg}}{\text{m}^2\text{h}(\Delta Y')} ;$$

$$t_1 = \left( \frac{W_0}{a} \right) \left( \frac{x_c}{N_c} \right) \ln \frac{x_c}{x_c/10} \text{ for linear falling rate}$$

$$t_2 = \left( \frac{W_0}{a} \right) \left( \frac{x_c^2}{N_c} \right) \left( \frac{1}{x_c/10} - \frac{1}{x_c} \right) \Rightarrow t_1/t_2 = 0.256$$

$$⑨ (ii) \rightarrow (N_t G)_d = \ln \frac{90-33}{70-33} = 0.432 ; ⑩ (i) \rightarrow \ln \frac{T_{G_i} - T_s}{T_{G_0} - T_s} = 35,$$

$$(\Delta T)_{LM} = (T_{G_i} - T_{G_0}) / \ln \left( \frac{T_{G_i} - T_s}{T_{G_0} - T_s} \right) = 35/3.5 = 10 ; ⑪ (ii) ;$$

- ⑫ (ii), since the temperature of the solid remains low; ⑬ (iii);  
 ⑭ yes; ⑮ (ii), (iii) and (vi); ⑯ (i); ⑰ (i); ⑱ (ii);

- ⑲ (i) spray dryer, (ii) drum dryer, (iii) direct heat rotary (hot air, not flue gas, may be used), (iv) rotary, (v) spray, (vi) belt or conveyor, (vii) belt or conveyor/tray; (viii) rotary, (ix) tunnel, (x) tunnel, (xi) indirect heat rotary;

- ⑳ If the surface is completely moist (at the wet bulb temp. of the drying gas), constant rate drying flux,  $N_c = k_{Y'} \Delta Y'$   
 $= (120)(0.037 - 0.018) = 2.28 \text{ kg/h m}^2$  ( $Y'_s = 0.037$ )  $\Rightarrow$  the fraction of surface wetted patches =  $(1.5 \text{ kg/h m}^2) / (2.28 \text{ kg/h m}^2)$   
 $\Rightarrow 65.3\%$  (Note: drying also occurs at the dry patches on the top surface also by vapor diffusion; this has been neglected here.); ⑳ (ii); ㉑ (iii);  
 ㉒ (iii); ㉓ (iii); ㉔ See at the end; ㉕ (c)  $\rightarrow$  so long as

(11.2)

The surface of the solid remains moist, the rate of drying, and hence the amount of heat supplied by the gas, remains constant  $\Rightarrow T_i - T_0$  remains constant. After this the solid gradually warms up, the rate of heat absorption by the solid decreases,  $T_0$  increases and  $T_i - T_0$  decreases; (26) see at the end; (27) (i);

(28) (a)  $\rightarrow$  (i), (b)  $\rightarrow$  (ii), (c)  $\rightarrow$  (ii); (29) (iii), since there is radiant heating, the solid temperature will be above the adiabatic saturation temperature of the gas;

$$(30) (ii) \rightarrow \frac{W_s}{\alpha} \cdot \frac{x_i - x_c}{N_c} = \frac{W_s}{\alpha} \cdot \frac{x_c - x^*}{N_c} \cdot \ln \frac{x_c - x^*}{x_f - x^*}$$

given -  $x_i = 0.33$ ,  $x_f = 0.01$ ,  $x^* \approx 0 \Rightarrow x_c = 0.1$ ;

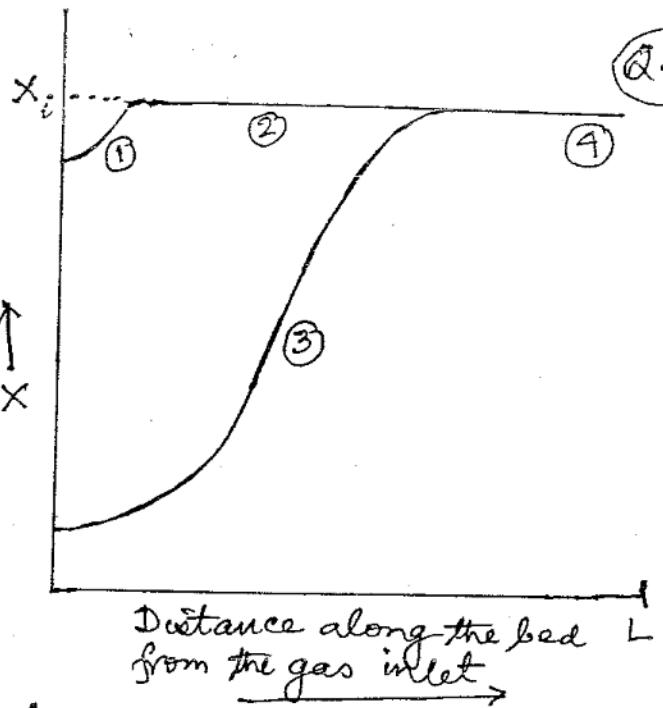
(31) (ii) and (iii); (32) (ii); (33) (ii)

(24) Curve A: There is a single linear falling rate period. A linear falling rate occurs when portions of the surface gets dried up while the rest remains moist. The fractional moist area depends upon the moisture content (see Q.20). This is "unsaturated surface drying". This is observed for a bed of high porosity, big pores, nonhygroscopic solids and a shallow bed.

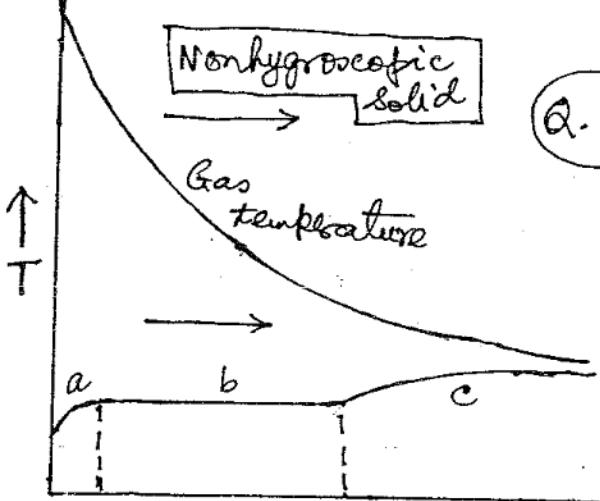
Curve B: The shape of the drying curve indicates two falling rate zones — one linear and another nonlinear. The solid has both large and small pores. Unsaturated surface drying occurs over a part of the falling rate period (the linear part) because of rapid transport of moisture through the bigger pores. The lower nonlinear part indicates substantial diffusional resistance probably because of diffusion through regions having small pores.

Curve C: The entire falling rate zone is nonlinear. The solid has a fine porous structure creating substantial resistance to liquid diffusion all through the falling rate. This behavior also occurs for a hygroscopic solid.

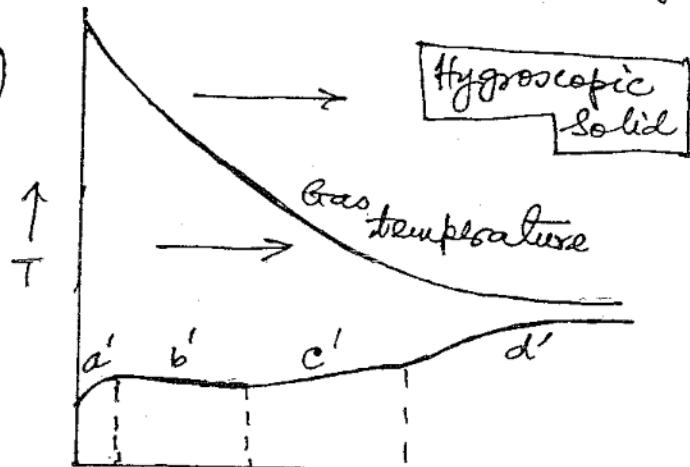
11.3



Distance along the bed  $L$   
from the gas inlet



- Zone a: heating of the solid to the wet bulb temperature,  
Zone b: loss of moisture while the solid is at the 'wet bulb temperature'.  
Zone c: heating of the solid with a small moisture content.



- Zone a': initial heating of the  
Zone b': solid at the 'wet bulb temperature'  
Zone c': the solid gets heated while most of the bound moisture is lost.  
Zone d': further heating of the solid with a small moisture content.

- (35) In a direct heat rotary dryer, a large quantity of heating gas has to be passed since the source of heat is the hot gas alone. Also the exit gas temperature should be considerably above the solid temperature so that a reasonable temperature driving force is available. Thus the drying gas leaves with a large quantity of heat in it. In an indirect heat unit, on the other hand, a small quantity of inert gas flows through the dryer for carrying away the vapor generated during drying. Its quantity and temperature both remain substantially smaller than in the direct heat unit. As a result the indirect heat dryer is more energy efficient.

(5)  $L_s = (H_{tG})_m \cdot (N_{tG})_m = (H_{tG})_h \cdot (N_{tG})_h ;$   $\begin{matrix} h \rightarrow \text{heat transfer} \\ m \rightarrow \text{mass transfer} \end{matrix}$

$$(H_{tG})_m = \frac{G_s}{k_y a} ; \quad (H_{tG})_h = \frac{G_s \cdot C_H}{h_c a}$$

$$\frac{(N_{tG})_m}{(N_{tG})_h} = \frac{L_s / (H_{tG})_m}{L_s / (H_{tG})_h} = \frac{G_s C_H}{h_c a} \cdot \frac{k_y a}{G_s} = \frac{k_y C_H}{h_c} \approx 1 \rightarrow (iii)$$

for the air-water system.

- (6) Since there is some additional heat gain by the solid through the tray floor by conduction, the solid-surface will be above the wet-bulb temperature of the air  $\Rightarrow T_s > T_w \rightarrow (iii)$

- (36) We use a prime ('') to indicate the quantities in SI unit.
- $L_t (\text{feet}) = L'_t (\text{metres}) / 0.3048 ; \quad C_H (\text{Btu}/(\text{lb} \cdot \text{F})) = \frac{C'_H \text{ kJ}/(\text{kg} \cdot \text{C})}{4.187}$
- $d (\text{feet}) = d' (\text{m}) / 0.3048 ; \quad G' (\text{kg}/\text{ft}^2 \cdot \text{h}) = (G')' / 4.8825 \text{ kg}/(\text{m}^2 \cdot \text{h})$
- $\Rightarrow L'_t / 0.3048 = (0.1) \left( \frac{d'}{0.3048} \right) \left( \frac{C'_H}{4.187} \right) \left[ \frac{(G')'}{4.8825} \right]^{0.84}$
- $\Rightarrow L'_t = 0.0063 C'_H \cdot d' [(G')']^{0.84} \rightarrow \text{Eq. (11.44)}$
- where the quantities are expressed in the SI unit.

PROBLEMS

(11.5)

- (11.1) Since the falling rate of drying is linear in the moisture concentration,  $x$ , the drying time can be calculated directly from Eq.(11.5).

Given: Solid loading,  $(W_s/a) = \underline{35 \text{ kg/m}^2 \text{ (dry solid)}}$ ;  $x_i = \text{initial moisture content (dry basis)} = \underline{0.30}$ ;  $x_c = \underline{0.1}$ ;  $x^* = \text{equilibrium moisture} = 0.2\% = \underline{0.002}$ ; final moisture,  $x_f = 1\% = \underline{0.01}$ .  
 $N_c = \underline{4.5 \text{ kg/m}^2 \text{ h}}$

$$\text{Constant rate drying time}, t_c = \frac{(W_s)(x_i - x_c)}{N_c} = \frac{(35)(0.3 - 0.1)}{4.5} = \underline{1.55 \text{ h}}$$

$$\begin{aligned} \text{Falling rate drying time}, t_f &= \frac{(W_s)(x_c - x^*)}{N_c} \ln \frac{x_c - x^*}{x_f - x^*} \\ &= \frac{(35)(0.1 - 0.002)}{4.5} \ln \frac{0.1 - 0.002}{0.01 - 0.002} = \underline{1.91 \text{ h}} \end{aligned}$$

$$\text{Total time}, t_c + t_f = 1.55 + 1.91 = \underline{3.46 \text{ h}} \leftarrow \text{Ans to (i)}$$

- (ii) The drying rate in the falling rate period is  $N = pX + q$

$$\text{At } X = x_c = 0.1, N = N_c = 4.5, \text{ and } X = x^* = 0.002, N_c = 0$$

$$\Rightarrow p = 45.92 \text{ and } q = -0.092$$

$$\text{When } X = 5\% = 0.05, \text{ drying rate } N = (45.92)(0.05) - 0.092$$

$$* * * = \underline{2.2 \text{ kg/m}^2 \text{ h}}$$

- (11.2) We first derive the working equation for the problem.

The initial moisture content =  $x_i$ ; equilibrium moisture,  $x^* \approx 0$   
 $X \geq x_{c_1}$ , the drying rate is constant,  $N = N_c$ .

$x_{c_1} \geq X \geq x_{c_2}$ , first falling rate period,  $N_f = \alpha_1 X + \alpha_2$

$x_{c_2} \geq X \geq 0$ , second " " " ",  $N_f = \beta X^2$

The constants  $\alpha_1, \alpha_2$  and  $\beta$  are to be determined;  $N_c$  is given.

$$\text{At } X = x_{c_1}, \alpha_1 x_{c_1} + \alpha_2 = N_f = N_c \quad \text{--- (i)}$$

$$\text{At } X = x_{c_2}, \alpha_1 x_{c_2} + \alpha_2 = N_c; \text{ also at } X = x_{c_2}, N_c = \beta x_{c_2}^2$$

Also we assume continuity of  $\frac{dN}{dx}$  at  $X = x_{c_2}$

$$\Rightarrow \frac{d}{dx}(N_{f_1}) = \frac{d}{dx}(N_{f_2}) \text{ at } X = x_{c_2}$$

$$\Rightarrow \alpha_1 = 2\beta x_{c_2} \quad \text{--- (ii); equating drying rates at } X = x_{c_2}, \underline{\alpha_1 x_{c_2} + \alpha_2 = \beta x_{c_2}^2} \quad \text{--- (iii)}$$

(11.2) contd...

(11.6)

Solving eqs. (i), (ii) and (iii),  $\alpha_1 = \frac{2N_c}{2x_{c_1} - x_{c_2}} \Rightarrow \alpha_2 = -\frac{N_c x_{c_2}}{2x_{c_1} - x_{c_2}}$

Drying time in different periods and  $\beta = \frac{N_c}{x_{c_2}(2x_{c_1} - x_{c_2})}$

Constant rate period;  $t_c = \left(\frac{W_s}{a}\right) \left(\frac{x_i - x_{c_1}}{N_c}\right)$  (i)

First falling rate period [see Eq.(11.1)]

$$t_{f_1} = \frac{W_s}{a} \int_{x_{c_2}}^{x_{c_1}} \frac{dx}{N} = \frac{W_s}{a} \int_{x_{c_2}}^{x_{c_1}} \frac{dx}{\alpha_1 x + \alpha_2} = \frac{W_s}{a} \cdot \frac{1}{\alpha_1} \ln \frac{\alpha_1 x_{c_1} + \alpha_2}{\alpha_1 x_{c_2} + \alpha_2}$$

$$= \frac{W_s}{a} \cdot \frac{2x_{c_1} - x_{c_2}}{2N_c} \cdot \ln \frac{N_c_1}{N_c_2}$$

$$\boxed{\begin{aligned} N_c_1 &= N_c \\ N_c_2 &= \alpha_1 x_{c_2} + \alpha_2 = \frac{N_c x_{c_2}}{2x_{c_1} - x_{c_2}} \end{aligned}}$$

Second falling rate period:

$$t_{f_2} = \frac{W_s}{a} \int_{x_f}^{x_{c_2}} \frac{dx}{N} = \frac{W_s}{a} \int_{x_f}^{x_{c_2}} \frac{dx}{\beta x^2} = \frac{W_s}{a} \cdot \frac{1}{\beta} \cdot \left( \frac{1}{x_f} - \frac{1}{x_{c_2}} \right)$$

$$= \frac{W_s}{a} \cdot \frac{x_{c_2}(2x_{c_1} - x_{c_2})}{N_c} \cdot \left( \frac{1}{x_f} - \frac{1}{x_{c_2}} \right)$$

Numerical:  $x_i = 0.32$ ,  $x_{c_1} = 0.183$ ,  $x_{c_2} = 0.097$ ,  $x_f = 0.01$ ,  $N_c = 4 \frac{\text{kg}}{\text{hm}^2}$

From (i),  $t_c = (30) \left( \frac{0.32 - 0.183}{4} \right) = 1.027 \text{ hr}; \quad \left( \frac{W_s}{a} \right) = 30 \frac{\text{kg}}{\text{m}^2}$

From (ii),  $t_{f_1} = (30) \cdot \frac{(2)(0.183) - 0.097}{(2)(4)} \ln \frac{2x_{c_1} - x_{c_2}}{x_{c_2}} = 1.029 \text{ hr}$

From (iii),  $t_{f_2} = (30) \cdot \frac{(0.097)[(2)(0.183) - 0.097]}{4} \left( \frac{1}{0.01} - \frac{1}{0.097} \right)$

$$= 17.552 \text{ hr}$$

Total time,  $t = t_c + t_{f_1} + t_{f_2} = 1.027 + 1.029 + 17.552 = 19.6 \text{ hr}$   
Assumption made: continuity of  $dx/dx$  at  $x = x_{c_2}$ .

(11.3)

Case 1, laboratory test, use Eq.(11.5)

Drying time,  $t_1 = \frac{W_s}{a} \cdot \frac{x_i - x_c}{N_c} + \frac{W_s}{a} \cdot \frac{x_c - x^*}{N_c} \ln \frac{x_c - x^*}{x_f - x^*}$

$$\Rightarrow t_1 = \frac{W_s}{a \cdot N_c} \left[ (0.28 - 0.1) + (0.1 - 0.005) \ln \left( \frac{0.1 - 0.005}{0.02 - 0.005} \right) \right]$$

$$\Rightarrow 8.5 = \frac{W_s}{a \cdot N_c} (0.18 + 0.175) \Rightarrow \frac{W_s}{a N_c} = 23.92$$

use this value in further calculations

$$\boxed{\begin{aligned} x_i &= 0.28, x_c = 0.1 \\ x^* &= 0.005, x_f = 2\% \\ t &= 8.5 \text{ hr} = 0.02 \end{aligned}}$$

(11.3) contd...

Case 2, drying time to be calculated, use (11.5) again

$$t_2 = \frac{W_s}{a N_c} \left[ (0.25 - 0.1) + (0.1 - 0.005) \ln \frac{0.1 - 0.005}{0.015 - 0.005} \right]; \text{ here } x_f = 0.015 \text{ (1.5%)}$$

$$= (23.92) [0.15 + 0.214] = \underline{\underline{8.7 \text{ hr}}}$$

The highest drying rate : it occurs in the constant rate period.

$$\frac{W_s}{a N_c} = 23.92 \Rightarrow N_c = \left( \frac{W_s}{a} \right) / 23.92 = 20 / 23.92 = 0.836 \frac{\text{kg}}{\text{m}^2 \text{h}}$$

The lowest drying rate : it occurs when  $x = x_f$

$$\text{At } x = x_c = 0.1, N = N_c = p x_c + q \Rightarrow 0.1p + q = 0.836 \quad p = 8.8$$

$$\text{At } x = x^* = 0.005, N = 0 \Rightarrow 0.005p + q = 0 \quad q = -0.044$$

Lowest drying rate (at  $x_f = 0.015$ ) is  $N = (8.8)(0.015) - 0.044$

$$* * * = \underline{\underline{0.088 \frac{\text{kg}}{\text{m}^2 \text{h}}}} *$$

(11.4) Test drying results :  $x_i = \frac{0.25}{1 - 0.25} = \underline{\underline{0.333}}; x_c = \frac{0.107}{0.893} = \underline{\underline{0.12}}$

$$x_f = \frac{0.0152}{0.9848} = \underline{\underline{0.0154}}; x^* = \frac{0.005}{0.9848} \text{ (65% dry basis); } W_s = 10 \text{ kg; drying time} = \underline{\underline{2 \text{ hr}}}$$

$$\text{Area of drying (consider both sides), } a = 2 \times (0.3 \times 0.18 \text{ m}^2) = \underline{\underline{0.108 \text{ m}^2}}$$

$$\text{Substituting in Eq (11.5), } t = 2 = \frac{W_s}{a N_c} \left[ (0.333 - 0.12) + (0.12 - 0.0154) x \right]$$

$$\Rightarrow 2 = \frac{10}{(0.108) N_c} (0.4644) \Rightarrow N_c = 2.15 \frac{\text{kg}}{\text{m}^2 \text{h}} \quad \ln \frac{0.12 - 0.005}{0.0154 - 0.005}$$

Time of drying of the 1m x 1m board

Since the thickness of the board is the same as that of the above,

$$\frac{W_s}{a} \text{ remains the same} \Rightarrow W_s/a = \frac{10 \text{ kg}}{0.108} = \underline{\underline{9.26 \frac{\text{kg}}{\text{m}^2}}}$$

Drying time to reduce the moisture content from  $x_i = \frac{0.35}{0.65}$

$$= \underline{\underline{0.5385}} \text{ to } x_f = \frac{1}{99} = \underline{\underline{0.0101}}$$

$$\text{Time} = \frac{9.26}{2.15} \left[ (0.5385 - 0.12) + (0.12 - 0.005) \ln \frac{0.12 - 0.005}{0.0101 - 0.005} \right]$$

$$= \frac{9.26}{2.15} (0.7975) = \underline{\underline{3.346 \text{ hr}}}$$

\* \* \* \*

(11.5) In order to solve the problem, the following quantities will be first calculated in sequence:  $R_e \rightarrow h_c \rightarrow k_y \rightarrow (H_{tG})_m$  and  $(N_{tG})_m \rightarrow N_c \rightarrow$  drying time.

(11.8)

Drying gas  $\rightarrow T_g = 90^\circ C$ ,  $Y_i = 0.018 \Rightarrow T_w = 36.5^\circ C$ ,  $Y_s = 0.0417$

Since the outlet humidity is not yet known, calculation of  $R_e$  will be done on the basis of inlet gas flow rate.

$$\text{Humid volume, } V_H = \left( \frac{1}{28.97} + \frac{0.018}{18} \right) (22.4) \left( \frac{36.3}{273} \right) = 1.058 \frac{m^3}{\text{kg dry gas}}$$

$$\text{Density of the inlet gas, } \rho_g = \frac{1+0.018}{1.058} = 0.962 \frac{\text{kg}}{m^3}$$

$$\text{Viscosity, } \mu = 2.1 \times 10^{-5} \frac{\text{N.s}}{m}; \quad U = 0.8 \frac{m}{s};$$

$$G' = (0.8 \times 3600)(0.962) = 2770 \frac{\text{kg}}{m^2 \text{h}}, \text{ based on inlet gas flow;}$$

\* [Since the gas picks up moisture, the outlet gas flow rate will be larger and an average gas rate effective diameter of a pellet,  $d_p = (d_{elec} + 0.5 d_c)^{1/2}$  should be used for better accuracy]

$$= [(5)(5) + 0.5(5)^2]^{1/2} = 6.124 \text{ mm} = \frac{6.124 \times 10^{-3} \text{ m}}{12.13}$$

$$R_e = d_p U \rho_g / \mu = (6.124 \times 10^{-3})(0.8)(0.962) / 2.1 \times 10^{-5} = 224.4$$

$$\text{Calculate } h_c \text{ from Eq. (2.24)} \Rightarrow h_c = 0.214 (2770)^{0.49} / (6.124 \times 10^{-3})^{0.51}$$

$$\text{Calculate } k_y \text{ from the relation } = 140 \frac{\text{W}}{\text{m}^2 \text{C}}$$

$$k_y = h_c / C_H; \quad C_H = \text{humid heat} = [1.005 + (1.88)(0.018)] \frac{\text{kJ}}{\text{kg} \text{C}}$$

$$\Rightarrow k_y = 140 / 1040 = 0.1346 \frac{\text{kg}}{\text{m}^2 \text{s} (\Delta T)} = 1040 \frac{\text{J}}{\text{kg} \text{C}}$$

$$(H_{tG})_m = \frac{G'}{k_y \bar{a}} = \frac{2770}{(0.1346)(3600) \cdot (400)} = 0.0143 \text{ m} \quad [\bar{a} = \frac{400 \text{ m}^2}{\text{m}^3}, \text{ given}]$$

$$\text{Bed height, } l_s = 30 \text{ mm} = 0.03 \text{ m} \Rightarrow (N_{tG})_m = l_s / (H_{tG})_m$$

$$\text{Change of humidity of the gas across the bed} \quad [ \text{Eq. (11.18 b)} ] = 0.03 / 0.0143 = 2.1$$

$$Y_o - Y_i = (Y_s - Y_i) [1 - \exp(-N_{tG})_m] = (0.0417 - 0.018) [1 - e^{-2.1}] = 0.0208$$

$$\text{Rate of flow of 'dry' gas, } G'_s = G'(1 - Y_i) = (2770)(1 - 0.018)$$

$$\text{Rate of moisture removal (per } m^2 \text{ of bed cross-section) in the constant rate period, } N_c = G'_s (Y_o - Y_i) = (2720)(0.0208)$$

$$\text{this is the initial drying rate} \quad \leftarrow = 56.58 \frac{\text{kg}}{\text{m}^2 \text{h}} \text{ Ans to (1)}$$

(11.5) contd.. Mass of dry solid per unit bed cross-section,  
 $(W_s/a)^0 = (1 \text{ m}^2)(0.03 \text{ m})(600 \text{ kg/m}^3) = 18 \text{ kg (per m}^2\text{ bed)}$

11.9

Moisture contents :  $x_i = 0.36$ ,  $x_c = 0.12$ ,  $x^* = 0$ ,  $x_f = 0.002$ .

Total drying time [eq(11.5)],  $t = \frac{W_s}{aN_c} [(x_i - x_f) + (x_c - x^*) \ln \frac{x_c - x^*}{x_f - x^*}]$   
 $= \frac{18}{56.58} \left[ (0.36 - 0.002) + (0.12 - 0.0) \ln \frac{0.12 - 0}{0.002 - 0} \right] = 0.233 \text{ h} = 14 \text{ min.}$

Note :  $R_e$  has been calculated on the basis of inlet gas velocity. Ans. to (ii)

Once the outlet humidity,  $Y_o$ , is known, the outlet gas velocity and density (these depend upon  $Y_o$ ) can be calculated. An average  $R_e$  can now be obtained and used for estimation of  $h_c$  and then of  $k_g$ . However, this will not change the results appreciably since the change in humidity of the gas is small.

(iii) Time required to heat the solid (from the wet bulb temperature of air,  $36.5^\circ\text{C}$ ) to  $70^\circ\text{C}$ :

The bed being thin, the gas temperature will remain practically constant while heating the dry bed. The following differential heat balance can be written over the bed :  $l_s P_s C_{ps} \frac{dT}{dt} = (l_s \cdot i) \bar{a} \cdot h_c (T_G - T)$

$$\Rightarrow \ln \frac{T_G - T_1}{T_G - T_2} = \frac{\bar{a} h_c}{P_s \cdot C_{ps}} \cdot t \quad \text{Given: } P_s = 600 \text{ kg/m}^3, C_{ps} = 400 \text{ J/kg}^\circ\text{C}$$

$$T_1 = 36.5^\circ\text{C}, T_2 = 70^\circ\text{C}$$

$$\Rightarrow t = \frac{(600)(400)}{(400)(140)} \ln \frac{90 - 36.5}{90 - 70} = 4.28$$

(11.6) Drying gas  $\rightarrow T_G = 100^\circ\text{C}$ , wet bulb temp  $= 35^\circ\text{C}$ ,  $Y = 0.0092 \frac{\text{kg}}{\text{kg}}$

Follow the following sequence of calculation :  $v_H \rightarrow P_G \rightarrow$

$G' \rightarrow h_c \rightarrow N_c \rightarrow$  constant and falling rate drying time.

Humid volume of the gas,  $v_H = \left(\frac{1}{28.97} + \frac{0.0092}{18}\right)(22.4)\left(\frac{373}{273}\right) = 1.072 \frac{\text{m}^3}{\text{kg}}$

$$P_G = \frac{(1+0.0092) \text{ kg}}{1.072 \text{ m}^3} = 0.9413 \text{ kg/m}^3; u = 2 \text{ m/s.}$$

$$G' = (2)(3600)(0.9413) = 6777 \text{ kg/m}^2\text{h}$$

Heat transfer coefficient [eq(11.17)],  $h_c = (6.0204)(6777)^{0.8} = 2307 \frac{\text{W}}{\text{m}^2\text{C}}$

Solid temperature = wet bulb temp of the gas =  $T_w = 35^\circ\text{C}$

(11-10)

(11.6) Contd.. Constant drying rate,  $N_c = \frac{h_c(T_g - T_w)}{\lambda_w} = \frac{(23.7)(100-35)}{(2400)(1000)} = 6.42 \times 10^{-4} \text{ kg/m}^2\text{s} = 2.31 \text{ kg/m}^2\text{h}$   
 $(\lambda_w = \text{latent heat of vaporization})$

It is assumed that the gas temperature remains constant.

Moisture contents :  $X_i = \frac{0.3}{0.7} = 0.4286$ ,  $X_e = 0.1$ ,  $X^* = 0.005$ ,  $X_f = 0.01$

Solid loading =  $W_s/a = \frac{(10 \text{ kg wet solid})(0.7)}{0.5 \text{ m}^2} = 14 \frac{\text{kg dry solid}}{\text{m}^2}$

Constant rate drying time,  $t_c = 14(0.4286 - 0.1)/N_c$   
 $= 4.6/2.31 = 1.991 \text{ hr}$

Falling rate drying time (see eq 11.5)

$$t_f = \frac{W_s}{a N_c} \cdot (X_e - X^*) \ln \frac{X_e - X^*}{X_f - X^*} = \frac{14}{2.31} (0.1 - 0.005) \ln \frac{0.1 - 0.005}{0.01 - 0.005}$$

Total drying time,  $t = 1.991 + 1.695 = 3.7 \text{ hr}$

\* \* \*

(11.7) Refer to the expressions for the times of drying in the two falling rate periods derived in Problem 11.2.

$$t_{f1} = \frac{W_s}{a} \cdot \frac{2X_{c1} - X_{c2}}{2N_c} \ln \frac{2X_{c1} - X_{c2}}{X_{c2}} ; t_{f2} = \frac{W_s}{a} \cdot \frac{X_{c2}(2X_{c1} - X_{c2})}{N_c} \left( \frac{1}{X_f} - \frac{1}{X_{c2}} \right)$$

Given:  $X_{c1} = 0.1$ ,  $X_{c2} = 0.05$ ,  $X^* = 0$ ,  $X_f = 0.005$ .

$$\frac{t_{f1}}{t_{f2}} = \frac{\ln \left( \frac{2X_{c1} - X_{c2}}{X_{c2}} \right)}{2X_{c2} \left( \frac{1}{X_f} - \frac{1}{X_{c2}} \right)} = \frac{\ln \left( \frac{0.2 - 0.05}{0.05} \right)}{(2)(0.05) \left( \frac{1}{0.005} - \frac{1}{0.05} \right)} = 0.061.$$

(11.8) Refer to Eq (11.5), put  $t_c = t_f$  for part (i)

$$\Rightarrow \frac{W_s}{a} \cdot \frac{X_i - X_e}{N_c} = \frac{W_s}{a} \cdot \frac{(X_e - X^*)}{N_c} \cdot \ln \frac{X_e - X^*}{X_f - X^*} \quad \begin{cases} X_i = 0.3, X^* = 0 \\ X_f = 0.003, X_e = ? \end{cases}$$

$$\Rightarrow (0.3 - X_e) = X_e \ln \frac{X_e}{0.003} \Rightarrow X_e = 0.072$$

(ii) In this case  $(0.3 - X_e) = \frac{1}{2} \left[ X_e \ln \frac{X_e}{0.003} \right] \Rightarrow X_e = 0.108$

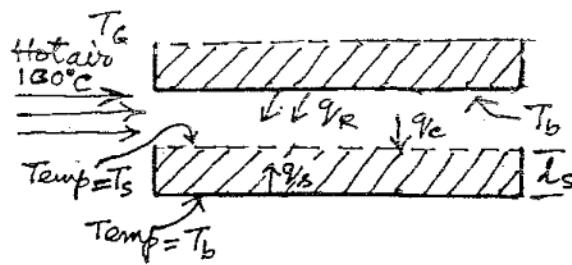
\* \* \*

- (11.9) Two consecutive trays are shown in the sketch.

- Heat input through the four vertical edges of a tray is neglected.

- Heat input occurs at the top open surface by convection and radiation.

It also occurs from the bottom of a tray by conduction through the layer of solid. (If heat inputs by radiation from above and by conduction from below are neglected, the surface temperature of the moist solid is near the wet bulb temperature.)



Steps to be followed to solve the problem: (i) write down the expressions for heat input both from above and from below; (ii) calculate the heat transfer coefficient and therefrom the mass transfer coefficient,  $k_y$ ; (iii) calculate the tray bottom temperature, then the drying rate and drying time (see example 11.3).

Heat input rate to the tray bottom by convection can be equated to the sum of the conduction and radiation heat fluxes at steady state.

$$-h_c(T_G - T_b) = \frac{k_e}{l_s}(T_b - T_s) + \alpha(T_b^4 - T_s^4) = \frac{k_e}{l_s}(T_b - T_s) + h_R(T_b - T_s) \quad (a)$$

$$\text{Heat input to the open surface} = h_c(T_G - T_s) \quad (b)$$

Some of the notations are given in the diagram above;  $h_R$  is the heat transfer coefficient for radiation. Since the open solid surface receives heat by convection, we write the following heat balance at steady state.

$$h_c(T_G - T_b) + h_c(T_G - T_s) = k_y(Y_s - Y_G)A_w \quad (c)$$

The first term on the left has to be evaluated taking help of eq (a) above. Convective heat transfer coefficient,  $h_c = 20 \text{ W/m}^2\text{°C}$ , given.

Hot gas  $\rightarrow T_G = 100^\circ\text{C}$ ,  $T_w = 35^\circ\text{C} \Rightarrow Y_G = 0.0092 \text{ kg/kg dry air}$ ;

Humid heat,  $C_H = 1.005 + (1.88)(0.0092) = 1.022 \text{ kJ/kg}^\circ\text{C}$

Use the relation,  $k_y = h_c/C_H = 20/1.022 = 0.01957 \text{ kg/m}^2\text{°C}^{-1}$

(11.9) Contd.. The adiabatic saturation humidity,  $Y_s$  is related to  $T_s$  through the vapor pressure equation:

$$Y_s = \frac{P^v}{1.013 - P^v} \cdot \frac{18}{29}, \ln P^v = 11.96481 - \frac{3984.923}{T_s + 233.3}, \frac{P^v \text{ in bar}}{T_s \text{ in } ^\circ\text{C}}$$

Eq(c) above has to solved by trial. Assume a value of  $T_s$ , calculate  $T_b$  from eq(c) and check if eq(a) is satisfied. It is seen that  $T_s = 46^\circ\text{C}$  and  $T_b = 84^\circ\text{C}$  satisfy the equations with reasonable accuracy. For  $T_s = 46^\circ\text{C}$ ,  $Y_s = 0.069$ .

The drying rate in the constant rate period

$$= k_r (Y_s - Y_f) = (0.01957)(0.069 - 0.0092) = 1.17 \times 10^{-3} \frac{\text{kg}}{\text{m}^2 \text{s}} \\ = \frac{4.213}{\text{kg/m}^2 \text{hr}} = N_c$$

Moisture concentration terms:  $x_i = 0.35$ ;  $x_c = 0.12$ ;  $x^* = 0.005$ ,  $x_f = 0.01$

The constant rate drying time,  $t_c = \frac{W_0}{A} \left( \frac{x_i - x_c}{N_c} \right)$  [given:  $(0.03)(1200) = W_0$ ]  
 $\Rightarrow t_c = (36) \left( \frac{0.35 - 0.12}{4.213} \right) = 1.965 \text{ hr}$   $\equiv 36 \frac{\text{kg dry solid}}{\text{m}^2}$

The falling rate time,  $t_f = (36) \left( \frac{0.12 - 0.005}{4.123} \right) \ln \frac{0.12 - 0.005}{0.01 - 0.005}$

Total time of drying  $= 1.965 + 3.148 = 5.1 \text{ hr.}$

(11.10) Here the rate of drying in the constant rate period is given as kg moisture per kg dry solid per hour, not on the basis of area of drying. A simpler version of Eq (11.5) can be derived for this case.

Given, constant rate of drying,  $N' = \frac{\text{kg moisture}}{(\text{kg dry solid})(\text{hr})} = -\frac{dx}{dt}$

Integrating over the constant rate period,

$$t_c = - \int \frac{dx}{N'} = \frac{x_i - x_c}{N'} \quad (i)$$

In the falling rate period,  $-\frac{dx}{dt} = r(x - x^*) \quad (ii)$

Integrating,  $t_f = \frac{1}{r} \ln \frac{x_c - x^*}{x_f - x^*} \quad (iii)$

When  $x = x_c$ , Eq (iii) gives  $-\frac{dx}{dt} = N' = r(x_c - x^*)$   
 $\Rightarrow r = N'/x_c$

(11.10) contd...

$$\text{Substituting for } \text{Eq (iii)}, t_f = \frac{x_c - x^*}{\gamma} \ln \frac{x_c - x^*}{x_f - x^*} \quad (iv)$$

Given:  $x_i = 1$ ,  $x_c = 0.15$ ,  $x^* = 0.001$ ,  $x_f = 0.003$ ,  $N = 0.5 \frac{\text{kg}}{\text{m}^2 \cdot \text{hr}}$

$$\begin{aligned} \text{Drying time}, t &= \frac{x_i - x_c}{N} + \frac{x_c - x^*}{N} \ln \frac{x_c - x^*}{x_f - x^*} \\ &= \frac{1 - 0.15}{0.5} + \frac{0.15 - 0.001}{0.5} \ln \frac{0.15 - 0.001}{0.003 - 0.001} = \underline{\underline{3 \text{ hours}}} \end{aligned}$$

Note: Since the drying rate is given on the basis of unit mass of dry solid, the area of drying and the total mass of dry solid do not appear in the calculations.

\* \* \* \*

(11.11) We derive the working equation first. Let  $N_c$  = drying rate in the constant rate period;  $N_{f_1}$  = first falling drying rate;  $N_{f_2}$  = second falling drying rate. Both  $N_{f_1}$  and  $N_{f_2}$  are linear in  $x$ . Given, equil. moisture,  $x^* = 0$

$$x \geq x_{c_1}, N = N_c \dots (i); x_{c_1} \geq x \geq x_{c_2}, N_{f_1} = \alpha_1 x + \beta_1 \dots (ii)$$

$$\text{and } x_{c_2} \geq x \geq x^*, N_{f_2} = \alpha_2 x + \beta_2 \quad (iii)$$

$$\text{At } x = x^* = 0, N_{f_2} = 0 \Rightarrow \beta_2 = 0 \quad (iv)$$

$$\text{At } x = x_{c_2}, N_{f_1} = N_{f_2} \Rightarrow \alpha_1 x_{c_2} + \beta_1 = \alpha_2 x_{c_2} + \beta_2 = \alpha_2 x_{c_2} \dots (v)$$

$$\text{At } x = x_{c_1}, N_{f_1} = N_c \Rightarrow \alpha_1 x_{c_1} + \beta_1 = N_c \quad (vi)$$

Time of drying over a concentration range (see Eq 11.2)

$$t = \frac{w_s}{a} \left( \frac{dx}{N} \right); \frac{w_s}{a} = \frac{7 \text{ kg}}{0.5 \text{ m}^2} = \underline{\underline{14 \frac{\text{kg dry solid}}{\text{m}^2}}}$$

Given, drying time  $t$  to reduce the moisture from  $x_{c_2} = 0.1$  to  $x_f = 0.03$  (in the second falling rate period) =  $1.68 \text{ hr}$

$$\Rightarrow t_{f_2} = - \frac{w_s}{a} \int_{x_{c_2}}^{x_f} \frac{dx}{N_{f_2}} = - 14 \int_{x_{c_2}}^{x_f} \frac{dx}{\alpha_2 x} \quad [\text{from (iii), putting } \beta_2 = 0]$$

$$\Rightarrow 1.68 = \frac{14}{\alpha_2} \ln \frac{x_{c_2}}{x_f} = \frac{14}{\alpha_2} \ln \frac{0.1}{0.03} \Rightarrow \underline{\underline{\alpha_2 = 10.03}}$$

$$\text{From Eq (v), } \alpha_1(0.1) + \beta_1 = (10.03)(0.1) \Rightarrow 0.1\alpha_1 + \beta_1 = 1.003 \quad (vii)$$

$$\text{From Eq (vi), } \alpha_1(x_{c_1}) + \beta_1 = N_c \Rightarrow 0.18\alpha_1 + \beta_1 = 2.5 \quad (viii)$$

(11.13)

Given:  
 $x_{c_1} = 0.18, x_{c_2} = 0.1$   
 $N_c = 2.5 \frac{\text{kg}}{\text{m}^2 \cdot \text{hr}}$

(11.14)

(11.11) contd.. Solving Eqs (vii) and (viii),  $\alpha_1 = 18.71$ ,  $\beta_1 = -0.868$

It is required to calculate the drying time for  $x_i = 0.35$

Constant rate drying time,  $t_c = \frac{W/a}{N_c} (x_i - x_{c1})$  to  $x_f = 0.01$

$$= \frac{14}{2.5} (0.35 - 0.18) = 0.952 \text{ hr}$$

First falling rate drying time:

$$t_{f1} = -\frac{W}{a} \int_{x_{c1}}^{x_f} \frac{dx}{\alpha_1 x + \beta_1} = \frac{W}{a \cdot \alpha_1} \ln \frac{\alpha_1 x_{c1} + \beta_1}{\alpha_1 x_f + \beta_1}$$

$$= \frac{14}{18.71} \ln \left[ \frac{(18.71)(0.18) - 0.868}{(18.71)(0.10) - 0.868} \right] = 0.683 \text{ hr}$$

Second falling rate drying time:

$$t_{f2} = -\frac{W}{a} \int_{x_{c2}}^{x_f} \frac{dx}{\alpha_2 x} = \frac{W}{a \cdot \alpha_2} \ln \frac{x_{c2}}{x_f} = \frac{14}{10.03} \ln \frac{0.10}{0.01} = 3.214 \text{ hr}$$

Total drying time,  $t = t_c + t_{f1} + t_{f2} = 0.952 + 0.683 + 3.214$

$$* * * = 4.85 \text{ hr} *$$

(11.12) The steps to be followed are (see Prob. 11.9): (i) calculate the convective heat transfer coefficient ( $h_c$ ) using Eq 11.17; (ii) calculate the overall coefficient for heat flow from the bottom,  $h_b$ ; (iii) write down the equation balancing heat flow to the top open surface and calculate  $T_s$ ; (iv) calculate  $N_c$  and then the drying time.

Drying gas :  $T_G = 90^\circ C$ ,  $Y_G = 0.015$ ,  $\rho_G = 0.964 \text{ kg/m}^3$   
 $u = 3 \text{ m/s}$  (calculation not shown)

$$G' = (u \rho_G)(3600) = (3)(0.964)(3600) = 10411 \text{ kg/m}^2 \text{ s}$$

$$h_c = 0.0204 (10411)^{0.8} = 33.4 \text{ W/m}^2 \text{ }^\circ C$$

Calculation of the coefficient  $h_b$  for heat flow through the bottom  
given:  $k_s = 2 \frac{W}{m \cdot ^\circ C}$ ,  $l_s = 0.03 \text{ m}$   
Eq (11.7)  $\rightarrow h_b = \frac{h_c k_s}{k_s + h_c l_s} = \frac{(33.4)(2)}{2 + (33.4)(0.03)} = 22.25 \frac{W}{m^2 \text{ }^\circ C}$

Balance of heat flow to the open top surface, Eq (11.8)

$$(h_c + h_b)(T_G - T_s) = k_r(Y_s - Y_G) \cdot 2w \quad (i)$$

(11-12) Contd... Humid heat of the drying air,  $c_H = 1033 \text{ J/(kg dry air)/}^{\circ}\text{C}$

$$\text{Calculate } k_Y = h_c/c_H = \frac{33.4}{1033} = 0.0323 \frac{\text{kg}}{\text{m}^2 \Delta Y}$$

$$T_G = 90^{\circ}\text{C}, Y_G = 0.015, \lambda_w = (2400)(1000) \text{ J/kg}; T_s = \text{Surface temp of solid}$$

$$\text{From Eq.(i) above: } (33.4 + 22.25)(90 - T_s) = 0.0323 (Y_s - 0.015)$$

$$\Rightarrow T_s = 110.9 - 1393 Y_s \quad (\text{ii})$$

Eq.(ii) can be solved, as in Example 11-3, graphically. It can also be solved by trial — assume  $T_s$ , calculate the vapor pressure of water,  $P_v$ , using the vapor press. eq. (see Prob 11-9),

$$\text{and corresponding humidity, } Y_s = \left( \frac{P_v}{1.013 - P_v} \right) \left( \frac{18}{28.97} \right)$$

and check if eq.(ii) is satisfied.

$$\text{Solution: } T_s = 40.7^{\circ}\text{C} \text{ and } Y_s = 0.0503$$

$$\text{The constant drying rate} = N_c = k_Y (Y_s - Y_G)$$

$$= (0.0323)(0.0503 - 0.015)(3600) = 4.105 \frac{\text{kg}}{\text{m}^2 \text{h}}$$

$$\text{Given: } X_i = 0.4, X_c = 0.15, X^* = 0.008, X_f = 0.012$$

$$\text{Dry solid loading, } W_s/a = \lambda_g \cdot P_s = (0.03)(700) = 21 \frac{\text{kg dry solid}}{\text{m}^2}$$

$$\begin{aligned} \text{Drying time} &= \frac{W_s}{aN_c} \left[ (X_i - X_c) + (X_c - X^*) \ln \frac{X_c - X^*}{X_f - X^*} \right] \\ &= \frac{21}{4.105} \left[ (0.4 - 0.15) + (0.15 - 0.008) \ln \frac{0.15 - 0.008}{0.012 - 0.008} \right] = 3.87 \text{ hr} \end{aligned}$$

Part(ii) An outline of the solution is only given. ( $a = 1.5 \times 3 \text{ m}^2/\text{h}$ )

$$Q' = (0.964)(1.5 \times 3)(3600) = 15617 \text{ kg/m}^2 \text{ s} \Rightarrow h_c = 46.2 \text{ W/m}^2 \text{ }^{\circ}\text{C};$$

$$h_b = 27.3 \text{ W/m}^2 \text{ }^{\circ}\text{C}; \quad k_Y = 46.2/1033 = 0.0447 \text{ kg/m}^2 \text{ s}(\Delta Y)$$

$$\text{Heat balance: } (46.2 + 27.3)(90 - T_s) = (0.0447)(Y_s - 0.015)(2400)(1000)$$

$$\Rightarrow T_s = 112 - 1459.6 Y_s \Rightarrow T_s = 40.3^{\circ}\text{C} \text{ & } Y_s = 0.04916 \frac{\text{kg}}{\text{kg}}$$

$$N_c = (0.0447)(0.04916 - 0.015)(3600) = 5.5 \frac{\text{kg}}{\text{m}^2 \text{h}}$$

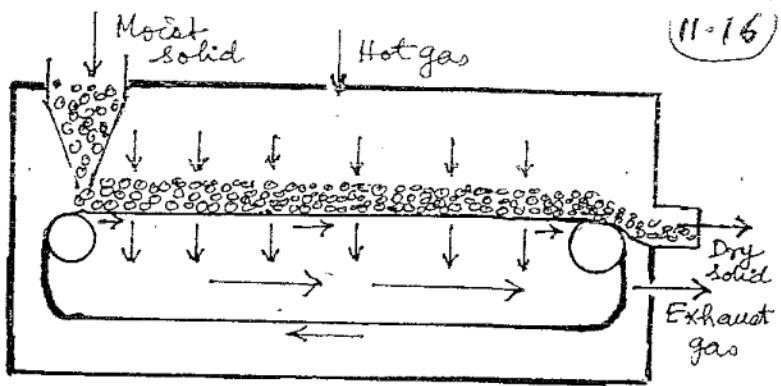
$$\text{Drying time} = 2.9 \text{ hr. Reduction of time} = \frac{3.87 - 2.9}{3.87} = 25\%.$$

Note: (i) The temperature of the solid surface does not change much;

(ii) If there is no heat flow by conduction through the solid, the top surface temp is the wet bulb temp.  $\star \star \star$  of air.

11.13

Since the solid is in 'plug flow' and the drying gas flows through the bed all along the length of the belt, the time of drying is the same as that of batch drying for the same thickness of the bed for through circulation of the drying gas.



11.16

Calculation of batch drying time (see Prob 11.5)

Density of the hot inlet gas,  $\rho_{in} = 0.964 \text{ kg/m}^3$  (see Prob 11.12)

The temperature and humidity of the gas at the exit to the bed will be different, but not known. If the gas leaves at its adiabatic saturation temperature, its humidity is  $Y_s = 0.038$  (from the humidity chart).

Take the humidity of the drying gas in the bed =  $Y = 0.025$

Superficial vel. =  $1 \text{ m/s}$  (this will be used for calculation of

Mass flowrate of the inlet gas =  $(\mu \rho_{in})(3600) = (1)(0.964)(3600)$  flowrate and density of the gas only)

If the average humidity is taken, =  $3470 \text{ kg/m}^2 \text{ h}$

mass flowrate of the gas,  $G' = (3470)(1 - 0.015)(1 + 0.025)$   
(Inlet gas humidity =  $0.015$ ) =  $3503 \text{ kg/m}^2 \text{ h}$

To select the correlation for calculation of the heat transfer coefficient,  $h$ , we need to calculate the Reynolds number

$$G' = 3503 \text{ kg/m}^2 \text{ h}, d_p = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}, \mu = 2.13 \times 10^{-5} \text{ Ns/m}$$

$$\text{Reynolds number, } Re = \frac{d_p G'}{\mu} = \frac{(5 \times 10^{-3})(3503)}{(2.13 \times 10^{-5})(3600)} = 228$$

For this value of  $Re$ , use Eq (11.24) to calculate  $h_c$ .

$$h_c = 0.214 (3503)^{0.49} / (5 \times 10^{-3})^{0.51} = 174 \text{ W/m}^2 \text{ °C}$$

$$\text{Mass transfer coefficient, } k_f = h_c / C_H = \frac{174}{1052} = 0.165 \text{ kg/m}^2 \text{ s} (\Delta Y)$$

(The humid heat  $C_H = 1052 \text{ J/kg}^\circ\text{C}$ )

(11.13) Contd...

The following quantities are given :  $x_i = 0.6$ ,  $x_c = 0.2$ ,  $x^* = 0.01$ ,  $x_f = 0.013$ ; inlet gas humidity,  $y_i = 0.015$ ; specific surface area of the solid,  $a = 500 \text{ m}^2/\text{m}^3$ ; thickness of the bed,  $l_s = 3.5 \text{ cm} = 0.035 \text{ m}$ ; rate of flow of the gas on 'dry basis',  $G'_s = 3470 (1 - 0.015) = 3418 \text{ kg/m}^2\text{h}$ .

Now shall calculate the height of a gas-phase transfer unit,  $(H_{tG})_m$  and then  $(N_{tG})_m$  in order to calculate the drying rate.

$$(H_{tG})_m = \frac{G'}{k_y a} = \frac{3503}{(0.165)(3600)(500)} = 0.0118 \text{ m}; (N_{tG})_m = l_s / (H_{tG})_m = 0.035 / 0.0118 = 2.97$$

The change in humidity of the gas across

$$\text{the bed [eq (11.18 b)]} = Y_o - Y_i = (Y_s - Y_i) [1 - \exp(-N_{tG})_m] \\ = (0.038 - 0.015) [1 - \exp(-2.97)] = 0.0218 \frac{\text{kg}}{\text{kg dry air}}$$

The constant rate of drying,  $N_c = G'_s (0.0218) = (3418)(0.0218)$

$$\text{Dry solid loading} = (0.035)(900) = 31.5 \text{ kg/m}^2 = \frac{W_s}{a} = 78.6 \text{ kg/m}^2$$

Time of drying,  $t =$

$$\frac{W_s}{N_c a} \left[ (x_i - x_c) + (x_c - x^*) \ln \frac{x_c - x^*}{x_f - x^*} \right] = \frac{31.5}{78.6} \left[ (0.6 - 0.2) + (0.2 - 0.01) \ln \frac{0.2 - 0.01}{0.013 - 0.01} \right] \\ = 0.476 \text{ hour}$$

The belt length,  $L = 10 \text{ m}$ ; belt velocity =  $\frac{10 \text{ m}}{0.476 \text{ hr}} = 21 \text{ m/h} = \frac{0.35 \text{ m}}{\text{min}}$

Breadth of the belt =  $2 \text{ m}$

$$\text{Production rate} = \frac{\text{rel. of belt}}{\text{bed depth}} \frac{\text{bed breadth}}{\text{breadth of product}} \frac{\text{density}}{\text{with 1.3% moisture}} = (21 \text{ m/h})(0.035 \text{ m})(2 \text{ m})(900 \text{ kg/m}^3) = 1323 \text{ kg/hour.}$$

$$\text{Production rate (bone dry solid)} = (1323)(1 - 0.013) = 1306 \text{ kg/h}$$

$$\text{Total rate of air supply} = (3470)(2 \times 10 \text{ m}^2) = 69400 \text{ kg/h with a belt area humidity, } y_i = 0.015$$

$$\text{Air supply (dry basis)} = (69400)(1 - 0.015) = 68359 \text{ kg/h}$$

$$\text{Volumetric rate of air supply} = 69400 / 0.964 = 71992 \text{ m}^3/\text{h at } 90^\circ\text{C}$$

Moisture removed from the solid per hour

$$= (1306 \text{ kg/h, bone dry}) (x_i - x_f) = (1306)(0.6 - 0.013) \\ = 766.6 \text{ kg/h}$$

$$\text{Humidity of exhaust air } y_i + \frac{766.6}{68359} = 0.015 + 0.0112$$

$$= 0.0262 \frac{\text{kg}}{\text{kg dry air}}$$

(11-14) The problem is similar to Example 11-5. Refer to Fig Ex 11-5 for notations used.

(11-18)

$$T_{G_1} = 28^\circ\text{C}; Y_1 = 0.013 \text{ kg/kg dry air}; T_{G_2} = 120^\circ\text{C}; Y_2 = 0.023; T_{G_3} = 65^\circ\text{C}; \\ Y_3 = 0.043; T_{S_i} = 26^\circ\text{C}; T_{S_o} = 45^\circ\text{C}; C_{P8} = 1.2 \text{ kJ/kg°C}; \\ W_s = (1200)(1-0.4) = 720 \text{ kg/h}; X_i = 0.4/0.6 = 0.6667; X_o = \frac{0.01}{0.99} = 0.0101.$$

Calculated enthalpy values (Ref. temp. = 0°C):

$$H'_{S_i} = [C_{P8} + (X_i)(4.18)](T_{S_i} - 0) = [1.2 + (0.6667)(4.18)](26) = \frac{103.7 \text{ kJ}}{\text{kg dry air}}$$

$$H'_{S_o} = [1.2 + (0.0101)(4.18)](45) = \frac{55.9 \text{ kJ}}{\text{kg liquid}}$$

$$H'_{G_1} = [1.005 + (1.88)(0.013)](28) + (0.013)(2500) = \frac{61.3 \text{ kJ/kg}}{\text{Basis: 1-hour}}$$

$$H'_{G_2} = \frac{183.3 \text{ kJ/kg}}{\text{Basis: 1-hour}}; H'_{G_3} = \frac{178.1 \text{ kJ/kg}}{\text{Basis: 1-hour}}$$

$$\text{Moisture balance over the dryer: } W_s(X_i - X_o) = G_{S_2}(Y_3 - Y_2)$$

$$\Rightarrow (720)(0.6667 - 0.0101) = G_{S_2}(0.043 - 0.023) \Rightarrow G_{S_2} = \frac{23638 \text{ kg}}{\text{dry basis}}$$

$$\text{Moisture balance at the heater inlet: } G_{S_1}Y_1 + (G_{S_2} - G_{S_1})Y_3 = G_{S_2}Y_2$$

$$\Rightarrow G_{S_1}(0.013) + (23638 - G_{S_1})(0.043) = (23638)(0.023) \Rightarrow G_{S_1} = \frac{15758 \text{ kg}}{\text{dry basis}}$$

$$\text{Air recirculated} = G_{S_2} - G_{S_1} = 23638 - 15758 = 7880 \text{ kg}$$

$$\text{Percentage recirculated} = 7880/23638 \Rightarrow 33.3\%.$$

$$\text{Energy balance over the heater: } G_{S_1}H'_{G_1} + (G_{S_2} - G_{S_1})H'_{G_3} + Q_H = G_{S_2}H'_{G_2}$$

$$\Rightarrow (15758)(61.3) + (7880)(178.1) + Q_H = (23638)(183.3) \Rightarrow Q_H = 1.963 \times 10^6 \text{ kJ/h}$$

$$\text{Energy balance over the dryer: } G_{S_2}H'_{G_2} + W_sH'_{S_i} = G_{S_2}H'_{G_3} + W_sH'_{S_o} + Q'$$

$$\Rightarrow (23638)(183.3) + (720)(103.7) = (23638)(178.1) + (720)(55.9) + Q'$$

$$\Rightarrow Q' = \frac{157333 \text{ kJ/hr}}{\text{rate of}} \rightarrow \text{heat loss from the dryer.}$$

Heater area calculation: We first calculate the temperature of the gas (it is a mixture of fresh air and recirculated gas) at the heater inlet.

Enthalpy balance for the gas mixture:

$$G_{S_1}H'_{G_1} + (G_{S_2} - G_{S_1})H'_{G_3} = G_{S_2} \cdot (H'_{G_2})_{\text{mix}} \Rightarrow \text{calculate } (H'_{G_2})_{\text{mix}} = 100.2 \frac{\text{kJ}}{\text{kg}}$$

$$\text{Temperature of the mixed gas} = (T_G)_{\text{mix}}, Y = Y_2 = 0.023$$

$$(H'_{G_2})_{\text{mix}} = 100.2 = [1.005 + (1.88)(0.023)](T_G)_{\text{mix}} + (2500)(0.023) \Rightarrow (T_G)_{\text{mix}} = 41^\circ\text{C}$$

$$\text{At the heater inlet, } (\Delta T)_{in} = 155 - 41 = 134^\circ\text{C}; \text{ outlet, } (\Delta T)_{out} = 155 - 120 = 35^\circ\text{C}$$

$$(\Delta T)_{LM} = 67^\circ\text{C}. \text{ Area} = Q_H / h_{LM}(\Delta T)_{LM} = (1.963 \times 10^6)(1000) / (20)(3600)(67) = \frac{271 \text{ m}^2}{\text{Steam temp.}, 4 \text{ bar absolute, saturated}} = 155^\circ\text{C}; \text{ heat trans. coeff} = 30 \text{ W/m}^2\text{°C}$$

(11.19)

(11.15) Refer to Fig. Ex 11.6. We have to consider zones I and II only, since no heating of the solid occurs.

Hot gas : temperature,  $T_G = T_{G_3} = 150^\circ\text{C}$ ,  $Y = Y_3 = 0.02$ . The solid is at the wet bulb temperature of the drying gas, i.e.  $T_w = T_s = 45.5^\circ\text{C}$ , corresponding saturation humidity,  $Y_s = 0.0662$ .

We first calculate the temperature and humidity of the gas at boundaries 1 and 2 by moisture and energy balance. Since the dryer is 'adiabatic' the solid remains at  $T_s = 45.5^\circ\text{C}$  all through. Given,  $X_1 = 0.3/0.7 = 0.4286$ ;  $X_c = 0.12/0.88 = 0.1364$ ;  $X_3 = 0.008/0.992 = 0.00806$ ; equil. moisture,  $X^* = 0$ ;  $c_p = 0.8 \text{ kJ/kgK}$

Enthalpy values of the gas and the solid at the boundaries are:

$$\underline{\text{Boundary 3}} : H'_{G_3} = [1.005 + (1.88)(0.02)](150) + 6.02(2500) = 206.4 \text{ kJ/kg}$$

$$\underline{\text{Solid}} : H'_{S_3} = (c_{ps} + c_{pw} \cdot X_3)(T_s - 0) = [0.8 + (0.00806)(4.187)](45.5) = 38 \text{ kJ/kg dry solid}$$

$$\underline{\text{Boundary 2}} : X = X_c, T = T_s = 45.5^\circ\text{C}; H'_{S_2} = 62.4 \text{ kJ/kg dry solid}$$

$$\underline{\text{Boundary 1}} : X_1 = 0.4286, T_s = 45.5^\circ\text{C}, H'_{S_1} = 118 \text{ kJ/kg}; H'_{G_1} = ?$$

$$\underline{\text{Overall moisture balance}} : L_s(X_1 - X_3) = G_s(Y_1 - Y_3); \frac{L_s}{G_s} = \frac{1500 \text{ kg/hr}}{15000 \text{ "}}$$

$$\Rightarrow 1500(0.4286 - 0.00806) = 15000(Y_1 - 0.02) \Rightarrow Y_1 = 0.06205$$

$$\underline{\text{Overall energy balance}} : L_s(H'_{S_1} - H'_{S_3}) = G_s(H'_{G_1} - H'_{G_3})$$

$$215.6 = [1.005 + (0.06205)(1.88)] T_{G_1} \Rightarrow H'_{G_1} = 215.6 \text{ kJ/kg dry air.}$$

$$+ (0.06205)(2500) \Rightarrow T_{G_1} = 54^\circ\text{C} = \text{temp. of exit gas.}$$

$$\underline{\text{Zone II}} : \underline{\text{Moisture balance}}, L_s(X_2 - X_3) = G_s(Y_2 - Y_3), X_2 = X_c = 0.1364$$

$$\Rightarrow 1500(0.1364 - 0.00806) = 15000(Y_2 - 0.02) \Rightarrow Y_2 = 0.0328$$

$$\underline{\text{Energy balance}} : L_s(H'_{S_2} - H'_{S_3}) = G_s(H'_{G_2} - H'_{G_3}) \Rightarrow H'_{G_2} = 210 \text{ kJ/kg}$$

$$[1.005 + (0.0328)(1.88)](T_{G_2} - 0) + (0.0328)(2500) = 210 \Rightarrow T_{G_2} = 120^\circ\text{C}$$

Use Eq. (11.33) to calculate the drying time in zone I.

$$t_I = \frac{G_s}{k_y(L_s a)} \ln \frac{Y_s - Y_2}{Y_3 - Y_1} = \frac{15,000}{(115)(1500 \times 0.08)} \ln \frac{0.0662 - 0.0328}{0.0662 - 0.06205} = 2.267 \text{ hr}$$

Drying time in zone II, Eq. (11.37)

$$t_{II} = \frac{(15,000)(0.1364)}{(115)(1500 \times 0.08)} \cdot \frac{1}{0.00806 + \frac{15000}{1500}(0.0662 - 0.02)} \cdot \ln \frac{(0.0662 - 0.02) \cdot (0.1364)}{(0.0662 - 0.0328)(0.00806)}$$

$$= 0.995 \text{ hr}$$

$$\text{Total drying time} = 2.267 + 0.995 = 3.26 \text{ hr}$$

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(11-20)

11-16 Refer to fig Ex 11-6. Since the dry solid is further heated, all the three zones are to be considered.

Given:  $G_s = 12,000 \text{ kg/h}$  (dry basis);  $L_s = 1000 \text{ kg/h}$  (dry basis);  $X_1 = 0.20$ ;  $X_c = 0.08$ ;  $X_f = 0.01 = X_4 (= X_3, \text{ we assume no drying in zone III})$ ;  $X^* \approx 0$ ;  $T_{G4} = 140^\circ\text{C}$ ;  $Y_4 = 0.017$ ;  $c_{ps} = 1.1 \text{ kJ/kg dry solid \cdot K}$ .

The solid temperature at boundary 3 (which is equal to the wet bulb temperature of the gas at that point) is to be calculated by trial. The dry solid leaves after it is heated to  $65^\circ\text{C}$ . (this is to be checked)

Zone III: Assume  $T_s (= T_w) = 44^\circ\text{C}$ , Saturation humidity,  $Y_s = 0.0625$ ;  $H'_{s3} = 50.2 \text{ kJ/kg dry solid}$  (Calculation not shown);  $H'_{G4} = \frac{187.7 \text{ kJ/kg}}{H'_{s4} = 74.2 \text{ kJ/kg}}$

$$\text{Energy balance : } L_s (H'_{s4} - H'_{s3}) = G_s (H'_{G4} - H'_{G3}) \Rightarrow H'_{G3} = 185.7 \text{ kJ/kg}$$

Since  $Y_3 = 0.017$  and  $H'_{G3} = 185.7 \text{ kJ/kg}$ ,  $T_{G3} = 138^\circ\text{C}$

The wet bulb temp of the gas at boundary 3,  $T_w = 44^\circ\text{C}$  which is the assumed value. It matches.

Heat load in zone III,  $Q_3 = L_s (H'_{s4} - H'_{s3}) = (1000)(24) = 24000 \text{ kJ/h}$

Temperature driving force:  $(\Delta T)_3 = 138-44 = 94^\circ\text{C}$ ,  $(\Delta T)_4 = 140-65 = 75^\circ\text{C}$

$(\Delta T)_m = (94-75)/\ln(94/75) = 84^\circ\text{C}$ ;  $f_c = k_f c_H = (110)(1.037) = 31.7 \text{ W/m}^2\text{C}$

Heating time,  $t_{III} = \frac{Q}{(L_s a')(f_c)(\Delta T)_M} = \frac{24000}{(1000 \times 0.0357)(31.7)(84)} = 0.252 \text{ hr}$

Calculation of  $a'$ : belt breadth = 2 m; solid depth = 4 cm = 0.04 m; mass of solid per meter length of the belt =  $1 \text{ m} \times 2 \text{ m} \times 0.04 \text{ m} \times 700 \text{ kg/m}^3$  (dry solid) = 56 kg; drying area =  $1 \times 2 = 2 \text{ m}^2$ ; drying area per kg =  $= 2 \text{ m}^2 / 56 \text{ kg} = 0.0357 \text{ m}^2/\text{kg}$

Zone I:  $L_s (X_1 - X_4) = G_s (Y_1 - Y_4) \Rightarrow (1000)(0.2 - 0.01) = 12,000(Y_1 - 0.017)$

$L_s (H'_{s4} - H'_{s1}) = G_s (H'_{G4} - H'_{G1}) \Rightarrow H'_{G1} = 188.6 \text{ kJ/kg} \Rightarrow Y_1 = 0.0328$

$H'_{s1} = [1.1 + (0.2)(4.187)](44-0) = 85.2 \text{ kJ/kg dry solid}$ ; this value is used in the above equation]

Exit gas:  $H'_{G1} = 188.6 \text{ kJ/kg}$ ,  $Y_1 = 0.0328 \Rightarrow T_{G1} = 100^\circ\text{C}$ .

Calculate  $Y_2$  by moisture balance:  $L_s (X_1 - X_2) = G_s (Y_1 - Y_2)$ ;  $X_2 = X_c = 0.08$

$$1000(0.2 - 0.08) = 12,000(0.0328 - Y_2) \Rightarrow Y_2 = 0.0228$$

Drying time,  $t_I = \frac{12,000}{(110)(1000 \times 0.0357)} \ln \frac{0.0625 - 0.0228}{0.0625 - 0.0328} = 0.887 \text{ h}$  [Eq 11-33]

Drying time,  $t_{II}$  from Eq 11-37  $\rightarrow t_{II} = 0.975 \text{ h}$  (calculation not shown)

Total time,  $t = t_I + t_{II} + t_{III} = 0.887 + 0.975 + 0.252 = 2.114 \text{ hr}$

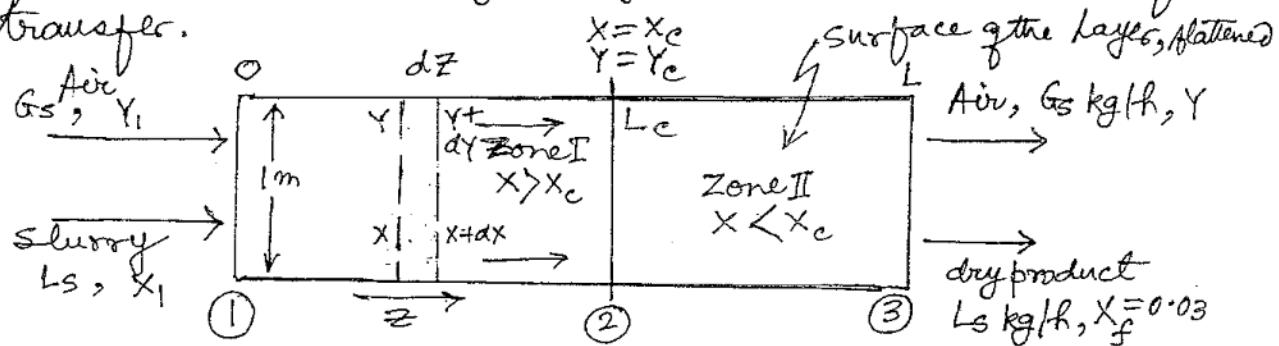
(11.21)

- (11.16) contd... Solid loading per meter belt length = 56 kg/m (dry basis)  
 Desired drying rate = 1000 kg/h  
 $\text{Belt speed} = (1000 \text{ kg/h}) / (56 \text{ kg/m}) = 17.86 \text{ m/h}$   
 $\text{Drying time} = 2.114 \text{ hr.}$   
 $\text{Belt length} = (17.86 \text{ m/h})(2.114 \text{ h}) = \underline{\underline{37.75 \text{ m}}}$
- \* \* \* \*

(11.17) The slurry and the hot drying air move in the same direction while transport of moisture from the surface of the thin layer of material on the rotating drum to the gas occurs. Thus it is a case of cocurrent continuous drying. As suggested in the problem, we neglect heat transfer from the gas to the slurry layer. It is assumed that the temperature of the material is maintained at  $75^\circ\text{C}$  by heat flow from the condensing steam within the drum. It is further assumed that the partial pressure of moisture at the drying surface is the same as vapor pressure of water at  $75^\circ\text{C}$  (note that this will not be true for solutions).

The problem can be solved by using suitably modified forms of Eqs. (11.33) and (11.37) for a countercurrent continuous dryer.

However, it will be first solved using alternative equations derived by using the basic principles of mass transfer.



The moving layer of the slurry, flattened

As in Section 11.12.4, we consider two zones of drying of the layer of slurry on the rotating drum. The moisture content reduces from  $x_1$  to  $x_c$  (critical moisture) in zone I, and the final moisture content is attained at the end of zone II.

(11.22)

(11.17) contd... Let us consider a thin strip of the layer of width  $dz$  over which the moisture concentration in the slurry layer changes by  $dx$  ( $dx$  is inherently negative) and that in the gas layer changes by  $dY$ . Then,

$$-L_s dx = k_y \cdot dz (Y_s - Y) = G_s dY \quad (i)$$

$Y_s$  is the saturation humidity of air at the temperature of the layer (which remains constant at  $75^\circ\text{C}$ ) and  $k_y$  is the mass transfer coefficient. Integrate from  $Y = Y_1$  to  $Y_c$

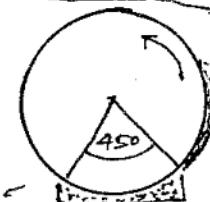
$$\frac{G_s}{k_y} \int_{Y_1}^{Y_c} \frac{dY}{Y_s - Y} = \int_{z=0}^{L_c} dz \Rightarrow L_c = \frac{G_s}{k_y} \ln \frac{Y_s - Y_1}{Y_s - Y_c} \quad \dots \quad (ii)$$

$Y = Y_c$  is the humidity of the bulk gas at  $z = L_c$  where the critical moisture content,  $X = X_c$ , of the slurry layer is reached.

In zone II, the flux is proportional to  $(X - X^*)$ , besides  $Y_s - Y$ .

$$-L_s dx = k_y dz \cdot (Y_s - Y) \cdot \left( \frac{X - X^*}{X_c - X^*} \right) = G_s dY$$

$$\therefore X^* = 0, \int_{L_c}^L dz = \frac{G_s X_c}{k_y} \int_{Y_c}^{Y_2} \frac{dY}{(Y_s - Y) X}$$



The local moisture concentrations  $X$  and  $Y$  are related by the moisture balance :  $L_s(X_1 - X) = G_s(Y - Y_1) \Rightarrow X = X_1 - \frac{G_s}{L_s}(Y - Y_1)$  (iii)

$$\text{i.e. } L - L_c = \frac{G_s X_c}{k_y} \int_{Y_c}^{Y_2} \frac{dY}{(Y_s - Y)} \left[ X_1 - \frac{G_s}{L_s} (Y - Y_1) \right]$$

$$= \frac{G_s X_c}{k_y} \cdot \frac{1}{\frac{G_s}{L_s} (Y_s - Y_1) - X_1} \cdot \ln \left[ \frac{Y_s - Y_c}{Y_s - Y_2} \cdot \frac{X_1 - \frac{G_s}{L_s} (Y_c - Y_1)}{X_1 - \frac{G_s}{L_s} (Y_2 - Y_1)} \right]$$

$$\begin{cases} X_c = X_1 - \frac{G_s}{L_s} (Y_c - Y_1) \\ \text{and} \\ X_2 = X_1 - \frac{G_s}{L_s} (Y_2 - Y_1) \end{cases}$$

$$\text{i.e. } L - L_c = \frac{G_s X_c}{k_y} \cdot \frac{1}{\frac{G_s}{L_s} (Y_s - Y_1) - X_1} \cdot \ln \left[ \frac{Y_s - Y_c}{Y_s - Y_2} \cdot \frac{X_c}{X_2} \right] \quad (iv)$$

Then lengths of zone I and zone II can be calculated.

### Numerical

from Eqs. (ii) and (iv).

Given :  $L_s = 50 \text{ kg/h}$ ;  $G_s = 7 L_s = 350 \text{ kg/h}$ ;  $X_1 = 0.8$ ;  $X_2 = 0.03$ ;  $X_c = 0.15$ ;  $X^* \approx 0$ ;  $Y_1 = 0.015$ ;  $Y_c = 0.108$ ,  $Y_2 = 0.125$  [these can be obtained from the moisture balance eq.(iii)];  $k_y = 160 \text{ kg/m}^2 \text{ (A)} \text{ (B)}$ ; and  $Y_s = 0.383$  (saturation humidity at  $75^\circ\text{C}$ ).

(11-17) contd... Putting the values of different quantities,

$$\text{Eq(ii)}, L_c = \frac{350}{160} \cdot \ln \frac{0.383 - 0.015}{0.383 - 0.108} = 0.637 \text{ m} \quad [\text{on the basis of } 1 \text{ m breadth of the layer}]$$

$$\text{Eq(iv)}, L - L_c = \frac{(350)(0.15)}{160} \cdot \frac{1}{\left[ \frac{350}{50} (0.383 - 0.015) - 0.8 \right]} \cdot \ln \left[ \frac{0.383 - 0.108 - 0.15}{0.383 - 0.125 - 0.03} \right]$$

$$\text{Total length} = 0.637 + 0.309 \text{ m} = \underline{\underline{0.946 \text{ m}}} = 0.946 \text{ m}$$

Drum diameter: If the part of the drum dipped in the slurry makes an angle  $45^\circ$  at the axis, fraction of the area dipped =  $\frac{45}{360} = 0.125$   
 If  $d$  is the drum diameter (length of drum = 1 m),  $= \underline{\underline{0.125}}$

$$(\pi d)(1 - 0.125) = 0.946 \Rightarrow d = \underline{\underline{0.344 \text{ m}}}, \text{ say } \underline{\underline{0.35 \text{ m}}}$$

Drum r.p.m.: It will depend upon the thickness of the film of slurry drawn by the rotating drum. Given: the thickness of the layer is  $h_s = 0.002 \text{ m}$ ; Solids in the slurry is  $714 \text{ kg per m}^3$  of slurry; the production rate =  $50 \text{ kg dry solid/h}$  i.e.  $\frac{50}{1.03} \text{ kg}$  solid =  $51.5 \text{ kg product (with 3% moisture, dry basis)}$ .

$$\text{Speed of the drum} = \frac{50 \text{ kg (bone dry)/h}}{(0.002)(0.946)(714 \text{ kg solid/m}^3)} = \underline{\underline{37 \text{ r.p.h}}} \\ \text{kg/rotation}$$

The same result can be obtained by using modified forms of Eqs. (11-33) and (11-37).

\*

\*

\*

11.20 Given  $t$ - $W$  data can be plotted straightaway [see Fig Prob 11.20(a)]. The drying rate <sup>at</sup> any time and for the corresponding moisture content can be calculated from the following equation.

$$N = \frac{W_1 - W_2}{t_2 - t_1} \cdot \frac{1}{a} \Rightarrow a = 0.1 \text{ m}^2 \text{ (given)}; \text{ and } X = \frac{W_1 - W_2}{W_s} = \frac{W_1 - W_2}{3 \text{ kg}}$$

The calculated values of  $X$  and  $N$  are given below.

$t$ (hr)	0	0.25	0.50	0.75	1.0	1.25	1.50	1.75
$W$ (kg)	4.2	4.145	4.091	4.035	3.978	3.92	3.688	3.811
$X$ (kg/kg)	0.382	0.3637	0.345	0.326	0.307	0.29	0.27	
$N$ (kg/m²·hr)	2.2	2.25	2.24	2.36	2.32	2.08	2.28	

$t$ (hr)	2.0	2.25	2.50	2.75	3.0	3.25	3.50	3.75	4.0
$W$ (kg)	3.756	3.704	3.653	3.605	3.558	3.516	3.480	3.448	3.421
$X$ (kg/kg)	0.252	0.235	0.218	0.202	0.186	0.172	0.16	0.15	0.1403
$N$ (kg/m²·hr)	2.2	2.08	2.04	1.92	1.88	1.68	1.44	1.28	1.08

$t$ (hr)	4.25	4.50	4.75	5.0	5.5	6.0	6.5	7.0	7.5	8.0
$W$ (kg)	3.394	3.368	3.345	3.323	3.285	3.253	3.226	3.202	3.183	3.166
$X$ (kg/kg)	0.131	0.1227	0.115	0.1076	0.095	0.0843	0.0753	0.0673	0.061	0.0553
$N$ (kg/m²·hr)	1.08	1.04	0.92	0.88	0.76	0.64	0.54	0.48	0.38	0.34

The drying rate curve,  $N$  against  $X$ , is plotted on Fig Prob 11.20(b). The falling rate of drying is found to be linear in moisture content,  $X$ .

- (i) From Fig. Prob 11.20(b), critical moisture content,  $X_c = 0.224 \frac{\text{kg}}{\text{kg dry solid}}$ , equilibrium moisture content,  $X^* = 0.0211 \frac{\text{kg}}{\text{kg dry solid}}$ , constant drying rate,  $N_c = 2.27 \frac{\text{kg}}{\text{m}^2 \cdot \text{hr}}$ ; falling rate of drying,  $N = 10.17X - 0.215$ .

11.25

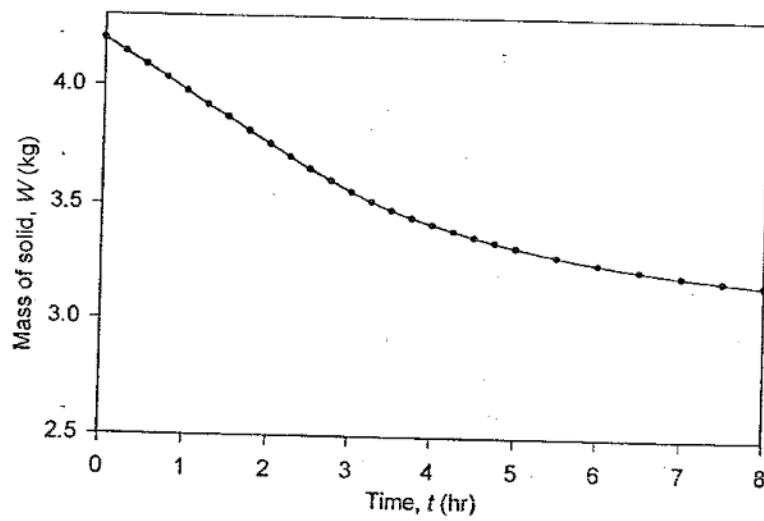


Fig Prob 11.20(a). Plot of the mass of wet solid against time.

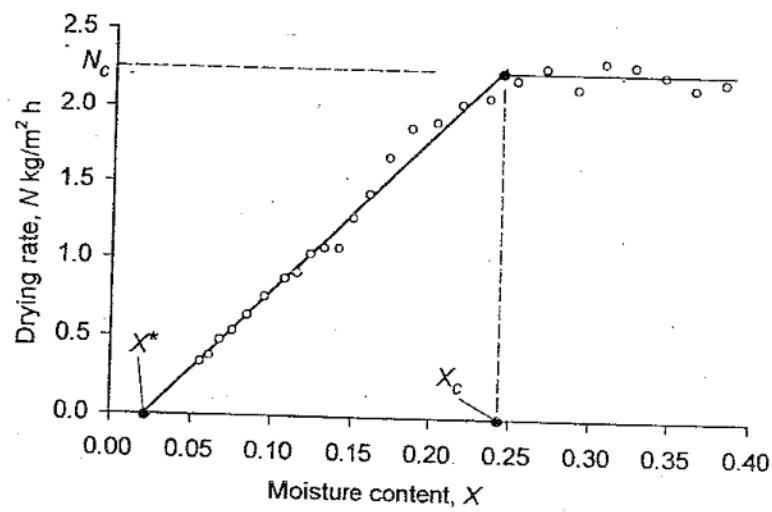


Fig Prob 11.20(b) Drying rate plot, linear falling rate, critical and equilibrium moisture shown.

(ii) Calculation of mass transfer coefficient,  $k_G$

Surface temperature of the wet solid =  $46^\circ\text{C}$ ; vapor pressure of water at  $46^\circ\text{C}$ ,  $p_s = 0.1 \text{ bar}$ ; partial pressure of moisture in the drying air (humidity =  $0.015 \text{ kg/kg dry air}$ , given) is

$$p_b = \frac{(0.015/18)(1.013 \text{ bar})}{(0.015/18)+(0.985/28.92)} = 0.0242 \text{ bar.}$$

Drying rate,  $N_c = 2.27 \frac{\text{kg}}{\text{m}^2 \cdot \text{h}} = 2.27 \frac{\text{kmol}}{18} = 0.126 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}} = k_G(p_s - p_b) = k_G(0.1 - 0.0242)$

$$\Rightarrow k_G = \frac{0.126}{0.1 - 0.0242} = 1.664 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h} \cdot \text{bar}}$$

(iii) When  $X = 0.15$ , drying rate,  $N = (0.17)(0.15) - 0.215 = 1.31 \frac{\text{kg}}{\text{m}^2 \cdot \text{h}}$   
If the solid surface temperature remains as above ( $46^\circ\text{C}$ ), the drying rate from the moist areas of the surface also remains  $2.27 \text{ kg/m}^2 \cdot \text{h}$ .

Fraction of the surface covered by dry patches

$$= \frac{2.27 - 1.31}{2.27}, \text{ ie: } 42.3\%$$

(iv) Calculation of drying time for  $X_i = \frac{0.25}{0.75} = 0.333$ ,

Use Eq.(11-5).  $\frac{W_s}{a} = \frac{3}{0.1} = 30 \frac{\text{kg}}{\text{m}^2}$   $\left| \begin{array}{l} \text{to } X_f = \frac{0.03}{0.97} = 0.0309 \\ N_c = 2.27 \frac{\text{kg}}{\text{m}^2 \cdot \text{h}}; X_c = 0.244, X^* = 0.0211 \end{array} \right.$

$$t = t_c + t_f = \frac{W_s}{a \cdot N_c} (X_i - X_c) + \frac{W_s}{a N_c} (X_c - X^*) \ln \frac{X_c - X^*}{X_f - X^*}$$

$$= \frac{30}{2.27} (0.333 - 0.244) + \frac{30}{2.27} (0.244 - 0.0211) \ln \frac{0.244 - 0.0211}{0.0309 - 0.0211}$$

$$= \underline{\underline{10.38 \text{ hr.}}} \quad * \quad * \quad *$$

11.27

(11.21) The drying time can be calculated directly from Eq (11.50). We first calculate  $v_w$ , which is the volume of material that contains 1 kg of frozen water. No volume change occurs on drying.

Mass of dry material that contains 1 kg water =  $v_w \cdot (400 \text{ kg/m}^3)$ .

$$\text{Moisture content} = \frac{1 \text{ kg}}{400 \cdot v_w \text{ kg dry solid}} = \frac{130}{100} \quad (\text{given: } 130\% \text{ moisture on dry basis})$$

$$\Rightarrow v_w = 1.923 \times 10^{-3} \text{ m}^3/\text{kg}$$

Other data:  $L = 1 \text{ cm} = 10^{-2} \text{ m}$ ;  $\lambda_w = 620 \text{ cal/gm} = 2596 \text{ kJ/kg}$ ;  
 $T_f - T_s = -8 - (-30) = 22^\circ\text{C}$ ;  $k_g = 1.25 \times 10^{-5} \text{ kcal/s.m.K}$   
 $= 0.1884 \text{ kJ/s.h.K}$ ;  $h_c = 70 \text{ kJ/m}^2 \cdot \text{h.K}$ .

Substitute in Eq (11.50).

$$t_f = \frac{2596}{(1.923 \times 10^{-3})(22)} \cdot \left[ \frac{(10^{-2})^2}{(8)(0.1884)} + \frac{0.01}{(2)(70)} \right]$$

$$= 6.136 \times 10^4 \cdot (6.635 \times 10^{-5} + 7.143 \times 10^{-5}) \text{ h} = \underline{\underline{8.45 \text{ h}}}$$

## CHAPTER 12: ADSORPTION

12.1

Short and MCQ's

(12.1)  $q = \frac{6.4p}{1+1.53p} = (4.18) \cdot \frac{1.53p}{1+1.53p} \Rightarrow q_m = 4.18 \text{ millimol/gm adsorbent}$   
 $\Rightarrow q_m = (4.18)(65) \text{ mg/gm} = \underline{\underline{0.272 \text{ kg/kg adsorbent}}}$

(12.2) Henry's law constant for adsorption equilibrium, Eq (12.6):

$$K_H = \left[ \frac{dq}{dp} \right]_{p \rightarrow 0} = \frac{d}{dp} \left[ \frac{14.7p}{(11.2 + p^{0.424})^{2.36}} \right] \text{ for } p \rightarrow 0$$

$$= \left[ \frac{14.7}{(11.2 + p^{0.424})^{2.36}} + \frac{(14.7p)(-2.36)(0.424)p^{-0.576}}{(11.2 + p^{0.424})^{3.36}} \right]_{p \rightarrow 0} = 0.0491 \frac{\text{mmol}}{\text{gm} \cdot \text{kPa}}$$

(12.3) Use Eq (12.12).

$$\begin{aligned} q_A &= q_{Am} \frac{K_A p_A}{1 + K_A p_A + K_B p_B} \\ &= \frac{1.72 p_A}{1 + 0.92 p_A + 0.18 p_B} \\ &= \frac{(1.72)(50)}{1 + (0.92)(50) + (0.18)(50)} \\ &= 1.536 \frac{\text{mmol}}{\text{gm}} = \underline{\underline{76.8 \frac{\text{mg}}{\text{gm}}}} \end{aligned}$$

$$\begin{aligned} q_B &= q_{Bm} \frac{K_B p_B}{1 + K_A p_A + K_B p_B} \\ &= \frac{0.33 p_B}{1 + 0.92 p_A + 0.18 p_B} \\ &= \frac{(0.33)(50)}{1 + (0.92)(50) + (0.18)(50)} \\ &= 0.295 \frac{\text{mmol}}{\text{gm}} = \underline{\underline{10.6 \frac{\text{mg}}{\text{gm}}}} \end{aligned}$$

$p_A = p_B$   
 $= 50 \text{ kPa}$   
 Mol. wt.  
 $M_A = 50$   
 $M_B = 36$

Total adsorbent loading at equilibrium  
 $= 76.8 + 10.6 = \underline{\underline{87.4 \text{ mg/gm adsorbent}}}$

(12.4) At  $p_A = 60 \text{ kPa}$ ,  $p_B = 40 \text{ kPa}$ ,  $q_A = 1.628 \frac{\text{mmol}}{\text{gm}}$  and  $q_B = 0.208 \frac{\text{mmol}}{\text{gm}}$   
 $\Rightarrow x_A = \frac{1.628}{1.628 + 0.208} = \underline{\underline{0.887}} ; x_B = \underline{\underline{0.113}} ; y_A = \underline{\underline{0.6}} ; y_B = \underline{\underline{0.4}}$

$$\alpha_{AB} = \frac{x_A/x_B}{y_A/y_B} = \frac{0.887/0.113}{0.6/0.4} = \underline{\underline{5.23}}$$

(12.5) (iii) ; (12.6) (i), since the adsorbed phase is more orderly;

(12.7) (i) ; (12.8) (iii) ; (12.9) (iii) ; (12.10) (i) molecular sieve,

(ii) silica gel, (iii) active carbon, (iv) active carbon ; (12.11) (i) ;

(12.12) (i) ; (12.13) (i) ; (12.14) (iii) ; (12.16) (i) ; (12.17) (i) ;

(12.18) (ii) ; (12.19) (iii) ; (12.20) (iii) ; (12.21) (i) ; (12.22) ;

(12.23)

CH<sub>12</sub> HClO F<sub>2</sub>

(12.2)

(12.24) (i); (12.25) (i); (12.26) (ii); (12.28) (ii);

(12.1) Let  $C_i$  = initial concentration of a solute;  $C_e$  = equil. concentration in solution;  $V_L$  = volume of solution;  $m_s$  = mass of the adsorbent used.

Amount of solute removed from solution =  $V_L(C_i - C_e)$

Then  $\eta_e = \frac{V_L(C_i - C_e)}{m_s} = K'(C_e)^{\frac{1}{n}}$ . Consistent units are to be used.

### (a) Adsorption of 2,4-DCP

$$V_L \rightarrow 1000 \text{ kg} \approx 1000 \text{ liter}; C_i = 200 \text{ ppm} = 200 \text{ gm}/10^6 \text{ gm} \approx 200 \text{ gm}/\text{m}^3$$

$$\Rightarrow \frac{1000 \text{ liter} (200 - C_e) \text{ mg/liter}}{1000 \text{ gm}} = K'(C_e)^{\frac{1}{n}} = 502 (C_e)^{\frac{1}{7.87}}$$

$$\Rightarrow C_e = \frac{7.15 \times 10^{-4} \text{ mg/liter}}{(502)^{\frac{1}{7.87}}}$$

### (b) Adsorption of 2,6-DCP

$$\frac{1000 (100 - C_e)}{1000} = 177 (C_e)^{\frac{1}{4.83}} \Rightarrow C_e = 0.0632 \text{ mg/liter}$$

(12.2) Freundlich isotherm :  $\eta_e = K' C_e^{\frac{1}{n}} \Rightarrow \ln \eta_e = \ln K' + \frac{1}{n} \ln C_e$

$\eta_e$  and  $C_e$  values are given in the table. Calculate  $\ln \eta_e$  and  $\ln C_e$  and make a plot. Fit the data by the least square line. The plots are shown in Fig Prob 12.2.

Results : Glycine -  $\eta_e = 5.21 \times 10^{-4} C_e^{0.9736}$

Phenylalanine -  $\eta_e = 4.37 \times 10^{-3} C_e^{0.952}$  ; correlation coeff.  $r^2 = 0.999$

The correlation coefficients are very high and the fitting is very good.

(12.3)(a) Four sets of adsorption equilibrium data ( $\eta$  vs  $P$ ) are given. Use Eq. 12.7 to fit the Langmuir isotherm.

Calculate and plot  $\eta/P$  against  $P$  [See fig. Prob 12.3(a)]. The 'least square' straight line fitting is shown in figure. In each plot two data points at low pressure coincide.

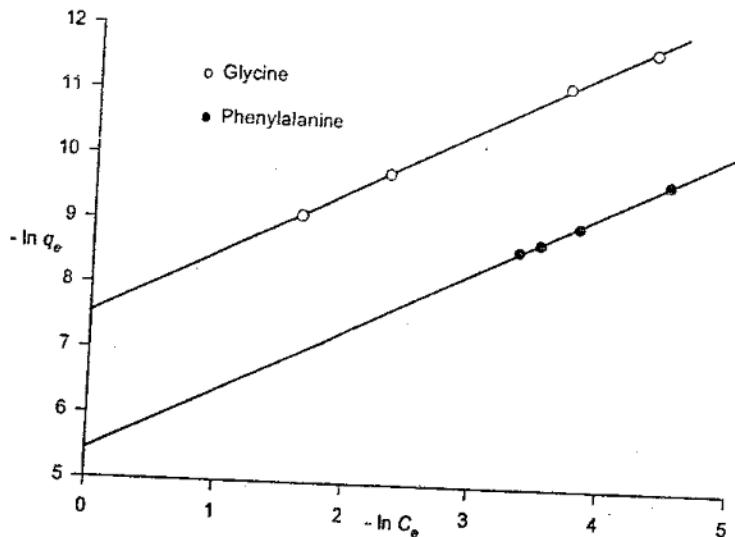


Fig. Prob 12.2. Freundlich isotherm plots -  
adsorption of glycine and phenylalanine on Amberlite XAD.

12.4

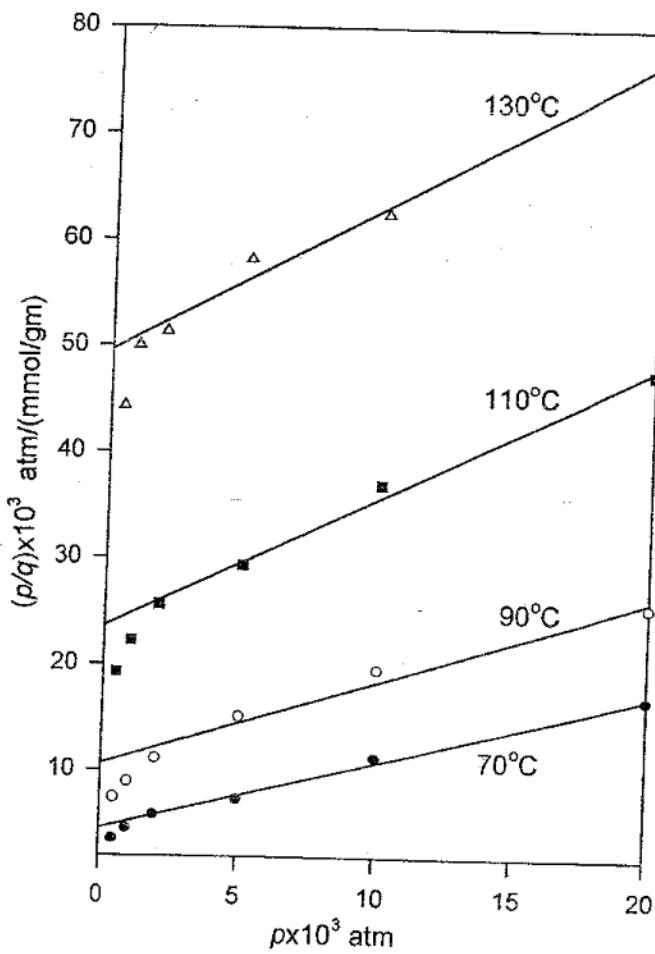


Fig Prob 12.3(a)

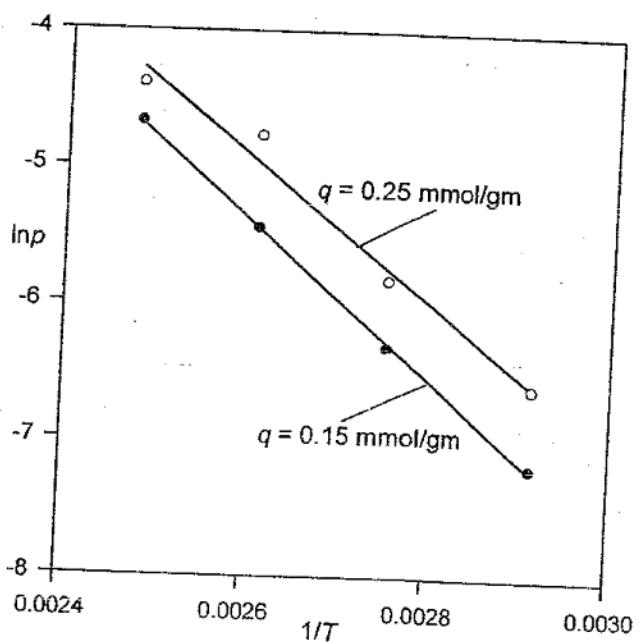


Fig Prob 12.3(b)

(12.5)

Substantially and have been discarded while drawing the least square lines. The fitting is reasonably good for the remaining four data points. The correlation coefficients are also shown on the graph against individual plots. The Langmuir isotherm equations at the four temperatures are given below.

$$T = 70^\circ\text{C} \rightarrow q = (1.591) \cdot \frac{0.137 \times 10^3 p}{1 + 0.137 \times 10^3 p}; T = 90^\circ\text{C} \rightarrow q = (1.284) \cdot \frac{0.073 \times 10^3 p}{1 + 0.073 \times 10^3 p}$$

$$T = 110^\circ\text{C} \rightarrow q = (0.816) \cdot \frac{0.052 \times 10^3 p}{1 + 0.052 \times 10^3 p}; T = 130^\circ\text{C} \rightarrow q = (0.734) \cdot \frac{0.0275 \times 10^3 p}{1 + 0.0275 \times 10^3 p}$$

Fitting of Freundlich isotherm, Eq(12.8)  $\rightarrow \ln q = \ln K' + \frac{1}{n} \ln p$

Values of  $\ln q$  is plotted against  $\ln p$  at each temperature, see Fig Prob 12.3(c). The fitting is much better than for Langmuir isotherm as indicated by the 'correlation coefficient' shown against individual plots. The values of  $\ln K'$  and  $1/n$  are obtained from the intercept and slope of the 'least square' line at a particular temperature. The isotherm equations are:

$$70^\circ\text{C} \rightarrow q = (17.74) \cdot p^{1/1.578}; 90^\circ\text{C} \rightarrow q = (10.82) \cdot p^{1/1.507};$$

$$110^\circ\text{C} \rightarrow q = (8.893) \cdot p^{1/1.307}; 130^\circ\text{C} \rightarrow q = (8.18) \cdot p^{1/1.242}$$

### (b) Isotherms and the heat of adsorption

An isotherm is the plot of  $\ln p$  against  $1/T$  at a constant value of  $q$ . We arbitrarily select two  $q$ -values,  $q = 0.15 \text{ mmol/gm}$ , and  $q = 0.25 \text{ mmol/gm}$ . For each  $q$ -value, we calculate the value of  $p$  at each of the temperatures. The calculated values are given below. (Obtained from the Langmuir isotherm)

$$q = 0.15 \text{ mmol/gm}$$

$$q = 0.25 \text{ mmol/gm}$$

$T(K)$	343	363	383	403	343	363	383	403
$10^3 \times p, \text{ atm}$	0.76	1.812	4.33	9.34	1.36	2.96	8.5	12.4
$10^3 \times 1/T$	2.915	2.755	2.611	2.48	2.915	2.755	2.611	2.48
$\ln p$	-7.182	-6.313	-5.442	-4.673	-6.6	-5.822	-4.767	-4.39

The plots of  $\ln p$  against  $1/T$  are shown in Fig Prob 12.3(b). The goodness of fit is indicated by the correlation coefficients shown on the graph. The heat of adsorption is obtained from

$$q = 0.15, \text{ slope} = -5791 = \Delta H/R \rightarrow \Delta H = (-5791)(1.987) = 11.5 \text{ kcal/gmol}$$

$$q = 0.25, \text{ slope} = -5319 = \Delta H/R \rightarrow \Delta H = (-5319)(1.987) = 10.56 \text{ kcal/gmol}$$

Prob 12.3 contd... The exercise is repeated by calculating  $p$  for  $q = 0.15$  and  $0.25 \text{ mmol/gm}$  from the Freundlich isotherm equation at each temperature. The calculated values are given below.

$$q = 0.15 \text{ mmol/gm}$$

T (K)	343	363	383	403
$p \times 10^3, \text{ atm}$	0.536	1.584	4.754	6.967
$1/T \times 10^3$	2.915	2.755	2.611	2.48
$\ln p$	-7.532	-6.448	-5.349	-4.966

$$q = 0.25 \text{ mmol/gm}$$

343	363	383	403
1.2	3.42	9.268	13.1
2.915	2.755	2.611	2.48
-6.726	-5.678	-4.681	-4.332

The least square straight line plots of  $q$  against  $\ln p$  for  $q = 0.15$  and  $q = 0.25$  are shown in fig Prob 12.3(d). The fit is good; the correlation coefficients are shown. The heat of adsorption is to be calculated from the slope of a plot.

$$q = 0.15, \text{slope} = -6106 = \Delta H/R \Rightarrow \Delta H = (-6106)(1.987) = -12.13 \text{ kcal/gmol}$$

$$q = 0.25, \text{slope} = -5680 = \Delta H/R \Rightarrow \Delta H = (-5680)(1.987) = -11.3 \text{ " "}$$

Note that The heat of vaporization (or condensation) of benzene in the given temperature range is about 7 kcal/gmol. The estimated heat of adsorption is considerably larger than this for the variety of silica gel.

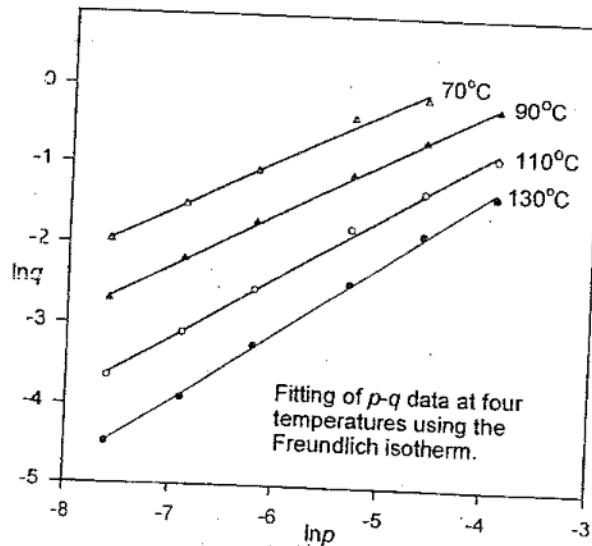


Fig Prob 12.3(c)

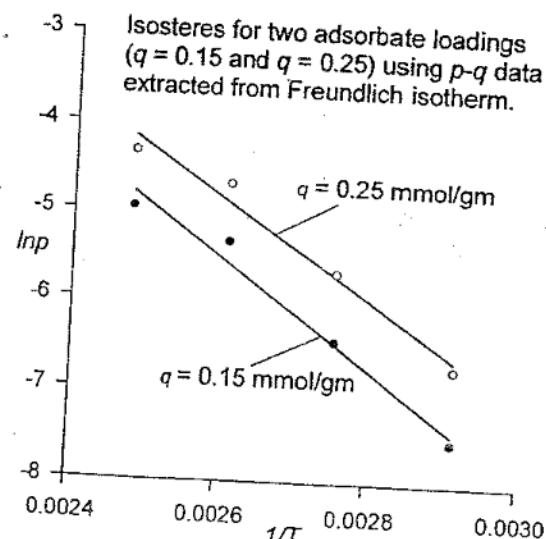


Fig Prob 12.3(d)

12.7

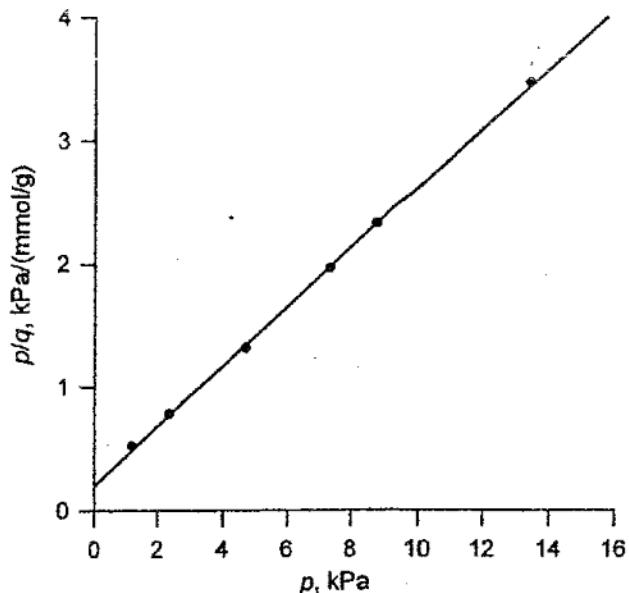


Fig. Prob 12.6. Fitting of the Langmuir Isotherm.

12.4

The equilibrium adsorption data were fitted in the Langmuir isotherm as before [see figure Prob 12.4(a)]. The results are :

$$T = 310.9 \text{ K} ; q = (5.464) \frac{0.0474p}{1+0.0474p} = \frac{0.259p}{1+0.0474p} ; r^2 = 0.999$$

$$T = 338.7 \text{ K} ; q = (5.32) \frac{0.02p}{1+0.02p} = \frac{0.1064p}{1+0.02p} ; r^2 = 0.992$$

$$T = 366.5 \text{ K} ; q = (4.425) \frac{0.02216p}{1+0.02216p} = \frac{0.098p}{1+0.02216p} ; r^2 = 0.996$$

The fitting is pretty good at each temperature.

Values of  $p$  against  $T$  for a constant loading ( $q = 4 \frac{\text{mmol}}{\text{gm}}$ , selected arbitrarily) obtained by interpolation are

$$q = 4 \frac{\text{mmol}}{\text{gm}}$$

$p$ , kPa	65	200	500
$T$ , K	310.9	338.7	366.5

The plot of  $\ln p$  against  $1/T$  (isotherm) is shown in Fig Prob 12.4(b). Slope =  $-4183 = \Delta H/R \Rightarrow \Delta H = -8.312 \text{ kcal/gmol}$

(12.8)

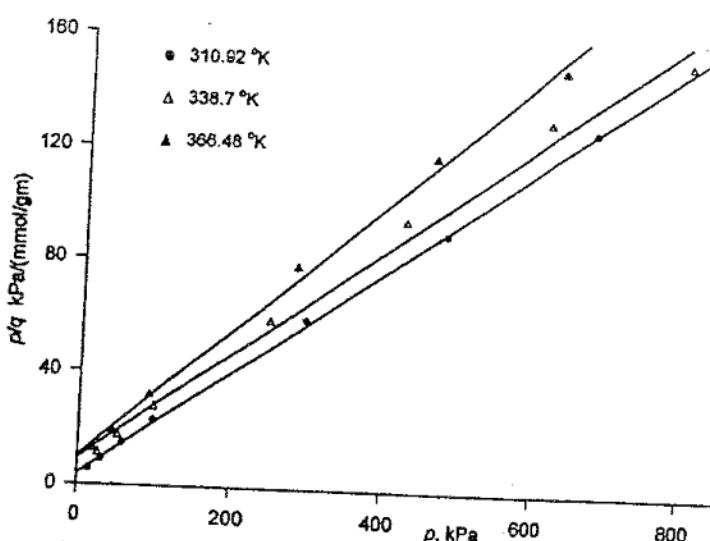


Fig. Prob 12.4(a) Fitting of Langmuir isotherm at three temperatures

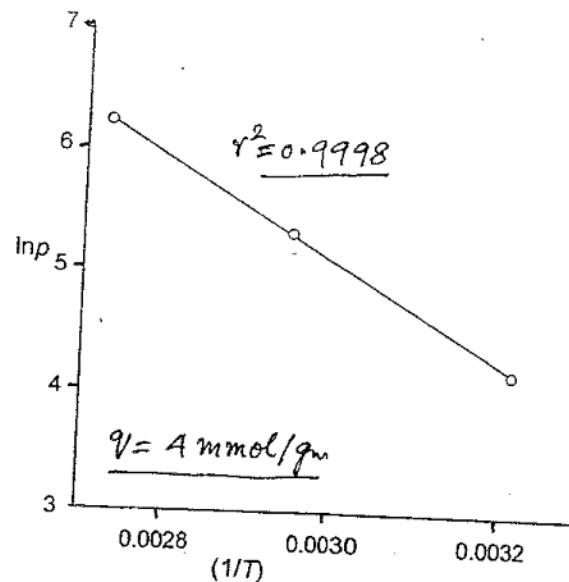


Fig. Prob. 12.4(b). Isosters for the determination of heat of adsorption.

(12.5) The individual adsorption equilibrium data for ethane (1) and propane (2) given in the problem can be fitted in the Langmuir isotherm (details not given)

$$\text{Ethane : } q_1 = \frac{0.342 p_1}{1 + 0.152 p_1} \rightarrow q_1 \text{ in } \frac{\text{mmol}}{\text{gm}}, p_1 \text{ in kPa}$$

$$\text{Propane : } q_2 = \frac{1.57 p_2}{1 + 0.802 p_2} \rightarrow q_2 \text{ and } p_2 \text{ have units as above.}$$

If the Langmuir's isotherm is applied to adsorption of a mixture of the gases [see Eq. 12.12]

$$\text{for ethane, } q_{1,\text{mix}} = \frac{0.342 p_1}{1 + 0.152 p_1 + 0.802 p_2} \quad \text{--- (i)}$$

$$\text{for propane, } q_{2,\text{mix}} = \frac{1.57 p_2}{1 + 0.152 p_1 + 0.802 p_2} \quad \text{--- (ii)}$$

In the given mixture, in equilibrium with the adsorbent,  $p_1 = 50 \text{ kPa}$  (for ethane), and  $p_2 = 50 \text{ kPa}$  (for propane)

Putting these values in Eqs (i) and (ii)

$$q_{1,\text{mix}} = \frac{(0.342)(50)}{1 + (0.152)(50) + (0.802)(50)} = \frac{0.351 \text{ mmol ethane/gm}}{= 0.351 \text{ mol/kg}}$$

$$\text{and } q_{2,\text{mix}} = \frac{1.57(50)}{1 + 0.152(50) + 0.802(50)} = \frac{1.612 \text{ mmol/gm propane}}{= 1.612 \text{ mol/kg}}$$

(12.9)

12.6 If the adsorption equilibrium data fits the Langmuir isotherm, it is possible to determine the adsorbate loading corresponding to formation of a monomolecular layer. The given data fitted to the Langmuir isotherm (see fig Prob 12.6) yields  $\theta_m = \underline{4.14 \text{ mmol/gm}}$  obtained from the slope of the plot (see page 2).

Specific surface area,  $S_g = \theta_m N_A$ , Eq (12.8)

$$\begin{aligned} \text{Eq. (12.7)} \rightarrow \alpha &= 1.09(M/N_A)^{2/3} = 1.09 \left[ \frac{(78)}{(6.023 \times 10^{23})(0.875)} \right]^{2/3} \\ S_g &= \left( 4.14 \times 10^{-3} \text{ gmol/gm} \right) \left( \frac{6.023 \times 10^{23} / \text{gmol}}{3.05 \times 10^{-16} \text{ cm}^2} \right) \\ &= \underline{76.05 \text{ m}^2/\text{gm}} \end{aligned}$$

12.7 (a) The minimum quantity of carbon corresponds to attainment of adsorption equilibrium. If  $C_e$ , mg/l, is the equilibrium concentration in solution,

$$\frac{(1000 \text{ liter})(C_i - C_e) \text{ mg/l}}{m_s'} = 68 C_e^{0.43}$$

Initial concentration,  
 $C_i = 10 \text{ ppm}$   
 $m_s' = \text{mass of carbon, gm} = 10 \text{ mg/liter}$

$$\Rightarrow \frac{(1000)(10 - 0.01)}{m_s'} = 68(0.01)^{0.43} \Rightarrow m_s' = 1.064 \text{ kg}$$

Amount of solution = 1 m<sup>3</sup>  
 $= 1000 \text{ liter}$   
 $C_e = 0.01 \text{ ppm}$

(b) If 1.3 times the minimum quantity ( $m_s'$ ), mass of carbon used,  $m_s = 1.3 \times 1.064 = 1.383 \text{ kg}$ . It is required to calculate the time for attaining a concentration 0.01 ppm.

At any time  $t$ , let the bulk concentration of the solute be  $C$  (ppm or mg/liter).

Loading of the adsorbent at that time,  $q_t = \frac{(1000)(10 - C) \text{ mg}}{1383 \text{ gm}}$

Corresponding equilibrium concentration of the solute in the solution,  $C_e'$ , is given by  $\frac{(1000)(10 - C)}{1383} = 68(C_e')^{0.43}$

[Note that the quantity  $C_e'$  is time-dependent since  $C$  is time-independent]  $\Rightarrow C_e' = 2.575 \times 10^{5} (10 - C)^{2.3256}$

(12.10)

12.7 contd... Instantaneous driving force for transport of the solute from the bulk liquid to the solid surface =  $C - C_e'$ . An unsteady state solute balance gives

$$-V_L \frac{dC}{dt} = k_L \cdot (m_s \cdot a') (C - C_e'); C \text{ changes from } 10 \text{ to } 0.01 \text{ ppm}$$

Integrating, put  $C_e'$  from (1) [Given:  $k_L = 1.8 \times 10^5 \text{ m/s}$ ;  $a' = 200 \text{ cm}^2/\text{gm adsorbent}$ ;  $V = 1 \text{ m}^3$ ]

$$-\int_{10}^{0.01} \frac{dc}{c - 2.575 \times 10^{-5} (10 - c)^{2.3256}} = \int_0^t \frac{(1.8 \times 10^5 \frac{\text{m}}{\text{s}}) (200 \times 10^{-4} \frac{\text{m}^2}{\text{g}} \times 1383 \text{ gm}) dt}{1 \text{ m}^3}$$

The value of the integral = 7.684 (obtained by using Mathcad Plus 6.0)

$$\Rightarrow 7.684 = 4.98 \times 10^{-4} t \Rightarrow t = 15,4308 = 4.3 \text{ hr}$$

12.8 Follow the procedure of Example 12.3(b). Consider the first stage of contact.

If  $Y$  (units/kg) is the color concentration in the oil at any time  $t$ , amount of color adsorbed by the clay per kg is

$$x = \frac{(1000 \text{ kg oil}) (50 - Y)}{m_s} \frac{\text{unit}}{\text{kg}} \frac{(1000)(50 - Y)}{12.36} = 80.91(50 - Y) \quad [m_s = 12.36 \text{ kg}]$$

Corresponding 'equilibrium color concentration' in the oil would have been,  $Y_e = 4.2 \times 10^{-4} x = (4.2 \times 10^{-4})(80.91)(50 - Y) = 0.034(50 - Y)$

Unsteady color balance:  $-\frac{d}{dt}(1000Y) = k_L(m_s \cdot a')(Y - Y_e) \cdot \text{oil}$

$$\Rightarrow -\frac{dY}{dt} = \frac{(5.2 \times 10^{-6} \text{ m/s})(12.36 \text{ kg} \times 25 \frac{\text{m}^2}{\text{kg}})(950 \text{ kg/m}^3)}{1000 \text{ kg}} [Y - 0.034(50 - Y)]$$

$$\Rightarrow -\frac{dY}{dt} = 1.526 \times 10^{-3} (1.034Y - 1.7) \quad \text{Contact time} = 30.6 \text{ min.} = 1836 \text{ s}$$

$$-\int_{50}^{Y_1} \frac{dY}{(1.034Y - 1.7)} = 1.526 \times 10^{-3} \int_0^{1836} dt \quad \text{Integrate from } t=0, Y=50 \quad t=1836, Y=?$$

$$\Rightarrow \frac{1}{1.034} \ln \frac{(1.034)(50) - 1.7}{(1.034)(Y_1) - 1.7} = 1.526 \times 10^{-3} \frac{-3}{1836}$$

$$\Rightarrow Y_1 = 4.313 \text{ unit/kg oil} \quad [\text{Note that more than 90% of the color gets adsorbed when half the amount of clay of Ex. 12.3 is used}]$$

Second stage of contact

Use the same quantity of clay, 12.36 kg. Starting color concentration in the oil,  $Y_1 = 4.313 \text{ units/kg}$

Follow the above procedure, write the unsteady state color balance. On integration the final color concentration in the oil can be found to be  $Y_2 = 0.372 \text{ unit/kg}$ .

(12.11)

(12.9) (a) The mass transfer wave front is drawn by plotting the given breakthrough data. For  $C/C_0 = 0.025$ ,  $t = 19.5 \text{ min.} = t_b$   
 $\Rightarrow$  breakthrough time  $= 19.5 \text{ min.}$

The MTZ is not symmetric. The stoichiometric time,  $t_s$ , is determined by matching the areas above the breakthrough curve for  $t < t_s$ , and that above the curve for  $t > t_s$  (see Fig. Prob. 12.9).

$$t_s = 46 \text{ min. Velocity of the wave front, } u_s = \frac{L}{t_s} = \frac{0.2 \text{ m}}{46 \text{ min}} = 0.00435 \frac{\text{m}}{\text{min}}$$

The equilibrium time,  $t_e$ , is taken as the time for  $C/C_0 = 0.095$

$$t_e = 86 \text{ min. Length of the MTZ} = u_s(t_s - t_b) \\ LUB = u_s(t_s - t_b) = (0.00435)(46 - 19.5) = 0.29 \text{ m} = 29 \text{ cm}$$

- (b) To design an adsorber bed for treating  $3000 \text{ m}^3/\text{hr}$  of gas, (i) we use the same superficial velocity as in the experimental feed ( $u_0 = 0.29 \text{ m/s}$ )  
(ii) we assume that the mass transfer front maintains the same shape as it progresses through the bed.

$$\text{Diameter of the bed} \\ = \left[ \frac{4}{\pi} \cdot \frac{(3000/3600) \text{ m}^3/\text{s}}{0.29 \text{ m/s}} \right]^{1/2} = 1.9 \text{ m}$$

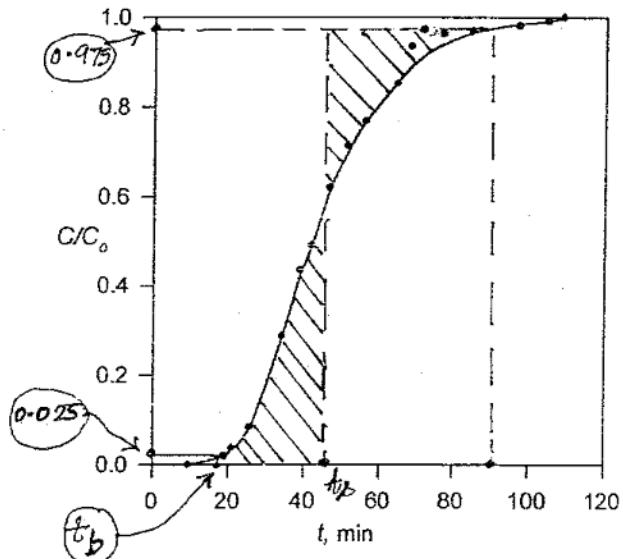


Fig. Prob. 12.9. Mass transfer wave front

### Bed height

The adsorption half cycle has a duration of 8 hr.

$$\text{Bed height} = (8 \text{ hr}) (u_s) + LUB = (8 \times 60 \text{ min}) (0.00435 \frac{\text{m}}{\text{min}}) + 0.115 \text{ m} \\ = 2.02 \text{ m}$$

Loading of the bed at breakthrough: All the solute entering the column in 8 hr is adsorbed. Considering unit area of the bed, moles of MEK entering  $= (8 \text{ hr} \times 3600 \frac{\text{sec}}{\text{hr}}) (0.29 \text{ m/s}) (1 \text{ m}^2) (0.11 \text{ g mol/l m}^3)$   
 $= 91.9 \text{ g mole}$

12-12

12.9 contd... Total volume of the bed =  $(2.2 \text{ m})(1 \text{ m}^2) = 2.2 \text{ m}^3$   
 Mass of the adsorbent =  $(2.2 \text{ m}^3)(P_b) = (2.2 \text{ m}^3)(700 \text{ kg/m}^3) = 1540 \text{ kg}$   
Average adsorbent loading =  $\frac{919 \text{ g/mol}}{1540 \text{ kg}} = 0.5967 \frac{\text{g/mol}}{\text{kg}}$

Maximum possible adsorbent loading: At the end of 8 hr. of adsorbent cycle time, a bed length =  $(8 \text{ hr.})(0.00435 \times 60)$   
 $= 2.088 \text{ m}$  of the bed gets effectively saturated.

Maximum loading =  $\frac{(8 \times 3600)(0.29)(0.11)}{(2.088)(700)} \frac{\text{g/mol}}{\text{kg}} = 0.6287 \frac{\text{g/mol}}{\text{kg}}$

12-10 The breakthrough data are plotted; the mass transfer wave front is reasonably symmetric. Take the breakthrough concentration as  $C/C_0 = 0.025$  and the equilibrium concentration as  $C/C_0 = 0.97$ . (i) Breakthrough time (from the plot),  $t_b = 191 \text{ min}$ .

Since the wave front is almost symmetric, take the stoichiometric time,  $t_m = 232 \text{ min}$  when  $C/C_0 = 0.5$ .

(ii) Velocity of the MTZ,  $U_s = \frac{L}{t_m} = \frac{50 \text{ cm}}{232 \text{ min}} = 0.215 \text{ cm/min}$

$$\text{LUB} = L \left(1 - \frac{t_b}{t_s}\right) = L \left(1 - \frac{t_b}{t_m}\right) = 50 \left(1 - \frac{191}{232}\right) \text{ cm} = 7.6 \text{ cm}$$

(iii) Fraction of the bed utilized =  $\frac{L - \text{LUB}}{L} = \frac{50 - 7.6}{50} = 84.5\%$

Evaluation of LUB by the integration technique

Take the breakthrough concentration  $C/C_0 = 0.005$  when  $t_b = 191 \text{ min}$

Equilibrium time,  $t_e = 270 \text{ min}$ , when  $C/C_0 = 0.992$ .

$$\begin{aligned} \text{Eq. (12.24)}, \omega_1 &= U_0 C_0 \int_{t_b=191}^{t_e=270} (1 - C/C_0) dt = U_0 C_0 \int_0^{270} (1 - C/C_0) dt \\ &= U_0 C_0 \int_0^{191} (1 - C/C_0) dt + \int_{191}^{270} (1 - C/C_0) dt = 191 \cdot U_0 C_0 + 41.55 U_0 C_0 \\ \omega_2 &= \int_0^{191} (1 - C/C_0) dt = 191 U_0 \Rightarrow \text{LUB} = \left(1 - \frac{\omega_2}{\omega_1}\right) L = \left(1 - \frac{191}{232.5}\right)(50 \text{ cm}) \end{aligned}$$

(by graphical integration)

The difference occurred probably due to the error in graphical integration and due to the asymmetry of the wave front.

Fig. → p. 13

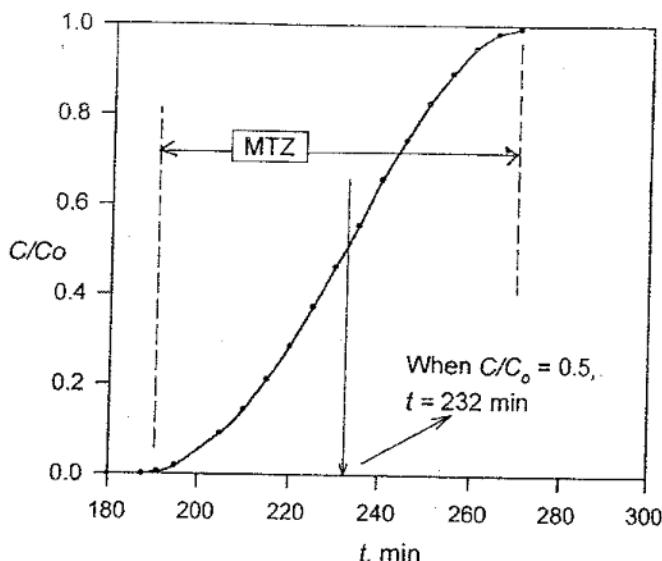


Fig. Prob. 12.10. Mass transfer wave front from experimental breakthrough data.

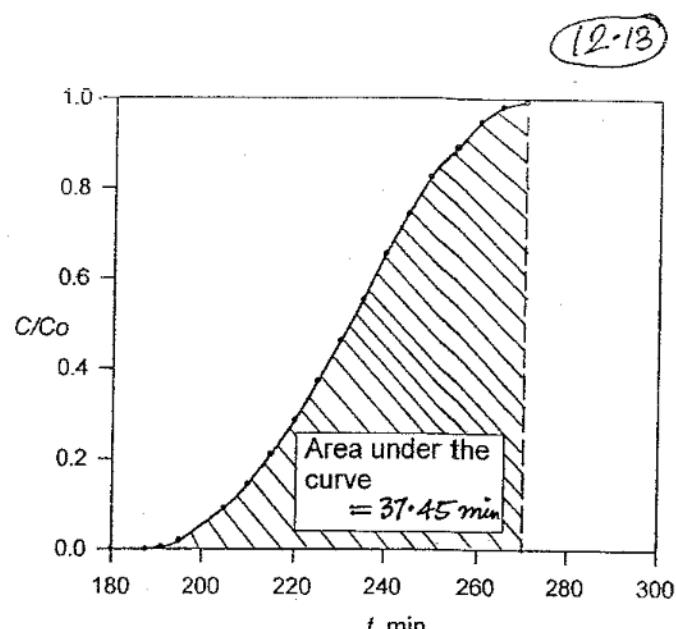


Fig. Prob. 12.10(b). Graphical integration.

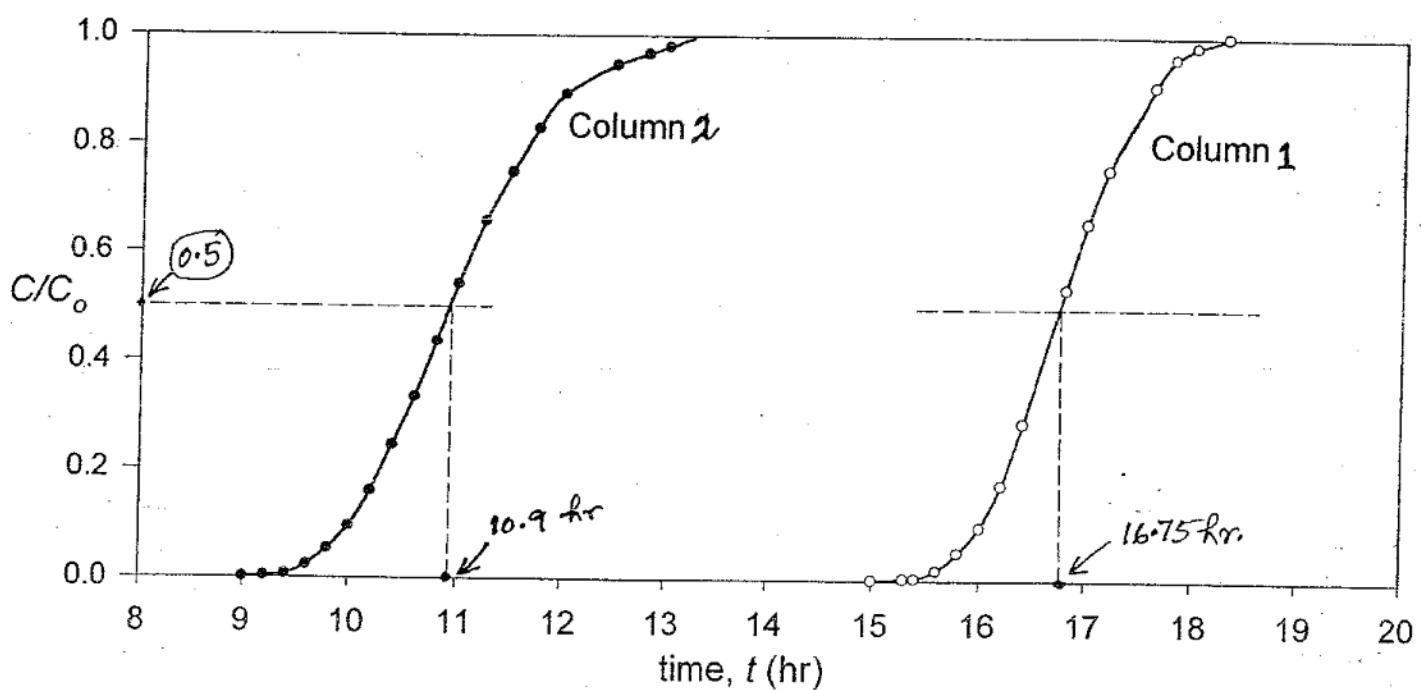


Fig. Prob. 12.11. Breakthrough data for two columns of different lengths.

(12.11) The breakthrough data for both the columns are plotted in Figure Prob 12.11.

For column 1, breakthrough time (for 1 ppm moisture in the effluent)  
 $t_{b1} = 10.9 \text{ hr}$

12.11 Contd... The mass transfer wave front is almost symmetric. For  $C/C_0 = 0.5$ ,  $t_s = t_m = \underline{16.75 \text{ hr}}$ . (12.14)

$$(a) L_{UB} = L \left(1 - \frac{t_b}{t_{s1}}\right) = (44 \text{ cm}) \left(1 - \frac{15.5}{16.75}\right) = \underline{3.3 \text{ cm}}$$

$$(b) \text{From column-1 data, } u_s = \frac{L_1}{t_{s1}} = \frac{44 \text{ cm}}{16.75 \text{ hr}} = \underline{0.0438 \text{ cm/min}}$$

$$\text{Taking } L_{UB} = 3.3 \text{ for column-2 also, } L_{s2} = L_2 - L_{UB} \\ = 27 - 3.3 = \underline{23.7 \text{ cm}}$$

$$\text{Breakthrough time for column-2 (calculated from column-1 data)} \\ t_{b2} = \frac{L_{s2}}{u_s} = \frac{23.7 \text{ cm}}{0.0438 \text{ cm/min}} = 540 \text{ min} = \underline{9.02 \text{ h}}$$

This appears to be pretty accurate in <sup>reflect 98</sup> column-2 data.

(c) Bed-1, saturation concentration:

$L_{s1} = L_1 - L_{UB} = 44 - 3.3 = \underline{40.7 \text{ cm}}$ . This length of the bed gets saturated. Consider unit area of the bed.

Total amount of moisture in the bed at

Moisture in the feed = 1490 ppm (v/v) =  $0.00149$  mole fraction.

Av. Molecular weight of the 'moist' air =  $\underline{29.9}$

$$\text{Moles air entering} = \left(\frac{3992}{29.9}\right)(0.00149) = 0.199 \text{ kmol/m}^2 \\ = \underline{3.58 \text{ kg/m}^2}$$

$$\text{Amount of moisture absorbed till the breakpoint} \\ \text{time } (t_{b1} = 15.5 \text{ hr}) = (3.58 \text{ kg/m}^2)(15.5 \text{ h}) = \underline{55.52 \text{ kg}}$$

$$\text{Amount of moisture} \\ = (L)(1 \text{ m}^2)(P_b)(0.01) = (0.44)(1)(715)(0.01) = 3.15 \text{ kg}$$

$$\text{Total moisture in the bed at breakthrough} \\ = 3.15 + 55.52 = \underline{58.67 \text{ kg}}$$

(15)

12-11 contd...

(d) Velocity of the MTZ from data from both the columns.

$$\text{Use Eq. (12. ) , } u_s = \frac{L_2 - L_1}{t_{S_2} - t_{S_1}}.$$

For  $C/C_0 = 0.5$ ,  $t_{S_1} = 16.75 \text{ hr}$ ;  $t_{S_2} = 10.9 \text{ hr}$ .

$$u_s = \frac{L_2 - L_1}{t_{S_2} - t_{S_1}} = \frac{0.44 - 0.27}{16.75 - 10.9} = 2.906 \text{ cm/hr} = 0.0484 \frac{\text{cm}}{\text{min}}$$

which is about 10% larger than the value calculated from column-1 data only.

12-12

Toth isotherm [Eq.(12.10)]:  $\vartheta = \vartheta_m \cdot \frac{P}{(b + P^n)^{1/n}}$ 

Given: total pressure,  $P = 10 \text{ bar}$ ;  $\text{CO}_2$  concentration in the feed gas = 3%  $\Rightarrow P_{\text{CO}_2} = (1000 \text{ kPa})(0.03) = 30 \text{ kPa}$ .

$b = 22.87$  in kPa unit;  $n = 0.628$   $\therefore \vartheta_m = 7.935 \text{ gmol/kg}$

$\Rightarrow$  Adsorbent loading at saturation (or equilibrium),

$$\vartheta = (7.935) \cdot \frac{30}{[22.87 + (30)]^{0.628}} = 0.989 \text{ gmol/kg}$$

Consider  $1 \text{ m}^2$  of bed area.  $L_s = L - L_{UB} = 2 \text{ m} - 0.15 \text{ m} = 1.85 \text{ m}$

$\text{CO}_2$  loading at breakthrough

$$= L_s \cdot 1 \text{ m}^2 \cdot P_b \cdot \vartheta = (1.85 \text{ m})(1 \text{ m}^2)(500 \text{ kg/m}^3)(0.989 \text{ gmol/kg})$$

$$= 914.5 \text{ gmol} = (914.5)(44) \text{ gm} = 40.24 \text{ kg}$$

Residual  $\text{CO}_2$  present in the regenerated bed

$$= L \cdot 1 \text{ m}^2 \cdot P_b \cdot \vartheta_i = (2 \text{ m})(1 \text{ m}^2)(500 \text{ kg/m}^3)(1 \text{ gm/kg}) = 1000 \text{ gm}$$

$\text{CO}_2$  removed in one adsorption cycle  $= 1 \text{ kg}$

$$= 40.24 - 1 = \underline{\underline{39.24 \text{ kg}}}$$

12.13

### External mass transfer coefficient

12.16

The benzene concentration in  $N_2$  is low (0.1%). Use the properties of  $N_2$  only in calculations. Temperature =  $25^\circ C = 298 K$ .

Density of the gas,  $\rho_G = \left(\frac{28}{22.4}\right) \left(\frac{273}{298}\right) = 1.145 \text{ kg/m}^3$ ;

$\mu_G = 1.85 \times 10^{-5} \text{ kg/m.s}$  (given); diffusivity of benzene in

$N_2 = 0.102 \text{ cm}^2/\text{s}$  at  $311.3 K \Rightarrow D$  at  $298 K = (0.102) \left(\frac{298}{311.3}\right)^{0.75} = 0.0945 \text{ cm}^2/\text{s}$

Particle size,  $d_p = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Gas velocity through the bed,  $u = \frac{u_0}{\epsilon_b} = \frac{0.28 \text{ m/s}}{0.43} = 0.651 \text{ m/s}$

Reynolds number,  $Re = \frac{d_p \rho_G u}{\mu_G} = \frac{(3 \times 10^{-3})(1.145)(0.651)}{1.85 \times 10^{-5}} = 121$

Schmidt number,  $Sc = \frac{\mu_G}{\rho_G \cdot D} = \frac{1.85 \times 10^{-5}}{(1.145)(0.0945 \times 10^{-4})} = 1.71$

Use Eq (12.74a)  $\rightarrow Sh = 2 + 1.1(Re)^{0.6}(Sc)^{1/3}$

$$\Rightarrow \frac{k_c d_p}{D} = 2 + 1.1(121)^{0.6}(1.71)^{1/3} = 25.37$$

External mass transfer coefficient,  $k_e = \frac{(25.37)(0.0945 \times 10^{-4})}{3 \times 10^{-3}} = 0.08 \text{ m/s}$

### Effective diffusivity in an adsorbent pellet

Use Eq (12.74c). Neglect surface diffusion.

Estimate Knudsen diffusivity using the following equation:

$$D_K = 9700 \gamma_p (T/M)^{1/2}; \gamma_p = 15/2 \text{ nm} = 7.5 \times 10^{-7} \text{ cm} ; M = 78 \quad (\text{see Section 2.8.1})$$

$$D_K = (9700)(7.5 \times 10^{-7})(298/78)^{1/2} = 0.0142 \text{ cm}^2/\text{s}$$

$$\text{Effective diffusivity, } D_e = \frac{\epsilon_p}{\tau} \left[ \frac{1}{(1/D) + (1/D_K)} \right]$$

$$\Rightarrow D_e = \frac{0.45}{2.8} \left[ \frac{1}{(1/0.0945) + (1/0.0142)} \right] = 0.00198 \text{ cm}^2/\text{s}$$

12.14

Time to reach  $C/C_0 = 0.5$  at the bed exit,  $t_m = 60 \text{ min}$ .

Bed length,  $L = 0.5 \text{ m} = 50 \text{ cm} \Rightarrow u_s = 50/60 = 0.833 \text{ cm/min}$

It takes  $15 \text{ min}$  for the mass transfer wave front to move through its own length (taken as  $C/C_0 = 0.02$  to  $0.98$ ).

(12-14)

(12-14) contd... Length of the MTZ =  $(0.833 \text{ cm/min})(15 \text{ min}) = 12.5 \text{ cm}$   
 $\Rightarrow L_{UB} = 12.5/2 = 6.25 \text{ cm}$  (assuming a symmetric wave front).

For 90% bed utilization,  $\frac{L - L_{UB}}{L} = 0.9$

$\Rightarrow 1 - \frac{6.25}{L} = 0.9 \Rightarrow L = 62.5 \text{ cm}$ , 62.5 cm long bed to be used.

Breakthrough time,  $t_b = \frac{L_s}{u_s} = \frac{(62.5 - 6.25) \text{ cm}}{0.833 \text{ cm/min}} = 67.5 \text{ min}$

(12-15) Working equation, Eq. (12-45):

$$\frac{1+KC_0}{KC_0} \ln \frac{C}{C_1} + \frac{1}{KC_0} \ln \frac{1-C_1/C_0}{1-C/C_0} = \frac{k_m \bar{a}}{(1-\epsilon_b) \cdot P_p} \cdot \frac{C_0}{y_0} (t - t_1)$$

Given: influent concentration,  $C_0 = \frac{P_0 / RT}{(1-2)(0.02)} = \frac{(0.08317)(299)}{9.65 \times 10^{-4} \text{ gmol/m}^3 \text{ atm}}$

i.e.,  $C_0 = 0.965 \text{ gmol/m}^3$ ;  $\frac{C_0}{y_0} = \frac{1+KC_0}{y_m K}$ .

$$\Rightarrow \frac{C_0}{y_0} = \frac{1 + (0.12 \text{ m}^3/\text{gmol})(0.965 \text{ gmol/m}^3)}{(6.2 \text{ gmol/kg})(0.12 \text{ m}^3/\text{gmol})} = \frac{1.5 \text{ kg/m}^3}{; KC_0 = 0.116}$$

$$k_m \bar{a} = 35 \text{ sec}^{-1}, \epsilon_b = 0.38$$

$$\Rightarrow (0.116) \ln \frac{C}{C_1} + \ln \frac{1-C_1/C_0}{1-C/C_0} = \frac{(0.116)(35)}{(1-0.38)(700)} \cdot (1.5) \cdot (t - t_1) ; P_p = 700 \text{ kg/m}^3$$

$$\Rightarrow 1.116 \frac{C}{C_1} + \ln \frac{1-C_1/C_0}{1-C/C_0} = 0.014 (t - t_1)$$

Time required for change of the concentration of the effluent from  $C/C_0 = 0.025$  to  $C/C_0 = 0.975$  is

$$t_{MTZ} = (1/0.014) \left[ 1.116 \ln \frac{0.975}{0.025} + \ln \frac{1-0.025}{1-0.975} \right] = 55.38 = 9.2 \text{ min}$$

Approximate velocity of the wave front, Eq.(12-37)

$$u_s = \frac{u_0}{0.38 + (1-0.38)(700)(\Delta q/\Delta C)_c_0} = \frac{0.32}{(0.38) + (0.62)(700)(0.667)} = \frac{0.0011 \text{ m}}{\text{s}}$$

Length of the MTZ =  $t_{MTZ} \cdot u_s = (9.2 \times 60)(0.0011) = 0.61 \text{ m}$

$L_{UB} = \frac{1}{2} MTZ = 30.5 \text{ cm}$  (for a symmetric wave front)

$L_s = L - L_{UB} = 2 \text{ m} - 0.305 \text{ m} = 1.695 \text{ m}$

$t_b = \frac{L_s}{u_s} = \frac{1.695 \text{ m}}{0.0011 \text{ m/s}} = 1541.8 = 25.7 \text{ min}$

12.18

12.16 Basis of calculation: 1 liter solution  
Amount of sodium acetate in solution = 110 gm  
(Eq wt = 99)

$$\text{Ca}^{++} \text{ in solution (eq. wt. = 20)} = \frac{110}{99} = 1.111 \text{ eq/liter}$$

$$\text{Ca}^{++} \text{ in the final solution (90% removal)} = 0.1 \text{ eq/liter}$$

$$\text{Equivalent fraction of Ca}^{++} \text{ in solution}, x_D = \frac{0.01}{1.111 + 0.01} = 0.00892$$

$$\text{From Eq (12.56), } \frac{y_D}{(1-y_D)^2} = K_{DB} \frac{\bar{C}_e}{C_e} \cdot \frac{x_D}{(1-x_D)^2}; \frac{\bar{C}_e}{C_e} = 2 \text{ eq/liter}$$

$$\Rightarrow \frac{y_D}{(1-y_D)^2} = (2.6) \cdot \frac{2}{1.111} \cdot \frac{0.00892}{(1-0.00892)^2} \Rightarrow y_D = 0.0035$$

i.e. Concentrating Ca<sup>++</sup> in the resin phase at equilibrium  
= 0.0035 eq/liter

Fraction of ionexchange sites occupied by Ca<sup>++</sup> ions

$$= \frac{y_D}{\bar{C}_e} = \frac{0.0035}{2} = 0.00175, \text{ i.e., } 0.175\%$$

This is too low a fraction of utilization and the resin does not appear to be commercially attractive for the above purpose.

Amount of resin required: Equivalent Ca<sup>++</sup> absorbed by the resin at equilibrium = (0.1)(0.9) = 0.09 equivalent

Amount of Ca<sup>++</sup> in the resin, y<sub>D</sub> = 0.0035 equivalent/liter

$$\text{Resin required} = \frac{0.09 \text{ eq.}}{0.0035 \text{ eq/liter}} = 25.7 \text{ liters per liter}$$

Resin required for m<sup>3</sup> solution = 2.57 × 10<sup>4</sup> liters

(this is an exceptionally large value and the resin should not be considered for this particular use)

(12.17) Volume of solution = 100 ml;  $C_i = 488 \text{ mg/liter}$  (12.19)

$V_e = 0.1 \text{ liter}; C_e = 3.2 \text{ mg/liter}; m_2 = 0.9 \text{ gm.}$

$$q_t = V_e (C_i - C)/m_2$$

Time $t$ (min)	0	5	10	15	20	25
$C, \text{mg/L}$	488	328.5	247	175	127	100
$q_t, \text{mg/l}$	0	17.7	26.8	34.8	40.1	43.1
$q_t/(q_t - q_e)$	0	0.536	1.12	2.19	3.8	5.702

Time $t, \text{min}$	30	40	50	60
$C, \text{mg/L}$	79	52	40	32
$q_t, \text{mg/g}$	45.4	48.4	49.8	50.7 = $q_e$
$q_t/(q_t - q_e)$	8.63	21.4	57.9	57.9

$$\text{Rate of adsorption}, \frac{dq_t}{dt} = k_2 (q_e - q_t)^2 \Rightarrow \int_0^{q_t} \frac{dq_t}{(q_e - q_t)^2} = k_2 t$$

$$\Rightarrow \frac{1}{q_e - q_t} - \frac{1}{q_e} = k_2 t. \text{ Rearranging we get, } \frac{1}{q_t} = \frac{1}{q_e} + \frac{1}{k_2 q_e^2 \cdot t}$$

Make a plot of  $1/q_t$  against  $t$

$t$	5	10	15	20	25	30	40	50
$1/t$	0.2	0.1	0.0667	0.05	0.04	0.0333	0.025	0.02
$1/q_t$	0.0565	0.0373	0.0287	0.025	0.0232	0.022	0.0207	0.02

$$\text{Intercept} = 0.0153 = 1/q_e$$

$$\text{Slope} = 0.2073 = \frac{1}{k_2 q_e^2} \quad (\text{g/mg} \cdot \text{min})$$

$$k_2 = \frac{1}{0.2073} \cdot q_e^2$$

It is better to use the experimental value of  $q_e$   
 $= 50.7 \frac{\text{mg}}{\text{g}}$

$$k_2 = 1.87 \times 10^{-3} \frac{\text{g}}{\text{mg} \cdot \text{min}}$$

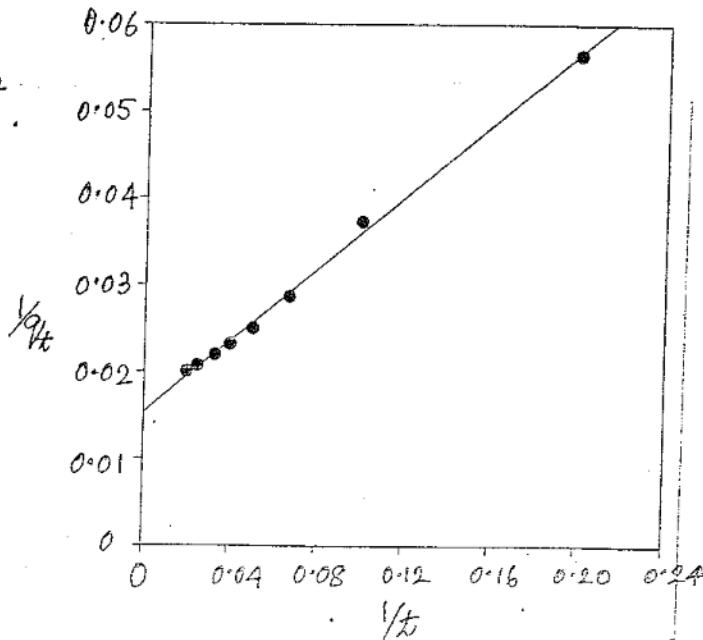


Fig. Prob 12.17. The plot of  $1/q_t$  against  $1/t$

## CHAPTER 13: CRYSTALLIZATION

Short and MCQ's

13.1

- (2) See Section 13.3.2. (3) No. Needs a seed crystal or a particle to grow upon; otherwise a very high supersaturation is required. (4) Generally used for organic compounds - p-xylene, dichlorobenzene, naphthalene, bisphenol, etc.
- (7) A small crystal has a higher solubility because of a larger surface energy. (8) (ii)  $m^{-3} \cdot hr^{-1}$ , (9) (i) 6.

$$(10) v_p = \Phi_v L^3 \Rightarrow \frac{dV_p}{dt} = 3L^2 \Phi_v \frac{dL}{dt} \Rightarrow \frac{dL}{dt} = G = \frac{(dV_p/dt)}{3L^2 \Phi_v}$$

$$= \frac{5.2 \times 10^{-4} m^3/s}{(3)(1 \times 10^{-3} m)^2 (0.42)} = \frac{4.13 \times 10^{-8}}{m/s}$$

$$(11) \Phi_v \text{ for a sphere} = \frac{\pi}{6} = 0.523 \rightarrow (\text{ii}) . (12) (\text{i}) \frac{1}{6} .$$

- (13) Assume that 'McCabe DL law' is valid.

Case 1:  $\frac{L_{t_1} - L_{81}}{t_1} = G = \frac{0.1 \text{ mm}}{2R} = 0.05 \frac{\text{mm}}{R}$ ; Case 2:  $L_{t_2} = L_{82} + G t_2$   
 $\Rightarrow L_{t_2} = 0.8 \text{ mm} + (0.05 \text{ mm/R})(4R) = 1 \text{ mm} \rightarrow (\text{iii}).$

- (14) (i) -10.3 kcal, heat release occurs.

$$(15) 1000 \text{ kg saturated solution at } 80^\circ\text{C. Mass of } CuSO_4 = \frac{1000 \times 55}{155}$$

$$= 355 \text{ kg, water} = 1000 - 355 = 645 \text{ kg. } CuSO_4 \rightarrow 159.5$$

$$\text{Mass of } CuSO_4 \cdot 5H_2O \text{ produced on cooling} = M_c \text{ kg}$$

$$CuSO_4 \text{ in the product} = M_c \cdot \frac{159.5}{249.5} = 0.64 M_c; \text{ water} = 0.36 M_c$$

$$\text{Water left in the cooled solution (30°)} = (645)(0.99) - 0.36 M_c$$

$$(1\% \text{ loss})$$

$$CuSO_4 \text{ left in the solution at } 30^\circ\text{C} = \frac{638.5 - 0.36 M_c}{(638.5 - 0.36 M_c)(25/100)}$$

$$CuSO_4 \text{ balance: } 355 = 0.64 M_c + \frac{159.6 - 0.09 M_c}{(159.6 - 0.09 M_c)}$$

$$\Rightarrow M_c = \frac{355}{kg \ CuSO_4 \cdot 5H_2O}$$

- (16) (iii) contact nucleation.

$$(17) \text{ Given: } n = 5.543 \times 10^{12} \times 10^{-4.63L} = 5.543 \times 10^{12} \exp(-10.66L) m^{-4}; L \text{ in mm.}$$

(13.2)

$$\textcircled{18} \text{ Contd... } \frac{1}{GT} = 10.66 \text{ mm}^{-1}. \text{(a) Dominant size, } L_D = 3GT = \underline{0.28 \text{ mm}} \rightarrow \text{(ii)}$$

$$\text{(b) Put } T = 200 \text{ min} \Rightarrow G = \frac{1}{(10.66)(200)} = 4.69 \times 10^{-4} \text{ mm/min} = \underline{7.82 \times 10^{-7} \text{ cm/s}} \rightarrow \text{(i)}$$

$$\text{(c) Nucleation rate, } B^o = n^o G = 5.543 \times 10^{12} \text{ m}^{-4} \times 7.82 \times 10^{-7} \frac{\text{cm}}{\text{s}} = \underline{1.56 \times 10^8 \text{ m}^{-3} \text{ s}^{-1}} \rightarrow \text{(iii)}$$

$$\textcircled{19} \text{ (ii) } \text{cm}^{-1} \text{m}^{-3}. \textcircled{20} \text{ } n = 8.23 \times 10^n \exp(-9.58L) \text{ m}^{-4}; \text{ L in mm.}$$

$$GT = \frac{1}{9.58} \text{ mm} = 0.1044 \text{ mm. (a) Mass average size, } \bar{L}_m = \sqrt[3]{6(GT)} = \underline{0.19 \text{ mm}} \rightarrow \text{(iii)}$$

$$\text{(b) Median size, } \bar{L}_M = 3.67 GT = (3.67)(0.1044 \text{ mm}) = \underline{0.38 \text{ mm}} \rightarrow \text{(iii)}$$

$$\text{(c) C.V.} = 52\%. \text{ (iii), see Section 13.5.3.}$$

$$\textcircled{21} \text{ (ii) D.T.B. } \textcircled{22} \text{ Average growth rate} = \frac{1}{1 \text{ mm}} \int_0^1 G(L) dL,$$

$$= (9.85 \times 10^{-7} \text{ m/s}) \cdot \frac{1}{(1 \text{ mm})(1.55)(0.16)} \left[ 1 + 0.16L \right]_0^1 = \underline{1.03 \times 10^{-6} \text{ m/s}} \rightarrow \text{(iii)}$$

\textcircled{23} (i) Yes, since it depends on mass transfer which in turn depends on particle size.

\textcircled{24} (ii) Surface reaction. \textcircled{25} (iii)  $10^{-6} \text{ m/min}$  ( $= 1.67 \times 10^{-8} \text{ m/s}$  lies in the usual range)

\textcircled{26} (ii) a low nucleation rate. \textcircled{27} (i) cooling surface.

\textcircled{28} (iii) Steel, since it is the hardest of the three materials.

\textcircled{30} (i) for a substance whose solubility does not change much with temperature.

$$\textcircled{31} \text{ Put } W(x) = 1 - e^{-x} \left( 1 + x + \frac{x^2}{2} + \frac{x^3}{6} \right) = 0.1 \Rightarrow x = 1.74 = L/GT.$$

$$L = 200 \mu\text{m} \Rightarrow GT = \frac{200 \mu\text{m}}{1.74} = \underline{114.9 \mu\text{m}}.$$

$$L_1 = 0.4 \text{ mm} = 400 \mu\text{m} \Rightarrow x_1 = L_1/GT = 3.48 \Rightarrow W(x_1) = 0.46$$

$$L_2 = 0.6 \text{ mm} = 600 \mu\text{m} \Rightarrow x_2 = L_2/GT = 5.22 \Rightarrow W(x_2) = 0.764$$

$$\text{Required fraction, } W(x_2) - W(x_1) = 0.764 - 0.46 = \underline{0.304}$$

$$\textcircled{32} \text{ } m_c = p_c L^3 \Rightarrow \frac{dm_c}{dt} = p_c \cdot 3L^2 \frac{dL}{dt} = k_L \cdot A_c \cdot \Delta C$$

$$\Rightarrow p_c \cdot 3L^2 \cdot G = k_L \cdot 6L^2 \cdot \left( \frac{C - Cs}{Cs} \right) Cs \Rightarrow G = \frac{(2)(2.5 \times 10^{-5} \text{ m/s})(0.07)(150 \frac{\text{kg}}{\text{m}^3})}{1800 \text{ kg/m}^3}$$

$$\Rightarrow G = \underline{2.9 \times 10^{-7} \text{ m/s}} = \text{crystal growth rate}$$

$$\textcircled{33} \text{ } T_p = V/Q_p = 50 \text{ gallon/(1 gal/min)} = 50 \text{ min} \rightarrow \text{(iii)}$$

**PROBLEMS**

13.3

(13.1)

$$\text{Use Eq. (13.1)} \rightarrow \ln(C/C_s) = 2\alpha M_w / RT r \cdot P_c$$

Put  $C/C_s = 1.05$ ,  $\alpha = 0.03 \text{ J/m}^2$ ,  $M_w = 74.5 \text{ kg/kmol}$ ,  $R = 8.317 \text{ kJ/kmol.K}$   
 $T = 293.2 \text{ K}$ ,  $P_c = 1990 \text{ kg/m}^3$ ,  $C/C_s = 1.05$ .

$$\Rightarrow \ln(1.05) = \frac{(2)(3 \times 10^{-5} \text{ kJ/m}^2)(74.5 \text{ kg/kmol})}{(8.317 \text{ kJ/kmol.K})(293.2 \text{ K}) \cdot (1990 \text{ kg/m}^3)} \Rightarrow r = 1.9 \times 10^{-8} \text{ m}$$

A spherical particle of radius  $r = 19 \text{ nm}$  or less will not grow in a solution of  $C = 1.05C_s$ .

\* \* \*

(13.2)

The free energy of formation of a nucleus of radius  $r$  can be considered to be the sum of its surface free energy and the free energy of phase transformation.

$$\Delta G = 4\pi r^2 \alpha + \frac{4}{3}\pi r^3 \Delta G_{tr} \quad (\text{i})$$

Surface free energy      free energy of phase transformation

If  $\Delta G$  has a critical or minimum value, put  $\frac{d(\Delta G)}{dr} = 0$

$$\Rightarrow 8\pi r_c \alpha + 4\pi r_c^2 \Delta G_{tr} = 0 \Rightarrow r_c = -\frac{2\alpha}{\Delta G_{tr}} \quad (\text{ii}) \quad \text{for } r=r_c$$

From eq.(i) and (ii), the free energy change at  $r=r_c$  is

$$\Delta G_{cr} = 4\pi r_c^2 \alpha + \frac{4}{3}\pi r_c^3 \left(-\frac{2\alpha}{r_c}\right) = \frac{4}{3}\pi r_c^2 \alpha \quad (\text{iii})$$

Now consider the Gibbs-Thomson eq. (13.1).

$$\ln \frac{C}{C_s} = \ln S = \frac{2\alpha M}{RT r \cdot P_c} = \frac{2\alpha v_m}{RT r}, \quad v_m = \text{volume of a molecule.}$$

$$\text{For } r=r_c, \quad \ln S = \frac{2\alpha v_m}{RT r_c} \Rightarrow r_c^2 = \left(\frac{2\alpha v_m}{RT \ln S}\right)^2$$

$$\text{From eq.(iii) above, } (\Delta G)_{cr} = \frac{4}{3}\pi \alpha \left(\frac{2\alpha v_m}{RT \ln S}\right)^2 = \frac{16\pi \alpha^3 v_m^2}{3(RT \ln S)^2}$$

Nucleation rate,  $B^\circ = A' \exp \left[ -\frac{\Delta G_{cr}}{kT} \right]$

$$\Rightarrow B^\circ = A' \exp \left[ -\frac{16\pi \alpha^3 v_m^2}{3(kT)^3 (\ln S)^2} \right]; \quad S = \frac{C}{C_s} = \frac{C - C_s}{C_s} + 1 = \delta + 1.$$

For the change of  $\delta$  from 1.2 to 1.21,

$$\frac{(B^\circ)_{S=1.21}}{(B^\circ)_{S=1.20}} = \frac{45,000}{\dots}$$

Take  $\alpha = 0.025 \text{ J/m}^2$ ;  
 $P_c = 2200 \text{ kg/m}^3$ ;  $v_m = 4.415 \times 10^{-29} \text{ m}^3$   
 $k = 1.38 \times 10^{-23} \text{ J/K}$   
 $T = 343 \text{ K}$

(13.2) contd...

$$\text{If } S \text{ changes from } 1.25 \text{ to } 1.26, \frac{(B^o)_{S=1.26}}{(B^o)_{S=1.25}} = \underline{\underline{300}}$$

(13.4)

Thus, at a lower supersaturation the rate increase in homogeneous nucleation is more.

\* \* \* \*

(13.3) Population density data of the crystals are calculated from the given sieve analysis data.

$$\text{Given: } P_c = 1.4 \text{ gm/cm}^3 \\ T_o = 0.42$$

Mesh	Av. size L, mm	$\Delta L, \text{mm}$	$\Delta W, \text{g/liter}$	$n, (\text{cm})^{-4}$	$\ln n$
12/14	1.3	0.22	28.5	211	5.35
14/20	1.015	0.35	29.2	1357	7.21
20/28	0.718	0.246	37.5	7004	8.85
28/35	0.507	0.175	27.0	20194	9.91
35/48	0.358	0.123	24.7	74433	11.22

Smaller than 48 mesh  $\rightarrow$  rest

$$\begin{aligned} & \text{sample calculation, 12/14 mesh} \\ & 0.0285 \text{ gm/cm}^3 \\ & n = \frac{(1.4 \text{ gm/cm}^3)(0.42)(0.13 \text{ cm})^3(0.022 \text{ cm})}{0.0285 \text{ gm/cm}^3} = 211 \end{aligned}$$

Write L for  $\bar{L}$  and make a plot of  $\ln(n)$  against L. The least square line is drawn; slope =  $-6.005$ ; intercept on

Crystal size distribution function (CSD)

the ordinate = 13.19

$$\ln(n) = 13.19 - 6.005L \Rightarrow n = 5.35 \times 10^5 \exp(-6.005L)$$

Figure Prob 13.3 below

$$L \text{ is in mm} \Rightarrow GT = \frac{1}{6.005} \text{ mm}$$

$$\Rightarrow GT = 0.1653 \text{ mm}$$

$$\text{Holding time, } T = \frac{V}{Q} = \frac{200 \text{ liters}}{250 \text{ liter/hr}}$$

$$\text{Crystal growth rate, } G = \frac{0.8 \text{ hr}}{0.1653 \text{ mm}} = 5.74 \times 10^{-8} \text{ m/s}$$

$$\text{Zero size population density, } n^o = 5.35 \times 10^5 \text{ (cm)}^{-4}$$

Rate of nucleation,  $B^o$

$$= G n^o = (5.74 \times 10^{-8} \text{ m/s})(5.35 \times 10^5) \text{ cm}^{-4}$$

$$= 3.09 \times 10^5 \text{ nuclei/(m}^3\text{)(s)}$$

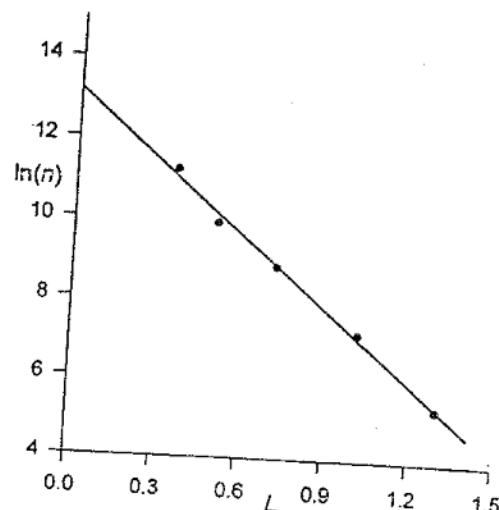


Fig. Prob 13.3. Plot of  $\ln(n)$  against  $L, \text{mm}$

\*

\*

\*

\*

(13.4) Given:  $\rho_c = 1.77 \text{ gm/cm}^3$ ,  $\Phi_{\text{re}} = 0.471$ . Calculated population data:

<u>Size range</u>	<u><math>L, \text{mm}</math></u>	<u><math>\Delta L, \text{mm}</math></u>	<u><math>\Delta W, \text{gm}</math></u> (out of 100 gm)	<u><math>\Delta W, \text{gm/lits}</math></u>	<u><math>n, \text{cm}^{-4}</math></u>	<u><math>\ln n</math></u>
0.71 to 0.5	0.605	0.21	8.46 gm	12.01 gm/lits.	3098	8.04
0.5 to 0.355	0.427	0.145	19.18	27.23	28933	10.27
0.355 to 0.25	0.302	0.095	30.11	42.76	138030	11.84
0.25 to 0.18	0.215	0.07	22.21	31.54	543818	13.21
0.18 to 0.125	0.152	0.055	11.92	16.93	740267	13.51
0.125 to 0.09	0.107	0.035	5.29	7.51	2101000	14.56
0.09 to 0.063	0.0765	0.027	1.87	2.66	2639608	14.78

Mass of sample sieve analyzed = 100 gm. Crystals per lits = 142 lits

Example  $\Delta W(\text{gm})$ , column IV, is converted to  $\Delta W, \text{gm/lits}$ , column V.  
For the first row,  $(\Delta W) \text{ gm/lits} = (8.46)(\frac{142}{100}) = 12.01 \text{ gm/lits}$

Sample calculation form:

$$\text{Last row, } n = \frac{2.66 \times 10^{-3} \text{ gm/cm}^3}{(1.77 \text{ gm/cm}^3)(0.471)(0.00765 \text{ cm})^3(0.0027)} = 2639608 \text{ numbers/cm}^4$$

The calculated data are plotted as  $\ln(n)$  against  $L(\text{mm})$  and the least square line is fitted. Slope =  $-12.8$ , intercept on vertical axis =  $15.76$

Crystal size distribution (CSD):

$$n = e^{15.76 - 12.8L} = 7 \times 10^6 e^{-12.8L}$$

$$\text{Residence time} = \frac{4 \text{ lits}}{8 \text{ lits/h}} = 0.5 \text{ h.}$$

$$Gt = \frac{1}{12.8} \text{ mm; } G = \frac{1}{(12.8)(0.5)} \text{ mm/h}$$

$$\Rightarrow G = 4.34 \times 10^{-8} \text{ m/h} = \text{crystal growth rate}$$

Zero-size population density,

$$n^o = 7 \times 10^6 (\text{cm})^{-4}$$

Nucleation rate,

$$B^o = Gn^o = (4.34 \times 10^{-8} \text{ m/h})(7 \times 10^6 \text{ cm}^{-4})$$

$$= 3.04 \times 10^7 \text{ nuclei} (\text{m}^3) (\text{s})$$

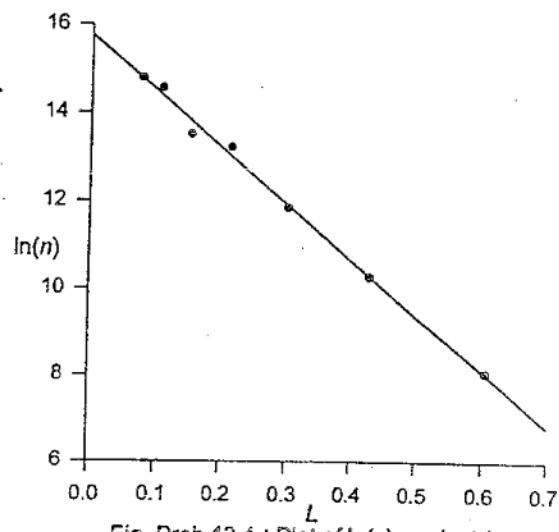


Fig. Prob 13.4.1 Plot of  $\ln(n)$  against  $L, \text{mm}$

13.6

(13.4) Contd...

The nucleation and growth rate data for three set of experiments at three different holding times are tabulated.

$\Omega \text{ (lit/hr)}$	$T(h)$	$B^\circ \frac{\text{number}}{\text{m}^3 \cdot \text{s}}$	$G, \text{m/s}$
24	0.167	$1.4 \times 10^8$	$1.3 \times 10^{-7}$
16	0.25	$7.1 \times 10^7$	$8.6 \times 10^{-8}$
8	0.50	$3.04 \times 10^7$	$4.34 \times 10^{-8}$

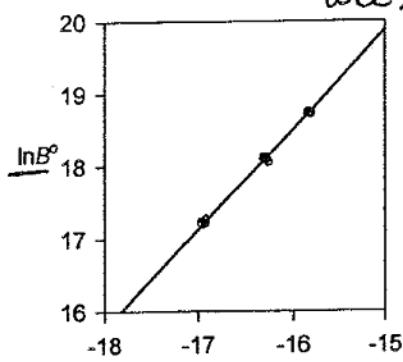
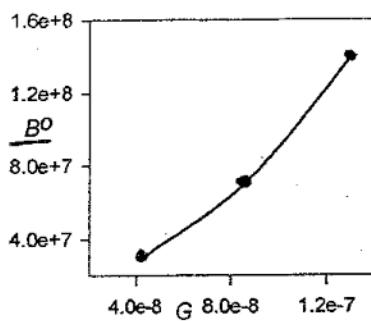


Fig. Prob 13.4.3

Fig. Prob 13.4.2. Plot of  $B^\circ$  against  $G$ .

The plot of  $B^\circ$  against  $G$  is nonlinear—see Fig. Prob 13.4.2.

However a plot on logarithmic scale (Fig. Prob 13.4.3) is linear with a slope of  $1.38$ . So the nucleation rate depends upon  $G$  as given below

$$B^\circ \propto G^{1.38} \text{ in the given case.}$$

(13.5) Mass average crystal size,  $L_m = \sqrt[3]{6 \cdot GT}$  [eq.(ix), Example 13.1]

$$\Rightarrow \sqrt[3]{6 \cdot GT} = 1 \text{ mm} \Rightarrow GT = 0.55 \text{ mm}$$

$$\text{Holding time, } T = V/\Omega = \frac{5 \text{ m}^3}{4 \text{ m}^3/\text{h}} = 1.25 \text{ h}$$

$$\text{Crystal growth rate, } G = 0.55 \text{ mm}/1.25 \text{ h} = 1.222 \times 10^{-7} \text{ m/s}$$

$$\text{Magma density, } M_T = 150 \text{ kg/m}^3 = 6 \text{ phfc } n^\circ (GT)^4, \text{ eq.(viii), Ex 13.1}$$

$$\Rightarrow 150 = (6)(0.6)(1900) \cdot n^\circ \cdot (5.5 \times 10^{-4})^4$$

$$\Rightarrow n^\circ = 2.4 \times 10^{11} \text{ number/m}^3 = \text{zero-size population density.}$$

$$\text{Secondary nucleation rate, } B^\circ = (2.4 \times 10^{11} \text{ m}^{-4})(1.222 \times 10^{-7} \text{ m/s})$$

$$= 2.933 \times 10^4 \text{ number/m}^3 \cdot \text{s}$$

(13.6) Working volume of the crystallizer,  $V = 5 \text{ m}^3; T = 1.8 \text{ h}$

$$\text{Product withdrawal rate, } Q_p = V/T = 5/1.8 = 2.778 \text{ m}^3/\text{h.}$$

$$\text{Suspension density, } M_T = 150 \text{ kg crystal/m}^3$$

$$\text{Production rate} = Q_p \cdot M_T = (2.778 \frac{\text{m}^3}{\text{h}})(150 \frac{\text{kg}}{\text{m}^3}) = 417 \text{ kg/h}$$

$$\text{Use Eq.(13.28). Median size, } L_M = 0.8 \text{ mm} = 3.67 GT; T = 1.8 \text{ h}$$

$$G = \frac{L_M}{3.67 T} = \frac{8 \times 10^{-4} \text{ m}}{(3.67)(1.8 \text{ h})} = 3.364 \times 10^{-8} \text{ m/s}$$

$$\text{Use Eq.(viii), Example 13.1. } M_T = 6 \text{ phfc } n^\circ (GT)^4$$

$$\Rightarrow 150 \frac{\text{kg}}{\text{m}^3} = (6)(0.65)(2500 \text{ kg/m}^3) \cdot n^\circ \cdot (8 \times 10^{-4} \text{ m/s})^4 \Rightarrow n^\circ = 6.814 \times 10^{12} \text{ m}^{-4}$$

= zero-size population density.

(13-7)

(13-6) Contd... Secondary nucleation rate,  $B^o = G \cdot n^o$

$$= (3.364 \times 10^{-8} \text{ m/s})(6.814 \times 10^{12} \text{ m}^{-3}) = 2.292 \times 10^5 \frac{\text{nuclei}}{\text{m}^3 \cdot \text{s}}$$

Mass fraction and number fraction of crystals upto  $L = 0.2 \text{ mm}$

$$\text{Dimensionless size, } x = \frac{L}{GT} = \frac{0.2 \text{ mm}}{0.218 \text{ mm}} = 0.917 \quad [GT = \frac{0.8 \text{ mm}}{3.67} = 0.218 \text{ mm}]$$

Cumulative mass fraction of crystals upto size  $x$ , eq(13.25b)

$$W(x) = 1 - e^{-x} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) \Rightarrow \text{for } x = 0.917, W = 0.014 \text{ or } 1.4\%$$

Number fraction of crystals upto a dimensionless size  $x = 0.917$

$$\frac{N}{N_T} = \frac{\int_0^L n^o e^{-L/GT} dL}{\int_0^\infty n^o e^{-L/GT} dL} = \frac{n_0 GT \int_0^{0.917} e^{-x} dx}{n_0 GT \int_0^\infty e^{-x} dx} = 0.6 \Rightarrow 60\%$$

Note that crystals upto a size of  $0.2 \text{ mm}$  constitute 60% of the population but have a mass of only 1.4% of the total mass.

\* \* \*

(13-7) Given:  $G = 2.31 \times 10^{-8} \text{ m/s}$ ;  $T = 2h = 7200s \Rightarrow GT = 1.663 \times 10^{-4} \text{ m}$

(a) Cumulative mass distribution function is given by eq(13.25b)

$$W(x) = 1 - e^{-x} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right), x = \frac{L}{GT} = \frac{6.013 \times 10^3 L}{1.663 \times 10^{-4} \text{ m}}, L \text{ in meter.}$$

$$\text{Dominant size, } L_D = 3 \cdot GT = (3)(1.663 \times 10^{-4} \text{ m}) = 0.5 \text{ mm}$$

$$\text{Median size, Eq(13.28), } L_M = 3.67 GT = (3.67)(1.663 \times 10^{-4} \text{ m}) = 0.61 \text{ mm}$$

(b) To achieve a dominant size,  $L_D = 0.6 \text{ mm}$ ,

$$GT = \frac{L_D}{3} = 0.6 \text{ mm}/3 = 0.2 \text{ mm}; G = 2.31 \times 10^{-8} \text{ m/s}$$

$$T = \frac{0.2 \text{ mm}}{2.31 \times 10^{-5} \text{ mm/s}} = 8660s = 2.4 \text{ h}$$

Volumetric rate of product withdrawal,

$$Q_p = V/T = \frac{2 \text{ m}^3}{2.4 \text{ h}} = 0.833 \text{ m}^3/\text{h}$$

$$\text{If we take } L = 0.2 \text{ mm}, x = \frac{L}{GT} = \frac{0.2 \text{ mm}}{0.2 \text{ mm}} = 1$$

Mass fraction of crystals below  $L = 0.2 \text{ mm}$ , i.e.  $x = 1$  is

$$W(x=1) = 1 - e^{-1} \left(1 + 1 + \frac{1}{2} + \frac{1}{6}\right) = 1 - 0.981 = 0.019 \Rightarrow 1.9\%$$

\* \* \*

(13.8)

(a) Start with the population balance equation (13.18)

$$\frac{d(nG)}{dL} + \frac{n}{\tau} = 0 ; \text{ for } L=0, n=n^o ; G=G_0 (1+rL)^b$$

$$\Rightarrow \frac{d(nG)}{dL} + \frac{nG}{\tau} = 0 \Rightarrow \frac{d(nG)}{nG} = -\frac{dL}{\tau} = -\frac{dL}{\tau G_0 (1+rL)^b}$$

Integrating,  $\ln(nG) = -\frac{1}{\tau G_0} \int \frac{dL}{(1+rL)^b} + K' = -\frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)} (1+rL)^{1-b} + K'$

Put  $L=0, n=n^o, G=G_0$  to get

$$\ln(n^o G_0) = -\frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)} + K' \Rightarrow K' = \ln(n^o G_0) + \frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)}$$

The population distribution function becomes

$$\ln\left(\frac{nG}{n^o G_0}\right) = -\frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)} \cdot (1+rL)^{1-b} + \frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)} = \frac{1-(1+rL)^{1-b}}{\tau G_0 \cdot r(1-b)}$$

$$\Rightarrow nG = n^o G_0 \exp\left[\frac{1-(1+rL)^{1-b}}{\tau G_0 \cdot r(1-b)}\right]$$

$$\Rightarrow n(L) = n^o (1+rL)^{-b} \exp\left[\frac{1-(1+rL)^{1-b}}{\tau G_0 \cdot r(1-b)}\right] \quad \dots \dots \dots \quad (i)$$

(b) Given:  $G_0 = 2.22 \times 10^9 \text{ m}^3/\text{s}$ ;  $\tau = 1\text{-h}$ ;  $r = 1.25 \times 10^4 \text{ m}^{-1}$ ;  $b = 0.2$

$$\phi_v = 0.48; P_c = 1460 \text{ kg/m}^3; n^o = 8 \times 10^{14} \text{ m}^{-4}$$

Putting the concerned values in eq (i) above, the population density distribution function becomes

$$n(L) = 8 \times 10^{14} (1 + 1.25 \times 10^4 L)^{-0.2} \cdot \exp\left[1.25 \left\{1 - (1 + 1.25 \times 10^4 L)^{0.8}\right\}\right]; \text{ in } \frac{\text{m}}{\text{s}}$$

Magnitude density,  $M_T = \int (\phi_v L^3 P_c) n(L) dL$

Put the expression for  $n(L)$  and numerically integrate to get  $M_T = 288 \text{ kg/m}^3$

(13.9)

(13.9) Given :  $G = \frac{5 \times 10^8 \text{ m}^3/\text{s}}{\text{m}^3}$ ;  $V = 10 \text{ m}^3$ ;  $Q_p = 4 \text{ m}^3/\text{h}$ ;  $T = \frac{V}{G} = \frac{2.5 \text{ hr}}{\text{m}}$   
 $n^o = 5 \times 10^{10} / (\text{cm})(\text{litter}) = \frac{5 \times 10^{15} \text{ m}^{-4}}{\text{m}^4}$ ;  $GT = (5 \times 10^8)(2.5 \times 3600) = 4.5 \times 10^{12} \text{ m}^{-3}$

(a) Total number of crystals in the vessel at any time,

$$N_T = n^o GT = (5 \times 10^{15} \text{ m}^{-4})(4.5 \times 10^{12} \text{ m}^{-3}) \xrightarrow{\text{Eq (iii), Ex. 13.1}} 2.25 \times 10^{12} \text{ m}^{-3}$$

(b) For  $L = L_1 = 0.5 \text{ mm}$ , dimensionless size,  $x = \frac{0.5 \times 10^{-3} \text{ m}}{4.5 \times 10^{-4} \text{ m}} = 1.11$   
For  $L = L_2 = 1 \text{ mm}$ ,  $x = 2.22$

Mass fraction lying between  $x = 1.11$  to  $x = 2.22$

$$= W(x_2) - W(x_1) = 0.973 - 0.8154 \Rightarrow 15.8\%$$

(c) Coefficient of variation = 52%, see Section 13.5.3.

\* \* \* \*

(13.10)  $B^o = n^o G \Rightarrow 7.33 \times 10^7 M_T^{0.6} G^{0.5} = 9.2 \times 10^{12} G$

Put  $M_T = 20 \text{ kg/m}^3$  to get  $G = \frac{2.31 \times 10^{-9} \text{ m}^3/\text{s}}{\text{dt}} = \frac{dL}{dt}$

Rate of increase in the mass of a crystal can be

written as  $\frac{dm_c}{dt} = \frac{d}{dt} (\rho_v L^3 \rho_c) = 3 \rho_v L^2 \rho_c \cdot \frac{dL}{dt}$

Put  $\rho_v = 0.6$ ;  $L = 5 \times 10^{-4} \text{ m}$ ;  $\rho_c = 2700 \text{ kg/m}^3$  to get

$$\frac{dm_c}{dt} = (3)(0.6)(5 \times 10^{-4})^2(2700)(2.31 \times 10^{-9}) = \frac{2.8 \times 10^{-12} \text{ kg/s}}{* * * *}$$

(13.11)

Given :  $V = \text{m}^3$ ;  $G = 2.31 \times 10^{-8} \text{ m}^3/\text{s}$ ;  $Q_i = 1 \text{ m}^3/\text{h}$  (since  $T = 2 \text{ h}$ )

clear liquid advance rate =  $Q_o \text{ m}^3/\text{h}$ ; underflow =  $Q_p \text{ m}^3/\text{h}$ .

Assume  $Q_i = Q_o + Q_p$ .

Use Eq (13.42b), put  $L_f = 0$  since the overflow does not have fines.  
 $n = n^o \exp(-L/G\tau_p) \Rightarrow w(x) = 1 - e^{-x} (1 + x + x^2/2 + x^3/6)$ , Eq (13.25b)

so mass% of the crystals are above 0.8 mm  $\Rightarrow w(x) = 0.2$

$$\Rightarrow x = 2.31 = \frac{1}{G\tau_p} = \frac{0.8 \text{ mm}}{(2.31 \times 10^{-8} \text{ m}^3/\text{s}) \tau} \Rightarrow \tau_p = \underline{4.17 \text{ hr}}$$

(13.11) Contd...  $Q_p = V/\tau_p = 2m^3/4.17h = 0.48m^3/h$ ; i.e.  $Q_0 = \underline{0.52 m^3/h}$

Case 2: Fines are removed in the overflow up to a size of  $80\mu\text{m}$ .

Since the fines removed constitute a small mass fraction of the crystals, the distribution function of the former case applies.

The answer remains the same, i.e.  $\underline{Q_0 = 0.52 m^3/h}$

(13.12) Given:  $V = 3.5 m^3$ ;  $Q_0 = 10 m^3/h$ ;  $Q_p = 3 m^3/h$ ;  $\Phi_0 = 0.55$ ;  $G = 1.45 \text{ mm}/h = 4.03 \times 10^{-7} \text{ m}/s$ ;  $M_T = 150 \text{ kg}/m^3$ ;  $P_c = 1993 \text{ kg}/m^3$ ;  $L_F = 7 \times 10^{-5} \text{ m}$

Use Eq.(i), Example 13.6, to calculate  $n^\circ$ .  $\tau_F = 3.5/10 = \underline{0.35 \text{ h}}$

$$M_T = 6 \Phi_0 P_c n^\circ \exp\left[-\frac{L_F}{G\tau_F}\right] (G\tau_p)^4 \quad \tau_p = 3.5/3 = \underline{1.167 \text{ h}}$$

$$\Rightarrow 150 \frac{\text{kg}}{m^3} = (6)(0.55)(1990 \text{ kg}/m^3) \cdot n^\circ \cdot \exp\left[-\frac{7 \times 10^{-5} \text{ m}}{4.03 \times 10^{-7} \frac{\text{m}}{\text{s}} \times (0.35 \times 3600 \text{ s})}\right] \times$$

$$\Rightarrow n^\circ = \underline{3.192 \times 10^9 \text{ m}^{-4}} \quad (4.03 \times 10^{-7} \text{ m/s} \times 1.167 \times 3600 \text{ s})^4$$

$$\text{Nucleation rate, } B^\circ = n^\circ G = (3.192 \times 10^9 \text{ m}^{-4})(4.03 \times 10^{-7} \text{ m/s}) \\ = \underline{1280 \text{ nuclei}/m^3 \cdot s}$$

Mass fraction of crystals removed as fines ( $L \leq L_F$ ) at the top.

Fines per unit volume,

$$M_F = \int_0^{L_F} (\Phi_0 L^3 P_c) \cdot n(L) dL; \quad n(L) \text{ is given by Eq. (13.42a)}$$

$$= (\Phi_0 P_c n_0) (G\tau_F)^4 \int_0^{\xi_F} \xi^3 e^{-\xi} d\xi; \quad \xi = L/G\tau_F \\ \xi_F = L_F/G\tau_F = 0.138$$

$$\Rightarrow M_F = (0.55)(1990)(3.192 \times 10^9)(5.078 \times 10^{-4})^4 (0.001) = \underline{2.323 \times 10^{-4} \frac{\text{kg}}{m^3}}$$

Mass fraction of crystals withdrawn as fines at the top

$$= \frac{M_F}{M_T} \cdot \frac{Q_0}{Q_p} = \frac{2.323 \times 10^{-4} \text{ kg}/m^3}{150 \text{ kg}/m^3} \cdot \frac{10 \text{ m}^3/h}{3 \text{ m}^3/h} = \underline{5.16 \times 10^{-6}}, \text{ a very small fraction.}$$

13.11

(13.13) Refer to Fig Prob 13.13 (a). If there are no crystals in the feed, the population balance equation can be written as:

$$-VG \frac{dn}{dL} = zQ_p n + (R'-1)Q_p \cdot n \cdot [1 - H(L_F - L)] - (z-1)Q_p \cdot n \cdot [1 - H(L - L_c)].$$

Significances of the terms on the r.h.s.:

$zQ_p n$  = rate of removal of crystals at the bottom

$(R'-1)Q_p \cdot n$  = rate of removal of fines at the top ( $L \leq L_F$ )

$(z-1)Q_p \cdot n$  = rate of return of the crystals of size  $L \leq L_c$   
through the elutriation leg.

The equation reduces to the following forms for the three size ranges.

$$\text{For } L \leq L_F, -VG \frac{dn}{dL} = zQ_p n + (R'-1)Q_p \cdot n - (z-1)Q_p \cdot n$$

$$\Rightarrow -VG \frac{dn}{dL} = R'Q_p \cdot n \Rightarrow n = n^* \exp\left(-\frac{R'}{GT} \cdot L\right), T = V/Q_p \dots (i)$$

$$\text{For } L_F \leq L \leq L_c, -VG \frac{dn}{dL} = zQ_p n - (z-1)Q_p \cdot n = Q_p \cdot n$$

$$\Rightarrow n = c' \exp(-L/GT) \dots (ii)$$

$$\text{For } L_c \leq L, -VG \frac{dn}{dL} = zQ_p n \Rightarrow n = c'' \exp\left(-\frac{zL}{GT}\right) \dots (iii)$$

Using the condition of "continuity of population density" at  $L = L_F$  and at  $L = L_c$ , the constants  $c'$  and  $c''$  can be determined. The resulting density functions are shown under "Hints" to Prob 13.13.

\* \* \* \*

(13.14) Feed = 2500 kg solution at  $80^\circ\text{C}$ , 125 kg anhydrous salt per 1000 kg water.

$$\text{Mass of } \text{Na}_2\text{CrO}_4 = (2500) \left( \frac{125}{1000+125} \right) = \underline{\underline{277.8 \text{ kg}}}; \text{ Water} = \underline{\underline{222.2 \text{ kg}}}$$

3% of the water evaporates  $\Rightarrow (2222.2)(0.03) = \underline{\underline{66.7 \text{ kg water lost}}}$

$$\text{Let the mass of } \text{Na}_2\text{CrO}_4 \cdot 10\text{H}_2\text{O} \quad \begin{bmatrix} \text{Molar } \text{Na}_2\text{CrO}_4 = 162 \\ \text{Na}_2\text{CrO}_4 \cdot 10\text{H}_2\text{O} = 342 \end{bmatrix}$$

deposited at the end =  $m$  kg

$$\text{Na}_2\text{CrO}_4 \text{ deposited} = \frac{162}{342} \cdot m = \underline{\underline{0.4737 \cdot m \text{ kg}}}; \text{ water} = \underline{\underline{0.5263 \cdot m \text{ kg}}}$$

$$\text{Mass of water in the final slurry} = \frac{2222.2 - 66.7 - 0.5263m}{2155.5 - 0.5263m}$$

13.12

(13.14) contd... Solubility of  $\text{Na}_2\text{CrO}_4$  at  $30^\circ\text{C}$  =  $88.7 \text{ kg}/1000 \text{ kg water}$ .

Mass of  $\text{Na}_2\text{CrO}_4$  in solution at the end

$$= (2155.5 - 0.5263m) \left( \frac{88.7}{1000} \right) = \underline{\underline{191.2 - 0.0467m \text{ kg}}}$$

$\text{Na}_2\text{CrO}_4$  balance:  $(277.8 - 0.4737m) = (191.2 - 0.0467) \Rightarrow m = \underline{\underline{203 \text{ kg}}}$

Quantity of Crystals =  $203 \text{ kg} + 1.2 \text{ kg (seed)} = \underline{\underline{204.2 \text{ kg}}}$

Crystal size calculation: Seed size =  $L_8 = 70 \mu\text{m} = \underline{\underline{7 \times 10^{-5} \text{ m}}}$

Mass of a seed =  $\phi_v L_s^3 P_c = (0.45)(70 \times 10^{-6})^3 (1480) = \underline{\underline{2.284 \times 10^{-10} \text{ kg}}}$

Number of seed crystals =  $\frac{1.2 \text{ kg}}{2.284 \times 10^{-10} \text{ kg}} = \underline{\underline{5.254 \times 10^9}}$

Mass of a product crystal =  $\frac{204.2 \text{ kg}}{5.254 \times 10^9} = \underline{\underline{3.87 \times 10^{-8} \text{ kg}}}$

If  $L$  is the average size of a product crystal,

$$\phi_v L^3 P_c = 3.87 \times 10^{-8} \Rightarrow (0.45) \cdot L^3 \cdot (1480) = 3.87 \times 10^{-8} \Rightarrow L = \underline{\underline{3.87 \times 10^{-4} \text{ m}}}$$

$$\text{Residence time} = 6h = \frac{6 \times 3600}{\underline{\underline{0.387 \text{ mm}}}} = \underline{\underline{0.387 \text{ mm}}}$$

Average growth rate,

$$G = \frac{L - L_8}{t} = \frac{3.87 \times 10^{-4} - 70 \times 10^{-6}}{6 \times 3600} = \underline{\underline{1.47 \times 10^{-8} \text{ m/s}}}$$

\*                   \*                   \*                   \*

13.15

Calculation of the quantity of feed solution per batch.

Initial solution concentration ( $80^\circ\text{C}$ ) =  $0.07 + (0.00132)(80)$

$$= 0.176 \text{ kg } \text{K}_2\text{SO}_4/\text{kg solution} = \frac{0.176}{1 - 0.176} = \underline{\underline{0.213 \text{ kg/kg water}}}$$

Final solution concentration ( $25^\circ\text{C}$ ) =  $0.07 + (0.00132)(25)$

$$= 0.103 \text{ kg/kg solution} = \underline{\underline{0.115 \text{ kg per kg water}}}$$

Neglect the mass of seeds added.

Product per kg water =  $0.213 - 0.115 = \underline{\underline{0.098 \text{ kg/kg water}}}$

Mass of product required = 400 kg

$$\begin{aligned} \text{Mass of feed solution} &= (400 \text{ kg}) \times \frac{1 \text{ kg water}}{0.098 \text{ kg crystal}} \times \frac{1.213 \text{ kg soln.}}{1 \text{ kg water}} \\ &= \frac{4950}{1200} = \underline{\underline{4.125 \text{ m}^3 \text{ solution}}} \end{aligned}$$

(13.15) contd... Final volume of slurry

(13.13)

$$= \text{volume of crystals} + \text{volume of solution}$$

$$= \frac{400}{2660} + (400 \text{ kg crystal}) \left( \frac{1 \text{ kg water}}{0.098 \text{ kg crystal}} \right) \left( \frac{1.115 \text{ kg soln}}{1 \text{ kg water}} \right) \left( \frac{1}{1200 \text{ kg/m}^3} \right)$$

$$= 0.15 \text{ m}^3 + 3.79 \text{ m}^3 = 3.94 \text{ m}^3 \text{ slurry.}$$

$$\text{Suspension density} = \frac{400 \text{ kg}}{3.94 \text{ m}^3 \text{ slurry}} = \frac{101.5 \text{ kg crystal}}{\text{m}^3 \text{ slurry}}$$

which is less than the allowable limit of 120.  
If the crystallization tank is 75% full,

$$\text{volume of crystallizer} = \frac{4.125 \text{ m}^3 (\text{feed})}{0.7} = \underline{5.9 \text{ m}^3, \text{ say } 6 \text{ m}^3}$$

$$\text{Mass of a product crystal } (L = 0.8 \text{ mm} = 8 \times 10^{-4} \text{ m})$$

$$= \rho_w L^3 P_c = (0.9)(8 \times 10^{-4})^3 (2660) = 1.226 \times 10^{-6} \text{ kg}$$

$$\text{Number crystals (in 400 kg product)} = \frac{400 \text{ kg}}{1.226 \times 10^{-6} \text{ kg}} = \underline{3.263 \times 10^8}$$

From this figure it can be shown that the mass of seed crystals is about 0.1% of the product.

$$\text{Average growth rate, } G = 5 \times 10^{-8} \text{ m/s; } L_s = \underline{8 \times 10^{-7} \text{ m}}$$

$$\Rightarrow \text{residence time} = \frac{L - L_s}{G} = \frac{8 \times 10^{-4} - 8 \times 10^{-5}}{5 \times 10^{-8}} = \underline{14,400 \text{ s}}$$

Cooling rate calculation = 4 hr

$$\text{Solubility, } C = 0.07 + 0.001320$$

Initial cooling rate [See Eq.(13.48)], use Eq.(13.48)

$$-\frac{d\theta}{dt} = \frac{3 M_s}{a_2 V} \cdot \frac{G}{L_s} (Z+1)^2 ; M_s = 0.4 \text{ kg} \text{ (calculate it!)}$$

$$\Rightarrow -\frac{d\theta}{dt}_i = \frac{(3)(0.4)}{(1.32)(4.125)} \cdot \frac{5 \times 10^{-8}}{8 \times 10^{-5}} = \underline{1.3 \times 10^{-3} \text{ }^\circ\text{C/s.}}$$

Final cooling rate,  $-\frac{d\theta}{dt}_f = 0.13 \text{ }^\circ\text{C/s}$  (calculate it!)

(13-16)

(13-14)

Product crystal size =  $1.1 \text{ mm}$ , mass of product =  $500 \text{ kg/batch}$

$$\text{Number of crystals, } N = \frac{500}{\varphi_b L^3 P_c} = \frac{500}{(0.95)(1.1 \times 10^{-3})^3 (1750)} = 2.26 \times 10^8$$

$$\text{Mass of seed crystals} = \varphi_b L^3 P_c N = (0.95)(1.1 \times 10^{-3})^3 (1750)(2.26 \times 10^8)$$

Batch time (growth time of crystals),  $= 0.129 \text{ kg}$

$$t = \frac{L - L_s}{G} = \frac{1.1 \times 10^{-3} - 7 \times 10^{-5} \text{ m}}{6 \times 10^{-8} \text{ m/s}} = 17160.8 = 4.77 \text{ hr}$$

The evaporation profile can be calculated from Eq.(13-45).

$$-\frac{dV}{dt} = \frac{3 M_s}{C_s} \cdot \frac{G}{L_s} \left(1 + \frac{Gt}{L_s}\right) = \frac{(3)(0.129)}{275} \cdot \frac{6 \times 10^{-8}}{7 \times 10^{-5}} \left(1 + \frac{6 \times 10^{-8}}{7 \times 10^{-5}} t\right)^2$$

$$= 1.206 \times 10^{-6} \left(1 + 8.57 \times 10^{-4} t\right)^2 \rightarrow 1.206 \times 10^{-6} \text{ m}^3/\text{s} \quad \text{at } t = 0$$

$$\text{and } -\frac{dV}{dt} = 1.894 \times 10^{-5} \text{ m}^3/\text{s} \quad \text{at } t = 4.77 \text{ hr.}$$

Volume of slurry: Take maximum allowable slurry density,

$$\text{Final Slurry volume} = \frac{500 \text{ kg crystal}}{1750 \text{ kg/m}^3} + \frac{500 \text{ kg crystal}}{\frac{180 \text{ kg/m}^3 \text{ soln.}}{180 \text{ kg crystal/m}^3}} = 3.064 \text{ m}^3$$

$$\text{Feed solution} \rightarrow 3.064 \text{ m}^3 + 500 \left(1 + \frac{1}{0.275}\right) \times (1.150) \text{ m}^3 = 5.08 \text{ m}^3$$

To take into account, bubbling, possible foaming and a guard against entrainment a crystallizer volume of  $10 \text{ m}^3$  may be considered.

- (14.1) (iii); (14.2) (i) polysulfone, the other two polymers are not soluble in any solvent; (14.4) (ii); (14.5) (i) cross-linked PVA;
- (14.6) (i) gas separation; (14.7) (iii) pervaporation; (14.8) (i);
- (14.9) (i) very large; (14.10) (i) fully permeable; (14.11) (i) increases;
- (14.12) (iii) track etching; (14.13) (i) water, CA is not soluble in water;
- (14.14) (i)  $\text{Na}^+$ , since the membrane has 'fixed' negative charges;
- (14.15) (i)  $1 - 100 \text{ nm}$ ; (14.17) (i) pore mouth adsorption; (14.18)  $1 - \frac{0.32}{8} = 0.96$ , (iii); (14.19)  $\frac{8\%}{4\%} = 2$ , (i); (14.20)  $R_m + R_g = \frac{\Delta P}{J_w \cdot \mu}$
- $$\Rightarrow R_m + R_g = \frac{4 \times 10^5}{(0.052)(9 \times 10^4)} = 8.55 \times 10^{-9} \text{ m}^{-1} \Rightarrow R_g = (8.55 - 2.4) \times 10^{-9}$$
- $= 6.15 \times 10^{-9} \text{ m}^{-1}$ , (i); (14.21) (iii)  $12,000 \text{ m}^2/\text{m}^3$ ; (14.22) (ii) hemodialysis;
- (14.23) (i) diffusivity increases; (14.24) (iii) pervaporation, since the solvent does not have any affinity for water (water in MIBK has a high fugacity); (14.25) (i) 1.2 Barrer;
- (14.26) (iii) rubbery; (14.27) (i) + (ii); (14.28) (iii) 13.5 Barrer;
- (14.29) Pol. Mod. =  $C_m/C_b = \exp(J_w/k_L) = \exp(\frac{0.042/3600 \text{ m}^2/\text{s}}{5 \times 10^{-6} \text{ m}^2}) = 10.3$ , (ii);
- (14.30) (ii) CA, polyimide, (iii) PDMS, (iv) PDMPS; (v) asymmetric UF membrane, (vii) crosslinked PVA, (viii) asymmetric UF, polysulfone, (x) PDMPS; (xiii) ceramic membrane, (xiv) CA;
- (14.31) (i) increases; (14.32) (iii) convection; (14.33) (i)  $1 \times 10^{16}$ ;
- (14.34) (iii)  $\text{m}^{-1}$ ; (14.35) (i) increases; (14.36)
- (14.37) Transport occurs by a combination of convection and diffusion; (14.38) reduced flux because of osmotic pressure, higher salt permeation rate, 'scaling' on the membrane surface; (14.39) 'Apparent' rejection =  $1 - C_p/C_b = 0.975$ ,  $C_b = 0.02$
- $$\Rightarrow C_p = 5 \times 10^{-4}, J_w(C_b - C_p) = k_L(C_m - C_b) \Rightarrow 1.8 \times 10^{-4} (0.02 - 5 \times 10^{-4})$$
- $= 4.4 \times 10^{-5} (C_m - 0.02) \Rightarrow C_m = 0.1$
- , true rejection =
- $1 - \frac{C_p}{C_m} = 1 - \frac{5 \times 10^{-4}}{0.1} = 0.995$
- (14.40) biological attack, hydrolysis under...; (14.41)  $J_A = J_B \Rightarrow$
- $$(\hat{P}_A/l_m) \cdot (P_1 x_A - P_2 y_A) = (\hat{P}_B/l_m) [P_1 (1-x_A) - P_2 (1-y_A)], \text{ given, } y_A = 0, P_1 = 12.5,$$
- $P_2 = 1.5, \hat{P}_A/\hat{P}_B = 3.15, \text{ solve to get } x_A = \underline{0.212}, x_B = \underline{0.788}.$

(14.2)

(14.42) (i)  $\alpha_{AB} = 1$ ; (14.43) (ii) decreases; (14.44) The alternative arrangement will not work satisfactorily.

Brine is fed into the cathode compartment and the product NaOH is also withdrawn from this compartment. As a result the product will have a lot of NaCl in it.

(14.45)  $\hat{E} = 1 - e^{-N_T}$  [see Eq.(14.52), take  $\varsigma \approx 0$ ]  $\Rightarrow 0.85 = 1 - e^{-N_T} \Rightarrow N_T = 1.9$   
 Calculate the length of the hollow fibers :  $L = a_m / n \cdot B'$ ;  $B' = \frac{\pi d}{4}$   
 $\Rightarrow L = 1.2 \text{ m}^2 / n \cdot (\pi \times 300 \times 10^{-6} \text{ m}) = 0.255 \text{ m}$ .  
 'Height of a transfer unit'  $= L/N_T = 0.255 \text{ m} / 1.9 = \underline{0.134 \text{ m}}$

### PROBLEMS

(14.1) Use Eq.(14.4). Plot  $J_w (\text{kg/m}^2\text{h})$  against  $\Delta P (\text{kPa})$ , Fig. Prob 14.1.  
 Slope =  $12.5 \text{ kg/hm}^2\text{kPa}$

$$= \underline{0.0125 \text{ m}^3/\text{hm}^2\text{kPa}}$$

$$\Rightarrow \frac{\epsilon d^2}{32\mu k_m T} = 0.0125 \frac{\text{m}^3}{\text{hm}^2\text{kPa}}$$

$$\text{Given: } \epsilon = 0.32, l_m = 100 \mu\text{m} = \underline{10^{-4} \text{ m}}$$

$$d = 1.0 \mu\text{m} = 1.0 \times 10^{-6} \text{ m}$$

$$\mu = 0.9 \text{ cP} = 9 \times 10^{-4} \text{ kg/m s}$$

$$\tau = \frac{(0.32)(1.0 \times 10^{-6})^2 (3600)(100)}{(32)(9 \times 10^{-4})(10^{-4})(0.0125)}$$

$$= \underline{3.2}$$

\* \* \*

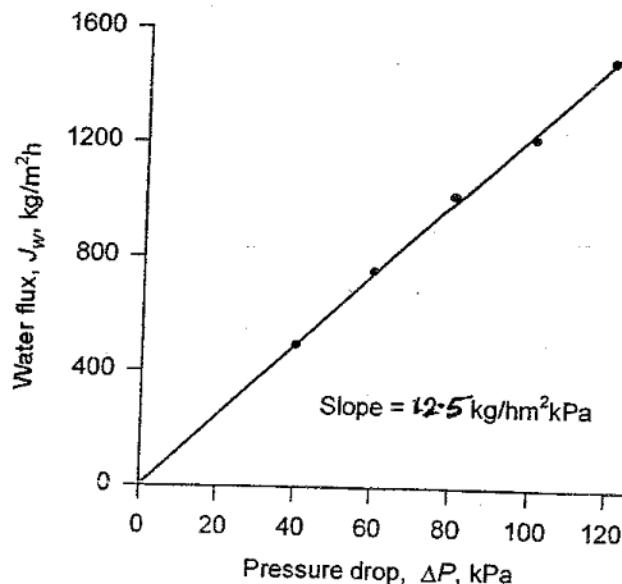


Figure Prob 14.1: Plot of  $J_w$  against  $\Delta P$

(14.2) Solute rejection,  $R' = 0.96$ ; no concentration polarization

$$\Rightarrow C_m = C_b = 0.015 \text{ molar}; C_p = (0.015)(1-0.96) = 6 \times 10^{-4} \text{ molar}$$

'Effective' pressure driving force =  $\Delta P - \Delta T T = 2.9 - 0.357 = \underline{2.543 \text{ bar}}$

$$[\Delta P = 4 - 1.1 = 2.9 \text{ bar}; \Delta T T = \Delta C \cdot R T = (0.015 - 6 \times 10^{-4})(0.0821)(298) = \underline{0.357 \text{ bar}}]$$

(14.3)

(14.2) contd... 2nd part: Polarization modulus =  $C_m/C_p = 5.5$

$$\Rightarrow C_m = (5.5)(0.015) = 0.0825 \text{ molar. } C_p = (0.0825)(1 - 0.96) = 0.0033.$$

$$\Delta T = (0.0825 - 0.0033)(0.0821)(298)(1.013) = 1.963 \text{ bar}$$

$$\text{'Effective' pressure driving force} = \Delta P - \Delta T = 2.9 - 1.963 = 0.937 \text{ bar}$$

(14.3) Follow the procedure of Example 14.4.

Initial volume of solution =  $V_0$ , number of moles =  $n_0$ ; at time  $t$ , volume =  $V$ , number of moles =  $n$  (since the rejection is not 100%); change of solution volume =  $dV$  in time  $dt$ .

Moles of solute leaving with the permeate in time  $dt$

$$= -\left(\frac{n}{V}\right) \cdot dV(1-R') = -dn \Rightarrow (1-R') \ln \frac{V}{V_0} = \ln \frac{n}{n_0} \Rightarrow n = n_0 \left(\frac{V}{V_0}\right)^{1-R'}$$

'Effective' driving force at any time,  $t$ ,

$$= \Delta P - \frac{RT \cdot n}{V} \cdot R' = \Delta P - \frac{RT \cdot R' \cdot n_0 (V/V_0)^{1-R'}}{V} = \Delta P - RT \cdot \frac{R' n_0}{V^{(1-R')}} \cdot \frac{1}{VR'}$$

$$J_w = -\frac{L}{a_m} \frac{dV}{dt} = L_p \left[ \Delta P - \frac{\beta \cdot \Delta P}{V^{R'}} \right]; \quad t=0, V=V_0 \quad = \Delta P - \frac{\beta \cdot \Delta P}{V^{R'}} \\ - \int_{V_0}^V \frac{V^{R'}}{V^{R'} - \beta} \cdot dV = L_p \cdot \Delta P \cdot a_m \cdot t = (V_0 - V) + \beta \ln \frac{V_0 - \beta}{V - \beta} \dots (i) \quad \beta = \frac{RT \cdot R' n_0}{\Delta P \cdot V_0^{(1-R')}}$$

Given:  $V_0 = 800 \text{ gal} = (800)(3.78)/1000 = 3.024 \text{ m}^3$ ;  $\frac{\text{Molal. of solute}}{1213} = 1213$

$$n_0 = (3.024)(14/1213) = 0.0349 \text{ kmol}; \text{ final concentration} =$$

$$n/V = 84 \text{ kg/m}^3 = 84/1213 = 0.0692 \text{ kmol/m}^3 = \left(\frac{n_0}{V}\right)(V/V_0)^{1-R'}$$

$$\Rightarrow 0.0692 = \frac{0.0349}{V} \cdot \frac{V^{1-R'}}{(3.024)^{1-R'}} \Rightarrow V = 0.477 \text{ m}^3 \text{ (putting } R' = 0.97)$$

Now consider Eq (i) to calculate the membrane area,  $a_m$ , after putting the values of the different quantities.

$$\text{Pure water flux (given)} = L_p \Delta P = 11.5 \text{ gal/ft}^2 \text{ h} = \frac{(11.5)(3.78)(10^{-3})}{(0.3048)^2 (3600)}$$

$$= \frac{1.3 \times 10^{-4}}{m^3/m^2 s} ; \quad \beta = \frac{RT \cdot R' n_0}{\Delta P \cdot V_0^{1-R'}} = \frac{(0.0821)(298)(0.97)(0.0349)}{(3/1.013)(3.024)^{0.03}}$$

$$\Rightarrow \beta = 0.2706 ; \quad t = 2 \text{ hr} = 7200 \text{ s};$$

$$\text{Put in Eq (i) to get } a_m = 3.47 \text{ m}^2$$

A more accurate calculation may be done by putting  $R' = 0.97$  and evaluating the integral in Eq (i) numerically.

\* \* \* \*

14.4

(14.4) From Eq (14.4), membrane resistance,  $R_m = \frac{32\tau h_m}{\epsilon d^2}$

Given:  $\tau = 2.7$ ;  $h_m = 0.3 \times 10^{-6} \text{ m}$ ;  $\epsilon = 0.35$ ;  $d = 15 \times 10^{-9} \text{ m}$ .

$$R_m = \frac{(32)(2.7)(0.3 \times 10^{-6})}{(0.35)(15 \times 10^{-9})^2} = \underline{\underline{3.3 \times 10^{11} \text{ m}^{-1}}}$$

$$\Delta P = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2; \mu = 9 \times 10^{-4} \text{ kg/m.s}; J_w = 1.2 / 3600 = \frac{3.333 \times 10^{-4} \text{ m}^3}{\text{m}^2 \cdot \text{s}}$$

$$J_w = \frac{\Delta P}{(R_m + R_g)\mu} \Rightarrow R_m + R_g = \frac{4 \times 10^5}{(3.333 \times 10^{-4})(9 \times 10^{-4})} = \underline{\underline{1.333 \times 10^{12} \text{ m}^{-1}}}$$

$$R_g = 1.333 \times 10^{12} - 3.3 \times 10^{11} = 1.003 \times 10^{12} = \frac{R_g = \text{gel resistance}}{180(1 - \epsilon_g)^2 \cdot l_g}$$

Put gel porosity,  $\epsilon_g = 0.5$ ;  $d_g = 30 \text{ nm} = 30 \times 10^{-9} \text{ m}$ ;  $\epsilon^2 \cdot \epsilon^3$

$$\text{Gel layer thickness, } l_g = \frac{(1.003 \times 10^{12})(30 \times 10^{-9})^2 (0.5)^3}{(180)(1-0.5)^2} = \underline{\underline{2.507 \times 10^{-6} \text{ m}}} = 2.507 \mu\text{m}$$

(14.5) Given:  $C_b = 0.0152$  mass fraction;  $C_m = C_g = 0.15$

(a)  $J_w = 0.032 \text{ m/h}$ ,  $R' = 0.98$

$$\text{Use Eq (14.10). } \frac{0.15}{0.0152} = \frac{\exp(-0.032/k_L)}{0.98 + (1-0.98) \cdot \exp(-0.032/k_L)}$$

$$\Rightarrow k_L = 1.285 \times 10^{-2} \text{ m/h} = 3.57 \times 10^{-6} \text{ m/s}$$

Calculation of the permeability of the gel layer - first calculate  $R_g$

$$\text{Pure water flux, } J_w = \frac{\Delta P}{\mu R_m} \Rightarrow 0.8 \frac{\text{m}^3}{\text{m}^2 \cdot \text{h}} = \frac{6 \text{ bar}}{\mu R_m}$$

$$\Rightarrow \mu R_m = 7.5 \frac{\text{bar} \cdot \text{m}^2 \cdot \text{R}}{\text{m}^3} \dots (i)$$

$$\text{For ultrafiltration, flux} = 0.032 \frac{\text{m}^3}{\text{m}^2 \cdot \text{h}} = \frac{6 \text{ bar}}{\mu(R_m + R_g)} \dots (ii)$$

$$\text{From (i) and (ii), } \mu R_g = 180 \frac{\text{bar} \cdot \text{m}^2 \cdot \text{R}}{\text{m}^3} \dots (ii)$$

$$\text{Permeability of the gel layer, } L_{P,g} = \frac{1}{\mu R_g} = \frac{1}{180} = 5.55 \times 10^{-3} \frac{\text{m}^3}{\text{m}^2 \cdot \text{bar}}$$

(The effect of osmotic pressure has been neglected.)

(b) If there is a sudden increase in the pressure difference, there will be sudden rise in flux that will decline with time. Instantaneous flux is

$$J_w = (0.032) \left( \frac{8 \text{ bar}}{6 \text{ bar}} \right) = \underline{\underline{0.0427 \text{ m}^3/\text{m}^2 \cdot \text{h}}}$$

(14.5)

14.5) Contd... Since the resistance to flow offered by the membrane is pretty small, we assume that the UF flux of  $0.032 \text{ m}^3/\text{m}^2\cdot\text{h}$  is nearly the 'limiting flux'.

→ The steady state flux at  $\Delta P = 8 \text{ bar}$  remains the same.

Fractional increase in the gel layer

$$\frac{\text{thickness}}{*} = \frac{8-6}{6} = \frac{2}{6} = \underline{\underline{33.3\%}}$$

14.6) First we relate the bulk concentration,  $C_b$ , with the concentration at the membrane surface,  $C_m$ , considering both concentration polarization and effect of osmotic pressure on effective pressure driving force.

$$J_w = k_L \ln \frac{C_m}{C_b} \quad [\text{Eq. (4.11)}] ; \text{ and } J_w = L_p \Delta P \left(1 - \frac{\Delta \Pi}{\Delta P}\right) = L_p \Delta P \left[1 - \frac{C_m RT}{\Delta P}\right]$$

$$\Rightarrow J_w = k_L \ln \frac{C_m}{C_b} = L_p \cdot \Delta P \cdot \left[1 - \frac{C_m RT}{\Delta P}\right] \xrightarrow{(i)} \text{i.e., } C_m \text{ can be expressed in terms of } C_b.$$

Now consider the change in volume and concentration of the solution as filtration proceeds.  $V_0, C_0 \rightarrow$  initial volume and concentr.

$V, C_b \rightarrow$  volume and concentration at time  $t$ .

$$\Rightarrow V_0 C_0 = V C_b \Rightarrow V = \frac{V_0 C_0}{C_b} \Rightarrow \frac{dV}{dt} = \frac{V_0 C_0}{C_b^2} \cdot \frac{dC_b}{dt}.$$

$$\text{Instantaneous flux, } J_w = -\frac{1}{A_m} \frac{dV}{dt} = -\frac{V_0 C_0}{C_b^2 A_m} \cdot \frac{dC_b}{dt} \dots (ii)$$

$$\text{From (i) and (ii)} \quad -\frac{V_0 C_0}{C_b^2 A_m} \cdot \frac{dC_b}{dt} = k_L \ln \frac{C_m}{C_b}; \quad t=0, C_b = C_0$$

The above equation can be integrated to obtain the membrane area to get a 'specified solution' concentration in the retentate within a specified time. The concentration  $C_m$  has to be expressed in terms of  $C_b$  (before integration can be done) from Eq.(i).

A numerical strategy has to be adopted.

\* \* \* \*

14.7) To determine  $R_m$  (membrane resistance) and  $\chi_c$  (specific cake resistance) from the experimental flux data, plot

(14.6)

14.7 Contd... Plot  $\frac{1}{J_w^2}$  against time ( $t$ ) and determine the intercept and slope of the 'least square' line; Eq.(14.21).

$t, \text{sec}$	10	20	30	40	50	60	70	80
$\frac{1}{J_w^2} \times 10^{-2}$	2.31	3.73	5.05	6.61	8.07	9.35	10.6	12.22

$J_w$  in liter/m<sup>2</sup>.s

$$\text{Intercept of the line} = 90.7 \left( \frac{\text{m}^2 \cdot \text{s}}{\text{liter}} \right)^2$$

$$\text{Slope} = 14.03 \left( \frac{\text{m}^2 \cdot \text{s}}{\text{liter}} \right)^2 \cdot \frac{1}{\text{s}}$$

$$\text{Given: } \mu = 9 \times 10^{-4} \text{ kg/m.s}$$

$$C_b = 20 \text{ kg/m}^3; \Delta P = 30 \times 10^3 \text{ N/m}^2; R' = 1$$

$$\frac{R_m^2 \cdot \mu^2}{(\Delta P)^2} = 90.7 \left( \frac{\text{m}^2 \cdot \text{s}}{\text{liter}} \right)^2$$

$$\Rightarrow R_m^2 = \frac{90.7 \times 10^6 \cdot \frac{\text{m}^4 \cdot \text{s}^2}{\text{m}^6}}{(9 \times 10^{-4} \text{ kg/m.s})^2} \cdot (30 \times 10^3 \text{ kg.m.s}^2 \cdot \frac{1}{\text{m}^2})^2 \Rightarrow R_m = 3.17 \times 10^{11} \text{ m}^{-1}$$

$$\text{Slope, } \frac{2\mu C_b \cdot r_c \cdot R'}{P_c \cdot \Delta P} = 14.03 \left( \frac{\text{m}^2 \cdot \text{s}}{\text{liter}} \right)^2 \cdot \frac{1}{\text{s}}$$

$$\Rightarrow \left( \frac{r_c}{P_c} \right) = \frac{14.03 \times 10^6 \left( \frac{\text{m}^2 \cdot \text{s}}{\text{m}^3} \right)^2 \cdot \frac{1}{\text{s}} \cdot 30 \times 10^3 \frac{\text{kg.m}}{\text{s}^2} \cdot \frac{1}{\text{m}^2}}{(2)(9 \times 10^{-4} \text{ kg/m.s})(20 \text{ kg/m}^3)(1)} = 1.17 \times 10^{13} \text{ m/kg}$$

14.8  $V_p$  = Accumulated volume of permeate  $\Rightarrow \frac{dV_p}{dt} = a_m \cdot J_w$ ,

$$\Rightarrow \frac{dV_p}{dt} = a_m \cdot (2.5 \times 10^{-5}) V_p^{-0.2}; t=0, V_p=0$$

Feed = 1 m<sup>3</sup>,  $V_p=0.5$ , Permeate = 0.5 m<sup>3</sup>, time  $t = 1\text{-hr} = 3600\text{s}$ .

$$\text{Integrate to get } \int V_p^{0.2} dV_p = a_m \cdot (2.5 \times 10^{-5}) \int dt$$

$$\Rightarrow \frac{1}{1.2} [V_p^{1.2}]_0^a = a_m \cdot (2.5 \times 10^{-5}) \cdot 3600 \Rightarrow a_m = \frac{(0.5)^{1.2}}{(1.2)(2.5 \times 10^{-5})(3600)}$$

$$\Rightarrow a_m = 4 \text{ m}^2$$

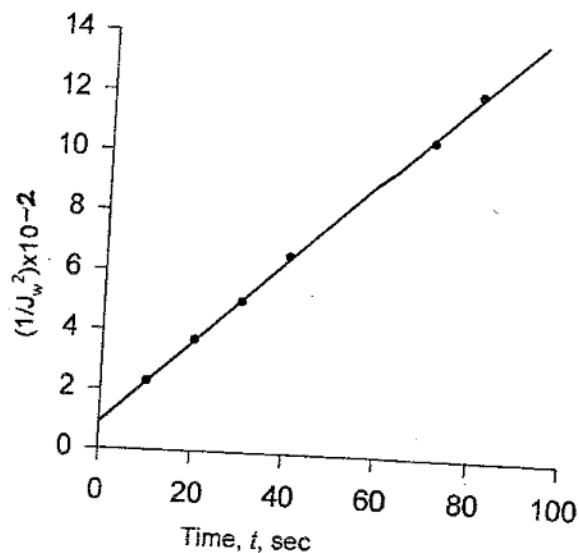


Figure Prob 14.7.

(14.7)

(14.9) Eq. (14.25) for the limiting flux, osmotic pressure model:

$$J_w = \frac{\Delta P - 5C_b^n \exp(n \cdot J_w / k_L)}{\mu R_m}$$

Given:  $5 = 120$ ;  $C_b = 0.006 \text{ kmol/m}^3$

$n = 2$ ;  $\mu R_m = 6 \times 10^5 \text{ atm}$

$$k_L = 3 \times 10^{-6} \text{ m/s}$$

Calculated values,  $J_w$  against  $\Delta P$

$J_w, \frac{m^3}{m^2 s} \times 10^5$	0.2	0.5	0.7	1.0
$\Delta P, \text{atm}$	1.216	3.121	4.66	9.39

$J_w \times 10^5$	1.1	1.2	1.25	1.3	1.35	1.4	1.5	1.55
$\Delta P$	13.21	20.1	25.5	32.8	43.1	57.25	104.1	142.1

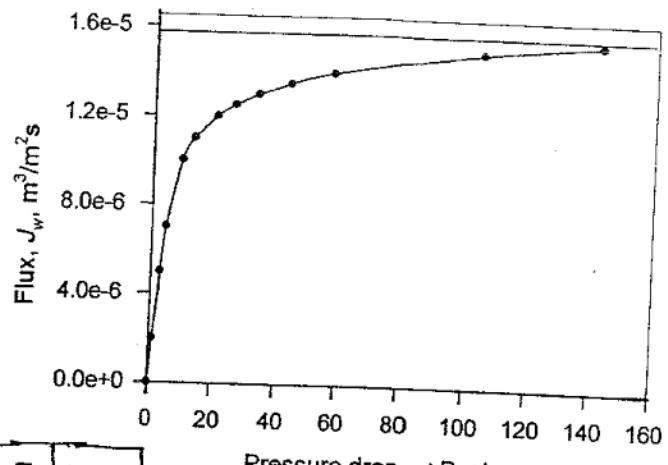


Figure Prob 14.9

The values are plotted in Fig Prob 14.8. The limiting flux according to the osmotic pressure model is about  $1.6 \times 10^{-5} \frac{\text{m}^3}{\text{m}^2 \cdot \text{s}}$

(14.10) Membrane resistance to flow,  $R_m = \frac{32 T \ln}{\epsilon d^2}$ .

Put  $T = 1.7$ ;  $d_m = 0.45 \times 10^{-6} \text{ m}$ ;  $\epsilon = 0.35$ ; and  $d = 20 \times 10^{-9} \text{ m}$ .

$$R_m = \frac{(32)(1.7)(0.45 \times 10^{-6})}{(0.35)(20 \times 10^{-9})^2} = 1.748 \times 10^{11} \text{ m}^{-1}$$

Calculation of  $k_L$ : Use the correlation  $Sh = 0.025 Re^{0.75} Sc^{1/3}$ .

$$Re = 6000 \text{ (given)}; Sc = \mu/P_D = \frac{9.5 \times 10^3}{(1050)(3.5 \times 10^{-11})} = 2.635 \times 10^5$$

$$\Rightarrow Sh = (0.025)(6000)^{0.75} (2.635 \times 10^5)^{1/3} = 1150 = \frac{k_L \cdot L}{D}; L = 7 \times 10^{-3} \text{ m}$$

$$k_L = (1150)(3.5 \times 10^{-11}) / 7 \times 10^{-3} = 5.8 \times 10^{-6} \text{ m/s} \quad \underline{\text{given}}$$

$$\text{Use (14.8). } k_L \cdot \ln \frac{C_g - C_p}{C_b - C_p} = \frac{\Delta P - \Delta \Pi}{\mu(R_m + R_g)} \quad \dots (i)$$

$$C_g - C_p = 0.99 C_g = (0.99)(0.12) \Rightarrow C_p = 1.2 \times 10^{-3}; C_g - C_p = \frac{120 - 1.2}{5000} \frac{\text{kmol}}{\text{m}^3}$$

$$C_b - C_p = 0.013 - 0.0012 = 0.0118 \text{ kg/m}^3$$

$$= 0.02376 \frac{\text{kmol}}{\text{m}^3}$$

$$\Delta \Pi = (C_g - C_p)RT = (0.02376)(0.0821)(300 \text{ K}) = 0.585 \text{ atm} = 0.593 \text{ bar}$$

(14.8)

$$(14.10) \text{ Contd... Put in Eq(i)} \quad 5.8 \times 10^{-6} \ln \frac{(0.12)(0.99)}{0.0118} = \frac{(3 - 0.593) \times 10^5}{\mu (R_m + R_g)}$$

Put  $\mu = 8.8 \times 10^{-4} \text{ N.S/m}^2$  for the solvent  $\Rightarrow R_m + R_g = 2.042 \times 10^{13} \text{ m}^{-1}$

$$R_g = 2.042 \times 10^{13} - R_m = 2.042 \times 10^{13} - 1.748 \times 10^{11} = 2.022 \times 10^{13} \text{ m}^{-1}$$

The membrane resistance is about 1% of the total resistance. Calculate the gel layer thickness from

$$R_g = \frac{(180)(1-\epsilon_g)^2 \lambda_g}{d_s^2 \epsilon_g^3} = 2.022 \times 10^{13}$$

Put  $\epsilon_g = 0.5$ , and  $d_s = 40 \text{ nm} = 40 \times 10^{-9} \text{ m}$  to get  $\lambda_g = 90 \mu\text{m}$

(b) The pressure drop (and corresponding flux) at which the gel layer starts forming, the solute concentration at the membrane surface reaches  $c_g = 0.12$  mass fraction. There is no gel layer resistance ( $R_g = 0$ ) since the gel is yet to form. Put the relevant values in Eq(i) and take  $R_g = 0$ .

$$5.8 \times 10^{-6} \ln \frac{(0.12)(0.99)}{0.0118} = \frac{(\Delta P - 0.593) \times 10^5}{(8.8 \times 10^{-4})(1.748 \times 10^{11})} \Rightarrow \Delta P = 0.62 \text{ bar}$$

Thus gel formation starts when  $\Delta P$  is only slightly larger than 0.62 bar.

$$(14.11) \text{ Rejection} = 100\% \Rightarrow V_o C_o = V_f C_f \quad (f = \text{final}) ; V_o = 600 \text{ gal} \\ \text{Use Eq(i) of Example 14.4} \quad V_f = (600) \left( \frac{1.4}{10.5} \right) \text{gal} = \frac{2.268 \text{ m}^3}{0.302 \text{ m}^3}$$

$$\text{Put } L_p \cdot \Delta P = \text{pure water flux} = 9.5 \text{ gal/h ft}^2 = \frac{(9.5)(0.00378)}{(3600)(0.3048)^2}$$

$$a_m = 50 \text{ ft}^2 = 4.645 \text{ m}^2;$$

$$n_o = \frac{(2.268 \text{ m}^3)(14 \text{ kg/m}^3)}{2020 \text{ kg/kmol}} = 0.0157 \text{ kmol} ; \Delta P = 3 \text{ bar} = \frac{1.074 \times 10^{-4} \text{ m}^3}{8 \text{ m}^2} = 2.961 \text{ atm}$$

$$RT n_o / \Delta P = (0.0821)(300)(0.0157) / 2.961 = \frac{0.1306 \text{ m}^3}{\text{bar}}$$

$$t = \frac{1}{a_m L_p \Delta P} \left[ (V_o - V_f) + \frac{RT n_o}{\Delta P} \ln \frac{V_o - (RT n_o / \Delta P)}{V_f - (RT n_o / \Delta P)} \right]$$

$$\Rightarrow t = \frac{1}{(4.645)(1.074 \times 10^{-4})} \left[ (2.268 - 0.302) + (0.1306) \ln \frac{2.268 - 0.1306}{0.302 - 0.1306} \right]$$

$$= 4600 \text{ s} = \underline{\underline{1.28 \text{ h}}} \quad * \quad * \quad *$$

(14.9)

14.12

$$\text{Rate of production of water} = q_m \cdot L_p (\Delta P - \Delta \Pi); q_m = 20 \text{ m}^2.$$

$$\Delta P = 70 - 1 = 69 \text{ atm}; \Delta \Pi = (5.2)(39.5/5) = 41.1 \text{ atm} [\text{at } 5\% \text{ salt conc., } \Pi = 39.5 \text{ atm}]$$

$$\text{Theoretical production rate} = (20 \text{ m}^2) \left( 7.5 \frac{\text{m}^3}{\text{m}^2 \cdot \text{h} \cdot \text{atm}} \times 10^4 \right) (69 - 41.1) \text{ atm}$$

$$= 0.4185 \text{ m}^3/\text{h} = 418.5 \text{ liter/h.}$$

The actual production rate (300 liter/h) is less than this.  
So there is concentration polarization.

$$\text{Put } L_p \cdot q_m (\Delta P)_{\text{eff.}} = 300 \text{ liter/h} = 0.3 \text{ m}^3/\text{h}$$

$$\Rightarrow (\Delta P)_{\text{eff.}} = \frac{0.3 \text{ m}^3/\text{h}}{(20 \text{ m}^2) (7.5 \times 10^4 \text{ m}^3/\text{m}^2 \cdot \text{h} \cdot \text{atm})} = 20 \text{ atm} = \Delta P - \Delta \Pi$$

$$\text{Actual } \Delta \Pi = \Delta P - 20 = 69 - 20 = 49 \text{ atm.}$$

Corresponding salt concentration at the membrane surface,  
 $C_m = (49)(5/39.5) = 6.2\%$ .

$$\text{Concentration polarization modulus} = \frac{C_m}{C_b} = \frac{6.2}{5.2} = 1.2$$

14.13 Feed-side concentration = 3.2%; no concentration polarization.

$$\text{Osmotic pressure on the feed side} = (3.2)(39.5/5) = 25.3 \text{ atm} = \Delta \Pi$$

$$\text{Effective pressure differential} = \Delta P - \Delta \Pi = (40 - 1.5) - 25.3 = 13.2 \text{ atm} = (\Delta P)_{\text{eff.}}$$

$$\text{Water flux, } J_w = L_p \cdot (\Delta P)_{\text{eff.}} = 9 \times 10^{-4} \frac{\text{m}^3}{\text{m}^2 \cdot \text{h} \cdot \text{atm}} \times 13.2 \text{ atm} = 1.188 \times 10^{-2} \frac{\text{m}^3}{\text{m}^2 \cdot \text{h}}$$

Salt flux [Eq. 14.29(a) and (b); take  $\alpha = 1$ ]

$$J_s = P_s (C_b - C_p) = 3.2 \times 10^{-8} \frac{\text{m}}{\text{s}} (32 \text{ kg/m}^3 - 0) = 1.024 \times 10^{-6} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

$$\text{Salt concentration in the permeate} = \frac{J_s}{J_w} = \frac{1.024 \times 10^{-6} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}}{1.188 \times 10^{-2} \frac{\text{m}^3}{\text{m}^2 \cdot \text{h}}} \times 3600 = 310 \text{ ppm}$$

This is above the allowable limit,  $\underline{200 \text{ ppm}}$ .

If the salt flux remains as above (assume that the salt flux is pressure independent), required water flux to maintain the quality is  $J_w = (1.188 \times 10^{-2})(310/200) = 1.841 \times 10^{-2} \frac{\text{m}^3}{\text{m}^2 \cdot \text{h}}$

$$L_p (\Delta P - 25.3) = 1.841 \times 10^{-2} \Rightarrow \Delta P = 20.5 + 25.3 = 45.8 \text{ atm}$$

$$\text{Feed side pressure required} = 45.8 + 1.5 = \underline{47.3 \text{ atm}}$$

(14.10)

14.14 Yield of permeate,  $\frac{Q_p}{Q_{fi}} = 0.3 = \theta_{wo}$ ; rejection,  $R' = 0.98$

$$\text{Eq. (14.45)} \rightarrow C_{fo} = C_{fi}(1 - \theta_{wo})^{-\beta'} = (3.3)(1 - 0.3)^{-0.98} = 4.68\%$$

Take the exit concentration of the mixed permeate  $= 300 \text{ ppm}$

$$\text{Polarization modulus} = 1.2 \quad = 0.03\%$$

Feed side concentration: Inlet  $\rightarrow C_{mi} = (1.2)C_{fi} = (1.2)(3.3)$

$$\text{Outlet} \rightarrow C_{mo} = (4.68)(1.2) = 5.62\% \quad = 3.96\%$$

$$(\Delta C)_i = 3.96 - 0.03 = 3.93\%; (\Delta C)_o = 5.62 - 0.03 = 5.59\%$$

Osmotic pressure difference at the

$$\text{Feed end}, (\Delta \Pi)_i = \left(\frac{39.5}{5}\right)(3.93) = 31.05 \text{ atm}; (\Delta \Pi)_o = \left(\frac{39.5}{5}\right)(5.59) = 44.16 \text{ atm}$$

Take the average value,  $\Delta \Pi = \frac{31.05 + 44.16}{2} = 37.6 \text{ atm}$

$$\text{Effective pressure driving force, } (\Delta P)_{eff} = (75 - 1) - (37.6) = 36.4 \text{ atm}$$

$$\text{Water flux} = L_p \cdot (\Delta P)_{eff} = \frac{(36.4)(1.013)}{10} = 3.687 \text{ MPa}$$

$$= (70 \text{ litres/m}^2 \text{ day})(3.687 \text{ MPa}) = 0.258 \text{ m}^3/\text{m}^2 \text{ day}$$

$$\text{Required capacity} = 20,000 \text{ m}^3/\text{day}$$

$$\text{Membrane area} = \frac{20,000}{(24)(0.258)} = \frac{3230 \text{ m}^2}{*}$$

14.15 Inlet concentration of Na-acetate ( $M_{NaAc} = 82$ ) = 3%  $\rightarrow \frac{30 \text{ kg}}{\text{m}^3}$

Water (dialysate)-side concentration  $\approx 0$  (high flow rate).

$$(\Delta C)_{inlet} = \frac{30}{82} = 0.366 \text{ kmol/m}^3$$

$$\text{Exit concentration} = 500 \text{ ppm} = \frac{0.05 \text{ kg}}{\text{m}^3} = 0.0061 \text{ kmol/m}^3 = (\Delta C)_o$$

$$\text{Log mean driving force is taken; } (\Delta C)_m = \frac{0.366 - 0.0061}{\ln \frac{0.366}{0.0061}} = 0.088 \frac{\text{kmol}}{\text{m}^3}$$

Total resistance to mass transfer in the dialyzer

= fluid phase film resistance + resistance for diffusion

$$= 2.5 \times 10^5 \text{ s/m} + \frac{lm}{D \cdot \epsilon} ; lm = \text{membrane thickness} = 100 \times 10^{-6} \text{ m}$$

$$= 2.5 \times 10^5 + \frac{100 \times 10^{-6}}{(1.2 \times 10^{-9})(0.4)}$$

$$= 4.58 \times 10^5 \text{ s/m}$$

$$\epsilon = \text{porosity} = 0.4$$

$$D = 1.2 \times 10^{-9} \text{ m}^2/\text{s}$$

(14.15) Contd...

(14.11)

$$\text{Overall coefficient, } K_L = 1/(4.58 \times 10^5) \text{ m}/\text{s} = 2.18 \times 10^{-6} \text{ m}/\text{s}.$$

Rate of flow of the feed,  $Q_f = 200 \text{ litres/h} = 0.2 \text{ m}^3/\text{h}$

$$\text{Then } Q_f(C_f - C_0) = K_L a_m (\Delta C)_m$$

$$\Rightarrow 0.2(0.366 - 0.0061) = (2.18 \times 10^{-6})(3600) \cdot a_m \cdot (0.088) \Rightarrow a_m = 104 \text{ m}^2$$

Loss of the target substance (Mol. wt. = 520)

$$\text{Inlet concentration (8%), } C_{in} = \frac{80}{520} = 0.1538 \text{ kmol/m}^3$$

$$\text{Outlet conc.} = C_{out} \cdot \text{Mean driving force} = \frac{0.1538 - C_{out}}{\ln \frac{0.1538}{C_{out}}}$$

$$\text{Rate of transfer: } Q_f (0.1538 - C_{out}) = K'_L a_m \cdot \frac{0.1538 - C_{out}}{\ln \frac{0.1538}{C_{out}}} \quad \dots (i)$$

$$K'_L \text{ for the target substance} = K_L/10 = 2.18 \times 10^{-7} \text{ m}/\text{s}$$

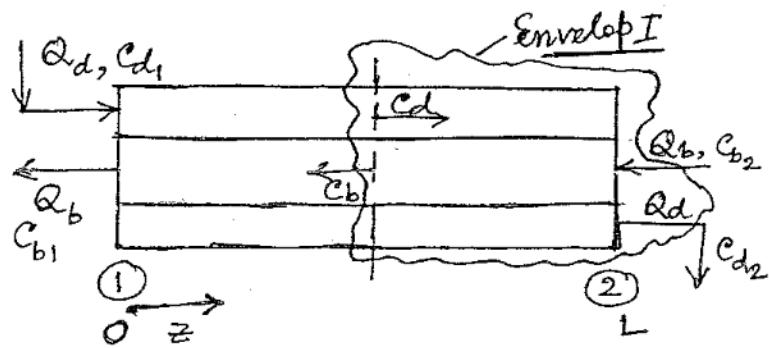
Since transport through the film or through a membrane and the diffusivity of the substance is  $\frac{1}{10}$  times that of Na-acetate. Put the values in Eq (i) above.

$$(0.2)(0.1538 - C_{out}) = (2.18 \times 10^{-7})(3600)(104) \cdot \frac{0.1538 - C_{out}}{\ln \frac{0.1538}{C_{out}}}$$

$$\Rightarrow C_{out} = 0.102$$

$$\text{Loss of the substance} = \frac{0.1538 - 0.102}{0.1538} = 33.7\%$$

(14.16) A hemodialyzer with countercurrent flow of blood and the dialyzate is shown in the sketch. The flow rates and concentrations are also shown.



Equations for axial concentration distribution:

$$\text{Blood: } Q_b \frac{dc_b}{dz} = K_L B' (C_b - C_d) \quad \dots (i) \quad | \begin{array}{l} K_L = \text{overall mass} \\ \text{transfer coeff.} \end{array}$$

$$\text{Dialyzate: } Q_d \frac{dc_d}{dz} = K_L B' (C_d - C_b) \quad \dots (ii) \quad | \begin{array}{l} B' = \text{total peri-} \\ \text{meter of the tubes.} \end{array}$$

(14-16) contd...

(14-12)

Total Solute balance over the dialyzer :  $Q_b (C_{b_2} - C_{b_1}) = Q_d (C_{d_2} - C_{d_1})$

Solute balance over envelop I :

$$Q_b (C_{b_2} - C_b) = Q_d (C_{d_2} - C_d)$$

$$\Rightarrow C_d (z) = C_{d_2} + \zeta (C_{b_2} - C_b) \dots (iv)$$

Consider Eq (i) above and use Eq (iv).  $Q_b \frac{dC_b}{dz} = K_L B' [C_b - C_{d_2} + \zeta C_{b_2} - \zeta C_b]$

$$\Rightarrow \int_{C_{b_1}}^{C_{b_2}} \frac{dC_b}{C_b(1-\zeta) + (\zeta C_{b_2} - C_{d_2})} = \int_0^L \frac{K_L B'}{Q_b} dz = \frac{K_L (B' L)}{Q_b} = \frac{K_L A_m}{Q_b} = N_T$$

$$\Rightarrow \frac{1}{(1-\zeta)} \ln \frac{C_{b_2}(1-\zeta) + (\zeta C_{b_2} - C_{d_2})}{C_{b_1}(1-\zeta) + (\zeta C_{b_2} - C_{d_2})} = N_T$$

$$\Rightarrow \ln \left[ \frac{C_{b_2} - C_{d_2}}{(C_{b_1} - C_{d_2}) + \zeta (C_{b_2} - C_{b_1})} \right] = (1-\zeta) N_T ; \quad \begin{aligned} & \text{Using Eq (iii),} \\ & (C_{b_1} - C_{d_2}) + \zeta (C_{b_2} - C_{b_1}) \\ & = C_{b_1} - C_{d_2} + C_{d_2} - C_{d_1} = C_{b_1} - C_{d_1} \end{aligned}$$

$$\Rightarrow \ln \left[ \frac{C_{b_2} - C_{d_2}}{C_{b_1} - C_{d_1}} \right] = (1-\zeta) N_T \Rightarrow \frac{C_{b_2} - C_{d_2}}{C_{b_1} - C_{d_1}} = \exp[(1-\zeta) N_T] \dots (v)$$

The 'extraction ratio' — see Eq.(14.52) — can be shown to be given as

$$\hat{E} = \frac{C_{b_2} - C_{b_1}}{C_{b_2} - C_{d_1}} = \frac{\exp[(1-\zeta) N_T] - 1}{\exp[(1-\zeta) N_T] - \zeta}$$

$$\begin{aligned} \text{Using (v), } \frac{\exp[(1-\zeta) N_T] - 1}{\exp[(1-\zeta) N_T] - \zeta} &= \frac{\frac{C_{b_2} - C_{d_2}}{C_{b_1} - C_{d_1}} - 1}{\frac{C_{b_2} - C_{d_2}}{C_{b_1} - C_{d_1}} - \zeta} = \frac{C_{b_2} - C_{d_2} - C_{b_1} + C_{d_1}}{C_{b_2} - C_{d_2} - \zeta C_{b_1} + \zeta C_{d_1}} \\ &= \frac{(C_{b_2} - C_{b_1}) - (C_{d_2} - C_{d_1})}{C_{b_2} - C_{d_2} - \zeta C_{b_1} + \zeta C_{d_1} + C_{d_1} - C_{d_1}} = \frac{(C_{b_2} - C_{b_1}) - \zeta (C_{b_2} - C_{b_1})}{C_{b_2} + \zeta C_{d_1} - C_{d_1} - [(C_{d_2} - C_{d_1}) + \zeta C_{b_1}]} \\ &= \frac{(C_{b_2} - C_{b_1})(\zeta - 1)}{C_{b_2} + \zeta C_{d_1} - C_{d_1} - \zeta C_{b_2}} = \frac{(C_{b_2} - C_{b_1})(\zeta - 1)}{C_{b_2}(1-\zeta) - C_{d_1}(1-\zeta)} = \frac{(C_{b_2} - C_{b_1})(\zeta - 1)}{(C_{b_2} - C_{d_1})(\zeta - 1)} \\ &= \frac{C_{b_2} - C_{b_1}}{C_{b_2} - C_{d_1}} = \hat{E} \end{aligned}$$

\*

\*

\*

\*

(14.13)

14.17 (a) Extraction ratio,  $\hat{E} = \frac{\exp[(1-\zeta)N_T] - 1}{\exp[(1-\zeta)N_T] - \zeta} = \frac{C_{b_2} - C_{b_1}}{C_{b_2} - C_{d_1}}$   
 for countercurrent dialysis, Eq(14.53)

Given:  $Q_d = 10 Q_b \Rightarrow \zeta = Q_b / Q_d = 0.1$ ;  $Q_b = 300 \text{ ml/min} = 5 \times 10^{-6} \text{ m}^3/\text{s}$ ;  
 $a_m = 1.2 \text{ m}^2$ ;  $K_L = 1.05 \times 10^6 \text{ m/s}$ .

$$N_T = \frac{K_L a_m}{Q_b} = \frac{(1.05 \times 10^6)(1.2)}{5 \times 10^{-6}} = 0.252$$

$$\hat{E} = \frac{\exp[(1-0.1)(0.252)] - 1}{\exp[(1-0.1)(0.252)] - 0.1} = 0.2205$$

(b) If the urea concentration in blood at any time is  $C_{b_2}$ , the change in concentration within a small time  $dt$  is given by [assume:  $C_{d_1} = \text{inlet conc. of dialyzate} = 0$ ]

$$-V_L \cdot dC_{b_2} = Q_b(C_{b_2} - C_{b_1}) \cdot dt = Q_b \cdot \hat{E} \cdot C_{b_2} \cdot dt$$

$$\text{Integrating, } -\int_{C_i}^{C_f} \frac{dC_{b_2}}{C_{b_2}} = \frac{Q_b \hat{E}}{V_L} \int_0^t dt \Rightarrow \ln \frac{C_i}{C_f} = \frac{Q_b \hat{E}}{V_L} \cdot t$$

The urea content in blood should change from 220 to 20 (mg%)

$$\Rightarrow \ln \frac{220}{20} = \frac{(5 \times 10^{-6} \text{ m}^3/\text{s})(0.2205)}{5.5 \times 10^{-3} \text{ m}^3} \cdot t; \text{ blood volume, } V_L = 5.5 \text{ liters} = 5.5 \times 10^{-3} \text{ m}^3$$

$$\Rightarrow t = 119608 = 200 \text{ min}$$

Time of dialysis = 200 min \*

14.18 Rate of transport of a gas through a film [Eq(14.70)]

$$= a_m \cdot \frac{\hat{P}_A}{l_m} (P_1 x_A - P_2 y_A) \cdot \begin{cases} P x_A = \text{vapor pressure of water at } 25^\circ C \\ = 24 \text{ mm Hg} \end{cases} \\ = (1000) \left( \frac{1.4 \times 10^{-10} \text{ cm}^3 (\text{STP}) \cdot \text{cm}}{0.003 \text{ cm}} \right) \times \begin{cases} P_2 y_A = (0.70)(24 \text{ mm Hg}); 70\% \text{ RH air} \\ \hat{P}_A = 1.4 \text{ Barriers}; l_m = 0.003 \text{ cm} \end{cases} \\ = 6.48 \times 10^{-5} \frac{\text{cm}^3 (\text{STP})}{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}} (2.4)(1-0.7) \text{ cm Hg}$$

$$= (6.48 \times 10^{-5} / 22414)(18)(3600) \text{ gm moisture/h} = 1.873 \times 10^{-4} \text{ gm/h}$$

Moisture to be lost for reducing the moisture content from 20% to 12% (wet basis) =  $(1 \text{ kg})(0.2) - \frac{(0.8)(0.12)}{0.88} \text{ kg} = 91 \text{ gm}$

(14.18) contd... Time for loss of 91 gm moisture

$$= \frac{91 \text{ gm}}{1.873 \times 10^{-4} \text{ gm hr}} = 485850 \text{ hr.} = 55.5 \text{ years}$$

(14.19)  $\text{CO}_2 \rightarrow A ; \text{N}_2 \rightarrow B$ ; membrane thickness: PDMS,  $l_m' = 5 \times 10^{-4} \text{ cm}$   
Given permeability values: CA,  $l_m'' = 2 \times 10^{-5} \text{ cm}$

$$\text{PDMS} - \hat{P}_A' = 4550, \hat{P}_B' = 351; \text{CA} - \hat{P}_A'' = 4.75, \hat{P}_B'' = 0.15 \text{ Barres}$$

Resistance to transport, Eq. 14.74 (Use resistance-in-series model)

$$R_A = R_A' + R_A'' = \frac{l_m'}{\hat{P}_A'} + \frac{l_m''}{\hat{P}_A''} = \frac{5 \times 10^{-4}}{4550 \times 10^{10}} + \frac{2 \times 10^{-5}}{4.75 \times 10^{10}} = 4.32 \times 10^4$$

$$\text{Total thickness of the composite membrane, } l_m = l_m' + l_m'' = 5 \times 10^{-4} + 2 \times 10^{-5} = 5.2 \times 10^{-4} \text{ cm}$$

$$\text{Permeability of CO}_2 \text{ in the composite membrane, } \hat{P}_A = \frac{l_m}{R_A} = 120.4 \text{ Barres}$$

Similarly for B,

$$R_B = R_B' + R_B'' = \frac{l_m}{\hat{P}_B'} + \frac{l_m''}{\hat{P}_B''} = \frac{5 \times 10^{-4}}{351 \times 10^{10}} + \frac{2 \times 10^{-5}}{1.5 \times 10^{11}} = 1.3476 \times 10^6 \frac{\text{cm}^2 \cdot \text{S} \cdot \text{cm Hg}}{\text{cm}^3 (\text{STP})}$$

$$\hat{P}_B = \frac{l_m}{R_B} = \frac{5.2 \times 10^{-4} \text{ cm}}{1.3476 \times 10^6} = 3.86 \times 10^{-10} \frac{\text{cm}^3 (\text{STP}) \text{ cm}}{\text{cm}^2 \cdot \text{S} \cdot \text{cm Hg}} \text{ or } 3.86 \text{ Barres}$$

$$\text{Ideal separation factor, } \alpha_{AB}^* = \hat{P}_A / \hat{P}_B = \frac{120.4}{3.86} = 31.2$$

(14.20) Flux equation for concentration-dependent diffusivity:

$$\frac{dJ_A}{dz} = 0 \text{ at steady state} \Rightarrow -D \frac{dc}{dz} = \text{constant} = K_1 \dots (i)$$

$$\text{Given: } D = D_0 e^{\beta c} \Rightarrow -D_0 e^{\beta c} \cdot \frac{dc}{dz} = K_1.$$

$$\text{Integrate to get } - \int_{c_1}^{c_2} e^{\beta c} dc = \frac{K_1}{D_0} \int_0^{l_m} dz \Rightarrow K_1 = \frac{D_0}{\beta l_m} [e^{\beta c_1} - e^{\beta c_2}] \quad (i)$$

$$\text{Calculation of } \beta c_1 \text{ and } \beta c_2 \mid c = 8.33 \frac{\text{m}^3 (\text{STP})}{\text{m}^3}, p \text{ in MPa} \quad (= \text{flux, see Eq. (i)})$$

$$\text{Upstream side, } p_1 = 2.3 \text{ MPa} \Rightarrow \beta c_1 = (0.07)(8.33 \times 2.3) = 1.341$$

$$\text{Downstream side, } p_2 = 1.5 \text{ MPa} \Rightarrow \beta c_2 = (0.07)(8.33 \times 1.5) = 0.8746$$

$$\text{Steady state flux, } J_A (= K_1) = \frac{4.53 \times 10^{-17} \text{ m}^2/\text{s}}{(0.07 \frac{\text{m}^3}{\text{m}^2 \text{STP}})(20 \times 10^{-6} \text{ m})} \left[ e^{1.341} - e^{0.8746} \right]$$

$$\Rightarrow J_A = 4.611 \times 10^{-11} \frac{\text{m}^3 (\text{STP})}{\text{m}^2 \text{s}} = 2.057 \times 10^{-12} \frac{\text{kmol}}{\text{m}^2 \text{s}}$$

14.15

(14.20) Contd... Eq.(4.70)  $\rightarrow$  flux =  $\frac{\hat{P}_A}{l_m} (P_1 - P_2) \Rightarrow \hat{P}_A = \frac{J_A \cdot l_m}{P_1 - P_2}$   
 $P_1 - P_2 = 2.3 - 1.5 \text{ MPa} = .8 \text{ bar} = \frac{8}{1.013} \times 76 = 600 \text{ cm Hg}$

Average permeability,  $\hat{P}_A = \frac{4.611 \times 10^{-11} \text{ m}^3/\text{m}^2\text{s} \cdot 20 \times 10^{-4} \text{ cm}}{600 \text{ cm Hg}}$   
 $\Rightarrow \hat{P}_A = 1.537 \times 10^{-14} \frac{\text{cm}^3(\text{STP}) \cdot \text{cm}}{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}} = 1.537 \times 10^{-4} \text{ Barrer}$

(14.21)  $O_2 \rightarrow A, N_2 \rightarrow B. R'_A, R'_B \rightarrow$  resistance to permeation, PDMS layer  
 $R''_A, R''_B$  = resistance to permeation, polysulfone backing.  
The feed-side mixture is equimolar, downstream pressure is small.

$$\Rightarrow J_A = \frac{\Delta P_A}{R'_A + R''_B} ; J_B = \frac{\Delta P_B}{R'_B + R''_A} ; \Delta P_A = \Delta P_B \Rightarrow R''_A = R''_B \text{ (given)}$$

$$\hat{P}'_A = \hat{P}'_B \cdot \alpha_{AB}^{*} = (270)(2.2) = 594 \text{ Barrers}$$

If  $l_m$  = thickness of the PDMS layer,  $R'_A = \frac{l_m}{594}$  ;  $R'_B = \frac{l_m}{270}$

Since  $J_A = 2J_B \Rightarrow \frac{\Delta P_A}{R'_A + R''_B} = 2 \cdot \frac{\Delta P_B}{R'_B + R''_A} \Rightarrow R'_B + R''_B = 2(R'_A + R''_A)$

$$\Rightarrow \frac{l_m}{270} + 10^3 = \frac{2 \times l_m}{594} + 2 \times 10^3 \Rightarrow l_m \left( \frac{1}{270} - \frac{2}{594} \right) = 10^3 \frac{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}}{\text{cm}^3(\text{STP}) \cdot \text{cm}}$$

$$\Rightarrow l_m = 10^3 \frac{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}}{\text{cm}^3(\text{STP}) \cdot \text{cm}} \times 2970 \times 10^{-10} \frac{\text{cm}^3(\text{STP}) \cdot \text{cm}}{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}}$$

$$= 2970 \times 10^{-7} \text{ cm} = 3 \text{ micron}$$

A 3  $\mu\text{m}$  layers of PDMS has to be applied on the porous polysulfone backing to achieve  $J_A = 2J_B$ .

(14.22) Given:  $\hat{P}_A = 30 \text{ Barrer}; \hat{P}_B = 0.12 \text{ Barrer}; P_1 = 30 \text{ bar}; P_2 = 1.1 \text{ bar}; \alpha_A = 0.5 = \alpha_B; l_m = 0.2 \mu\text{m} = 2 \times 10^{-5} \text{ cm}$ .

(a) Flux of A,  $J_A = \frac{\hat{P}_A}{l_m} \cdot (P_1 x_A - P_2 y_A)$  Assume  $P_2 \ll P_1$   
 $= \left( \frac{30 \times 10^{-10}}{2 \times 10^{-5}} \right) (30 \times 0.5) \left( \frac{76 \text{ cm Hg}}{1.013} \right) = 0.1688 \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}}$   
 $= 7.53 \times 10^{-6} \frac{\text{gmol}}{\text{cm}^2 \cdot \text{s}}$

(14.22) Contd...  $J_B = J_A (0.12/30)$  since  $x_A = x_B$

$$= (7.53 \times 10^{-6}) (0.12/30) = \frac{3.012 \times 10^{-8}}{\text{cm}^2 \text{s}} \frac{\text{gmol}}{\text{cm}^2 \text{s}}$$

Composition of the permeate:  $y_A = \frac{J_A}{J_A + J_B} = \frac{7.53}{7.53 + 0.03012} = 0.996$

(b) The 'ideal separation factor',  $\alpha_{AB}^* = \hat{P}_A / \hat{P}_B = 30/0.12 = 250$

The permeate side pressure is not neglected now.

Pressure ratio:  $r = 1/30 = 0.0367$ . Use Eq.(4.83) to calculate the actual separation factor.

$$\alpha = \alpha_{AB}^* \cdot \frac{x_A(\alpha-1) + 1 - r\alpha}{x_A(\alpha-1) + 1 - r} \Rightarrow \alpha = (250) \cdot \frac{(0.5)(\alpha-1) + 1 - 0.0367\alpha}{0.5(\alpha-1) + 1 - 0.0367}$$

Solving the quadratic equation,  $\alpha^2 - 230.7\alpha - 250 = 0$

we get,  $\underline{\alpha = 232}$  which is less than the ideal

\* \* \* separation factor,  $\underline{\alpha_{AB}^* = 250}$ .

(14.23) Refer to Fig 14.36(a), a 'well-mixed reactor'.

Product cut = 0,  $Q_p = Q_f \cdot \theta$ ;  $Q_f = 1 \text{ m}^3 (\text{STP})/\text{h}$ ;  $x_{Af} = 0.21$

Component A balance:  $Q_f x_{Af} = \theta Q_f y_A + (1-\theta) Q_f x_A \quad A \rightarrow \text{oxygen}$

$$\Rightarrow 0.21 = 0.25 y_A + 0.75 x_A \quad (\text{putting } \theta = 0.25),$$

$$\Rightarrow y_A = 0.84 - 3x_A \quad \dots \dots \text{(i)} \quad \text{and} \quad y_B = 1 - y_A$$

Fluxes:  $J_A = (\hat{P}_A / \text{lm})(P_1 x_A - P_2 y_A)$ ;  $J_B = (\hat{P}_B / \text{lm})(P_1 x_B - P_2 y_B)$

$$\Rightarrow \frac{J_A}{J_B} = \frac{\hat{P}_A}{\hat{P}_B} \cdot \frac{(P_1 x_A - P_2 y_A)}{(P_1 x_B - P_2 y_B)} = \frac{Q_p \cdot y_A}{Q_p \cdot y_B} = \frac{y_A}{y_B}; \quad \frac{\hat{P}_A}{\hat{P}_B} = \alpha_{AB}^* = 5.2$$

Put  $P_1 = 10 \text{ bar}$ ,  $P_2 = 1 \text{ bar}$ . Note that  $x_A$  and  $y_A$  are related by

$$\frac{0.84 - 3x_A}{1 - (0.84 - 3x_A)} = (5.2) \cdot \frac{10x_A - (1)(0.84 - 3x_A)}{10(1-x_A) - (1)(-0.84 + 3x_A)} \quad \text{Eq. (i) above.} \Rightarrow x_A = 0.1448$$

Feed rate,  $Q_f = 100 \text{ m}^3 (\text{STP})/\text{h}$ ;  $Q_p = \theta \cdot Q_f = (0.25)(100) = 25 \text{ m}^3/\text{h}$ .

Rate of permeation of A =  $y_A Q_p = a_m (\hat{P}_A / \text{lm})(P_1 x_A - P_2 y_A)$ ;  $\text{lm} = 0.15 \times 10^{-4} \frac{\text{cm}^3}{\text{cm}}$

$$\Rightarrow \frac{(0.4057)(25) \times 10^6}{3600} \frac{\text{cm}^3 (\text{STP})}{\text{s}} = a_m \left( \frac{1.7 \times 10^{-10}}{0.15 \times 10^{-4}} \right) [(10)(0.1448) - (1)(0.4057)]$$

$\Rightarrow$  Membrane area,  $a_m = 3.18 \times 10^6 \text{ cm}^2 = 318 \text{ m}^2 \quad \times (76/1.013) \text{ atm/Hg}$

(14-17)

(14.24)  $\text{CO}_2 \rightarrow A$ ,  $\text{CH}_4 \rightarrow B$ . Given:  $\hat{P}_A = 4570 \text{ Barres}$ ,  $\alpha_{AB}^* = 3.15$

$x_A = 0.2$ ;  $x_B = 0.8$ ;  $y_A = 0$ , since  $\Rightarrow \hat{P}_B = 4570/3.15 = 1451 \text{ Barres}$   
the permeate compartment is continuously swept with  $\text{CH}_4$ ;  $y_B = 1.0$ .  
 $P_1 = 12.5 \text{ atm}$ ;  $P_2 = 1.5 \text{ atm}$ ;  $l_m = 300 \mu\text{m} = 300 \times 10^{-4} \text{ cm}$ .

(a) Flux of  $\text{CO}_2$ ,  $J_A = (\hat{P}_A/l_m)(P_1 x_A - P_2 y_A) = \frac{4570 \times 10^{-10}}{300 \times 10^{-4}} \cdot [(12.5)(0.2) - 0] \quad (76)$

$$\Rightarrow J_A = 2.894 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}} = 1.27 \times 10^{-7} \text{ g mol/cm}^2 \cdot \text{s}$$

Flux of  $\text{CH}_4$ ,  $J_B = (\hat{P}_B/l_m)(P_1 x_B - P_2 y_B) = \left( \frac{1451 \times 10^{-10}}{300 \times 10^{-4}} \right) (0.8 \times 12.5 - 1.5 \times 1) \quad (76)$   
 $= 3.124 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}} = 1.394 \times 10^{-7} \text{ g mol/cm}^2 \cdot \text{s}$

(b) Permeance of  $\text{CO}_2$ ,  $\tilde{P}_A = \hat{P}_A/l_m = \frac{4570 \times 10^{-10}}{300 \times 10^{-4}} = 1.523 \times 10^{-5} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}}$

(c) Diffusivity of  $\text{CH}_4$  in PDMS, Use Eq.(14-71)

$$D_B = \hat{P}_B/S_B = \frac{1451 \times 10^{-10} \frac{\text{cm}^3(\text{STP}) \cdot \text{cm}}{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}}}{(0.0059 \times 10) \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{cm Hg}}} = 2.46 \times 10^{-6} \text{ cm}^2/\text{s}$$

(14.25) Using the given flux values of A and B,

$$\frac{J_A}{J_B} = \frac{1.245 \times 10^{-3}}{1.06 \times 10^{-3}} = 1.1745 \Rightarrow \frac{y_A}{y_B} = 1.1745 = \frac{y_A}{1-y_A} \Rightarrow y_A = 0.54$$

The feed compartment receives a large rate of air  $\Rightarrow x_A = 0.21$

$$\text{Separation factor, } \alpha = \frac{x_A/(1-y_A)}{x_A/(1-x_A)} = \frac{0.54/0.46}{0.21/0.79} = 4.42$$

Permeability of A is given by  $l_m = 2 \times 10^{-5} \text{ cm}$

$$J_A = (\hat{P}_A/l_m) \cdot (P_1 x_A - P_2 y_A) \Rightarrow 1.245 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}} = \frac{\hat{P}_A}{2 \times 10^{-5}} \left[ (15)(0.21) - (1.5) \times (0.54) \right] \quad (76)$$

$$\Rightarrow \hat{P}_A = 1.4 \times 10^{-10} \frac{\text{cm}^3(\text{STP}) \cdot \text{cm}}{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}} = 1.4 \text{ Barres}$$

Henry's law constant

$$\text{Equilibrium solubility of the gas, } C_A = \frac{10.4 \text{ cm}^3(\text{STP})}{10.5 \text{ gm}/1.02 \text{ gm/cm}^3} = \frac{1.01}{1.01} \frac{\text{cm}^3(\text{STP})}{\text{cm}^3 \text{ polymer}}$$

$$\hat{P}_A = (5 \text{ atm})(0.21) = 1.05 \text{ atm. Solubility coeff., } S_A = \frac{C_A/p_A}{1.01} = \frac{1.01}{1.05}$$

$$\text{Average diffusivity, } D_A = \hat{P}_A/S_A = 1.4 \times 10^{-8} \text{ cm}^2/\text{s} = 0.962 \frac{\text{cm}^3 \text{ STP}}{(\text{cm}^3 \text{ polymer})(\text{atm})}$$

(14.26) Refer to FigProb 14.26.  $x, y \rightarrow$  mole fractions of oxygen  
 Pressure ratio (equal in the two stages),  $r = \frac{1.103}{1.15} = 0.96$  (more permeating)  
 Retentate from the second stage (96% N<sub>2</sub>) =  $x_{2,0} = 1 - 0.96 = 0.04$   
 Write down Eq.(14.80) for the second stage to calculate  $y_2$ .

$$\frac{y_2}{1-y_2} = \alpha^* \cdot \frac{x_{2,0} - r y_2}{(1-x_{2,0}) - r(1-y_2)} \Rightarrow \frac{y_2}{1-y_2} = (4.3) \cdot \frac{(0.04) - (0.0881)y_2}{(1-0.04) - (0.0881)(1-y_2)} \Rightarrow y_2 = 0.1244$$

Now make a material balance on stage 2 to relate  $x_{2,0}$  with  $\theta$ .

$$Q_f(1-\theta) \cdot x_{2,0} = [Q_f(1-\theta) - 100] y_2 + (100)(0.04)$$

Product from the second stage =  $100 \text{ m}^3/\text{h} = Q_f(1-\theta)^2$

$$\Rightarrow x_{1,0} = \theta y_2 + (1-\theta)(0.04) = 0.1244\theta + (1-\theta)(0.04)$$

$$\Rightarrow x_{1,0} = 0.04 + 0.0844\theta \quad \dots \text{(i)}$$

Write down Eq.(14.84) for stage 1 (this is also the A-balance equation).

$$x_{1,0} = \frac{0.21 - \theta y_1}{1-\theta} \Rightarrow \theta y_1 = 0.21 - (1-\theta)x_{1,0}; x_{1,0} \text{ given by Eq.(i).}$$

$$\Rightarrow \theta y_1 = 0.21 - (1-\theta)(0.04 + 0.0844\theta) = 0.17 - 0.0444\theta + 0.0844\theta^2$$

Now write down Eq.(14.80) for stage 1.

$$\frac{y_1}{1-y_1} = (4.3) \frac{x_{1,0} - 0.088y_1}{(1-x_{1,0}) - 0.088(1-y_1)} \Rightarrow y_1 = \frac{4.3x_{1,0} - 0.3784y_1}{0.912 + 3.3x_{1,0} - 0.2904y_1} \dots \text{(iii)}$$

Eliminate  $x_{1,0}$  and  $y_1$  from (iii) using their expressions for  $\theta$  from Eqs. (i) and (ii). We get a biquadratic equation in  $\theta$ .

$$\theta^4 - 11.84\theta^3 - 9.212\theta^2 + 11.52\theta - 0.393 = 0 \Rightarrow \text{solution, } \theta = 0.66$$

Putting the values of  $\theta$  in Eq.(i) and (ii),  $x_{1,0} = 0.0957; y_1 = 0.269$

Stage 2 : Oxygen flux,  $J_{A2} = (\hat{P}_A / l_m) (P_1'' x_{2,0} P_2'' y_2) \quad \hat{P}_A = 48 \text{ Bar}, l_m = 5 \times 10^{-5} \text{ cm}$

$$\Rightarrow J_{A2} = \frac{48 \times 10^{-10}}{5 \times 10^{-5}} \cdot \left[ \frac{(1.15)(0.04)}{1.013} - (1)(0.1244) \right] (76)$$

$$= 2.405 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}} = 0.0866 \text{ m}^3(\text{STP}) / \text{m}^2 \cdot \text{h}$$

Permeation rate of A =  $(100) \left( \frac{\theta}{1-\theta} \right) \cdot y_2 = (100) \left( \frac{0.66}{0.34} \right) (0.1244) = 24.15 \frac{\text{m}^3}{\text{h}}$

$$\text{Membrane area, } A_{m_2} = \frac{24.15 \text{ m}^3/\text{h}}{0.0866 \text{ m}^3/\text{h m}^2} = 280 \text{ m}^2$$

(14.19)

(14.26) Contd... Membrane area in Stage 1

See Fig Prob 14.26. Retentate from Stage 2,  $100 \text{ m}^3/\text{h} = Q_f(1-\theta)^2$

$$\Rightarrow \text{Rate of supply of feed air to Stage 1}, Q_f = \frac{100}{(1-0.66)^2} = 865 \text{ m}^3/\text{h}$$

$$\text{Permeate from Stage 1} = Q_f \cdot \theta = (865)(0.66) = 571 \text{ m}^3/\text{h}$$

$$\text{Flux of A in Stage 1}, J_{A1} = (\hat{P}_A / l_m) [P_1' x_{1,0} - P_2' y_1]$$

$$\Rightarrow J_{A1} = \frac{48 \times 10^{-10}}{5 \times 10^{-5}} \left[ \frac{11.5}{5.013} (0.095) - (1)(0.269) \right] (76) = 5.964 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}}$$

$$\text{Membrane area, } Q_m = \frac{Q_f \theta \cdot y_1}{J_{A1}} = \frac{(571)(0.269)}{0.215} = 715 \text{ m}^2$$

\* \* \* \*

(14.27) The problem can be solved following Example 14.15.

A reasonable average value of  $\alpha$  has to be selected first.

At the inlet to the permeator,  $x = 0.21$  for oxygen.

Pressure ratio,  $r = 1/8 = 0.0555$ ;  $\alpha_{AB}^* = 4.5$ .  $y$  is given by

$$\frac{y}{1-y} = (4.5) \cdot \frac{0.21(\alpha-1) + 1 - 0.0555\alpha}{0.21(\alpha-1) + 1 - 0.0555} \Rightarrow \alpha = 4.02$$

Take a guess value  $x_R = 0.08$  to have a value at the exit.

$$\frac{y}{1-y} = (4.5) \cdot \frac{0.08(\alpha-1) + 1 - 0.0555\alpha}{0.08(\alpha-1) + 1 - 0.0555} \Rightarrow \alpha = 3.88$$

Take the average  $\alpha = 3.93$  when  $x_R = 0.112$

After another we select an average  $\alpha = 3.97$  and  $x_R = 0.112$

Use Eq (i) of Example 14.15 to determine the membrane area.

Use the values -  $x_R = 0.112$ ;  $\theta = 0.3$ ;  $l_m = 10^{-5} \text{ cm}$ ;  $P_1 = 18 \text{ atm}$ ;

$r = 0.0555$ ,  $\hat{P}_A = 12 \text{ Barres}$ ;  $Q_f = (300 \times 10^6 / 3600) \text{ cm}^3/\text{sec}$

$$Q_m = \frac{10^{-5}}{12 \times 10^{-10}} \cdot \frac{\frac{(300 \times 10^6)}{3600} (1-0.3) (3.97)}{(18 \times 76) (3.97-1)} (0.112)^{-0.337} (0.888)^{1.346} \cdot I$$

$I = \text{integral.} = 0.6077$

(14.27) contd...

Membrane area,  $A_m = 51.5 \text{ m}^2$

(14.20)

The mixed permeate concentration can be obtained by material balance.

$$(Q_f)(0.2) = Q_f(1-\theta)(0.112) + Q_f \cdot \theta (y_p), \theta = 0.3$$

$$\Rightarrow y_p = \underline{0.44} ; Q_p = \text{permeate rate} = \theta \cdot Q_f = \underline{90 \text{ m}^3/\text{hr}}$$

\* \* \* \*

(14.28) Check for any resistance to oxygen diffusion through the wall/ga fibers.

Refer to Fig Prob 14.28.

Diffusion of oxygen through the wall occurs through 'non-diffusing' nitrogen. Following equation applies at steady state

$$A_{LM} \frac{\epsilon D_K P}{RT \tau \ln P_{BM}} (P_{A_i} - P_{A_0}) = k_L A_o (C_o - C_b) \dots \dots (i)$$

The left side expression is the rate of transport through the porous wall;  $A_{LM} = 2\pi \gamma_{LM} r L$  - logmean area of the fiber of length L,  $\gamma_{LM} = (r_o - r_i)/\ln(r_o/r_i)$ ,  $r_o$  and  $r_i$  are outside and inside radius of the fiber wall;  $\epsilon$  = porosity of the wall;  $D_K$  = Knudsen diffusivity of  $O_2$  through the pores of the wall;  $\epsilon$  = wall porosity;  $P$  = total pressure;  $P_{A_i}$ ,  $P_{A_0}$  are the partial pressure of  $O_2$  at the inside and outside of the wall;  $P_{BM}$  = logmean partial pressure of  $N_2$  (non-diffusing species);  $k_L$  = liquid film mass transfer coefficient, outside the wall;  $A_o$  = outside wall area,  $2\pi r_o L$ ;  $C_o$  = oxygen concentration in the liquid in equilibrium with oxygen at partial pressure  $P_{A_0}$  at the outside of the wall ( $r=r_o$ );  $C_b$  = oxygen concentration in the bulk liquid on the shell side. Take  $C_b=0$  to check for gas phase resistance. [ $\tau$  = tortuosity of the pores on the wall]

(14.28) Contd..

14.21

Solubility relation:  $C = 37.4 \text{ ppm} = 37.4 \text{ g/m}^3 = 1.17 \times 10^{-3} \frac{\text{kmol}}{\text{m}^3}$

Henry's law constant:

$$H = 1.17 \times 10^{-3} \frac{\text{kmol}}{\text{m}^3 \text{ atm}}$$

for  $P_{O_2} = 1 \text{ atm}$ .

$$C_{O_2} = H P_{O_2}$$

Now consider Eq. (i) above.

$$\gamma_o = 8.75 \times 10^{-5} \text{ m}, \gamma_i = 4.75 \times 10^{-5} \text{ m}, \gamma_{LM} = 6.548 \times 10^{-5} \text{ m};$$

$$D_L = 5 \times 10^{-6} \text{ m}^2/\text{s}; P = 1 \text{ atm}; P_i = 0.21 \text{ atm}; k_L = 5 \times 10^{-6} \text{ m/s};$$

$$R = 0.0821 \text{ m}^3 \cdot \text{atm}/\text{K} \cdot \text{kmol}; T = 303 \text{ K}; \epsilon = 0.4; \tau = 2.55$$

Put the values in Eq. (i)

$$\frac{(2\pi L) \cdot (6.548 \times 10^{-5}) \cdot (0.4) \cdot (5 \times 10^{-6}) \cdot (1)}{(0.0821)(303)(2.55)(4 \times 10^{-5})} \cdot \ln \frac{1 - P_o}{1 - 0.21}$$

$$= (5 \times 10^{-6})(8.75 \times 10^{-5})(1.17 \times 10^{-3}) \cdot P_o$$

The above equation can be solved by trial for  $P_o$ , the partial pressure of oxygen at  $r = r_o$ .

Solution:  $P_o \approx 0.21 \text{ atm} \Rightarrow$  there is practically no drop of oxygen partial pressure across the wall of the tube  $\Rightarrow$  there is virtually no gas-phase mass transfer resistance  $\Rightarrow$  the liquid phase diffusion controls.

Mass transfer calculation is now greatly simplified.

If  $C_b$  is the concentration of dissolved oxygen in the liquid at any time  $t$ ,

$$V_L \frac{dC_b}{dt} = k_L a_m (C^* - C_b); a_m = \text{area of the module.}$$

$$\text{Integrating, } \ln \frac{C^* - C_{bi}}{C^* - C_{bf}} = \frac{k_L a_m}{V_L} \cdot t \quad \dots \dots \text{(ii)}$$

Assume that oxygen partial pressure remains uniform at  $P_{O_2} = 0.21 \text{ atm} \Rightarrow C^* = H P_{O_2} = (1.17 \times 10^{-3})(0.21)$

$$C_{bi} = 0 \text{ (initially the water was oxygen-free)} \quad = 2.457 \times 10^{-4} \frac{\text{kmol}}{\text{m}^3} = 7.85 \text{ ppm}$$

$$C_{bf} = 5 \text{ ppm}; V_L = 200 \text{ ml} = 2 \times 10^{-4} \text{ m}^3$$

(14.28) Contd...

$$\text{Area, } a_m = n \cdot \pi d_0 L = (500) \cdot \pi (1.75 \times 10^4) (0.25) = 0.0687 m^2$$

(14.22)

Putting the values in Eq (ii),

$$t = \frac{2 \times 10^{-4} m^3}{(5 \times 10^6 m/s)(0.0687 m^2)} \cdot \ln \frac{7.85}{7.85 - 5} = \frac{5908}{*}, \text{ i.e. about } \frac{10 \text{ mins.}}{*}$$

(14.29)