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Q1

- Welch method: This method aims at reducing variance of the estimate by averaging the periodogram.

The N-length signal is segmented into overlapping segments. These segments are further multiplied by tapered window prior to computation of periodogram. These modified periodograms are averaged to get Welch power spectrum estimate.

mean: $E[P_{xx}^{w}(f)] = E(\tilde{P}_{xx}^{(L)}(f))$

Variance: $V[P_{xx}^{w}(f)] = \frac{1}{L}\Gamma_{xx}^{2}(f)$ } for no overlap

$= \frac{9}{8L}\Gamma_{xx}^{2}(f)$ } for 50% overlap & triangular window

→ Computational requirement:

FFT length $= M = \frac{1.28}{\Delta F}$

No. of FFT $= 2\frac{N}{M}$ (for 50% overlap)

No. of computations $= N\log_2\left(\frac{1.28}{\Delta F}\right)$

Total multiplications $= N\log_2\left(\frac{5.12}{\Delta F}\right)$

⇒ AR model estimation methods are:

(i) Yule-walker method (ii) Burg method

(iii) Least-square method (iv) Sequential estimation method

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Q2

Barlett's method for Power Spectrum Estimation: This method aims at reducing the variance by averaging the periodogram, but at the expense of increased spectral width.

Steps:

(i) The N-point sequence is divided into K number of non-overlapping segments of length m each

(ii) For each segment, periodogram is computed as $P_{xx}^{(i)}(f)$.

(iii) These are averaged over all K segments to get Barlett Power spectrum estimate

$$P_{xx}^{(B)}(f) = \frac{1}{K} \sum_{i=0}^{k-1} P_{xx}^{(i)}(f)$$

$$E[P_{xx}^{B}] = E[P_{xx}^{(i)}(f)] \qquad var[P_{xx}^{B}] = \frac{1}{K} var[P_{xx}^{(i)}(f)]$$

→ Computational Requirement: K number of m-length FFT to be computed. No. of computation for m-length of FFT is $\frac{m}{2} \log_2(m)$

→ Total computation = $\frac{N}{2} \cdot \log_2\left(\frac{0.9}{\Delta f}\right)$.

→ Advantages of Non-parametric methods:

(i) Low variance

(ii) Simple and easy to compute using FFT

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Q3 Blackman - Turkey method : This method aims at improving the quality of PS estimate by smoothing the periodogram. The periodogram is Fourier Transform of autocorrelation of signal. The autocorrelation of m-length sequence is first multiplied by window to get biased autocorrelation. ($r_{xx}(m)$).

The mathematical expression for PSD estimation:

$$P_{xx}^{BT}(f) = \sum_{m=-(m-1)}^{(m-1)} r_{xx}(m)\, w(m)\, e^{-j2\pi f m}$$

$$r_{xx}(m) = \frac{1}{N} \sum_{n=0}^{N-m-1} x(n)\, x(n+m)$$

$$Q_{BT} = 2.34 N \Delta f$$

No. of computation $= N \log_2 \left(\frac{1.28}{\Delta f} \right)$

- Spectral Leakage :
  Spectral leakage occurs when energy from one frequency "leaks" into adjacent frequencies in a spectral estimate. This happens due to discontinuities at the boundaries of finite duration signals or when signals do not complete an integer number of cycles within the observation window.

- Spectral Resolution :
  Spectral resolution refers to the ability to distinguish closely spaced frequencies in a spectral estimate. Limited resolution arises due to finite observation time.

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Q4 Periodogram method of power spectrum estimation: is one of the simplest non-parametric methods for estimating the power spectral density of a signal. It is based on the squared magnitude of the DFT of the signal.

For a discrete time signal $x[n]$ of length N, the periodogram estimate of PSD is defined as:

$$P(f_k) = \frac{1}{N} |X(f_k)|^2$$

$X(f_k)$ is the DFT of $x[n]$, given by:

$$X(f_k) = \sum_{n=0}^{N-1} x[n] e^{-j 2\pi f_k n} \qquad f_k = \frac{k}{N} \quad k = 0, 1, \ldots, N-1$$

- Limitations of Periodogram method:

(i) High variance
(ii) Spectral leakage
(iii) Poor Frequency Resolution

Several non-parametric methods have been developed to overcome the limitations of Periodogram:

(i) Bartlett's method: Reduces variance by dividing the signal into K non-overlapping segments and ~~overlapping~~ averaging their periodograms

(ii) Welch's method: Extends Bartlett's method by allowing overlapping segments and applying a window function to each segment.

(iii) Blackman-Tuckey: Estimates PSD by applying a window to the autocorrelation function before taking ~~the~~ ~~its~~ Fourier transform

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Q5 The auto correlation function measures the similarity between a signal and its time-delayed version. PSD can be estimated using the autocorrelation function because they form a Fourier Transform pair.

Auto correlation at lag m is given by

$$r_{xx}(m) = \frac{1}{N} \sum_{n=0}^{N-m-1} x[n]\,x[n+m]$$

→ The Blackman-Tukey method estimates PSD by applying a window to the autocorrelation function before performing Fourier transform.

$$P_{BT}(F) = \sum_{m=-M+1}^{M-1} w(m)\, r_{xx}(m)\, e^{-j2\pi F m}$$

→ Some important system models and corresponding system function for parametric power spectrum estimation:

(i) Auto-Regressive model (AR)

$$H(z) = \frac{1}{1 + \sum_{k=1}^{p} a_k z^{-k}}$$

(ii) Auto Regressive moving Average (ARMA) model.

$$H(z) = \frac{\sum_{k=0}^{q} b_k z^{-k}}{1 + \sum_{k=1}^{p} a_k z^{-k}}$$

(iii) moving Average model:

$$H(z) = \sum_{k=0}^{q} b_k z^{-k}$$