Linear Phase FIR Filter Frequency sampling method

Dr. Sampath Kumar

Associate Professor

Department of ECE

MIT, Manipal

Methods for design of FIR filters:

1) using window functions
2) using frequency sampling technique.

Frequency-sampling method

$H_d(\omega)$ – desired frequency response

sampled at M frequency points, $\omega_k = \frac{2\pi}{M} k$, $k = 0, 1, \cdots M-1$

$$H(k) = H_d(\omega_k) = H_d\left(\frac{2\pi}{M} k\right)$$

Take IDFT to find M filter coefficients

$$h(n) = \frac{1}{M} \sum_{k=0}^{M-1} H(k)\, e^{j\frac{2\pi}{M}kn} \quad \text{———} \quad (1)$$

In case of linear phase FIR filter,

$$\omega_k = \frac{2\pi}{M} k \quad , \begin{cases} k = 0, 1, \ldots \frac{M-1}{2} & \text{for } M = \text{odd} \\ k = 0, 1, \ldots \frac{M}{2} - 1 & \text{for } M = \text{even} \end{cases}$$

$$H_d(\omega) = H_r(\omega) \cdot e^{-j\omega\left(\frac{M-1}{2}\right)}$$

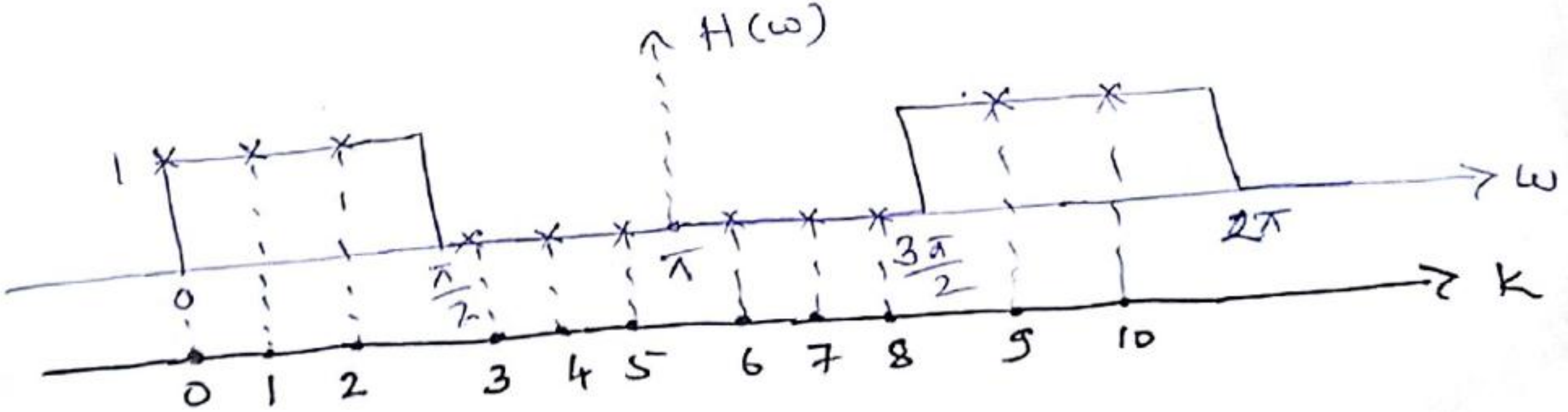
$$H(k) = H_d(\omega_k) = H_d\left(\frac{2\pi}{M} k\right) = H_r\left(\frac{2\pi}{M} k\right) \cdot e^{-j\frac{2\pi}{M} k\left(\frac{M-1}{2}\right)}$$

Substitute in eq(1),

$$h(n) = \frac{1}{M} \sum_{k=0}^{M-1} H_r\left(\frac{2\pi}{M} k\right) \cdot e^{j\frac{2\pi}{M} k\left(n - \frac{M-1}{2}\right)} \quad \text{——— (2)}$$

Example:

Consider an ideal LPF with cutoff freq at $\frac{\pi}{2}$ and $M=11$.



$$H(k) = H_d(k) = \begin{cases} 1 & , k = 0, 1, 2, 9, 10 \\ 0 & , k = 3, 4, 5, 6, 7, 8 \end{cases}$$

Imposing linear phase condition,

$$H(k) = \begin{cases} 1 \cdot e^{-j\frac{2\pi}{M}k\left(\frac{M-1}{2}\right)} = e^{-j\frac{2\pi}{11}\times 5k} & , \text{for } k = 0, 1, 2, 9, 10 \\ 0 & , \text{otherwise} \end{cases}$$

Q1. Determine the coefficients of linear phase FIR filter of length 15, which has symmetric impulse response h(n), that satisfies the condition,

$$H_r(k) = H_r\left(\frac{2\pi}{15}k\right) = \begin{cases} 1, & k = 0, 1, 2, 3 \\ 0.4, & k = 4 \\ 0, & k = 5, 6, 7 \end{cases}$$

Soln: Since h(n) is symmetric $H_r(k) = H_r(M-k)$

i.e. $$H_r\left(\frac{2\pi}{15}k\right) = \begin{cases} 1, & k = 12, 13, 14 \\ 0.4, & k = 11 \\ 0, & k = 8, 9, 10 \end{cases}$$

From eqn (2),

$$h(n) = \frac{1}{M} \sum_{k=0}^{M-1} H_r\left(\frac{2\pi}{M}k\right) \cdot e^{j\frac{2\pi}{M}k\left(n-\frac{M-1}{2}\right)}$$

$$= \frac{1}{15} \sum_{k=0}^{14} H_r\left(\frac{2\pi}{15}k\right) \cdot e^{j\frac{2\pi}{15}k(n-7)}$$

$$= \frac{1}{15}\Big[1 + 1\cdot e^{j\frac{2\pi}{15}(n-7)} + 1\cdot e^{j\frac{4\pi}{15}(n-7)} + 1\cdot e^{j\frac{6\pi}{15}(n-7)}$$

$$+ 0.4\, e^{j\frac{8\pi}{15}(n-7)} + 0.4\, e^{j\frac{\overset{-8\pi}{\cancel{22\pi}}}{15}(n-7)} + 1\cdot e^{j\frac{\overset{-6\pi}{\cancel{24\pi}}}{15}(n-7)}$$

$$+ 1\cdot e^{j\frac{\overset{-4\pi}{\cancel{26\pi}}}{15}(n-7)} + 1\cdot e^{j\frac{\overset{-2\pi}{\cancel{28\pi}}}{15}(n-7)}\Big]$$

We know, $\frac{22\pi}{15} - 2\pi = \frac{-8\pi}{15}$

Grouping $e^{j\theta} + e^{-j\theta}$ terms and subs as $2\cos\theta$,

$$\therefore h(n) = \frac{1}{15}\left[1 + 2\left\{\cos\frac{2\pi}{15}(n-7) + \cos\frac{4\pi}{15}(n-7) + \cos\frac{6\pi}{15}(n-7) + 0.4\cos\frac{8\pi}{15}(n-7)\right\}\right]$$

Substitute for $n = 0$ to 7, to get values of $h(n)$

$h(0) = -0.014$

$h(1) = -0.002$

$h(2) = 0.04$

$h(3) = 0.012$

$h(4) = -0.091$

$h(5) = -0.018$

$h(6) = 0.313$

$h(7) = 0.52$

Since symm, $h(M-1-n) = h(n)$

$h(8) = h(6) = 0.313$

$h(9) = h(5) = -0.018$

$h(10) = h(4) = -0.091$

$h(11) = h(3) = 0.012$

$h(12) = h(2) = 0.04$

$h(13) = h(1) = -0.002$

$h(14) = h(0) = -0.014$

Another equation for linear phase condition,

$$H(k) = H_r(k) \cdot e^{-j\frac{2\pi}{M}k\left(\frac{M-1}{2}\right)}$$

$$= H_r(k) \cdot e^{-j\frac{2\pi}{M}k\frac{M}{2}} \cdot e^{j\frac{2\pi}{M}k\frac{1}{2}}$$

$$= \underbrace{H_r(k)\,(-1)^k}_{G(k)}\; e^{j\frac{\pi k}{M}} \qquad , \; k = 0, 1, \dots M-1$$

Imposing symmetric condition, $H(k) = H^*(M-k)$

$$h(n) = \begin{cases} \dfrac{1}{M}\left[G(0) + 2\displaystyle\sum_{k=1}^{\frac{M}{2}-1} G(k)\cos\frac{2\pi}{M}k\left(n+\frac{1}{2}\right)\right] & \text{for } M = \text{even} \\[2ex] \dfrac{1}{M}\left[G(0) + 2\displaystyle\sum_{k=1}^{\frac{M-1}{2}} G(k)\cos\frac{2\pi}{M}k\left(n+\frac{1}{2}\right)\right] & \text{for } M = \text{odd} \end{cases} \qquad (3)$$

Some important points :

1. Similar analysis can be done for antisymm case too.

2. In Q1, we obtained $H_r(\omega_k) = 0.4$ at $k = 4$.

There is a transition band introduced.

3. Since $H_r(\omega)$ is sampled, the designed filter will have ripples in the passband and stopband.

Ripples can be suppressed by increasing filter length, M.

4. The main advantage of frequency-sampling method is that an efficient frequency sampling structure can be realized using this.

HW Qn: Obtain and draw the frequency sampling structure for the filter considered in Q1.

Frequency-sampling method

Type I sampling

$$\omega_k = \frac{2\pi}{M} k$$

$$\alpha = 0$$

Type II sampling

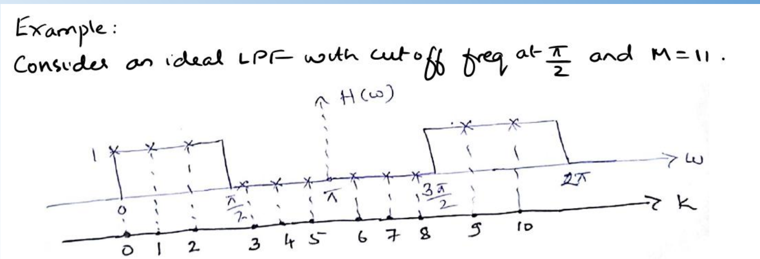
$$\omega_k = \frac{2\pi}{M}(k+\alpha)$$

$$\alpha = 1/2$$

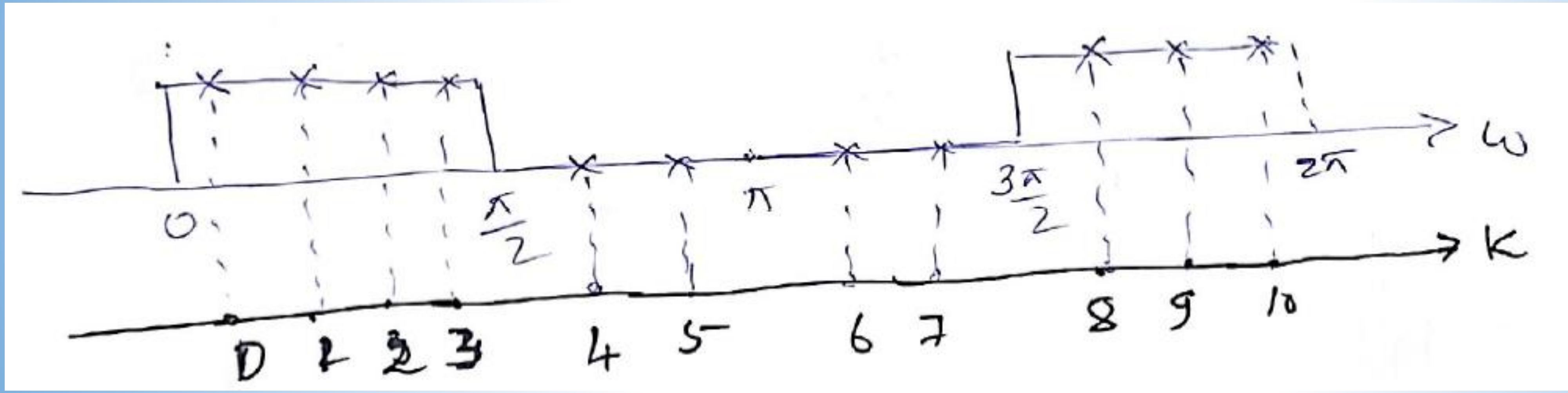
For Type II sampling,

$$H(k+\alpha) = H_r\left(\frac{2\pi}{M}(k+\alpha)\right) e^{-j\frac{2\pi}{M}(k+\alpha)\left(\frac{M-1}{2}\right)} \quad , k=0,1,\ldots M-1$$

In example 1, if we take $\alpha = 1/2$, we get



$$H\left(k+\tfrac{1}{2}\right) = \begin{cases} 1, & k = 0, 1, 2, 3, 8, 9, 10 \\ 0, & k = 4, 5, 6, 7 \end{cases}$$



Similar analysis to find h(n) values can be done here.

Reference

- Proakis J. G, Manolakis D. G. Mimitris D., “Introduction to Digital Signal Processing” Prentice Hall, India, 2007.


Thank you