Exam Date & Time: 11-Jun-2022 (09:00 AM - 12:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester B.Tech. (Aeronautical)
End Semester Examination June 2022

ENGINEERING MATHEMATICS - IV [MAT 2251]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) In a college where boys and girls are in equal in proportion, it was found that 10 out of 100 boys and 25 out of 100 girls were using the same brand of a two wheeler. If a student using that was selected at random what is the probability of being boy? (3)

A)

B) A man tosses a coin and throws a die, beginning with coin. What is the probability that he will get a head before he gets a '5' or '6' on the die? (3)

C) Suppose that a 2 -dimensional continuous random variable (X,Y) has the joint p. d. f.

$$f(x,y)=\begin{cases} kx(x-y); & |y|<x, \quad 0<x<2 \\ 0; & otherwise \end{cases}$$

. (i) Find k (ii) Marginal p. d. f. of Y. (4)

(ii) Find the conditional p. d. f. of X given Y.

2) If $X,Y,$ and Z are uncorrelated random variable with standard deviation $5, 12, 9$ respectively. If

A) $U=X+Y$ and $V=Y+Z$. Evaluate the correlation coefficient between U and V. (3)

B) Apply the Chebyshev's inequality to calculate

i) $P(5<X<15)$
ii) $P(|X-10|\geq 3)$
iii) $P(|X-10|<3)$ (3)

for a random variable X with $\mu=10$ and $\sigma^2=4$.

C) Derive mean and variance of chi square distribution. (4)

3) The heights of 500 soldiers are found to have normal distribution. Of them 258 are found to be within 2 cm from the mean height 170 cm. Find the standard deviation of X. (3)

A)

B) If X_1 and X_2 are two independent random variables having standard normal distribution, then find the p. d. f of $Z = \frac{X_1}{X_2}$. (3)

C) For the following data given below, find the equation to the best fitting curve of the form $y = ax^2 + bx + c$ and hence estimate y at $x = 6$

x	1	2	3	4	5
y	10	12	13	16	19

(4)

4) If $M_X(t)$ is the mgf of a random variable X. Then show that mgf of $Y = aX + b$ where and b are constants is $M_y(t) = e^{bt}M_X(at)$. If $M_X(t) = (0.3e^t + 0.7)^8$, find the mgf of $Y = 5X + 3$. (3)

A)

B) Suppose that $X_j, j = 1,2, \ldots 50$ are independent random variables each having a poisson distribution with $m = 0.03$. Let $S = X_1 + X_2 + \cdots + X_{50}$. Find $P(S \geq 3)$. (3)

C) Find the missing values in the following distribution, given that the mean and the median are 62.7 and 66 respectively.

Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	1	3	?	8	?	30	38

(4)

5) $\frac{d}{dx}\{x^{-n}J_n(x)\} = -x^{-n}J_{n+1}(x)$.

Prove that Also expand $J_1(x)$ in terms of series. (3)

A)

B) Find the series solution of $y'' + xy = 0$. (3)

C) Obtain the series solution of the equation $4xy'' + 2y' + y = 0$ using Frobenius method. (4)

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