

MATH I.A.-2

a)

Given: Bag with 3 white & 2 Red marble drawn without replacement until all marble are drawn.

$X = \text{no. of draws required to get 1st white marble}$

$Y = \text{no. of draws required to get 2nd white marble}$

(a) Joint Probability Distribution of $X \& Y$

<u>X</u>	<u>Y</u>	<u>P</u>
1	1	3/10
1	2	3/10
1	3	3/10
2	1	3/10
2	2	2/10
3	1	1/10

$$P(x, y) = \begin{cases} 3/10, & (1, 1) \\ 3/10, & (1, 2), (1, 3) \\ 3/10, & (2, 1) \\ 1/10, & (2, 2), (3, 1) \\ 0, & \text{otherwise} \end{cases}$$

(b) Correlation function:

$$\rho_Y = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$$E_x = 1 \times 0.6 + (2 \times 0.4) = 1.4$$

$$E_y = (1 \times 0.7) + (2 \times 0.3) = 1.3$$

$$E_{XY} = \sum (x_i y_i \times P(x_i, y_i)) = 1.8 - 5$$

Variance \Rightarrow

$$\text{Var}(x) = \mathbb{E}(x^2) - (\mathbb{E}(x))^2 = 0.64$$

$$\text{Var}(y) = \mathbb{E}(y^2) - (\mathbb{E}(y))^2 = 0.61$$

Covariance \Rightarrow

$$\mathbb{E}(xy) = \mathbb{E}(x)\mathbb{E}(y) = 0.03$$

$$P(x,y) = \frac{0.03}{\sqrt{0.64} \sqrt{0.61}} \approx \underline{0.05} \quad (0.04801)$$

Q2

Given: $f(x,y) = k$ $0 \leq x \leq 1$

$$x \leq y \leq 4-x^2$$

15 $k = ?$

$$\int_0^1 \int_x^{4-x^2} k \, dy \, dx = 1$$

$$\int_0^1 k \left[\frac{(4-x^2) - x}{2} \right] dx = 1$$

20 $k = \frac{40}{729} = 0.055$

→ marginal Density Function:

$$f_x(x) = \int_x^{4-x^2} k \, dy$$

25 $f_x(x) = \frac{40}{729} [15x - 8x^2 + x^3]$

$$f_y(y) = \frac{40}{729} \times \frac{y^2}{2}$$

Q3

$$f(x) = 1 \quad 0 \leq x \leq 1$$

Final moment Generating Function

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$$m_x(t) = E(e^{tx}) = \int_0^1 e^{tx} \cdot f(x) dx$$

$$m_x(t) = \frac{e^t - 1}{t}, \quad t \neq 0$$

For $t=0$:

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$$m_x(0) = \lim_{t \rightarrow 0} \frac{e^t - 1}{t} = \underline{\underline{1}} = m_x(0)$$

$$\underline{\underline{m_x(0) = 1}}$$

Variance = $V(x)$

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$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_0^1 x^2 dx = 1/3$$

$$V(x) = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \underline{\underline{\frac{1}{12}}}, \text{ ans.}$$