Table 3 THE STANDARD NORMAL DISTRIBUTION

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = P(X \le x)$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5369
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9131	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Probability

Addition rule

If A and B are two events of an experiment having sample space S, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

The conditional probability of an event B, given that the event A already taken place is

$$P(B / A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) > 0.$$

Baye's Theorem

Let $B_1, B_2, \ldots B_k$ are the partitions of S with $P(B_i) \neq 0, i = 1, 2, \ldots k$ and A be any event of S, then

$$P(B_i / A) = \frac{P(A / B_i) P(B_i)}{\sum_{i=1}^{k} P(A / B_i) P(B_i)}.$$

The multiplicative rule of probability : $P(A \cap B) = \begin{cases} P(A)P(B|A), if\ P(A) \neq 0 \\ P(B)P(A|B), if\ P(B) \neq 0 \end{cases}$

If $P(A \cap B) = P(A)P(B)$, then A and B are independent.

Random Variable: Let S be the sample of space of a random experiment. Suppose with each element s of S, a unique real number X is associated according to some rule then X is called random variable. There are two types of random variable, i) Discrete and ii) Continuous.

Discrete Random Variable: A random variable X is said to be discrete, if the number of possible values of X is finite or countably infinite. The probability distribution function (pdf) is named as probability mass function (PMF). The Probability mass function is defined as, let X be a random variable, hence the range space R_X of consists of atmost a countably infinite number of values. The probability mass function is defined as

$p(x_i) = \Pr\{X = x_i\}$, satisfying the conditions i) $p(x_i) \geq 0\ for\ all\ i$

ii) $\sum_{i=1}^{k} p(x_i) = 1.$

Continuous Random Variable: A random variable X is said to be continuous if it can take all possible values between certain limits, here the range space of X is infinite. Therefore the probability distribution function named for such random variable is Probability density function (PDF), which is defined as the pdf of X is a function $f(x)$ satisfying the following properties i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

iii) $\Pr\{a \leq X \leq b\} = \int_a^b f(x)dx$ for any a, b such that $-\infty < a < b < \infty$.

Note: 1. If X is a continuous random variable with pdf f(x), then

$$P(a < X < b) = P(a \leq X < b) = P(a < X \leq b) = P(a \leq X \leq b) = \int_a^b f(x)dx.$$

2. $P(X = a) = 0$, if X is a continuous random variable.

Cumulative distribution function: Let X be random variable (discrete or continuous), we define F to be the cumulative distribution function of a random variable X given by $F(x) = \Pr\{X \leq x\}$.

Case i) If X is discrete random variable then

$$F(t) = \Pr\{X \leq t\} = P(x_1) + P(x_2) + \cdots + P(t)$$

Case ii) If x is a continuous random variable then $F(x) = \Pr\{X \leq x\} = \int_{-\infty}^{x} f(x)dx$.

Two dimensional random variable: Let E be an experiment and S be a sample space associated with E. Let X=X(s) and Y=Y(s) be two functions each assigning a real number to each outcome s of S. We call (X, Y) to be two dimensional random variable.

Discrete 2D: If the possible values of (X, Y) are finite or countably infinite then (X, Y) is called discrete and it is defined as $P(x_i, y_j)$ satisfying the following condition,

i) $P(x_i, y_j) \geq 0$ and

ii) $\sum_{j=1}^{\infty} \sum_{i=1}^{\infty} P(x_i, y_j) = 1$. The function $P(x_i, y_j)$ defined is called as Joint probability distribution function (Jpdf).

Continuous 2D: If (X, Y) is a continuous random variable assuming all values in some region R of the Euclidean plane, then the Joint probability density function $f(x, y)$ is a function satisfying the following conditions

i) $f(x, y) \geq 0$ for all (x, y)ϵR

ii) $\iint f(x, y)dx\, dy = 1$ over the region R.

Marginal Probability distribution: The marginal probability distribution is defined as

Case i) In the discrete (X, Y), it is defined as $p(x_i) = P\{X = x_i\} = \sum_{j=1}^{\infty} P(x_i, y_j)$ is the marginal probability distribution of X. Similarly $q(y_j) = P\{Y = y_j\} = \sum_{i=1}^{\infty} P(x_i, y_j)$ is the marginal probability distribution of Y.

Case ii) In the continuous (X, Y), it is defined as the marginal probability function of X is defined as $g(x) = \int_{-\infty}^{\infty} f(x,y)dy$ and the marginal probability function of Y is defined as $h(y) = \int_{-\infty}^{\infty} f(x,y)dx$.

To calculate the conditional probability:

Case i) Discrete: Probability of x_i given y_j is defined as $= \frac{P(x_i, y_j)}{q(y_j)}$, $q(y_j) > 0$

Probability of y_j given x_i is defined as $= \frac{P(x_i, y_j)}{p(x_i)}$, $p(x_i) > 0$

Case ii) Continuous: The pdf of X for given Y=y is $= \frac{f(x,y)}{h(y)}, h(y) > 0$

The pdf off Y for given X=x is $= \frac{f(x,y)}{g(x)}, g(x) > 0$.

Independent Random variable: If X and Y are independent random variable then two dimensional random variable in case of discrete is defined as $P(x_i, y_j) = p(x_i).q(y_j)$ for all the values of i and j. In case of Continuous it is defined as $f(x,y) = g(x).h(y)$.

Mathematical Expectation: If X is a discrete random variable with pmf p(x), then the expectation of X is given by $E(X) = \sum_x xp(x)$, provided the series is absolutely convergent.

If X is continuous with pdf f(x), then the expectation of X is given by $E(X) = \int xf(x)dx$, provided $\int |x|f(x)dx < \infty$.

Variance of X is given by $V(X) = E(X - E(X))^2 = E(X^2) - \left(E(X)\right)^2$.

Distribution	PMF/PDF	Mean	Variance
Binomial distribution $X \sim B(n,p)$	$P(x) = {}^{n}C_{k} p^{k}(1-p)^{n-k} \quad , k = 0,1,2,\ldots,n$	$E(x) = np$	$V(x) = np(1-p)$
Poisson's Distribution $X \sim P(\alpha)$	$P(x) = \dfrac{e^{-\alpha}\alpha^{k}}{k!}, \ k = 0,1,2,\ldots, \alpha > 0$	$E(x) = \alpha = np$	$V(x) = \alpha = np$
Uniform Distribution $X \sim U(a,b)$	$f(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & otherwise \end{cases}$	$E(x) = \dfrac{b+a}{2}$	$V(x) = \dfrac{(b-a)^{2}}{12}$
Normal Distribution $X \sim N(\mu, \sigma^{2})$	$f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\frac{(x-\mu)^{2}}{\sigma^{2}}}, \quad -\infty < x, \mu < \infty, \ \sigma > 0$	$E(x) = \mu$	$V(x) = \sigma^{2}$
Exponential Distribution $X \sim E(\lambda)$	$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & otherwise \end{cases}$	$E(x) = \dfrac{1}{\lambda}$	$V(X) = \dfrac{1}{\lambda^{2}}$
Chi-square Distribution $X \sim \chi^{2}(n)$	$f(x) = \begin{cases} \dfrac{x^{\frac{n}{2}-1} e^{-\frac{x}{2}}}{\Gamma(n/2) 2^{\frac{n}{2}}}, & x > 0 \\ 0, & elsewhere \end{cases}$	$E(x) = n$	$V(x) = 2n$

Uniform distribution on a two dimensional set: If R is a set in the two-dimensional plane, and R has a finite area, then we may consider the density function equal to the reciprocal of the area of R inside R, and equal to 0 otherwise:

$$f(x, y) = \begin{cases} \frac{1}{area\ R}; & if\ (x, y) \in R \\ 0 & Otherwise \end{cases}.$$

Covariance:

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

Correlation coefficient:

$$\rho_{xy} = \rho = \frac{E(XY) - E(X)E(Y)}{\sqrt{V(X)V(Y)}}$$

Properties:

1. $E(c) = c$, where c is a constant.
2. $V(c) = 0$, where c is a constant.
3. If $E(XY) = 0$ then X and Y are orthogonal.
4. $V(AX + B) = A^2V(X)$ when AX+B is linear function of X.
5. If $\rho_{xy} = 0$ then X and Y are un correlated.
6. $V(AX + BY) = A^2V(X) + B^2V(Y) + 2ABCOV(X,Y)$

FUNCTIONS OF ONE DIMENSIONAL RANDOM VARIABLES

Theorem: Let X be a continuous random variable with p.d.f. $f(x)$ where $f(x) > 0$ for $a < x < b$. Suppose that $Y = H(X)$ is strictly monotonic function on $[a, b]$. Then the p.d.f. of the random variable $Y = H(X)$ is given by

$$g(y) = f(x)\left|\frac{dx}{dy}\right|$$

If $Y = H(X)$ is strictly increasing then $g(y) > 0$ for $H(a) < y < H(b)$.

If $Y = H(X)$ is strictly decreasing then $g(y) > 0$ for $H(b) < y < H(a)$.

Theorem: Let X be a continuous random variable with p.d.f. $f(x)$. Let $Y = X^2$ then the p.d.f. of Y is

$$g(y) = \frac{1}{2\sqrt{y}}\left[f(\sqrt{y}) + f(-\sqrt{y})\right]$$

MOMENT GENERATING FUNCTION (M.G.F.) OF ONE DIMENSIONAL RANDOM VARIABLES

Let X be any one dimensional random variable then the mathematical expectation $E(e^{tX})$ if exists then it is called the moment generating function (m.g.f.) of X.

i.e., $M_X(t) = E(e^{tX})$

In particular, if X is discrete then, $M_X(t) = \sum_{i=1}^{i=\infty} e^{tx_i} P(X = x_i)$.

If X is continuous then, $M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx$.

MGF of some standard distributions:

1. **Binomial Distributions:** $\boldsymbol{M_X(t)} = M_X(t) = (pe^t + q)^n$
2. **Poisson Distributions:** $\boldsymbol{M_X(t)} = e^{\propto(e^t - 1)}$
3. **Normal Distributions:** $\boldsymbol{M_X(t)} = e^{t\mu + \frac{\sigma^2 t^2}{2}}$
4. **Exponential Distributions:** $\boldsymbol{M_X(t)} = \frac{\propto}{\propto - t}$
5. **Gamma Distributions:** $\boldsymbol{M_X(t)} = \frac{\propto^r}{(\propto - t)^r}$
6. **Chi square Distributions:** $\boldsymbol{M_X(t)} = (1 - 2t)^{-n/2}$