DFT - convolution

Dr. Sampath Kumar

Associate Professor

Department of ECE

MIT, Manipal

Perform the circular convolution of the following two sequences:

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

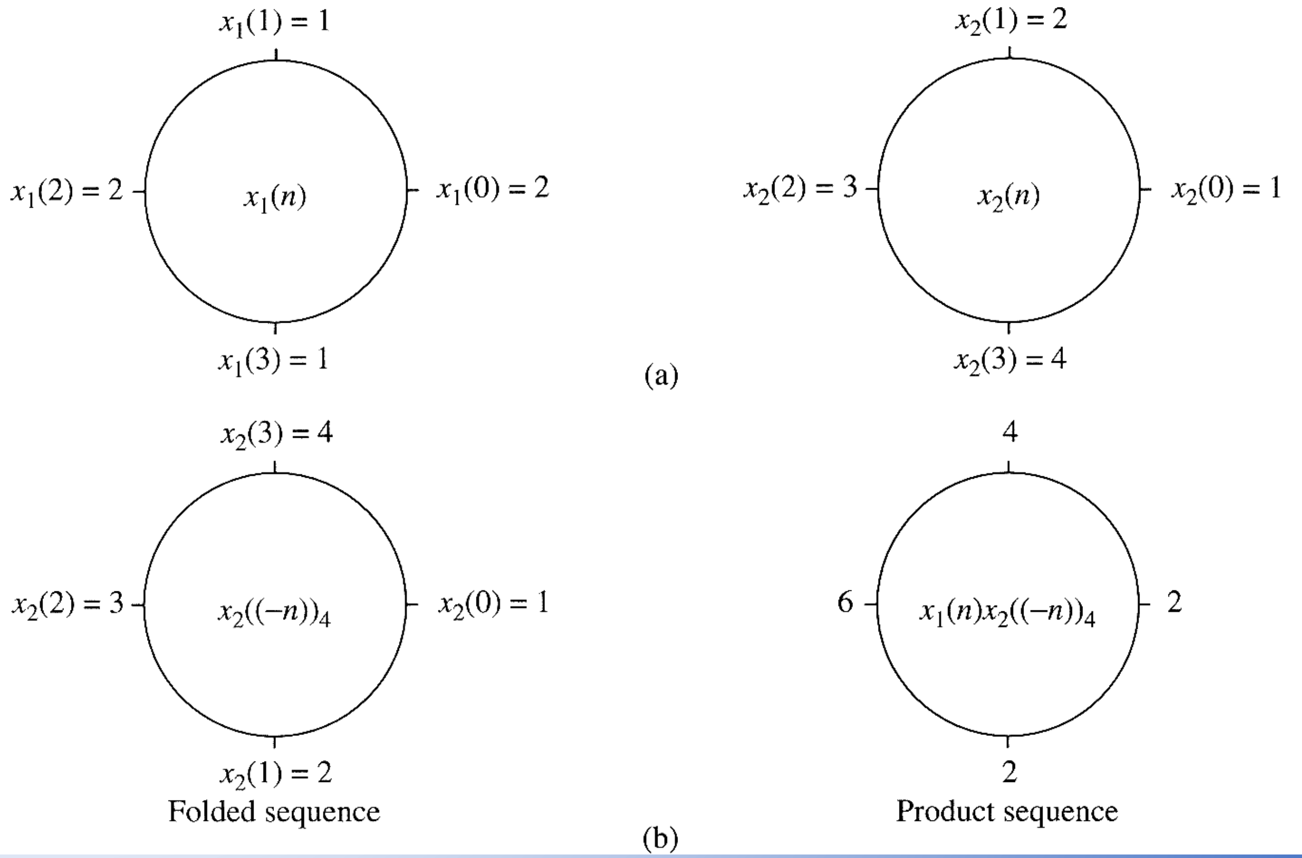
$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

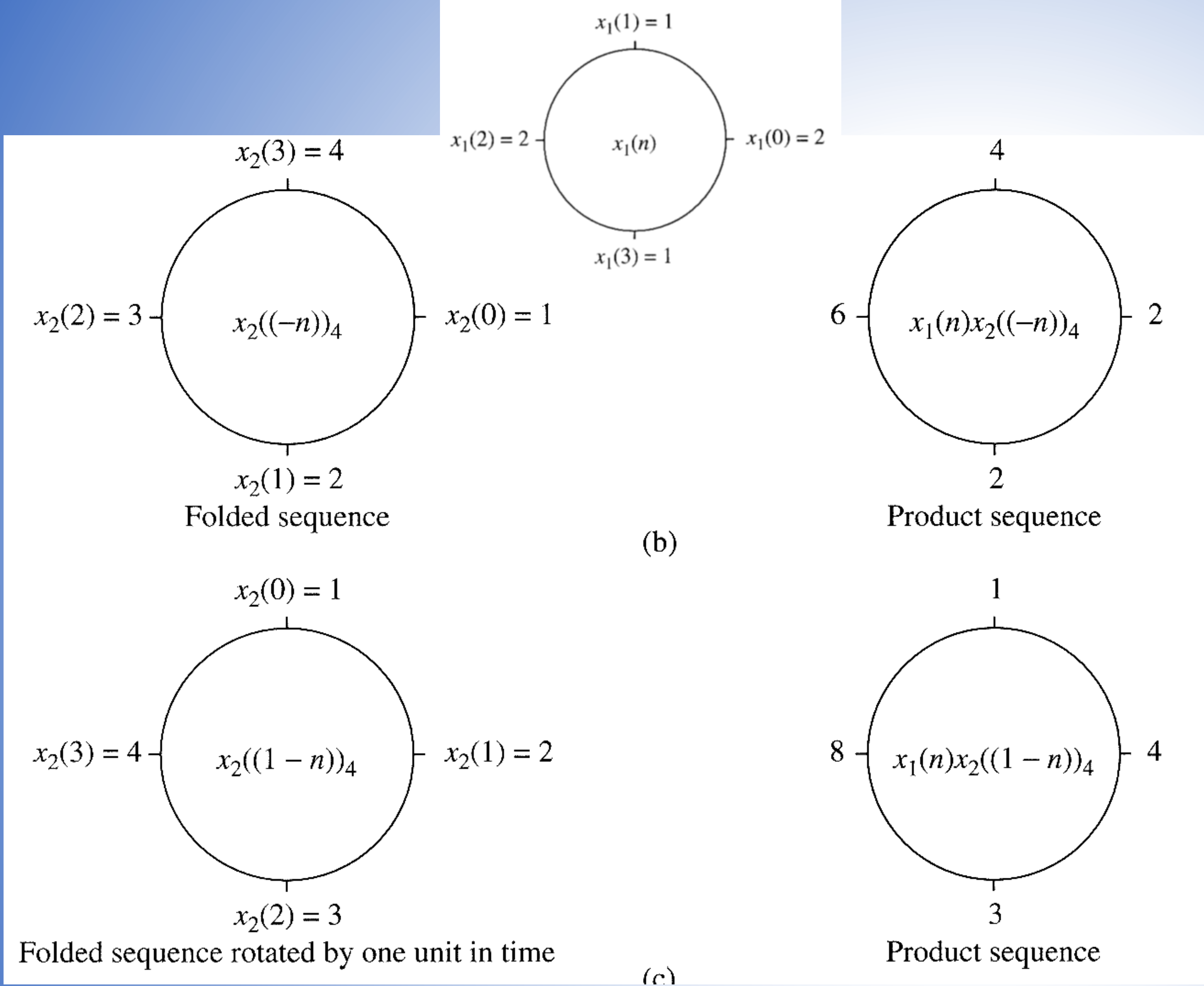
- Method 1: Circular convolution – Time domain approach

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)x_2((m-n))_N, \qquad m = 0, 1, \ldots, N-1$$

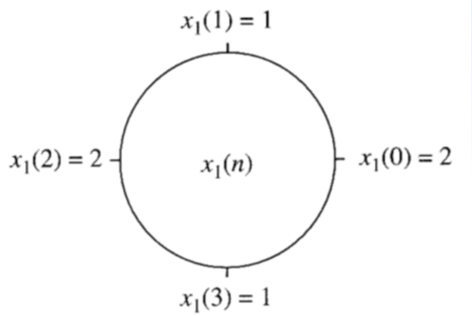
$$x_3(0) = \sum_{n=0}^{3} x_1(n)x_2((-n))N$$

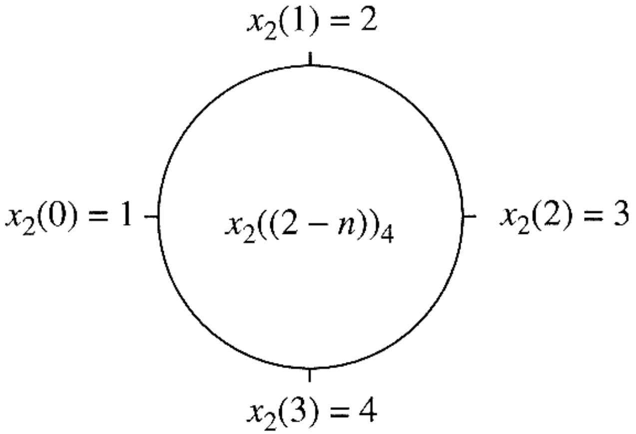
$$x_3(0) = 14$$



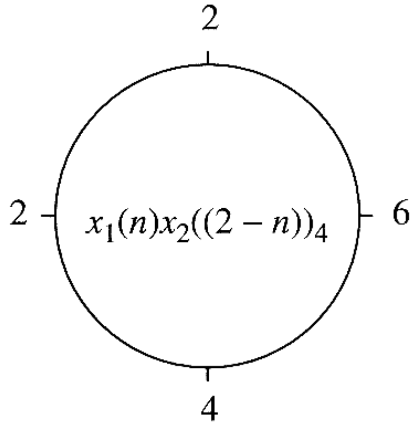

$x_1(1) = 1$
$x_1(2) = 2$
$x_1(n)$
$x_1(0) = 2$
$x_1(3) = 1$
$x_2(3) = 4$
$x_2(2) = 3$
$x_2((-n))_4$
$x_2(0) = 1$
$x_2(1) = 2$
Folded sequence
4
6
$x_1(n)x_2((-n))_4$
2
2
Product sequence
(b)
$x_2(0) = 1$
$x_2(3) = 4$
$x_2((1-n))_4$
$x_2(1) = 2$
$x_2(2) = 3$
Folded sequence rotated by one unit in time
1
8
$x_1(n)x_2((1-n))_4$
4
3
Product sequence
(c)

$$x_3(1) = 16$$





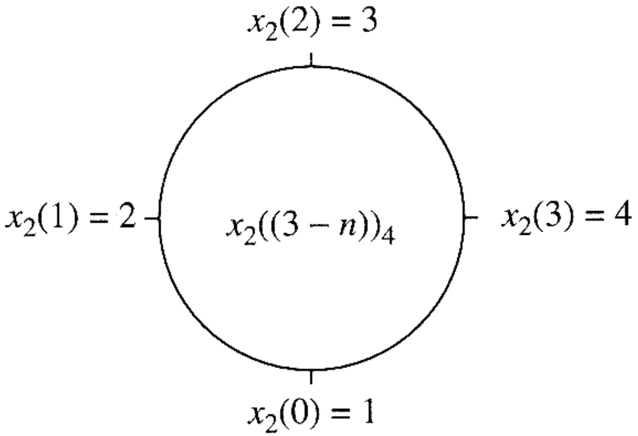
Folded sequence rotated by two units in time



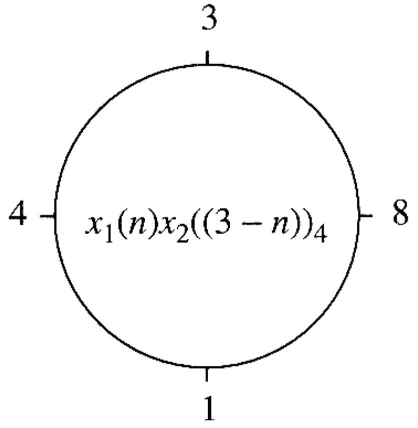
Product sequence

(d)

$$x_3(2) = 14$$



Folded sequence rotated by three units in time



Product sequence

(e)

$$x_3(3) = 16$$

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}$$

- Same 4 operations of convolution – Fold, shift, multiply and add
- Folding and shifting are circular in nature.

- Method 2: Circular convolution: DFT – IDFT approach

Perform the circular convolution of the following two sequences:

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

$$X_1(k) = \sum_{n=0}^{3} x_1(n)e^{-j2\pi nk/4}, \qquad k = 0, 1, 2, 3$$

$$= 2 + e^{-j\pi k/2} + 2e^{-j\pi k} + e^{-j3\pi k/2}$$

$$X_1(0) = 6, \qquad X_1(1) = 0, \qquad X_1(2) = 2, \qquad X_1(3) = 0$$

$$X_2(k) = \sum_{n=0}^{3} x_2(n)e^{-j2\pi nk/4}, \qquad k = 0, 1, 2, 3$$

$$= 1 + 2e^{-j\pi k/2} + 3e^{-j\pi k} + 4e^{-j3\pi k/2}$$

$$X_2(0) = 10, \qquad X_2(1) = -2 + j2, \qquad X_2(2) = -2, \qquad X_2(3) = -2 - j2$$

$$X_3(k) = X_1(k)X_2(k)$$

$$X_3(0) = 60, \qquad X_3(1) = 0, \qquad X_3(2) = -4, \qquad X_3(3) = 0$$

Now, the IDFT of $X_3(k)$ is

$$x_3(n) = \frac{1}{N}\sum_{k=0}^{3} X_3(k)e^{j2\pi nk/4}, \qquad n = 0, 1, 2, 3$$

$$= \frac{1}{4}(60 - 4e^{j\pi n})$$

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}$$

Circular convolution using matrix method

$$x_1(n) = \{1, -1, -2, 3, -1\}$$

$$x_2(n) = \{1, 2, 3, 0, 0\}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 2 & 1 \end{bmatrix}}_{x_2(n)} \underbrace{\begin{bmatrix} 1 \\ -1 \\ -2 \\ 3 \\ -1 \end{bmatrix}}_{x_1(n)} = \underbrace{\begin{bmatrix} 8 \\ -2 \\ -1 \\ -4 \\ -1 \end{bmatrix}}_{y(n)}$$

$$y(n) = \{8, -2, -1, -4, -1\}$$


Thank you