Fast Fourier Transform Algorithms - DITFFT

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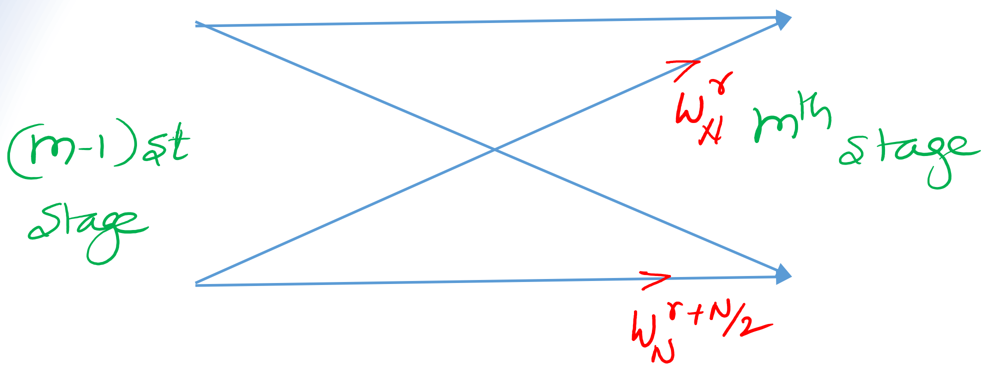
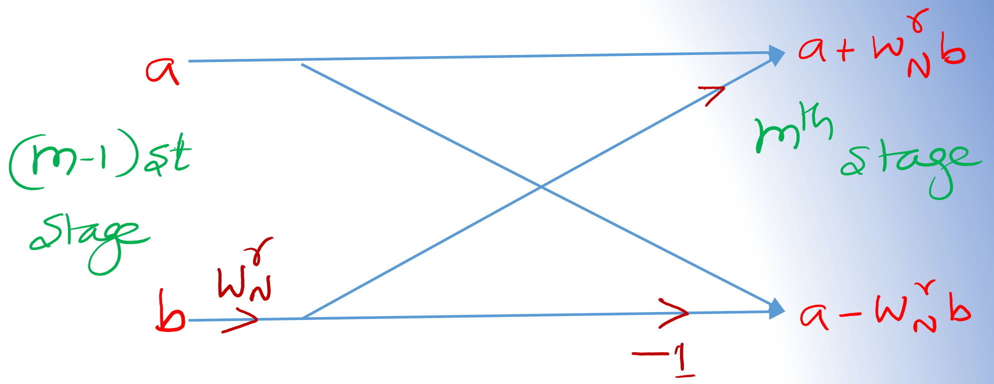
Butterfly Computation





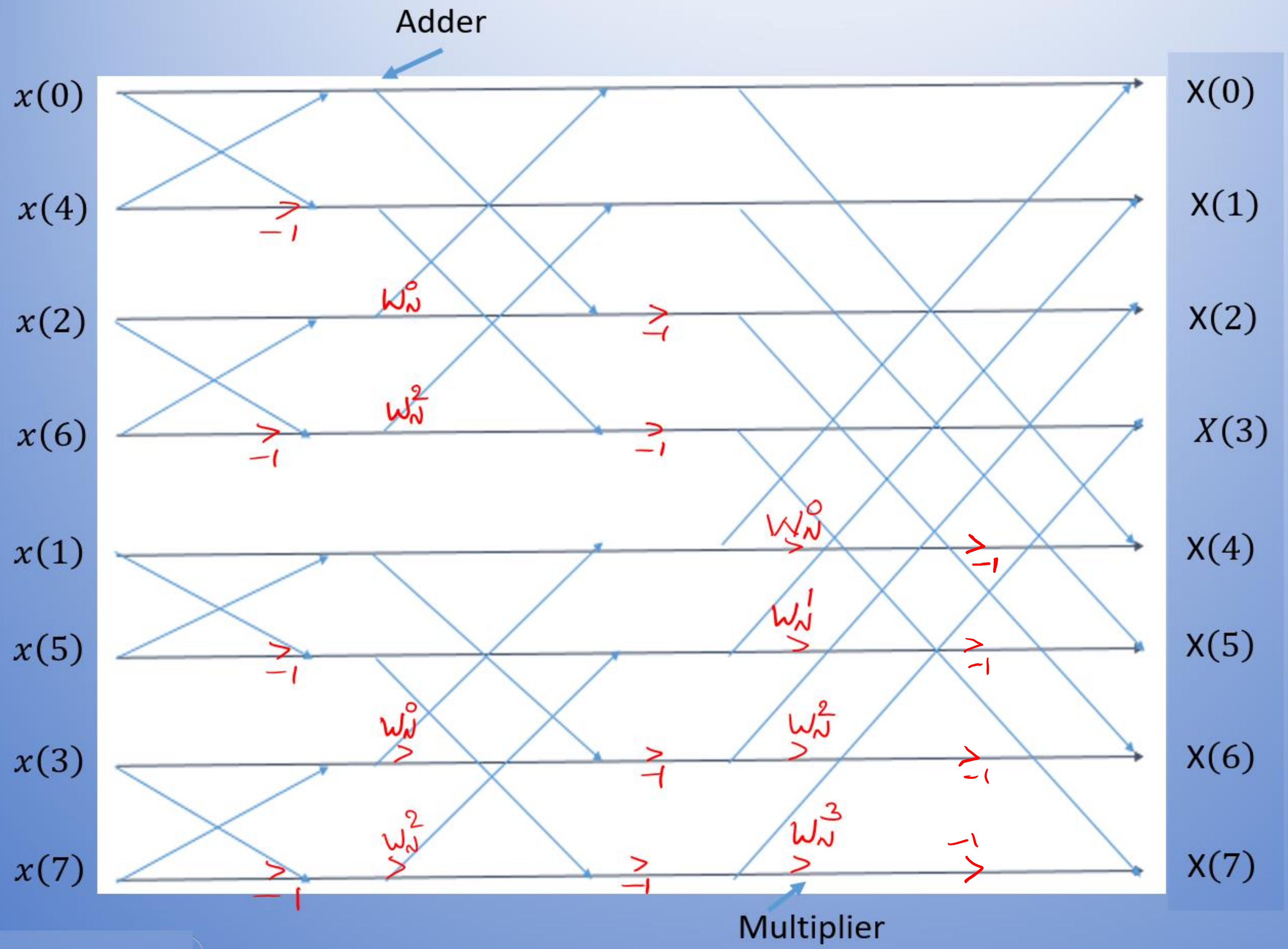
We can write $W_N^{r+N/2} = W_N^{N/2} \cdot W_N^r$

$$= e^{-j2\pi \frac{N/2}{N}} \cdot W_N^r$$

$$= e^{-j\pi} W_N^r$$

$$\therefore W_N^{r+N/2} = -W_N^r$$

Complete signal flow graph for DIT FFT Algorithm for 8-pt DFT



Binary	Bit Reverse	Decimal
000	000	0
001	100	4
010	010	2
011	110	6
100	001	1
101	101	5
110	011	3
111	111	7

Complex phase factors for $N = 8$

$$W_N^0 = e^{-j\frac{2\pi \times 0}{N}} = 1$$

$$W_N^1 = e^{-j\frac{\pi}{4}} = \cos(\pi/4) - j\sin(\tfrac{\pi}{4}) = \frac{1-j}{\sqrt{2}} = \underline{\underline{0.707 - j0.707}}$$

$$W_N^2 = e^{-j\pi/2} = -j$$

$$W_N^3 = e^{-j3\pi/4} = \cos(\tfrac{3\pi}{4}) - j\sin(\tfrac{3\pi}{4}) = \frac{-1-j}{\sqrt{2}}$$

$$= \underline{\underline{-0.707 - j0.707}}$$

$$W_N^4 = e^{-j\pi} = -1$$

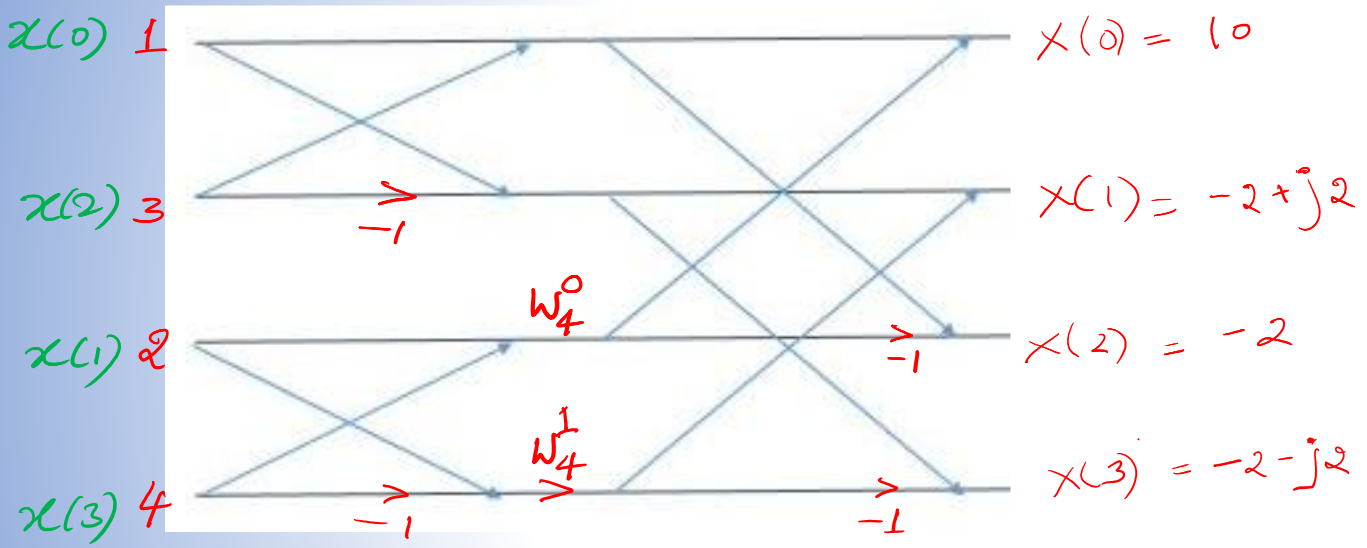
for $N = 4$

$$W_4^0 = 1$$

$$W_4^1 = e^{-j\frac{\pi}{2}} = -j$$

$$W_4^2 = e^{-j\pi} = -1$$

Problem: Compute 4-pt DFT of a sequence $x(n) = \{1, 2, 3, 4\}$



Try {2,1,2,1}.
Ans:{6,0,2,0}

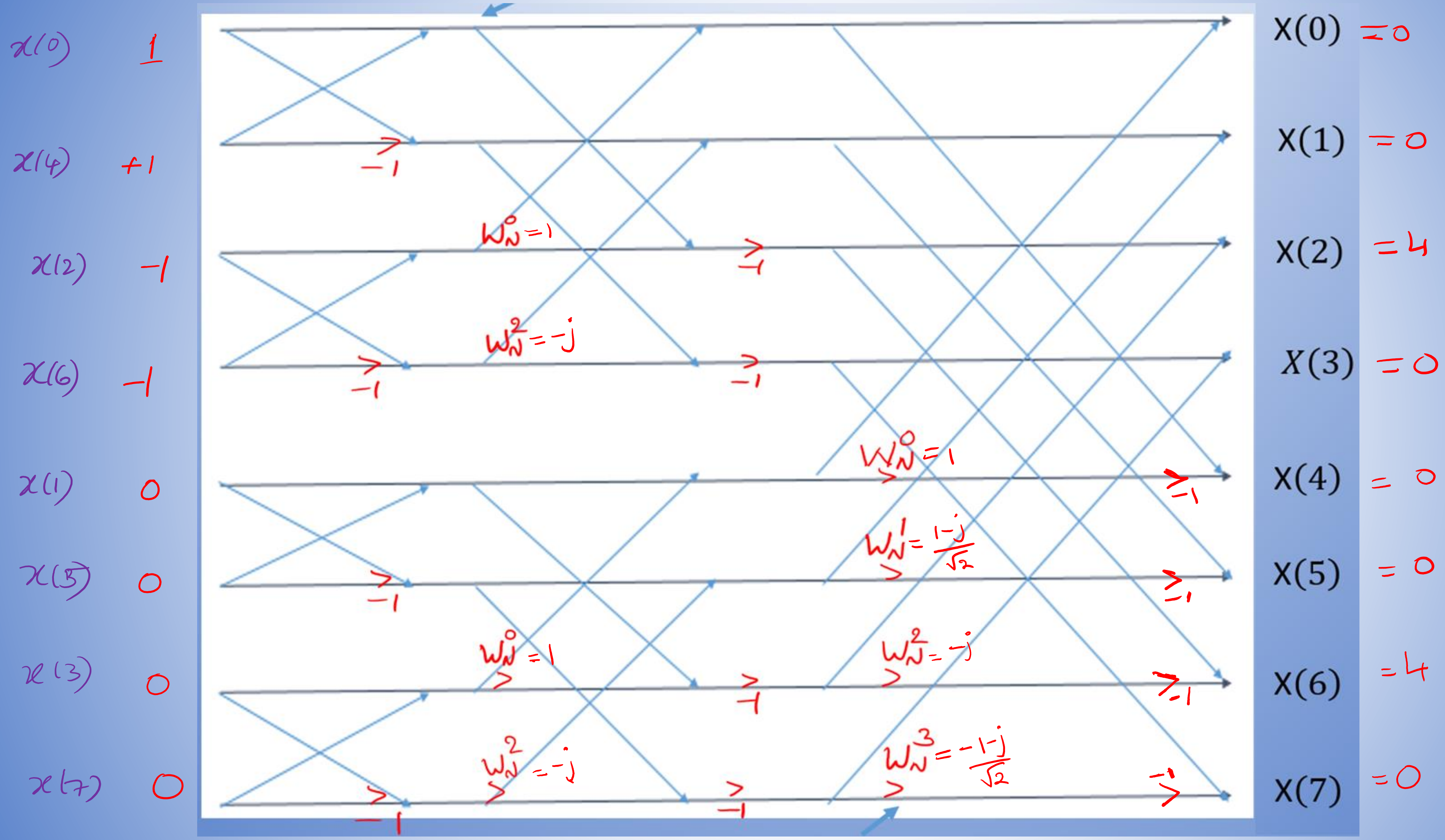
Compute 8-pt DFT of the following sequence $x(n) = \cos\frac{n\pi}{2}$ $0 \le n \le 7$

$= 0$ elsewhere

$$x(n) = \{\underset{x(0)}{1}, 0, \underset{x(2)}{-1}, 0, \underset{x(4)}{1}, 0, \underset{x(6)}{-1}, 0\}$$

$n \Rightarrow$ 0 1 2 3 4 5 6 7

$x(n) = \{1, 0, -1, 0, 1, 0, -1, 0\}$



x(0) 1
x(4) +1
x(2) -1
x(6) -1
x(1) 0
x(5) 0
x(3) 0
x(7) 0
-1
$W_N^0 = 1$
$W_N^2 = -j$
$W_N^0 = 1$
$W_N^1 = \frac{1-j}{\sqrt{2}}$
$W_N^2 = -j$
$W_N^3 = \frac{-1-j}{\sqrt{2}}$
X(0) = 0
X(1) = 0
X(2) = 4
X(3) = 0
X(4) = 0
X(5) = 0
X(6) = 4
X(7) = 0

Verify this using DIT-FFT algorithm

$x(n) = \{1,1,1,0,0,0,0,0\}$

$X(k) = \{3, 1.71-j1.71, -j, 0.29+j0.29, 1, 0.29-j0.29, j, 1.71+j1.71\}$


Thank you