# Linear Filtering based on DFT

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An LTI system has impulse response $h(n) = \cos\left(\frac{n\pi}{2}\right)$ & i/p $x(n) = 2^n$. using DFT - IDFT approach compute the response of LTI system. choose $0 \le n \le 3$.

solution

$h(n) = [1, 0, -1, 0]$

$x(n) = [1, 2, 4, 8]$

$N \ge L+M-1$ $\quad N \ge 7$, $\quad N = 8$

$X(k) \Rightarrow$ 8 Point DFT of $x(n)$

$H(k) \Rightarrow$ 8 Point DFT of $h(n)$

$Y(k) = X(k)\, H(k)$

$$Y(k) \xrightarrow[\text{IDFT}]{\text{8 point}} y(n)$$

$\downarrow$

Response of LTI system

# Linear convolution using matrix method (Time domain)

$$\boldsymbol{h} = [1,\ 2, -1,\ 1]$$
$$\boldsymbol{x} = [1,\ 1,\ 2,\ 1,\ 2,\ 2,\ 1,\ 1]$$

$$\boldsymbol{y} = H\boldsymbol{x} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \\ 2 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 5 \\ 3 \\ 7 \\ 4 \\ 3 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

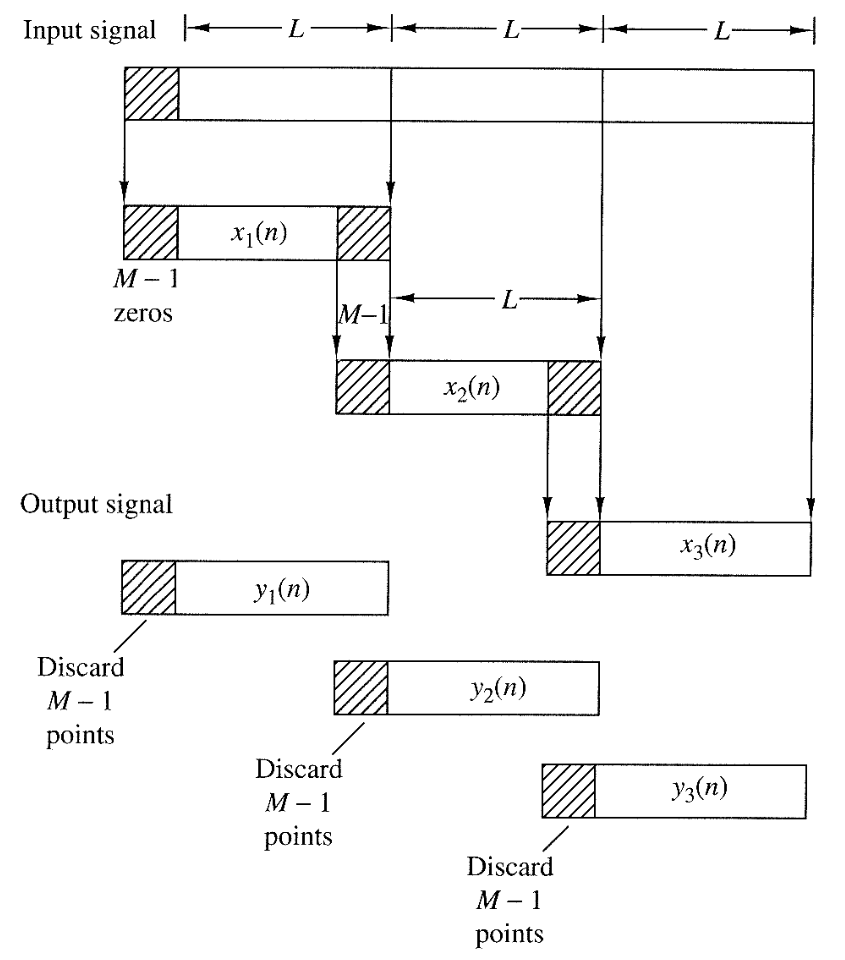
# Filtering of Long Data Sequences

Overlap save method

1) The i/p sequence $x(n)$ is subdivided into block of length L (L samples in a block)

2) Let the FIR $h(n)$ has a length M.
For N-point circular convolution $N = L+M-1$

3) For the first block, the first N samples of $x(n)$ are taken & left of it is padded with M-1 zeros. This sequence is circularly convolved with $h(n)$ after padding as many zeros to its right, (N-M) zero to make it of length N.

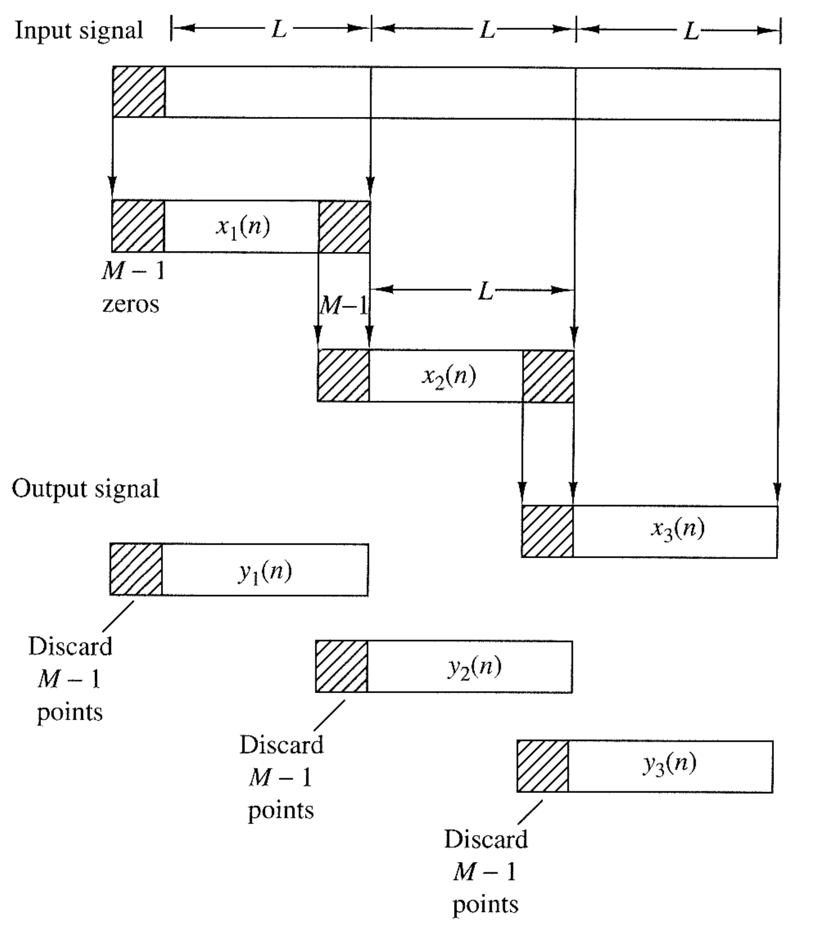


4) From the result of convolution the first (M-1) elements are discarded & the remaining is saved.

5) For the second block, next L samples of x(n) are selected. To its left the (M-1) elements of the previous block are added. This is overlapping of data of x(n)

6) The convolution is performed. From this first (M-1) elements corresponding to the overlapped data are rejected and the remaining L elements are saved (To overcome aliasing)

This is continued till all blocks are convolved. Finally the saved result are cascaded to get the final result y(n).



Using **overlap save method** find the output sequence y(n). The input sequence is x(n)=[3, 2, 1, 1, 2, 2, 0, 1, 2, 0, 1, 3] and h(n)= [1, 1, 1].

Let $L=4$, $M=3$ $\therefore N = L+M-1 = 6$

$x_1(n) = \{0, 0, 3, 2, 1, 1\}$

$x_2(n) = \{1, 1, 2, 2, 0, 1\}$

$x_3(n) = \{0, 1, 2, 0, 1, 3\}$

$x_4(n) = \{1, 3, 0, 0, 0, 0\}$

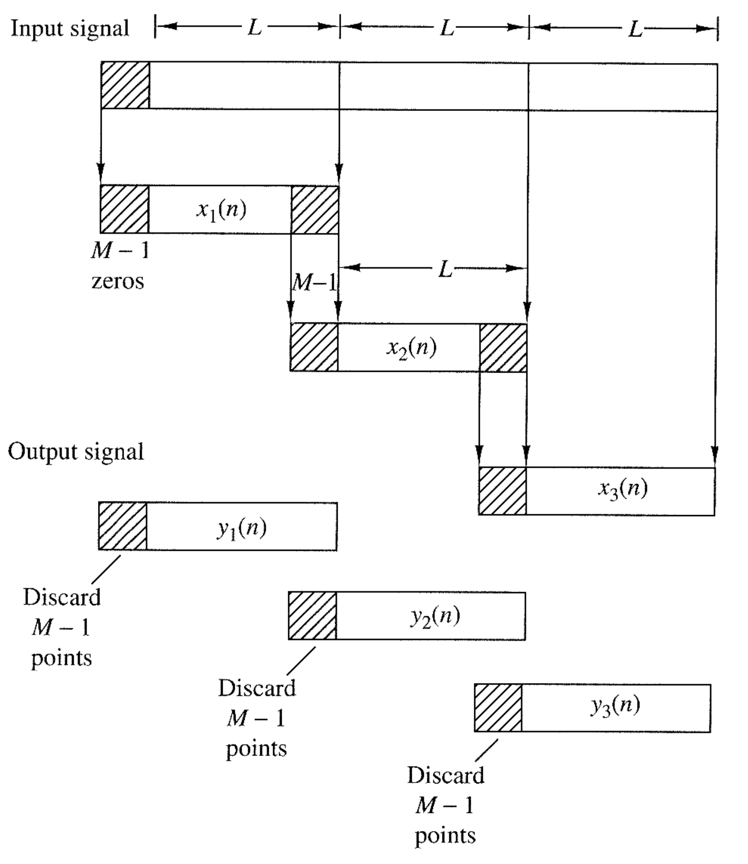
$h(n) = \{1, 1, 1, 0, 0, 0\}$

$x_1(n) \circledast_N h(n) = \{\cancel{2}, \cancel{1}, 3, 5, 6, 4\}$

$x_2(n) \circledast_N h(n) = \{\cancel{2}, \cancel{3}, 4, 5, 4, 3\}$

$x_3(n) \circledast_N h(n) = \{\cancel{4}, \cancel{4}, 3, 3, 3, 4\}$

$x_4(n) \circledast_N h(n) = \{\cancel{1}, \cancel{4}, 4, 3, 0, 0\}$



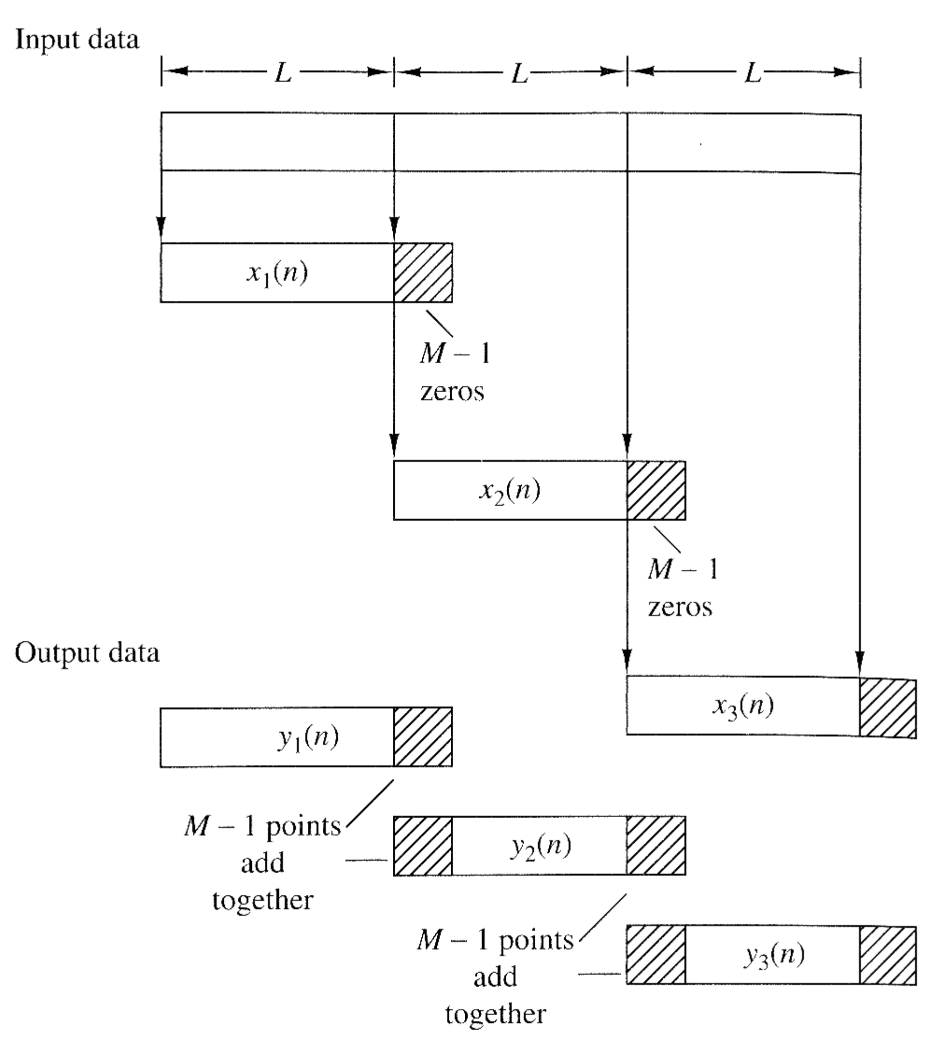

Use DFT – IDFT method for finding circular convolution

Try with L = 8

Ans: [3,5,6,4,4,5,4,3,3,3,3,4,4,3]

# Overlap add method

1) The input sequence $x(n)$ is subdivided into blocks of L samples.
2) Let the FIR $h(n)$ has M-samples. For N point circular convolution $N = L + M - 1$
3) For the first block $x(n)$ the first L samples are taken & M-1 zeros are padded to its right.
4) The sequence $h(n)$ is also order of length N by padding to its right (N - M) zeros.
5) Circular convolution is performed to first block.

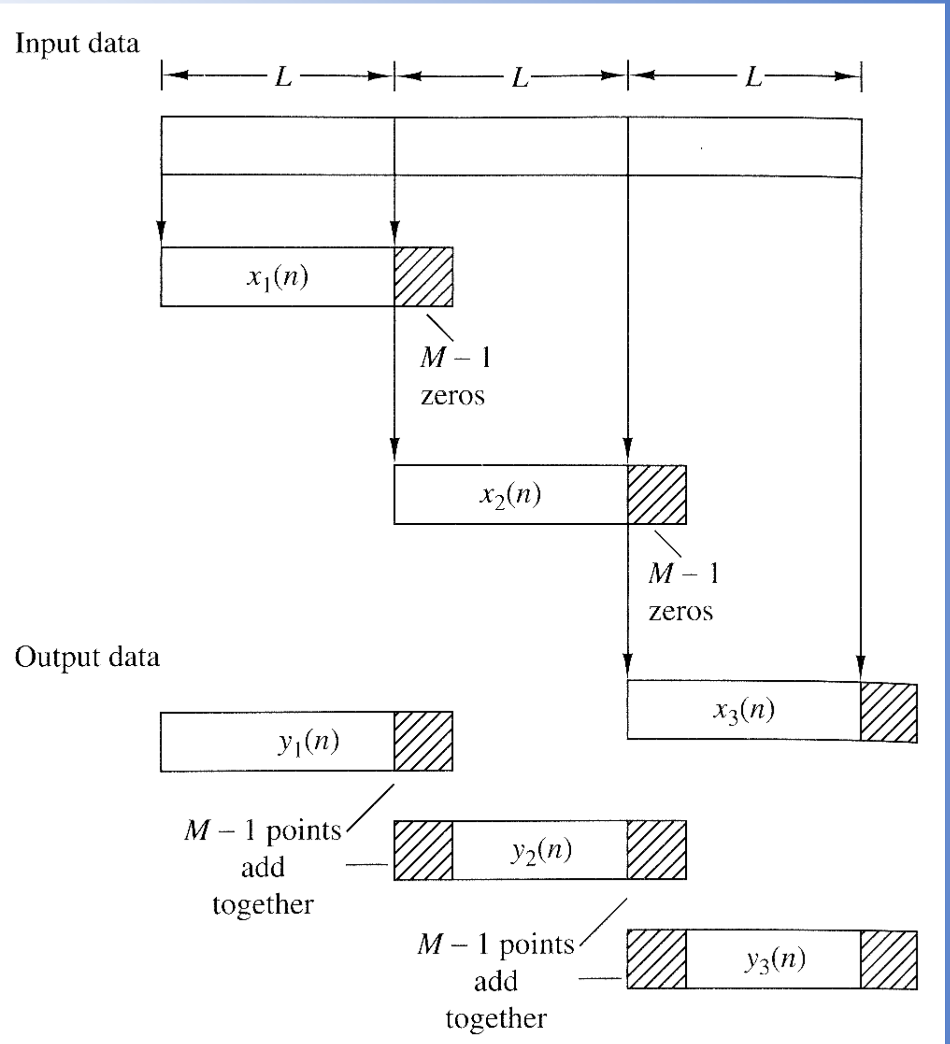


6) For the second block the rest L samples are taken and to its right (M-1) zeros are added

7) The circular convolution of the samples of second block and h(n) is performed.

8) From this result of second convolution, first (M-1) elements are overlapped and added to the last (M-1) samples of the result of first block convolution. This generates the first 2L samples of final result.

9) This procedure is continued untill all blocks are convolved & final result is obtained.



Using **overlap add method** find the output sequence y(n). The input sequence is x(n)=[3,2,1,1,2,2,0,1,2,0,1,3] and h(n)=[1,1,1].

## Homework

Take L = 4

M = 3

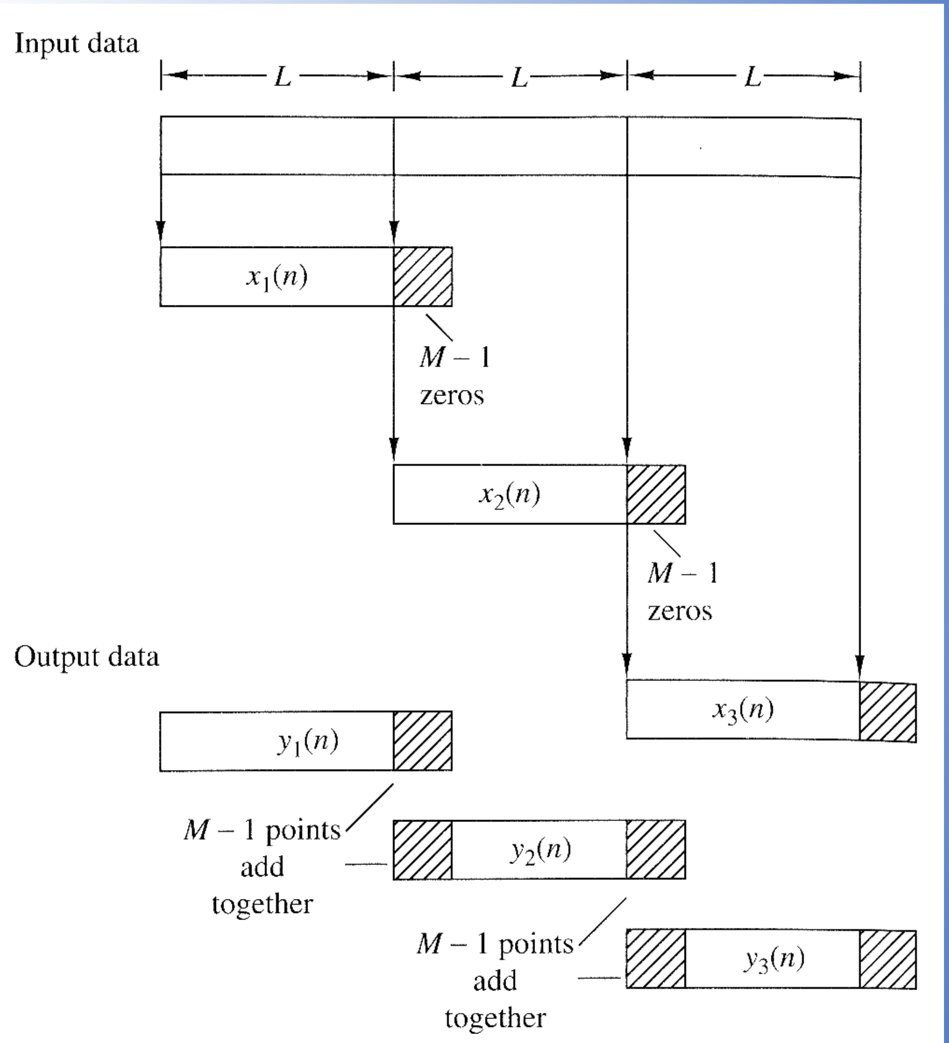
N = 6

$x_1(n)$, $x_2(n)$, $x_3(n)$ - Add 2 zeros

h(n) – Add 3 zeros

Compute 6 point circular convolutions

Overlap and add to get y(n)




Ans: [3,5,6,4,4,5,4,3,3,3,3,4,4,3]


Thank you