SIMPLEX METHOD

1 Introduction

SImplex method is the most popular method used for the solution of LPP.

- It is the search procedure that shifts through the set of basic feasible solutions, one at a time until the optimal basic feasible solution is identified.
- It deals only with a small and unique set of feasible solutions, the set of vertex points (i.e. extreme points or corner points) of the convex feasible space that contain the optimal solution
- All the inequalities of the constraint should be converted to equalties with the help of slack or surplus variables. It is not used to examine all feasible solutions and all the resource values or constraints should be non negative
- The number of basic feasible solutions of LP problem is finite and at the most ${}^{n}C_{m}$, where n is the number of decision variables and m is the number of constraints in the problem.

Definition 1.1 : **Slack and Surplus variable:** *If a constraint has an inequality(≤) then to make it on equality ground, the positive term added on LHS o the constraint is called the slack variable and if the constraint has an inequality(≥), the term subtracted to LHS to make the constraint equal is called the surplus variable. For example:*

(i) *If* $2x + 3y \leq 1, \quad 8x + 4y \leq 3, x, y \geq 0$ *are the two constraints, we add the slack variables* s_1 *and* s_2 *such that* $2x+3y+s_1 = 1, \quad 8x+4y+s_2 = 3, x, y, s_1, s_2 \geq 0$

(ii) *If* $2x + 3y \geq 1, \quad 8x + 4y \geq 3, x, y \geq 0$ *are the two constraints, we add the slack variables* s_1 *and* s_2 *such that* $2x+3y+s_1 = 1, \quad 8x+4y+s_2 = 3, x, y, s_1, s_2 \geq 0$

Definition 1.2 :**Feasible solution** *Any set* $X = \{x_1, x_2, x_3, ..., x_m\}$ *of variables is called a feasible solution of L.P. problem, if it satisfies the set of constraints as well as non-negativity restrictions.*

Definition 1.3 :**Basic solution** *For a system of m simultaneous linear equations in n variables (n>m), a solution obtained by setting (n-m) variables equal to zero and solving for the remaining variables is called a basic solution. Such m variables (of course, some of them may be zero) are called basic variables and remaining (n-m) zero-valued variables are called non-basic variables.*

Definition 1.4 :**Basic feasible solution** *is a basic solution which also satisfies the non-negativity restrictions, that is all basic variables are non-negative. Basic solutions are of two types*

(a) Non-degenerate: A non-degenerate basic feasible solution is the basic feasible solution which has exactly m positive X_j

(b) Degenerate : A basic feasible solution is called degenerate, if one or more basic variables are zero-valued.

Definition 1.5 :**Optimum basic feasible solution** *A basic feasible solution is said to be optimum, if it also optimizes (maximizes or minimizes) the objective function.*

2 Methodology

Step 1: Formulate the linear programming model. Maximize or minimize objective function subject to the constraints.

Step 2: Express LP problem in standard form by adding slack variables/surplus variables to the contraints and assign zero coefficient to these variables in objective function.

Step 3: Determine a starting basic feasible solution that is by setting $x_1 = x_2 = x_3, \cdots, x_n = 0$.

Step 4: Construct the starting simplex tableau. Select an entering variable using the optimality conditions. Stopif there is no variable; the last solution is optimal. Else go to the step 5.

Step 5: Select a leaving variable using the feasibility condition.

Step 6: Determine the new basic solution by using the appropraite Gauss-Jordan comutations, then go to Step 4.

2.1 Gauss Jordan Row Operations

1 Pivot row: (a) Replace the leaving variable in the basic column with the entering variable
(b) New pivot row= Current row divided by Pivot element.

2 All other rows including z is obtained by: New row= Current row-(pivot column co-efficient)x New pivot row

Problem 1: Reddy Mikks Model: Maximize: $z = 5x + 4y$ subject to
$6x + 4y \leq 24$,
$x + 2y \leq 6$,
$-x + y \leq 1$,
$y \leq 2$, $x, y \geq 0$.

Solution: 1) Convert into a standard LPP with surplus/slack variables: Objective function $z = 5x + 4y + 0s_1 + 0s_2 + 0s_3$ subject to $6x + 4y + s_1 = 24$,
$x + 2y + s_2 = 6$,
$-x + y + s_3 = 1$,
$y + s_4 = 2$, $x, y, s_1, s_2, s_3, s_4 \geq 0$.

One obvious feasible solution that satisfies all the constraints is $x = y = 0$ and $s_1 = 24$, $s_2 = 6$, $s_3 = 1$, $s_4 = 2$ and $z = 0$.

2) x has the most positive co-efficient, which is the entering variable. By feasibility condition s_1 is the leaving variable, that is by ratio rule (24/6, 6/1, 1/-1,2/0) identifying the minimum(24/6).

Basic	z	x	y	s_1	s_2	s_3	s_4	Solution
z	1	-5	-4	0	0	0	0	0
s_1	0	6	4	1	0	0	0	2.4
s_2	0	1	2	0	1	0	0	6
s_3	0	-1	1	0	0	1	0	1
s_4	0	0	1	0	0	0	1	2

Table 1

3) With the new row and pivot row formula discussed above, prepare the new table 2 . Z still has negative co-efficients, so look for the next entering variable that is y with most negative co-efficient. s_2 leaves the basic solution and the new value of y is 1.5.

4)None of the z-row co-efficients associated with the non basic variables s_1 and s_2 are negative. Hence the last table 3 is optimal.

Basic	z	x	y	s_1	s_2	s_3	s_4	Solution
z	1	0	-2/3	5/6	0	0	0	20
x	0	1	2/3	1/6	0	0	0	4
s_2	0	0	4/3	-1/6	1	0	0	2
s_3	0	0	5/3	1/6	0	1	0	5
s_4	0	0	1	0	0	0	1	2

Table 2

Basic	z	x	y	s_1	s_2	s_3	s_4	Solution
z	1	0	0	3/4	1/2	0	0	21
x	0	1	0	1/4	-1/2	0	0	3
y	0	0	1	-1/8	3/4	0	0	3/2
s_3	0	0	0	3/8	-5/4	1	0	5/2
s_4	0	0	0	1/8	-3/4	0	1	1/2

Table 3

Hence the optimal solution: $x = 3, y = 3/2$, $maximumvaluez = 21$

Problem 2: Maximize $z = 13x + 11y$ subject to $4x + 5y \leq 1500$,
$5x + 3y \leq 1575$,
$x + 2y \leq 420$,
$x, y \geq 0$.

Problem 3: Minimize $z = 5x + 3y$ subject to $x + y \leq 2$,
$5x + 2y \leq 10$,
$3x + 8y \leq 12$,
$x, y \geq 0$.