DFT - convolution

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1) Find the 4 Point DFT of $x(n)$

$$x(n) = \{2, 1, 2, -1\}$$

$$X(k) = \sum_{n=0}^{4-1} x(n) e^{\frac{-j2\pi kn}{4}}, \quad 0 \le k \le 4-1$$

$$= 2 + e^{-j\pi k/2} + 2e^{-j\pi k} - e^{-j3\pi k/2}$$

Alternate method (using Twiddle factor)

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^4 & W_4^6 \\ 1 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

- Ans: 4, -2j, 4 , 2j

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Det the 4 point IDFT of $X(K) = [\quad]$

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{\frac{j2\pi nk}{N}}, \quad n = 0, 1, \dots N-1$$

Alternate method (Twiddle factor)

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, W_N^{*nK}, \quad n = 0, 1 \dots N-1$$

(29) If we have $x(n) = [\boxed{8}, 1, 2, 2]$

In this case find 4 point DFT $X(k)$

then $x(-n) = x(-n + N)$ → N point DFT

So in this example

$x(-1) = 8$ so $x(-1 + 4) = x(3) = 8$

Now $x(n) = [1, 2, 2, 8]$

Then proceed to find 4 - Point DFT $X(k)$.

Applications of DFT – IDFT tools

- Frequency (spectrum) analysis
- Power spectrum estimations
- Linear filtering
- Presence of computationally efficient algorithms

Multiplication of two DFT and circular convolution

- Let

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n)e^{-j2\pi nk/N}, \qquad k = 0, 1, \ldots, N-1$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n)e^{-j2\pi nk/N}, \qquad k = 0, 1, \ldots, N-1$$

We have

$$X_3(k) = X_1(k)X_2(k), \qquad k = 0, 1, \ldots, N-1$$

Let us determine the relationship between $x_3(n)$ and the sequences $x_1(n)$ and $x_2(n)$.

The IDFT of $\{X_3(k)\}$ is

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1} X_3(k) e^{j2\pi km/N}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X_1(k) X_2(k) e^{j2\pi km/N}$$

$$x_3(m) = \frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{n=0}^{N-1} x_1(n) e^{-j2\pi kn/N}\right]\left[\sum_{l=0}^{N-1} x_2(l) e^{-j2\pi kl/N}\right] e^{j2\pi km/N}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l)\left[\sum_{k=0}^{N-1} e^{j2\pi k(m-n-l)/N}\right]$$

$$x_3(m) = \frac{1}{N}\sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \left[\sum_{k=0}^{N-1} e^{j2\pi k(m-n-l)/N}\right]$$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & a = 1 \\ \dfrac{1-a^N}{1-a}, & a \neq 1 \end{cases}$$

where a is defined as

$$a = e^{j2\pi(m-n-l)/N}$$

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N, & l = ((m-n))_N \\ 0, & \text{otherwise} \end{cases}$$

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n) x_2((m-n))_N, \qquad m = 0, 1, \ldots, N-1$$

circular convolution.

$$x_1(n) \; \text{Ⓝ} \; x_2(n) \; \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} \; X_1(k)X_2(k)$$


Thank you