Structure for IIR systems - 2

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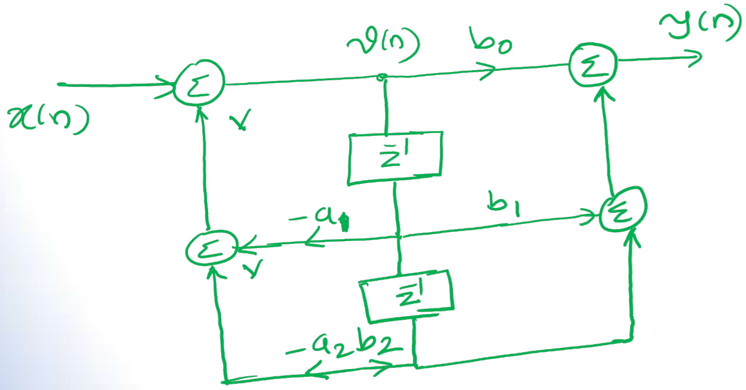
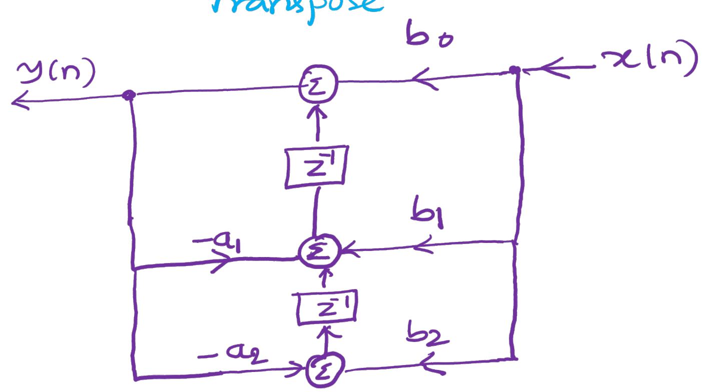
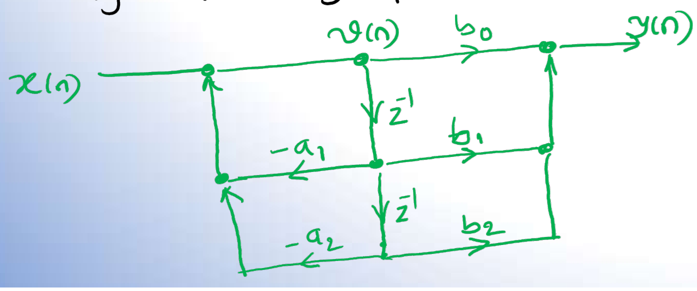
Transpose structure

- First draw DF 2 structure and form SFG
- Interchange i/p and o/p
- Reverse the direction of all branches
- Summing points become branching points and branching points become summing points

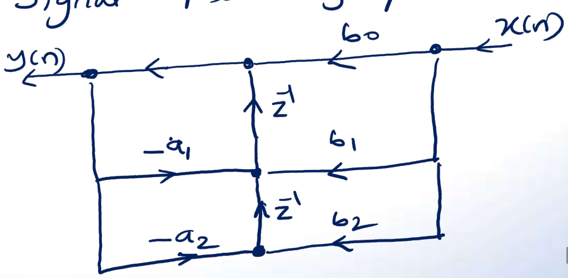




Signal flow graph



Signal Flow graph



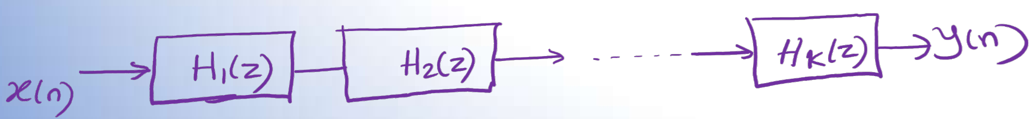
Cascaded form structures

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

Assume $N \geq M$

This system can be cascaded into second order subsystems

$$H(z) = \prod_{k=1}^{K} H_k(z)$$



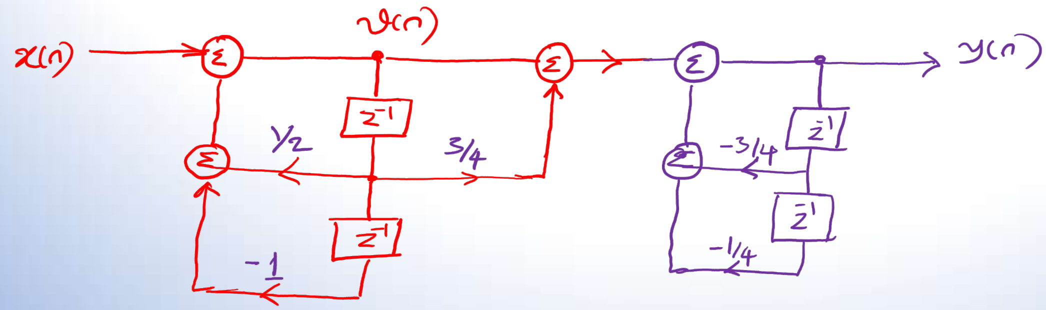
Where each $H_k(z)$ to have denominator polynomial of degree 2.

Ex: Consider the system function $H(z) = \dfrac{1+\frac{3}{4}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+z^{-2}\right)\left(1+\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}\right)}$

Realize the given function in cascaded form.

Let $$H(z) = \frac{1+\frac{3}{4}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+z^{-2}\right)\left(1+\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}\right)} = \underbrace{\frac{1+\frac{3}{4}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+z^{-2}\right)}}_{H_1(z)} \times \underbrace{\frac{1}{\left(1+\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}\right)}}_{H_2(z)}$$

$$H(z) = H_1(z)\cdot H_2(z)$$



Parallel Form structures

Parallel form structures can be obtained by performing partial fraction of H(z)

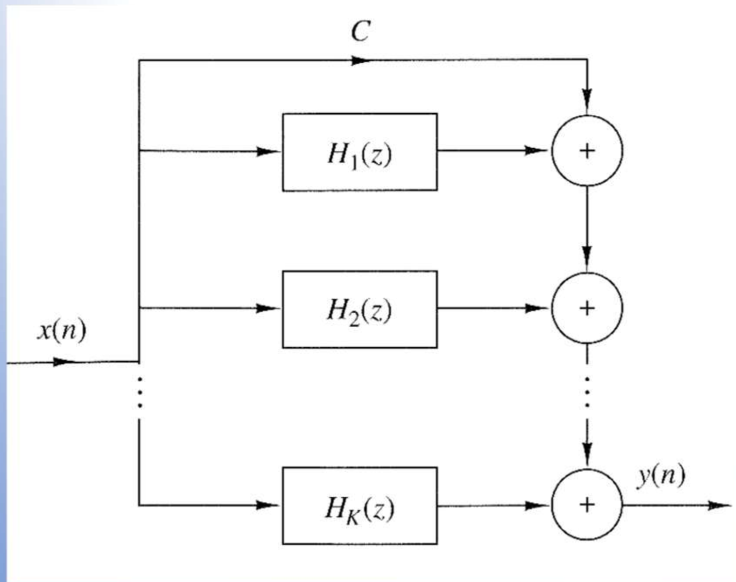
Assume $N \geq M$

constant $= \frac{b_N}{a_N}$

residues in the partial fraction

$$H(z) = C + \sum_{k=1}^{N} \frac{A_k}{1 - p_k z^{-1}}$$

pole (Distinct)



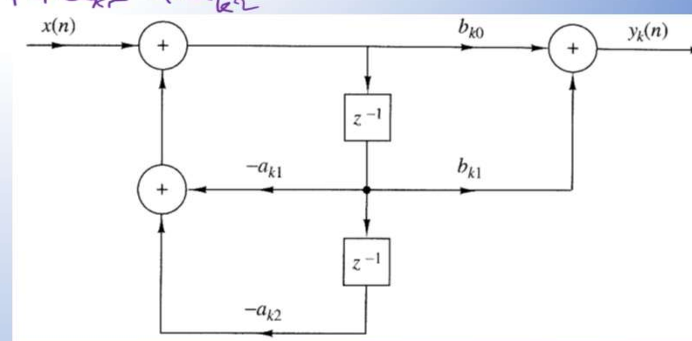
Note:

For complex valued poles corresponding A_k are also complex.

To avoid multiplication by complex numbers we can form two pole subsystem by combining pair of conjugate poles.

Such subsystems are of the form

$$H_k(z) = \frac{b_{k0} + b_{k1} z^{-1}}{1 + a_{k1} z^{-1} + a_{k2} z^{-2}}$$



Example: Realize the given function in parallel form.

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$$

$$\begin{array}{r|l} & \quad 8 \\ 1-0.75z^{-1}+0.125z^{-2} & \quad 1+2z^{-1}+z^{-2} \\ & \quad 8-6z^{-1}+z^{-2} \\ \hline & \quad -7+8z^{-1} \end{array}$$

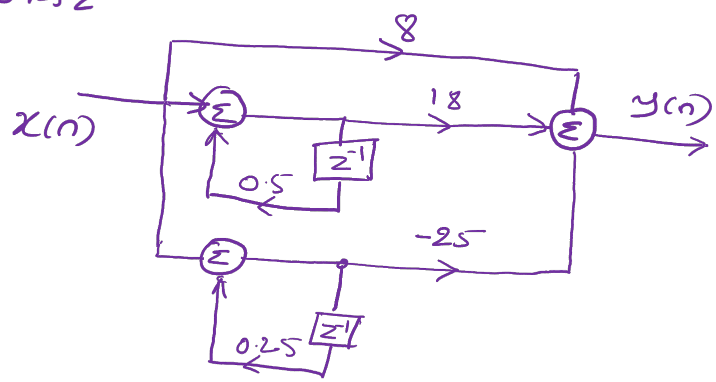
$$\therefore \; H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}} = 8 + \frac{-7+8z^{-1}}{1-0.75z^{-1}+0.125z^{-2}}$$

$$\frac{-7+8z^{-1}}{1-0.75z^{-1}+0.125z^{-2}} = \frac{A}{(1-0.5z^{-1})} + \frac{B}{(1-0.25z^{-1})}$$

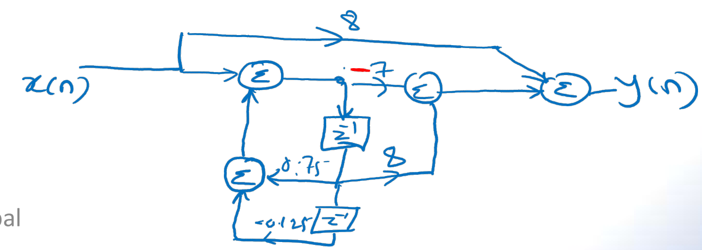
$$\therefore \; (-7+8z^{-1}) = A(1-0.25z^{-1}) + B(1-0.5z^{-1})$$

$A = 18, \; B = -25$

$$H(z) = 8 + \frac{18}{1-0.5z^{-1}} - \frac{25}{1-0.25z^{-1}}$$

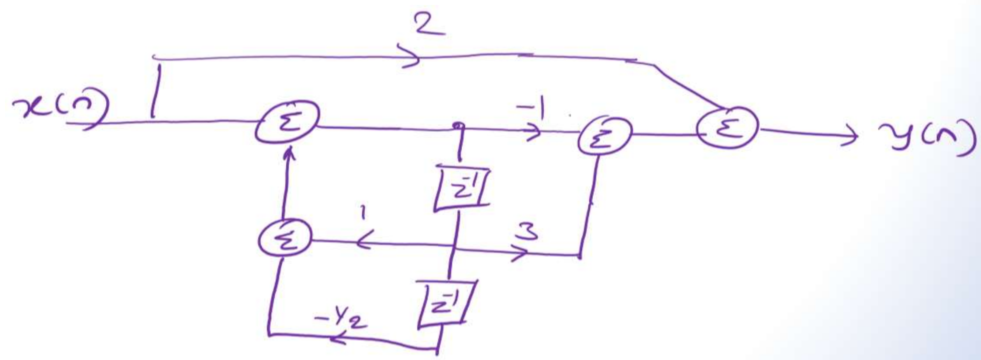


OR



Ex: Realize the given function in parallel form : $H(z) = \dfrac{1+z^{-1}+z^{-2}}{1-z^{-1}+\frac{1}{2}z^{-2}}$

$$H(z) = 2 + \frac{-1+3z^{-1}}{1-z^{-1}+\frac{1}{2}z^{-2}}$$



H.W.

$$H(z) = \frac{1+\frac{1}{4}z^{-1}}{\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}\right)} = \frac{A}{\left(1+\frac{1}{2}x\right)} + \frac{Bx+C}{\left(1+\frac{1}{2}x+\frac{1}{4}x^{2}\right)}$$


Thank you