FERMI ENERGY

EXAMPLE: Assume the Fermi energy level is 0.30 eV below the conduction band energy E_c. Assume $T = 300$ K. (a) Determine the probability of a state being occupied by an electron at $E = E_c + kT/4$. (b) Repeat part (a) for an energy state at $E = E_c + kT$.

[Ans. (a) 7.26×10^{-6}; (b) 3.43×10^{-6}]

EXAMPLE:

The Fermi energy at T = 300 K is 7.0 eV.

(a) Find the probability of an energy level at 7.15 eV being occupied by an electron.
(b) Repeat part (*a*) for T =1000 K.
(c) Repeat part (*a*) for E = 6.85 eV and T = 300 K.
(d) Determine the probability of the energy state at $E = EF$ being occupied at T =300 K and at T = 1000 K.

[Ans. a) 0.304% , b) 14.96% , c) 99.7% , d) 0.5]

FERMI ENERGY

Assume that the Fermi energy level is 0.35 eV above the valence band energy. Let $T = 300$ K. (a) Determine the probability of a state being empty of an electron at $E = E_v - kT/2$. (b) Repeat part (a) for an energy state at $E = E_v - 3kT/2$.

[Ans. (a) 8.20×10^{-7}; (b) 3.02×10^{-7}]

FERMI ENERGY

Assume that the Fermi energy level for a particular material is 6.25eV. Electrons in this material follow the Fermi-Dirac distribution function. Calculate the temperature at which there is 1% probability that a state 0.30eV below the Fermi energy level will not contain an electron. $\{k=8.6173303\times 10^{-5}\ eVK^{-1}\}$

The probability that a state is empty is

$$1 - f_F(E) = 1 - \frac{1}{1 + \exp(E - E_F/k.T)}$$

$$0.01 = 1 - \frac{1}{1 + \exp(5.95 - 6.25/kT)}$$

Solving for kT we find $kT = 0.06529$eV, so that the temperature is $T = 756\ K$.

The Fermi probability function is a strong function of temperature.

FERMI ENERGY

EXAMPLE Assume that E_F is 0.3 eV below E_c. Determine the temperature at which the probability of an electron occupying an energy state at $E = (E_c + 0.025)$ eV is 8×10^{-6}.

(Ans. $T = 321$ K)

- For an intrinsic semiconductor, the concentration of electrons in CB is equal to the concentration of holes in the VB. [$n_0=p_0$]

- The Fermi energy level for the intrinsic semiconductor is called the intrinsic Fermi energy, or $E_F=E_{Fi}$

- The concentration of electrons in the conduction band is the effective density of states (Nc) times the probability of occupancy at Ec.

- Similarly, the concentration of holes in the valence band is the effective density of states (Nv)) times the probability of non occupancy of electron at Ev

$$n_0 = Nc \times f(Ec)$$

$$f(\mathrm{Ec}) = \frac{1}{1+\exp\left[\frac{(E_C-E_{Fi})}{kT}\right]}$$

$$f(\mathrm{Ec}) = \exp\left[\frac{-(E_C-E_{Fi})}{kT}\right]$$

$$n_0 = N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right]$$

$$p_0 = Nv \times \{1- f(Ev)\}$$

$$f(Ev) = 1- \frac{1}{1+\exp\left[\frac{(Ev-E_{Fi})}{kT}\right]}$$

$$f(Ev) = \exp\left[\frac{-(E_{Fi}-E_v)}{kT}\right]$$

$$p_0 = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

Taking product of the above two equations

$$n_0 p_0 = n_i^{\,2} = N_C N_v \exp\left[\frac{-(E_C - E_v)}{kT}\right] = N_C N_v \exp\left[\frac{-Eg}{kT}\right]$$

where Eg is the band gap energy. For a given semiconductor material at a constant temperature, the value of n_i is a constant, and independent of the Fermi energy.

- N_C and N_v are constant for a given material (effective mass) and temperature.

- Position of Fermi energy is important
  - If E_F is closer to E_C than to E_v, $n > p$
  - If E_F is closer to E_v than to E_C, $n < p$

Effective density of states function and density of states effective mass values

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

Commonly accepted values of n_i at $T = 300$ K

Silicon	$n_i = 1.5 \times 10^{10}$ cm^{-3}
Gallium arsenide	$n_i = 1.8 \times 10^{6}$ cm^{-3}
Germanium	$n_i = 2.4 \times 10^{13}$ cm^{-3}

EXAMPLE:To calculate the intrinsic carrier concentration in Gallium arsenide at T=*300K* and at *T = 450K.*

The values of N_C and N_V, at 300 K for gallium arsenide are 4.7 x 10^{17} cm^{-3} and 7.0 x $10^{18}$$cm^{-3}$, respectively.

Assume the band gap energy of gallium arsenide is 1.42eV and does not vary with temperature (over this range).

$$For\ T = 300K$$

$$n_i^2 = (4.7\times10^{17})(7.0\times10^{18})\exp\left[\frac{-1.42}{0.0259}\right]$$

$$n_i^2 = 5.09\times10^{12}$$

$$n_i = 2.26\times10^{6}\,cm^{-3}$$

$$For\ T = 450K$$

$$n_i^2 = (4.7\times10^{17})(7.0\times10^{18})\left(\frac{450}{300}\right)^3\exp\left[\frac{-1.42}{0.0388}\right]$$

$$n_i^2 = 1.48\times10^{21}$$

$$n_i = 3.85\times10^{10}\,cm^{-3}$$

The intrinsic carrier concentration is a very strong function of temperature.

EQUILIBRIUM DISTRIBUTION OF ELECTRONS AND HOLES

Objective: Calculate the intrinsic carrier concentration in silicon at $T = 250$ K and at $T = 400$ K.

The values of N_c and N_v for silicon at $T = 300$ K are 2.8×10^{19} cm^{-3} and 1.04×10^{19} cm^{-3}, respectively. Both N_c and N_v vary as $T^{3/2}$. Assume the bandgap energy of silicon is 1.12 eV and does not vary over this temperature range.

■ Solution

Using Equation (4.23), we find, at $T = 250$ K

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{250}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(250/300)}\right]$$

$$= 4.90 \times 10^{15}$$

or

$$n_i = 7.0 \times 10^7 \text{ cm}^{-3}$$

At $T = 400$ K, we find

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{400}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(400/300)}\right]$$

$$= 5.67 \times 10^{24}$$

or

$$n_i = 2.38 \times 10^{12} \text{ cm}^{-3}$$

■ Comment

We may note from this example that the intrinsic carrier concentration increased by over 4 orders of magnitude as the temperature increased by 150°C.

THE INTRINSIC FERMI LEVEL

We have qualitatively argued that the Fermi energy level is located near the center of the forbidden band gap for the intrinsic semiconductor.

Since the electron and hole concentrations are equal,

$$n_0 = p_0$$

$$N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right] = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

$$E_{Fi} = \tfrac{1}{2}(E_C + E_v) + \tfrac{3}{4}kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$We\ define\ \ E_{midgap} = \tfrac{1}{2}(E_C + E_v)$$

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \qquad N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

THE INTRINSIC FERMI LEVEL

$$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

If the electron and hole effective masses are equal so that then the intrinsic Fermi level is exactly in the center of the band gap.

If $m_p^* > m_n^*$ the intrinsic Fermi level is slightly above the center.
If $m_p^* < m_n^*$ the intrinsic Fermi level is slightly below the center.

THE INTRINSIC FERMI LEVEL

EXAMPLE: To calculate the position of the intrinsic Fermi level with respect to the center of the bandgap in silicon at T = *300 K* .

The density of states effective carrier masses in silicon are $m_n^* = 1.08m_0$ *and* $m_p^* = 0.56m_0$.

$$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4}(0.0259)\ln\left(\frac{0.56}{1.08}\right)$$

$$E_{Fi} - E_{midgap} = -0.0128eV$$

THE INTRINSIC FERMI LEVEL

Determine the position of the intrinsic Fermi level at $T = 300$ K with respect to the center of the bandgap for (*a*) GaAs and (*b*) Ge.

[Ans. (*a*) +38.25 meV; (*b*) −7.70 meV]

Determine the position of the intrinsic Fermi level with respect to the center of the bandgap in silicon at (*a*) $T = 200$ K and (*b*) $T = 400$ K. Assume the effective masses are constant over this temperature range.

[Ans. (*a*) −8.505 meV; (*b*) −17.01meV]

$$or \quad n_0 = N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right] \exp\left[\frac{(E_F - E_{Fi})}{kT}\right]$$

The intrinsic carrier concentration is given as

$$n_i = N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right]$$

$$\Rightarrow n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$Similarly \quad p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

$$\boxed{n_0 p_0 = n_i^{\,2}}$$

The product of n_0 and p_0 is always a constant for a given semiconductor material at a given temperature.

$$n_o + N_a = p_o + N_d$$

From $n_{o*}p_o = n_i^2$

$$n_o + N_a = \frac{n_i^2}{n_0} + N_d$$

$$\boxed{n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}}$$

- Electron concentration is given as function of donors and acceptors concentrations.
- Although above equation was derived for a compensated semiconductor, the equation is also valid for $Na=0$.
- Similarly

$$\boxed{p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}}$$

CHARGE NEUTRALITY

EXAMPLE : Determine the thermal-equilibrium electron and hole concentrations in silicon at T =300 K for given doping concentrations.

(a) Let $N_d = 10^{16}$ cm^{-3} and $N_a = 0$. (b) Let $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{15}$ cm^{-3}. Use $n_i = 1.5 \times 10^{10}$ cm^{-3} in silicon at T = 300 K.

a).

- $Electrons$

$$n_o = \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right)\left(\frac{10^{16}}{2}\right) + (1.5 \times 10^{10})^2}$$

$$\approx 10^{16} cm^{-3}$$

- $Holes$

$$p_o = \frac{{n_i}^2}{n_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}}$$

$$= 2.25 \times 10^4 cm^{-3} \longrightarrow$$

Reduced below Intrinsic conc^n

b).

- $Electrons$

$$n_o = \frac{(5-2)\times 10^{15}}{2} + \sqrt{\left(\frac{((5-2)\times 10^{15}}{2}\right)\left(\frac{((5-2)\times 10^{15}}{2}\right) + (1.5\times 10^{10})^2}$$

$$= 3\times 10^{15} cm^{-3}$$

$Holes$

$$p_o = \frac{{n_i}^2}{n_0} = \frac{(1.5\times 10^{10})^2}{3\times 10^{15}}$$

$$= 7.5\times 10^4 cm^{-3}$$

Reduced below Intrinsic conc^n

CHARGE NEUTRALITY

EXAMPLE: Calculate the thermal-equilibrium electron and hole concentrations in a compensated p-type semiconductor.

Consider a silicon semiconductor at T =300 K in which $N_a=10^{16}$ cm^{-3} and $N_d=3\text{x}10^{15}$ cm^{-3}. Assume $n_i = 1.5 \text{ x}10^{10}$ cm^{-3}.

Since $Na > Nd$, the compensated semiconductor is p-type and the thermal-equilibrium majority carrier hole concentration is given by

$$p_0 = \frac{10^{16} - 3\times10^{15}}{2} + \sqrt{\left(\frac{10^{16} - 3\times10^{15}}{2}\right)^2 + (1.5\times10^{10})^2}$$

$$p_0 \approx 7\times10^{15}\,cm^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{7\times10^{15}} = 3.21\times10^4\,cm^{-3}$$

Find the thermal-equilibrium electron and hole concentrations in silicon with doping concentrations of $Nd = 7 \times 10^{15}$ cm^{-3} and $Na = 3 \times 10^{15}$ cm^{-3} for (*a*) T =250 K and (*b*) T= 400 K.

[Ans. (*a*) $n_0 = 4 \times 10^{15}$ cm^{-3}, $p_0 = 1.225$ cm^{-3}; (*b*) $n_0 = 4 \times 10^{15}$ cm^{-3}, $p_0 = 1.416 \times 10^{9}$ cm^{-3}]

Consider silicon at $T = 300$ K. Calculate the thermal-equilibrium electron and hole concentrations for impurity concentrations of (*a*) $N_a = 4 \times 10^{16}$ cm^{-3}, $N_d = 8 \times 10^{15}$ cm^{-3} and (*b*) $N_a = N_d = 3 \times 10^{15}$ cm^{-3}.

[Ans. (*a*) $p_0 = 3.2 \times 10^{16}$ cm^{-3}, $n_0 = 7.03 \times 10^{3}$ cm^{-3}; (*b*) $p_0 = n_0 = 1.5 \times 10^{10}$ cm^{-3}]

EXAMPLE: Find the thermal-equilibrium electron and hole concentrations in silicon with doping concentrations of $Nd = 7 \times 10^{15}$ cm^{-3} and $Na = 3 \times 10^{15}$ cm^{-3} for (*a*) T =250 K and (*b*) T= 400 K.

[Ans. (*a*) $n_0 = 4 \times 10^{15}$ cm^{-3}, $p_0 = 1.225$ cm^{-3}; (*b*) $n_0 = 4 \times 10^{15}$ cm^{-3}, $p_0 = 1.416 \times 10^9$ cm^{-3}]

Determine the position of the Fermi level with respect to the valence-band energy in p-type GaAs at $T = 300$ K. The doping concentrations are $N_a = 5 \times 10^{16}\ \text{cm}^{-3}$ and $N_d = 4 \times 10^{15}\ \text{cm}^{-3}$.

(Ans. $E_F - E_v = 0.130$ eV)

Calculate the position of the Fermi energy level in n-type silicon at $T = 300$ K with respect to the intrinsic Fermi energy level. The doping concentrations are $N_d = 2 \times 10^{17}\ \text{cm}^{-3}$ and $N_a = 3 \times 10^{16}\ \text{cm}^{-3}$.

(Ans. $E_F - E_{Fi} = 0.421$ eV)

Consider silicon at $T = 300$ K with doping concentrations of $N_d = 8 \times 10^{15}$ cm^{-3} and $N_a = 5 \times 10^{15}$ cm^{-3}. Determine the position of the Fermi energy level with respect to E_c.

(Ans. $E_c - E_F = 0.2368$ eV)

Silicon at $T = 300$ K contains an acceptor impurity concentration of $N_a = 10^{16}$ cm^{-3}. Determine the concentration of donor impurity atoms that must be added so that the silicon is n type and the Fermi energy is 0.20 eV below the conduction-band edge.

$$N_d = 2.24 \times 10^{16} \text{ cm}^{-3}$$

Consider germanium with an acceptor concentration of $N_a = 10^{15}$ cm^{-3} and a donor concentration of $N_d = 0$. Consider temperatures of $T = 200$, 400, and 600 K. Calculate the position of the Fermi energy with respect to the intrinsic Fermi level at these temperatures.

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

For Germanium:

$T(^\circ K)$	$kT(eV)$	$n_i\left(cm^{-3}\right)$
200	0.01727	$2.16x10^{10}$
400	0.03454	$8.6x10^{14}$
600	0.0518	$3.82x10^{16}$

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2} \text{ and } N_a = 10^{15}\ cm^{-3}$$

$T(^\circ K)$	$p_o\left(cm^{-3}\right)$	$E_{Fi} - E_F\ (eV)$
200	$1.0x10^{15}$	0.1855
400	$1.49x10^{15}$	0.01898
600	$3.87x10^{16}$	0.000674

Consider silicon at $T = 300$ K with donor concentrations of $N_d = 10^{14}$, 10^{15}, 10^{16}, and 10^{17}, cm^{-3}. Assume $N_a = 0$. (*a*) Calculate the position of the Fermi energy level with respect to the conduction band for these donor concentrations. (*b*) Determine the position of the Fermi energy level with respect to the intrinsic Fermi energy level for the donor concentrations given in part (*a*).

(a) $$E_c - E_F = kT \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{N_d}\right)$$

For $10^{14}\,\text{cm}^{-3}$, $E_c - E_F = 0.3249\,\text{eV}$

$10^{15}\,\text{cm}^{-3}$, $E_c - E_F = 0.2652\,\text{eV}$

$10^{16}\,\text{cm}^{-3}$, $E_c - E_F = 0.2056\,\text{eV}$

$10^{17}\,\text{cm}^{-3}$, $E_c - E_F = 0.1459\,\text{eV}$

(b) $$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{N_d}{1.5\times10^{10}}\right)$$

For $10^{14}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.2280\,\text{eV}$

$10^{15}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.2877\,\text{eV}$

$10^{16}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.3473\,\text{eV}$

$10^{17}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.4070\,\text{eV}$

Silicon at T = 300 K contains acceptor atoms at a concentration of Na= 5 x 10^{15} cm^{-3}). Donor atoms are added forming an n-type compensated semiconductor such that the Fermi level is 0.215 eV below the conduction band edge. What concentration of donor atoms are added?

$$n_o = N_d - N_a = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

so

$$N_d = 5\times10^{15} + \left(2.8\times10^{19}\right)\exp\left(\frac{-0.215}{0.0259}\right)$$

$$= 5\times10^{15} + 6.95\times10^{15}$$

or

$$N_d = 1.2\times10^{16}\ \mathrm{cm}^{-3}$$

(a)Determine the position of the Fermi level with respect to the intrinsic Fermi level in silicon at T = 300 K that is doped with phosphorus atoms at a concentration of 10^{15} cm^{-3}. *(b)*Repeat part *(a)*if the silicon is doped with boron atoms at a concentration of 10^{15} cm^{-3}? (c) Calculate the electron concentration in the silicon for parts (a) and (b)

(a) $$E_F - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right) = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right)$$

or

$$\underline{E_F - E_{Fi} = 0.2877\ eV}$$

(b)

$$E_{Fi} - E_F = kT\ln\left(\frac{N_a}{n_i}\right) = 0.2877\ eV$$

(c)

For (a), $\underline{n_O = N_d = 10^{15}\ cm^{-3}}$

For (b)

$$n_O = \frac{n_i^2}{p_O} = \frac{\left(1.5x10^{10}\right)^2}{10^{15}} \Rightarrow$$

$$\underline{n_O = 2.25x10^5\ cm^{-3}}$$

Assume that the Fermi energy level is exactly in the center of the bandgap energy of a semiconductor at T= 300 K. Calculate the probability that an energy state in the bottom of the conduction band is occupied by an electron for Si, Ge, and GaAs.

$$E_F = E_{midgap}$$

$$E = E_C$$

$$E - E_F = E_C - E_{midgap} = E_g/2$$

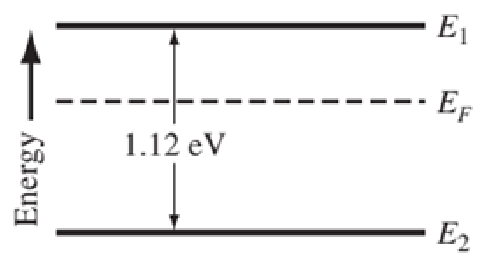
$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_g}{2kT}\right)}$$

For Silicon $E_g = 1.12eV; f(E) = 4.07 \times 10^{-10}$

For Germanium $E_g = 0.66eV; f(E) = 2.93 \times 10^{-6}$

For GaAs $E_g = 1.42eV; f(E) = 1.24 \times 10^{-12}$

Consider the energy levels shown in the figure below. Let T=300 K. Determine the probability that an energy state at $E = E_1$ is occupied by an electron and the probability that an energy state at $E=E_2$ is empty. If (i) $E_1 - E_F = 0.30$ eV (ii) $E_F - E_2 = 0.40$ eV.



(a) For $E = E_1$

$$f(E) = \frac{1}{1+\exp\left(\frac{E_1 - E_F}{kT}\right)} \cong \exp\left[\frac{-(E_1 - E_F)}{kT}\right]$$

Then

$$f(E_1) = \exp\left(\frac{-0.30}{0.0259}\right) = 9.32\times10^{-6}$$

For $E = E_2$, $E_F - E_2 = 1.12 - 0.30 = 0.82$ eV

Then

$$1 - f(E) = 1 - \frac{1}{1+\exp\left(\frac{-0.82}{0.0259}\right)}$$

or

$$1 - f(E) \cong 1 - \left[1 - \exp\left(\frac{-0.82}{0.0259}\right)\right]$$

$$= \exp\left(\frac{-0.82}{0.0259}\right) = 1.78\times10^{-14}$$

(b) For $E_F - E_2 = 0.4$ eV,

$$E_1 - E_F = 0.72 \text{ eV}$$

At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-0.72}{0.0259}\right)$$

or

$$f(E) = 8.45\times10^{-13}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right]$$

$$= \exp\left(\frac{-0.4}{0.0259}\right)$$

or

$$1 - f(E) = 1.96\times10^{-7}$$

(a) Silicon at $T = 300$ K is doped with donor impurity atoms at a concentration of $N_d = 6 \times 10^{15}$ cm^{-3}. (i) Determine $E_c - E_F$. (ii) Calculate the concentration of additional donor impurity atoms that must be added to move the Fermi energy level a distance kT closer to the conduction band edge. (b) Repeat part (a) for GaAs if the original donor impurity concentration is $N_d = 1 \times 10^{15}$ cm^{-3}.

(a) Silicon

(i) $E_c - E_F = kT \ln\left(\frac{N_c}{N_d}\right)$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{6\times10^{15}}\right) = 0.2188 \text{ eV}$$

(ii) $E_c - E_F = 0.2188 - 0.0259 = 0.1929$ eV

$$N_d = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= (2.8\times10^{19})\exp\left[\frac{-0.1929}{0.0259}\right]$$

$$N_d = 1.631\times10^{16} \text{ cm}^{-3} = N_d' + 6\times10^{15}$$

$$\Rightarrow N_d' = 1.031\times10^{16} \text{ cm}^{-3} \text{ Additional donor atoms}$$

(b) GaAs

(i) $E_c - E_F = (0.0259)\ln\left(\frac{4.7\times10^{17}}{10^{15}}\right)$

$$= 0.15936 \text{ eV}$$

(ii) $E_c - E_F = 0.15936 - 0.0259 = 0.13346$ eV

$$N_d = (4.7\times10^{17})\exp\left[\frac{-0.13346}{0.0259}\right]$$

$$= 2.718\times10^{15} \text{ cm}^{-3} = N_d' + 10^{15}$$

$$\Rightarrow N_d' = 1.718\times10^{15} \text{ cm}^{-3} \text{ Additional donor atoms}$$

For a particular semiconductor, $E_g = 1.50$ eV, $m_p^* = 10\, m_n^*$, $T = 300$ K, and $n_i = 1 \times 10^5$ cm^{-3}. (*a*) Determine the position of the intrinsic Fermi energy level with respect to the center of the bandgap. (*b*) Impurity atoms are added so that the Fermi energy level is 0.45 eV below the center of the bandgap. (*i*) Are acceptor or donor atoms added? (*ii*) What is the concentration of impurity atoms added?

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4}(0.0259)\ln(10)$$

or

$$E_{Fi} - E_{midgap} = +0.0447 \text{ eV}$$

(b) Impurity atoms to be added so

$$E_{midgap} - E_F = 0.45 \text{ eV}$$

(i) p-type, so add acceptor atoms

(ii) $E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947$ eV

$$p_o = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

$$= (10^5)\exp\left(\frac{0.4947}{0.0259}\right)$$

or

$$p_o = N_a = 1.97 \times 10^{13} \text{ cm}^{-3}$$

A new semiconductor material is to be "designed." The semiconductor is to be p type and doped with 5×10^{15} cm^{-3} acceptor atoms. Assume complete ionization and assume $N_d = 0$. The effective density of states functions are $N_c = 1.2 \times 10^{19}$ cm^{-3} and $N_v = 1.8 \times 10^{19}$ cm^{-3} at $T = 300$ K and vary as T^2. A special semiconductor device fabricated with this material requires that the hole concentration be no greater than 5.08×10^{15} cm^{-3} at $T = 350$ K. What is the minimum bandgap energy required in this new material?

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$5.08 \times 10^{15} = \frac{5 \times 10^{15}}{2} + \sqrt{\left(\frac{5 \times 10^{15}}{2}\right)^2 + n_i^2}$$

$$\left(5.08 \times 10^{15} - 2.5 \times 10^{15}\right)^2 = \left(2.5 \times 10^{15}\right)^2 + n_i^2$$

$$6.6564 \times 10^{30} = 6.25 \times 10^{30} + n_i^2$$

$$\Rightarrow n_i^2 = 4.064 \times 10^{29}$$

$$n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

$$kT = (0.0259)\left(\frac{350}{300}\right) = 0.030217 \text{ eV}$$

$$N_c = \left(1.2 \times 10^{19}\right)\left(\frac{350}{300}\right)^2 = 1.633 \times 10^{19} \text{ cm}^{-3}$$

$$N_v = \left(1.8 \times 10^{19}\right)\left(\frac{350}{300}\right)^2 = 2.45 \times 10^{19} \text{ cm}^{-3}$$

$$4.064 \times 10^{29} = \left(1.633 \times 10^{19}\right)\left(2.45 \times 10^{19}\right) \times \exp\left[\frac{-E_g}{0.030217}\right]$$

So

$$E_g = (0.030217) \ln\left[\frac{\left(1.633 \times 10^{19}\right)\left(2.45 \times 10^{19}\right)}{4.064 \times 10^{29}}\right]$$

$$\Rightarrow E_g = 0.6257 \text{ eV}$$

Silicon at $T = 300$ K is doped with boron atoms such that the concentration of holes is $p_0 = 5 \times 10^{15}$ cm^{-3}. (*a*) Find $E_F - E_v$. (*b*) Determine $E_c - E_F$. (*c*) Determine n_0. (*d*) Which carrier is the majority carrier? (*e*) Determine $E_{Fi} - E_F$.

(a) $$E_F - E_\upsilon = kT \ln\left(\frac{N_\upsilon}{p_o}\right)$$
$$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{5\times10^{15}}\right)$$
$$= 0.1979 \text{ eV}$$

(b) $$E_c - E_F = E_g - (E_F - E_\upsilon)$$
$$= 1.12 - 0.19788 = 0.92212 \text{ eV}$$

(c) $$n_o = (2.8\times10^{19})\exp\left[\frac{-0.92212}{0.0259}\right]$$
$$= 9.66\times10^3 \text{ cm}^{-3}$$

(d) Holes

(e) $$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$
$$= (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$$
$$= 0.3294 \text{ eV}$$

Two semiconductor materials have exactly the same properties except material A has a bandgap energy of 0.90 eV and material B has a bandgap energy of 1.10 eV. Determine the ratio of n_i of material B to that of material A for (*a*) $T = 200$ K, (*b*) $T = 300$ K, and (*c*) $T = 400$ K.

$$\frac{n_i(B)}{n_i(A)} = \frac{exp\left(\frac{-1.10}{2kT}\right)}{exp\left(\frac{-0.90}{2kT}\right)} = exp\left(\frac{-0.10}{kT}\right)$$

For $T = 200\,\text{K}, \quad kT = 0.017267$ eV

For $T = 300\,\text{K}, \quad kT = 0.0259$ eV

For $T = 400\,\text{K}, \quad kT = 0.034533$ eV

For T=200K

$$\frac{n_i(B)}{n_i(A)} = exp\left(\frac{-0.10}{0.017267}\right) = 3.05 \times 10^{-3}$$

For T= 300K

$$\frac{n_i(B)}{n_i(A)} = exp\left(\frac{-0.10}{0.0259}\right) = 0.02104$$

For T= 400K

$$\frac{n_i(B)}{n_i(A)} = exp\left(\frac{-0.10}{0.034533}\right) = 0.05526$$

In a particular semiconductor material, the effective density of states functions are given by $N_c = N_{c0} \cdot (T/300)^{3/2}$ and $N_v = N_{v0} \cdot (T/300)^{3/2}$ where N_{c0} and N_{v0} are constants independent of temperature. Experimentally determined intrinsic carrier concentrations are found to be $n_i = 1.40 \times 10^2$ cm^{-3} at $T = 200$ K and $n_i = 7.70 \times 10^{10}$ cm^{-2} at $T = 400$ K. Determine the product $N_{c0} \cdot N_{v0}$ and the bandgap energy E_g. (Assume E_g is constant over this temperature range.)

$$\text{At } T = 200\,\text{K},\ kT = (0.0259)\left(\frac{200}{300}\right) = 0.017267\ \text{eV}$$

$$\text{At } T = 400\,\text{K},\ kT = (0.0259)\left(\frac{400}{300}\right) = 0.034533\ \text{eV}$$

$$\frac{n_i^2(400)}{n_i^2(200)} = \frac{\left(7.70\times10^{10}\right)^2}{\left(1.40\times10^{2}\right)^2} = 3.025\times10^{17}$$

$$= \frac{\left(\frac{400}{300}\right)^3}{\left(\frac{200}{300}\right)^3} \times \frac{\exp\left[\frac{-E_g}{0.034533}\right]}{\exp\left[\frac{-E_g}{0.017267}\right]}$$

$$= 8\exp\left[\frac{E_g}{0.017267} - \frac{E_g}{0.034533}\right]$$

$$3.025\times10^{17} = 8\exp\left[E_g(57.9139 - 28.9578)\right]$$

$$E_g(28.9561) = \ln\left(\frac{3.025\times10^{17}}{8}\right) = 38.1714$$

or $E_g = 1.318$ eV

Now

$$\left(7.70\times10^{10}\right)^2 = N_{co}N_{\upsilon o}\left(\frac{400}{300}\right)^3 \times \exp\left(\frac{-1.318}{0.034533}\right)$$

$$5.929\times10^{21} = N_{co}N_{\upsilon o}(2.370)\left(2.658\times10^{-17}\right)$$

so $N_{co}N_{\upsilon o} = 9.41\times10^{37}$ cm^{-6}