FISAC-2
Engineering Mathematics-IV (MAT 2227)

Answer all the questions.

1. There are 2 white marbles in box A and 3 red marbles in box B. At each step of the process a marble is selected from each box and the two marbles selected are interchanged. Let X_i be the number of red marbles in the box A. (a) Find the transition matrix P . (b) What is the probability that there are 2 red marbles in box A after 3 steps (c) In the long run, what is the probability that there are 2 red marbles in box A.

2. Use simplex method to solve: Max $Z = 4x_1 - x_2$ subject to $x_1 + 2x_2 \leq 4, 2x_1 + 3x_2 \leq 12, x_1 - x_2 \leq 3, x_1, x_2 \geq 0$.

3. Determine a minimum of $f(x) = 9x_1^2 + x_2^2 + 18x_1 - 4x_2$ starting from $x_0 = (2, 4)$.

4. Find the directional derivative of the function xyz along the direction of the normal to the surface $xy^2 + yz^2 - 2zx^2 = 4$ at the point (1,1,-1). Also find the equation of the tangent plane and the normal line to this surface.

5. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,-1,2).

6. If $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$, then prove that $\nabla \cdot (r^n \vec{r}) = (n + 3)r^n$.

7. Prove that $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$.