

# Carrier Transport Phenomena

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## DRIFT CURRENT DENSITY

Drift applicable to electrons can be expressed as:

$$J_{drf} = \rho v_d \quad J_{n|drf} = v_{dn} (-en)$$

where  $J_{n|drf}$  is the drift current density due to electrons and  $v_{dn}$  is the average drift velocity of electrons. The net charge density of electrons is negative.

Since the electron is negatively charged, the net motion of the electron is opposite to the electric field direction.

$$v_{dn} = -\mu_n E$$
$$J_{n|drf} = (-en)(-\mu_n E) = e\mu_n nE \quad \text{same direction as the applied electric field}$$

The conventional drift current due to electrons is also in the same direction as the applied electric field even though the electron movement is in the opposite direction.

Similarly

$$J_{p|drf} = (ep)v_{dp} = e\mu_p pE$$

Since both electrons and holes contribute to the drift current, the total *drift current density* is the sum of the individual electron and hole drift current densities, so we may write

$$J_{drf} = e(\mu_n n + \mu_p p)E$$

## DRIFT CURRENT DENSITY

Consider a gallium arsenide sample at 300 K with doping concentrations of  $N_a = 0$  and  $N_d = 10^{16} \text{ cm}^{-3}$ . Assume complete ionization and assume electron and hole mobilities given. Calculate the drift current density if the applied electric field is  $E = 10 \text{ V/cm}$ .

Typical mobility values at  $T = 300 \text{ K}$  and low doping concentrations

	$\mu_n (\text{cm}^2/\text{V-s})$	$\mu_p (\text{cm}^2/\text{V-s})$
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

$$n = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \approx 10^{16} \text{ cm}^{-3} \quad p = \frac{n^2}{n} = \frac{(1.8 \times 10^6)^2}{10^{16}} = 3.24 \times 10^{-4} \text{ cm}^{-3}$$

$$J_{n|drf} = e\mu_n n E$$

$$J_{p|drf} = e\mu_p p E$$

$$J_{drf} = e(\mu_n n + \mu_p p)E$$

Significant drift current densities can be obtained in a semiconductor applying relatively small electric fields. We may note from this example that the drift current will usually be due primarily to the majority carrier in an extrinsic semiconductor.

A drift current density of  $J_{drf} = 75 \text{ A/cm}^2$  is required in a device using p-type silicon when an electric field of  $E = 120 \text{ V/cm}$  is applied. Determine the required impurity doping concentration to achieve this specification.

$$(\text{Ans. } N_a = 8.14 \times 10^{15} \text{ cm}^{-3})$$

Consider a sample of silicon at  $T = 300 \text{ K}$  doped at an impurity concentration of  $N_d = 10^{15} \text{ cm}^{-3}$  and  $N_a = 10^{14} \text{ cm}^{-3}$ . Calculate the drift current density if the applied electric field is  $E = 35 \text{ V/cm}$ .

$$(\text{Ans. } 6.80 \text{ A/cm}^2)$$

## CONDUCTIVITY

The drift current density, may be written as

$$J_{drf} = e(\mu_n n + \mu_p p)E = \sigma E$$

where  $\sigma$  is the *conductivity* of the semiconductor material. The conductivity is given in units of  $(\text{ohm}\cdot\text{cm})^{-1}$  and is a function of the electron and hole concentrations and mobilities.

For intrinsic material, the conductivity can be written as  $\sigma_i = e(\mu_n + \mu_p)n_i$

The reciprocal of conductivity is *resistivity*, which is denoted by  $\rho$  and is given in units of **ohm-cm**. We can write the formula for resistivity as

$$\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}$$

The concentration of donor impurity atoms in silicon is  $N_d = 10^{15} \text{ cm}^{-3}$ . Assume an electron mobility of  $\mu_n = 1300 \text{ cm}^2/\text{V-s}$  and a hole mobility of  $\mu_p = 450 \text{ cm}^2/\text{V-s}$ .

(a) Calculate the resistivity of the material. (b) What is the conductivity of the material?

$$\rho = \frac{1}{e\mu_n N_d} = \frac{1}{(1.6 \times 10^{-19})(1300)(10^{15})} = 4.808 \Omega \cdot \text{cm}$$
$$\sigma = \frac{1}{\rho} = \frac{1}{4.8077} = 0.208 (\Omega \cdot \text{cm})^{-1}$$

A p-type silicon material is to have a conductivity of  $\sigma = 1.80 (\Omega \cdot \text{cm})^{-1}$ . If the mobility values are  $\mu_n = 1250 \text{ cm}^2/\text{V-s}$  and  $\mu_p = 380 \text{ cm}^2/\text{V-s}$ , what must be the acceptor impurity concentration in the material?

$$\sigma = e\mu_p N_a$$
$$N_a = \frac{\sigma}{e\mu_p} = \frac{1.80}{(1.6 \times 10^{-19})(380)} = 2.96 \times 10^{16} \text{ cm}^{-3}$$

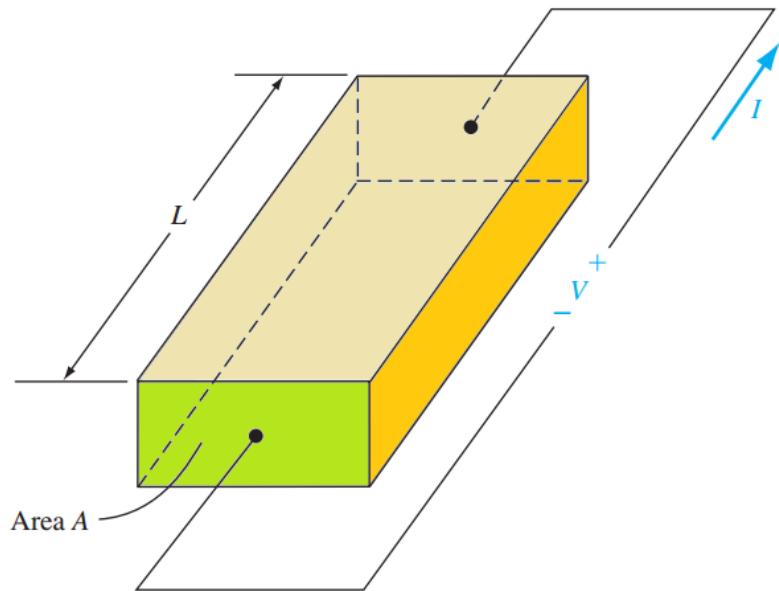
## CONDUCTIVITY

If we have a bar of semiconductor material as shown in Figure with a voltage applied that produces a current  $I$ , then we can write

$$J = \frac{I}{A} \quad \text{and} \quad E = \frac{V}{L}$$

$$J = \sigma E \Rightarrow \frac{I}{A} = \sigma \left( \frac{V}{L} \right)$$

$$V = \left( \frac{L}{\sigma A} \right) I = \left( \frac{\rho L}{A} \right) I = IR$$



Above equation is Ohm's law for a semiconductor. The resistance is a function of resistivity, or conductivity, as well as the geometry of the semiconductor.

## CONDUCTIVITY

A silicon semiconductor at 300 K is initially doped with donors at a concentration of  $N_d = 5 \times 10^{15} \text{ cm}^{-3}$ . The resistor is to have a resistance of 10K Ohm and handle a current density of 50 A/cm<sup>2</sup> when 5V is applied. E= 100V/cm

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$$I = \frac{V}{R} = \frac{5}{10} = 0.5 \text{ mA}$$

If the current density is limited to 50 A/cm<sup>2</sup>, then the cross-sectional area is

$$A = \frac{I}{J} = \frac{0.5 \times 10^{-3}}{50} = 10^{-5} \text{ cm}^2$$

Electric field = 100 V/cm, then the length of the resistor is

$$L = \frac{V}{E} = \frac{5}{100} = 5 \times 10^{-2} \text{ cm}$$

In a particular semiconductor material,  $\mu_n = 1000 \text{ cm}^2/\text{V-s}$ ,  $\mu_p = 600 \text{ cm}^2/\text{V-s}$ , and  $N_C = N_V = 10^{19} \text{ cm}^{-3}$ . These parameters are independent of temperature. The measured conductivity of the intrinsic material is  $\sigma = 10^{-6} (\Omega\text{-cm})^{-1}$  at  $T = 300 \text{ K}$ . Find the conductivity at  $T = 500 \text{ K}$ .

$$\sigma_i = en_i(\mu_n + \mu_p)$$

Then

$$10^{-6} = (1.6 \times 10^{-19})(1000 + 600)n_i$$

or

$$n_i(300 \text{ K}) = 3.91 \times 10^9 \text{ cm}^{-3}$$

Now

$$n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$

or

$$\begin{aligned} E_g &= kT \ln\left(\frac{N_c N_v}{n_i^2}\right) \\ &= (0.0259) \ln\left[\frac{(10^{19})^2}{(3.91 \times 10^9)^2}\right] \end{aligned}$$

which gives

$$E_g = 1.122 \text{ eV}$$

$$\begin{aligned} n_i^2(500 \text{ K}) &= (10^{19})^2 \exp\left[\frac{-1.122}{(0.0259)(500/300)}\right] \\ &= 5.15 \times 10^{26} \end{aligned}$$

or

$$n_i(500 \text{ K}) = 2.27 \times 10^{13} \text{ cm}^{-3}$$

Then

$$\sigma_i = (1.6 \times 10^{-19})(2.27 \times 10^{13})(1000 + 600)$$

which gives

$$\sigma_i(500 \text{ K}) = 5.81 \times 10^{-3} (\Omega\text{-cm})^{-1}$$

(a) Calculate the conductivity at  $T = 300$  K of intrinsic (i) silicon, (ii) germanium, and (iii) gallium arsenide. (b) If rectangular semiconductor bars are fabricated using the materials in part (a), determine the resistance of each bar if its cross-sectional area is  $85 \mu\text{m}^2$  and length is  $200 \mu\text{m}$ .

(a) (i) Silicon:  $\sigma_i = en_i(\mu_n + \mu_p)$

$$\sigma_i = (1.6 \times 10^{-19})(1.5 \times 10^{10})(1350 + 480)$$

or

$$\sigma_i = 4.39 \times 10^{-6} (\Omega \cdot \text{cm})^{-1}$$

(ii) Ge:

$$\sigma_i = (1.6 \times 10^{-19})(2.4 \times 10^{13})(3900 + 1900)$$

or

$$\sigma_i = 2.23 \times 10^{-2} (\Omega \cdot \text{cm})^{-1}$$

(iii) GaAs:

$$\sigma_i = (1.6 \times 10^{-19})(1.8 \times 10^6)(8500 + 400)$$

or

$$\sigma_i = 2.56 \times 10^{-9} (\Omega \cdot \text{cm})^{-1}$$

(b)  $R = \frac{L}{\sigma A}$

(i) Si:

$$R = \frac{200 \times 10^{-4}}{(4.39 \times 10^{-6})(85 \times 10^{-8})} = 5.36 \times 10^9 \Omega$$

(ii) Ge:

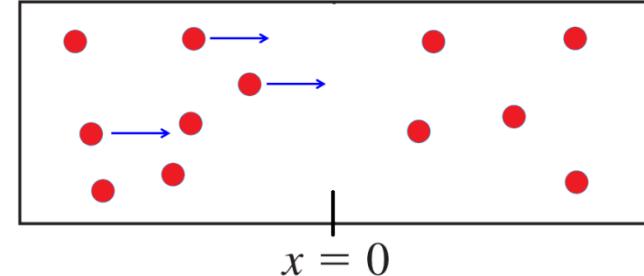
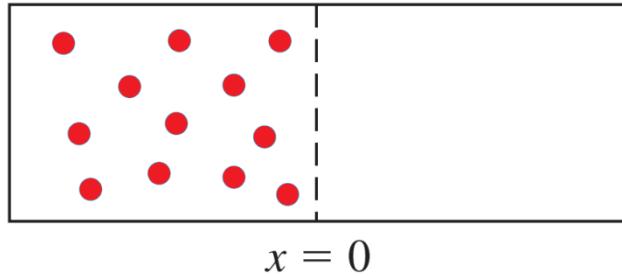
$$R = \frac{200 \times 10^{-4}}{(2.23 \times 10^{-2})(85 \times 10^{-8})} = 1.06 \times 10^6 \Omega$$

(iii) GaAs:

$$R = \frac{200 \times 10^{-4}}{(2.56 \times 10^{-9})(85 \times 10^{-8})} = 9.19 \times 10^{12} \Omega$$

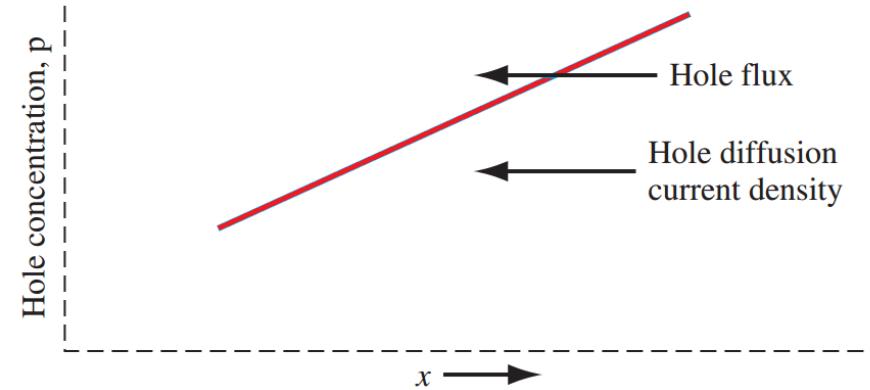
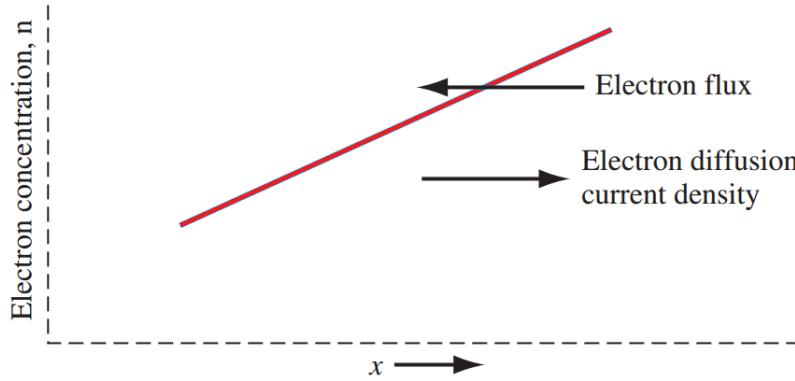
## CARRIER DIFFUSION

- There is a second mechanism, in addition to drift, that can induce a current in a semiconductor.
- We may consider a classic physics example in which a container, as shown in Figure is divided into two parts by a membrane.
- The left side contains gas molecules at a particular temperature and the right side is initially empty.
- The gas molecules are in continual random thermal motion so that, when the membrane is broken, the gas molecules flow into the right side of the container.



*Diffusion* is the process whereby particles flow from a region of high concentration toward a region of low concentration. If the gas molecules were electrically charged, the net flow of charge would result in a *diffusion current*.

## DIFFUSION CURRENT DENSITY



$$J_{nx|dif} = eD_n \frac{dn}{dx}$$

$D_n$  is called the *electron diffusion coefficient*, has units of  $\text{cm}^2/\text{s}$ , and is a +ve quantity.

The hole diffusion current density is proportional to the hole density gradient and to the electronic charge, so we may write

$$J_{px|dif} = -eD_p \frac{dp}{dx}$$

$D_p$  is called the *hole diffusion coefficient*, has units of  $\text{cm}^2/\text{s}$ , and is a positive quantity.

## DIFFUSION CURRENT DENSITY

**EXAMPLE:** Calculate the diffusion current density given a density gradient. Assume that, in an n-type gallium arsenide semiconductor at 300 K, the electron concentration varies linearly from  $1 \times 10^{18}$  to  $7 \times 10^{17} \text{ cm}^{-3}$  over a distance of 0.10 cm. Calculate the diffusion current density if  $D_n = 225 \text{ cm}^2/\text{s}$

$$J_{nx|dif} = eD_n \frac{dn}{dx} \approx eD_n \frac{\Delta n}{\Delta x}$$

$$J_{nx|dif} = (1.6 \times 10^{-19})(225) \left( \frac{1 \times 10^{18} - 7 \times 10^{17}}{0.10} \right) = 108 \text{ A/cm}^2$$

A significant diffusion current density can be generated in a semiconductor material with only a modest density gradient.

The hole density in silicon is given by  $p(x) = 10^{16} e^{-(x/L_p)}$  ( $x \geq 0$ ) where  $L_p = 2 \times 10^{-4} \text{ cm}$ . Assume the hole diffusion coefficient is  $D_p = 8 \text{ cm}^2/\text{s}$ . Determine the hole diffusion current density at (a)  $x = 0$ , (b)  $x = 2 \times 10^{-4} \text{ cm}$ , and (c)  $x = 10^{-3} \text{ cm}$ .

[Ans. (a)  $J_p = 64 \text{ A/cm}^2$ ; (b)  $J_p = 23.54 \text{ A/cm}^2$ ; (c)  $J_p = 0.431 \text{ A/cm}^2$ ]

Consider a sample of silicon at  $T = 300$  K. Assume that the electron concentration varies linearly with distance, as shown in Figure P5.29. The diffusion current density is found to be  $J_n = 0.19$  A/cm<sup>2</sup>. If the electron diffusion coefficient is  $D_n = 25$  cm<sup>2</sup>/s, determine the electron concentration at  $x = 0$ .

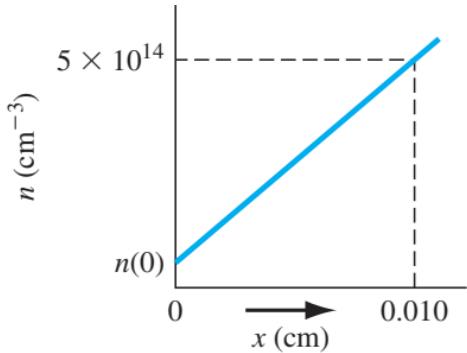


Figure P5.29

$$J_n = eD_n \frac{dn}{dx} = eD_n \left( \frac{5 \times 10^{14} - n(0)}{0.01 - 0} \right)$$

$$0.19 = (1.6 \times 10^{-19})(25) \left( \frac{5 \times 10^{14} - n(0)}{0.010} \right)$$

Then

$$\frac{(0.19)(0.010)}{(1.6 \times 10^{-19})(25)} = 5 \times 10^{14} - n(0)$$

which yields

$$n(0) = 0.25 \times 10^{14} \text{ cm}^{-3}$$

The steady-state electron distribution in silicon can be approximated by a linear function of  $x$ . The maximum electron concentration occurs at  $x = 0$  and is  $n(0) = 2 \times 10^{16} \text{ cm}^{-3}$ . At  $x = 0.012 \text{ cm}$ , the electron concentration is  $5 \times 10^{15} \text{ cm}^{-3}$ . If the electron diffusion coefficient is  $D_n = 27 \text{ cm}^2/\text{s}$ , determine the electron diffusion current density.

$$J_n = eD_n \frac{dn}{dx} = eD_n \frac{\Delta n}{\Delta x}$$

$$J_n = (1.6 \times 10^{-19})(27) \left[ \frac{2 \times 10^{16} - 5 \times 10^{15}}{0 - 0.012} \right]$$

$$J_n = -5.4 \text{ A/cm}^2$$

The electron diffusion current density in a semiconductor is a constant and is given by  $J_n = -2 \text{ A/cm}^2$ . The electron concentration at  $x = 0$  is  $n(0) = 10^{15} \text{ cm}^{-3}$ . (a) Calculate the electron concentration at  $x = 20 \mu\text{m}$  if the material is silicon with  $D_n = 30 \text{ cm}^2/\text{s}$ . (b) Repeat part (a) if the material is GaAs with  $D_n = 230 \text{ cm}^2/\text{s}$ .

$$(a) J_n = eD_n \frac{dn}{dx} = eD_n \frac{\Delta n}{\Delta x}$$

$$-2 = (1.6 \times 10^{-19})(30) \left[ \frac{10^{15} - n(x_1)}{0 - 20 \times 10^{-4}} \right]$$

$$4 \times 10^{-3} = 4.8 \times 10^{-3} - 4.8 \times 10^{-18} n(x_1)$$

which yields

$$n(x_1) = 1.67 \times 10^{14} \text{ cm}^{-3}$$

$$(b) -2 = (1.6 \times 10^{-19})(230) \left[ \frac{10^{15} - n(x_1)}{0 - 20 \times 10^{-4}} \right]$$

$$4 \times 10^{-3} = 3.68 \times 10^{-2} - 3.68 \times 10^{-17} n(x_1)$$

$$n(x_1) = 8.91 \times 10^{14} \text{ cm}^{-3}$$

In silicon, the electron concentration is given by  $n(x) = 10^{15}e^{-x/L_n} \text{ cm}^{-3}$  for  $x \geq 0$  and the hole concentration is given by  $p(x) = 5 \times 10^{15}e^{+x/L_p} \text{ cm}^{-3}$  for  $x \leq 0$ . The parameter values are  $L_n = 2 \times 10^{-3} \text{ cm}$  and  $L_p = 5 \times 10^{-4} \text{ cm}$ . The electron and hole diffusion coefficients are  $D_n = 25 \text{ cm}^2/\text{s}$  and  $D_p = 10 \text{ cm}^2/\text{s}$ , respectively. The total current density is defined as the sum of the electron and hole diffusion current densities at  $x = 0$ . Calculate the total current density.

For electrons:

$$J_n = eD_n \frac{dn}{dx} = eD_n \frac{d}{dx} [10^{15} e^{-x/L_n}] \\ = \frac{-eD_n (10^{15}) e^{-x/L_n}}{L_n}$$

At  $x = 0$ ,

$$J_n = \frac{-(1.6 \times 10^{-19})(25)(10^{15})}{2 \times 10^{-3}} = -2 \text{ A/cm}^2$$

For holes:

$$J_p = -eD_p \frac{dp}{dx} = -eD_p \frac{d}{dx} [5 \times 10^{15} e^{+x/L_p}] \\ = \frac{-eD_p (5 \times 10^{15}) e^{+x/L_p}}{L_p}$$

For  $x = 0$ ,

$$J_p = \frac{-(1.6 \times 10^{-19})(10)(5 \times 10^{15})}{5 \times 10^{-4}} = -16 \text{ A/cm}^2$$

$$J_{Total} = J_n(x=0) + J_p(x=0) \\ = -2 + (-16) = -18 \text{ A/cm}^2$$

The hole concentration in p-type GaAs is given by  $p(x) = 10^{16}(1 + x/L)^2 \text{ cm}^{-3}$  for  $-L \leq x \leq 0$  where  $L = 12 \mu\text{m}$ . The hole diffusion coefficient is  $D_p = 10 \text{ cm}^2/\text{s}$ . Calculate the hole diffusion current density at (a)  $x = 0$ , (b)  $x = -6 \mu\text{m}$ , and (c)  $x = -12 \mu\text{m}$ .

$$J_p = -eD_p \frac{dp}{dx} = -eD_p \frac{d}{dx} \left[ 10^{16} \left( 1 + \frac{x}{L} \right)^2 \right] \quad (\text{b}) \text{ For } x = -6 \mu\text{m},$$

$$= -eD_p \cdot \frac{10^{16}}{L} \cdot 2 \left( 1 + \frac{x}{L} \right)$$

(a) For  $x = 0$ ,

$$J_p = \frac{-\left(1.6 \times 10^{-19}\right)(10)(10^{16})(2)}{12 \times 10^{-4}}$$

$$= -26.7 \text{ A/cm}^2$$

$$J_p = \frac{-\left(1.6 \times 10^{-19}\right)(10)(10^{16})(2)\left(1 - \frac{6}{12}\right)}{12 \times 10^{-4}}$$

$$= -13.3 \text{ A/cm}^2$$

(c) For  $x = -12 \mu\text{m}$ ,

$$J_p = 0$$

## TOTAL CURRENT DENSITY

We now have four possible independent current mechanisms in a semiconductor.

Electron Drift

Electron Diffusion

Hole Drift

Hole Diffusion

The total current density is the sum of these four components.

$$J = \underbrace{en\mu_n E_x}_{\substack{\text{Electron} \\ \text{Drift}}} + \underbrace{ep\mu_p E_x}_{\substack{\text{Hole} \\ \text{Drift}}} + \underbrace{eD_n \frac{dn}{dx}}_{\substack{\text{Electron} \\ \text{Diffusion}}} - \underbrace{eD_p \frac{dp}{dx}}_{\substack{\text{Hole} \\ \text{Diffusion}}}$$

This equation may be generalized to three dimensions as

$$J = en\mu_n E + ep\mu_p E + eD_n \nabla n - eD_p \nabla p$$

- The mobility gives an indication of how well a charge carrier moves in a semiconductor as a result of the force of an electric field.
- The diffusion coefficient gives an indication of how well a charge carrier moves in a semiconductor as a result of a density gradient.
- The electron mobility and diffusion coefficient are not independent parameters.

- As the mobilities are strong functions of temperature because of the scattering processes, the diffusion coefficients are also strong functions of temperature.
- The specific temperature dependence given in Equation is only a small fraction of the real temperature characteristic.

Typical mobility and diffusion coefficient values at  
 $T = 300 \text{ K}$  ( $\mu = \text{cm}^2/\text{V}\cdot\text{s}$  and  $D = \text{cm}^2/\text{s}$ )

	$\mu_m$	$D_n$	$\mu_p$	$D_p$
Silicon	1350	35	480	12.4
Gallium arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2

The total current in a semiconductor is constant and equal to  $J = -10 \text{ A/cm}^2$ . The total current is composed of a hole drift current and electron diffusion current. Assume that the hole concentration is a constant and equal to  $10^{16} \text{ cm}^{-3}$  and assume that the electron concentration is given by  $n(x) = 2 \times 10^{15} e^{-x/L} \text{ cm}^{-3}$  where  $L = 15 \mu\text{m}$ . The electron diffusion coefficient is  $D_n = 27 \text{ cm}^2/\text{s}$  and the hole mobility is  $\mu_p = 420 \text{ cm}^2/\text{V-s}$ . Calculate (a) the electron diffusion current density for  $x > 0$ , (b) the hole drift current density for  $x > 0$ , and (c) the required electric field for  $x > 0$ .

$$\begin{aligned}
 (a) J_n &= eD_n \frac{dn}{dx} = eD_n \frac{d}{dx} [2 \times 10^{15} e^{-x/L}] \\
 &= \frac{-eD_n (2 \times 10^{15}) e^{-x/L}}{L} \\
 &= \frac{-(1.6 \times 10^{-19})(27)(2 \times 10^{15}) e^{-x/L}}{15 \times 10^{-4}} \\
 &= -5.76 e^{-x/L}
 \end{aligned}$$

$$\begin{aligned}
 (b) J_p &= J_{Total} - J_n = -10 - (-5.76 e^{-x/L}) \\
 &= [5.76 e^{-x/L} - 10] \text{ A/cm}^2 \\
 (c) \text{ We have } J_p &= \sigma E = (e\mu_p p_o) E \\
 5.76 e^{-x/L} - 10 &= (1.6 \times 10^{-19})(420)(10^{16}) E \\
 \text{So } E &= [8.57 e^{-x/L} - 14.88] \text{ V/cm}
 \end{aligned}$$

## THE HALL EFFECT

The Hall effect is a consequence of the forces that are exerted on moving charges by electric and magnetic fields.

Applications of Hall Effect are :

- Used to distinguish whether a semiconductor is n type or p type.
- Used to measure the majority carrier concentration.
- Used to measure majority carrier mobility.
- Used in engineering applications as a magnetic probe.
- Used in Current sensing applications.

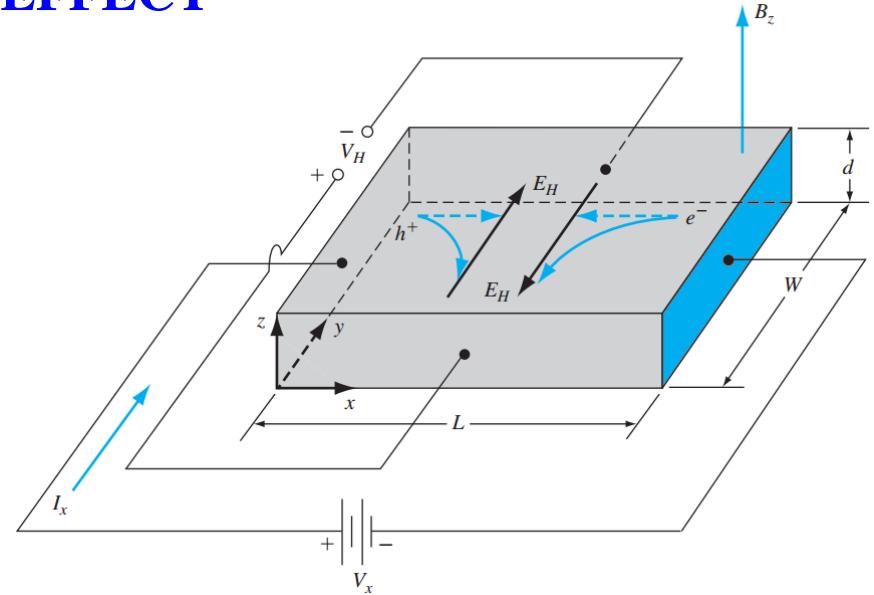
The force on a particle having a charge  $q$  and moving in a magnetic field is given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

where the **cross product** is taken between velocity and magnetic field so that the **force vector** is **perpendicular to both the velocity and magnetic field**.

# THE HALL EFFECT

- Figure illustrates the Hall effect.
- A semiconductor with a current  $I_x$  is placed in a magnetic field perpendicular to the current.
- In this case, the magnetic field is in the  $z$  direction.
- Electrons and holes flowing in the semiconductor will experience a force as indicated in the figure.
- The force on both electrons and holes is in the (-y) direction.
- In a **p-type** semiconductor ( $p_0 > n_0$ ), there will be a **buildup of positive charge** on the  $y = 0$  surface of the semiconductor
- In a **n-type** semiconductor ( $n_0 > p_0$ ), there will be a **buildup of negative charge** on the  $y = 0$  surface.

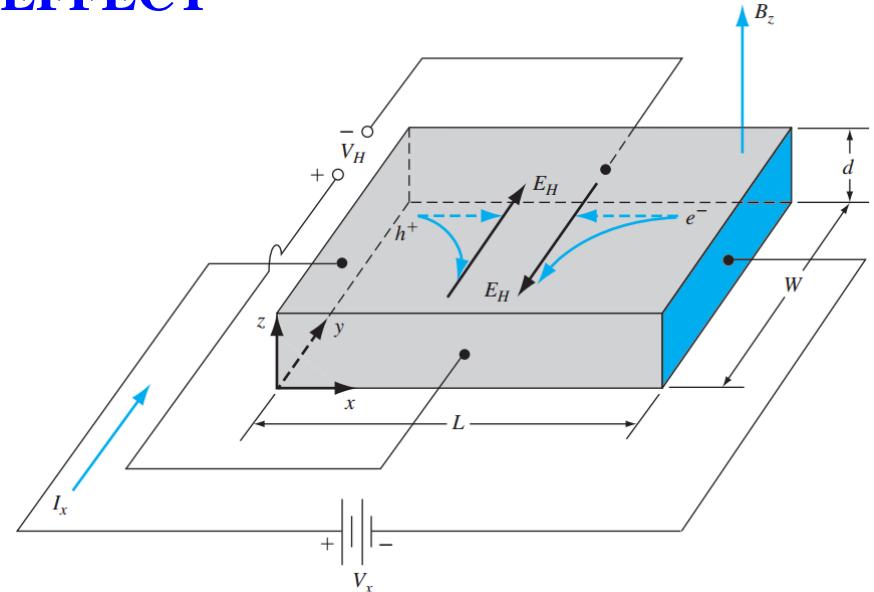


# THE HALL EFFECT

This net charge induces an electric field in the  $y$  direction as shown in the figure.

In steady state, the magnetic field force will be exactly balanced by the induced electric field force. This balance may be written as

$$\begin{aligned} F &= q[E + v \times B] = 0 \\ \Rightarrow qE_y &= qv_x B_z \end{aligned}$$



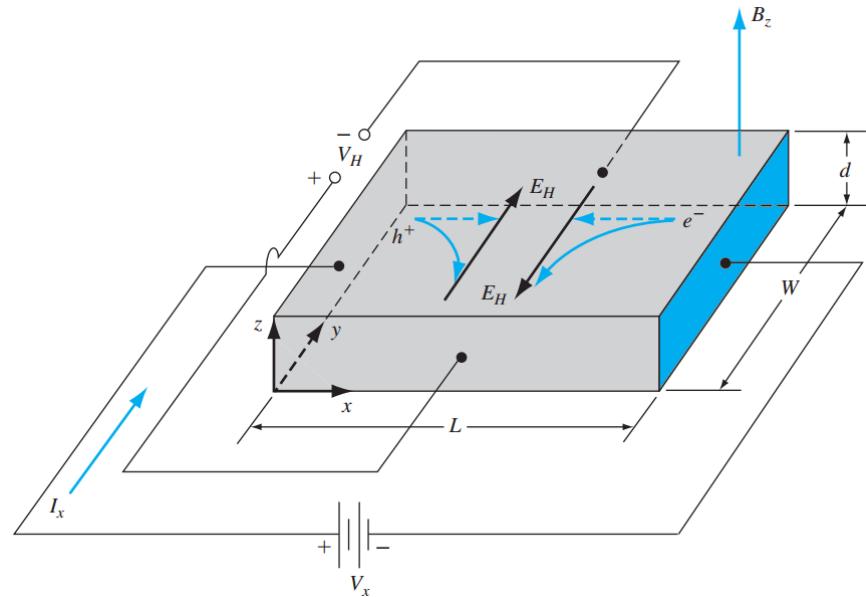
The induced electric field in the  $y$  direction is called the *Hall field*. The Hall field produces a voltage across the semiconductor which is called the *Hall voltage*. We can write

$$V_H = +E_H W$$

where  $E_H$  is assumed positive in the  $+y$  direction and  $V_H$  is positive with the polarity shown.

## THE HALL EFFECT

- In a p-type semiconductor, the Hall voltage will be positive as defined in Figure.
- In an n-type semiconductor, the Hall voltage will have the opposite polarity.
- The polarity of the Hall voltage is used to determine whether an extrinsic semiconductor is n type or p type.



$$\text{From } qE_y = qv_x B_z$$

$$\text{and } V_H = +E_H W$$

$$\Rightarrow V_H = v_x W B_z$$

For a p-type semiconductor, the drift velocity of holes can be written as

$$v_{dx} = \frac{J_x}{ep} = \frac{I_x}{(ep)(W \cdot d)}$$

# THE HALL EFFECT

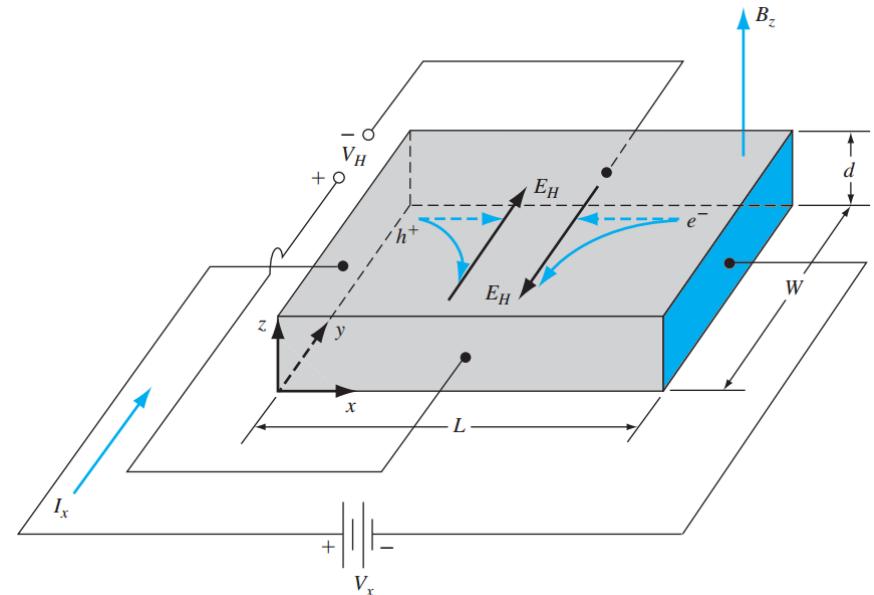
$$V_H = \frac{I_x B_z}{epd}$$

$$p = \frac{I_x B_z}{edV_H}$$

For n-type semiconductor, the  $V_H$  is given by

$$V_H = -\frac{I_x B_z}{ned}$$

$$n = -\frac{I_x B_z}{edV_H}$$



Note that the **Hall voltage is negative for the n-type semiconductor**; therefore, the electron concentration determined from Equation is actually positive quantity.

## THE HALL EFFECT

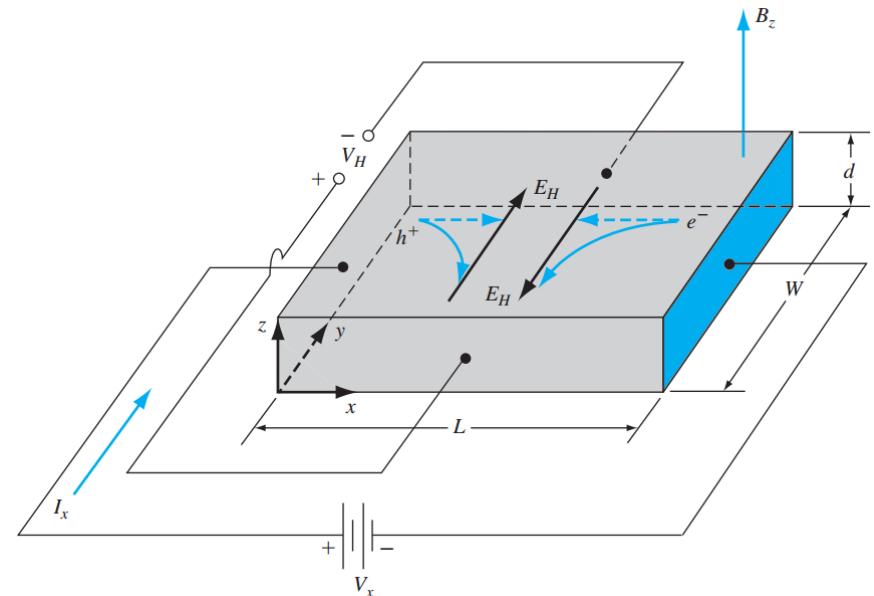
Once the majority carrier concentration has been determined, we can calculate the low-field majority carrier mobility. For a p-type semiconductor, we can write

$$J_x = ep\mu_p E_x$$

$$\frac{I_x}{Wd} = ep\mu_p \frac{V_x}{L}$$

$$\mu_p = \frac{I_x L}{epWdV_x}$$

$$\mu_n = \frac{I_x L}{enWdV_x}$$



## THE HALL EFFECT

**EXAMPLE:** Determine the majority carrier concentration and mobility, given Hall effect parameters.

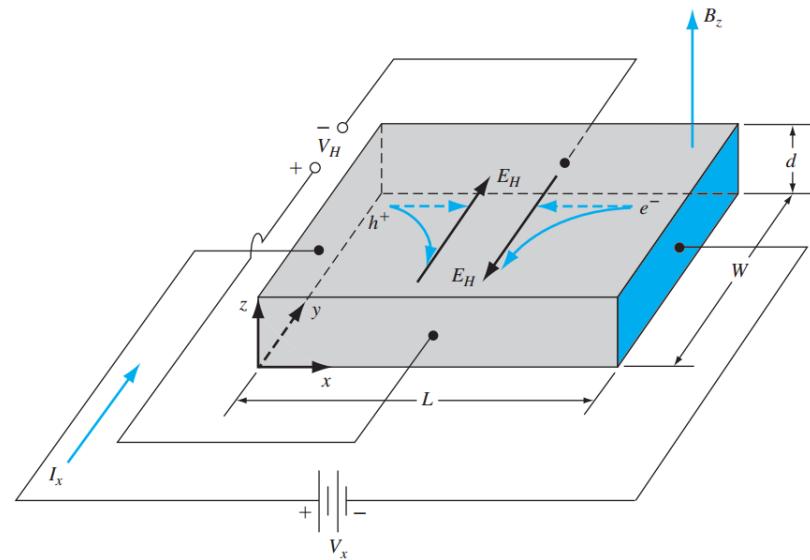
Consider the geometry shown in Figure. Let  $L=10^{-1}$  cm,  $W=10^{-2}$  cm, and  $d = 10^{-3}$  cm. Also assume that  $I_x = 1.0$  mA,  $V_x = 12.5$  V,  $B_z = 500$  gauss =  $5 \times 10^{-2}$  tesla, and  $V_H = -6.25$  mV.

A negative Hall voltage for this geometry implies that we have an n-type semiconductor.

$$n = -\frac{I_x B_z}{e d V_H}$$

$$n = -\frac{(10^{-3})(5 \times 10^{-2})}{(1.6 \times 10^{-19})(10^{-5})(-6.25 \times 10^{-3})}$$

$$n = 5 \times 10^{21} m^{-3} = 5 \times 10^{15} cm^{-3}$$



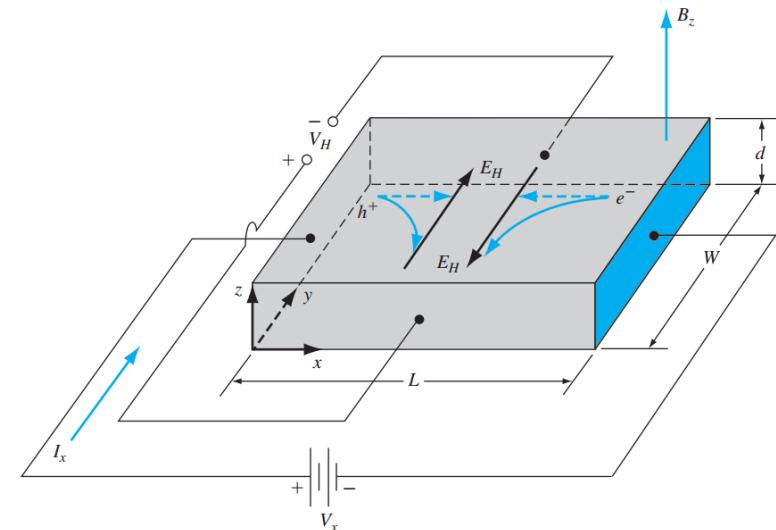
## THE HALL EFFECT

The electron mobility is then determined from Equation as

$$\mu_n = \frac{I_x L}{e n W d V_x}$$

$$\mu_n = \frac{(10^{-3})(10^{-3})}{(1.6 \times 10^{-19})(5 \times 10^{21})(12.5)(10^{-4})(10^{-5})}$$

$$\mu_n = 1000 \text{ cm}^2 / \text{V} - \text{s}$$



It is important to note that the MKS units must be used consistently in the Hall effect equations to yield correct results.

A p-type silicon sample with the geometry shown below has parameters L= 0.2 cm, W=10<sup>-2</sup> cm, and d=8×10<sup>-4</sup> cm. The semiconductor parameters are p=10<sup>16</sup> cm<sup>-3</sup> and  $\mu_p$ =320 cm<sup>2</sup>/V-s. For V<sub>x</sub> =10 V and B<sub>z</sub> =500 gauss=5×10<sup>-2</sup> tesla, determine I<sub>x</sub> and V<sub>H</sub>.

$$I_x = \frac{ep\mu_p V_x W d}{L}$$

$$I_x = \frac{1.6 \times 10^{-19} \cdot 10^{22} \cdot 320 \times 10^{-4} \cdot 10 \cdot 10^{-4} \cdot 8 \times 10^{-6}}{0.2 \times 10^{-2}}$$

$$I_x = 0.2048mA$$

$$V_H = \frac{I_x B_z}{ped}$$

$$V_H = \frac{0.2048 \times 10^{-3} \cdot 5 \times 10^{-2}}{10^{22} \cdot 1.6 \times 10^{-19} \cdot 8 \times 10^{-6}}$$

$$V_H = 0.80mV$$

Germanium is doped with  $5 \times 10^{15}$  donor atoms per  $\text{cm}^3$  at  $T = 300$  K. The dimensions of the Hall device are  $d = 5 \times 10^{-3}$  cm,  $W = 2 \times 10^{-2}$  cm, and  $L = 10^{-1}$  cm. The current is  $I_x = 250 \mu\text{A}$ , the applied voltage is  $V_x = 100$  mV, and the magnetic flux density is  $B_z = 500$  gauss =  $5 \times 10^{-2}$  tesla. Calculate (i) the Hall voltage, (ii) the Hall field, and (iii) the carrier mobility.

(i) Hall Voltage

$$V_H = \frac{-I_x B_z}{ped}$$

$$V_H = \frac{-(250 \times 10^{-6}) \cdot (5 \times 10^{-2})}{(5 \times 10^{21}) \cdot (1.6 \times 10^{-19}) \cdot (5 \times 10^{-5})}$$

$$V_H = -0.3125 \text{ mV}$$

(ii) Hall Field

$$E_H = \frac{V_H}{W} = \frac{-0.3125 \times 10^{-3}}{2 \times 10^{-2}} = -1.56 \times 10^{-2} \text{ V/cm}$$

(iii) Carrier mobility

$$\mu_n = \frac{I_x L}{enV_x Wd}$$

$$\mu_n = \frac{(250 \times 10^{-6}) \cdot (10^{-3})}{(1.6 \times 10^{-19}) \cdot (5 \times 10^{21}) \cdot (0.1) \cdot (2 \times 10^{-4}) \cdot (5 \times 10^{-5})}$$

$$\mu_n = 0.3125 \text{ m}^2/\text{V-s or } 3125 \text{ cm}^2/\text{V-s}$$

Consider a gallium arsenide sample at  $T = 300$  K. A Hall effect device has been fabricated with the following geometry:  $d = 0.01$  cm,  $W = 0.05$  cm, and  $L = 0.5$  cm. The electrical parameters are:  $I_x = 2.5$  mA,  $V_x = 2.2$  V, and  $B_z = 2.5 \times 10^{-2}$  tesla. The Hall voltage is  $V_H = -4.5$  mV. Find: (a) the conductivity type, (b) the majority carrier concentration, (c) the mobility, and (d) the resistivity.

(a)  $V_H = \text{negative} \Rightarrow \text{n-type}$

$$\begin{aligned} \text{(b)} \quad n &= \frac{-I_x B_z}{e d V_H} \\ &= \frac{-(2.5 \times 10^{-3})(2.5 \times 10^{-2})}{(1.6 \times 10^{-19})(0.01 \times 10^{-2})(-4.5 \times 10^{-3})} \end{aligned}$$

or

$$n = 8.68 \times 10^{20} \text{ m}^{-3} = 8.68 \times 10^{14} \text{ cm}^{-3}$$

$$\begin{aligned} \text{(c)} \quad \mu_n &= \frac{I_x L}{e n V_x W d} \\ &= \left[ \frac{(2.5 \times 10^{-3})(0.5 \times 10^{-2})}{(1.6 \times 10^{-19})(8.68 \times 10^{20})(2.2)} \right] \\ &\quad \times \left[ \frac{1}{(0.05 \times 10^{-2})(0.01 \times 10^{-2})} \right] \end{aligned}$$

or

$$\mu_n = 0.8182 \text{ m}^2/\text{V-s} = 8182 \text{ cm}^2/\text{V-s}$$

$$\begin{aligned} \text{(d)} \quad \sigma &= \frac{1}{\rho} = e \mu_n n \\ &= (1.6 \times 10^{-19})(8182)(8.68 \times 10^{14}) \end{aligned}$$

or

$$\rho = 0.88 (\Omega \cdot \text{cm})$$