Lattice Structure

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We know that $y(n) = \sum_{k=0}^{M-1} h(k)\, x(n-k)$ for FIR systems

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$$

$$h(n) = \begin{cases} b_n & 0 \le n \le M-1 \\ 0 & \text{otherwise} \end{cases} \longrightarrow b_0 = 1 \Rightarrow h_m(0) = 1$$

Let us define the FIR filter in the following form

$$H_m(z) = A_m(z), \quad m = 0, 1, 2, \cdots M-1$$

Then $A_m(z) = 1 + \sum_{k=1}^{m} \alpha_m(k)\, z^{-k} \quad m \geq 1$ —— (1)

$A_0(z) = 1$, $\quad \alpha_m(k) = h_m(k)$

Let $\{x(n)\}$ be the input sequence to the filter $A_m(z)$ and $\{y(n)\}$ be the output

- Lattice filter implementation is widely used in adaptive filtering. Assume that we have a filter with transfer function H(z). We can write,

$$H_m(z) = A_m(z), \qquad m = 0, 1, 2, \ldots, M-1$$

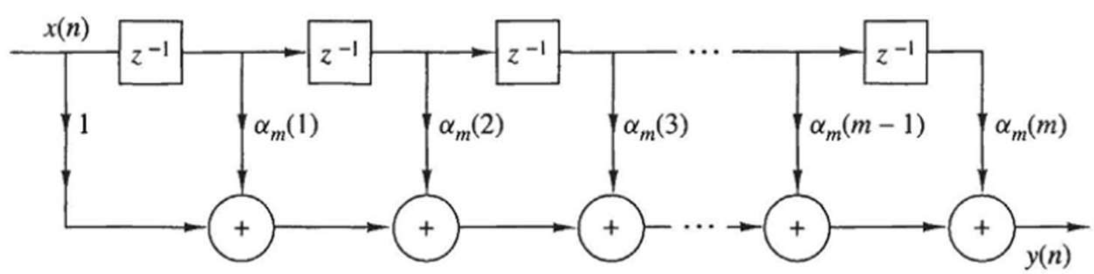
- where $A_m(z)$ is a polynomial with $A_0(z) = 1$

$$A_m(z) = 1 + \sum_{k=1}^{m} \alpha_m(k) z^{-k}, \qquad m \geq 1$$

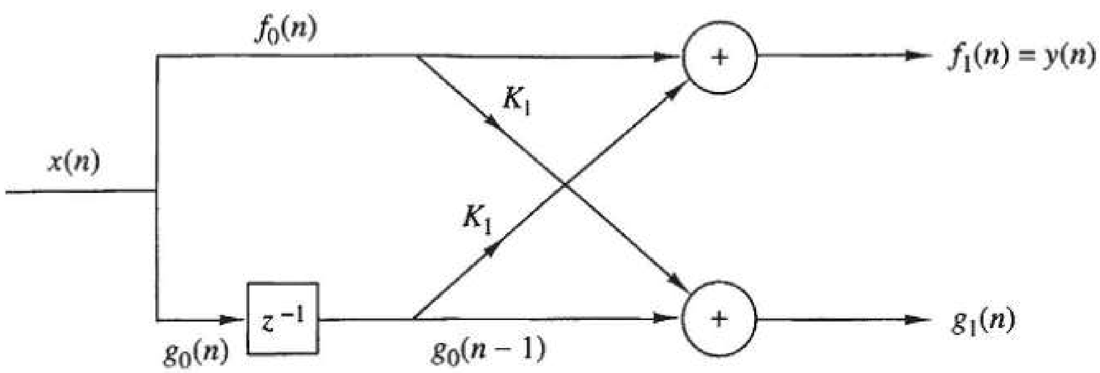
- Y[n] can be written as

$$y(n) = x(n) + \sum_{k=1}^{m} \alpha_m(k) x(n-k)$$

- The direct form implementation can be expressed as



- Let's consider a first order FIR filter, i.e., m=1: $y(n) = x(n) + \alpha_1(1)x(n-1)$
- Let the reflection coefficient $K_1 = \alpha_1(1)$. to get:



$$f_0(n) = g_0(n) = x(n)$$
$$f_1(n) = f_0(n) + K_1 g_0(n-1) = x(n) + K_1 x(n-1)$$
$$g_1(n) = K_1 f_0(n) + g_0(n-1) = K_1 x(n) + x(n-1)$$

- Now consider m=2:

$$y(n) = x(n) + \alpha_2(1)x(n-1) + \alpha_2(2)x(n-2)$$

- We cascade two lattice stages:
- The out put of the first stage is,

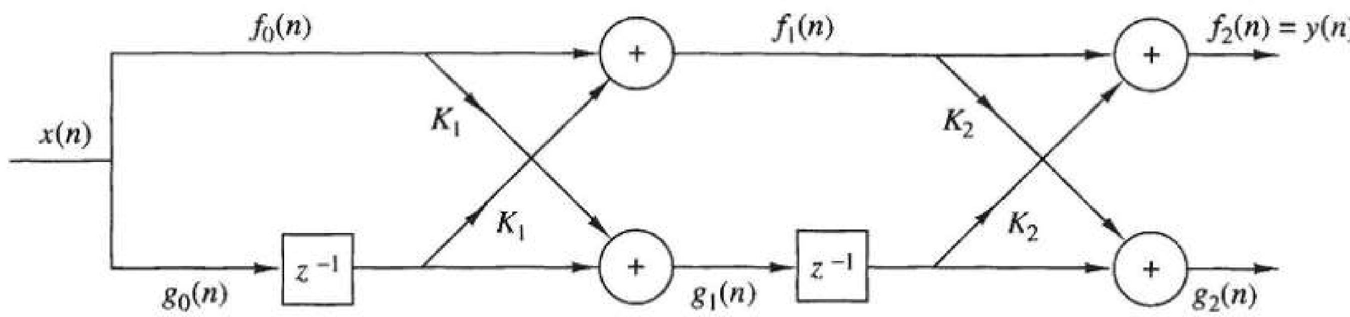
$$f_1(n) = x(n) + K_1 x(n-1)$$

$$g_1(n) = K_1 x(n) + x(n-1)$$

- And the output of the second stage is:

$$f_2(n) = f_1(n) + K_2 g_1(n-1)$$

$$g_2(n) = K_2 f_1(n) + g_1(n-1)$$



Two-stage lattice filter.

- Let's consider on $f_2[n]$:

$$f_2(n) = x(n) + K_1 x(n-1) + K_2[K_1 x(n-1) + x(n-2)]$$

$$= x(n) + K_1(1 + K_2)x(n-1) + K_2 x(n-2)$$

- $f_2[n]$ will be y[n] if:

$$\alpha_2(2) = K_2, \qquad \alpha_2(1) = K_1(1 + K_2)$$

- or, equivalently if:

$$K_2 = \alpha_2(2), \qquad K_1 = \frac{\alpha_2(1)}{1 + \alpha_2(2)}$$

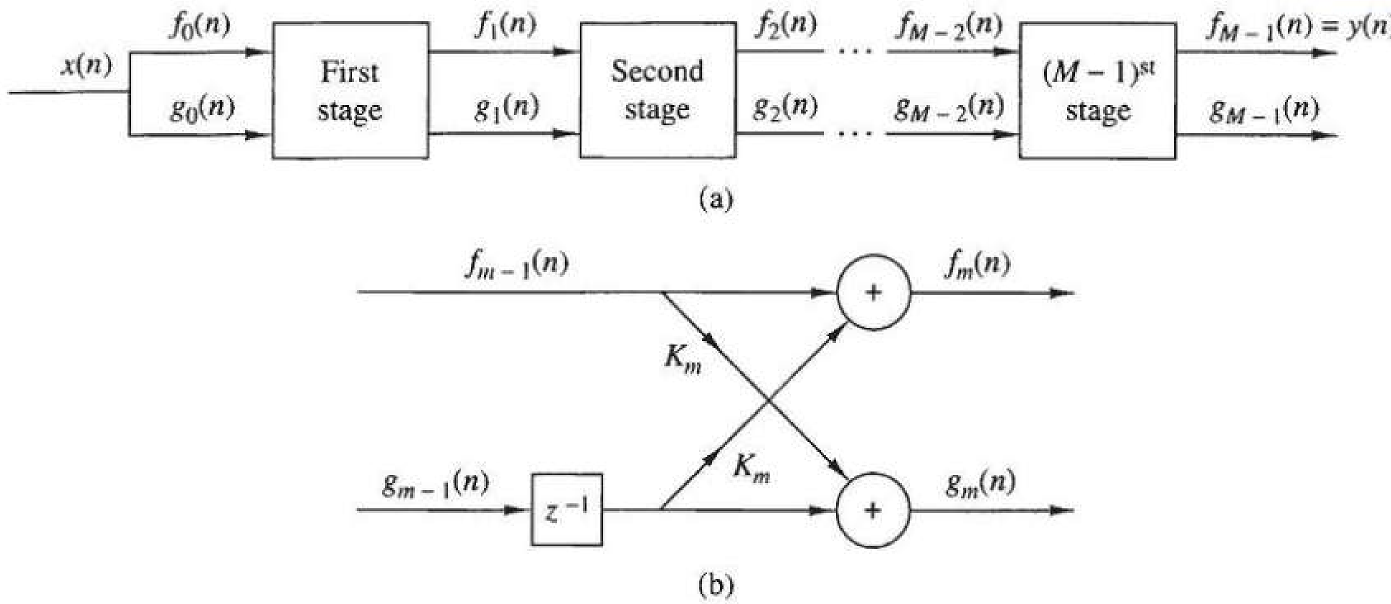
- In general: $f_0(n) = g_0(n) = x(n)$

$$f_m(n) = f_{m-1}(n) + K_m g_{m-1}(n-1), \qquad m = 1, 2, \ldots, M-1$$

$$g_m(n) = K_m f_{m-1}(n) + g_{m-1}(n-1), \qquad m = 1, 2, \ldots, M-1$$

- Then:

$y(n) = f_{M-1}(n)$



$(M-1)$-stage lattice filter.

Conversion of FIR taps to Lattice coefficients

$$\alpha_{m-1}(k) = \frac{\alpha_m(k) - \alpha_m(m)\,\alpha_m(m-k)}{1 - \alpha_m^2(m)}$$

Determine the lattice coefficients corresponding to the FIR filter with system function

$$y(n) = x(n) + \frac{1}{3}x(n-1) + \frac{1}{2}x(n-2) + \frac{1}{3}x(n-3)$$

$\alpha_3(0) = 1$

$\alpha_3(1) = 1/3$

$\alpha_3(2) = 1/2$

$\alpha_3(3) = 1/3$

$K_m = \alpha_m(m)$

$K_1 = \alpha_1(1)$

$K_2 = \alpha_2(2)$

$K_3 = \alpha_3(3)$.

$$\alpha_{m-1}(k) = \frac{\alpha_m(k) - \alpha_m(m)\,\alpha_m(m-k)}{1 - \alpha_m^2(m)}$$

$K_3 = \alpha_3(3) = 1/3$.

$K_2 = \alpha_2(2)$, $\underline{m = 3, k = 2}$

$$\alpha_2(2) = \frac{\alpha_3(2) - \alpha_3(3)\,\alpha_3(1)}{1 - \alpha_3^2(3)}$$

$K_2 = \frac{7}{16}$

$\alpha_2(1) =$ $m = 3$ $k = 1$

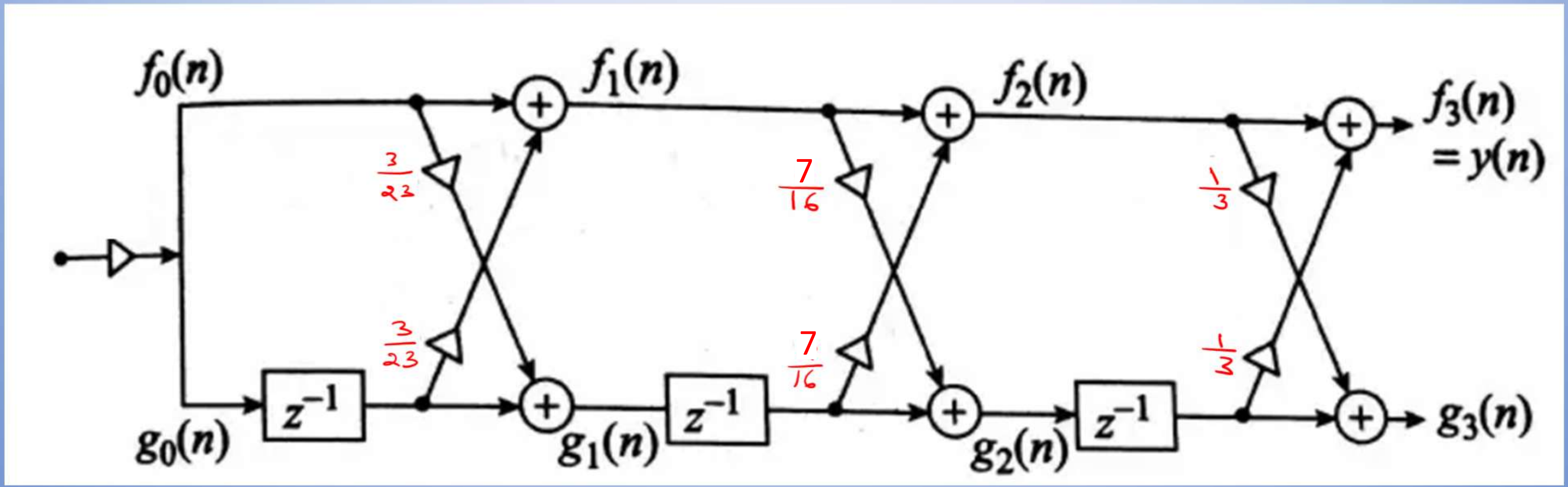
$$\alpha_2(1) = \frac{\alpha_3(1) - \alpha_3(3)\,\alpha_3(2)}{1 - \alpha_3^2(3)}$$

$\boxed{\alpha_2(1) = \frac{3}{16}}$

$K_1 = \alpha_1(1)|_{\underline{m=2, k=1}}$

$$\alpha_1(1) = \frac{\alpha_2(1) - \alpha_2(2)\,\alpha_2(1)}{1 - \alpha_2^2(2)}$$

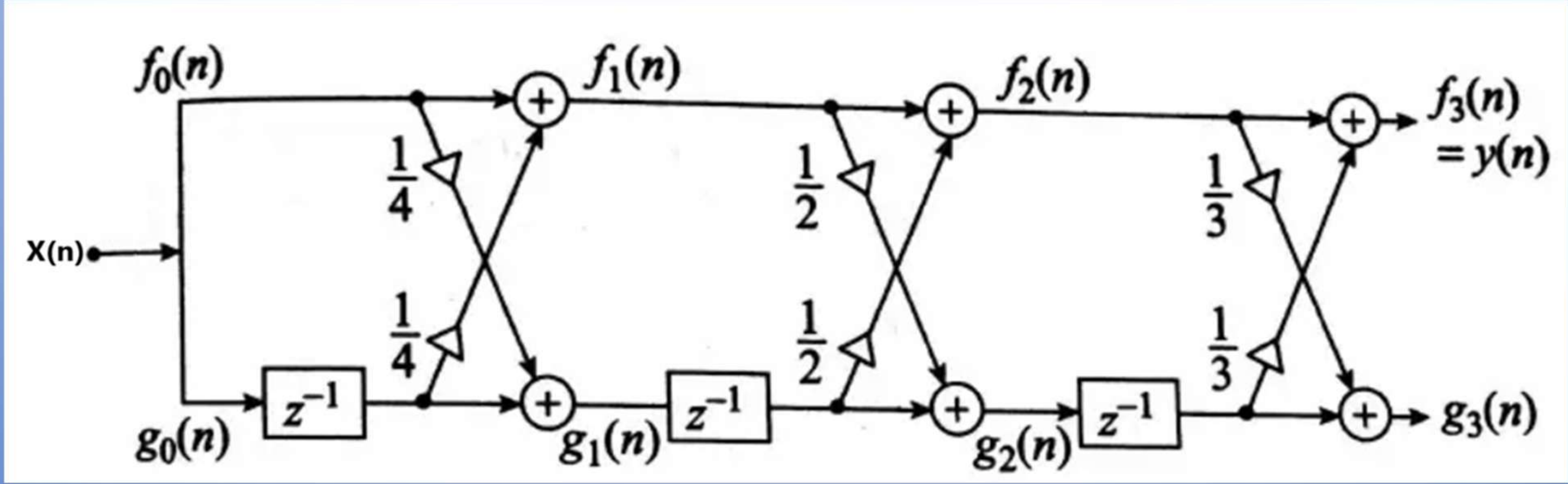
$= \frac{3}{23}$



$f_0(n)$
$f_1(n)$
$f_2(n)$
$f_3(n)$
$= y(n)$
$\frac{3}{23}$
$\frac{3}{23}$
$\frac{7}{16}$
$\frac{7}{16}$
$\frac{1}{3}$
$\frac{1}{3}$
z^{-1}
z^{-1}
z^{-1}
$g_0(n)$
$g_1(n)$
$g_2(n)$
$g_3(n)$

Determine the lattice coefficients corresponding to the FIR filter with system function

$$H(z) = A_3(z) = 1 + \frac{13}{24}z^{-1} + \frac{5}{8}z^{-2} + \frac{1}{3}z^{-3}$$



Consider a three – stage FIR lattice structure having the coefficients

$k1=0.65,\ k2=0.5\ \&\ k3=0.9$

Find its impulse response and direct form structure.

$\alpha_m(0) = 1$ ——— (I)

$\alpha_m(m) = k_m$ ——— (II)

$\alpha_m(k) = \alpha_{m-1}(k) + k_m \alpha_{m-1}(m-k)$ — (III)

$\boxed{m=3}$ $\boxed{\alpha_m(0)=1}$ $\boxed{\alpha_1(1)=0.65}$ $\boxed{\alpha_2(2)=0.5}$ $\boxed{\alpha_3(3)=0.9}$

To realize direct form structure we need to find

$\underline{\alpha_3(1)}$ $\underline{\alpha_3(2)}$ $\underline{\alpha_3(3)}$

$$\alpha_m(k) = \alpha_{m-1}(k) + k_m \alpha_{m-1}(m-k) \quad \text{--- (III)}$$

put $m=3, \ k=1$ in Eqn (III),

$$\alpha_3(1) = \alpha_2(1) + k_3 \times \alpha_2(2) \quad \text{--- (A)}$$

Now calculate $\alpha_2(1)$

put $m=2, \ k=1$ in Eqn (III)

$$\alpha_2(1) = \alpha_1(1) + k_2 \times \alpha_1(1) = 0.65 + 0.5 \times 0.65 = 0.975$$

Substitute $\alpha_2(1)$ in Eqn (A)

$$\therefore \quad \alpha_3(1) = 0.975 + 0.9 \times 0.5 = 1.425$$

Now Substitute $m=3, \ k=2$ in Eqn (III)

$$\alpha_3(2) = \alpha_2(2) + k_3 \times \alpha_2(1) = 0.5 + 0.9 \times 0.975 = 1.3775$$

$\alpha_3(1) = 1.425 \qquad \alpha_3(2) = 1.3775 \qquad \alpha_3(3) = 0.9$

$\therefore$ W.k.T

$$H(z) = 1 + \sum_{k=1}^{m} \alpha_m(k)\, z^{-k}$$

$$H(z) = 1 + \sum_{k=1}^{3} \alpha_m(k)\, z^{-k}$$

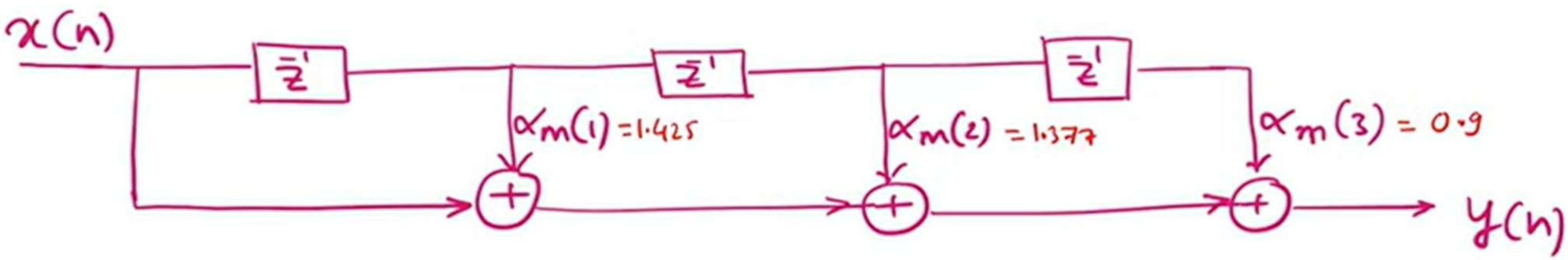
$$H(z) = 1 + \alpha_3(1) z^{-1} + \alpha_3(2) z^{-2} + \alpha_3(2) z^{-3}$$

$$\therefore H(z) = 1 + 1.425 z^{-1} + 1.3775 z^{-2} + 0.9 z^{-3}$$

$$\frac{Y(z)}{X(z)} = 1 + 1.425 z^{-1} + 1.375 z^{-2} + 0.9 z^{-3}$$

$$\boxed{y(n) = x(n) + 1.425\, x(n-1) + 1.375\, x(n-2) + 0.9\, x(n-3)}$$

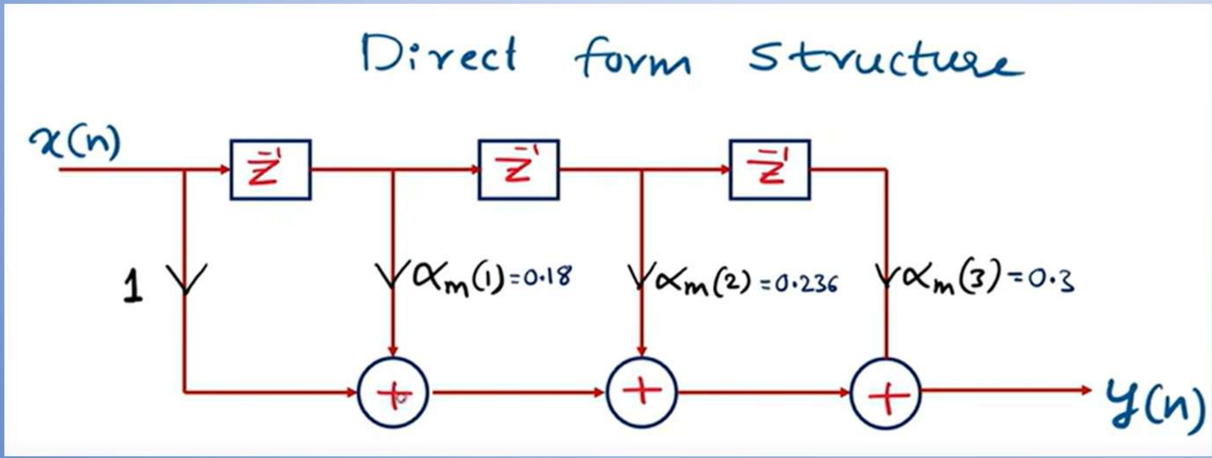
Direct form structure is shown below



Let the coefficients of a three stage FIR lattice structure be

$$K1=0.1,\ K2=0.2 \text{ and } K3=0.3.$$

Find the coefficients of the direct form 1 FIR filter and draw its block diagram




Thank you