Implementation of Discrete Time Systems (Realization): Structure for FIR systems

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FIR and IIR systems

- A LTI system is classified based on impulse response as below
  - FIR system
  - IIR system

⇒ FIR system:

- Finite impulse response system.
- impulse response is defined only for finite number of samples.

$$h(n) = \begin{cases} 1 ; & n = -1, 2 \\ 2 ; & n = 1 \\ 3 ; & n = 0, 3 \\ 0 ; & \text{otherwise} \end{cases}$$

⇒ IIR system:

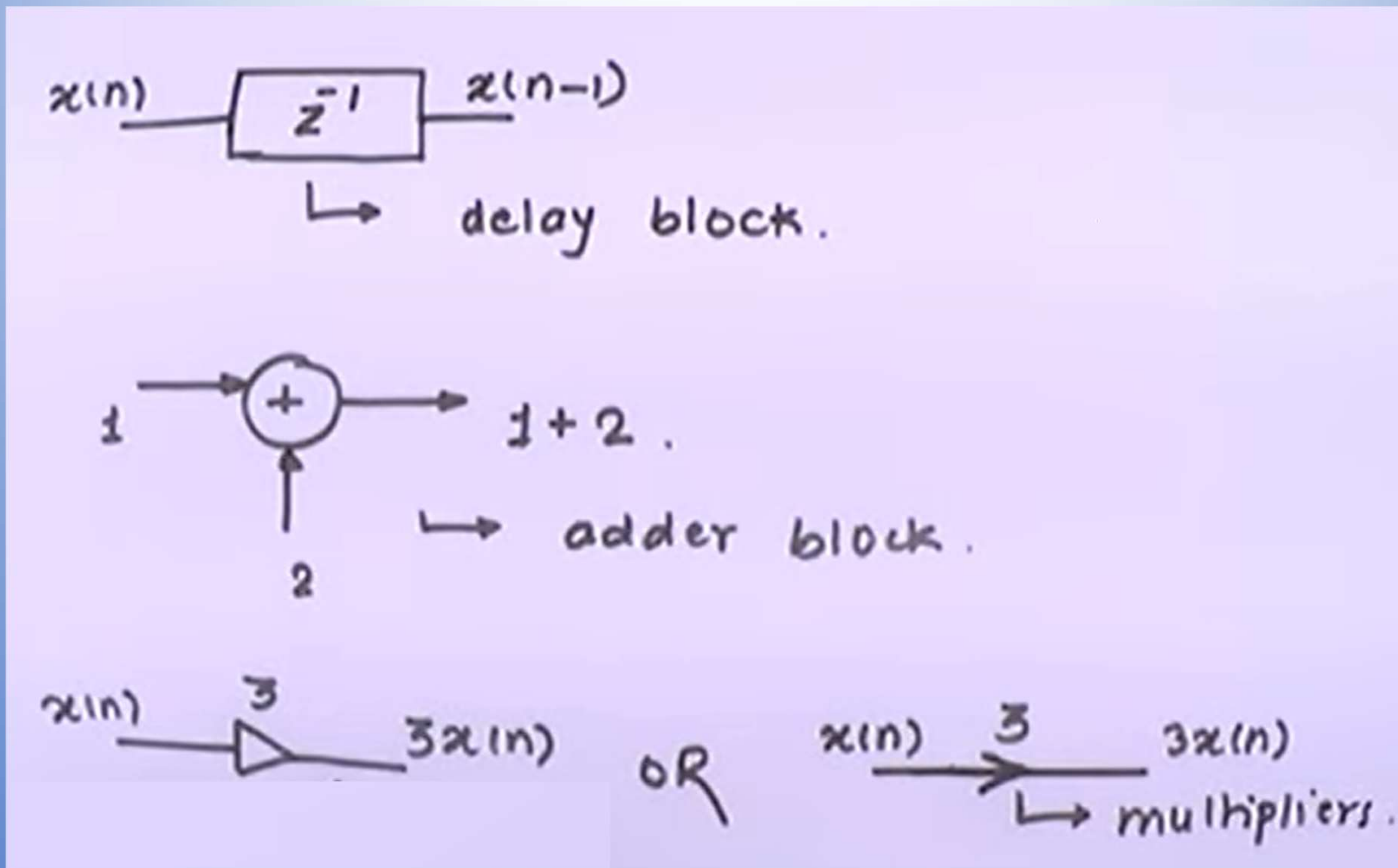
- infinite impulse response system
- impulse response have infinite number of sample.

$$h(n) = a^n u(n)$$

IIR filters are less powerful than FIR filters, & require less processing power and less work to set up the filters	FIR filters are more powerful than IIR filters, but also require more processing power and more work to set up the filters
It cannot implement linear-phase filtering.	It can implement linear-phase filtering.
It cannot be used to correct frequency-response errors in a loudspeaker	It can be used to correct frequency-response errors in a loudspeaker to a finer degree of precision than using IIRs
Usage is generally more easier than FIR filters.	Usage is generally more complicated and time-consuming than IIR filters
IIR filter uses current input sample value, past input and output samples to obtain current output sample value.	FIR filter uses only current and past input digital samples to obtain a current output sample value. It does not utilize past output samples.

Simple IIR equation is mention below.,y(n)= b(0)x(n) + b(1)x(n-1) + b(2)x(n-2) + b(3)x(n-3) + a(1)y(n-1) + a(2)y(n-2) + a(3)y(n-3)	Simple FIR equation is mention below. y(n)= h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + h(3)x(n-3) + h(4)x(n-4)
Transfer function of IIR filter will have both zeros and poles and will require less memory than FIR counterpart	Transfer function of FIR filter will have only zeros, need more memory
IIR filters are not stable as they are recursive in nature and feedback is also involved in the process of calculating output sample values.	FIR filters are preferred due to its linear phase response and also they are non-recursive. Feedback is not involved in FIR, hence they are stable

- Realization – hardware – adder, multiplier and delay


x(n)
z^-1
x(n-1)
delay block.
1
+
1+2.
2
adder block.
x(n)
3
3x(n)
OR
x(n)
3
3x(n)
multipliers.

Structures for FIR systems

FIR systems can be described by $y(n) = \sum_{k=0}^{M-1} b_k \cdot x(n-k)$

or $Y(z) = \sum_{k=0}^{M-1} b_k z^{-k} X(z)$

$$\frac{Y(z)}{X(z)} = H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$$

The unit sample response (impulse response), $h(n) = \begin{cases} b_n & 0 \le n \le M-1 \\ 0 & \text{otherwise} \end{cases}$

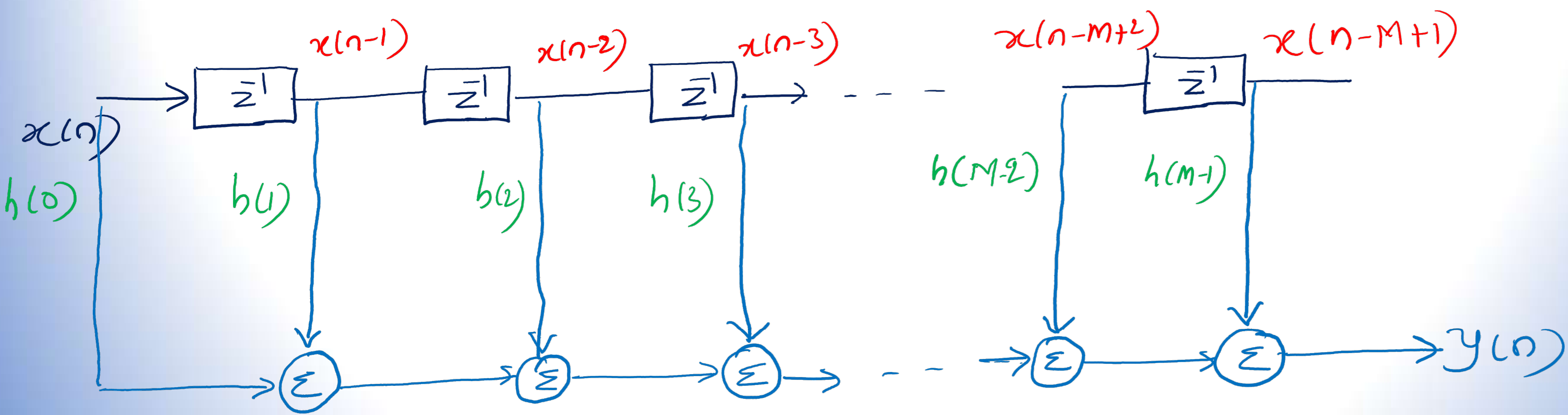
Structures:

1. Direct form
2. Cascaded form
3. Frequency Sampling
4. Lattice

Direct Form Structure

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

$$= h(0)\cdot x(n) + h(1)x(n-1) + \cdots + h(M-1)\cdot x(n-(M-1))$$



(M) multiplications and (M-1) additions

Tapped delay line structure

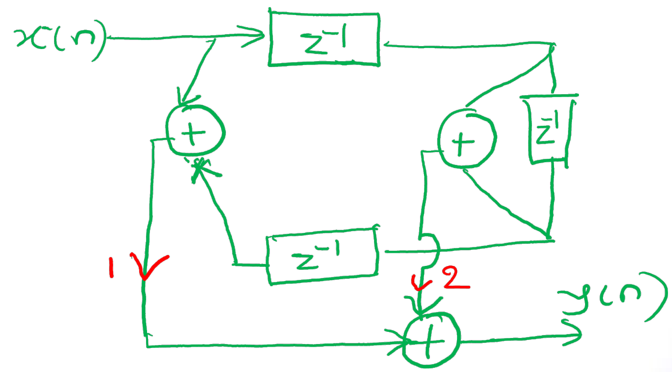
Linear Phase FIR filters:

An FIR filter is said to have linear phase if its impulse response satisfies either symmetry or asymmetry conditions

$$h(n) = \pm h(M-1-n)$$

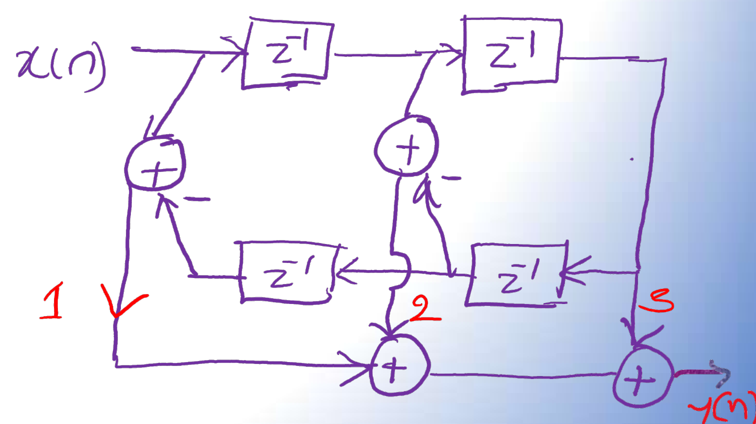
Ex: $h(n) = \{1,\ 2,\ 2,\ 1\}$ $M = 4$

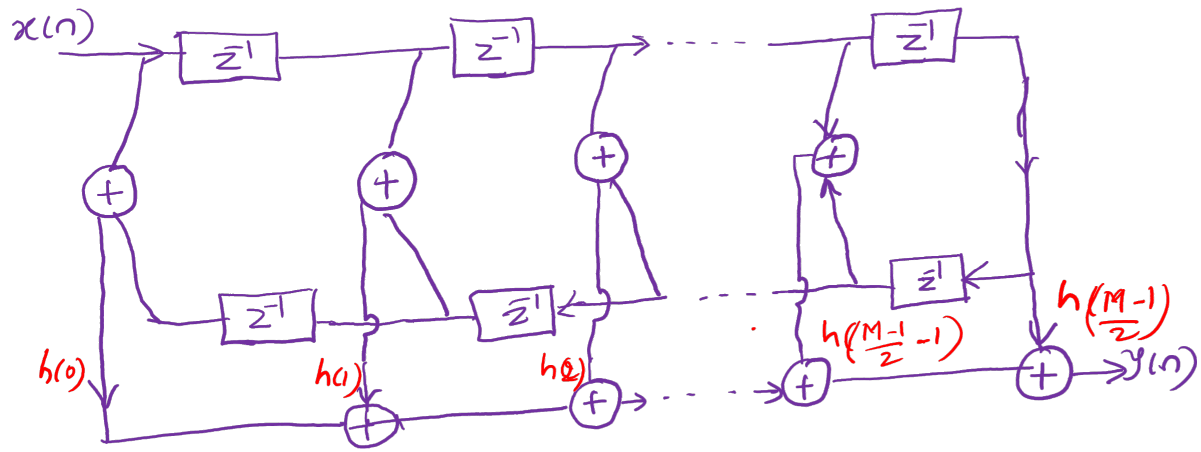
$y(n) = x(n) + 2x(n-1) + 2x(n-2) + x(n-3)$



$h(n) = \{1,\ 2,\ 3,\ -2,\ -1\}$ $M = 5$

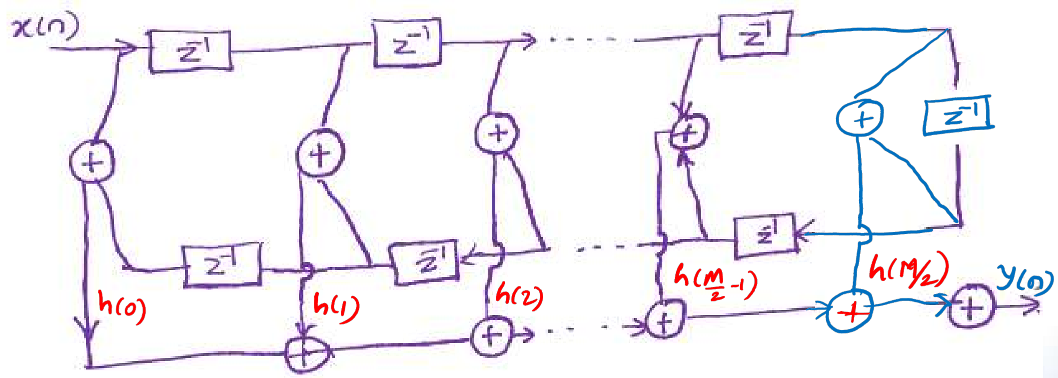
$y(n) = x(n) + 2x(n-1) + 3x(n-2) - 2x(n-3) - x(n-4)$





x(n)
z^{-1}
h(0)
h(1)
h(2)
$h\left(\frac{M-1}{2}-1\right)$
$h\left(\frac{M-1}{2}\right)$
y(n)

Odd M



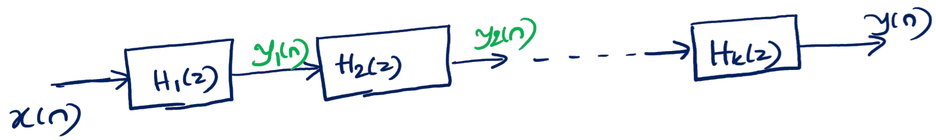
x(n)
z^{-1}
h(0)
h(1)
h(2)
$h\left(\frac{M}{2}-1\right)$
$h(M/2)$
y(n)

Even M

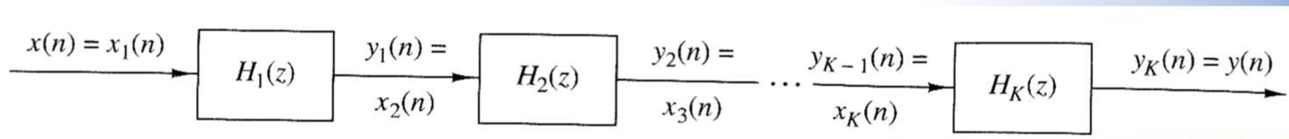
Cascade- Form structures

$$H(z) = \prod_{k=1}^{K} H_k(z)$$

$$H_k(z) = b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2} \quad , \quad k = 1, 2, \cdots K$$



Cascade- Form structures



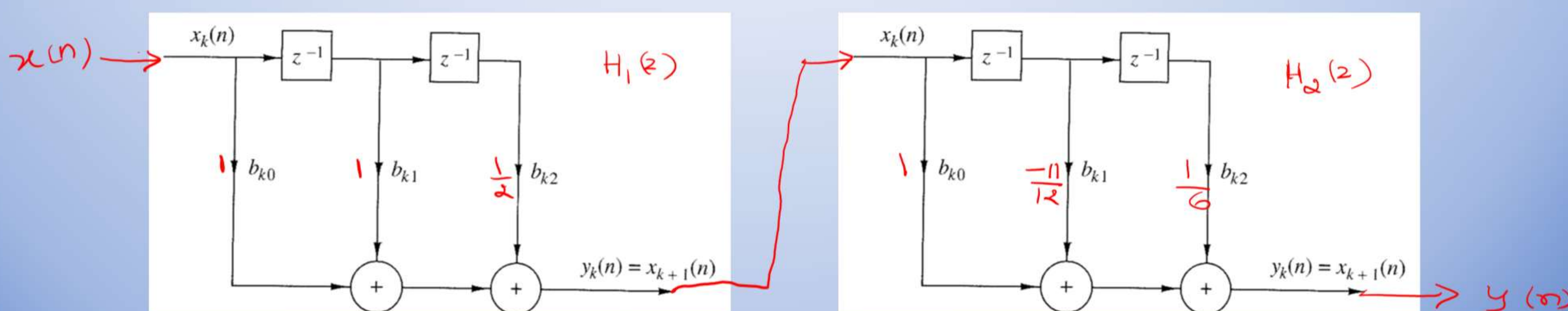
Example: Obtain the cascaded realization for

$$H(z) = \left[1 + \left(\tfrac{1}{2} + j\tfrac{1}{2}\right)z^{-1}\right]\left[1 + \left(\tfrac{1}{2} - j\tfrac{1}{2}\right)z^{-1}\right]\left[1 - \tfrac{2}{3}z^{-1}\right]\left[1 - \tfrac{1}{4}z^{-1}\right]$$

$$= \left[1 + z^{-1} + \tfrac{1}{2}z^{-2}\right]\left[1 - \tfrac{11}{12}z^{-1} + \tfrac{1}{6}z^{-2}\right]$$

$$H_1(z) = \left[1 + z^{-1} + \tfrac{1}{2}z^{-2}\right] \qquad \frac{Y_1(z)}{X(z)} = H_1(z)$$

$$H_2(z) = \left[1 - \tfrac{11}{12}z^{-1} + \tfrac{1}{6}z^{-2}\right] \qquad \frac{Y_2(z)}{Y_1(z)} = H_2(z)$$




Thank you