Structure for IIR systems

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Structure for IIR systems:

LTI discrete time systems can be characterized by Linear constant coefficient difference equation(LCCDE)

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

Zeros

Poles

Direct Form Structures:

- Direct Form I
- Direct Form II

Transposed

Cascaded

Parallel

Lattice-Ladder structure

Direct Form I structure realization:

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} = \sum_{k=0}^{M} b_k z^{-k} \text{ X } \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} = H_1(z)\, H_2(z) \quad \text{——(1)}$$

$$x(n) \rightarrow \boxed{H(z)} \rightarrow y(n) \quad = \quad x(n) \rightarrow \boxed{H_1(z)} \xrightarrow{v(n)} \boxed{H_2(z)} \rightarrow y(n)$$

i.e, $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{V(z)}{X(z)} \cdot \dfrac{Y(z)}{V(z)}$ ——(2)

$= H_1(z) \cdot H_2(z)$

$H_1(z) = \dfrac{V(z)}{X(z)} = \sum_{k=0}^{M} b_k z^{-k}$ ——(3) and $H_2(z) = \dfrac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$ ——(4)

Consider eqn(3)

$$H_1(z) = \frac{V(z)}{X(z)} = \sum_{k=0}^{M} b_k z^{-k} \quad \text{—} (3)$$

$$V(z) = \sum_{k=0}^{M} b_k X(z) z^{-k}$$

Taking inverse Z.T. we get

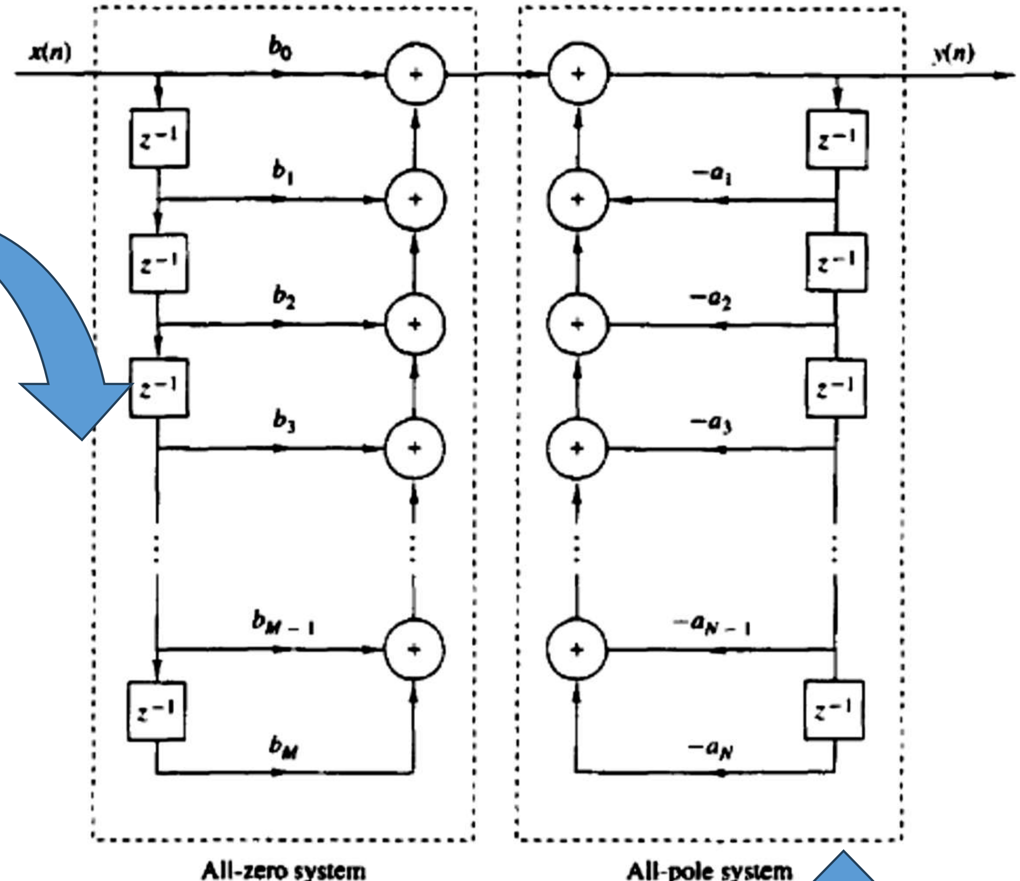
$$v(n) = \sum_{k=0}^{M} b_k x(n-k) \quad \text{—} (5)$$

Consider eqn(4) $H_2(z) = \frac{Y(z)}{V(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$

$$Y(z) + \sum_{k=1}^{N} a_k z^{-k} Y(z) = V(z)$$

$$Y(z) = V(z) - \sum_{k=1}^{N} a_k z^{-k} \cdot Y(z)$$

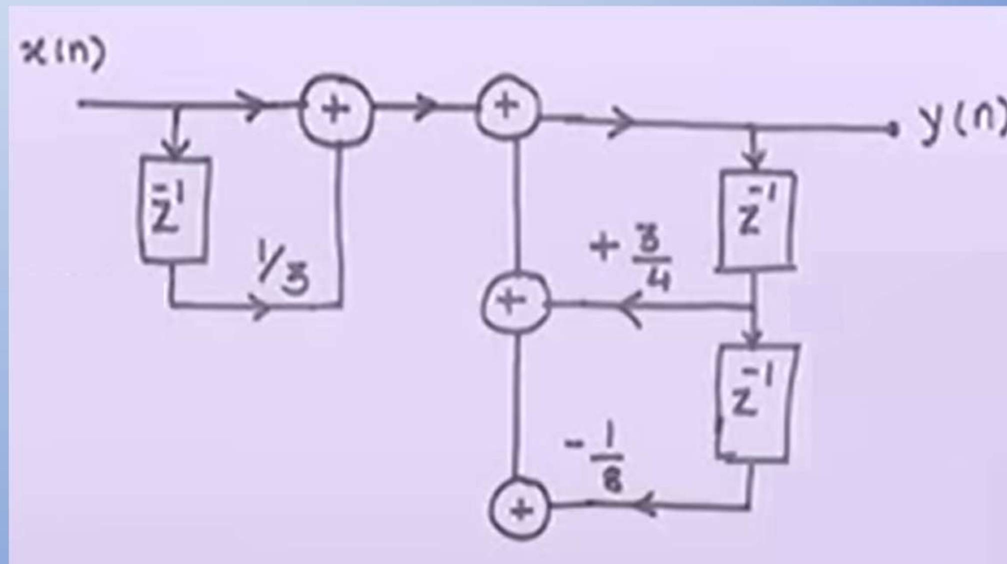
or $y(n) = v(n) - \sum_{k=1}^{N} a_k y(n-k) \quad \text{—} (6)$



Find Direct form I structure

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{3}x(n-1)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}.$$



Direct Form II structure realization:

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \times \sum_{k=0}^{M} b_k z^{-k} = H_1(z)\, H_2(z)$$

$$H_1(z) = \frac{V(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

and

$$H_2(z) = \frac{Y(z)}{V(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$V(z) + \sum_{k=1}^{N} a_k z^{-k} V(z) = X(z)$$

$$Y(z) = \sum_{k=0}^{M} b_k z^{-k} V(z)$$

Taking inverse Z.T.

$$v(n) = x(n) - \sum_{k=1}^{N} a_k v(n-k)$$

$$y(n) = \sum_{k=0}^{M} b_k v(n-k)$$

Let us rewrite

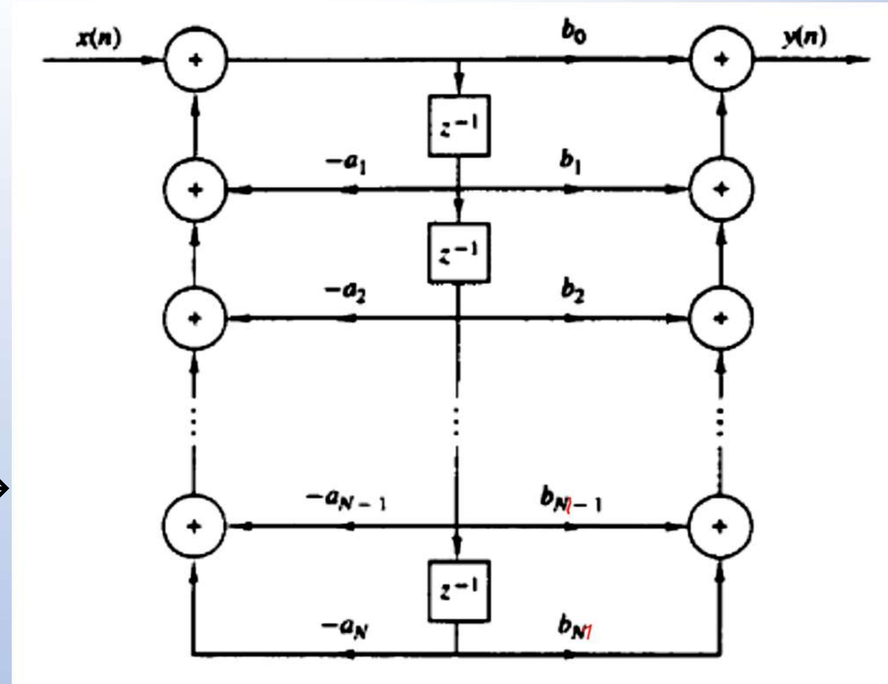
$$v(n) = x(n) - \sum_{k=1}^{N} a_k v(n-k)$$

$$v(n) = x(n) - a_1 v(n-1) - a_2 v(n-2) \cdots a_N v(n-N)$$

$$y(n) = \sum_{k=0}^{M} b_k v(n-k)$$

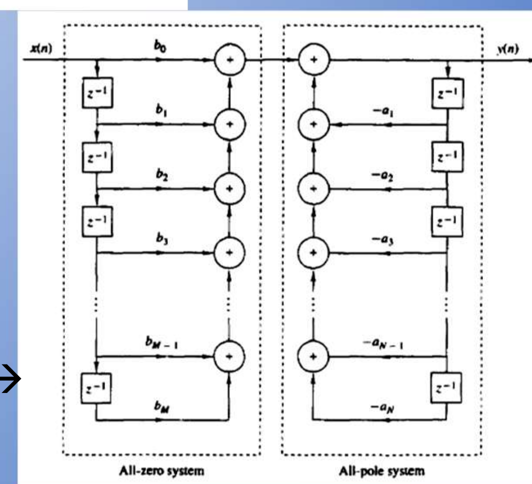
$$y(n) = b_0 v(n) + b_1 v(n-1) + \cdots b_M v(n-M)$$

DF 2→



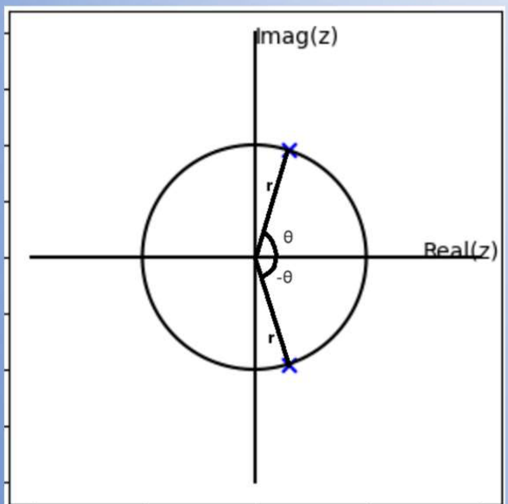
Delay blocks are reduced – Canonic form

DF 1 →



Two pole structures:

- If zero/pole is complex – difficult to implement (complex coefficients)
- This can be converted into structure with real coefficients – 2 pole structures



$$H(z) = \frac{1}{(1 - p_1 z^{-1})(1 - p_1^* z^{-1})}$$

$$H(z) = \frac{1}{(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})}$$

$$= \frac{1}{1 - (re^{j\theta} + re^{-j\theta})z^{-1} + r^2 z^{-2}}$$

$$H(z) = \frac{1}{1 - 2r\cos\theta z^{-1} + r^2 z^{-2}}$$

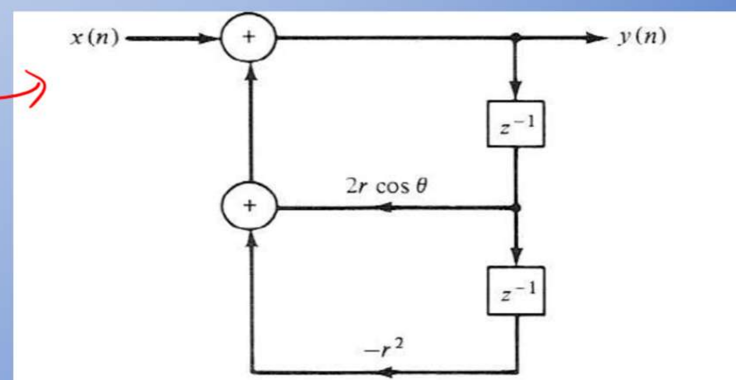


FIGURE 7.44 Realization of a two-pole IIR filter.

Recap – Goertzel algorithm:

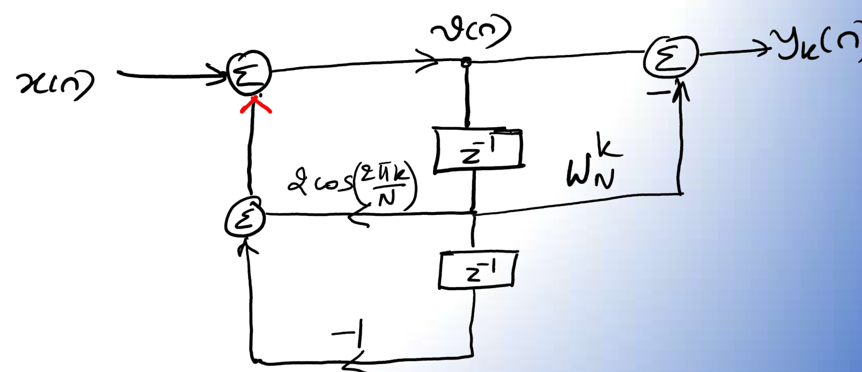
$$h(n) = W_N^{-nk} u(n) \Longleftrightarrow H(z) = \frac{1}{1 - W_N^{-k} z^{-1}} \quad \text{—} (5)$$

Now we can realize the structure for equation (5) as follows:

$$H_k(z) = \frac{1}{1 - W_N^{-k} z^{-1}} \frac{(1 - W_N^{k} z^{-1})}{(1 - W_N^{k} z^{-1})} = \frac{1 - W_N^{k} z^{-1}}{1 - (W_N^{k} + W_N^{-k}) z^{-1} + z^{-2}}$$

$$H_k(z) = \frac{1 - W_N^{k} z^{-1}}{1 - 2\cos\left(\frac{2\pi k}{N}\right) z^{-1} + z^{-2}} \quad \text{—} (6)$$

Let $H_k(z) = \dfrac{Y_k(z)}{X(z)} = \dfrac{1 - W_N^{k} z^{-1}}{1 - 2\cos\left(\frac{2\pi k}{N}\right) z^{-1} + z^{-2}}$



Direct form II realization for computing k^{th} DFT point


Thank you