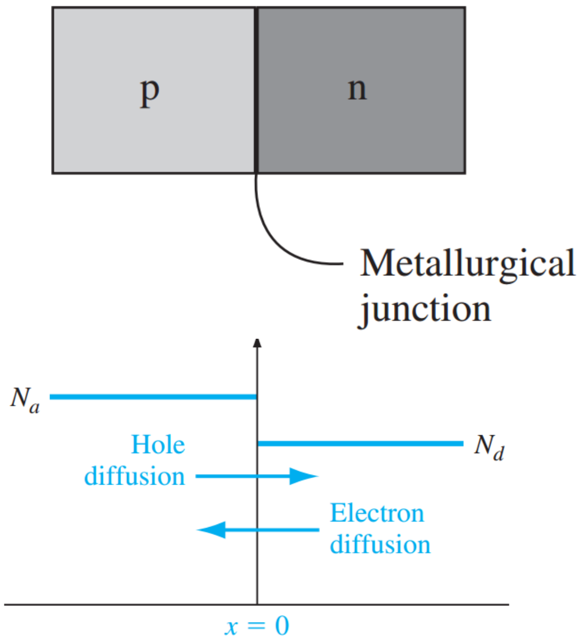
The p-n Junction

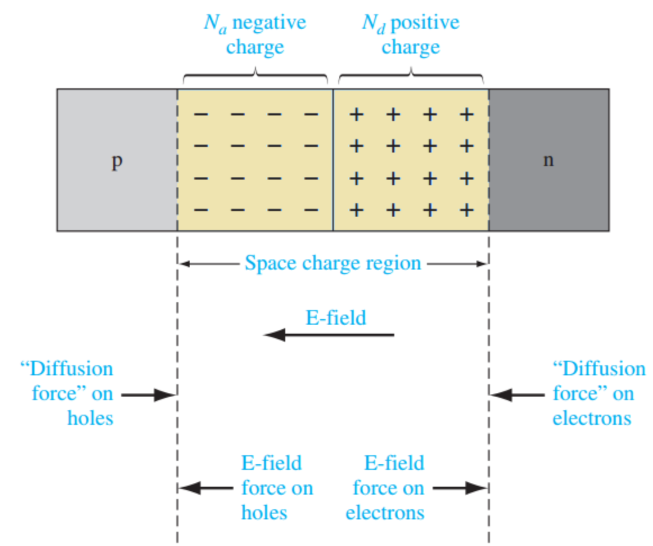
BASIC STRUCTURE OF THE pn JUNCTION



- It is important to realize that the entire semiconductor is a single-crystal material in which one region is doped with acceptor atoms and the adjacent region is doped with donor atoms.
- The interface separating regions is referred to as the *Metallurgical junction.*
- For simplicity, we will consider a *step junction* in which the doping concentration is uniform.
- Initially, at the metallurgical junction, there is a very large density gradient in both electron and hole concentrations.
- Majority carrier electrons in the n region will begin diffusing into the p region.
- Majority carrier holes in the p region will begin diffusing into the n region.

BASIC STRUCTURE OF THE pn JUNCTION

- If we assume there are no external connections to the semiconductor, then this diffusion process cannot continue indefinitely.
- As electrons diffuse from the n region, positively charged donor atoms are left behind.
- Similarly, as holes diffuse from the p region, they uncover negatively charged acceptor atoms.
- The net positive and negative charges in the n and p regions induce an electric field in the region near the metallurgical junction.
- The direction of the induced electric field is from the positive to the negative charge, or from the n to the p region.



BASIC STRUCTURE OF THE pn JUNCTION

- Regions with charged impurity atoms are referred to as the *space charge region.*

- Essentially all electrons and holes are swept out of the space charge region by the electric field.

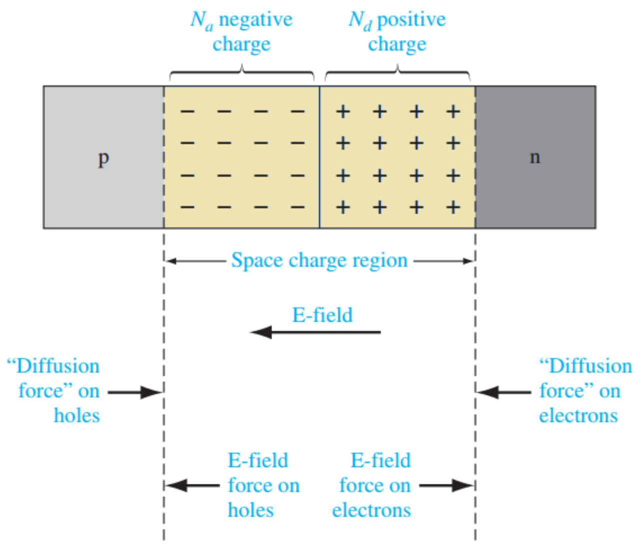
- Since the space charge region is depleted of any mobile charge, this region is also referred to as the *Depletion region*.

- Density gradients still exist in the majority carrier concentrations at each edge of the space charge region.

- Diffusion forces, acts on the electrons and holes at the edges of the space charge region.

- The electric field produces another force which is in the opposite direction to the diffusion forces.

- In thermal equilibrium, the diffusion force and the E-field force exactly balance each other.



BUILT-IN POTENTIAL BARRIER

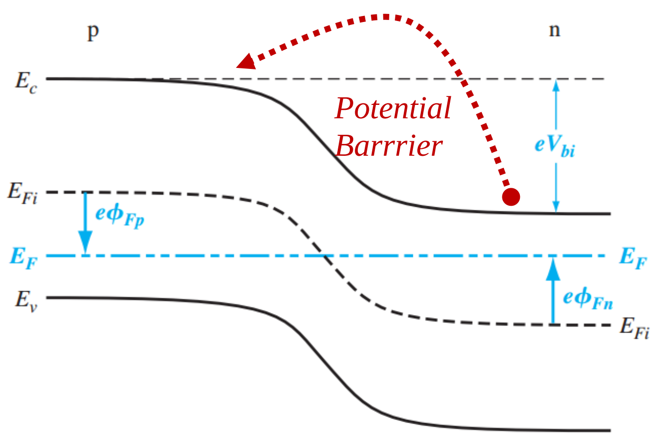
- If we assume that no voltage is applied across the pn junction, then the junction is in thermal equilibrium —the Fermi energy level is constant throughout the entire system.

- Figure shows the energy-band diagram for the pn junction in thermal equilibrium.

- The conduction and valance band energies must bend as we go through the space charge region, since the relative position of the conduction and valence bands with respect to the Fermi energy changes between p and n regions.

- Electrons in the conduction band of the n region see a potential barrier in trying to move into the conduction band of the p region.

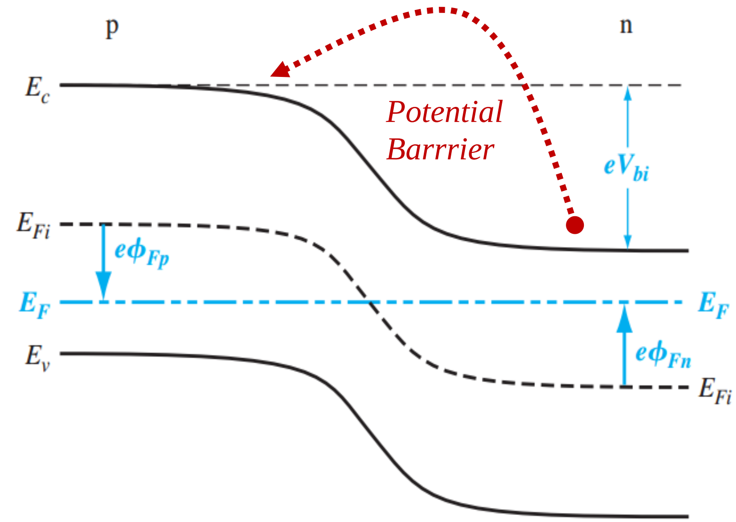
- This potential barrier is referred to as the *Built-in potential barrier* and is denoted by V_{bi}.



BUILT-IN POTENTIAL BARRIER

- The built-in potential barrier maintains equilibrium between electron in n side (majority) and p side (minority) and also between hole in n side (minority) and p side (majority).

- Is it possible to measure potential barrier using Voltmeter?



- The potential V_{bi} maintains equilibrium, so no current is produced by this voltage.

- The intrinsic Fermi level is equidistant from the conduction band edge through the junction.

- Thus, the built-in potential barrier can be determined as the difference between the intrinsic Fermi levels in the p and n regions.

$$V_{bi} = |e\varphi_{Fn}| + |e\varphi_{Fp}|$$

BUILT-IN POTENTIAL BARRIER

In the n region, the electron concentration in the conduction band is given by

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

which can also be written in the form

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

where n_i and E_{Fi} are the intrinsic carrier concentration and the intrinsic Fermi energy, respectively. We may define the potential ϕ_{Fn} in the n region as

$$e\phi_{Fn} = E_{Fi} - E_F$$

$$\Rightarrow n_0 = n_i \exp\left[\frac{-(e\phi_{Fn})}{kT}\right]$$

Taking the natural log of both sides of Equation, setting $n_0 = N_d$,

$$\phi_{Fn} = \frac{-kT}{e} \ln\left(N_d / n_i \right)$$

BUILT-IN POTENTIAL BARRIER

Similarly, In the p region

$$\varphi_{Fp} = \frac{+kT}{e} \ln \left({}^{N_a}/{}_{n_i}\right)$$

$$\boxed{V_{bi} = |\varphi_{Fn}| + |\varphi_{Fp}| = \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln \left(\frac{N_a N_d}{n_i^2}\right)}$$

where $V_t = kT/e$ and is defined as the Thermal voltage.

BUILT-IN POTENTIAL BARRIER

OBJECTIVE: Calculate the built-in potential barrier in a pn junction.
Consider a silicon pn junction at T = 300 K with doping concentrations of Na =2 x10^{17} cm^{-3} and Nd = 10^{15} cm^{-3}.

The built-in potential barrier is determined as

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left[\frac{(2\times10^{17})(10^{15})}{(1.5\times10^{10})^2}\right] = 0.713V$$

If we change the doping concentration in the p region of the pn junction such that the doping concentrations become N_a = 10^{16} cm^{-3} and N_d = 10^{15} cm^{-3}, then the built-in potential barrier becomes V_{bi} = 0.635 V

The built-in potential barrier changes only slightly as the doping concentrations change by orders of magnitude because of the logarithmic dependence.

If we change the doping concentration in the p region of the pn junction such that the doping concentrations become $N_a = 10^{16}$ cm^{-3} and $N_d = 10^{15}$ cm^{-3}, then the built-in potential barrier becomes ?

$$V_{bi} = 0.635 \text{ V}$$

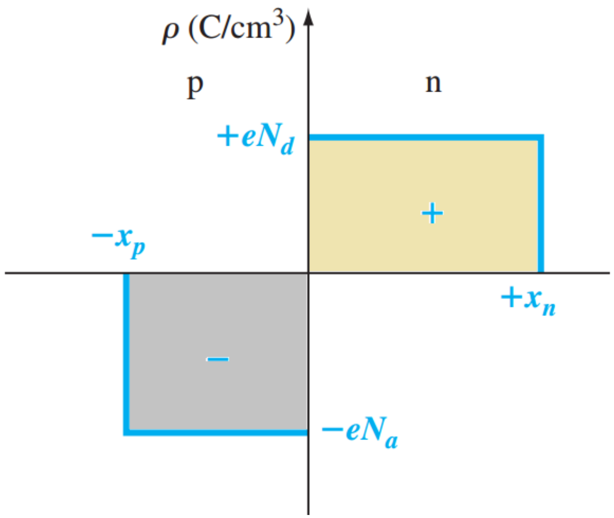
(a) Calculate the built-in potential barrier in a silicon pn junction at $T = 300$ K for (i) $N_a = 5 \times 10^{15}$ cm^{-3}, $N_d = 10^{17}$ cm^{-3} and (ii) $N_a = 2 \times 10^{16}$ cm^{-3}, $N_d = 2 \times 10^{15}$ cm^{-3}.
(b) Repeat part (a) for a GaAs pn junction.

[Ans. (a) (i) 0.736 V, (ii) 0.671 V; (b) (i) 1.20 V, (ii) 1.14 V]

Calculate the built-in potential barrier, V_{bi}, for Si, Ge, and GaAs pn junctions if they each have the following dopant concentrations at $T = 300$ K:

(a) $N_d = 10^{14}$ cm^{-3}	$N_a = 10^{17}$ cm^{-3}	Si: $V_{bi} = 0.635$ V	Si: $V_{bi} = 0.778$ V	Si: $V_{bi} = 0.814$ V
(b) $N_d = 5 \times 10^{16}$	$N_a = 5 \times 10^{16}$	Ge: $V_{bi} = 0.253$ V	Ge: $V_{bi} = 0.396$ V	Ge: $V_{bi} = 0.432$ V
(c) $N_d = 10^{17}$	$N_a = 10^{17}$	GaAs: $V_{bi} = 1.10$ V	GaAs: $V_{bi} = 1.25$ V	GaAs: $V_{bi} = 1.28$ V

ELECTRIC FIELD



- An electric field is created in the depletion region by the separation of positive and negative space charge densities.

- Figure shows the volume charge density distribution in the pn junction assuming step junction

- The electric field is determined from Poisson's equation, which, for a one dimensional analysis, is

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\varepsilon_s} = -\frac{dE(x)}{dx}$$

where $\phi(x)$ is the electric potential, E(x) is the electric field, $\rho(x)$ is the volume charge density, and ε_s is the permittivity of the semiconductor

- From Figure the charge densities (charge/volume) are

$$\rho(x) = -eN_a \qquad -x_p < x < 0 \qquad\qquad \rho(x) = eN_d \qquad 0 < x < x_n$$

ELECTRIC FIELD

- The electric field in the p region is found by poisson's equation as

$$E = \int \frac{\rho(x)}{\varepsilon_s} dx = \int \frac{-eN_a}{\varepsilon_s} dx = \frac{-eN_a}{\varepsilon_s} x + C_1$$

- The electric field is assumed to be zero in the neutral p region for $x < -x_p$ since the currents are zero in thermal equilibrium.

- Since there are no surface charge densities within the pn junction structure, the electric field is a continuous function.

- The constant of integration is determined by setting E=0 at $x=-x_p$. The electric field in the p region is then given by

$$E = \frac{-eN_a}{\varepsilon_s}\left(x + x_p\right) \qquad -x_p \le x \le 0$$

- In the n region, the electric field is determined from

$$E = \int \frac{eN_d}{\varepsilon_s} dx = \frac{eN_d}{\varepsilon_s} x + C_2$$

ELECTRIC FIELD

- C_2 is again a constant of integration and is determined by setting E=0 at $x = x_n$, since the E-field is assumed to be zero in the n region and is a continuous function. Then

$$E = \frac{-eN_d}{\varepsilon_s}\left(x_n - x\right) \qquad 0 \le x \le x_n$$

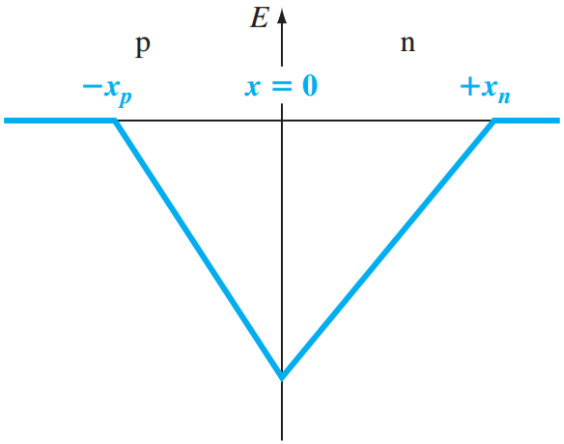
- The electric field is also continuous at the metallurgical junction, or at $x = 0$.

$$N_a x_p = N_d x_n$$

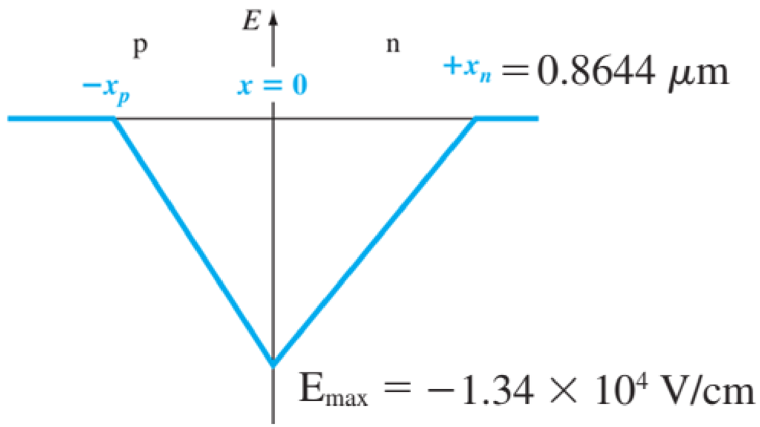
- Above equation states that the number of negative charges per unit area in the p region is equal to the number of positive charges per unit area in the n region.

- The electric field direction is from the n to the p region, or in the negative x direction for this geometry.

- For the uniformly doped pn junction, the E-field is a linear function of distance through the junction.

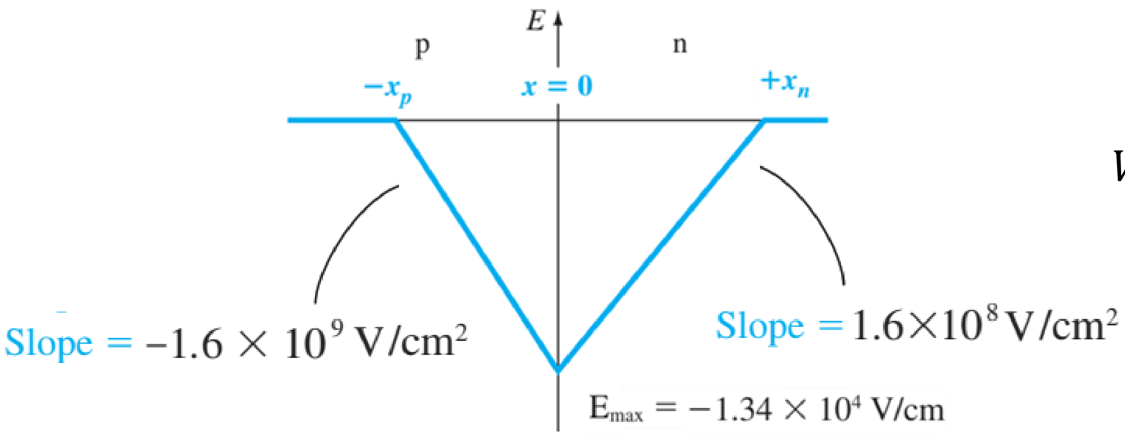


The electric profile for silicon p-n junction diode at T=300K is shown below, It has a depletion width of 0.951μm. Calculate the built-in potential.



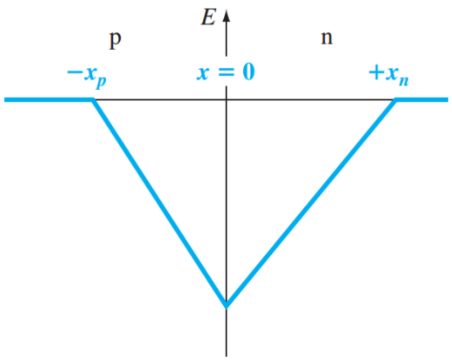
$V_{bi} = 0.635$ V.

The electric profile for silicon p-n junction diode at T=300K is shown below, Calculate the built-in potential and the depletion width.



$V_{bi} = 0.6368V$ and $W = 0.92125 \mu m$

POTENTIAL



- The maximum (magnitude) electric field occurs at the metallurgical junction.

- An electric field exists in the depletion region even when no voltage is applied between the p and n regions.

- The potential in the junction is found by integrating the electric field. In the p region then, we have

$$\phi(x) = -\int E(x)dx = \int \frac{eN_a}{\varepsilon_s}(x + x_p)dx$$

$$\phi(x) = \frac{eN_a}{\varepsilon_s}\left(\frac{x^2}{2} + x_p \cdot x\right) + C_1'$$

- The potential difference through the pn junction is the important parameter, rather than the absolute potential.

- We may arbitrarily set the potential equal to zero at $x = x_p$. The constant of integration is then found as

BUILT IN POTENTIAL

$$C_1' = \frac{eN_a}{2\varepsilon_s} x_p^2$$

so that the potential in the p region can now be written as

$$\phi(x) = \frac{eN_a}{2\varepsilon_s}(x + x_p)^2 \qquad \left(-x_p \le x \le 0\right)$$

We determine the potential in the n region by integrating the electric field

$$\phi(x) = \int \frac{eN_d}{\varepsilon_s}(x_n - x)dx$$

$$\phi(x) = \frac{eN_d}{\varepsilon_s}(x_n \cdot x - \frac{x^2}{2}) + C_2'$$

The potential is a continuous function, so at the metallurgical junction, or at x=0, gives

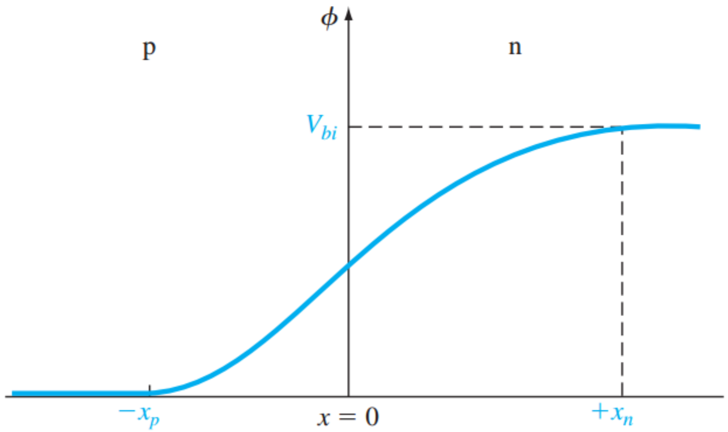
$$C_2' = \frac{eN_a}{2\varepsilon_s} x_p^2$$

$$\phi(x) = \frac{eN_d}{\varepsilon_s}(x_n \cdot x - \frac{x^2}{2}) + \frac{eN_a}{2\varepsilon_s} x_p^2 \qquad (0 \le x \le x_n)$$

POTENTIAL

- Figure is a plot of the potential through the junction and shows the quadratic dependence on distance.

- The magnitude of the potential at $x = x_n$ is equal to the built in potential barrier.



- Then we have

$$V_{bi} = |\phi(x = x_n)| = \frac{e}{2\varepsilon_s}(N_d x_n^2 + N_a x_p^2)$$

- The potential energy of an electron is given by $E = e\phi$, which means that the electron potential energy also varies as a quadratic function of distance through the space charge region.

SPACE CHARGE WIDTH

We can determine the distance that the space charge region extends into the p and n regions from the metallurgical junction. This distance is known as the space charge width.

$$N_a x_p = N_d x_n \Rightarrow x_p = \frac{N_d x_n}{N_a}$$

Then, substituting in the V_{bi} equation and solving for x_n, we obtain

$$\boxed{x_n = \left\{ \frac{2\varepsilon_s V_{bi}}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}}$$

Above equation gives the space charge width, or the width of the depletion region, x_n extending into the n-type region for the case of zero applied voltage.

Similarly solving for x_p

$$\boxed{x_p = \left\{ \frac{2\varepsilon_s V_{bi}}{e} \left[\frac{N_d}{N_a} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}}$$

where x_p is the width of the depletion region extending into the p region for the case of zero applied voltage.

SPACE CHARGE WIDTH

The total depletion or space charge width W is the sum of the two components, or

$$W = x_p + x_n$$

$$\boxed{W = \left\{ \frac{2\varepsilon_s V_{bi}}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}}$$

OBJECTIVE: Calculate the space charge width and electric field in a pn junction for zero bias.

Consider a silicon pn junction at T =300 K with doping concentrations of N_a=10^{16} cm^{-3} and N_d =10^{15} cm^{-3}.

In previous example we determined the built-in potential barrier as V_{bi} = 0.635 V.

The space charge width is

$$W = \left\{ \frac{2\varepsilon_s V_{bi}}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2} = \left\{ \frac{2(11.7)(8.85\times10^{-14})(0.635)}{1.6\times10^{-19}} \left[\frac{10^{16} + 10^{15}}{10^{16} \times 10^{15}} \right] \right\}^{1/2}$$

$$W = 0.951\times10^{-4}\,cm = 0.951\mu m$$

SPACE CHARGE WIDTH

We can also find x_n *and* x_p *as* x_n =0.8644 μm, and x_p = 0.0864 μm.

The peak electric field at the metallurgical junction is,

$$E = \frac{-eN_d}{\varepsilon_s}(x_n) = -\frac{(1.6\times10^{-19})(1\times10^{15})(0.8644\times10^{-4})}{(11.7)(8.85\times10^{-14})} = -1.34\times10^4 V/cm$$

- The peak electric field in the space charge region of a pn junction is quite large.
- We must keep in mind, however, that there is no mobile charge in this region; hence there will be no drift current.
- We may also note, that the width of each space charge region is a reciprocal function of the doping concentration
- The depletion region will extend further into the lower-doped region.

A silicon pn junction at $T = 300$ K with zero applied bias has doping concentrations of $N_d = 5 \times 10^{16}$ cm^{-3} and $N_a = 5 \times 10^{15}$ cm^{-3}. Determine x_n, x_p, W, and $|\mathrm{E}_{max}|$.

(Ans. $x_n = 4.11 \times 10^{-6}$ cm, $x_p = 4.11 \times 10^{-5}$ cm, $W = 4.52 \times 10^{-5}$ cm, $|\mathrm{E}_{max}| = 3.18 \times 10^4$ V/cm)

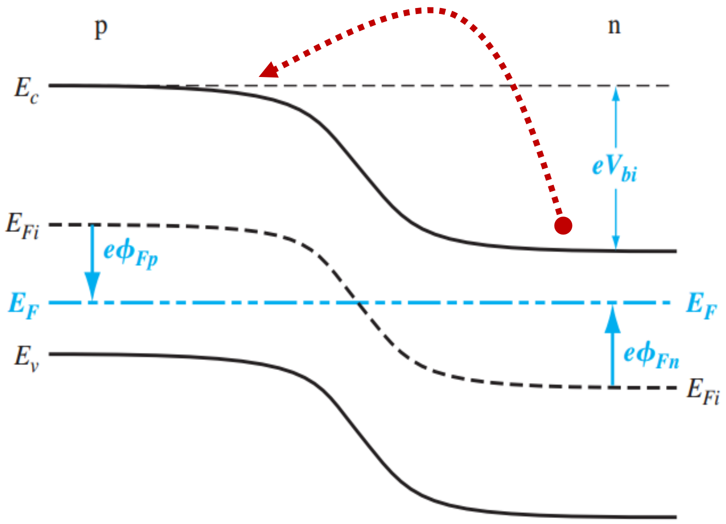
Calculate V_{bi}, x_n, x_p, W, and $|\mathrm{E}_{max}|$ for a silicon pn junction at zero bias and $T = 300$ K for doping concentrations of (*a*) $N_a = 2 \times 10^{17}$ cm^{-3}, $N_d = 10^{16}$ cm^{-3} and (*b*) $N_a = 4 \times 10^{15}$ cm^{-3}, $N_d = 3 \times 10^{16}$ cm^{-3}.

[Ans. (*a*) $V_{bi} = 0.772$ V, $x_n = 0.3085$ μm, $x_p = 0.0154$ μm, $W = 0.3240$ μm, $\mathrm{E}_{max}| = 4.77 \times 10^4$ V/cm; (*b*) $V_{bi} = 0.699$ V, $x_n = 0.0596$ μm, $x_p = 0.4469$ μm, $W = 0.5064$ μm, $|\mathrm{E}_{max}| = 2.76 \times 10^4$ V/cm]

An abrupt Si p-n junction has $N_a = 10^{18}\,\text{cm}^{-3}$ on one side and $N_d = 5 \times 10^{15}\,\text{cm}^{-3}$ on the other.

(a) Calculate the Fermi level positions at 300 K in the p and n regions.

(b) Draw an equilibrium band diagram for the junction and determine the contact potential V_0 from the diagram.



$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \qquad p_0 = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right]$$

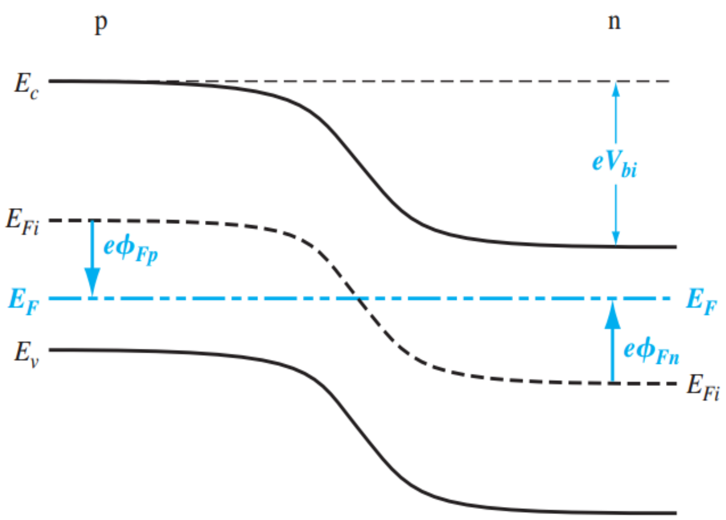
$$E_F - E_{Fi} = kT \ln \frac{n_0}{n_i} \qquad E_F - E_{Fi} = 0.329\text{eV}$$

$$E_{Fi} - E_F = kT \ln \frac{p_0}{n_i} \qquad E_{Fi} - E_F = 0.467\text{eV}$$

$$eV_{bi} = e|\varphi_{Fn}| + e|\varphi_{Fp}| \qquad eV_{bi} = 0.467\text{eV} + 0.329\text{ eV} = 0.796\text{eV}$$

$$eV_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.796\text{eV}$$

A silicon pn junction in thermal equilibrium at $T = 300$ K is doped such that $E_F - E_{Fi} = 0.365$ eV in the n region and $E_{Fi} - E_F = 0.330$ eV in the p region. (*a*) Sketch the energy-band diagram for the pn junction. (*b*) Find the impurity doping concentration in each region. (*c*) Determine V_{bi}.



$$eV_{bi} = e|\varphi_{Fn}| + e|\varphi_{Fp}|$$

$$eV_{bi} = 0.467\text{eV} + 0.329 \text{ eV} = 0.796\text{eV}$$

$$eV_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.796\text{eV}$$

$$N_d = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] \qquad N_D = 1.97 \times 10^{16}\ cm^{-3}$$

$$N_a = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right] \qquad N_a = 5.12 \times 10^{15}\ cm^{-3}$$

An abrupt silicon pn junction at zero bias has dopant concentrations of $N_a = N_d = 2 \times 10^{16}$ cm^{-3}, $T = 300$ K. (*a*) Calculate the Fermi level on each side of the junction with respect to the intrinsic Fermi level. (*b*) Sketch the equilibrium energy- band diagram for the junction and determine V_{bi} (*c*) Determine x_n, x_p, and the peak electric field for this junction.

(*a*) n side: $E_F - E_{Fi} = 0.3653$ eV,
p side: $E_{Fi} - E_F = 0.3653$ eV;

(*b*) $V_{bi} = 0.7306$ V;

(*c*) $x_n = 0.154\ \mu$m, $x_p = 0.154\ \mu$m,
$|E_{max}| = 4.75 \times 10^4$ V/cm

A silicon pn junction at $T = 300$ K with zero applied bias has doping concentrations of $N_d = 5 \times 10^{16}$ cm^{-3} and $N_a = 5 \times 10^{15}$ cm^{-3}. Determine $|E_{max}|$ and junction capacitance

$$E_{\max} = \left\{\frac{2e(V_{bi})}{\varepsilon_s}\left[\frac{N_a N_d}{N_a + N_d}\right]\right\}^{1/2}$$

$$C' = \left\{\frac{e\varepsilon_s N_a N_d}{2(V_{bi})(N_a + N_d)}\right\}^{1/2}$$

$$E_{max} = 3.18 \times 10^4\ V/cm$$

$$C' = 0.2289 nF/cm^2$$

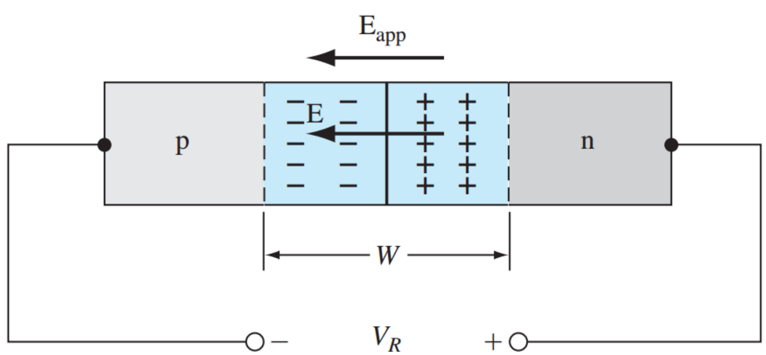
REVERSE APPLIED BIAS

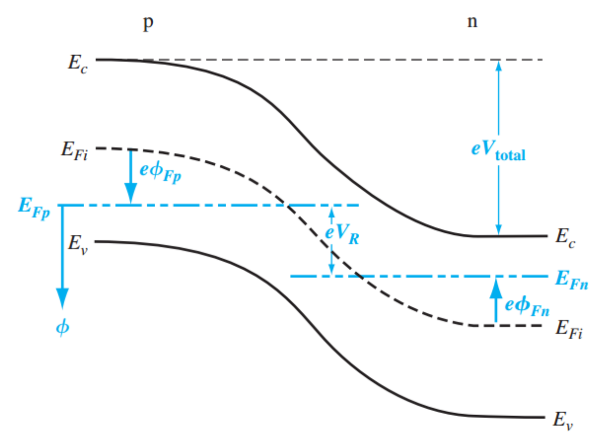
- If we apply a potential between the p and n regions, we will no longer be in an equilibrium condition—the Fermi energy level will no longer be constant through the system.

- Figure shows the energy-band diagram of the pn junction for the case when a positive voltage is applied to the n region with respect to the p region.

- As the positive potential is downward, the Fermi level on the n side is below the Fermi level on the p side.

- The difference between the two is equal to the applied voltage in units of energy.





REVERSE APPLIED BIAS

- The total potential barrier, indicated by V_{total}, has increased. The applied potential is the reverse-biased condition. The total potential barrier is now given by-

$$V_{Total} = |\phi_{Fn}| + |\phi_{Fp}| + V_R$$

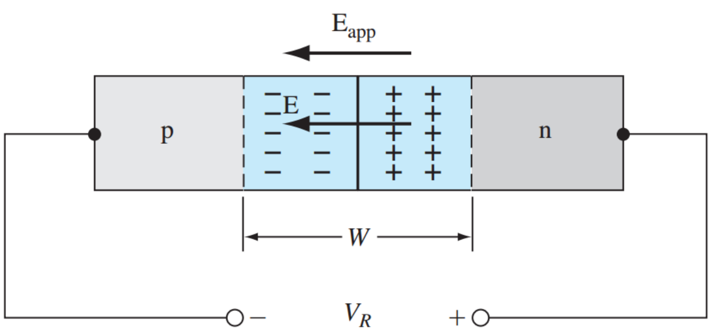
$$V_{Total} = V_{bi} + V_R$$

where V_R is the magnitude of the applied reverse-biased voltage.

SPACE CHARGE WIDTH AND ELECTRIC FIELD

- Figure shows the electric field in the space charge region and the electric field E_{app}, induced by the applied voltage.

- The electric fields in the neutral p and n regions are essentially zero, or at least very small.



- This means that the magnitude of the electric field in the space charge region must increase above the thermal-equilibrium value due to the applied voltage.

SPACE CHARGE WIDTH AND ELECTRIC FIELD

- The electric field originates on positive charge and terminates on negative charge; this means that the number of positive and negative charges must increase if the electric field increases.

- For given impurity doping concentrations, the number of positive and negative charges in the depletion region can be increased only if the space charge width W increases.

- The space charge width W increases, therefore, with an increasing reverse-biased voltage V_R.

- In all of the previous equations, the built-in potential barrier can be replaced by the total potential barrier.

$$\boxed{W = \left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{N_a+N_d}{N_aN_d}\right]\right\}^{1/2}}$$

The total space charge width increases as we apply a reverse-biased voltage.

$$\boxed{x_n = \left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a+N_d}\right]\right\}^{1/2}}$$

$$\boxed{x_p = \left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a+N_d}\right]\right\}^{1/2}}$$

SPACE CHARGE WIDTH AND ELECTRIC FIELD

OBJECTIVE: Calculate the width of the space charge region in a pn junction when a reverse biased voltage is applied.

Again consider a silicon pn junction at T=300K with doping concentrations of $N_a=10^{16}$cm^{-3} and $N_d=10^{15}$ cm^{-3}. Assume that $n_i= 1.5\text{x}10^{10}$ cm^{-3} and V_R =5V.

The built-in potential barrier was calculated in previous example for this case as V_{bi}=0.635 V.

The space charge width is determined as

$$W=\left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{N_a+N_d}{N_aN_d}\right]\right\}^{1/2}=\left\{\frac{2(11.7)(8.85\times10^{-14})(0.635+5)}{1.6\times10^{-19}}\left[\frac{10^{16}+10^{15}}{10^{16}\times10^{15}}\right]\right\}^{1/2}$$

$$W=2.83\times10^{-4}\,cm=2.83\,\mu m$$

The space charge width has increased from 0.951 μm at zero bias to 2.83 μm at a reverse bias of 5 V.

SPACE CHARGE WIDTH AND ELECTRIC FIELD

(a) A silicon pn junction at $T = 300$ K has doping concentrations of $N_a = 5 \times 10^{15}$ cm^{-3} and $N_d = 5 \times 10^{16}$ cm^{-3}. A reverse-biased voltage of $V_R = 4$ V is applied. Determine V_{bi}, x_n, x_p, and W. (b) Repeat part (a) for $V_R = 8$ V.

[Ans. (a) $V_{bi} = 0.718$ V, $x_n = 0.1054$ μm, $x_p = 1.054$ μm, $W = 1.159$ μm;
(b) $V_{bi} = 0.718$, $x_n = 0.1432$ μm, $x_p = 1.432$ μm, $W = 1.576$ μm]

SPACE CHARGE WIDTH AND ELECTRIC FIELD

- The magnitude of the electric field in the depletion region increases with an applied reverse-biased voltage.

- The electric field is still given by previously derived equation and is still a linear function of distance through the space charge region.

- Since x_n and x_p increase with reverse-biased voltage, the magnitude of the electric field also increases.

- The maximum electric field still occurs at the metallurgical junction.

- The maximum electric field at the metallurgical junction, is

$$E_{\max} = \frac{-eN_d x_n}{\varepsilon_s} = \frac{-eN_a x_p}{\varepsilon_s}$$

- Using $V_{total} = V_{bi} + V_R$ and equation of x_n in above equation

$$E_{\max} = \left\{ \frac{2e(V_R + V_{bi})}{\varepsilon_s} \left[\frac{N_a N_d}{N_a + N_d} \right] \right\}^{1/2}$$

SPACE CHARGE WIDTH AND ELECTRIC FIELD

OBJECTIVE: Design a pn junction to meet maximum electric field and voltage specifications.

Consider a silicon pn junction at T=300K with a p-type doping concentration of N_a= 2 x10^{17} cm^{-3}. Determine the n-type doping concentration such that the maximum electric field is $|E_{max}|$=2.5x10^5 V/cm at a reverse-biased voltage of V_R = 25 V.

Neglecting V_{bi} compared to V_R, we can write

$$|E_{\max}| \cong \left\{ \frac{2eV_R}{\varepsilon_s} \left[\frac{N_a N_d}{N_a + N_d} \right] \right\}^{1/2}$$

$$2.5\times10^5 \cong \left\{ \frac{2(1.6\times10^{-19})(25)}{(11.7)(8.85\times10^{-14})} \left[\frac{(2\times10^{17})N_d}{2\times10^{17} + N_d} \right] \right\}^{1/2}$$

$$N_d = 8.43\times10^{15}\,cm^{-3}$$

SPACE CHARGE WIDTH AND ELECTRIC FIELD

The maximum electric field in a reverse-biased GaAs pn junction at $T = 300$ K is to be limited to $|E_{max}| = 7.2 \times 10^4$ V/cm. The doping concentrations are $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 3 \times 10^{16}$ cm^{-3}. Determine the maximum reverse-biased voltage that can be applied.

(Ans. $V_R = 3.21$ V)

JUNCTION CAPACITANCE

The total depletion width W of the space charge region under reverse bias is

$$W = \left\{ \frac{2\varepsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

and

$$C' = \left\{ \frac{e\varepsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}$$

we find that we can write

$$C' = \frac{\varepsilon_s}{W}$$

same as the capacitance per unit area of a parallel plate capacitor

Keep in mind that the space charge width is a function of the reverse-biased voltage so that the junction capacitance is also a function of the reverse-biased voltage applied to the pn junction.

JUNCTION CAPACITANCE

OBJECTIVE: Calculate the junction capacitance of a pn junction.

Again consider a silicon pn junction at T=300K with doping concentrations of N_a=10^{16}cm^{-3} and N_d =10^{15} cm^{-3}. Assume that n_i= 1.5x10^{10} cm^{-3} and V_R =5V.

The junction capacitance is found as

$$C' = \left\{ \frac{e\varepsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2} = \left\{ \frac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})(10^{16})(10^{15})}{2(0.635+5)(10^{16}+10^{15})} \right\}^{1/2}$$

$$C' = 3.66\times10^{-9}\, F/cm^2$$

If the cross-sectional area of the p-n junction is, for example, A=10^{-4} cm^2 , then the total junction capacitance is

$$C = C' \cdot A = 3.66 \times 10^{-13} F = 0.366 pF$$

- The value of junction capacitance is usually in the pF, or smaller, range.

Consider a uniformly doped GaAs pn junction at $T = 300$ K. The junction capacitance at zero bias is $C_j(0)$ and the junction capacitance with a 10-V reverse-biased voltage is $C_j(10)$. The ratio of the capacitances is

$$\frac{C_j(0)}{C_j(10)} = 3.13$$

Also under reverse bias, the space charge width into the p region is 0.2 of the total space charge width. Determine (*a*) V_{bi} and (*b*) N_a, N_d.

$$\frac{C_j(0)}{C_j(10)} = \frac{\left[\dfrac{\in N_a N_d}{2V_{bi}(N_a + N_d)}\right]^{1/2}}{\left[\dfrac{\in N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}\right]^{1/2}}$$

or

$$\frac{C_j(0)}{C_j(10)} = 3.13 = \left(\frac{V_{bi} + V_R}{V_{bi}}\right)^{1/2}$$

For $V_R = 10\,V$, we find

$$(3.13)^2 V_{bi} = V_{bi} + 10$$

or

$$\underline{V_{bi} = 1.14\ V}$$

(b)

$$x_p = 0.2W = 0.2(x_p + x_n)$$

Then

$$\frac{x_p}{x_n} = 0.25 = \frac{N_d}{N_a}$$

Now

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \Rightarrow$$

so

$$1.14 = (0.0259)\ln\left[\frac{0.25N_a^2}{(1.8x10^6)^2}\right]$$

We can then write

$$N_a = \frac{1.8x10^6}{\sqrt{0.25}}\exp\left[\frac{1.14}{2(0.0259)}\right]$$

or

$$\underline{N_a = 1.3x10^{16}\ cm^{-3}}$$

and

$$N_d = 3.25x10^{15}\ cm^{-3}$$

Consider a silicon p-n junction at T=300K with doping concentrations of N_a=10^{16}cm^{-3} and N_d =10^{15} cm^{-3}. Assume that n_i= 1.5x10^{10} cm^{-3} . Calculate a) Vbi b) xn c) xp d) W e) Emax f) Cj, For V_R =0V & V_R =4V.

For V_R =0V	For V_R =4V
Vbi= 0.6350V	Vbi= 0.6350V
xn= 0.864μm	xn = 2.335μm
Xp= 0.0864μm	xp = 0.2335μm
W= 0.9505μm	W= 2.566μm
Emax=1.335×10^4 V/cm	Emax=9.75×10^4 V/cm
Cj = 10.89 nF/cm^2	Cj = 4.032 nF/cm^2

An abrupt silicon pn junction at $T = 300$ K is uniformly doped with $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 5 \times 10^{15}$ cm^{-3}. The cross-sectional area of the pn junction is 8×10^{-4} cm^2. An inductance is placed in parallel with the pn junction. (*a*) With a reverse-biased voltage of $V_R = 10$ V applied to the pn junction, the resonant frequency of the circuit is $f = 1.25$ MHz. What is the value of the inductance? (*b*) Using the results of part (*a*), what is the resonant frequency if the reverse-biased voltage is (*i*) $V_R = 1$ V and (*ii*) $V_R = 5$ V?

$$V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{17})(5\times10^{15})}{(1.5\times10^{10})^2}\right]$$

$$= 0.7543 \text{ V}$$

(a) $$C = AC' = A\left\{\frac{e\in_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right\}^{1/2}$$

$$= (8\times10^{-4})\left\{\frac{(1.6\times10^{-19})(11.7)(8.85\times10^{-14})}{2(0.7543+10)} \times \frac{(2\times10^{17})(5\times10^{15})}{(2\times10^{17}+5\times10^{15})}\right\}^{1/2}$$

$$C = 4.904\times10^{-12} \text{ F}$$

$$f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L = \frac{1}{C(2\pi f)^2}$$

$$L = \frac{1}{(4.904\times10^{-12})[2\pi(1.25\times10^6)]^2}$$

$$= 3.306\times10^{-3} \text{ H} = 3.306 \text{ mH}$$

(b)

(i) For $V_R = 1$ V, $C = 12.14$ pF

$$f = \frac{1}{2\pi[(3.306\times10^{-3})(12.14\times10^{-12})]^{1/2}}$$

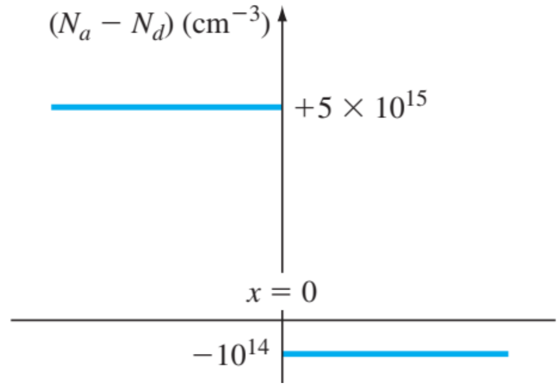
$$= 7.94\times10^5 \text{ Hz} = 0.794 \text{ MHz}$$

(ii) For $V_R = 5$ V, $C = 6.704$ pF

$$f = \frac{1}{2\pi[(3.306\times10^{-3})(6.704\times10^{-12})]^{1/2}}$$

$$= 1.069\times10^6 \text{ Hz} = 1.069 \text{ MHz}$$

A silicon pn junction at $T = 300$ K has the doping profile shown in Figure P7.28. Calculate (*a*) V_{bi}, (*b*) x_n and x_p at zero bias, and (*c*) the applied bias required so that $x_n = 30\ \mu$m.



(a) $$V_{bi} = (0.0259)\ln\left[\frac{(5\times10^{15})(10^{14})}{(1.5\times10^{10})^2}\right]$$

or

$$V_{bi} = 0.5574\ \text{V}$$

(b)

$$x_p = \left[\frac{2\in_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.5574)}{1.6\times10^{-19}}\right.$$

$$\left.\times\left(\frac{10^{14}}{5\times10^{15}}\right)\left(\frac{1}{10^{14}+5\times10^{15}}\right)\right]^{1/2}$$

$$x_p = 5.32 \times 10^{-6}\ \text{cm}$$

Also

$$x_n = \left[\frac{2\in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a + N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)(8.85 \times 10^{-14})(0.5574)}{1.6 \times 10^{-19}}\right.$$

$$\left. \times \left(\frac{5 \times 10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14} + 5 \times 10^{15}}\right)\right]^{1/2}$$

or

$$x_n = 2.66 \times 10^{-4}\ \text{cm}$$

(c) For $x_n = 30\ \mu\text{m}$, we have

$$30 \times 10^{-4} = \left[\frac{2(11.7)(8.85 \times 10^{-14})(V_{bi} + V_R)}{1.6 \times 10^{-19}}\right.$$

$$\left. \times \left(\frac{5 \times 10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14} + 5 \times 10^{15}}\right)\right]^{1/2}$$

which becomes

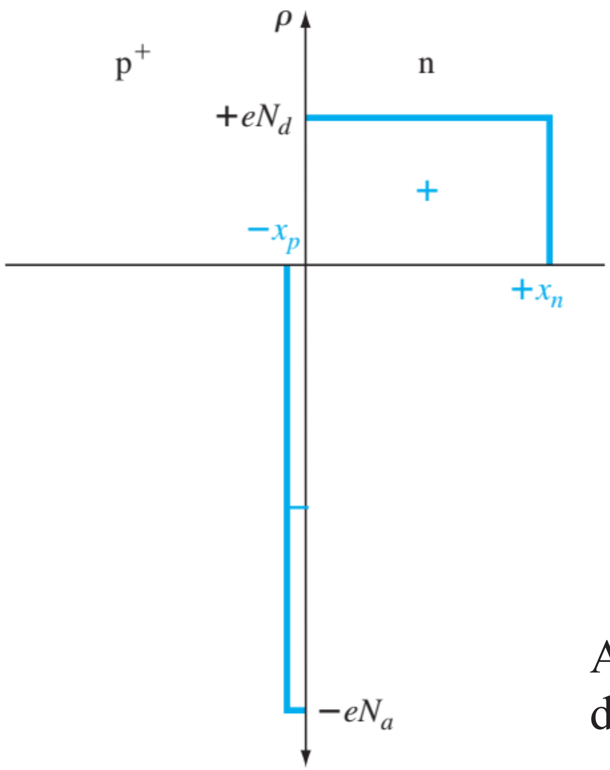
$$9 \times 10^{-6} = 1.269 \times 10^{-7}(V_{bi} + V_R)$$

We find

$$V_R = 70.4\ \text{V}$$

ONE-SIDED JUNCTION

- Special p-n junction called the one-sided junction. If *Na>Nd,* this junction is referred to as a p+n junction.



$$x_p \ll x_n$$

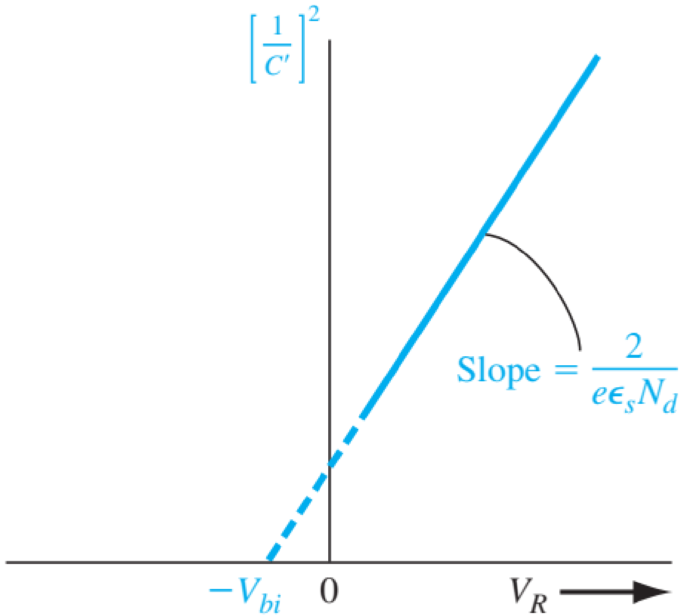
$$W \approx x_n$$

$$W \approx \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{eN_d} \right\}^{1/2}$$

Almost the entire space charge layer extends into the low-doped region of the junction

ONE-SIDED JUNCTION

- The junction capacitance of the p+n junction

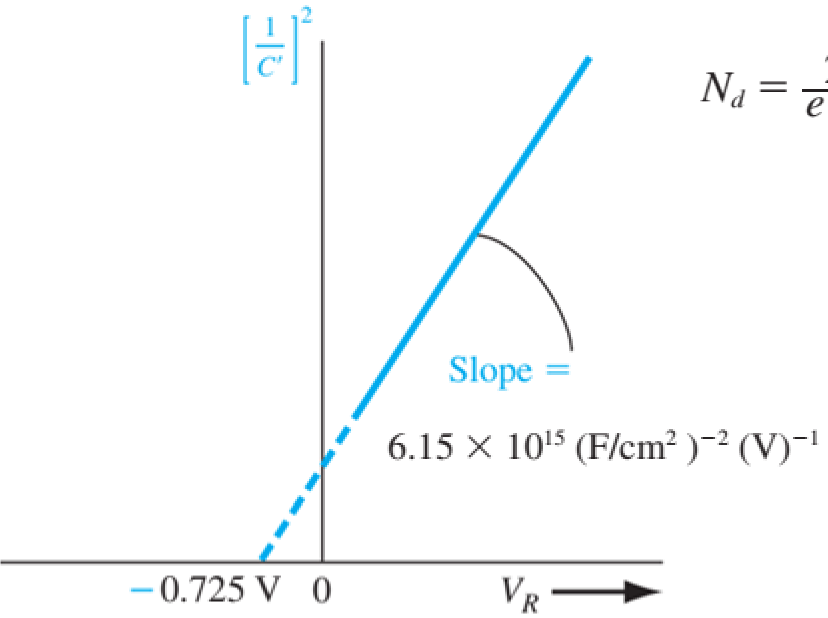


$$C' \approx \left\{\frac{e\epsilon_s N_d}{2(V_{bi} + V_R)}\right\}^{1/2}$$

$$\left(\frac{1}{C'}\right)^2 = \frac{2(V_{bi} + V_R)}{e\epsilon_s N_d}$$

The slope of the curve is inversely proportional to the doping concentration of the low-doped region in the junction

Determine the doping concentrations Na and Nd in p+n junction.



$$N_d = \frac{2}{e\,\epsilon_s} \cdot \frac{1}{slope} = \frac{2}{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(6.15 \times 10^{15})}$$

$$N_d = 1.96 \times 10^{15}\ \text{cm}^{-3}$$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

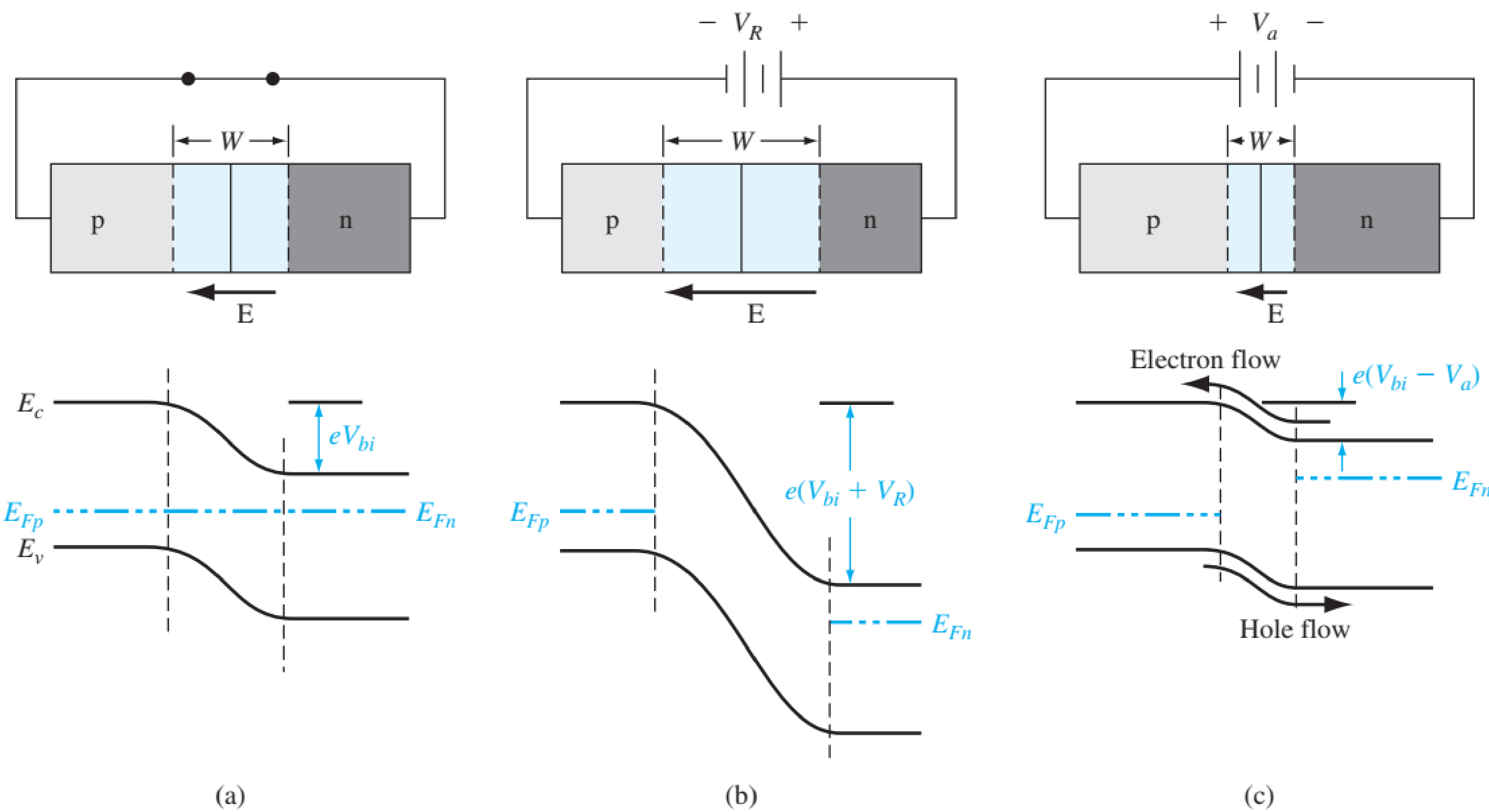
$$N_a = \frac{n_i^2}{N_d} \exp\left(\frac{V_{bi}}{V_t}\right) = \frac{(1.5 \times 10^{10})^2}{1.963 \times 10^{15}} \exp\left(\frac{0.725}{0.0259}\right)$$

$$N_a = 1.64 \times 10^{17}\ \text{cm}^{-3}$$

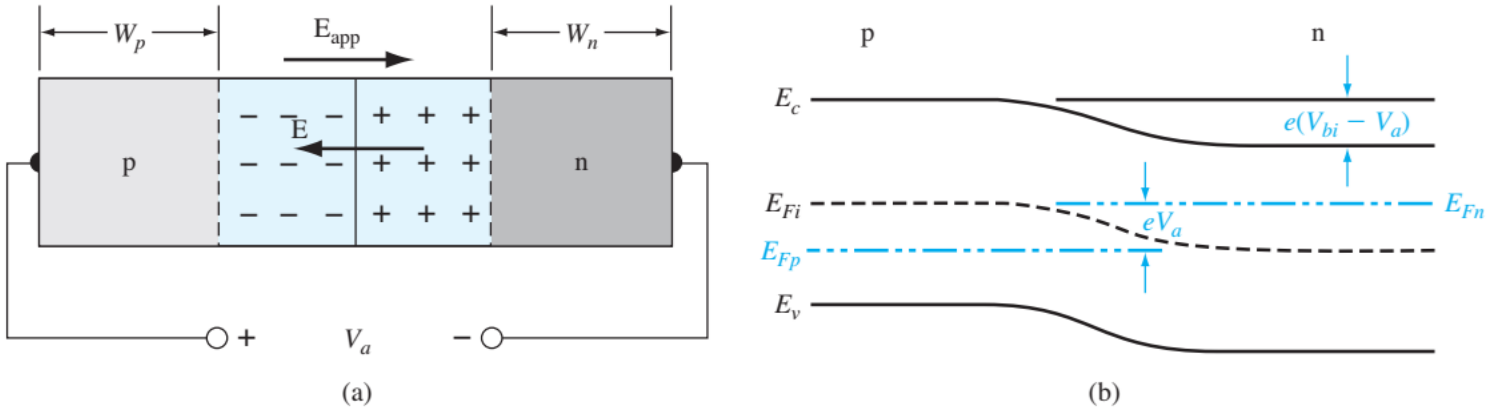
The experimentally measured junction capacitance of a one-sided silicon n^+p junction biased at $V_R = 3$ V and at $T = 300$ K is $C = 0.105$ pF. The built-in potential barrier is found to be $V_{bi} = 0.765$ V. The cross-sectional area is $A = 10^{-5}$ cm^2. Find the doping concentrations.

(Ans. $N_a = 5.01 \times 10^{15}$ cm^{-3}, $N_d = 3.02 \times 10^{17}$ cm^{-3})

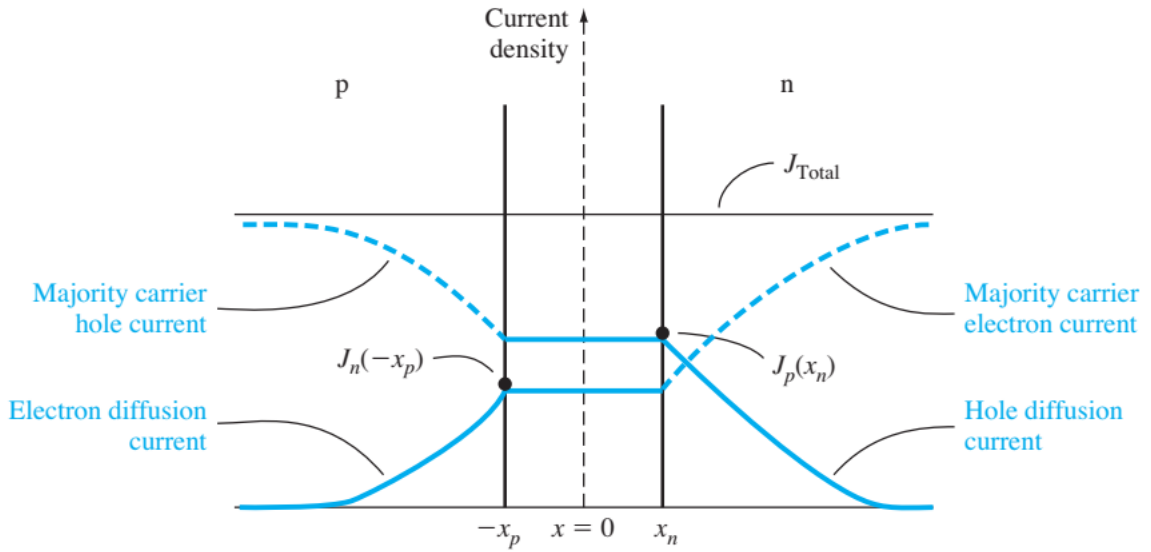
(*a*) Consider a uniformly doped silicon pn junction at $T = 300$ K. At zero bias, 25 percent of the total space charge region is in the n-region. The built-in potential barrier is $V_{bi} = 0.710$ V. Determine (*i*) N_a, (*ii*) N_d, (*iii*) x_n, (*iv*) x_p, and (*v*) $|E_{max}|$. (*b*) Repeat part (*a*) for a GaAs pn junction with $V_{bi} = 1.180$ V.



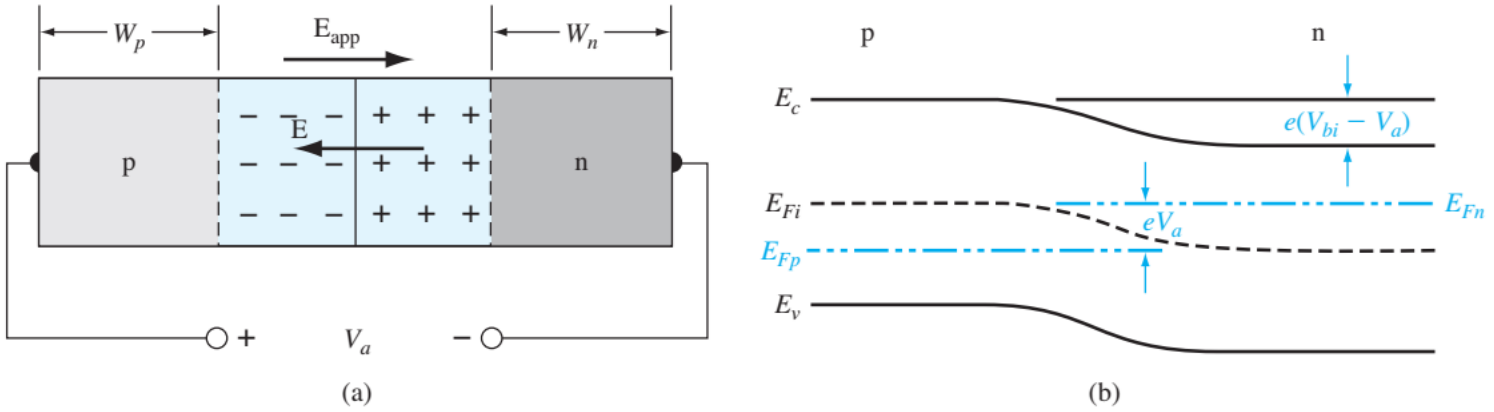
A pn junction and its associated energy-band diagram for (a) zero bias, (b) reverse bias, and (c) forward bias.



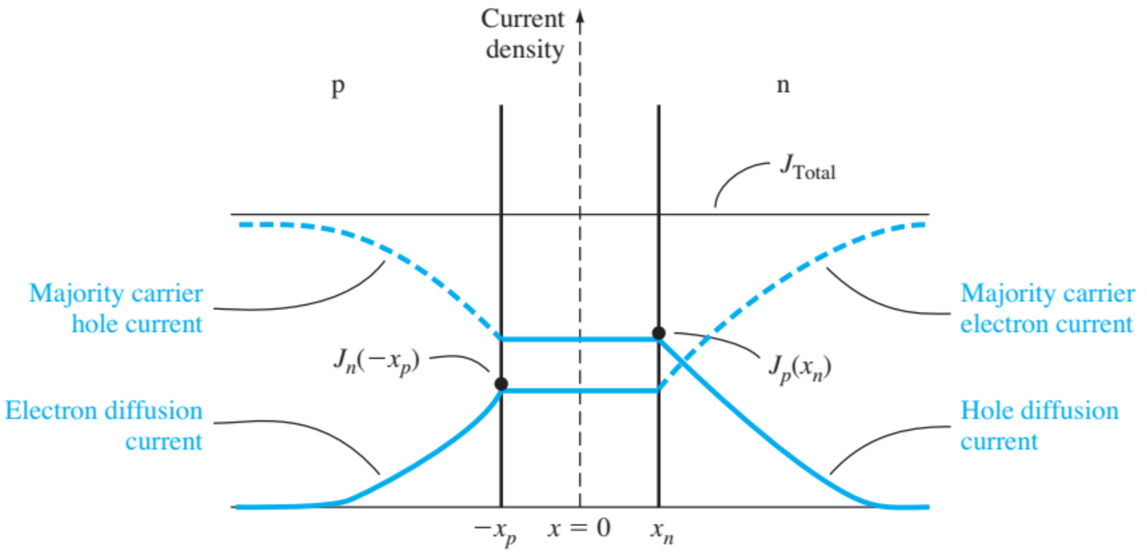
(a) A pn junction with an applied forward-bias voltage showing the directions of the electric field induced by V_a and the space charge electric field. (b) Energy-band diagram of the forward-biased pn junction.



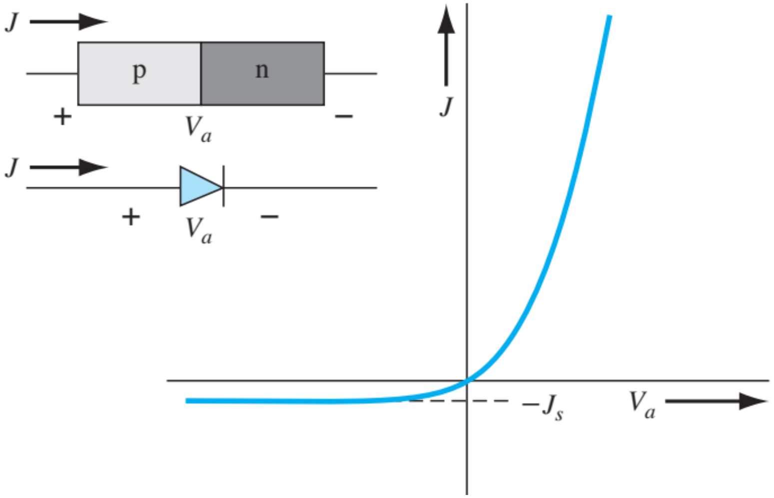
Ideal electron and hole current components through a pn junction under forward bias.



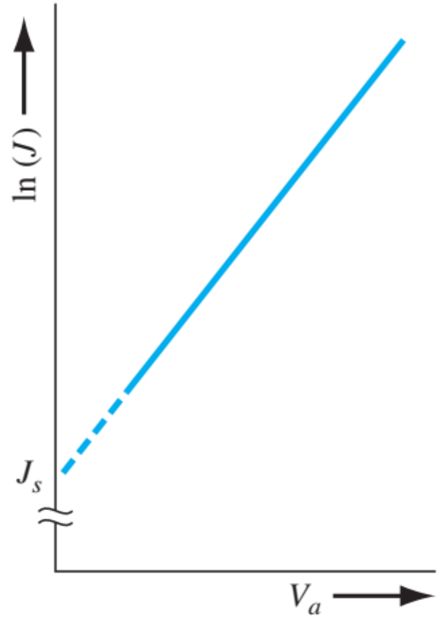
(a) A pn junction with an applied forward-bias voltage showing the directions of the electric field induced by V_a and the space charge electric field. (b) Energy-band diagram of the forward-biased pn junction.



Ideal electron and hole current components through a pn junction under forward bias.



$$\boxed{J = J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]}$$



JUNCTION BREAKDOWN

- Reverse-biased voltage may not increase without limit; at some particular voltage, the reverse-biased current will increase rapidly. The applied voltage at this point is called the *breakdown voltage.*

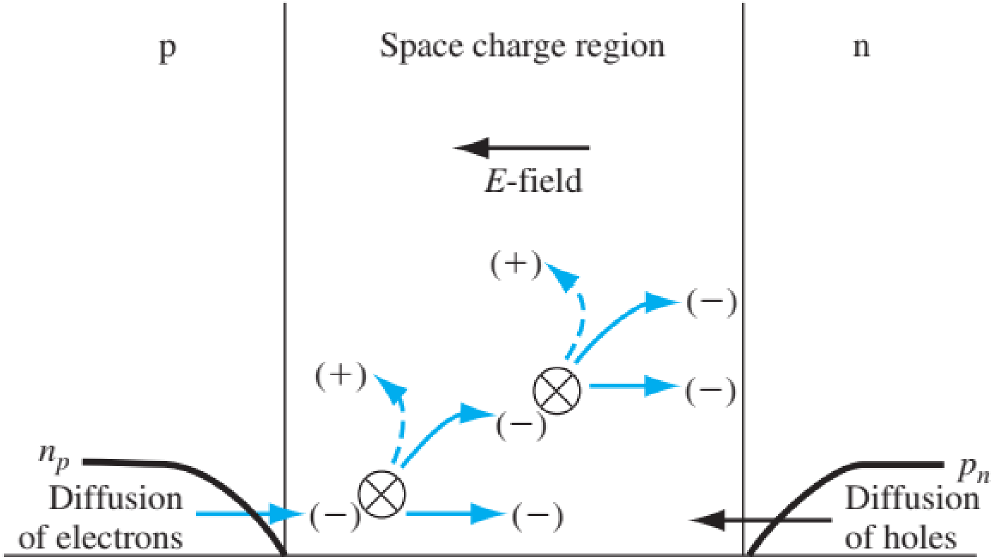
- Two physical mechanisms give rise to the reverse-biased breakdown in a pn junction: the *Zener effect* and the *Avalanche effect.*

- Zener breakdown occurs in highly doped pn junctions through a tunneling mechanism.

- Avalanche breakdown occurs in normally doped pn junctions through an avalanche multiplication.

AVALANCHE BREAKDOWN

- The avalanche breakdown process occurs when electrons and/or holes, moving across the space charge region, acquire sufficient energy from the electric field to create electron-hole pairs by colliding with atomic electrons within the depletion region.
- The newly created electrons and holes move in opposite directions due to the electric field and thereby create a reverse-biased current. In addition, the newly generated electrons and/or holes may acquire sufficient energy to ionize other atoms, leading to the avalanche process.

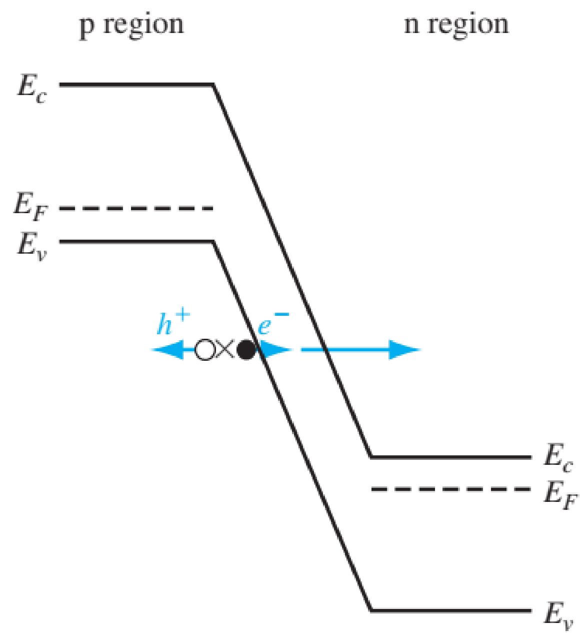


Avalanche breakdown process in a reverse-biased pn junction.

ZENER BREAKDOWN

- Zener breakdown occurs in highly doped pn junctions through a tunneling mechanism.

- In a highly doped junction, the conduction and valence bands on opposite sides of the junction are sufficiently close during reverse bias that electrons may tunnel directly from the valence band on the p side into the conduction band on the n side.



Zener breakdown mechanism in a reverse-biased pn junction

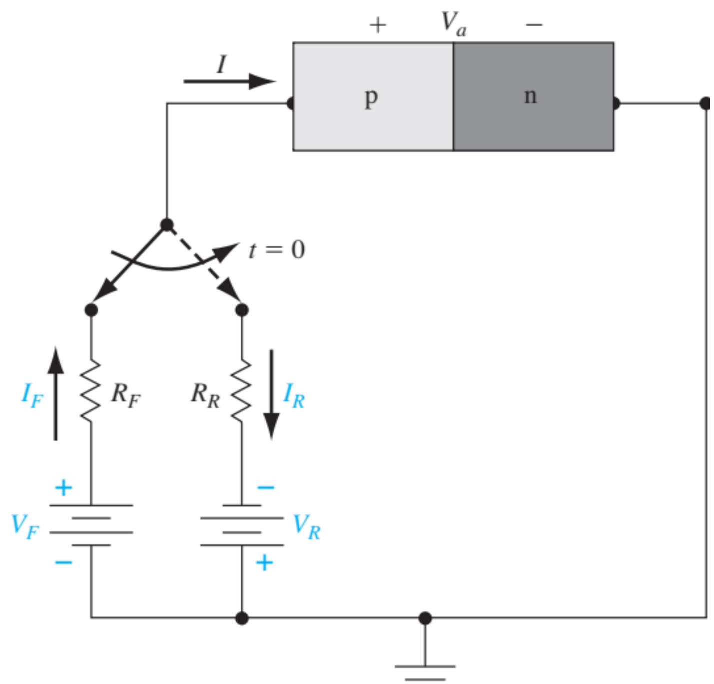
CHARGE STORAGE AND DIODE TRANSIENTS

- The pn junction is typically used as an electrical switch.

- In forward bias, referred to as the *on* state, a relatively large current can be produced by a small applied voltage; in reverse bias, referred to as the *off* state, only a very small current will exist.

- For $t < 0$, the forward-bias

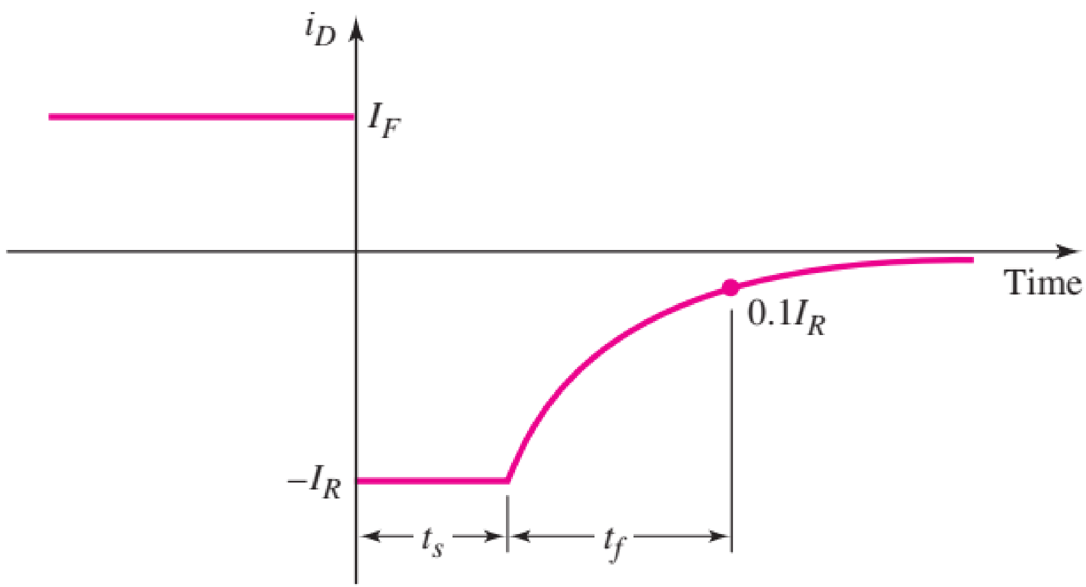
$$I = I_F = \frac{V_F - V_a}{R_F}$$



CHARGE STORAGE AND DIODE TRANSIENTS

- As the forward-bias voltage is removed, diode switches goes to reverse bias.
- Excess minority carrier electrons flow back across the junction into the n-region, and the excess minority carrier holes flow back across the junction into the p-region.
- The large reverse-bias current is initially limited by resistor R_R to approximately

$$I = -I_R \approx \frac{-V_R}{R_R}$$



CHARGE STORAGE AND DIODE TRANSIENTS

- The reverse current I_R is approximately constant for $0+ < t < t_s$, where t_s is the **storage time,** which is the length of time required for the minority carrier concentrations at the space-charge region edges to reach the thermal equilibrium values.

- After this time, the voltage across the junction begins to change. The fall time t_f is typically defined as the time required for the current to fall to 10 percent of its initial value.

- The total **turn-off time** is the sum of the storage time and the fall time.