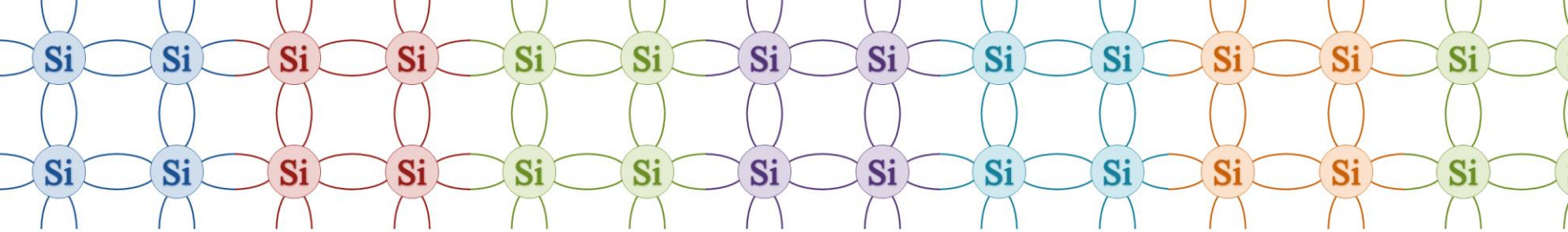
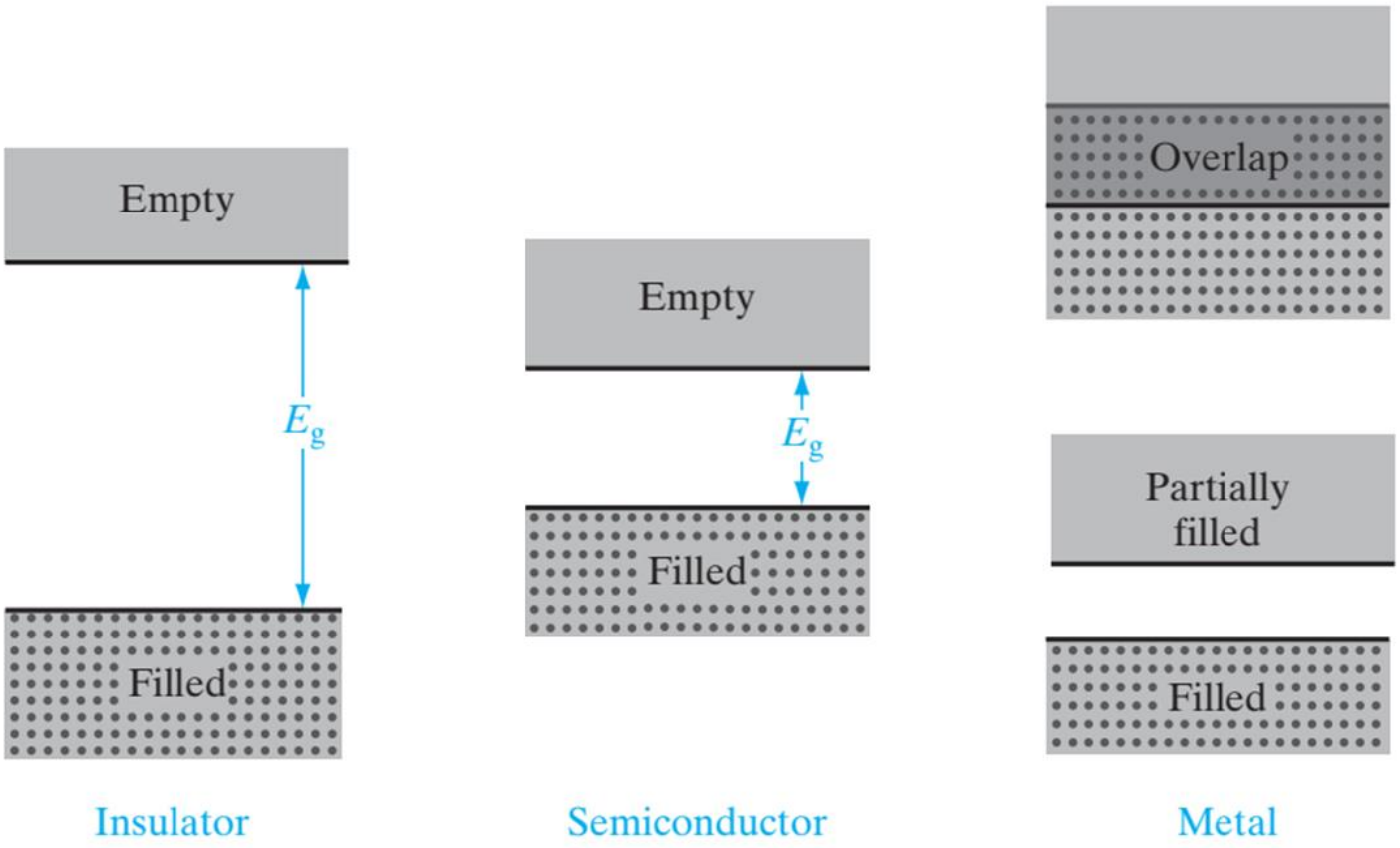


Physics of Semiconductor Devices

By Dr. Bhaskar Awadhiya

METAL, INSULATOR & SEMICONDUCTOR



Typical band structures at 0 K.

Semiconductor materials at 0K have the same structure as insulators—a filled valence band separated from an empty conduction band

FERMI-DIRAC PROBABILITY FUNCTION

We may write the most probable distribution function as

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

where

- E_F = Fermi energy.
- $f_F(E)$ = Fermi-Dirac distribution or probability function.

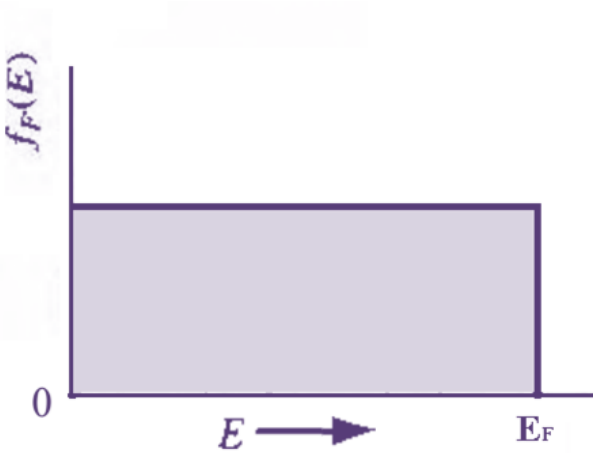
$f_F(E)$ gives the probability that a quantum state at the energy E will be occupied by an electron.

FERMI ENERGY

We plot the distribution function versus energy to understand the meaning of the distribution function and the Fermi Energy

Initially, let T = 0*K* and consider the case when $E < E_F$.

$$f_F(E < E_F) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k \cdot (0)}\right)} \Rightarrow \frac{1}{1 + \exp(-\infty)} = 1$$



Now, let T = 0*K* and consider the case when $E > E_F$.

$$f_F(E > E_F) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k \cdot (0)}\right)} \Rightarrow \frac{1}{1 + \exp(\infty)} = 0$$

This result shows that, for T = 0*K*, the electrons are in their lowest possible energy states. The probability of a quantum state being occupied is unity (100%) for $E < E_F$ and the probability of a state being occupied is zero for $E > E_F$. All electrons have energies below the Femi energy at T = 0*K*.

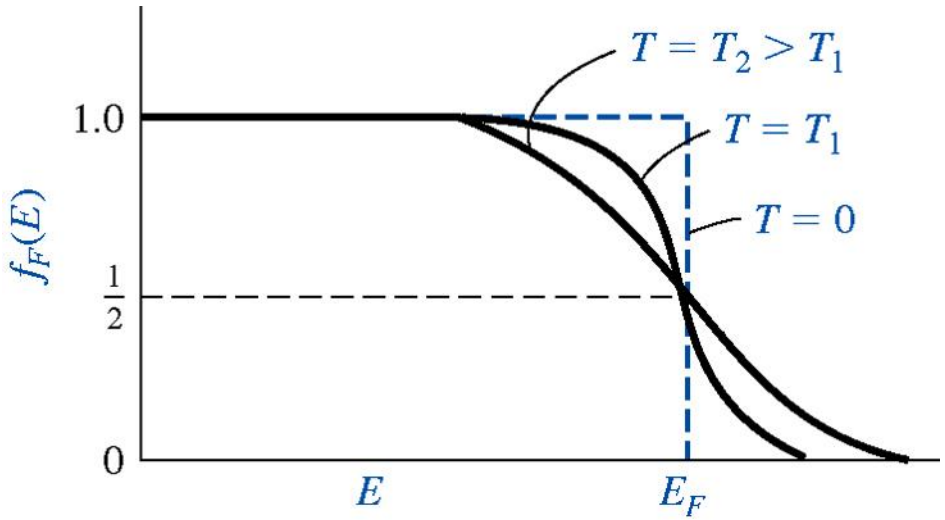
FERMI ENERGY

For T > 0K and consider the case when E = E_F.

$$f_F(E = E_F) = \frac{1}{1 + \exp(E - E_F/k.T)} \Rightarrow \frac{1}{1 + \exp(0)} = \frac{1}{2}$$

The probability of a state being occupied at $E = E_F$ is 0.5

Figure shows the Fermi-Dirac distribution function plotted for several temperatures, assuming the Fermi energy is independent of temperature.



As temperature increases the probability of finding electron beyond E_F increases.

FERMI ENERGY

EXAMPLE: To calculate the probability that an energy state above E_F is occupied by an electron.

Let T = 300 K. Determine the probability that an energy level 3kT above the Fermi energy is occupied by an electron.

$$f_F(E) = \frac{1}{1 + \exp(E - E_F/k.T)} = \frac{1}{1 + \exp(3kT/kT)}$$

$$f_F(E) = \frac{1}{1 + 20.09} = 0.0474 = 4.74\%$$

At energies above E_F, the probability of a state being occupied by an electron can become significantly less than unity or the ratio of electrons to available quantum states can be quite small.

FERMI ENERGY

EXAMPLE: Assume the Fermi energy level is 0.30 eV below the conduction band energy Ec. Assume $T = 300$ K. (a) Determine the probability of a state being occupied by an electron at $E = Ec + kT/4$. (b) Repeat part (a) for an energy state at $E = Ec + kT$.

[Ans. (a) 7.26×10^{-6}; (b) 3.43×10^{-6}]

FERMI ENERGY

EXAMPLE:

The Fermi energy at $T = 300$ K is 7.0 eV.

(a) Find the probability of an energy level at 7.15 eV being occupied by an electron.

(b) Repeat part (*a*) for $T = 1000$ K.

(c) Repeat part (*a*) for $E = 6.85$ eV and $T = 300$ K.

(d) Determine the probability of the energy state at $E = EF$ being occupied at $T = 300$ K and at $T = 1000$ K.

[Ans. a) 0.304% , b) 14.96% , c) 99.7% , d) 0.5]

FERMI ENERGY

Assume that the Fermi energy level is 0.35 eV above the valence band energy. Let $T = 300$ K. (a) Determine the probability of a state being empty of an electron at $E = E_v - kT/2$. (b) Repeat part (a) for an energy state at $E = E_v - 3kT/2$.

[Ans. (a) 8.20×10^{-7}; (b) 3.02×10^{-7}]

FERMI ENERGY

Assume that the Fermi energy level for a particular material is 6.25eV. Electrons in this material follow the Fermi-Dirac distribution function. Calculate the temperature at which there is 1% probability that a state 0.30eV below the Fermi energy level will not contain an electron. {$k=8.6173303\times10^{-5}\ eVK^{-1}$}

The probability that a state is empty is

$$1 - f_F(E) = 1 - \frac{1}{1 + \exp(E - E_F/k.T)}$$

$$0.01 = 1 - \frac{1}{1 + \exp(5.95 - 6.25/kT)}$$

Solving for kT we find kT = 0.06529eV, so that the temperature is T = 756 K.

The Fermi probability function is a strong function of temperature.

FERMI ENERGY

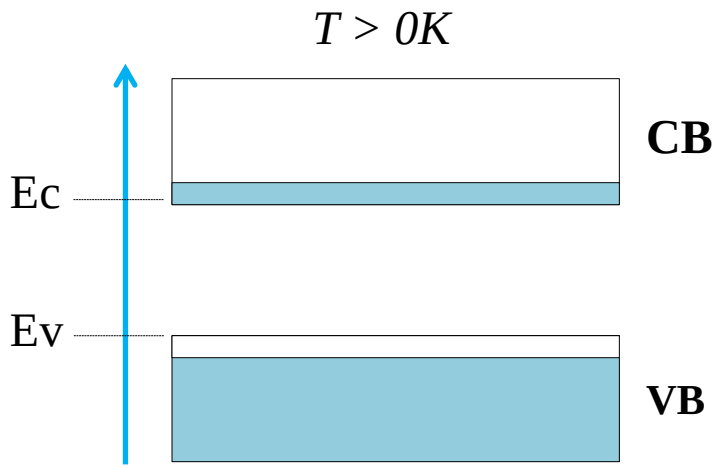
EXAMPLE Assume that EF is 0.3 eV below Ec. Determine the temperature at which the probability of an electron occupying an energy state at $E = (Ec + 0.025)$ eV is 8×10^{-6}.

(Ans. $T = 321$ K)

CHARGE CARRIERS IN SEMICONDUCTORS

Particles that can freely move and contribute to the current flow (conduction) are considered as Carriers

Semiconductors have 2 types of charge Carrier,
ELECTRON in CB,
HOLE in VB



The current in a semiconductor is determined largely by the number of carriers and hence an important characteristic of the semiconductor is the density of these charge carriers.

- For an intrinsic semiconductor, the concentration of electrons in CB is equal to the concentration of holes in the VB. [$n_0=p_0$]

- The Fermi energy level for the intrinsic semiconductor is called the intrinsic Fermi energy, or $E_F=E_{Fi}$

- The concentration of electrons in the conduction band is the effective density of states (*Nc)* times the probability of occupancy at *Ec*.

- Similarly, the concentration of holes in the valence band is the effective density of states (*Nv))* times the probability of non occupancy of electron at *Ev*

$$n_0 = Nc \times f(Ec)$$

$$f(\mathrm{Ec}) = \frac{1}{1+\exp\left[\frac{(E_C - E_{Fi})}{kT}\right]}$$

$$f(\mathrm{Ec}) = \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right]$$

$$n_0 = N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right]$$

$$p_0 = Nv \times \{1 - f(Ev)\}$$

$$f(Ev) = 1 - \frac{1}{1+\exp\left[\frac{(Ev - E_{Fi})}{kT}\right]}$$

$$f(Ev) = \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

$$p_0 = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

Taking product of the above two equations

$$n_0 p_0 = {n_i}^2 = N_C N_v \exp\left[\frac{-(E_C - E_v)}{kT}\right] = N_C N_v \exp\left[\frac{-Eg}{kT}\right]$$

where *Eg* is the band gap energy. For a given semiconductor material at a constant temperature, the value of n_i is a constant, and independent of the Fermi energy.

- N_C and N_v are constant for a given material (effective mass) and temperature.
- Position of Fermi energy is important
  - If E_F is closer to E_C than to E_v, n > p
  - If E_F is closer to E_v than to E_C, n < p

Effective density of states function and density of states effective mass values

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

Commonly accepted values of n_i at $T = 300$ K

Silicon	$n_i = 1.5 \times 10^{10}$ cm^{-3}
Gallium arsenide	$n_i = 1.8 \times 10^{6}$ cm^{-3}
Germanium	$n_i = 2.4 \times 10^{13}$ cm^{-3}

EQUILIBRIUM DISTRIBUTION OF ELECTRONS AND HOLES

EXAMPLE:To calculate the intrinsic carrier concentration in Gallium arsenide at T=*300K* and at *T = 450K*.

The values of N_C and N_V, at 300 K for gallium arsenide are 4.7 x 10^{17} cm^{-3} and 7.0 x $10^{18} cm^{-3}$, respectively.

Assume the band gap energy of gallium arsenide is 1.42eV and does not vary with temperature (over this range).

For $T = 300K$

$$n_i^2 = (4.7\times10^{17})(7.0\times10^{18})\exp\left[\frac{-1.42}{0.0259}\right]$$

$$n_i^2 = 5.09\times10^{12}$$

$$n_i = 2.26\times10^{6}\,cm^{-3}$$

For $T = 450K$

$$n_i^2 = (4.7\times10^{17})(7.0\times10^{18})\left(\frac{450}{300}\right)^3\exp\left[\frac{-1.42}{0.0388}\right]$$

$$n_i^2 = 1.48\times10^{21}$$

$$n_i = 3.85\times10^{10}\,cm^{-3}$$

The intrinsic carrier concentration is a very strong function of temperature.

EQUILIBRIUM DISTRIBUTION OF ELECTRONS AND HOLES

Objective: Calculate the intrinsic carrier concentration in silicon at $T = 250$ K and at $T = 400$ K.

The values of N_c and N_v for silicon at $T = 300$ K are 2.8×10^{19} cm^{-3} and 1.04×10^{19} cm^{-3}, respectively. Both N_c and N_v vary as $T^{3/2}$. Assume the bandgap energy of silicon is 1.12 eV and does not vary over this temperature range.

■ Solution

Using Equation (4.23), we find, at $T = 250$ K

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{250}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(250/300)}\right]$$

$$= 4.90 \times 10^{15}$$

or

$$n_i = 7.0 \times 10^7 \text{ cm}^{-3}$$

At $T = 400$ K, we find

$$n_i^2 = (2.8 \times 10^{19})(1.04 \times 10^{19})\left(\frac{400}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(400/300)}\right]$$

$$= 5.67 \times 10^{24}$$

or

$$n_i = 2.38 \times 10^{12} \text{ cm}^{-3}$$

■ Comment

We may note from this example that the intrinsic carrier concentration increased by over 4 orders of magnitude as the temperature increased by 150°C.

THE INTRINSIC FERMI LEVEL

We have qualitatively argued that the Fermi energy level is located near the center of the forbidden band gap for the intrinsic semiconductor.

Since the electron and hole concentrations are equal,

$$n_0 = p_0$$

$$N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right] = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

$$E_{Fi} = \tfrac{1}{2}(E_C + E_v) + \tfrac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$We\ define\ \ E_{midgap} = \tfrac{1}{2}(E_C + E_v)$$

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \qquad N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

THE INTRINSIC FERMI LEVEL

$$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

If the electron and hole effective masses are equal so that then the intrinsic Fermi level is exactly in the center of the band gap.

If $m_p^* > m_n^*$ the intrinsic Fermi level is slightly above the center.
If $m_p^* < m_n^*$ the intrinsic Fermi level is slightly below the center.

THE INTRINSIC FERMI LEVEL

EXAMPLE: To calculate the position of the intrinsic Fermi level with respect to the center of the bandgap in silicon at T = *300 K* .

The density of states effective carrier masses in silicon are $m_n^* = 1.08m_0$ *and* $m_p^* = 0.56m_0$.

$$E_{Fi} - E_{midgap} = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right) = \frac{3}{4}(0.0259)\ln\left(\frac{0.56}{1.08}\right)$$

$$E_{Fi} - E_{midgap} = -0.0128eV$$

THE INTRINSIC FERMI LEVEL

Determine the position of the intrinsic Fermi level at $T = 300$ K with respect to the center of the bandgap for (a) GaAs and (b) Ge.

[Ans. (a) +38.25 meV; (b) −7.70 meV]

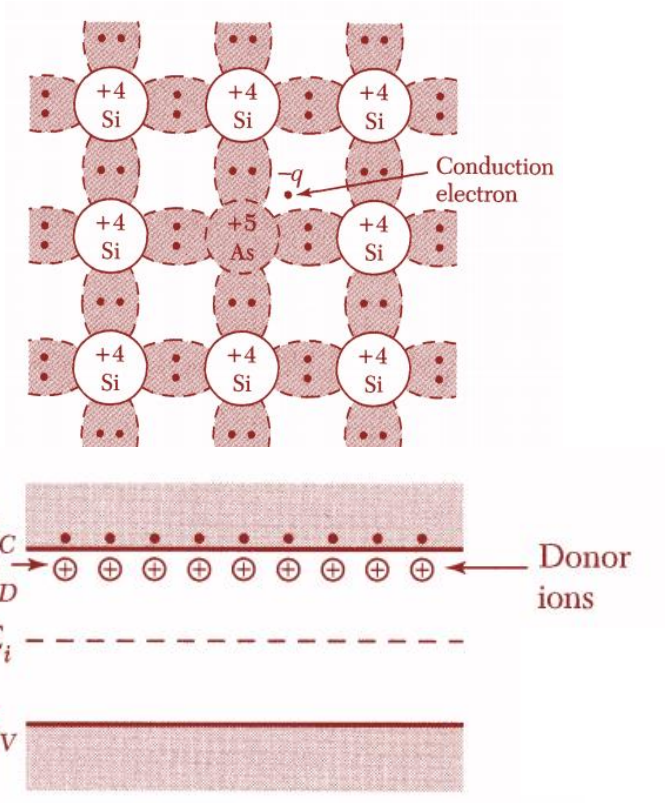
Determine the position of the intrinsic Fermi level with respect to the center of the bandgap in silicon at (a) $T = 200$ K and (b) $T = 400$ K. Assume the effective masses are constant over this temperature range.

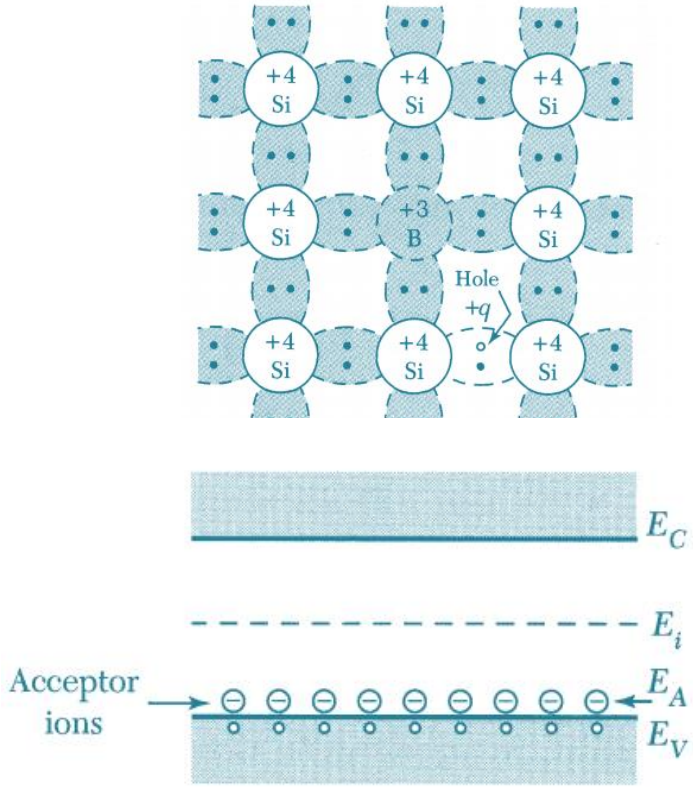
[Ans. (a) −8.505 meV; (b) −17.01meV]

DOPANT ATOMS AND ENERGY LEVELS

The real power of semiconductors is realized by adding small, controlled amounts of specific dopant or impurity atoms.

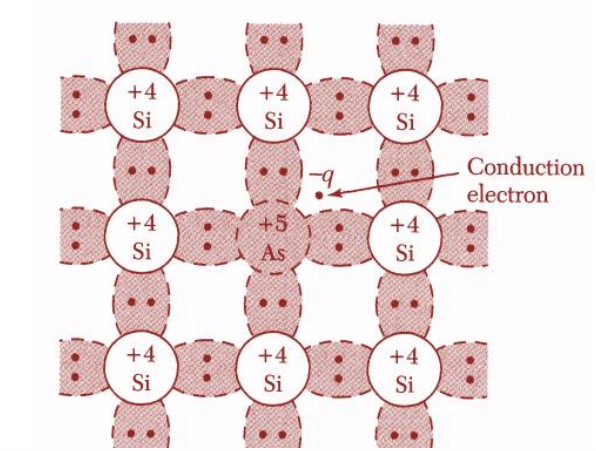
This doping process, can greatly alter the electrical characteristics of the semiconductor. The doped semiconductor, called an extrinsic material.





DOPANT ATOMS AND ENERGY LEVELS

- With a small amount of energy, such as thermal energy the donor electron, can be elevated into the conduction band, leaving behind a positively charged ion.
- The electron in the conduction band can now move through the crystal generating a current, while the positively charged ion is fixed.
- This type of impurity atom donates an electron to the conduction band and so is called a donor impurity atoms (Group 5 elements).
- The donor impurity atoms add electrons to the conduction band without creating holes in the valence band. The resulting material is referred to as a (*n-type* for the negatively charged electron).(What about neutrality ?????)



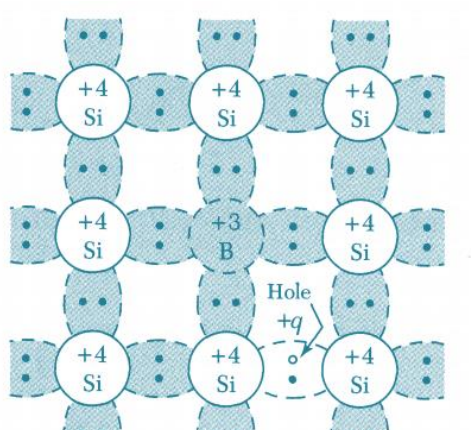
DOPANT ATOMS AND ENERGY LEVELS

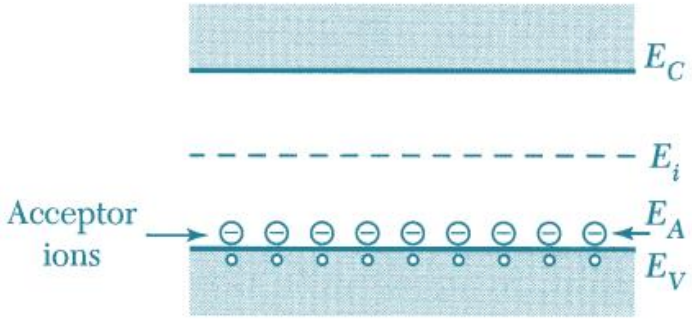
- If an electron were to occupy the "empty" position its energy would have to be greater than that of the valence electrons(since atom is negatively charged).

- Electrons may gain a small amount of thermal energy and move about in the crystal. The "empty“ position becomes occupied,and other valence electron positions become vacated.

- The hole can move through the crystal generating a current, while the negatively charged atom is fixed in the crystal.

- The acceptor atom can generate holes in the valence band without generating electrons in the conduction band. The resulting material is referred to as a (*p-type* for the positively charged hole).





EXTRINSIC SEMICONDUCTORS

- In the previous example, since $n_0 > p_0$, the semiconductor is n type.
- In an n-type semiconductor, electrons are referred to as the majority carrier and holes as the minority carrier. (comparing the relative values of n_0 and p_0)
- Similarly, in a p-type semiconductor where $p_0 > n_0$, holes are the majority carrier and electrons are the minority carrier.

$$n_0 = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

If we add and subtract an intrinsic Fermi energy in the exponent of Equation , we can write

$$n_0 = N_C \exp\left[\frac{-(E_C - E_{Fi}) + (E_F - E_{Fi})}{kT}\right]$$

EXTRINSIC SEMICONDUCTORS

$$or \quad n_0 = N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right]\exp\left[\frac{(E_F - E_{Fi})}{kT}\right]$$

The intrinsic carrier concentration is given as

$$n_i = N_C \exp\left[\frac{-(E_C - E_{Fi})}{kT}\right]$$

$$\Rightarrow n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

$$Similarly \quad p_0 = n_i \exp\left[\frac{-(E_F - E_{Fi})}{kT}\right]$$

$$n_0 p_0 = n_i^2$$

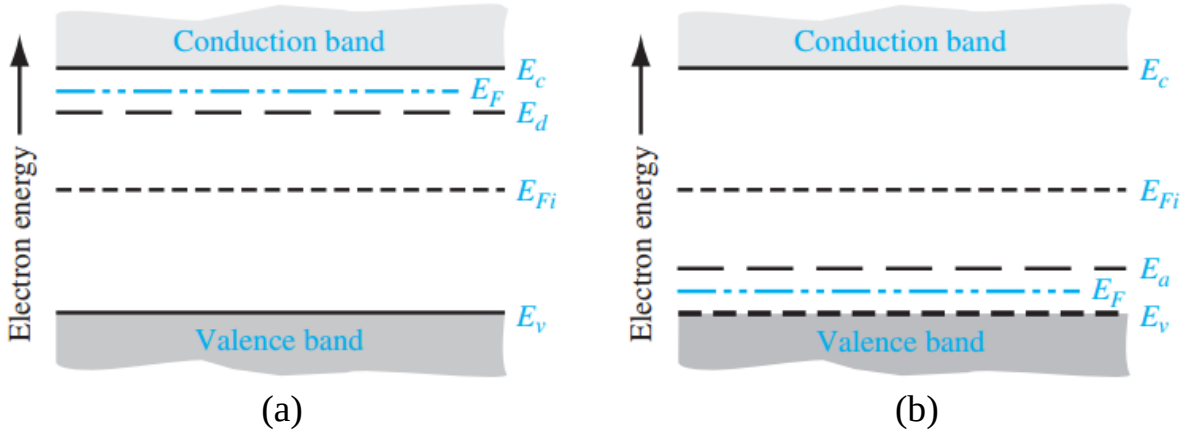
The product of n_0 and p_0 is always a constant for a given semiconductor material at a given temperature.

FREEZE-OUT

➢ At T = 0 K, the Fermi energy is halfway between *Ec* and *Ed* for the n-type material and halfway between *Ea* and *Ev* for the p-type material.

➢In an n-type semiconductor, no electrons from the donor state are thermally elevated into the conduction band; this effect is called *freeze-out.*

➢Similarly, in a p-type semiconductor, no electrons from the valance band are elevated into the acceptor states, this effect is also called freeze-out.



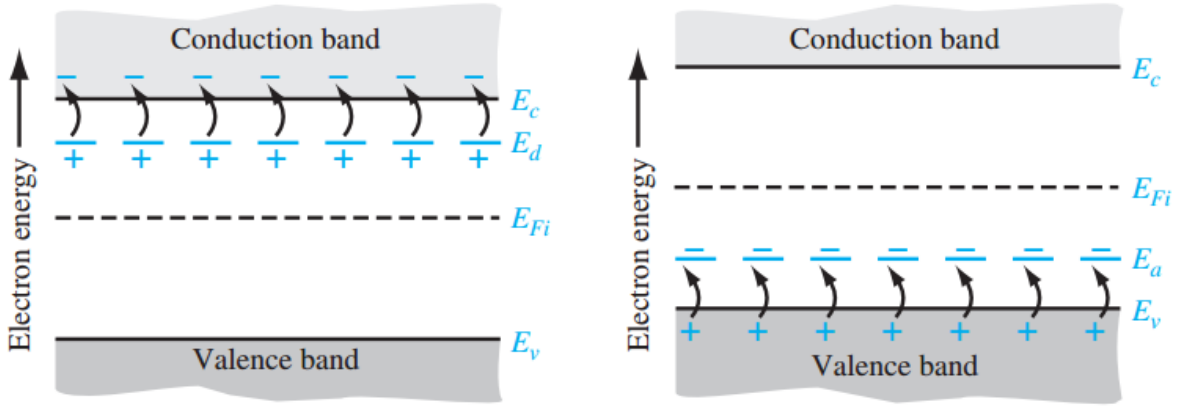
Energy-band diagram at T = 0 K for (a) n-type and (b) p-type semiconductors.

STATISTICS OF DONORS AND ACCEPTORS

COMPLETE IONIZATION

- At room temperature, all of the electrons from the donor states are in the conduction band, and all donor impurity atoms have donated an electron to the conduction band.

- At room temperature, each acceptor atom has accepted an electron from the valence band.



COMPENSATED SEMICONDUCTOR

- A ***COMPENSATED SEMICONDUCTOR*** is one that contains both donor and acceptor impurity atoms in the same region.

- A compensated semiconductor can be formed, by diffusing acceptor impurities into an n-type material or by diffusing donor impurities into a p-type material.

If $N_d > N_a$ → n-type compensated semiconductor
If $N_a > N_d$ → p-type compensated semiconductor
If $N_d = N_a$ → characteristics of an intrinsic semiconductor

$$n_o + N_a = p_o + N_d$$

From $n_{o*}p_o = n_i^2$

$$n_o + N_a = \frac{n_i^2}{n_0} + N_d$$

$$n_o = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

- Electron concentration is given as function of donors and acceptors concentrations.
- Although above equation was derived for a compensated semiconductor, the equation is also valid for $Na=0$.
- Similarly

$$p_o = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

CHARGE NEUTRALITY

EXAMPLE : Determine the thermal-equilibrium electron and hole concentrations in silicon at T =300 K for given doping concentrations.
(a) Let N_d =10^{16} cm^{-3}and N_a =0. (b) Let N_d =5 x 10^{15} cm^{-3} and N_a =2 x 10^{15} cm^{-3}. Use n_i =1.5 x 10^{10} cm^{-3} in silicon at T = 300 K.

a).

• *Electrons*

$$n_o = \frac{10^{16}}{2} + \sqrt{\left(\frac{10^{16}}{2}\right)\left(\frac{10^{16}}{2}\right) + (1.5 \times 10^{10})^2}$$

$$\approx 10^{16} cm^{-3}$$

• *Holes*

$$p_o = \frac{n_i^{\,2}}{n_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}}$$

$$= 2.25 \times 10^{4} cm^{-3}$$

→ Reduced below Intrinsic conc^{n}

CHARGE NEUTRALITY

b).

• $Electrons$

$$n_o = \frac{(5-2)\times 10^{15}}{2} + \sqrt{\left(\frac{((5-2)\times 10^{15}}{2}\right)\left(\frac{((5-2)\times 10^{15}}{2}\right) + (1.5\times 10^{10})^2}$$

$$= 3\times 10^{15} cm^{-3}$$

$Holes$

$$p_o = \frac{{n_i}^2}{n_0} = \frac{(1.5\times 10^{10})^2}{3\times 10^{15}}$$

$$= 7.5\times 10^{4} cm^{-3}$$

Reduced below Intrinsic conc^n

CHARGE NEUTRALITY

EXAMPLE: Calculate the thermal-equilibrium electron and hole concentrations in a compensated p-type semiconductor.

Consider a silicon semiconductor at T =300 K in which N_a=10^{16} cm^{-3} and N_d=3x10^{15} cm^{-3}. Assume n_i = 1.5 x10^{10} cm^{-3}.

Since *Na* > *Nd*, the compensated semiconductor is p-type and the thermal-equilibrium majority carrier hole concentration is given by

$$p_0 = \frac{10^{16} - 3\times10^{15}}{2} + \sqrt{\left(\frac{10^{16} - 3\times10^{15}}{2}\right)^2 + (1.5\times10^{10})^2}$$

$$p_0 \approx 7\times10^{15}\,cm^{-3}$$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.5\times10^{10})^2}{7\times10^{15}} = 3.21\times10^{4}\,cm^{-3}$$

Find the thermal-equilibrium electron and hole concentrations in silicon with doping concentrations of $Nd = 7 \times 10^{15}$ cm^{-3} and $Na = 3 \times 10^{15}$ cm^{-3} for (*a*) T =250 K and (*b*) T= 400 K.

[Ans. (*a*) $n_0 = 4 \times 10^{15}\ \text{cm}^{-3}$, $p_0 = 1.225\ \text{cm}^{-3}$; (*b*) $n_0 = 4 \times 10^{15}\ \text{cm}^{-3}$, $p_0 = 1.416 \times 10^{9}\ \text{cm}^{-3}$]

Consider silicon at $T = 300$ K. Calculate the thermal-equilibrium electron and hole concentrations for impurity concentrations of (*a*) $N_a = 4 \times 10^{16}\ \text{cm}^{-3}$, $N_d = 8 \times 10^{15}\ \text{cm}^{-3}$ and (*b*) $N_a = N_d = 3 \times 10^{15}\ \text{cm}^{-3}$.

[Ans. (*a*) $p_0 = 3.2 \times 10^{16}\ \text{cm}^{-3}$, $n_0 = 7.03 \times 10^{3}\ \text{cm}^{-3}$; (*b*) $p_0 = n_0 = 1.5 \times 10^{10}\ \text{cm}^{-3}$]

EXAMPLE: Find the thermal-equilibrium electron and hole concentrations in silicon with doping concentrations of $Nd = 7 \times 10^{15}$ cm^{-3} and $Na = 3 \times 10^{15}$ cm^{-3} for (*a*) T =250 K and (*b*) T= 400 K.

[Ans. (*a*) $n_0 = 4 \times 10^{15}\ \text{cm}^{-3}$, $p_0 = 1.225\ \text{cm}^{-3}$; (*b*) $n_0 = 4 \times 10^{15}\ \text{cm}^{-3}$, $p_0 = 1.416 \times 10^{9}\ \text{cm}^{-3}$]

POSITION OF FERMI ENERGY LEVEL

- We have discussed qualitatively how the electron and hole concentrations change as the Fermi energy level moves through the band gap energy.
- We also calculated the electron and hole concentrations as a function of donor and acceptor impurity concentrations.
- We can now determine the position of the Fermi energy level as a function of the doping concentrations and as a function of temperature.

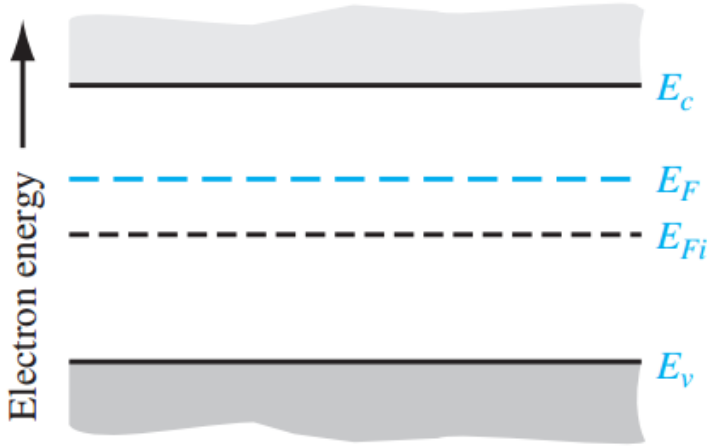
POSITION OF FERMI ENERGY LEVEL (N-TYPE SC)

If we assume the Boltzmann approximation to be valid, then from

$$n_0 = N_C \exp\left[\frac{-(E_C - E_F)}{kT}\right]$$

We obtain

$$E_C - E_F = kT \ln\left(\frac{N_C}{n_0}\right)$$



If we consider an n-type semiconductor in which $N_d >> n_i$,

$$E_C - E_F = kT \ln\left(\frac{N_C}{N_d}\right) \qquad n_0 \approx N_d$$

- The distance between the bottom of the conduction band and the Fermi energy is a logarithmic function of the donor concentration.
- As the donor concentration increases, the Fermi level moves closer to the conduction band.
- Conversely, if the Fermi level moves closer to the conduction band, then the electron concentration in the conduction band is increasing.

POSITION OF FERMI ENERGY LEVEL

If we have a compensated semiconductor, then the N_d term in above equation is simply replaced by $N_d - N_a$, or the net effective donor concentration.

$$n_0 = n_i \exp\left[(E_F - E_{Fi})/kT\right]$$

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right)$$

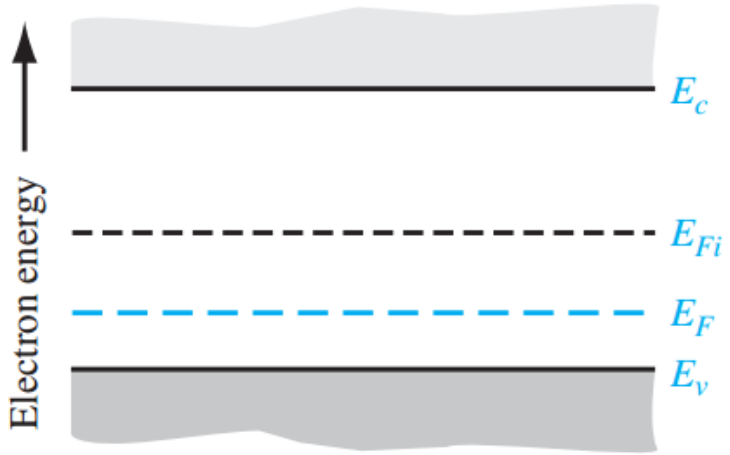
We may note that, if the net effective donor concentration is zero, that is, $N_d - N_a = 0$ then $n_0 = n_i$ and $E_F = E_{Fi}$.

A completely compensated semiconductor has the characteristics of an intrinsic material in terms of carrier concentration and Fermi-level position.

POSITION OF FERMI ENERGY LEVEL (P TYPE SC)

similarly from $p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$ we get

$$E_F - E_v = kT \ln\left(\frac{N_v}{p_0}\right)$$



Consider an p-type semiconductor ($N_a >> n_i$,)

$$E_F - E_v = kT \ln\left(\frac{N_v}{N_a}\right) \qquad p_0 \approx N_a$$

- The distance between the Fermi level and the top of the valence-band energy for a p-type semiconductor is a logarithmic function of the acceptor concentration
- As the acceptor concentration increases, the Fermi level moves closer to the valence band.

POSITION OF FERMI ENERGY LEVEL

If we have a compensated p-type semiconductor, then the N_a term in above equation is replaced by $N_a - N_d$, or the net effective acceptor concentration.

$$\therefore p_0 = n_i \exp\left[-(E_F - E_{Fi})/kT\right]$$

$$E_{Fi} - E_F = kT \ln\left(\frac{p_0}{n_i}\right)$$

Note:

For an n-type semiconductor, $n_0 > n_i$ and $E_F > E_{Fi}$. The Fermi level for an n-type semiconductor is above E_{Fi}.

For a p-type semiconductor, $p_0 > n_i$, and $E_{Fi} > E_F$. The Fermi level for a p-type semiconductor is below E_{Fi}.

Determine the position of the Fermi level with respect to the valence-band energy in p-type GaAs at $T = 300$ K. The doping concentrations are $N_a = 5 \times 10^{16}\ \text{cm}^{-3}$ and $N_d = 4 \times 10^{15}\ \text{cm}^{-3}$.

(Ans. $E_F - E_v = 0.130$ eV)

Calculate the position of the Fermi energy level in n-type silicon at $T = 300$ K with respect to the intrinsic Fermi energy level. The doping concentrations are $N_d = 2 \times 10^{17}\ \text{cm}^{-3}$ and $N_a = 3 \times 10^{16}\ \text{cm}^{-3}$.

(Ans. $E_F - E_{Fi} = 0.421$ eV)

Consider silicon at $T = 300$ K with doping concentrations of $N_d = 8 \times 10^{15}$ cm^{-3} and $N_a = 5 \times 10^{15}$ cm^{-3}. Determine the position of the Fermi energy level with respect to E_c.

(Ans. $E_c - E_F = 0.2368$ eV)

Silicon at $T = 300$ K contains an acceptor impurity concentration of $N_a = 10^{16}$ cm^{-3}. Determine the concentration of donor impurity atoms that must be added so that the silicon is n type and the Fermi energy is 0.20 eV below the conduction-band edge.

$$N_d = 2.24 \times 10^{16} \text{ cm}^{-3}$$

Consider germanium with an acceptor concentration of $N_a = 10^{15}$ cm^{-3} and a donor concentration of $N_d = 0$. Consider temperatures of $T = 200$, 400, and 600 K. Calculate the position of the Fermi energy with respect to the intrinsic Fermi level at these temperatures.

$$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

For Germanium:

$T(^\circ K)$	$kT(eV)$	$n_i(cm^{-3})$
200	0.01727	$2.16x10^{10}$
400	0.03454	$8.6x10^{14}$
600	0.0518	$3.82x10^{16}$

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2} \quad \text{and} \quad N_a = 10^{15}\ cm^{-3}$$

$T(^\circ K)$	$p_o(cm^{-3})$	$E_{Fi} - E_F\ (eV)$
200	$1.0x10^{15}$	0.1855
400	$1.49x10^{15}$	0.01898
600	$3.87x10^{16}$	0.000674

Consider silicon at $T = 300$ K with donor concentrations of $N_d = 10^{14}$, 10^{15}, 10^{16}, and 10^{17}, cm^{-3}. Assume $N_a = 0$. (*a*) Calculate the position of the Fermi energy level with respect to the conduction band for these donor concentrations. (*b*) Determine the position of the Fermi energy level with respect to the intrinsic Fermi energy level for the donor concentrations given in part (*a*).

(a) $$E_c - E_F = kT \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{N_d}\right)$$

For $10^{14}\,\text{cm}^{-3}$, $E_c - E_F = 0.3249\,\text{eV}$

$10^{15}\,\text{cm}^{-3}$, $E_c - E_F = 0.2652\,\text{eV}$

$10^{16}\,\text{cm}^{-3}$, $E_c - E_F = 0.2056\,\text{eV}$

$10^{17}\,\text{cm}^{-3}$, $E_c - E_F = 0.1459\,\text{eV}$

(b) $$E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{N_d}{1.5\times10^{10}}\right)$$

For $10^{14}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.2280\,\text{eV}$

$10^{15}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.2877\,\text{eV}$

$10^{16}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.3473\,\text{eV}$

$10^{17}\,\text{cm}^{-3}$, $E_F - E_{Fi} = 0.4070\,\text{eV}$

Silicon at T = 300 K contains acceptor atoms at a concentration of Na= 5 x 10^{15} cm^{-3}). Donor atoms are added forming an n-type compensated semiconductor such that the Fermi level is 0.215 eV below the conduction band edge. What concentration of donor atoms are added?

$$n_o = N_d - N_a = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

so

$$N_d = 5\times10^{15} + (2.8\times10^{19})\exp\left(\frac{-0.215}{0.0259}\right)$$

$$= 5\times10^{15} + 6.95\times10^{15}$$

or

$$N_d = 1.2\times10^{16}\ \text{cm}^{-3}$$

(a)Determine the position of the Fermi level with respect to the intrinsic Fermi level in silicon at T = 300 K that is doped with phosphorus atoms at a concentration of 10^{15} cm^{-3}. *(b)*Repeat part *(a)*if the silicon is doped with boron atoms at a concentration of 10^{15} cm^{-3}? (c) Calculate the electron concentration in the silicon for parts (a) and (b)

(a) $E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = (0.0259)\ln\left(\frac{10^{15}}{1.5x10^{10}}\right)$ (b)

or $E_{Fi} - E_F = kT \ln\left(\frac{N_a}{n_i}\right) = 0.2877\ eV$

$$\underline{E_F - E_{Fi} = 0.2877\ eV}$$

(c)

For (a), $\underline{n_o = N_d = 10^{15}\ cm^{-3}}$

For (b)

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5x10^{10}\right)^2}{10^{15}} \Rightarrow$$

$$\underline{n_o = 2.25x10^5\ cm^{-3}}$$

Assume that the Fermi energy level is exactly in the center of the bandgap energy of a semiconductor at T= 300 K. Calculate the probability that an energy state in the bottom of the conduction band is occupied by an electron for Si, Ge, and GaAs.

$$E_F = E_{midgap}$$

$$E = E_C$$

$$E - E_F = E_C - E_{midgap} = E_g/2$$

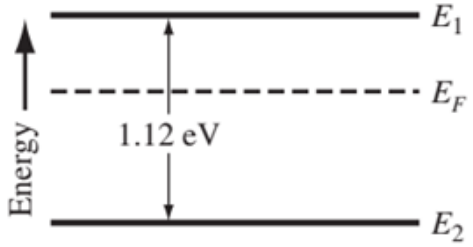
$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{E_g}{2kT}\right)}$$

For Silicon $E_g = 1.12eV; f(E) = 4.07 \times 10^{-10}$

For Germanium $E_g = 0.66eV; f(E) = 2.93 \times 10^{-6}$

For GaAs $E_g = 1.42eV; f(E) = 1.24 \times 10^{-12}$

Consider the energy levels shown in the figure below. Let T=300 K. Determine the probability that an energy state at E = E_1 is occupied by an electron and the probability that an energy state at E=E_2 is empty. If (i) $E_1 - E_F$ = 0.30 eV (ii) $E_F - E_2$ = 0.40 eV.



(a) For $E = E_1$

$$f(E) = \frac{1}{1+\exp\left(\dfrac{E_1 - E_F}{kT}\right)} \cong \exp\left[\frac{-(E_1 - E_F)}{kT}\right]$$

Then

$$f(E_1) = \exp\left(\frac{-0.30}{0.0259}\right) = 9.32\times10^{-6}$$

For $E = E_2$, $E_F - E_2 = 1.12 - 0.30 = 0.82$ eV

Then

$$1 - f(E) = 1 - \frac{1}{1+\exp\left(\dfrac{-0.82}{0.0259}\right)}$$

or

$$1 - f(E) \cong 1 - \left[1 - \exp\left(\frac{-0.82}{0.0259}\right)\right]$$

$$= \exp\left(\frac{-0.82}{0.0259}\right) = 1.78\times10^{-14}$$

(b) For $E_F - E_2 = 0.4$ eV,

$$E_1 - E_F = 0.72 \text{ eV}$$

At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-0.72}{0.0259}\right)$$

or

$$f(E) = 8.45\times10^{-13}$$

At $E = E_2$,

$$1 - f(E) = \exp\left[\frac{-(E_F - E_2)}{kT}\right]$$

$$= \exp\left(\frac{-0.4}{0.0259}\right)$$

or

$$1 - f(E) = 1.96\times10^{-7}$$

(a) Silicon at $T = 300$ K is doped with donor impurity atoms at a concentration of $N_d = 6 \times 10^{15}$ cm^{-3}. (i) Determine $E_c - E_F$. (ii) Calculate the concentration of additional donor impurity atoms that must be added to move the Fermi energy level a distance kT closer to the conduction band edge. (b) Repeat part (a) for GaAs if the original donor impurity concentration is $N_d = 1 \times 10^{15}$ cm^{-3}.

(a) Silicon

$$\text{(i) } E_c - E_F = kT \ln\left(\frac{N_c}{N_d}\right)$$

$$= (0.0259)\ln\left(\frac{2.8\times10^{19}}{6\times10^{15}}\right) = 0.2188 \text{ eV}$$

$$\text{(ii) } E_c - E_F = 0.2188 - 0.0259 = 0.1929 \text{ eV}$$

$$N_d = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

$$= (2.8\times10^{19})\exp\left[\frac{-0.1929}{0.0259}\right]$$

$$N_d = 1.631\times10^{16} \text{ cm}^{-3} = N_d' + 6\times10^{15}$$

$$\Rightarrow N_d' = 1.031\times10^{16} \text{ cm}^{-3} \text{ Additional donor atoms}$$

(b) GaAs

$$\text{(i) } E_c - E_F = (0.0259)\ln\left(\frac{4.7\times10^{17}}{10^{15}}\right)$$

$$= 0.15936 \text{ eV}$$

$$\text{(ii) } E_c - E_F = 0.15936 - 0.0259 = 0.13346 \text{ eV}$$

$$N_d = (4.7\times10^{17})\exp\left[\frac{-0.13346}{0.0259}\right]$$

$$= 2.718\times10^{15} \text{ cm}^{-3} = N_d' + 10^{15}$$

$$\Rightarrow N_d' = 1.718\times10^{15} \text{ cm}^{-3} \text{ Additional donor atoms}$$

For a particular semiconductor, $E_g = 1.50$ eV, $m_p^* = 10\, m_n^*$, $T = 300$ K, and $n_i = 1 \times 10^5$ cm^{-3}. (*a*) Determine the position of the intrinsic Fermi energy level with respect to the center of the bandgap. (*b*) Impurity atoms are added so that the Fermi energy level is 0.45 eV below the center of the bandgap. (*i*) Are acceptor or donor atoms added? (*ii*) What is the concentration of impurity atoms added?

(a) $$E_{Fi} - E_{midgap} = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

$$= \frac{3}{4}(0.0259)\ln(10)$$

or

$$E_{Fi} - E_{midgap} = +0.0447 \text{ eV}$$

(b) Impurity atoms to be added so

$$E_{midgap} - E_F = 0.45 \text{ eV}$$

(i) p-type, so add acceptor atoms

(ii) $$E_{Fi} - E_F = 0.0447 + 0.45 = 0.4947 \text{ eV}$$

$$p_o = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right)$$

$$= (10^5)\exp\left(\frac{0.4947}{0.0259}\right)$$

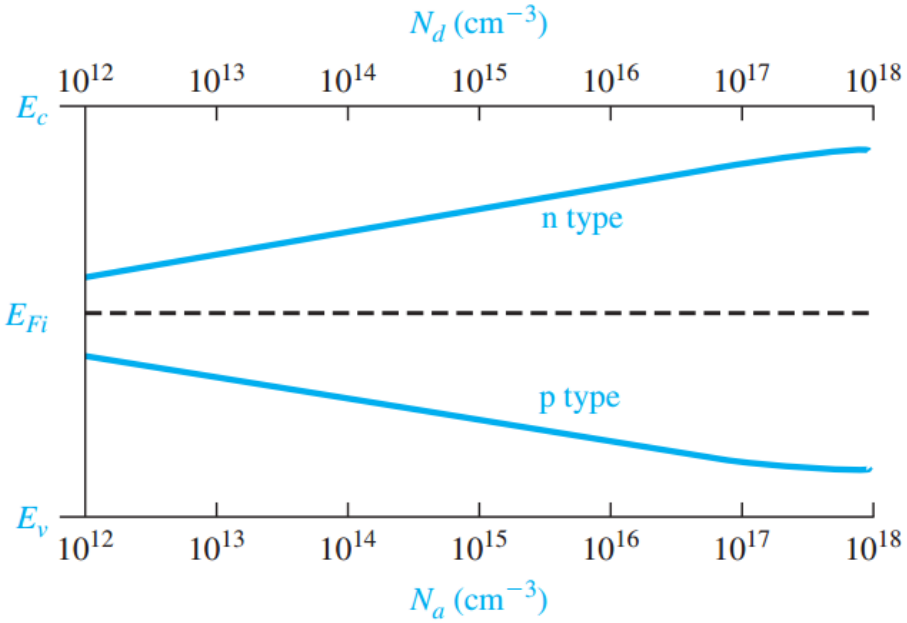
or

$$p_o = N_a = 1.97 \times 10^{13} \text{ cm}^{-3}$$

VARIATION OF E_F WITH DOPING CONCENTRATION

- Figure shows the Fermi energy level as a function of donor concentration (n-type) and as a function of acceptor concentration (p-type) for silicon at T=300 K.

- As the doping levels increase, the Fermi energy level moves closer to the conduction band for the n-type material and closer to the valence band for the p-type material.

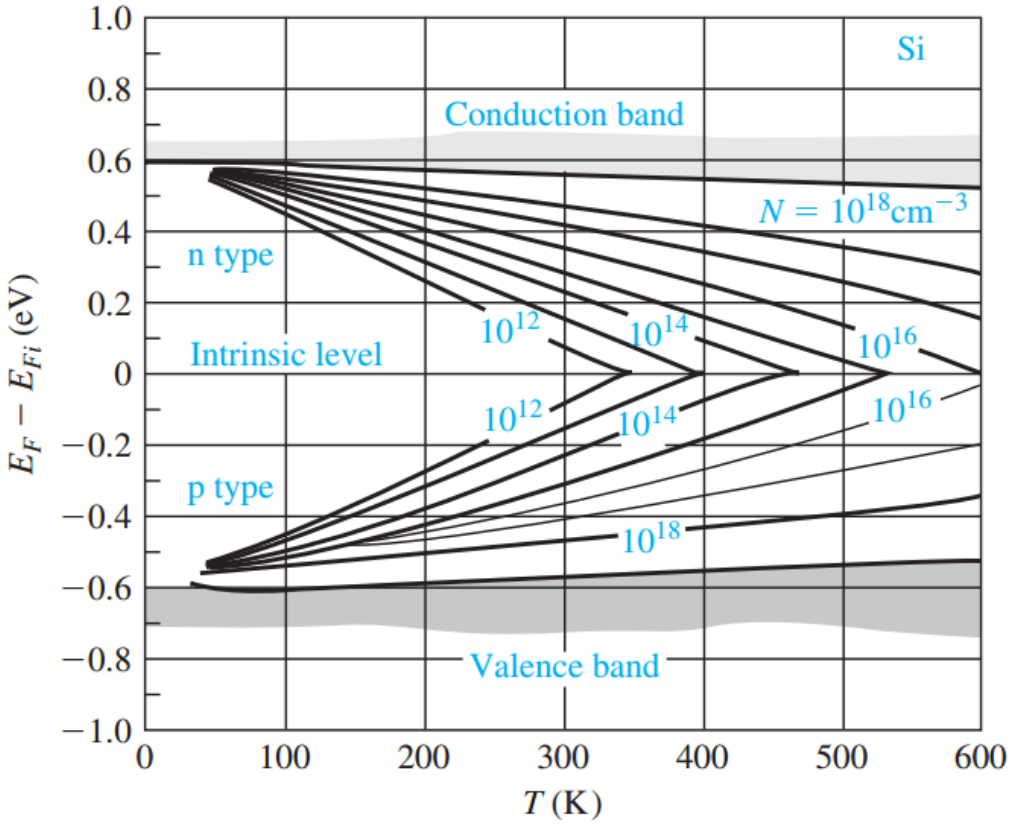


VARIATION OF E_F WITH TEMPERATURE

- As the temperature increases, n_i increases, and E_F moves closer to the intrinsic Fermi level.
- At high temperature, the semiconductor material begins to lose its extrinsic characteristics and begins to behave more like an intrinsic semiconductor.
- At absolute zero degrees, all energy states below E_F are full and all energy states above E_F are empty.

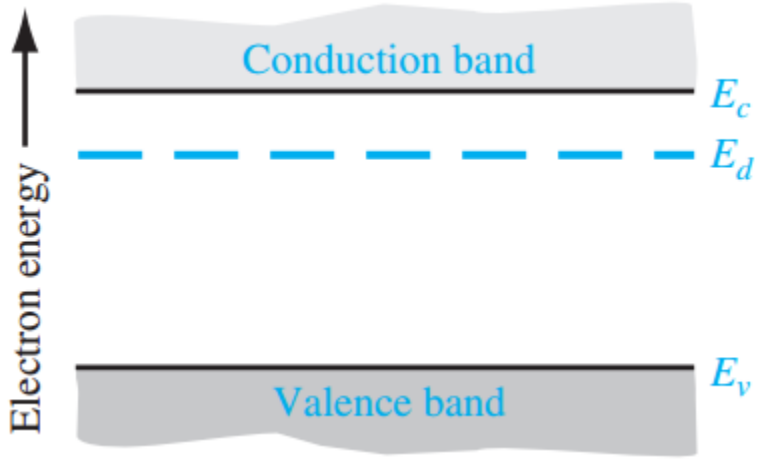
VARIATION OF E_F WITH TEMPERATURE

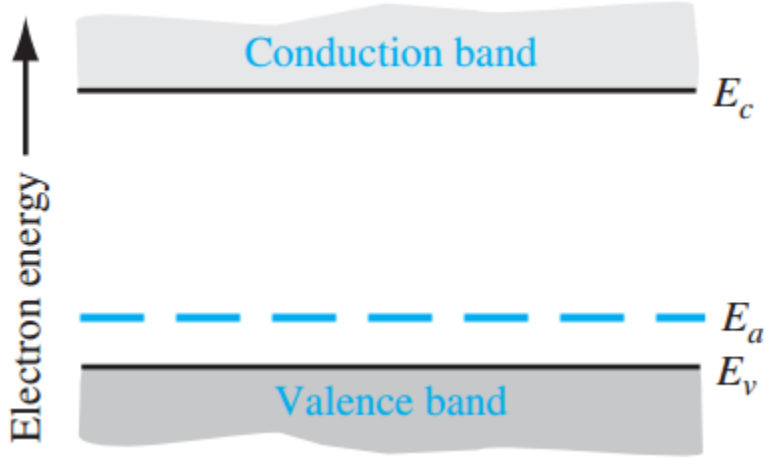
Figure shows the variation of the Fermi energy level in silicon with temperature for several donor and acceptor concentrations.



DEGENERATE AND NON-DEGENERATE SEMICONDUCTORS

- Till now we have implicitly assumed that the concentration of dopant atoms added is small when compared to the density of host or semiconductor atoms.
- The small number of impurity atoms are spread far enough apart so that there is no interaction between dopant energy states.
- These types of semiconductors are referred to as non-degenerate semiconductors.

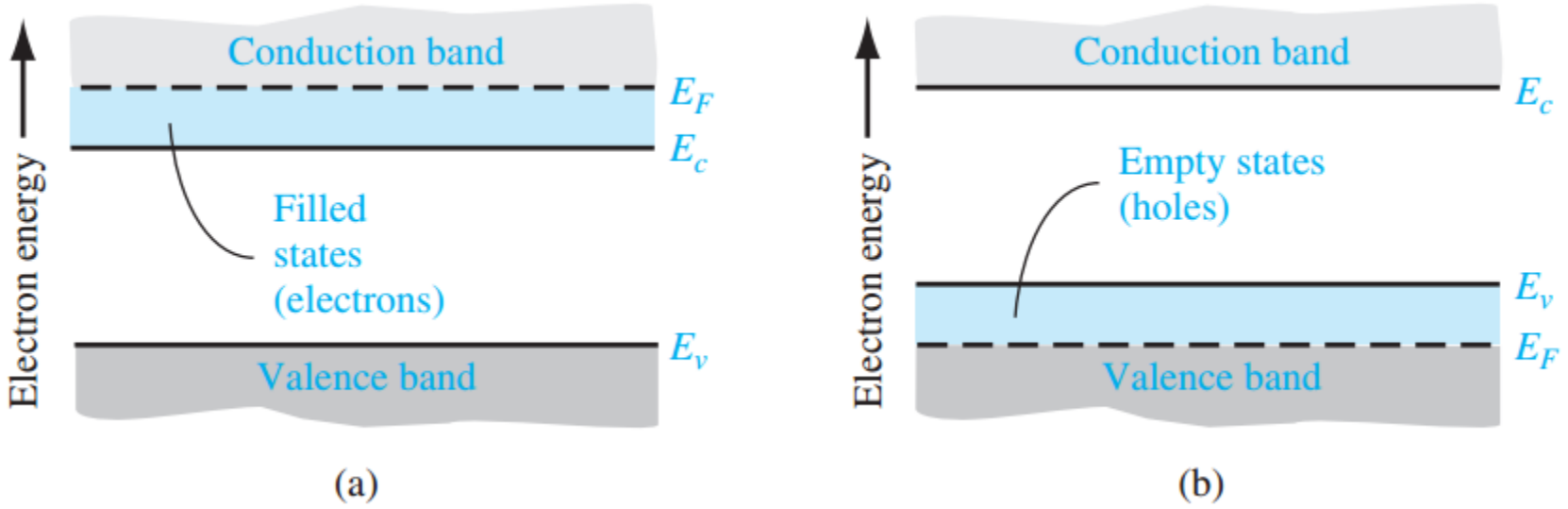




DEGENERATE AND NON-DEGENERATE SEMICONDUCTORS

- If the impurity concentration increases, the distance between the impurity atoms decreases and a point will be reached when donor or acceptor states will begin to interact with each other.

- As the donor concentration further increases, the band of donor states widens and may overlap the bottom of the conduction band. This overlap occurs when the donor concentration becomes comparable with the effective density of states.



A new semiconductor material is to be "designed." The semiconductor is to be p type and doped with 5×10^{15} cm^{-3} acceptor atoms. Assume complete ionization and assume $N_d = 0$. The effective density of states functions are $N_c = 1.2 \times 10^{19}$ cm^{-3} and $N_v = 1.8 \times 10^{19}$ cm^{-3} at $T = 300$ K and vary as T^2. A special semiconductor device fabricated with this material requires that the hole concentration be no greater than 5.08×10^{15} cm^{-3} at $T = 350$ K. What is the minimum bandgap energy required in this new material?

$$p_o = \frac{N_a}{2} + \sqrt{\left(\frac{N_a}{2}\right)^2 + n_i^2}$$

$$5.08\times10^{15} = \frac{5\times10^{15}}{2} + \sqrt{\left(\frac{5\times10^{15}}{2}\right)^2 + n_i^2}$$

$$\left(5.08\times10^{15} - 2.5\times10^{15}\right)^2 = \left(2.5\times10^{15}\right)^2 + n_i^2$$

$$6.6564\times10^{30} = 6.25\times10^{30} + n_i^2$$

$$\Rightarrow n_i^2 = 4.064\times10^{29}$$

$$n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

$$kT = (0.0259)\left(\frac{350}{300}\right) = 0.030217 \text{ eV}$$

$$N_c = \left(1.2\times10^{19}\right)\left(\frac{350}{300}\right)^2 = 1.633\times10^{19} \text{ cm}^{-3}$$

$$N_v = \left(1.8\times10^{19}\right)\left(\frac{350}{300}\right)^2 = 2.45\times10^{19} \text{ cm}^{-3}$$

$$4.064\times10^{29} = \left(1.633\times10^{19}\right)\left(2.45\times10^{19}\right) \times \exp\left[\frac{-E_g}{0.030217}\right]$$

So

$$E_g = (0.030217)\ln\left[\frac{\left(1.633\times10^{19}\right)\left(2.45\times10^{19}\right)}{4.064\times10^{29}}\right]$$

$$\Rightarrow E_g = 0.6257 \text{ eV}$$

Silicon at $T = 300$ K is doped with boron atoms such that the concentration of holes is $p_0 = 5 \times 10^{15}$ cm^{-3}. (*a*) Find $E_F - E_v$. (*b*) Determine $E_c - E_F$. (*c*) Determine n_0. (*d*) Which carrier is the majority carrier? (*e*) Determine $E_{Fi} - E_F$.

(a) $$E_F - E_\upsilon = kT \ln\left(\frac{N_\upsilon}{p_o}\right)$$

$$= (0.0259)\ln\left(\frac{1.04\times10^{19}}{5\times10^{15}}\right)$$

$$= 0.1979 \text{ eV}$$

(b) $$E_c - E_F = E_g - (E_F - E_\upsilon)$$

$$= 1.12 - 0.19788 = 0.92212 \text{ eV}$$

(c) $$n_o = (2.8\times10^{19})\exp\left[\frac{-0.92212}{0.0259}\right]$$

$$= 9.66\times10^{3} \text{ cm}^{-3}$$

(d) Holes

(e) $$E_{Fi} - E_F = kT \ln\left(\frac{p_o}{n_i}\right)$$

$$= (0.0259)\ln\left(\frac{5\times10^{15}}{1.5\times10^{10}}\right)$$

$$= 0.3294 \text{ eV}$$

Two semiconductor materials have exactly the same properties except material A has a bandgap energy of 0.90 eV and material B has a bandgap energy of 1.10 eV. Determine the ratio of n_i of material B to that of material A for (*a*) $T = 200$ K, (*b*) $T = 300$ K, and (*c*) $T = 400$ K.

$$\frac{n_i(B)}{n_i(A)} = \frac{exp\left(\frac{-1.10}{2kT}\right)}{exp\left(\frac{-0.90}{2kT}\right)} = exp\left(\frac{-0.10}{kT}\right)$$

For $T = 200$ K, $kT = 0.017267$ eV
For $T = 300$ K, $kT = 0.0259$ eV
For $T = 400$ K, $kT = 0.034533$ eV

For T=200K

$$\frac{n_i(B)}{n_i(A)} = exp\left(\frac{-0.10}{0.017267}\right) = 3.05 \times 10^{-3}$$

For T= 300K

$$\frac{n_i(B)}{n_i(A)} = exp\left(\frac{-0.10}{0.0259}\right) = 0.02104$$

For T= 400K

$$\frac{n_i(B)}{n_i(A)} = exp\left(\frac{-0.10}{0.034533}\right) = 0.05526$$

In a particular semiconductor material, the effective density of states functions are given by $N_c = N_{c0} \cdot (T/300)^{3/2}$ and $N_v = N_{v0} \cdot (T/300)^{3/2}$ where N_{c0} and N_{v0} are constants independent of temperature. Experimentally determined intrinsic carrier concentrations are found to be $n_i = 1.40 \times 10^2\ \text{cm}^{-3}$ at $T = 200$ K and $n_i = 7.70 \times 10^{10}\ \text{cm}^{-2}$ at $T = 400$ K. Determine the product $N_{c0} \cdot N_{v0}$ and the bandgap energy E_g. (Assume E_g is constant over this temperature range.)

$$\text{At } T = 200\,\text{K},\ kT = (0.0259)\left(\frac{200}{300}\right) = 0.017267\,\text{eV}$$

$$\text{At } T = 400\,\text{K},\ kT = (0.0259)\left(\frac{400}{300}\right) = 0.034533\,\text{eV}$$

$$\frac{n_i^2(400)}{n_i^2(200)} = \frac{\left(7.70\times10^{10}\right)^2}{\left(1.40\times10^2\right)^2} = 3.025\times10^{17}$$

$$= \frac{\left(\frac{400}{300}\right)^3}{\left(\frac{200}{300}\right)^3} \times \frac{\exp\left[\frac{-E_g}{0.034533}\right]}{\exp\left[\frac{-E_g}{0.017267}\right]}$$

$$= 8\exp\left[\frac{E_g}{0.017267} - \frac{E_g}{0.034533}\right]$$

$$3.025\times10^{17} = 8\exp\left[E_g(57.9139 - 28.9578)\right]$$

$$E_g(28.9561) = \ln\left(\frac{3.025\times10^{17}}{8}\right) = 38.1714$$

or $E_g = 1.318\,\text{eV}$

Now

$$\left(7.70\times10^{10}\right)^2 = N_{co}N_{vo}\left(\frac{400}{300}\right)^3 \times \exp\left(\frac{-1.318}{0.034533}\right)$$

$$5.929\times10^{21} = N_{co}N_{vo}(2.370)\left(2.658\times10^{-17}\right)$$

so $N_{co}N_{vo} = 9.41\times10^{37}\ \text{cm}^{-6}$