VATSAL JINDAL -58
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Vat

DSP - IA 4

1. $y[n] = 0.1x[n] - 0.9y[n-1]$.

$\Rightarrow y[n] + 0.9y[n-1] = 0.1x[n]$

$\Rightarrow Y(z)\{1+0.9z^{-1}\} = 0.1X(z)$.

$\Rightarrow H(z) = \dfrac{0.1}{1+0.9z^{-1}} = \dfrac{0.1z}{z+0.9}$

Let $z = e^{j\omega}$

$\Rightarrow H(e^{j\omega}) = \dfrac{0.1e^{j\omega}}{e^{j\omega}+0.9}$

$\Rightarrow |H(e^{j\omega})| = \dfrac{0.1}{|e^{j\omega}+0.9|}$

$= \dfrac{0.1}{\sqrt{1.81+1.8\cos\omega}}$

Squaring both sides,

$\Rightarrow 0.5 = \dfrac{0.01}{1.81+1.8\cos\omega}$

$\Rightarrow 1.81 + 1.8\cos\omega = 0.02$.

$\Rightarrow \cos\omega = -0.9944$.

$\Rightarrow \omega = 2.997$ rad/sample.

It is a low pass filter.

2. $f_p = 4kHz$; $f_s = 8kHz$; $A_p = 1dB$; $A_s = 40dB$; $f_{sa} = 24kHz$

$\Rightarrow 40 = -20\log(\delta_s)$.

$\Rightarrow \delta_s = 0.01$.

$1 = -20\log(1-\delta_p)$.

$\Rightarrow 1-\delta_p = 0.89125$

$\dfrac{1}{\sqrt{1+\varepsilon^2}} = 0.89125$.

$\Rightarrow \varepsilon = 0.5088$

$\Omega_p = \cancel{2\pi f_p = 8000\pi}\ 2\cdot f_{sa}\tan\left(\dfrac{\pi f_p}{f_{sa}}\right) = 27715.2$ rad/s.

$\Omega_s = \cancel{2\pi f_s = 16000\pi} = 2\cdot f_{sa}\tan\left(\dfrac{\pi f_s}{f_{sa}}\right) = 83136$ rad/s.

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$\therefore$ ~~$N = \dfrac{\log(\ldots)}{\log(\Omega_P/\Omega_S)}$~~ $\left\{ \delta = \dfrac{1}{\delta_s^2} - 1 \right\}$. $N = \dfrac{\log\left(\sqrt{1/A_P^2 - 1} \,/\, \sqrt{1/A_S^2 - 1}\right)}{\log(\Omega_P/\Omega_S)}$.

$\therefore$ ~~$N = \dfrac{\log(0.005)}{\log(\frac{1}{3}) \cdot \log(0.5)}$~~ ~~$= -2.3010$~~ $N = \dfrac{\log(0.005)}{\log(0.3337)}$.

$\therefore N = 4.823 \simeq 5.$

Let $\Omega_p = \Omega_c$.

$\therefore S_k = \Omega_c e^{j\pi/2} \cdot e^{j(2k+1)\frac{\pi}{2N}}$. $\forall\ k = 0, 1, \ldots \frac{N-1}{2} = 2.$

$\therefore S_0 = 27715.2\, e^{j\pi/2} e^{\pi/10}$

$\therefore S_1 = 27715.2\, e^{j\pi/2} e^{3\pi/10}$.

$\therefore S_2 = 27715.2\, e^{j\pi/2} e^{5\pi/10}$.

$\therefore H(s) = \dfrac{K}{(s-s_0)(s-s_1)\ldots(s-s_4)}$ $\longrightarrow$ (1)

$T = \dfrac{1}{f_{sa}} = 41.67\,\mu s.$

$s = \dfrac{2}{T}\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right).$

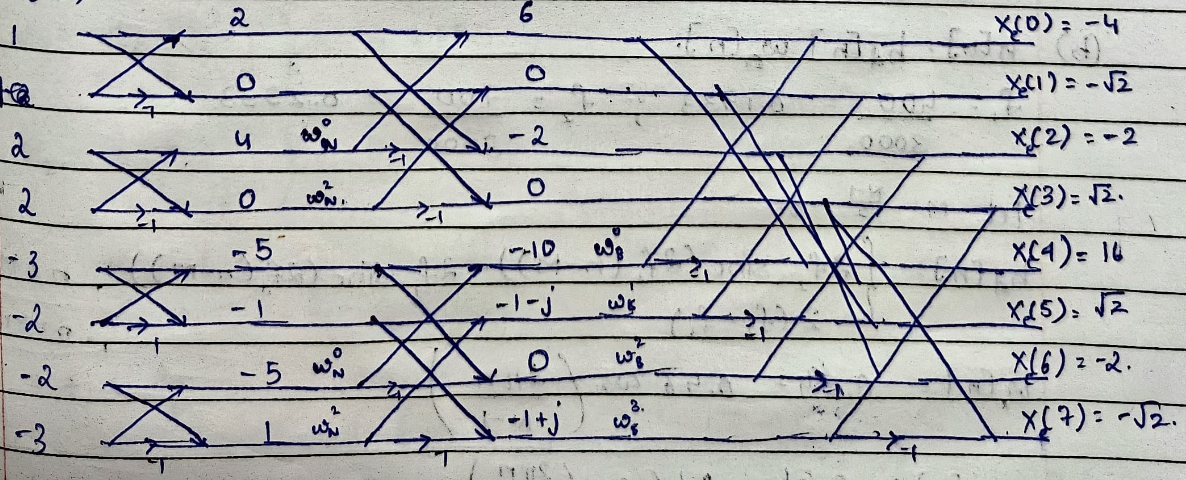
$\therefore$ To get $H(z)$ replace s with $48000\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right)$ in (1).

3. Split into 2 8 point sequences,

$x_e(n) = \{1, -3, 2, -2, 1, -2, 2, -3\}.$

$x_o(n) = \{2, 4, 3, 1, 1, 3, 4, 2\}.$

$X_e(k) =$



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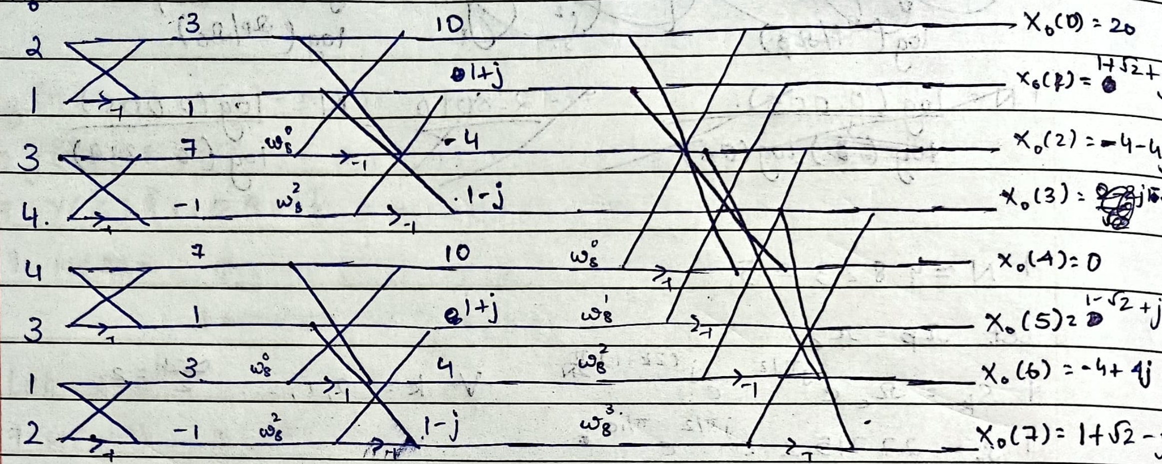
$X_o(k)$:



$X(k) = X_e(k) + W_{16}^k X_o(k), \quad 0 \le k < 8$

$X(k+8) = X_e(k) - W_{16}^k X_o(k), \quad 0 \le k < 8.$

$\therefore X(k) = \{16, 1.1989, -7.6569, 0.3318, 16, 2.4966, 3.6569, -4.0273, -24, -4.0273, 3.6569, 2.4966, 16, 0.3318, -7.6569, 1.1989\}.$

4. (a) $\Delta f = \min(400-100, 1000-700) = 300\,Hz$

$\Delta f_{norm} = \dfrac{300}{f_s/2} = 0.2.$

$N = \dfrac{3.3}{\Delta f_{norm}} = 16.5.$

$\therefore N = 17.$

(b) $h[n] = h_d[n]\, w_{\alpha}[n].$

$f_1 = \dfrac{400}{3000} = 0.1333 \; ; \; f_2 = \dfrac{700}{3000} = 0.2333.$

Let $M = \frac{N-1}{2} = 8.$

$h_d[n] = \begin{cases} 2f_2 \cdot \text{sinc}(2f_2(n-M)) - 2f_1 \cdot \text{sinc}(2f_1(n-M)) & ; n \ne M. \\ 2(f_2 - f_1) & ; n = M. \end{cases}$

$w_{\alpha}[n] = 0.54 - 0.46 \cos\left(\dfrac{2\pi n}{N-1}\right)$

$w[n] = 0.54 - 0.46 \cos\left(\dfrac{2\pi n}{16}\right).$

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$\therefore h[n] = \{-0.0049, -0.0023, 0.023, 0.054, -0.01158, -0.1580, -0.1468, 0.10467, 0.2711\}$

$\therefore h[n] = h[N-1-n]$

5. $0.8 \le |H(e^{j\omega})| \le 1$, $0 \le \omega \le 0.2\pi$

$|H(e^{j\omega})| \le 0.2$, $0.6\pi \le \omega \le \pi$

$T = 1\,sec.$

$\therefore A_p = 0.8$; $A_s = 0.2$; $\omega_p = 0.2\pi$; $\omega_s = 0.6\pi$.

$$\Omega_p = \frac{\omega_p}{T} = 0.2\pi$$

$$\Omega_s = \frac{\omega_s}{T} = 0.6\pi$$

$$d = \sqrt{\frac{\frac{1}{A_p^2} - 1}{\frac{1}{A_s^2} - 1}} = 0.1531$$

$$k = \frac{\Omega_p}{\Omega_s} = \frac{1}{3}$$

$$\therefore N = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)} = 1.4545 \simeq 2$$

$x_k = r_2 \cos\phi_k$

$y_k = r_1 \sin\phi_k$

$$\beta = \left[\frac{1 + \sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{1/2} \quad \left\{\varepsilon = \sqrt{\frac{1}{A_p^2} - 1} = 0.75\right\}$$

$\therefore \beta = 1.73205 \simeq \sqrt{3}$.

$$r_1 = \Omega_p\left(\frac{\beta^2+1}{2\beta}\right) = 0.7255$$

$$r_2 = \Omega_p\left(\frac{\beta^2-1}{2\beta}\right) = 0.36275$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k+1)\pi}{2N}$$

$\therefore \phi_0 = \frac{3\pi}{4}$ $\quad\Rightarrow \phi_k = \pm\frac{3\pi}{4}$

$\therefore \phi_1 = -\frac{3\pi}{4}$.

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$\mp x_k + j y_k = r_2 \cos\phi_k + j r_1 \sin\phi_k$

$= 0.36275 \cos\left(\frac{3\pi}{4}\right) \pm j\, 0.7255 \sin\left(\frac{3\pi}{4}\right)$

$= -0.2565 \pm j\, 0.513$

$$H(s) = \frac{C}{[s-(-0.2565+j0.513)][s-(-0.2565-j0.513)]}$$

$$= \frac{C}{(s+0.2565)^2 - (j\,0.513)^2}$$

$$= \frac{C}{s^2 + 0.513s + 0.3289}$$

@ $\Omega = 0$, $|H(0)| = A_p$ if N = even

$H(s)\big|_{s=0} = 0.8$

$\Rightarrow \frac{C}{0.3289} = 0.8 \Rightarrow C = 0.2631$

$$\therefore H(s) = \frac{0.26310}{s^2 + 0.513s + 0.3289}$$

$$H(s) = \frac{0.2631}{(s+0.2565)^2 + (0.513)^2}$$

$$H(z) = \frac{1}{2}\left\{\frac{0.513}{(s+0.2565)^2 + (0.513)^2}\right\}$$

$$\Rightarrow H(z) = \frac{1}{2}\left\{\frac{e^{-0.2565T} \sin 0.513T\, z^{-1}}{1-2e^{-0.2565T} \cos 0.513T\, z^{-1} + e^{-0.513T} z^{-2}}\right\}$$

$$\Rightarrow H(e^{j\omega}) = \frac{1}{2}\left\{\frac{0.37975}{e^{j\omega}\left((e^{j\omega})^2 - 0.6742 e^{j\omega} + 0.5987\right)}\right\}$$

$$H(s) = \frac{\frac{0.2631}{0.513} \times 0.513}{(s+0.2565)^2 + (0.513)^2}$$

$$= \frac{0.5128 \times 0.513}{(s+0.2565)^2 + (0.513)^2}$$

$$\Rightarrow H(z) = \frac{0.5128\, e^{-0.2565T} z^{-1} \sin 0.513T}{1-2e^{-0.2564} z^{-1} \cos 0.513T\, z + e^{-0.513} z^{-2}}$$

$$\boxed{\therefore H(z) = \frac{0.195 z^{-1}}{1 - 1.348 z^{-1} + 0.5987 z^{-2}}}$$