3.4. DECIMATION IN TIME (DIT) RADIX 2 FFT:

Decimation in Time (DIT) Radix 2 FFT algorithm converts the time domain N point sequence x(n) to a frequency domain N-point sequence X(k).In Decimation in Time algorithm the time domain sequence x(n) is decimated and smaller point DFT are performed. The results of smaller point DFTs are combined to get the result of N-point DFT.

In DIT radix -2 FFT the time domain sequence is decimated into 2-point sequences. For each 2-point sequence,2 -point DFT can be computed. From the result of 2-point DFT the 4-point DFT can be calculated. From the result of 4-point DFT the 8-point DFT can be calculated. This process is continued until we get N point DFT.This FFT algorithm is called radix-2 FFT.

In decimation in time algorithm the N point DFT can be realized from two numbers of N/2 point DFTs, The N/2 point DFT can be calculated from two numbers of N/4-point DFTs and so on.

Let x (n) be N sample sequence, we can decimate x (n) into two sequences of N/2 samples. Let the two sequences be f_1 (n) and f_2 (n).Let f_1 (n) consists of even numbered samples of x (n) and f_2(n) consists of odd numbered samples of x(n).

$f_1(n) = x(2n)$ for $n=0,1,2,3\ldots\ldots\ldots\frac{N}{2} - 1$

$f_2(n) = x(2n+1)$ for $n=0,1,2,3\ldots\ldots\ldots\frac{N}{2} - 1$

Let X (k) = N-point DFT of x (n)

F_1 (k) = N/2 point DFT of f_1(n)

F_2 (k) = N/2 point DFT of f_2(n)

By definition of DFT the N/2 point DFT of f_1(n) and f_2(n) are given by

$$F_1(k) = \sum_{n=0}^{\frac{N}{2}-1} f_1(n) W_{N/2}^{kn};$$

$$F_2(k) = \sum_{n=0}^{\frac{N}{2}-1} f_2(n) W_{N/2}^{kn}$$

Now-point DFT X (k), in terms of N/2 point DFTs F_1 (k) and F_2 (k) is given by

$$X(k) = F_1(k) + W_N^k F_2(k), \text{ where, } k= 0,1,2,\ldots\ldots\ldots(N-1)$$

Having performed the decimation in time once, we can repeat the process for each of the sequences f_1 (n) and f_2 (n).Thus f_1(n) would result in the two N/4 point sequences and f_2(n) would result in another two N/4 point sequences.

Let the decimated N/4 point sequences of f_1 (n) be V_{11} (n) and V_{12} (n).

$V_{11}(n) = f_1(2n)$; for n= 0, 1, 2,….. $\frac{N}{4} - 1$

$V_{12}(n) = f_1(2n+1)$; for n= 0,1,2,…..$\frac{N}{4} – 1$

Let the decimated N/4 point sequences of f_2(n) be V_{21}(n) and V_{22}(n).

$V_{21}(n) = f_1(2n)$; for n= 0,1,2,…..$\frac{N}{4} - 1$

$V_{22}(n) = f_1(2n+1)$; for n= 0,1,2,…..$\frac{N}{4} – 1$

Let $V_{11}(k)$ = N/4 point DFT of V_{11} (n);

$V_{12}(k)$ = N/4 point DFT of V_{12} (n)

$V_{21}(k)$ = N/4 point DFT of V_{21}(n)

$V_{22}(k)$ = N/4 point DFT of V_{22}(n)

Then like earlier analysis we can show that,

$$F_1(k) = V_{11}(k) + W^k_{N/2} V_{12}(k) \text{ ; for } k = 0,1,2,3,\ldots\ldots\ldots\ldots \frac{N}{2} - 1$$

$$F_2(k) = V_{21}(k) + W^k_{N/2} V_{22}(k) \text{ ; for } k = 0,1,2,3,\ldots\ldots\ldots\ldots \frac{N}{2} - 1$$

Hence the N/2 point DFTs are obtained from the results of N/4 point DFTs.

The decimation of the data sequence can be repeated again and again until the resulting sequences are reduced to 2-point sequences.

Flow graph for 8 point DFT using radix 2 DIT FFT

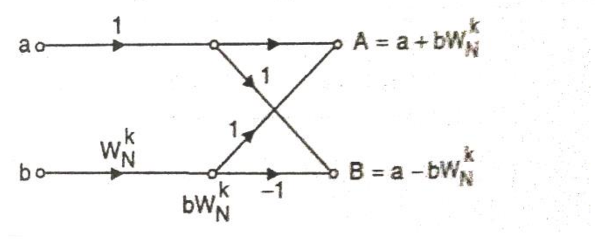

Fig.3.4.1.Basic Butterfly computation

[Source: 'Digital Signal Processing Principles, Algorithms and Applications' by J.G. Proakis and D.G. Manolakis page-460]

In each computation two complex numbers "a" and "b" are considered. The complex number "b" is multiplied by a phase factor "W_N^k "The product "b W_N^k" is added to complex number "a" to form new complex number "A". The product "b W_N^k" is subtracted from complex number "a" to form new complex number"B".

The input sequence is 8 point sequence.Therefore, N $=8=2^3=r^m$.Here r=2 and m=3.The sequence x (n) is arranged in bit reversed order and then decimated into two sample sequences.

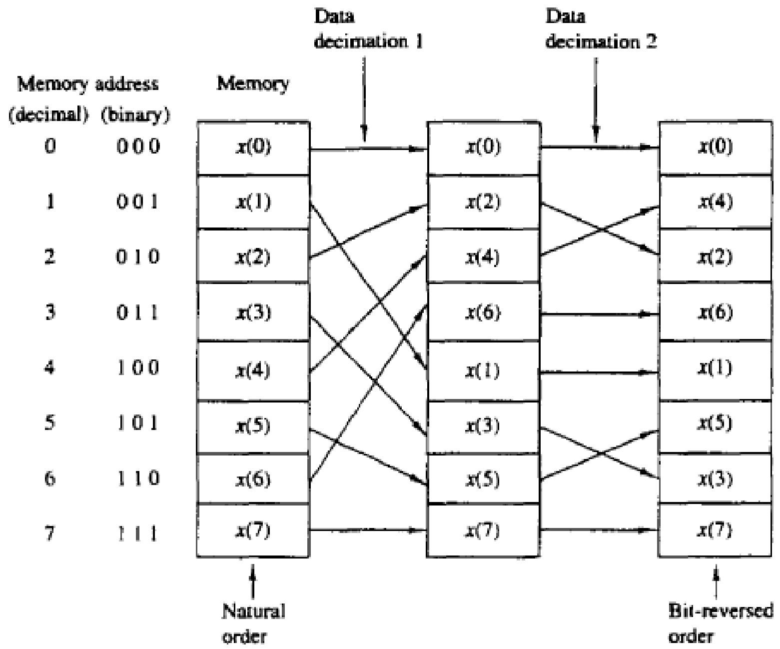

Fig.3.4.2.Bit reversed order

[Source: 'Digital Signal Processing Principles, Algorithms and Applications' by J.G. Proakis and D.G. Manolakis page-462]

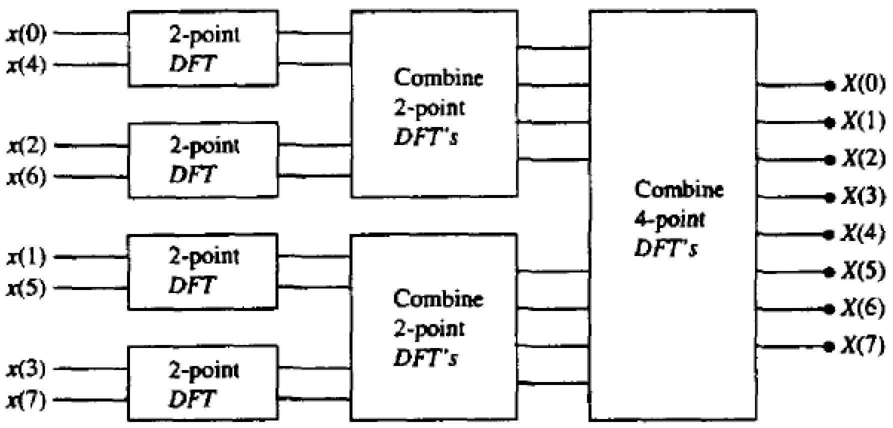

Fig 3.4.3. Three stages in the computation of an N = 8 point

[Source: 'Digital Signal Processing Principles, Algorithms and Applications' by J.G. Proakis and D.G. Manolakis page-459]

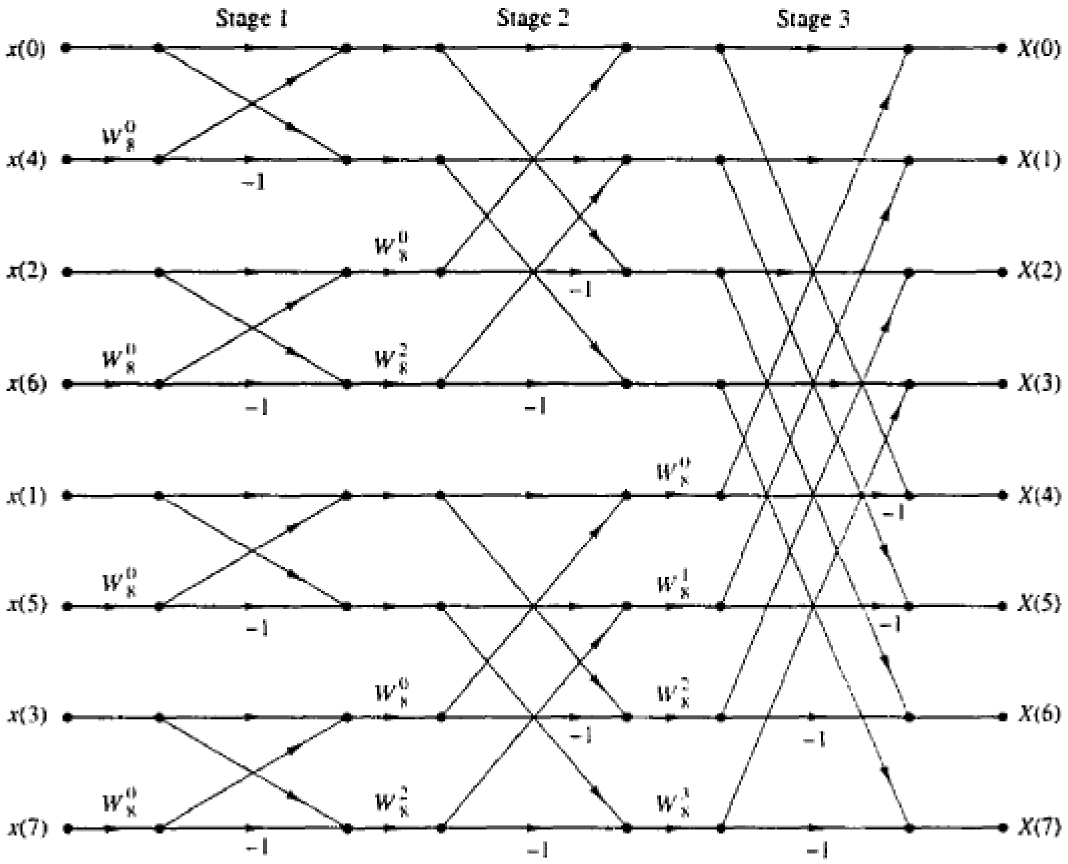

Fig 3.4.4. Eight point Decimation In Time-FFT

[Source: 'Digital Signal Processing Principles, Algorithms and Applications' by J.G. Proakis and D.G. Manolakis page-460]