

MATH I.A.-2

Q1

Given: Bag with 3 white & 2 Red marbles drawn without replacement until all marbles are drawn.

X = no. of draws required to get 1st white marble

Y = no. of draws required to get 2nd white marble

(a) Joint Probability Distribution of X & Y

X	Y	P
1	1	3/10
1	2	3/10
1	3	3/10
2	1	3/10
2	2	2/10
3	1	1/10

$$P(X, Y) = \begin{bmatrix} 3/10 & , & (1, 1) \\ 3/10 & , & (1, 2), (1, 3) \\ 3/10 & , & (2, 1) \\ 1/10 & , & (2, 2), (3, 1) \\ 0 & , & \text{otherwise} \end{bmatrix}$$

(b) Correlation function:

$$\rho_{xy} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$$E_X = 1 \times 0.6 + (2 \times 0.4) = 1.4$$

$$E_Y = (1 \times 0.2) + (2 \times 0.3) = 1.3$$

$$E_{XY} = \sum (x \cdot y \cdot P(x, y)) = 1.85$$

Variance \Rightarrow

$$\text{Var}(x) = E(x^2) - (E(x))^2 = 0.64$$

$$\text{Var}(y) = E(y^2) - (E(y))^2 = 0.61$$

Covariance \Rightarrow

$$E(xy) = E(x)E(y) = 0.03$$

$$P(x, y) = \frac{0.03}{\sqrt{0.64 \times 0.61}} \approx \underline{\underline{0.05}} \quad (0.04801)$$

Q2

Given: $f(x, y) = k$ $0 \leq x \leq 1$
 $x \leq y \leq 4-x^2$

 $k = ?$

$$\int_0^1 \int_x^{4-x^2} k \, dy \, dx = 1$$

$$\int_0^1 k \left[\frac{(4-x^2) - x}{2} \right] dx = 1$$

$$k = \frac{40}{729} = 0.055$$

 \rightarrow Marginal Density Function:

$$f_x(x) = \int_x^{4-x^2} k \, dy$$

$$f_x(x) = \frac{40}{729} [15x - 8x^2 + x^3]$$

$$f_y(y) = \frac{40}{729} \times \frac{y^2}{2}$$

Q3

$$f(x) = 1$$

$$0 \leq x \leq 1$$

Find moment Generating Function

$$m_X(t) = E(e^{tx}) = \int_0^1 e^{tx} \cdot f(x) dx$$

$$m_X(t) = \frac{e^t - 1}{t}, \quad t \neq 0$$

For $t=0$:

$$m_X(0) = \lim_{t \rightarrow 0} \frac{e^t - 1}{t} = \underline{\underline{1}} = m_X(0)$$

$$\underline{m_X(0) = 1}$$

Variance = $V(x)$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_0^1 x^2 dx = 1/3$$

$$V(x) = \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \underline{\underline{\frac{1}{12}}} \text{ ans.}$$