

Graphics and Visual Computing.

sites.google.com/iiita.ac.in/course-of-paran-iiita/gvc

* Pavan Chakraborty :-

$$\begin{aligned}\text{Screen size} &= 1920 \times 1080 \text{ px} \\ &= 2073600 \text{ pixels.}\end{aligned}$$

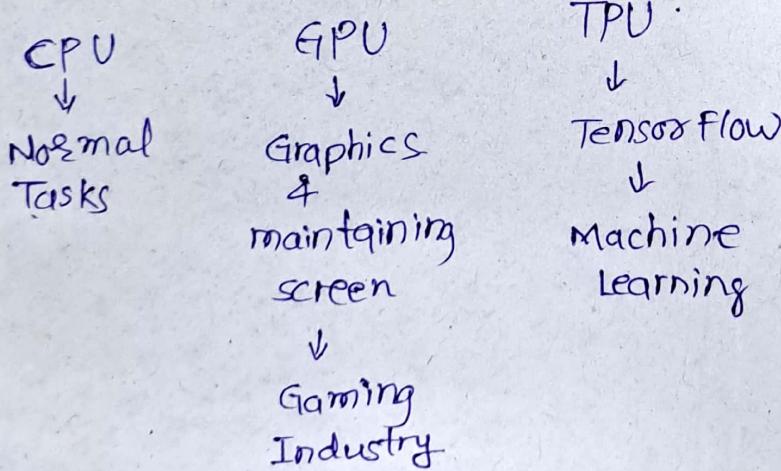
Each pixel has RGB, each colour having 1 byte size (8 bits)

$$\therefore \begin{array}{r} 2073600 \\ \times \quad 3 \\ \hline 6220800 \end{array} \text{ Byte.}$$

Also, to show the graphics per second, 50 frames are needed.

$$\therefore \begin{array}{r} 6220800 \\ \times \quad 50 \\ \hline 31,10,40,000 \approx 31 \text{ M Bytes/sec} \end{array}$$

For this, we need separate dedicated processing unit, for Graphics processing. GPU cards



openCV → images
OpenGL → Graphics.

Tensor :- has effect in all directions.

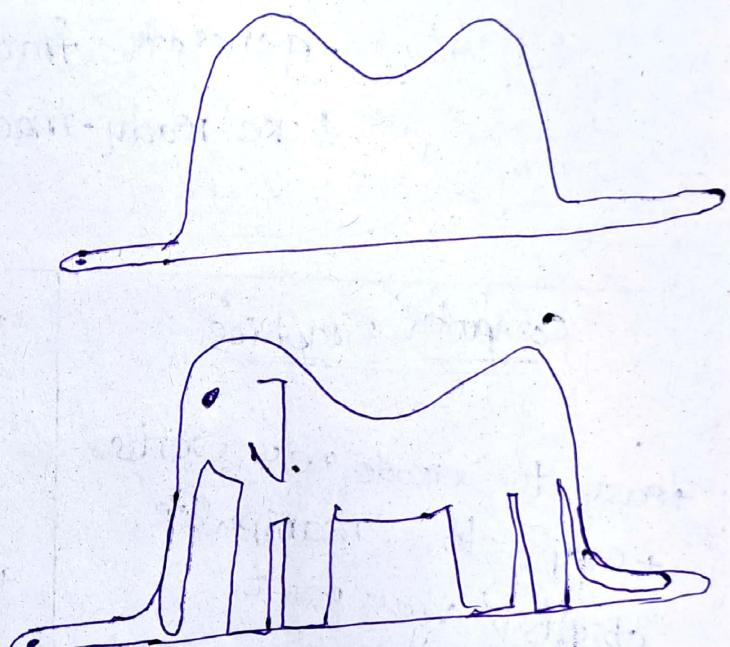
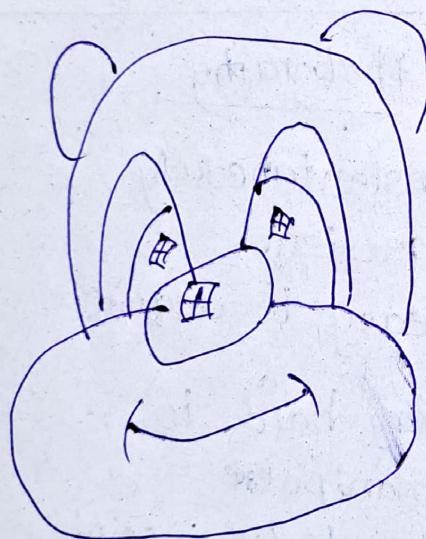
→ used by Einstein to develop Space-time curvature

Reflection Type → metallic
→ mirror-like

Green Apples > Red Apples.

Book:- Little Prince

Sir drawings



Lens Based Camera Obscura

* Traditional Computer Graphics

Image → wireframes
breakdown

A wireframe has a similar surface & texture.

Fractals - used to generate Trees

- but take a lot of time

- generate fractals offline

+ use ready-made vertices stored in device

Computer Graphics

+ easy to create new worlds.

+ easy to manipulate.

objects / viewpoint.

- very hard to look realistic.

Photography

+ instantaneously
realistic.

+ easy to acquire

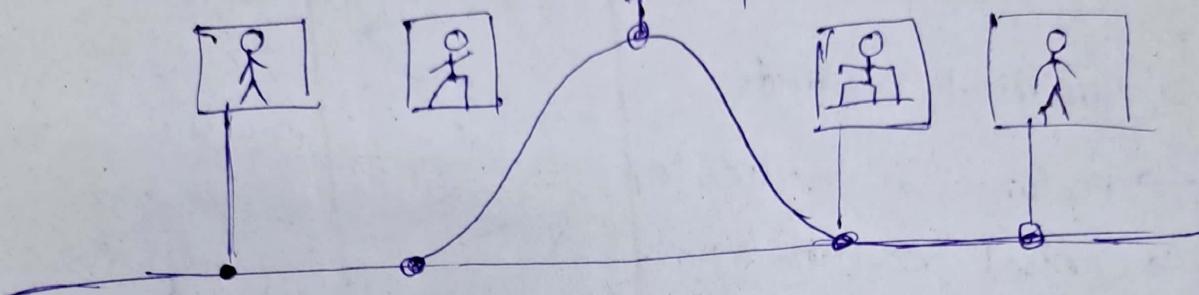
- very hard to
manipulate
objects / viewpoint.

Merging

Computer Graphics + Photography = visual computing

• Cheaper by Dozen - Pavan chakraborthy sir

Animation & Keyframing [♂]



Rigid Body Dynamics :- Learn torque & force.

Face transitioning ; Face warping & morphing.

coordinate systems

- Cylindrical coordinates
- Rectangular coordinates
- Conical coordinates
- Spherical Polar Coordinates

↳ use in GPS.

↳ has accuracy more than 10 metres.

India has Navik instead of GPS.

Object space

- local to each object.
- most natural coordinate system for the geometry object.
- Local coordinate system / modelling coordinate system.

World Space

- common to all objects.
- The global coordinate system shared by all objects in world.

Eye Space / Camera Space :-

- derived from view frustum

Screen Space

- indexed according

World Space \rightarrow Eye Space \rightarrow Screen space.

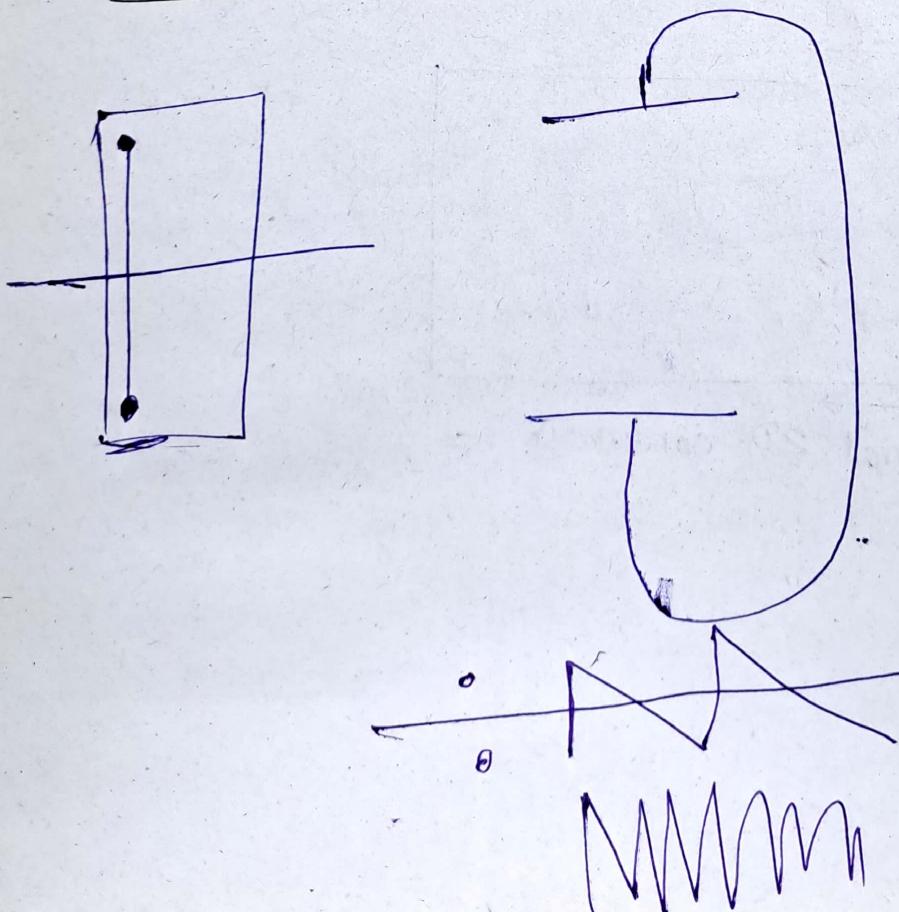
The world is matrix!

model matrix \rightarrow view matrix \rightarrow projection matrix

Raster :- derived from TV system for a row of pixels.

- Scanline.
- influence algorithms.
- is derivation of Rasterization or scanline algorithms.

Raster interlaced scanning:-



- colored CRT screen is achieved by hitting electron on fluorescent screen.

- LCD - Liquid crystal display, use view-voltage to change rotation of liquid crystal.
- OLED is better.

How 3D is achieved?

Ans- using 3D polarized glasses

• screen also polarizes image.

Hence multiple dimensions can be felt.

* used in PVR.

• blue part of sky is polarized light

• Also used in seeing Eclipse.

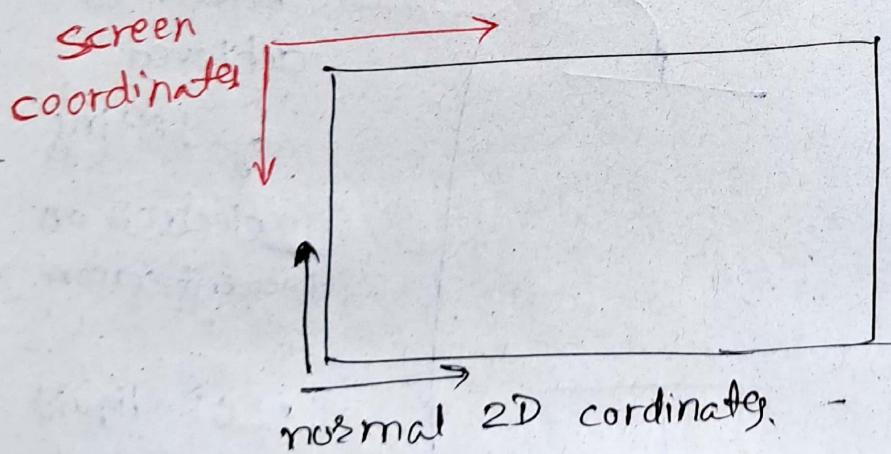
Graphics Pipeline

→ Geometry or 3D pipeline

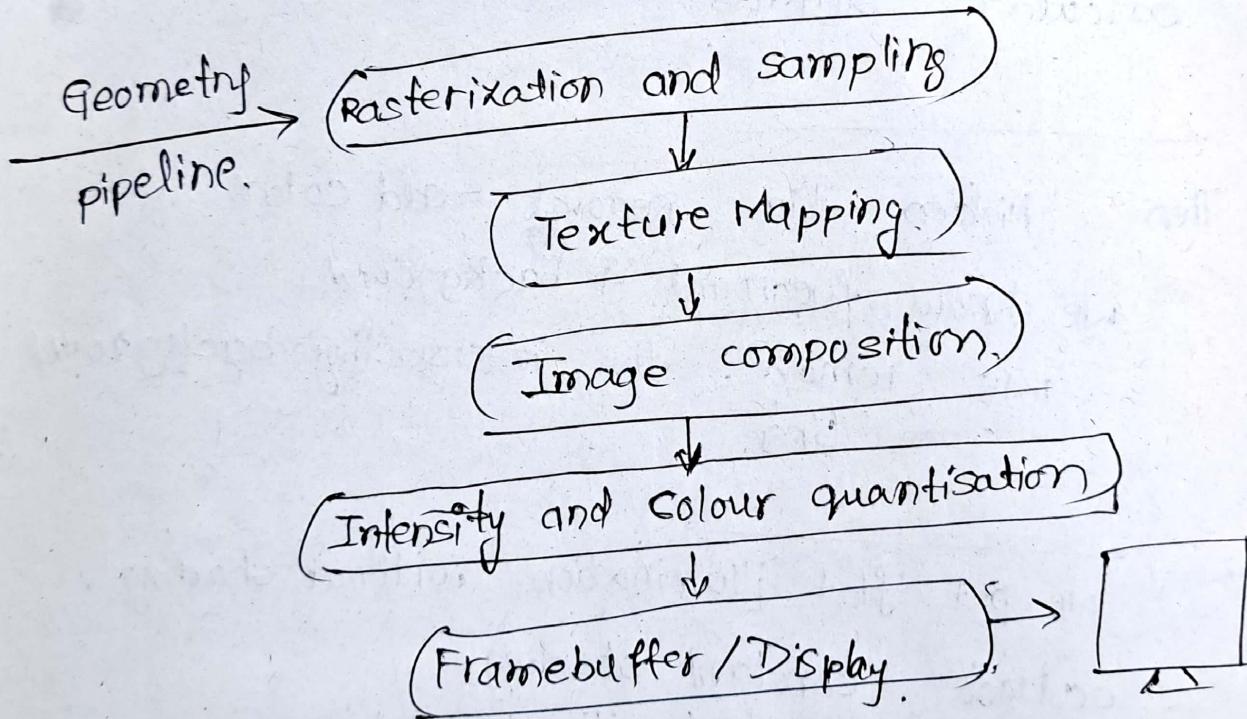
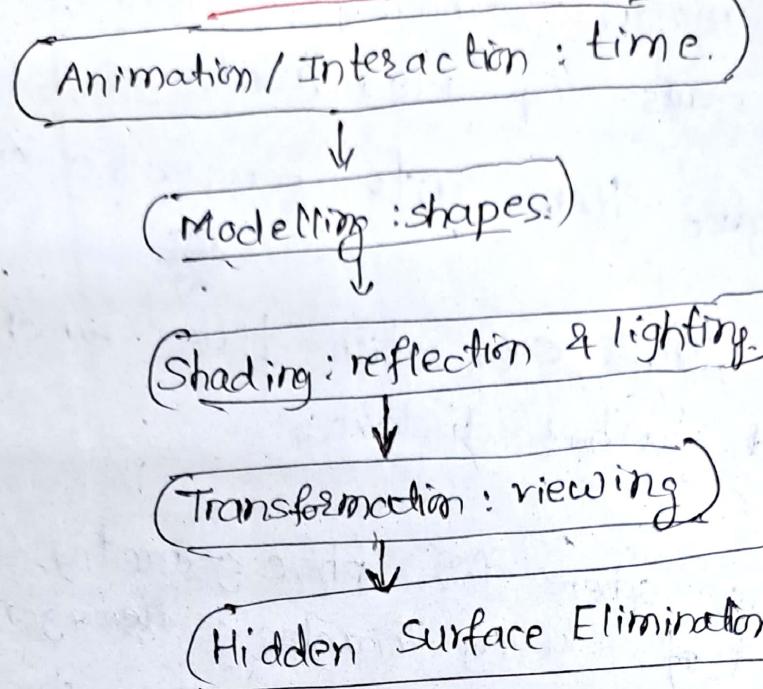
→ Imaging or 2D pipeline

• screen has pixels, which is quantized.

How to color, to make continuous image



Geometry Pipeline



This happens 50 times a second, to represent the fast view of screen.

Wireframe model - orthographic view.

3 views:- Top, side, front.

We, then place them into perspective view.

We do Depth cue, which tells which things are at what distance.

Q) Why not use sphere for sphere geometry,
instead of using Triangles & Hexagons.

Ans:- To plot good reflections, easy to calculate surfaces normal to light

Then Hidden Line Removal - add color

we know foreground & background.

we remove the intersecting background lines.

we put flat illumination without shading,

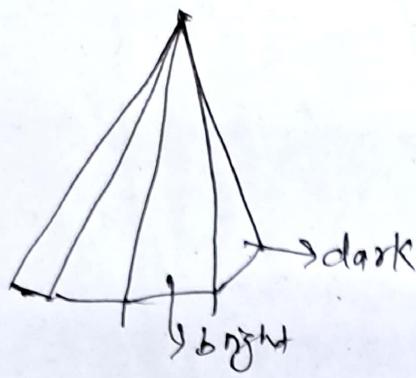
called constant shading.

• ambient illumination

- ext light coming from all directions equally

Faceted Shading - Flat.

• light surfaces according to direction of Light source. Angles are measured, normal to surfaces.



Gouraud Shading,
No specular Highlights.

- we do interpolating of shades between adjacent surfaces (smoothening).

Specular Highlights Added

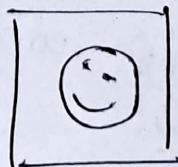
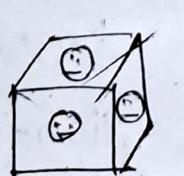
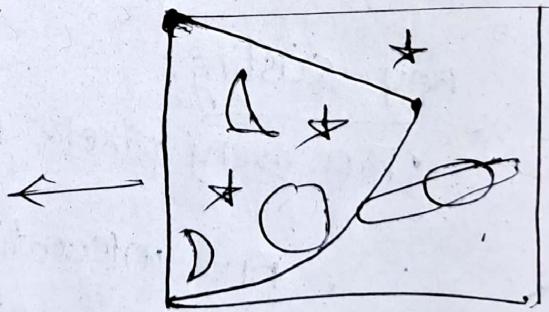
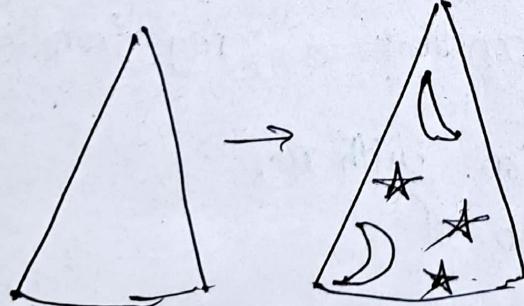
- add regions which reflect bright

Phong Shading

Texture Mapping-

- stick / map texture matrices onto the surfaces,

e.g.-



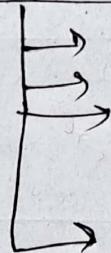
Finally polished & realistic images are made.

Why Computer Graphics?

- movies
- CAD-CAM
- medical imaging
- astrophysics.

Graphics APIs (OpenGL, WebGL),
modelling packages (3D Studio MAX)

Rendering pipeline



Texture and modelling

Ray casting

- For every pixel, construct a ray from eye
 - Find intersection with ray
 - Keep if closest.
- The objects which reach light source \rightarrow illuminated brightly
- Those not reaching \rightarrow ambiently illuminated.

Drawing Lines & Antialiasing

Jurrasic Park - done using OpenGL.

Line Equation

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$i_1 = \text{Int} (x_1 + 0.5)$$

$$i_2 = \text{Int} (x_2 + 0.5)$$

$$j_1 = \text{Int} (y_1 + 0.5)$$

$$j_2 = \text{Int} (y_2 + 0.5)$$

$\rightarrow 0.5$ added to increase
~~numbers~~ (or) Round-off the
numbers.

- * floor and ceiling achieved without using special function.

~~Let~~ Let $i = i_1$ and $j = j_1$

while ($i \leq i_2$) do

{ print (i,j);

$i = i + 1$

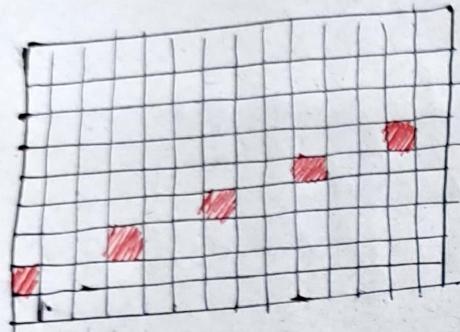
$y = m * i + c$

$j = \text{Int} (y + 0.5)$ }

a) Is this good?

ans:-

ex:- $y = 0.25x + 0$



x	y	j
0	0	0
1	0.25	1
2	0.5	1
3	0.75	1
4	1.0	1
5	1.25	1
6	1.5	2
7	1.75	2
8	2.0	2
9	2.25	2

Also if slope is more

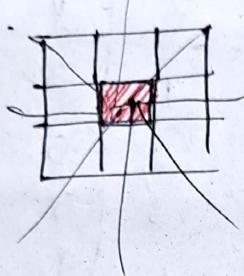
$$y = 10x + 0$$

we get dotted line
in this case.

x	y	j
0	0	0
1	10	10
2	20	20
3	30	30
4	40	40
5	50	50
6	60	60

If we make smaller increments like $y+0.01$?
 we will colour the same pixel many times

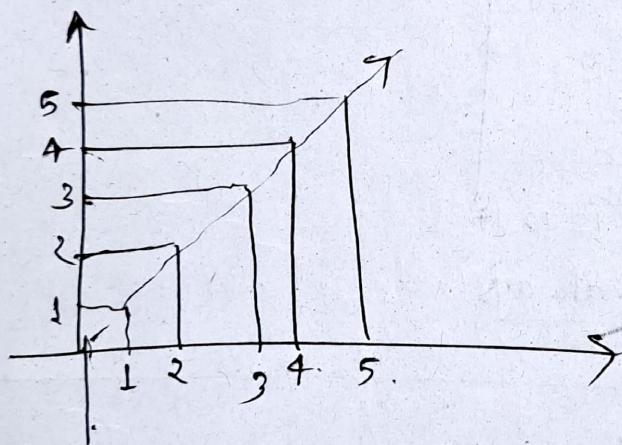
Solution :-



Just apply line equation
such that u find
adjacent out of 8 pixels.

Line drawing algo

$$y = mx + c$$



Line passing
through $(1,1)$
and $(6,6)$.

$$y = mx + c$$

$$1 = m(1) + c$$

$$6 = m(6) + c$$

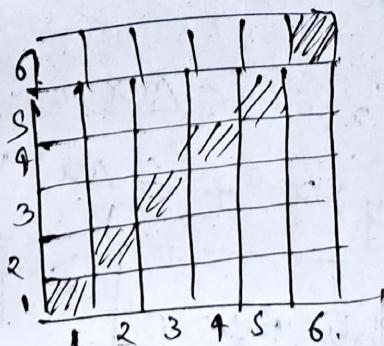
$$5 = 5m$$

$$\boxed{m=1}$$

x	y	eqn $y=x$
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	

$$1 = 1 + c$$

$$\boxed{c=0}$$



a) Find Line passing through $P_1(1, 3)$, $P_2(3, 7)$.

ans:

$$y = m + c$$

$$7 = 3m + c$$

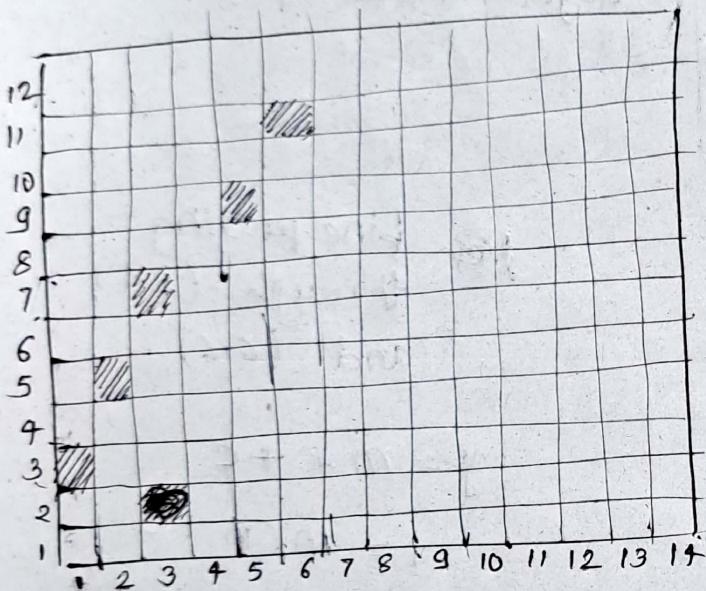
$$4 = 2m$$

$$\boxed{m=2}$$

$$3 = 2 + c$$

$$\therefore \boxed{c=1}$$

$$\boxed{y = 2x + 1}$$



x	y
1	3
2	5
3	7
4	9
5	11

• Not continuous line always.

Any efficient Algorithm:-

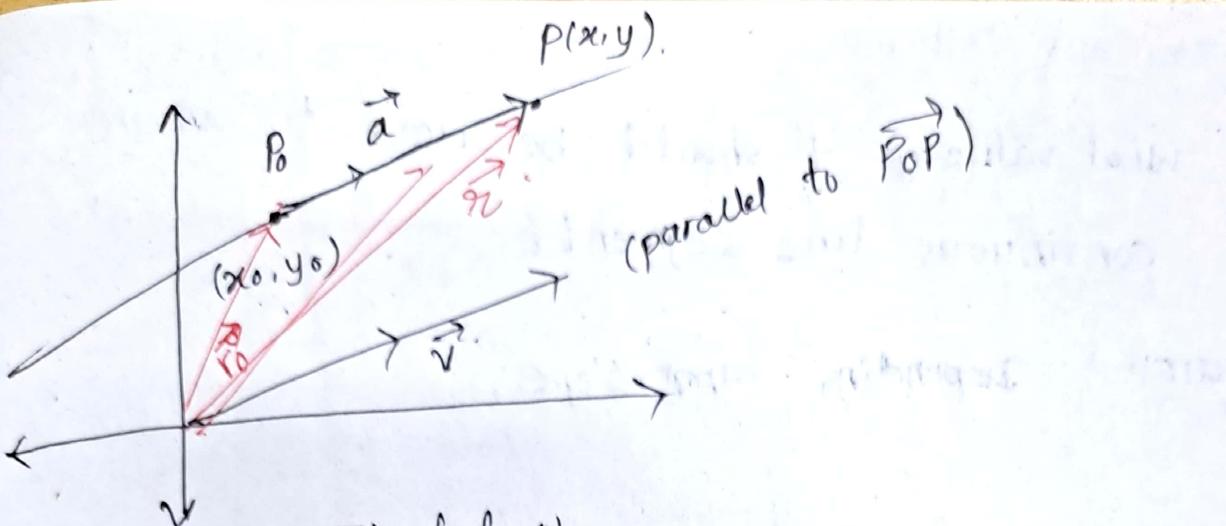
• Yes, uses parametric line form

$$y = mx + c$$

$$y = \frac{\Delta y}{\Delta x} x + c$$

$$\Delta xy = \Delta y x + c \Delta x$$
$$0 = (\Delta y) x + (\Delta x) y + c'$$

$$\boxed{f(x, y) = ax + by + c = 0}$$



Triangle law.

$$\vec{r}_0 + \vec{a} = \vec{r}$$

$$\therefore \boxed{\vec{a} = \vec{r} - \vec{r}_0}$$

Let constant 't'

$$\text{since } \vec{a} \parallel \vec{v} \quad \therefore \vec{a} = t \cdot \vec{v}$$

$$\therefore \boxed{\vec{r} = \vec{r}_0 + t \cdot \vec{v}}$$

DDA:-

$$\langle x_0, y_0, z_0 \rangle + t \langle a, b, c \rangle$$

$$\Rightarrow \langle x_0 + ta, y_0 + tb, z_0 + tc \rangle$$

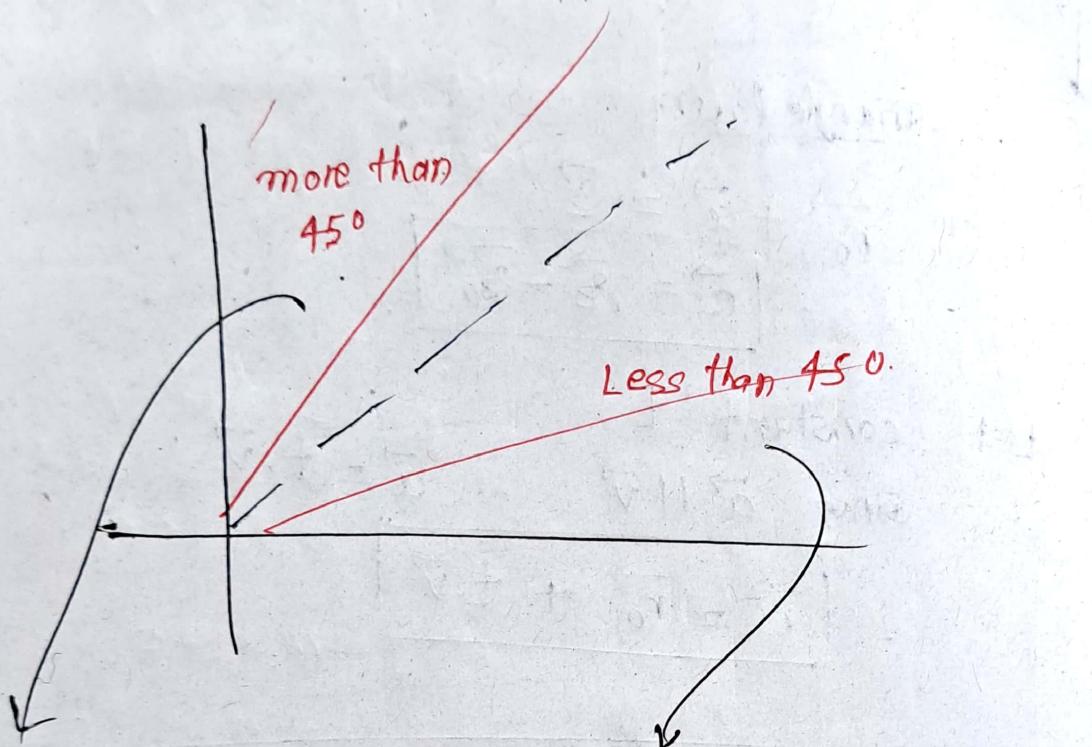
$$\left. \begin{array}{l} x = x_0 + ta \\ y = y_0 + tb \\ z = z_0 + tc \end{array} \right\} \quad \begin{array}{l} x = x_1 + t(x_2 - x_1) \\ y = y_1 + t(y_2 - y_1) \\ z = z_1 + t(z_2 - z_1) \end{array}$$

~~t < 0~~ ~~& t > 1~~

$$\left\{ \begin{array}{l} \frac{1}{dt} \\ t = \frac{1}{\Delta x} \\ t = \frac{1}{\Delta y} \end{array} \right\}$$

what value of 't' should be used to ensure continuous line segment?

answ Depending upon slope,



$$m > 1$$

$$y = y + 1.$$

$x = \text{compute.}$

$$x + \{ \}$$

$$x + \frac{dx}{dt} -$$

$$\Rightarrow x = x + \frac{dx}{dy}$$

$$\boxed{x = x + \frac{1}{m}}$$

and if

$$\begin{aligned} m &= 1 \\ x &= x + 1 \\ y &= y + 1 \end{aligned}$$

$$m < 1$$

$$x = x + 1$$

$y = \text{compute}$

$$y = y + \{ \} \rightarrow \frac{dy}{dt} = \frac{dy}{dx}$$

$$y = y + \frac{dy}{dx}$$

$$\boxed{(y = y + m)}$$

DDA algorithm :- (Digital differential analyser)

if $m < 1$

$$x = x + 1$$

$$y = y + m$$

if $m > 1$

$$x = x + \frac{1}{m}$$

$$y = y + 1$$

if $m = 1$

$$x = x + 1$$

$$y = y + 1$$

ex:- $P_1(1, 3)$
 $P_2(3, 7)$.

$m = 2$.

$m > 1$

$$x = x + \frac{1}{m}$$

$$y = y + 1.$$

x	y	pixel to be plotted.
1	3	(1, 3)
1.5	4	(2, 4)
2	5	(2, 5)
2.5	6	(3, 6)
3	7	(3, 7)

start :- $x = 1, y = 3$

$$x \Rightarrow x + \frac{1}{m} = 1 + \frac{1}{2} = 1.5.$$

$$y \Rightarrow 3 + 1 = 4$$

$$x = 1.5 + 0.5 = 2$$

$$y = 5$$

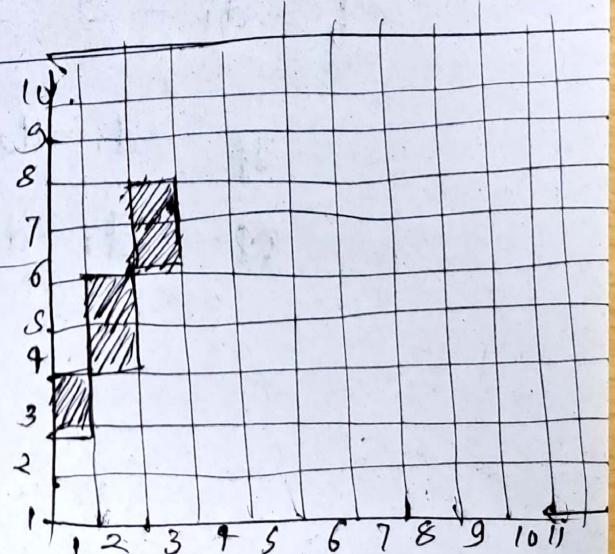
$$x = 2.5$$

$$y = 6.$$

$$x = 3$$

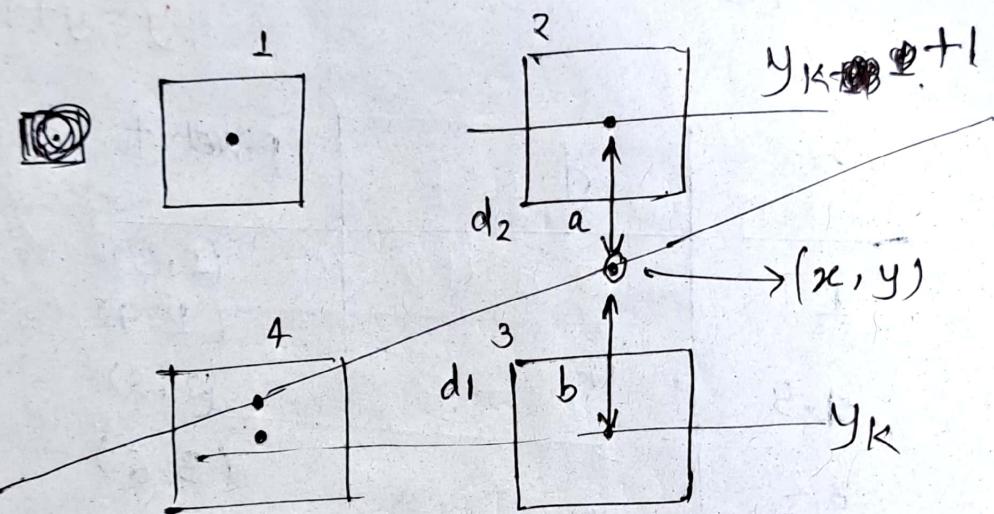
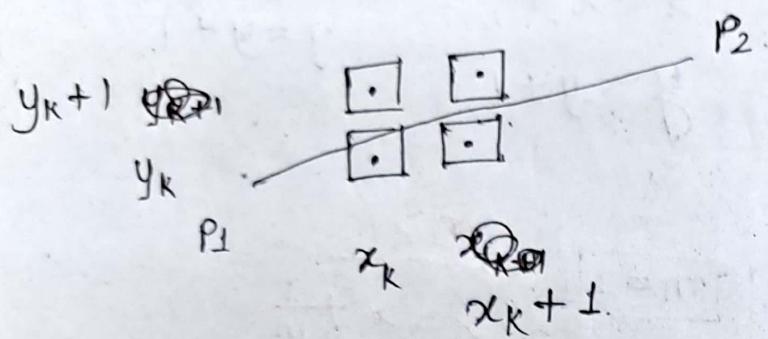
$$y = 7$$

! Simplest
Algorithm !



Bresenham Algorithm

4. @ closely spaced pixels.



$a < b \Rightarrow$ Glow 2

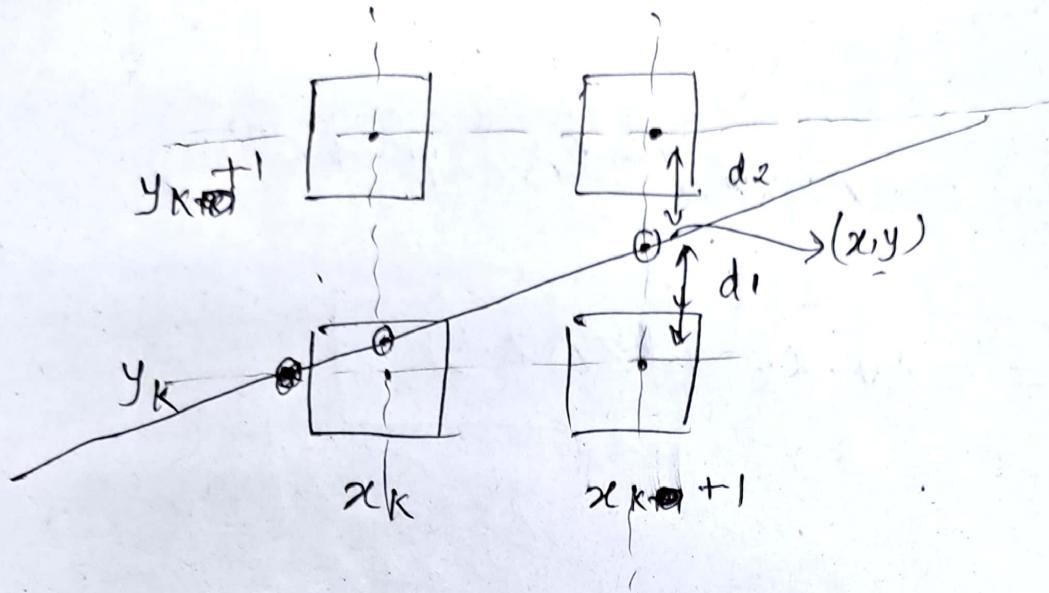
$a > b \Rightarrow$ Glow 3.

$$\therefore a = d_2 = y_{k+1} - y$$

$$b = d_1 = y - y_k.$$

If $d_1 - d_2 > 0 \Rightarrow$ Glow 2

@ $d_1 - d_2 < 0 \Rightarrow$ Glow 3.



$$d_1 = y - y_k$$

$$d_2 = y_{k+1} - y.$$

$$\begin{aligned} d_1 - d_2 &= (y - y_k) - (y_{k+1} - y) \\ &= y - y_k - y_{k+1} + y \\ \boxed{d_1 - d_2} &= 2y - y_k - y_{k+1} = 2y - 2y_k - 1 \end{aligned}$$

We are plotting for (x, y)
which is (x_{k+1}, y) .

$x \rightarrow x_{k+1}$

$$\begin{aligned} \therefore d_1 - d_2 &= 2mx + 2c - y_k - y_{k+1} \\ &= 2m(x_{k+1}) + 2c - y_k - y_{k+1}. \end{aligned}$$

$$\begin{aligned} &= 2m x_k + \frac{2m + 2c}{\Delta x} - y_k - y_{k+1} - 1 \\ &= 2m x_k + c' - 2y_k - 1 \end{aligned}$$

$$d_1 - d_2 = 2 \frac{\Delta y}{\Delta x} (x_k) + c' - 2y_k - 1$$

$$\Delta x(d_1 - d_2) = 2 \Delta y \cdot x_k + \frac{\Delta x \cdot c'}{\Delta x} - 2y_k \cdot \Delta x - \Delta x$$

$$\Delta x(d_1 - d_2) = 2 \Delta y \cdot x_k + c' - 2y_k \cdot \Delta x.$$

Finally

$$\Delta x(d_1 - d_2) = 2\Delta y \cdot x_k - 2\Delta x \cdot y_k + c''$$

δP_k

$$P_k = 2\Delta y \cdot x_k - 2\Delta x \cdot y_k$$

k, k_{next}

$$P_{next} = 2\Delta y \cdot x_{next} - 2\Delta x \cdot y_{next}$$

$$\therefore P_{next} - P_k = 2\Delta y (x_{next} - x_k) \\ - 2\Delta x (y_{next} - y_k)$$

D

$$\begin{aligned} &= 2\Delta y (x_k + 1 - x_k) \\ &\quad - 2\Delta x (y_k + 1 - y_k) \\ &= 2\Delta y - 2\Delta x. \end{aligned}$$

IF $P_{next} - P_k < 0$.

$$x_k \rightarrow x_k + 1$$

$$y_k \rightarrow y_k$$

Hence $P_{next} - P_k = 2\Delta y (x_k + 1 - x_k) - 2\Delta x (y_k - y_k)$

$$\therefore P_{next} = P_k + 2\Delta y$$

\therefore If $P_{next} - P_k \geq 0$.

$$Y_{next} = Y_k + 1$$

$$X_{next} = X_k + 1$$

$$\therefore \boxed{P_{next} = P_k + 2\Delta y - 2\Delta x}$$

$$P_i = 2\Delta y - \Delta x$$

$$P = 2dy - dx$$

$x++$

if ($P < 0$)

$$\{ \quad P = P + 2\Delta y$$

}

else {

$$P = P + 2dy - 2dx.$$

$y++$

}

$$Q] P = 2dy - dx$$

print (1,1)

print (2,5)

ans:-

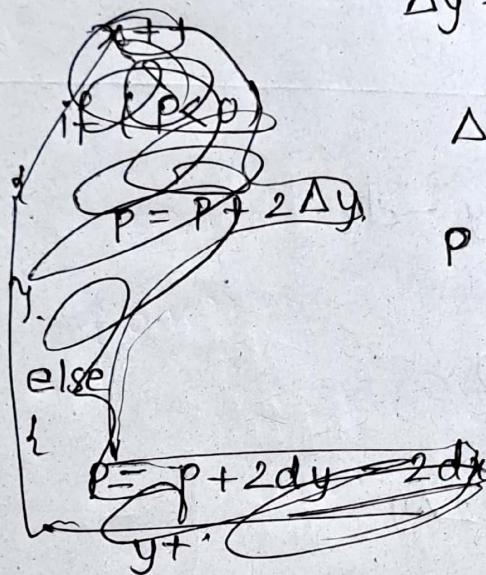
$$P = 2dy - dx$$

$$\Delta y = 5 - 1 \\ = 4$$

$$\Delta x = 8 - 1 = 7$$

$$P = 2 \times 4 - 7 \\ = 2 \cdot dy - dx \\ = 8 - 7$$

$$P = p + 2dy - 2dx. \quad \boxed{P=1}$$



$$P > 0$$

Hence

$y++ \rightarrow$

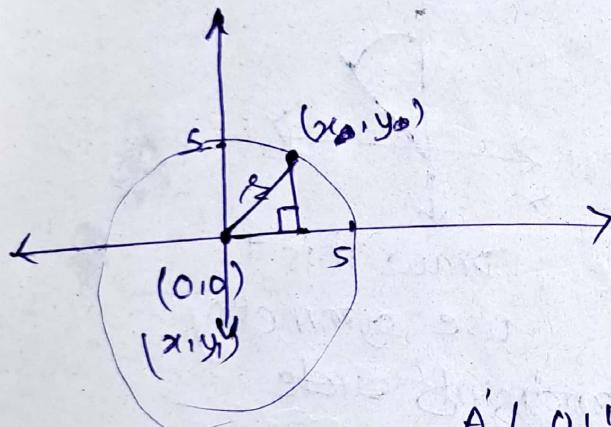
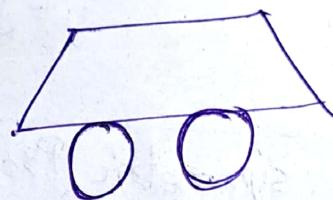
x	y	P
1	1	1
2	2	$P = 1 + 2 \times 4 - 2 \times 7$ $\boxed{P = -5}$
3		$(-5 < 0) \quad P = -5 + 2(4) - 2(7)$ $P = 3.$

Hence, x always ++

* we have to choose which y to glow based on P at each step.

This will be used for all lines.

Draw this



$$(x - x_1)^2 + (y - y_1)^2 = r^2$$

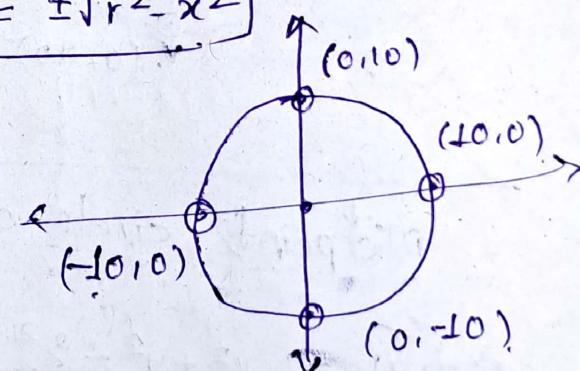
$$\begin{aligned} x^2 + y^2 &= r^2 \\ y &= \pm \sqrt{r^2 - x^2} \end{aligned}$$

$$A'(0, 10)$$

$$B(10, 0)$$

$$C(-10, 0)$$

$$D(0, -10)$$

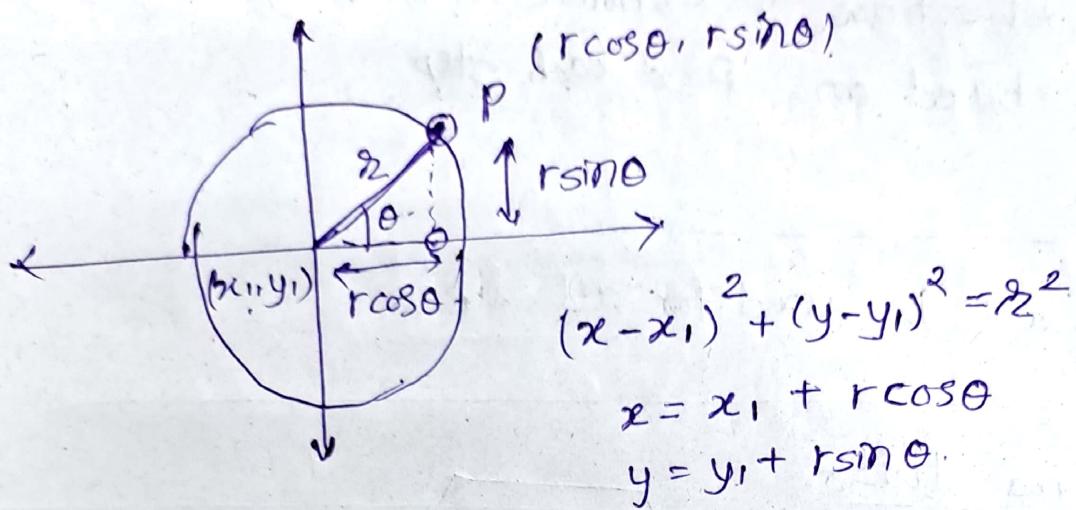


Drawback

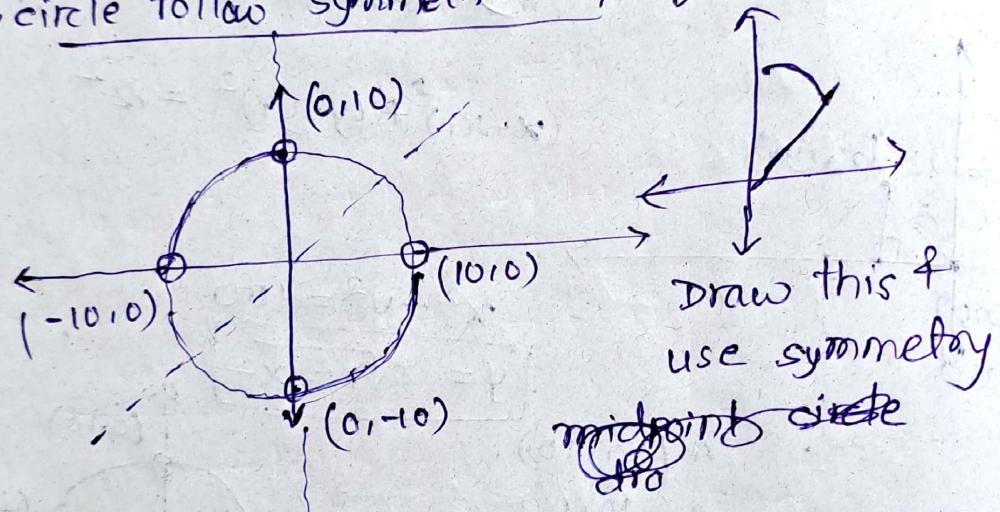
1. Floating point no-s

2. square points.

Parametric Form :-



circle follow symmetric Property



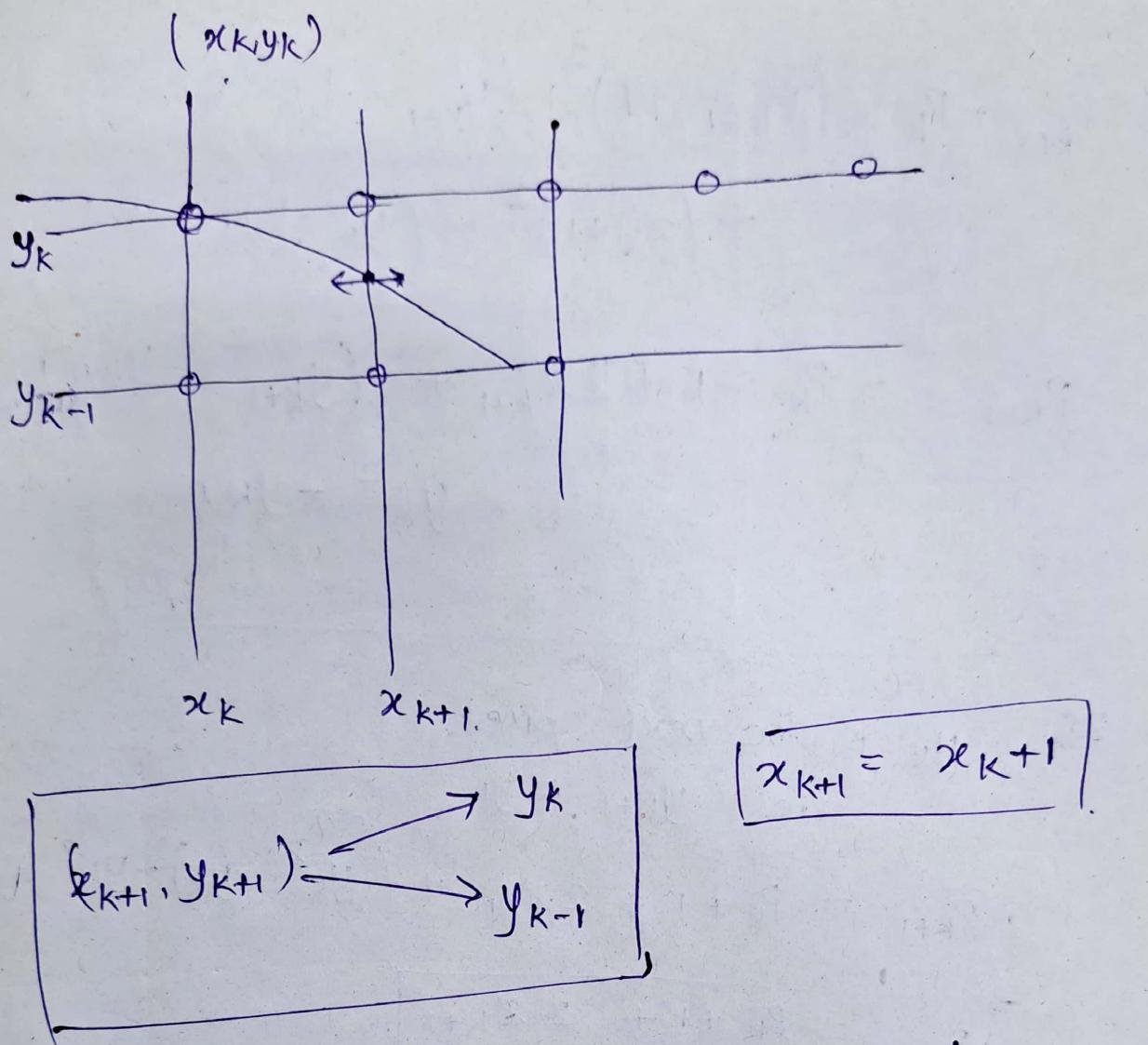
midpoint circle drawing algo

$$f(x,y) = x^2 + y^2 - r^2$$

$f(x,y) \leq 0 \rightarrow$ inside.

$f(x,y) > 0 \rightarrow$ outside.

$f(x,y) = 0 \rightarrow$ on the boundary.



$$(x_{k+1}, y_{k+1}) \rightarrow y_k \quad \boxed{x_{k+1} = x_{k+1}}$$

$$(x_{k+1}, y_{k-1}) \rightarrow y_{k-1}$$

$$(x_{k+1}, y_{k-\frac{1}{2}}) < 0 \rightarrow (x_{k+1}, y_k)$$

$$\left[\begin{array}{l} (x_{k+1}, y_{k+1}) \\ (x_{k+1}, y_{k-1}) \end{array} \right] \quad (x_{k+1}, y_{k-\frac{1}{2}}) > 0 \rightarrow (x_{k+1}, y_{k-\frac{1}{2}})$$

$$P_{k+1} = (x_{k+1} + 1)^2 + (y_{k+1} - \frac{1}{2})^2 - \gamma^2 \quad \text{--- (2)}$$

(2)-①

$$P_{k+1} - P_k = (x_{k+1} + 1)^2 + (y_{k+1} - \frac{1}{2})^2 - \gamma^2$$

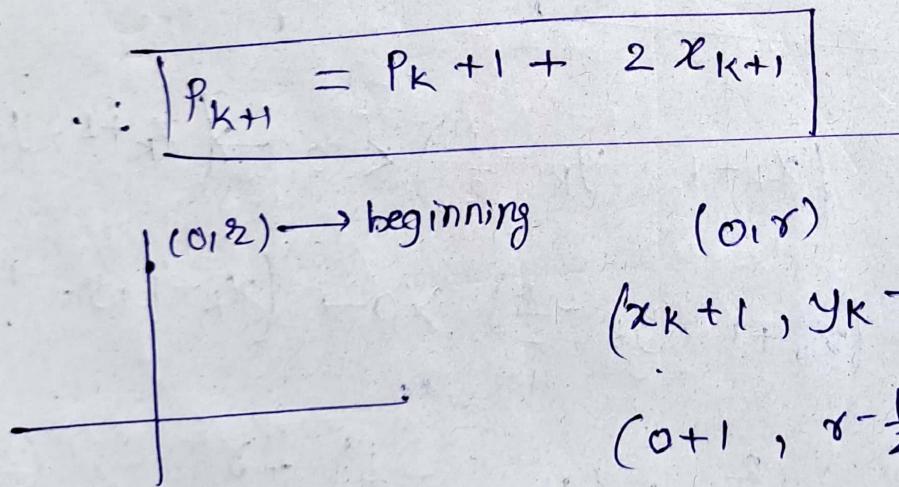
$$- \left[(x_{k+1})^2 + (y_{k-1})^2 - \gamma^2 \right]$$

$$P_{k+1} - P_k = (x_{k+1} + 1)^2 + (y_{k+1} - \frac{1}{2})^2 - (x_k + 1)^2 - (y_k - \frac{1}{2})^2$$

$$\begin{aligned} P_{k+1} &= P_k + 1 + 2x_{k+1} + (y_{k+1}^2 - y_{k+1}) \\ &\quad - (y_k^2 - y_k) \end{aligned}$$

If y_k is the next pixel.
ie $y_{k+1} = y_k$.

$$\therefore P_{k+1} = P_k + 1 + 2x_{k+1} + (y_k^2 - y_k) - y_k^2 + y_k$$



$$\begin{aligned} &x^2 + y^2 - r^2 \\ &1 + (r - \frac{1}{2})^2 - r^2 \end{aligned}$$

$$1 + r^2 + \frac{1}{4} - r - r^2$$

$$\frac{5}{4} - r = 0$$

$$\Rightarrow r = \frac{5}{4}$$

$$\textcircled{1} \quad P_0 = 1 - \gamma = \frac{5}{4} - \gamma$$

$$\textcircled{2} \quad P_{k+1} = P_k + 2x_{k+1}$$

$$\textcircled{3} \quad P_{k+1} = P_k + 1 + 2 \cdot (x_{k+1}) - 2y_{k+1}$$

concept: $P_0 = 1 - \gamma$

$$P_{k+1} = P_k + 1 + 2(x_{k+1}) \rightarrow P_k < 0 \quad (x++)$$

$$P_{k+1} = P_k + 1 + 2(x_{k+1}) - 2y_{k+1} \rightarrow P_k > 0.$$

start with $(0, 10) \rightarrow$ radius.

k	P_k	x_{k+1}, y_{k+1}
0	-9	(1, 10)
1	-6	(2, 10)
2	-1	(3, 10)
3	-6	(4, 9)
4	-3	(5, 9)
5	8	(6, 8)
6	5	(7, 7)

$(x \geq y)$

$$P_0 = 1 - \gamma = 1 - 10 = -9.$$

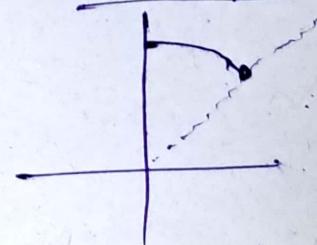
$P_0 < 0$. stop

$$P_{0+1} = P_0 + 1 + 2(x_0 + 1)$$

$$= -9 + 1 + 2(0 + 1)$$

$$\boxed{P_1 = -6}$$

Arc complete



Arc

(x_{k+1}, y_{k+1})

(y_{k+1}, x_{k+1})

$(1, 10)$

$(10, 1)$

$(2, 10)$

$(10, 2)$

$(3, 10)$

$(10, 3)$

$(4, 9)$

$(9, 4)$

$(5, 9)$

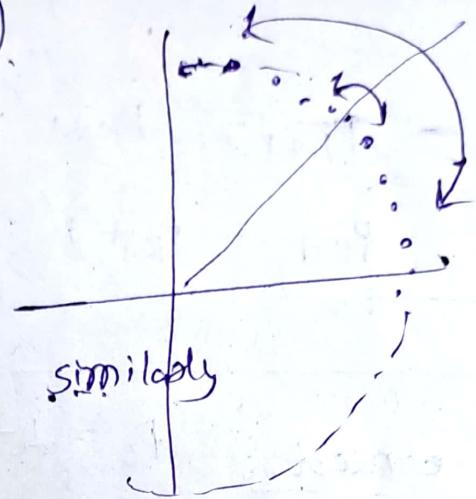
$(9, 5)$

$(6, 8)$

$(8, 6)$

$(7, 7)$

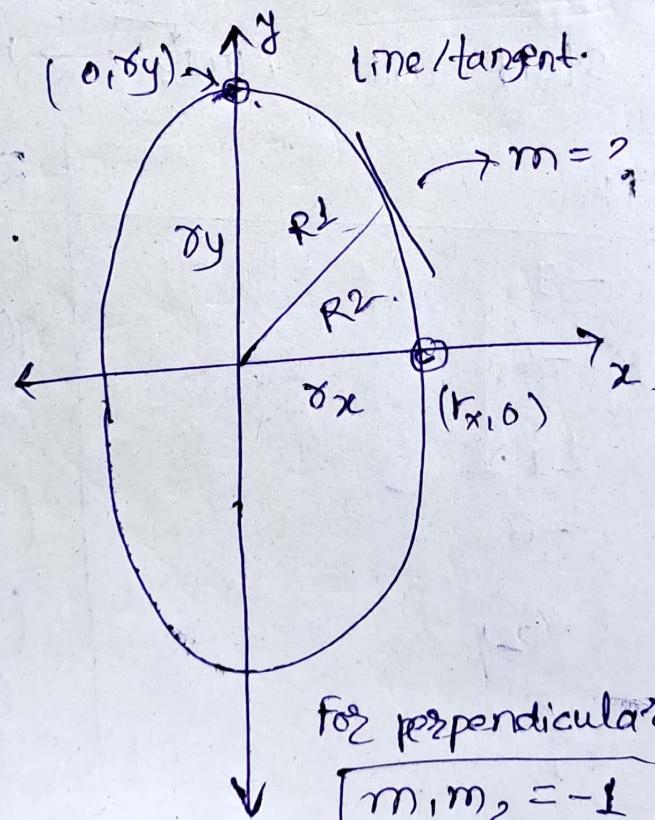
$(7, 7)$



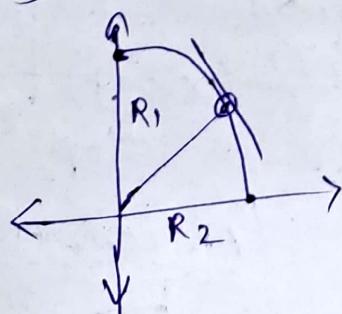
We use ellipse to draw this face.



$x_x \neq x_y$



By symmetry



For perpendicular lines:-

$$m_1, m_2 = -1$$

Regions are divided by $m=1$

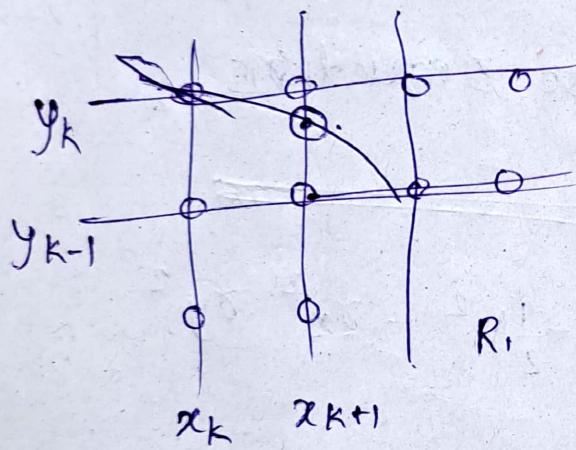
tangent slope = -1

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1$$

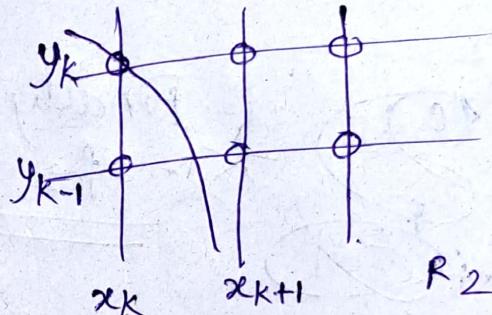
differentiate w.r.t x , to get slope.

$$\frac{dy}{dx} = -\frac{2x}{2y}$$

why 2 different regions?
ans due to difference in slope & point selection.



R_1	y_k
choice	y_{k-1}



R_2	x_k
choice	x_{k+1}

midpoint $(x_{k+1}, y_k - \frac{1}{2})$

midpoint $(x_k + \frac{1}{2}, y_{k-1})$

$$P_k' = f(x_k + 1, y_k - \frac{1}{2})$$

$$= \delta_y^2 (x_k + 1)^2 + \delta_x^2 (y_k - \frac{1}{2})^2 - \delta_x^2 \delta_y^2$$

$$\boxed{P_{k+1}' = P_k' + \delta_y^2 [1 + 2(x_k + 1)] + \delta_x^2 [y_{k+1}^2 - y_k^2 - (y_{k+1} - y_k)]}$$

$$\begin{cases} P_k' < 0 \\ P_{k+1}' = 1 \end{cases}$$

Finally use 3 equations

~~for R.L~~

$$P_0' = r_y^2 + \delta_x^2 (\delta_y - \frac{1}{2})^2 - \delta_x^2 \delta_y^2$$

$$P_k' < 0 \Rightarrow P_{k+1}' = P_k' + \delta_y^2 (1 + 2(x_k + 1))$$

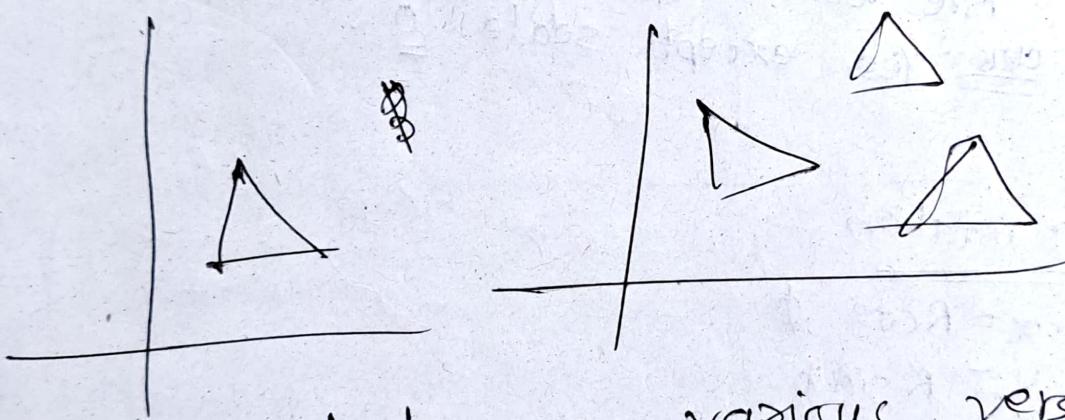
$$P_k' > 0 : P_{k+1}' = P_k' + \delta_y^2 (1 + 2(x_k + 1)) + 2(\delta_x^2 (1 - y_k))$$

Region 2.

$$P_0^2 = \gamma_y^2 (x_0 + \frac{1}{2})^2 + \gamma_x^2 (y_0 - 1)^2 - \gamma_x^2 \gamma_y^2.$$

$$P_k^2 = P_0^2 < 0 : (x_{k+1}, y_{k-1}).$$

$$P_{k+1}^2 = P_k^2 + 2\gamma_y^2(x_{k+1}) + \gamma_x^2(1 - 2(y_{k-1})).$$



If u want to copy various versions
a figure, use Transformations.

3D

$$x' = a_x x + b_x y + c_x z + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Translation

$$y' = a_y x + b_y y + c_y z.$$

$$z' = a_z x + b_z y + c_z z$$

2D

$$x' = a_x x + b_x y + \begin{bmatrix} t_p \\ t_q \end{bmatrix}$$

translation,

Transformation :- maps (x, y) in one coordinate system to points (x', y') in another coordinate system

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} ax & bx \\ ay & by \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}. \quad \underline{2D}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} ax & bx & cx \\ ay & by & cy \\ az & bz & cz \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} tx \\ ty \\ tz \end{bmatrix} \quad \underline{3D}$$

Are all transformations reversible?
Ans :- Yes, except scale $\equiv 0$.

2D rotation

$$x = R \cos \phi$$

$$y = R \sin \phi$$

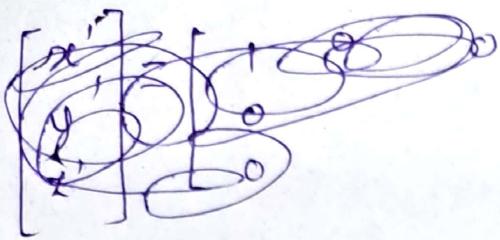
$$x' = R \cos (\phi + \theta)$$

$$y' = R \sin (\phi + \theta)$$

matrix :-

$$2D \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$3D \quad \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{Rotation around } z\text{-axis.}$$



Scaling is multiply.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

scale a in x
scale by b in y

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

Anti-Aliasing

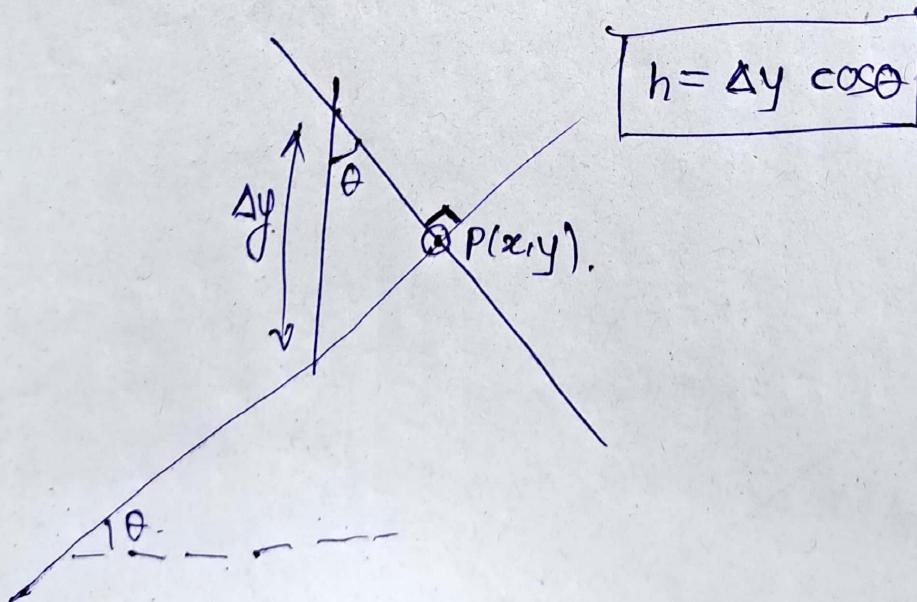
Gaussian cross section

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}} = \frac{1}{\sqrt{1+m^2}}$$

$$h = \frac{\Delta y}{\sqrt{m^2+1}} \quad \sigma' = \sigma \sqrt{m^2+1}$$

$$I(h) = I_0 \cdot e^{-\frac{h^2}{2\sigma'^2}}$$

$$= I_0 \cdot \exp \left(\frac{-\Delta y^2}{2\sigma^2(m^2+1)} \right)$$



$$I(\Delta y) = I_0 \cdot e^{-\frac{h^2}{2\sigma^2}} = I_0 \cdot e^{-\frac{\Delta y^2}{2\sigma^2(m^2+1)}} \\ = \frac{I_0}{I_0} \cdot \left(\frac{-\Delta y^2}{2\sigma^2} \right)$$

$$I(h) = I_0 (1 - \alpha h) \Rightarrow I(\Delta y) = I_0 \left(1 - \frac{\alpha \Delta y}{\sqrt{m^2+1}} \right).$$

- Linear Transformations :-

$x \in \mathbb{R}$.
const $a \in \mathbb{R}$.

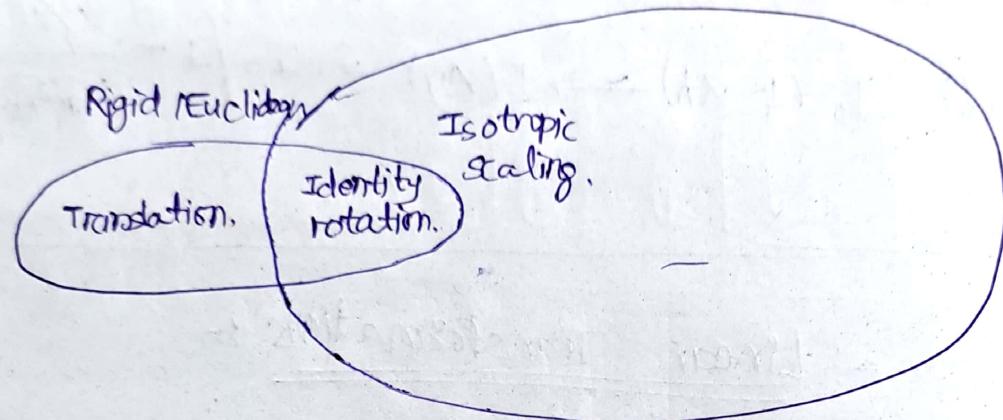
$$\underbrace{f(ax) = af(x)}_{x_1, x_2 \in \mathbb{R}} \quad \text{homogeneity condition.}$$

$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

■

$$\frac{d}{dx} (a_1 f_1(x) + a_2 f_2(x)) = a_1 \cdot \frac{d}{dx} f_1(x) + a_2 \cdot \frac{d}{dx} f_2(x).$$

Thursday 7 Quiz



Q] What is an n-dimensional transformation matrix?

change of orthonormal basis:-

$$x = (\hat{x} \cdot \hat{u})\hat{u} + (\hat{x} \cdot \hat{v})\hat{v} + (\hat{x} \cdot \hat{n})\hat{n}$$

$$y = (\hat{y} \cdot \hat{u})\hat{u} + (\hat{y} \cdot \hat{v})\hat{v} + (\hat{y} \cdot \hat{n})\hat{n}$$

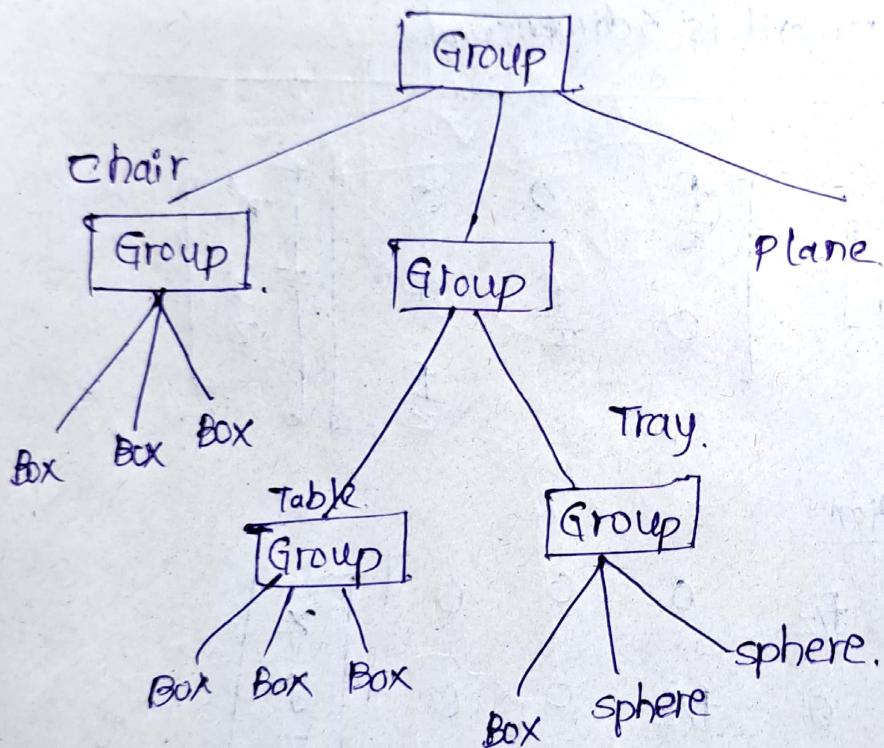
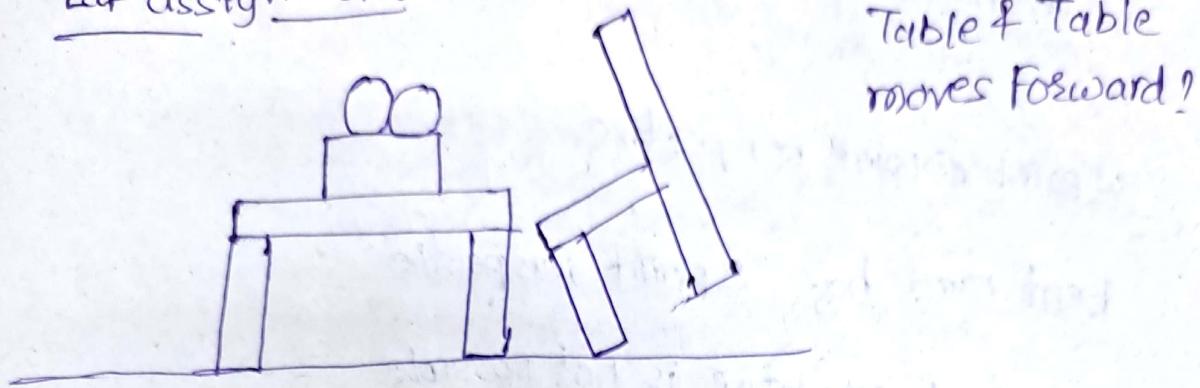
$$z = (\hat{z} \cdot \hat{u})\hat{u} + (\hat{z} \cdot \hat{v})\hat{v} + (\hat{z} \cdot \hat{n})\hat{n}$$

$$\begin{bmatrix} u \\ v \\ n \end{bmatrix} = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ n_x & n_y & n_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{where} \quad u_x = \hat{x} \cdot \hat{u} = \hat{x}_u$$

$$u_y = \hat{x} \cdot \hat{v} = \hat{y}_u$$

Group:- group of 3D shapes:-

Lab assignment



If chair hits Table, Table moves
All objects in hierarchical Tree of Table
should move.

Donald Hamm

Paron Chakraborty youtube video

2 point, 3 point perspective

bent road has 2 point perspective

4 point perspective is not possible

\because world is 3 dimensional.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} f/z & 0 & 0 \\ 0 & f/z & 0 \\ 0 & 0 & f/z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

homogenisation

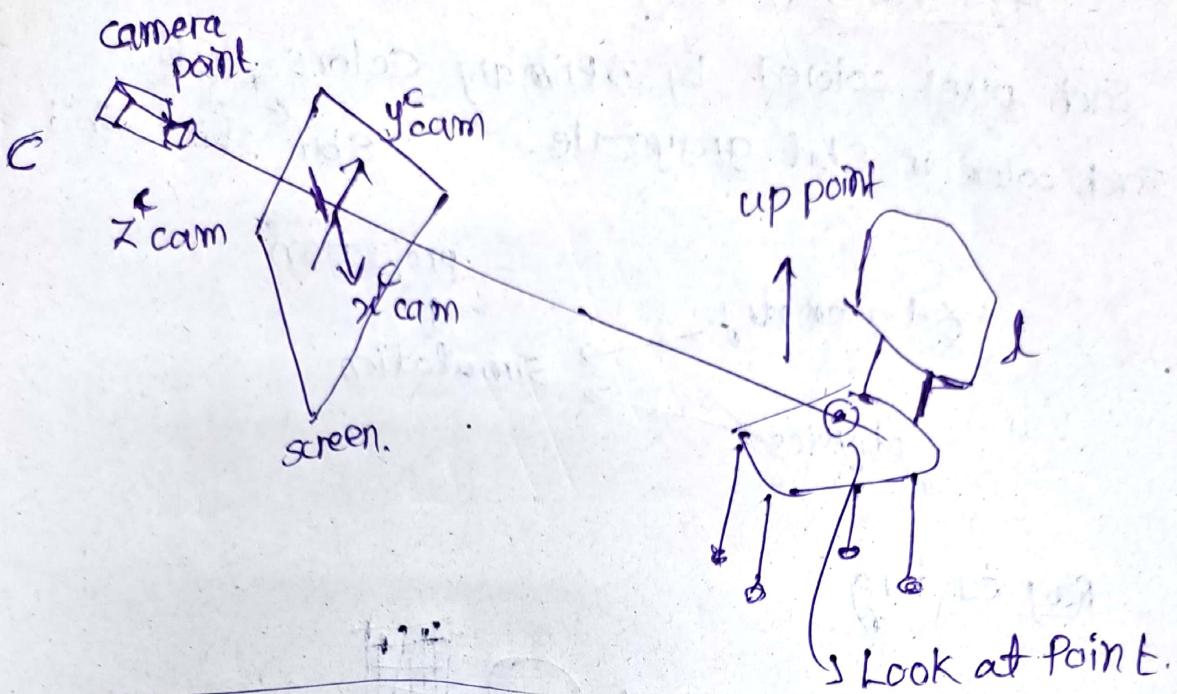
$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} f/z & 0 & 0 & 0 \\ 0 & f/z & 0 & 0 \\ 0 & 0 & f/z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$x' = x$$

$$y' = y$$

$$z' = z$$

$$w' = \frac{z}{f}$$



$$\vec{c} = \{x_1^c, x_2^c, x_3^c, \dots, x_n^c\}$$

$$\vec{l} = \{x_1^c, x_2^c, x_3^c, \dots, x_n^c\}$$

Look at vector $\vec{l} - \vec{c}$.

$$\vec{u} = \{x_1^n, x_2^n, \dots, x_n^n\}$$

n -dimensional vector can be represented in 2D screen.

tsne \rightarrow Large lang. models clustering