Fixed-Income Securities

Classifying Securities

Basic Types	Major Subtypes
Interest-bearing	Money market instruments, Fixed-income securities
Equities	Ordinary and Preferred Stocks
Derivatives	Options, Futures, Interest rate derivatives

- Classification of Fixed-Income Securities:
  1. Treasury Securities:
     - Govt. Treasury securities (bills, notes, bonds).
  2. Securities issued by State Governments (SDLs)
  3. Corporate Securities:
     - Commercial paper.
     - Medium-term notes
     - Corporate bonds
  4. Municipal Securities.
  5. Mortgage-Backed Securities.

Fixed-income securities

- A **debt** security is a financial security that is characterized by a principal amount (the amount borrowed), together with a plan for repayment of the principal plus interest.
- Fixed-income securities are financial claims with promised cash flows of fixed amount paid at fixed dates.
- Unlike a stock the payoffs for fixed income securities are fixed and known in advance.
- **Bond** is a piece of paper is an IOU. It says, “I owe you” an amount known as principal at a prescribed future date (called the maturity date).
- Bonds are less liquid. Bonds do not trade as frequently as equities and futures. Unlike stocks bonds are not traded on organized exchanges.
- Bonds are secured and debentures are unsecured.

Fixed-income market participants

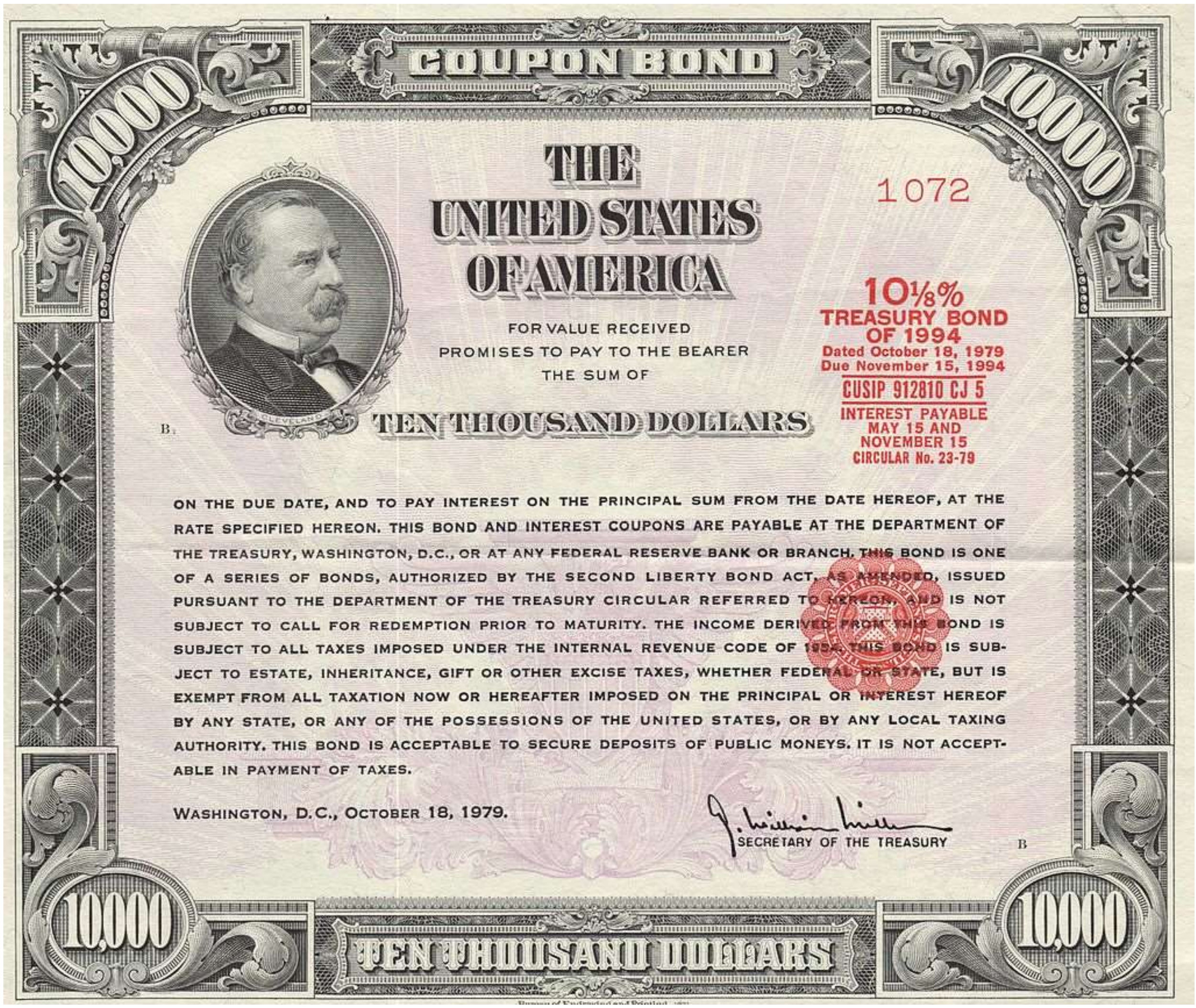
- The fixed income market participants fall into three groups-- issuers, intermediaries, and investors.

Issuers:
- Governments
- Corporations
- Commercial Banks
- States
- Municipalities
- SPVs
- Foreign Institutions

Intermediaries:
- Primary Dealers
- Other Dealers
- Investment Banks
- Credit-rating Agencies
- Credit Enhancers
- Liquidity Enhancers

Investors:
- Governments
- Pension Funds
- Insurance Companies
- Commercial Banks
- Mutual Funds
- Hedge Funds
- Foreign Institutions
- Individuals

- There are two types of market that issues bond
  - public offering: an issue of bonds sold to the general public that can then be traded on the secondary market.
  - private placement: an issue that usually is sold to one or a few institutional investors and is generally held to maturity.



COUPON BOND

10,000 10,000

THE UNITED STATES OF AMERICA

1072

FOR VALUE RECEIVED
PROMISES TO PAY TO THE BEARER
THE SUM OF

TEN THOUSAND DOLLARS

10⅛%
TREASURY BOND
OF 1994
Dated October 18, 1979
Due November 15, 1994
CUSIP 912810 CJ 5
INTEREST PAYABLE
MAY 15 AND
NOVEMBER 15
CIRCULAR No. 23-79

ON THE DUE DATE, AND TO PAY INTEREST ON THE PRINCIPAL SUM FROM THE DATE HEREOF, AT THE RATE SPECIFIED HEREON. THIS BOND AND INTEREST COUPONS ARE PAYABLE AT THE DEPARTMENT OF THE TREASURY, WASHINGTON, D.C., OR AT ANY FEDERAL RESERVE BANK OR BRANCH. THIS BOND IS ONE OF A SERIES OF BONDS, AUTHORIZED BY THE SECOND LIBERTY BOND ACT, AS AMENDED, ISSUED PURSUANT TO THE DEPARTMENT OF THE TREASURY CIRCULAR REFERRED TO HEREON, AND IS NOT SUBJECT TO CALL FOR REDEMPTION PRIOR TO MATURITY. THE INCOME DERIVED FROM THIS BOND IS SUBJECT TO ALL TAXES IMPOSED UNDER THE INTERNAL REVENUE CODE OF 1954. THIS BOND IS SUBJECT TO ESTATE, INHERITANCE, GIFT OR OTHER EXCISE TAXES, WHETHER FEDERAL OR STATE, BUT IS EXEMPT FROM ALL TAXATION NOW OR HEREAFTER IMPOSED ON THE PRINCIPAL OR INTEREST HEREOF BY ANY STATE, OR ANY OF THE POSSESSIONS OF THE UNITED STATES, OR BY ANY LOCAL TAXING AUTHORITY. THIS BOND IS ACCEPTABLE TO SECURE DEPOSITS OF PUBLIC MONEYS. IT IS NOT ACCEPTABLE IN PAYMENT OF TAXES.

WASHINGTON, D.C., OCTOBER 18, 1979.

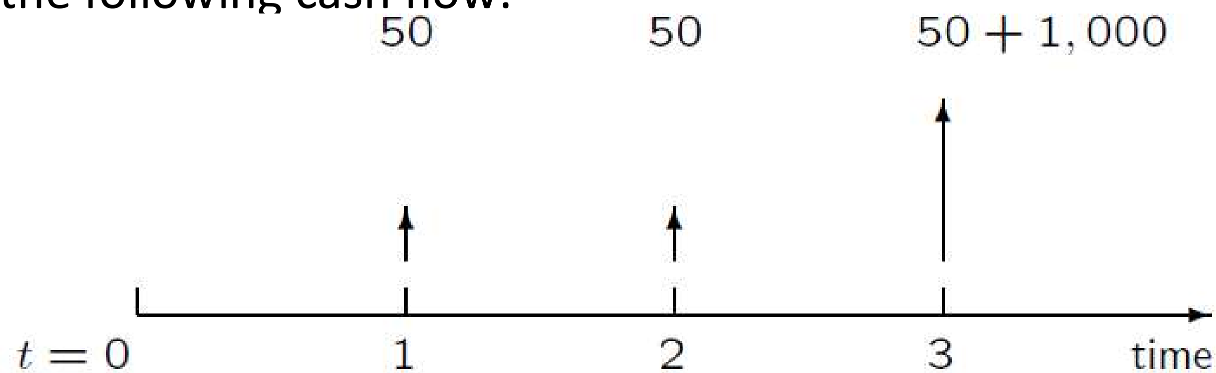
SECRETARY OF THE TREASURY

10,000 TEN THOUSAND DOLLARS 10,000

Valuation of Fixed-Income Securities

- A **coupon bond** pays a sequence of cash flows typically semiannually, quarterly, or annually till maturity. These are called coupon payments.
- When the bond matures, the issuer repays the debt by paying the bond's **par value** (equivalently, its **face value**).
- A bond have got a sequence of cash flows with
  - Maturity (normally ranging from 2 years to 30 years)
  - Principal
  - Coupon

- A 3-year bond with principal of \$1,000 and annual coupon payment of 5% has the following cash flow:



- Question-- what is this value worth at date zero?
  - It depends on the interest rate.

Components of Valuation

1. Time value of principal and coupons
2. Discount rate which depends on the risk factors:
   - Inflation
   - Credit risk
   - Timing (callability)
   - Liquidity risk
   - Currency fluctuations

- How do we figure out the market value of this bond today at times 0?
  - Compute the net present value, NPV.
  - Auction it off and figure out the price from the market.

- Valuation of bonds with no uncertainty:
  - Compute present values using discount rates that we get from the market, and time value of money.
  - For special cash flows like annuities and perpetuities, we use the same closed form solutions of time value of money.
- Valuation for riskless debt: Riskless in terms of default
- For now, we consider a riskless bond for valuation.

Valuation of Discount Bonds

- **Pure Discount Bond**
  - The most basic type of fixed-income security is a **zero-coupon** bond. This type of security make
    - No coupons, single payment of principal at maturity
    - Bond trades at a "discount" from the face value
      - Consider a one year, zero-coupon bond with face value \$100 and price \$96.61. The discount reflects the opportunity cost of capital – while the bond pays no "interest" as an investor you are compensated for the time value of money.
  - Every security with deterministic payments can be expressed as a portfolio of zero-coupon bonds.

- Valuation of zero-coupon bonds is straightforward application of NPV.
- The price of a discount on is simply equal to its face value (F) discounted to the present by the appropriate interest rate.

$$P_0 = \frac{F}{(1+r)^T}$$

- We can calculate r *given the price of a T-period zero-coupon bond:*

$$P_0 = \frac{F}{(1+r)^T} \Leftrightarrow r = \left(\frac{F}{P_0}\right)^{1/T} - 1$$

- Example: A zero-coupon bond pays 100 after 3 years and has a price of 90. – What is its *interest rate (yield) ?*

What if interest rates varies over time?

- In reality, interest rates vary through time.
- A single interest rate is used to discount all future cash flows that may change over time.
- **Spot Interest Rates**
  - Spot rate is the rate of interest between today and some other point in time.
  - **Spot interest rate, $r_{0,T}$,** *is the (annualized) interest rate* for maturity date *T. (e.g., a T- period zero coupon bond)*
  - At spot rate we have to pay or that we will earn on the spot, ie., $r_{0,T}$ *is for payments only on date T.*
  - It is a multi-year rates

- Example: On 2001.08.01., the spot interest rates are:

Maturity (Year)	¼	½	1	2	5	10	30
Interest Rate (%)	3.52	3.45	3.44	3.88	4.64	5.15	5.57

- **Future spot rate (short rate)**
  - Short rate are the interest rate that prevails over a specific time period at some future date.
  - Short rates are always one-year rates.
    - R_1 denotes the spot rate of interest between today and next year. R_3 denotes the spot rate of interest between years 2 and 3.
  - *In general,* $\boldsymbol{R_t}$ *is d*enoted as *the* ***one-year spot rate of interest*** between dates *t-1* and *t.*
  - Short rate provide us with the correct rate of discount to apply over a certain time period, e.g. the rate that prevailed between year one and year two.

Spot rate and short rate relationship

- Spot rate $r_{0,T}$ *is the "average" rate of interest between now and date T.*
- The one-year short rate, when accumulated by multiplying them together, will give the accumulated interest over the entire T-year period.
- For example, the 2-year spot rate is an average of today's short rate and next year's short rate. But because of compounding, that average is a geometric one.

$$(1 + r_{0,2})^2 = (1 + R_1)(1 + R_2)$$
$$r_{0,2} = [(1 + R_1)(1 + R_2)]^{1/2} - 1$$

Similarly, $r_{0,T} = [(1 + R_1)(1 + R_2) \cdots (1 + R_T)]^{1/T} - 1$

Therefore, if r varies over time for T−period,

$$P_0 = \frac{F}{(1+R_1)(1+R_2)\ldots(1+R_T)} = \frac{F}{(1+r_{0,T})^T}$$

- But we don’t observe the entire sequence of future spot rates today!
- What we observe is the price P_0, F that we get from the marketplace and the bond contract.
- Today’s T-year spot rate is an “average” of one-year future spot rates.
- $r_{0,T}$ as being equal to the geometric average of R’s.

$$r_{0,T} = \text{Yield} = \text{Today's T−year spot rate}$$

- Within the $r_{0,T}$ which we can observe, contains information about the future course of interest rates. It is today's expectations of what those big R's are going to be.

Example

- On 20010801, STRIPS (Separate Trading of Registered Interest and Principal Securities) are traded at the following prices:

Maturity (Year)	¼	½	1	2	5	10	30
Price	0.991	0.983	0.967	0.927	0.797	0.605	0.187

- For the 5-year STRIPS, we have the price today is 0.797, and that's equal to $1 paid five years later. So,

$$0.797 = \frac{1}{(1 + r_{0,5})^5} \Rightarrow r_{0,5} = \frac{1}{(0.797)^{1/5}} - 1 = 4.64\%$$

- Here, 4.64% is the rate of return, the cost of capital, the yield of that five-year horizon.
- Prices of discount bonds provide information about spot interest rates and vice versa.

Term structure of interest rates

- From a theoretical perspective, $r_{0,T}$, which we can observe, contains information about the future course of interest rates.
- This spot rate r is equal to the geometric average of one-year short rates.
- Suppose we observe several discount bond prices today

$$P_{0,1} = \frac{F}{(1+R_1)} \quad \rightarrow r_{0,1}$$

$$P_{0,2} = \frac{F}{(1+R_1)(1+R_2)} \quad \rightarrow r_{0,2}$$

$$P_{0,3} = \frac{F}{(1+R_1)(1+R_2)(1+R_3)} \quad \rightarrow r_{0,3}$$

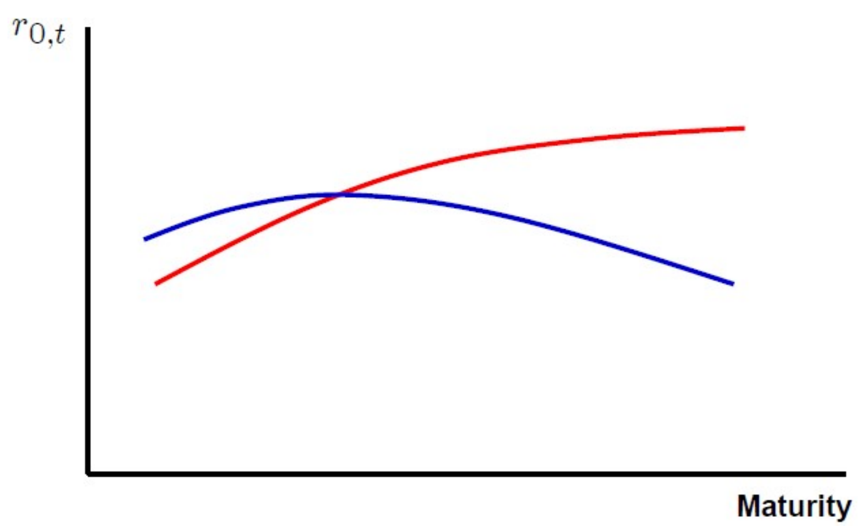
$$\vdots$$

$$P_{0,T} = \frac{F}{(1+R_1)(1+R_2)(1+R_3)\dots(1+R_T)} \quad \rightarrow r_{0,T}$$

$$= \frac{F}{(1+r_{0,T})^T}$$

- A plot of the $r_{0,T}$ as a function of time, is known as the **term structure of interest rates or the yield curve**.
- Yield curve gives a sense of where future interest rates would be.
- Term structure contain information about future interest rates.
- Term structure of interest rates or the yield curve,

$$\left\{P_{0,1}, P_{0,2}, \ldots, P_{0,T}\right\} \rightarrow \left\{r_{0,1}, r_{0,2}, \ldots, r_{0,T}\right\}$$

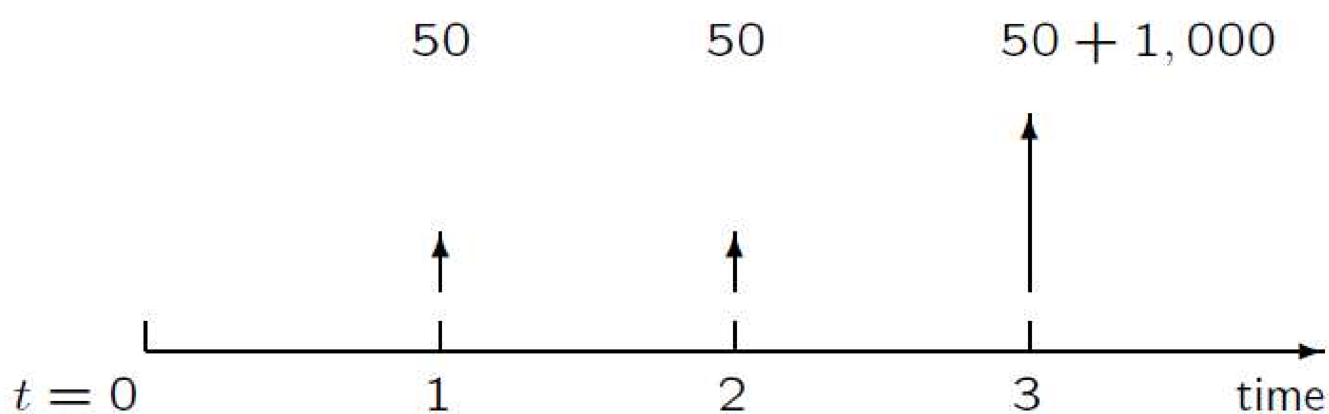


- If the curve is upward-sloping, it says that as for longer maturities, average yield, the geometric average of all the R's is getting bigger with time.
- In upward-sloping curve, the market's expectations are that the rates are going to go up.
- If it's downward-sloping, it suggests that future interest rates, future R's, are declining.
- The relationship between price and the r is inverse, means a low yield indicates price is really high.

Valuation of Coupon Bonds

- A coupon bond pays a stream of regular coupon payments and a principal at maturity.
- Coupon bonds can trade at discounts or premiums to face value
- A coupon bond is a collection of discount bonds at different maturities.
- Valuation is straightforward application of NPV
- **Example:**

  3-year bond of $1,000 par value with 5% coupon

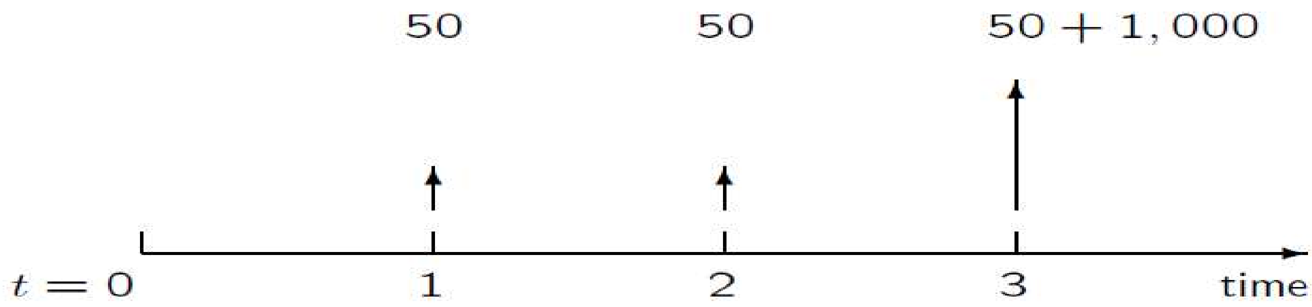


- A bond with coupon payments C and a principal F at maturity T is composed of
  - C units of discount bonds maturing at t, t = 1, . . . , T
  - F units of discount bond maturing at T.
- The price of a coupon bond must be

$$P_0 = \frac{C}{(1+R_1)} + \frac{C}{(1+R_1)(1+R_2)} + \ldots + \frac{C+F}{(1+R_1)\ldots(1+R_T)}$$

- Coupon bonds are often quoted with a single number that is a yield. Since short rates are unobservable, we summarize them with *y*

$$P_0 = \frac{C}{(1+y)} + \frac{C}{(1+y)^2} + \ldots + \frac{C+F}{(1+y)^T}$$



•Using the formula: $P_0 = \dfrac{C}{(1+y)} + \dfrac{C}{(1+y)^2} + \cdots + \dfrac{C+F}{(1+y)^T}$; coupon $= 5\%$

$$\text{Price} = \frac{50}{(1+0.06)} + \frac{50}{(1+0.06)^2} + \frac{1050}{(1+0.06)^3} = \$973.27 \quad if\ y = 6\%$$

$$\text{Price} = \frac{50}{(1+0.05)} + \frac{50}{(1+0.05)^2} + \frac{1050}{(1+0.05)^3} = \$1000 \quad if\ y = 5\%$$

$$\text{Price} = \frac{50}{(1+0.04)} + \frac{50}{(1+0.04)^2} + \frac{1050}{(1+0.04)^3} = \$1027.75 \quad if\ y = 4\%$$

- Bonds can be priced at
  - par value (when market interest rate=coupon rate)
  - premium (when market interest rate < coupon rate)
  - discount (when market interest rate > coupon rate)

- Why?

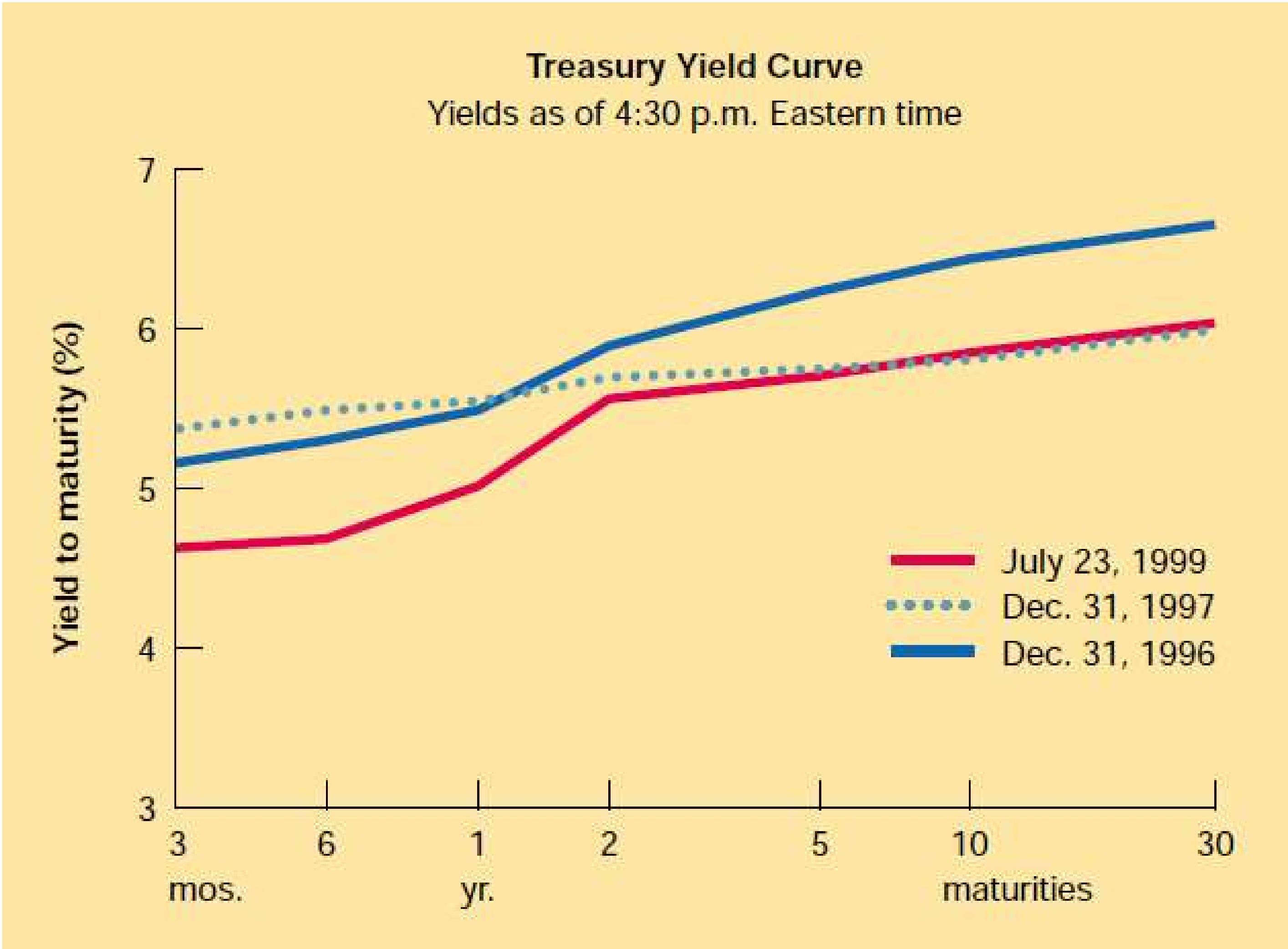
Yield-to-maturity (YTM)

$$P_0 = \sum_{t=1}^{T} \frac{C}{(1+y)^t} + \frac{F}{(1+y)^T}$$

- y is called the **yield-to-maturity (YTM)** of a bond. It is equal to the rate of return on the bond if
  - it is bought now at price P_0 and held to maturity, and
  - all coupons are reinvested at rate *y*.
- YTM is the single interest rate which, if interest rates were constant throughout time, would make the present value of all the coupons and principal equal to the current price.
- There is usually no closed-form solution for *y.* Given the price, a numerical methods must be used to compute it (T^{th}-degree polynomial).
- YTM is not a spot rate. It is a complex average of all future spot rates. For pure discount bonds, the YTM's are the current spot rates.

Yield curve

- The **yield curve** is the yield to maturity (YTM) on a T-year (pure) discount bond graphed as a function of *t*.
- Discount bonds of different maturities can have different yields to maturity. Given its maturity, the principal and the coupon rate, there is a one to one mapping between the price of a bond and its YTM.
- By looking at yield curves over different dates, you can get a sense of how the market's expectations are of the future.
- In the market, it is conventional to quote bond prices in YTM.


Treasury Yield Curve
Yields as of 4:30 p.m. Eastern time
Yield to maturity (%)
7
6
5
4
3
3
mos.
6
1
yr.
2
5
10
maturities
30
July 23, 1999
Dec. 31, 1997
Dec. 31, 1996

- For a semi-annual-pay coupon bond, the annual percentage rate (APR) is twice the 6-month period rate.
- **Example:** An 8% coupon, 30-year maturity bond with par value of $1,000 paying semi-annual coupon payments. Suppose that the interest rate is 10% annually. Find the value of the bond.

An 8% coupon annually is 4% semi-annually, which means $1000 X 0.04=$40 coupon payment in every 6-month period. There will be 30 X 2 =60 payments till maturity.

The interest rate is 10% annually, or r = 5% per 6-month period.

Then the value of the bond can be written as

$$P_0 = \sum_{t=1}^{60} \frac{\$40}{(1+0.05)^t} + \frac{\$1{,}000}{(1+0.05)^{60}}$$

$$= \$757.17 + \$53.54 = \$810.71$$

- **Example.** Current 1- and 2-year spot interest rates are 5% and 6%, respectively. Perhaps this inequality in interest rates occurs because inflation is expected to be higher over the second year than over the first year. The price of a 2-year Treasury coupon bond with par value of $100 and a coupon rate of 6% is

$$P_0 = \frac{6}{(1+0.05)} + \frac{106}{(1+0.06)^2} = 100.0539$$

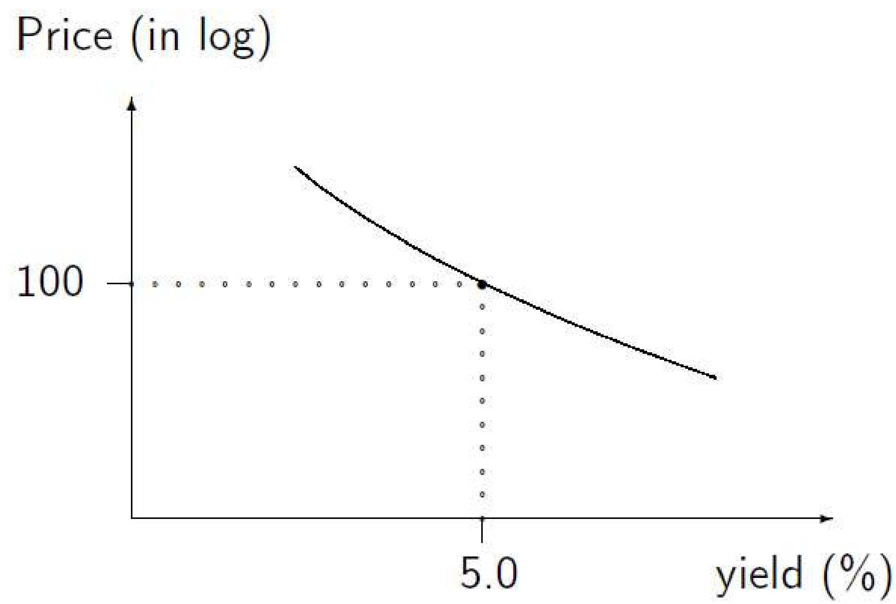
- If P_0 was observable and the YTM was not, we could determine the YTM using the PV formula. To find YTM,

$$P_0 = \frac{6}{(1+y)} + \frac{106}{(1+y)^2} = 100.0539 \Rightarrow y = 5.9706\%$$

- Note the difference between YTM definition and bond pricing formula.

Interest Rate Risk

- The value of a bond is subject to interest rate risk. A change in the interest rate results in:
  1. price risk
  2. re-investment risk
- As interest rates change, bond prices changes inversely. A bond's interest rate risk can be measured by its relative price change with respect to a change in yield.



Duration

- Duration is a measure of bond price sensitivity that means for a certain basis point change in the interest rate, how much the bond changes as a percentage of its current price?
- Duration named after Macaulay a measure of how sensitive the bond price is to changes in yield.

$$\text{Macaulay Duration is defined as } D_m = \sum_{k=1}^{T} k.\frac{PV(C_k)}{P}$$

$$\text{A Modified Duration} = D_m^* = -\frac{1}{P}\frac{\partial P}{\partial y}$$

P is the current price, y is yield. A small change (Δy) in YTM will cause bond price to change by approximately

$$\Delta P \approx -P \cdot MD \cdot \Delta y., \quad \text{or in percentage, } \frac{\Delta P}{P} \times 100 \approx -MD \times \Delta y \times 100.$$

- The negative number indicates that there's an inverse relationship between price and yield.
- Longer the maturity of a bond, the more sensitive is the bond price to the yield results more duration.
- Modified duration is the approximate percentage change in price for a 100 basis point change in yield.
- A 10-years maturity bond having duration 7 years means 7% change in price for 100 basis point change in yield.
- Example: Consider a bond whose modified duration is 11.54 with a yield of 10%. If the yield increases instantaneously from 10% to 10.1%, the approximate percentage price change will be $-11.54 \times 0.001 = -0.01154 = -1.154\%$.

Inflation Risk

- Most bonds give nominal payoffs. In the presence of inflation risk, real payoffs are risky even when nominal payoffs are safe.
- Example. Suppose that inflation next year is uncertain ex ante, with equally possible rate of 10%, 8% and 6%.
- The 1-year nominal interest rate will be (roughly) 10%.
- Consider the return from investing in a 1-year Treasury security:

Now, $r_{\text{real}} \approx r_{\text{nominal}} - r_{\text{inflation}}$

Year 0 value	Inflation rate (%)	Year 1 nom. payoff	Year 1 real payoff
1000	0.10	1100	1000
1000	0.08	1100	1020
1000	0.06	1100	1040

Credit Risk and Corporate Bonds

- Fixed-income securities have promised payoffs of fixed amount at fixed times. Excluding government bonds, other fixed-income securities, such as corporate bonds, carry the risk of failing to pay off as promised.
- Default risk (credit risk) refers to the risk that a debt issuer fails to make the promised payments (interest or principal).
- Credit ratings by rating agencies (e.g., Moody's and S&P) provide indications of the likelihood of default by each issuer.

Description	Moody’s	S&P
Gilt-edge	Aaa	AAA
Very high grade	Aa	AA
Upper medium grade	A	A
Lower medium grade	Baa	BBB
Low grade	Ba	BB
Very Poor	Caa	CCC

- Investment grade bonds: Aaa – Baa by Moody's or AAA – BBB by S&P.
- *Speculative (junk) bonds: Ba and below by Moody's or BB* and below by S&P.
- Listing of corporate bonds:

ISSUER NAME	SYMBOL	COUPON	MATURITY	RATING MOODY'S/S&P/ FITCH	HIGH	LOW	LAST	CHANGE	YIELD %
General Electric Capital	GE.HGW	3.000%	Dec 2011	Aaa/AAA/–	104.038	103.369	103.730	0.031	1.219
Citigroup Funding	C.HRU	1.375%	May 2011	Aaa/AAA/AAA	101.123	100.770	100.770	0.272	0.869
JPMorgan Chase & Co	JPM.LVC	6.300%	Apr 2019	Aa3/A+/AA–	111.753	109.459	111.045	1.600	4.836
Citigroup	C.GOS	5.500%	Oct 2014	A3/A/A+	103.497	101.580	101.792	0.425	5.086
Goldman Sachs Gp	GS.HRH	1.700%	Mar 2011	Aaa/AAA/AAA	101.412	101.412	101.412	0.112	0.685
Citigroup	C.HFV	8.125%	Jul 2039	A3/A/A+	117.308	113.188	116.961	1.711	6.789
General Electric Capital	GE.HJL	6.000%	Aug 2019	Aa2/AA+/–	107.390	103.161	105.010	1.415	5.335
Citigroup	C.HFF	6.000%	Aug 2017	A3/A/A+	102.250	99.784	100.107	0.536	5.981

- **Decomposition of Corporate Bond Yields**
- To compensate for the possibility of default, corporate bonds must offer a **default premium.**
- Default premium is the difference between promised yield and expected yield.
- Promised YTM is the yield if default does not occur.
- Expected YTM is the probability-weighted average of all possible yields.
- Bond risk premium is the difference between the expected yield on a risky bond and the yield on a risk-free bond of similar maturity and coupon rate. At this risk premium, investors are willing to bear the price risk associated with interest rate uncertainty.
- The default risk premium is an additional interest rate a borrower pays to lenders/investors for higher credit risk, calculated as the difference between bond interest rates and risk-free rates.

Example: Suppose all bonds have par value \$1,000 and

- 10-year Treasury STRIPS is selling at \$463.19, yielding 8%
- 10-year zero issued by XYZ Inc. is selling at \$321.97
- Expected payoff from XYZ's10-year zero is \$762.22

- For the 10-year zero issued by XYZ:

$$\text{Promised YTM} = \left(\frac{1000}{321.97}\right)^{1/10} - 1 = 12\%$$

$$\text{Expected YTM} = \left(\frac{762.22}{321.97}\right)^{1/10} - 1 = 9\%$$

- and
  - Default Premium = Promised YTM − *Expected YTM*
    = 12% − *9% = 3%*
  - Risk Premium = Expected YTM − *Default-free YTM*
    = 9%− *8% = 1%.*

- Yield-to-maturity for a risky bond