2. trimetric projection

↳ rotate by

① $\phi = 45°$ about y-axis

② $\theta = 45°$ about x-axis

③ Parallel projection onto $z=0$ plane

④ foreshortening factors f_x f_y f_z

⑤ DDA to display the object on the projection plane $z=0$.
(DDA code)

$$[X] = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0.5 & 1 & 1 \\ 0.5 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0.5 & 1 \end{bmatrix}$$

① $$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta + z\sin\theta \\ y \\ -x\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x\cos\theta + z\sin\theta \\ y \\ -x\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

① $[x] = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 1 \\ \sqrt{2} & 0 & 0 & 1 \\ \sqrt{2} & 0.5 & 0 & 1 \\ \frac{3}{2\sqrt{2}} & 1 & 1/2\sqrt{2} & 1 \\ 1/\sqrt{2} & 1 & 1/\sqrt{2} & 1 \\ 0 & 0 & 0 & 1 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} & 1 \\ 1/\sqrt{2} & 1 & -1/\sqrt{2} & 1 \\ 0 & 1 & 0 & 1 \\ 3/2\sqrt{2} & 1 & -1/2\sqrt{2} & 1 \end{bmatrix}$

② $$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y\cos\theta - z\sin\theta \\ y\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y\cos\theta - z\sin\theta \\ y\sin\theta + z\cos\theta \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 1 \\ \sqrt{2} & 0 & 0 & 1 \\ \sqrt{2} & 0.5 & 0 & 1 \\ \frac{3}{2\sqrt{2}} & 1 & \frac{1}{2\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 1 \\ \frac{1}{\sqrt{2}} & 1 & -\frac{1}{\sqrt{2}} & 1 \\ 0 & 1 & 0 & 1 \\ \frac{3}{2\sqrt{2}} & 1 & -\frac{1}{2\sqrt{2}} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{1}{2} & 1 \\ \sqrt{2} & 0 & 0 & 1 \\ \sqrt{2} & \frac{1}{2\sqrt{2}} & \frac{1}{2\sqrt{2}} & 1 \\ \frac{3}{2\sqrt{2}} & \frac{1}{\sqrt{2}} - \frac{1}{4} & \frac{1}{\sqrt{2}} + \frac{1}{4} & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} - \frac{1}{2} & \frac{1}{\sqrt{2}} + \frac{1}{2} & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} + \frac{1}{2} & \frac{1}{\sqrt{2}} - \frac{1}{2} & 1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1 \\ \frac{3}{2\sqrt{2}} & \frac{1}{\sqrt{2}} + \frac{1}{4} & \frac{1}{\sqrt{2}} - \frac{1}{4} & 1 \end{bmatrix}$$

(3) Parallel projection on $z=0$

put all z axis point values $=0$

$$\Rightarrow [X] = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2} & 0 & 1 \\ \sqrt{2} & 0 & 0 & 1 \\ \sqrt{2} & \frac{1}{2\sqrt{2}} & 0 & 1 \\ \frac{3}{2\sqrt{2}} & \frac{1}{\sqrt{2}} - \frac{1}{4} & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} - \frac{1}{2} & 0 & 1 \\ 0 & 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} + \frac{1}{2} & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 1 \\ \frac{3}{2\sqrt{2}} & \frac{1}{\sqrt{2}} + \frac{1}{4} & 0 & 1 \end{bmatrix}$$

④ $f_z = 0$

$\therefore f_x = \frac{1}{1}$ (length after projection / actual length along any axis) = 1

$f_x = 1$

similarly $f_y = 1$

[$f_x + f_y = 1$ means length of the cube hasn't changed on projection.