- 1Q from system types.
- Correlation (Auto or cross)
- Fourier series → 1 derivation → 1 numerical
- DTFT or CTFT

30 Marks

L-1

ॐ

WJMK

22/3/23
wednesday

POC (Dr. Ramesh Kumar Bhukya)

Q How to convert signal from time domain to Frequency domain?

Signal → physical quantity varies with time, space or any independent variable.

↳ can be manipulated,
can +, −, *, ⊕ %
↳ if possible
will give some meaningful info.

(*) multidimensional signal → if signal depends on more than 2 independent variables.

We'll mostly consider 1 dimensional signal.

Source → Destination

what happens here to signal?

we'll ~~consider~~ study that!

*when looking into photo → treat as 2D signal.
Sonograms → Ultra sound signal.
→all scanning signals are treated as 2D signal.

* If diff signals interfere → miscommunication will happen → to avoid it → 'Bandwidth'.

* Speech signal → we'll treat as 1D signal (won't consider amplitude part)
↳ signals u r looking into radio, TV or talking with each other.

[Sampled signals - ???.
sample of signals - ???]

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* compact disc Player → (CD player)
* class also signal (may be wrt age,
wrt marks,
wrt gender etc.)

⊙ Picture can be treated as 2D or 3D.
↳ mostly we'll treat as 2D.

[Sonograms → Ultrasound Scanning]

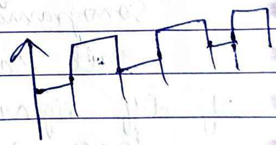
→ sometimes we need to discard some info (like background noise)
For them we used "filtering" — comes under electronic engineering

Signals have some patterns, ~~structures~~ res or themes!
↓
detect, to classify them.

⊙ diff 3 types of Domains!
1) continuous time domain
2) Discrete
3) ?

→ Flow of info, which can be displayed, manipulated etc.

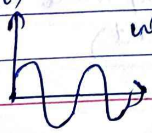
Signal → Digital signals



Continuous time (continuous in time & amplitude) — can be converted to → Discrete Time

Continuous time ↓ will give info at every time

$x(t)$



Discrete Time ↳ will give info at discrete time instants
→ discrete in time, continuous in amplitude
→ $x(n)$ → also known as "sequence of numbers"

DEPENDENT VARIABLE

$f(t_1, t_2, \ldots, t_n)$

independent variables

each t_i → representing 1 dimension.

n-dimensional SIGNAL.

1024×400 → Total no. of dimensions we've till now!

after sampling → we get sampled signals $\xrightarrow[\text{Quantization}]{\text{then}}$ get 'digital signals'

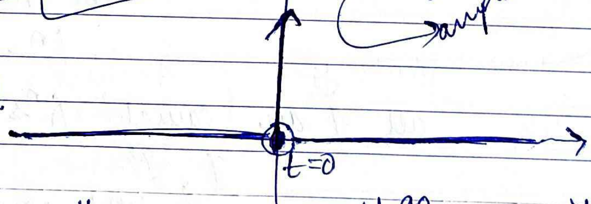
• Impulse signal → represented as $\delta(t)$

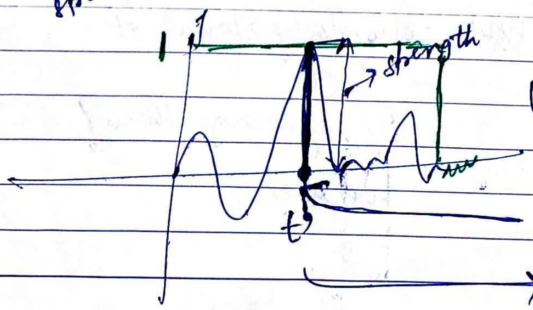
$$\delta(t) = \begin{cases} 1 ; & \text{for } t = 0 \\ 0 ; & \text{for } t \neq 0 \end{cases}$$

1 → or (∞) → (it depends on us)

Impulse fn. → used to calculate strength of signal, amplitude etc.

Used when want to see signal strength at particular location!





want to see strength of signal here!

(or consider it as strength) Consider it as area under curve at t'

apply $\delta(t)$ at $t = t'$ and see signal strength → (amplitude, area, width etc.)

* Consider whole signal as 1 unit sample.

↓

means we represent signal as unit step f^n. $(u(t))$

$$u(t) = \begin{cases} 1 & ; t \geq 0 \\ 0 & ; t < 0 \end{cases}$$

↑

means we are not considering negative part of signal or imaginary part of signal (can)

* Our Devices can detect past or present, not predict future of signal.

* Ramp f^n will be used for accumulation of SIGNAL.

* parabolic f^n → Integral part of Ramp f^n

↓

Integral part of unit step f^n

↓

integral part of Impulse f^n.

↓

all 4 are 'causal' f^ns !!

↳ ?? not considering signal at $t < 0$

↳ ignoring them!

↓

we are

when signal changes rapidly!

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* $\frac{d}{dt}$(Impulse f^n) $= 0 \quad \forall t$

* Analog communication → cont. in time & amplitude

Can be produced only, can't do analysis etc.

∴ ONLY DIGITAL SIGNALS KI KR SKTE HAIN.

Representation Ways of signals

- graphical
- Functional
- Tabular
- Sequence

$x(t) = x(n) = \{1, 2, 1, 2, 3\}$

Every signal can be represented as form of f^n

$x(t) = x(t \pm t_0)$

$x(t + t_0)$ → Advanced version

$x(t - t_0)$ → Delayed version

→ So when shifting signal with help of duration $(t \pm t_0)$ → then we'll get advanced or delayed version.

let's say pehle $a=1$, then, $a=2$ → amplitude will enhanced to 2 (expanding signal)

Scaling → [So Doing manipulation in signal]

- expansion → signals ki waves dur hongi
- compression → pass aayeingi

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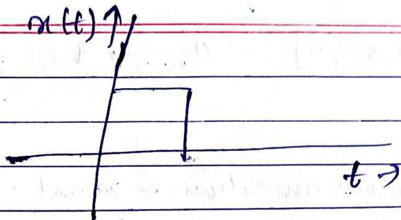
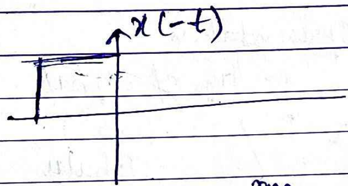
* Time Reversal :-



by time reversal:

same value will be reflected in negative half



Combination

↳ Combination of all 3 opᵐˢ

① shift

② scaling

③ Reversal

x(t) ± c ↳ amplitude shifting

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28/03/2023
Wednesday

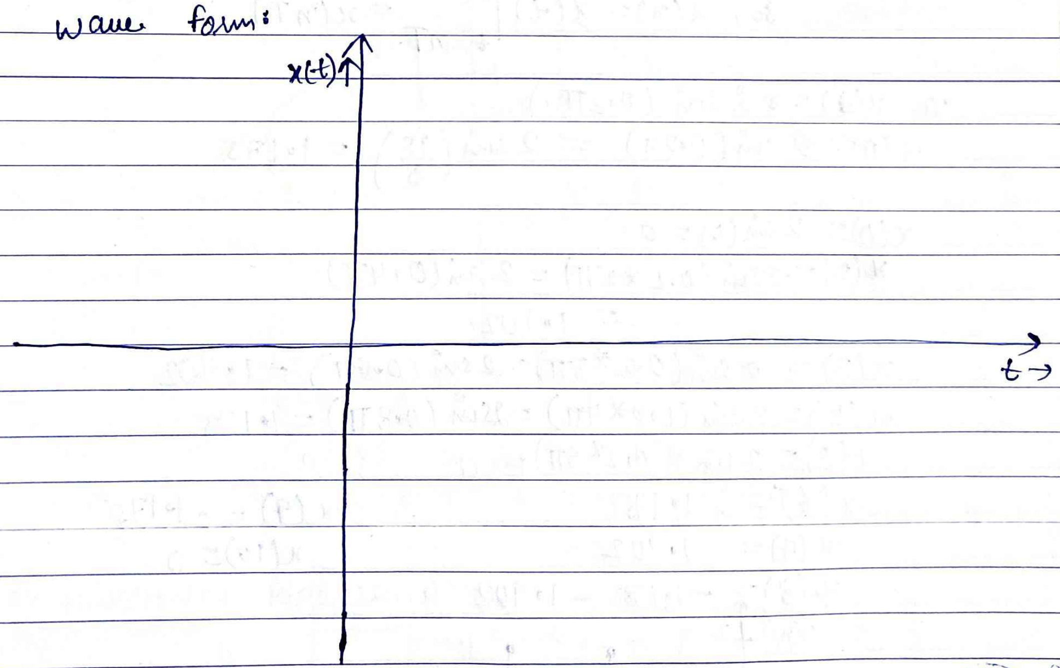
Tutorial - 1

Q1. $x(t) = \begin{cases} 2\sin(\pi t) & ; 0 \le t \le 2 \\ 0 & ; \text{Otherwise} \end{cases}$

$\Rightarrow$ $x(0) = 0$, $x(0.25) = \sqrt{2}$, $x(0.5) = 2$, $x(1) = 0$,
$x(1.25) = -\sqrt{2}$, $x(1.5) = -2$, $x(1.75) = -2\sin(0.75)$
$= -2\left(\frac{1}{\sqrt{2}}\right) = -\sqrt{2}$,

$x(2) = 0$

wave form:



For discrete time signal, we need to take sampling period

$\rightarrow x(n) = x(nT) = x(t)\Big|_{t=nT}$ $n \in \mathbb{Z}$

To convert cont. time signal to discrete time signal

$T = 0.2$ sec, then cont. changes to Discrete time signal.

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$x(0.2n) = x(t)\big|_{t=0.2n} = x(0.2n) = 2\sin(0.2\pi n)$

$n = 1, 2, 3, 4, 5, \ldots$

'T' will be given to us every time.

$nT \rightarrow$ 'n' varying at constant 'T'.

$\downarrow$

Sampling period

$\downarrow$

decides duration b/w any 2 consecutive points, it should be 'T'.

So, $x(n) = x(t)\big|_{t=nT} = x(nT)$

So, $x(n) = 2\sin(0.2\pi n)$

$x(1) = 2\sin(0.2\pi) = 2\sin\left(\frac{\pi}{5}\right) = 1.175$

$x(0) = 2\sin(0) = 0$

$x(2) = 2\sin(0.2 * 2\pi) = 2\sin(0.4\pi)$
$= 1.902$

$x(3) = 2\sin(0.2 * 3\pi) = 2\sin(0.6\pi) = 1.902$

$x(4) = 2\sin(0.2 * 4\pi) = 2\sin(0.8\pi) = 1.175$

$x(5) = 2\sin(0.2 * 5\pi) = 0$

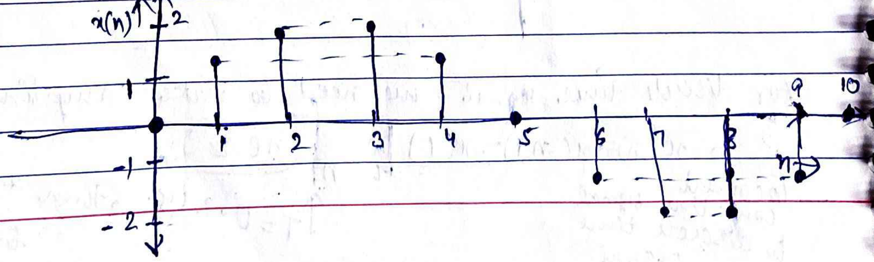
$x(6) = -1.175$

$x(7) = -1.902$

$x(8) = \cancel{-1.175}\ -1.902$

$x(9) = -1.175$

$x(10) = 0$



HW Q2

WJMK

continuous time

H/W Q2.

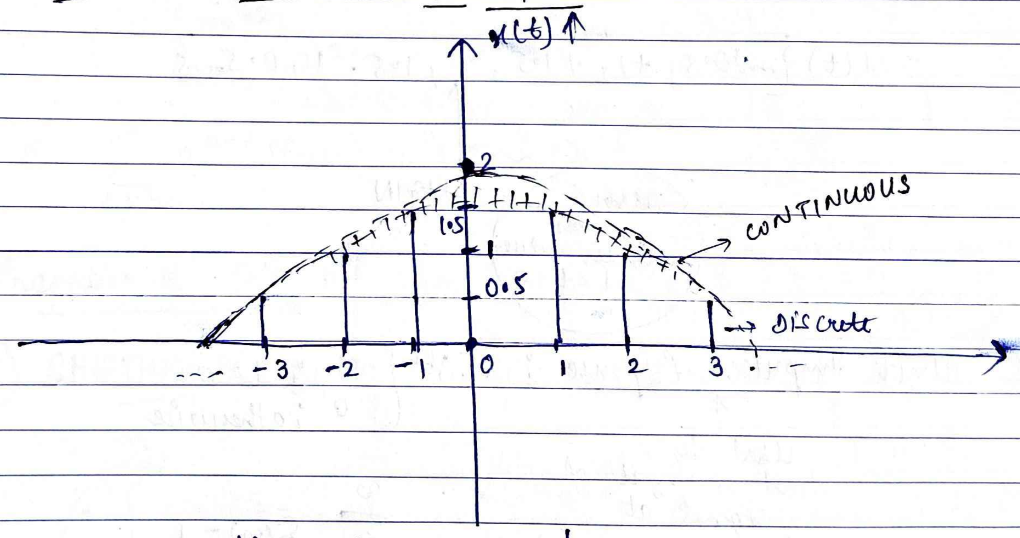
$$x(t) = \begin{cases} e^{-t} ; & 0 \le t \le 2 \\ 0 ; & \text{otherwise} \end{cases}$$

Represent this expression in CT and DT waveform and sample period $T = 0.2$ sec.

discrete time

* 4 ways to represent signal:-

(i) GRAPHICAL REPRESENTATION OF SIGNAL :-



$x(t)\big|_{t=0} = 2$, $x(t)\big|_{t=1,-1} = 1.5$

$x(t)\big|_{t=2,-2} = 1$ $x(t)\big|_{t=3,-3} = 0.5$

(ii) FUNCTIONAL REPRESENTATION:-

$$x(t) = \begin{cases} 2 ; & t = 0 \\ 1.5 ; & t = \pm 1 \\ 1 ; & t = \pm 2 \\ 0.5 ; & t = \pm 3 \\ 0 ; & \text{otherwise} \end{cases}$$

$$\delta(t) = \begin{cases} 1 ; & t = 0 \\ 0 ; & t \neq 0 \end{cases}$$

$$\delta(at) = \begin{cases} 1 ; & at = 0 \\ 0 ; & at \neq 0 \end{cases}$$

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① Tabular Representation :-

t	-3	-2	-1	0	1	2	3
$x(t)$	0.5	1	1.5	2	1.5	1	0.5

① SEQUENCE Representation :-

(Just directly show amplitude values of signal)

$$x(t) \{ \ldots +0.5, +1, +1.5, \underset{\uparrow}{2}, 1.5, 1, 0.5 \ldots \}$$

ORIGIN

arrow gives under value at $t=0$

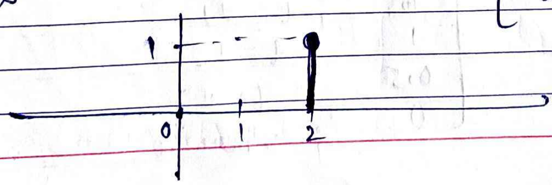
* Unit Impulse Response : $\delta(n) = \begin{cases} 1 ; & t = 0 \\ 0 ; & \text{otherwise} \end{cases}$

Used to find strength of signal at particular position

$$\sum_{n=-\infty}^{\infty} \delta(n) = 1$$

$$\delta(t) = \begin{cases} 1 ; & t = 0 \\ 0 ; & t \neq 0 \end{cases} \qquad \int_{-\infty}^{\infty} \delta(t)(dt) = 1$$

* $\sum_{n=-\infty}^{\infty} e^{2n} (\delta(n-2))$ $\qquad \delta(n-2) = \begin{cases} 1 ; & n = 2 \\ 0 ; & n \neq 2 \end{cases}$



$$\int_{-\infty}^{\infty} \delta(t)(dt) = u(t) = \begin{cases} 1 ; & \text{for } t \geq 0 \\ 0 ; & t < 0 \end{cases}$$

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$$\sum_{n=-\infty}^{\infty} e^{2n}(\delta(n-2)) = \sum_{n=2} e^{2n}\delta(n-2) = \sum_{n=2} e^{2n}(1)$$

$$= e^{2\times 2} = e^4$$

$$\sum_{n=-\infty}^{\infty} n^2(\delta(n+2)) \qquad \delta(n+2) = \begin{cases} 1 ; & n = -2 \\ 0 ; & n \neq -2 \end{cases}$$

$$= \sum_{n=-2} n^2 \, \delta(n+2)$$

$$= (-2)^2(1) = 4$$

$$\sum_{n=-\infty}^{\infty} a^{n-2}\delta(n+3) = a^{-3-2} = a^{-5}$$

- **Properties of C.T. unit Impulse fn:** → delta fn OR DIRAC delta fn OR Impulse fn,

① SHIFTING PROPERTY:- $\int_{-\infty}^{\infty} \delta(t)(x(t))(dt) = x(t)\Big|_{t=0} = x(0)$

- $\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)(dt)$ → shifting kitni ki, uska effect pdhta hai → as to indicate located khaan hai

$\delta(t-t_0) = \begin{cases} 1 ; & k = t_0 \\ 0 ; & \text{otherwise} \end{cases}$

$$= \int_{t_1}^{t_2} x(t_0)\,\delta(t-t_0)(dt)$$

→ kuch bhi ho! → no effect on impulse fn.

$$= x(t_0)$$

In general: **

$$\boxed{x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)(dt)} \qquad \delta(t-\tau) = \begin{cases} 1 ; & t = \tau \\ 0 ; & t \neq \tau \end{cases}$$

→ To find strength of wave at time = t

② **Scaling Property:-** $\delta(at) = \frac{1}{|a|}\delta(t)$

$$at = \tau \qquad \hookrightarrow \delta(\tau) = (1/|a|)(\delta(t))$$

$$t = \frac{\tau}{a} \to dt = \frac{1}{a}(d\tau) \Rightarrow \cancel{dt = \frac{1}{a}d(at)}$$

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If $a \geq 0 \Rightarrow$

$$\int_{-\infty}^{\infty} (\delta(at))(x(t))(dt) = \frac{1}{|a|} \int_{-\infty}^{\infty} \delta(t)\, x\left(\frac{\tau}{a}\right)(dt)$$

$$= \frac{1}{|a|} \int_{-\infty}^{\infty} \delta(\tau)\, x\left(\frac{\tau}{a}\right)(d\tau)$$

as impulse f^n mein convert kara, so, $x(t) = \delta(t)$

let $\delta(t) = x(t)$

$\Rightarrow \delta(t) = x(t)|_{t=0}$

$= x(0)$

$$= \frac{1}{|a|}\, x(0) = \frac{1}{|a|}\, \delta(t)$$

* Impulse function is an even function:

$$\delta(t) = \delta(t)$$

Substituting $a = -1$ in, $\delta(\overset{a}{t}) = \frac{1}{|a|}\delta(t)$

$$\Rightarrow \delta(-t) = \frac{1}{|-1|}\, \delta(t)$$

$$\Rightarrow \boxed{\delta(-t) = \delta(t)}$$

③ Multiplication Property:-

① If $x(t)$ is continuous at $t=0$, then:

$$\Rightarrow \int_{-\infty}^{\infty} [x(t)\, \delta(t)]\, \phi(t)(dt) = \int_{-\infty}^{\infty} \delta(t)[x(t)\, \phi(t)](dt)$$

$$= \int_{-\infty}^{\infty} x(0)\, \phi(t)\delta(t)(dt)$$

$$= x(0) \int_{-\infty}^{\infty} \phi(t)\delta(t)(dt) = x(0)\,\phi(0)$$

As, $x(t)\,\delta(t) = x(0)\,\delta(t)$

W JMK

If $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

If $x(t)$ is continuous at $t=t_0$, then:

$$\int_{-\infty}^{\infty} [x(t)\,\delta(t-t_0)]\,(\phi(t))\,(dt) = \int_{-\infty}^{\infty} \delta(t-t_0)\,[x(t)\,\phi(t)]\,(dt)$$

$$= \int_{-\infty}^{\infty} x(t_0)\,\delta(t-t_0)\,(\phi(t))\,(dt)$$

$$= x(t_0) \int_{-\infty}^{\infty} \phi(t)\,\delta(t-t_0)\,(dt)$$

$$= x(t_0)\,\delta(t-t_0) \left[\int_{-\infty}^{\infty} \phi(t)\,(dt) = 1 \right]$$

If $x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)$

(4) **Amplitude Reversal:-**

$$\int_{-\infty}^{\infty} [t\,\delta'(t)]\,\phi(t)\,(dt) = \int_{-\infty}^{\infty} \delta'(t)\,[t\,\phi(t)]\,(dt)$$

$\Longrightarrow$ $\boxed{\int \text{by parts}}$

$$\cancel{= -\int_{-\infty}^{\infty} \delta(t)\,[\phi(t) + t\,\phi(t)]\,(dt)}$$

$$= -\int_{-\infty}^{\infty} \delta(t)\left[\phi(t)\frac{d(t)}{dt} + \frac{t}{1}\frac{d}{dt}(\phi(t))\right](dt)$$

$$= -\int_{-\infty}^{\infty} \delta(t)\,[\phi(t) + t\,\phi'(t)]\,(dt)$$

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$$= -\int_{-\infty}^{\infty} [\phi(t) + t\,\phi'(t)]\,\delta(t)(dt)$$

$$= -\left[\phi(t) + t\,\phi'(t)\right]_{t=0}$$

$$= -\phi(0) = -\int_{-\infty}^{\infty} \phi(t)\,\delta(t)(dt)$$

$$\Rightarrow \boxed{t\,\delta'(t) = -\delta(t)}$$

H/w

Prove: * $\delta(n) = u(n) - u(n-1)$; $u(n) = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$

* $x(n)\,\delta(n-k) = x(k)\,\delta(n-k)$

* $\sum_{k=-\infty}^{\infty} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k) = u(n) = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$

* Derive r^n b/w $\delta(t)$, $u(t)$, $r(t)$

(Impulse fn) (unit step function) (ramp fn)

WJMK

24 March, 2023
Friday

Lab-1

* Matlab is high-level programming language.
  ↳ compatible for all operating system.
  ↳ available in both offline & online mode.
  ↳ pehle sirf yeh hota tha.

* Components in Matlab desktop:
  1) Command Window
  2) ——— History
  3) workspace
  4) Current directory
  5) Help Browser.
  6) Start Button

tells in which location u r working currently

>> → double arrow means where u can do oprns.

>> a=6*3 ← input by user
a=
    18 ← output by MATLAB
>> → waiting for next command

Figure window → where u can plot the figures / signals.

/ → Right division   6/3 → 2
\ → left "   3\6 → 2
^ → Exponentation
sqrt (x) → $\sqrt{x}$
log (x) = $\log_e(x)$
log10 (x) = $\log_{10}(x)$
exp (x) = ex
sin(x) = sin x
acos(x) = $\cos^{-1}(x)$

we use log fn to bring linearity in non-linear
↳ $\log_{10}(1000) = 3$
huge change

(~~tanh~~ atan(x) → hyperbolic ?!) WTMK

★

$a^n + b^m$

if a, b → variable

→ exp (a, n) + exp (b, m)

if $2^2 + 3^3$

= 2^2 + 3^3

>> → means ~~we~~ are on the home directory!

>> clc → clears all input and output from the command window display, giving u "clean screen".

↳ But still u ~~have~~ will have all this in command history.

>> clear

↳ ~~it will look like new start for programming~~

→ clear everything → figures u made, from command history etc!

>> format long ← will give upto 16 variables after decimal

>> —— short ← —— 4 ——

>> —— bank ← —— 2 ——

>> format rat ← —— in p/q form!

↑ means (rational)

→ round off element ~~towards~~ nearest Z

>> round (6.628)

ans =

7

→ fix element to nearest Z towards '0'

↳ means will remove fractional value.

>> fix (6.628)

ans =

6

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In matlab → no comma ~~also~~ needed.
give space → it will assume as comma

```
>> ceil (6.628)
ans =
     7
>> floor (6.628)
ans =
     6
```

① Array ~~cre~~ Creation :-

```
>> a = [ 3  5  7  -8  5  2]
```

↑ row vector.

← Just like sequence representation

$a = (3, 5, 7, -8, 5, 2)$

↑ [origin]

```
>> a = [ 3 ; 5 ; 7 ; -8 ; 5 ; 2]
```

↑ column vector

ans →

$$\begin{bmatrix} 3 \\ 5 \\ 7 \\ -8 \\ 5 \\ 2 \end{bmatrix}$$

→ OR

' → for Transpose

```
>> a = [3  5  7 -8  5 2]'
```

ans →

$$\begin{bmatrix} 3 \\ 5 \\ 7 \\ -8 \\ 5 \\ 2 \end{bmatrix}$$

a*b → matrix multiplication.

⇒ $\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix} [\cdot\ \cdot\ \cdot]$ → give only one value

a.*b

$$\begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} [a\ b\ c] \rightarrow \begin{bmatrix} a'a \\ b'b \\ c'c \end{bmatrix}$$ $\boxed{\text{give (3x1) matrix}}$ *

WJJMK

* CREATING ROW VECTOR :-

increment value → if not specified, then by default → '1'

>> a = 1 : 2 : 13

Initial value — Final value

{specify interval}

a =

1 3 5 7 9 11 13

>> b = 1 : 8

b =

1 2 3 4 5 6 7 8

→ input signal given hota, & 2nd f^n is already know from eq^n

↳ either: (1) unit step f^n, or

(2) ~~ramp~~

(3) parabolic ~~or~~

(4) impulse

>> v = linspace (a, b, n)

↑ length of linearly spaced vector from a to b

⇒ [a, b] → divided into n−1 equally spaced sub-intervals

>> a = linspace (0, 20, 5)

a =

0 5 10 15 20

$\boxed{\sim = \rightarrow \text{not equal to}}$

>> a = [2 4 2 5 7] means Jo value ≤4 → return 0

>> a > 4 → else → ——— 1

ans =

0 0 0 1 1

>> b = [5 7 3 5 3] → where not equal → return (0)

>> a == b → else → ——— (1)

ans = 0 0 0 1 0

WJMK

* For vectors and matrices sort(X) → sort in ascending order.

```
>> m=[28   3   5]
>> sort(m)
 ans=

      3    5    28
>>[n  ind]= sort(m)
 m =

      3    5    28
ind =

      2    3    1
```

→ means '28' → 1st position mein tha!

```
>> sort(m, 'descend')
```

↳ to ~~write~~ sort in 'descending order'.

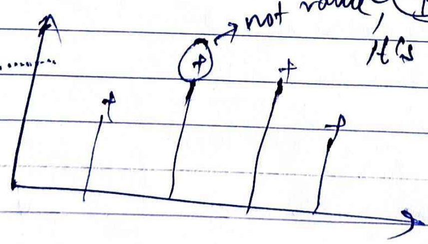
* No plotting in '1D' in Matlab
  only 2D, 3D, ...

plot (xdata , Y data , 'option')

'option' ← optional → not value, it's mark'.

$\underbrace{+, -, *, \circ,}$ ......

↓ any thing can be used as mark...



```
>> x = linspace (0, 2*pi)
>> y = sin (x)
>> plot (x, y)
```

• Plot $z = x^2 + y^2$ w $-3 \le x, y \le 3$

```
>> x = -3:0.1:3; y = x;
>> [x, y] = meshgrid(x, y)
>> z = x.^2 + y.^2
>> surf(x, y, z)
```

for 3D plot

→ plot → for 2D plot

WJMK

L-2

ॐ

29/3/2023
wednesday

SIGNALS :-

* A CT signal is f^n
* A DT signal is sequence.

special case of f^n where domain $= \mathbb{Z}$

using exclusively
(f^n which is not sequence)

$\boxed{f(t)}$ value of f^n 'f' at pt. (t)

In digital signal, There is no Time Domain.
very useful.

Prop. of signals:- → graph, symmetric abt origin

1. Even signals. f^n: $x(t) = x(-t) \ \forall t$
   seq: $x(n) = x(-n) \ \forall n$
2. odd signals → $x(t) = -x(-t) \ \forall t$ (f^n)
   $x(n) = -x(-n) \ \forall n$ (seq.)
   anti symmetric abot origin
3. Periodic signals.
   ↳ periodic with period (T) [T-periodic]
   ↳ for some $T > 0, T \in \mathbb{R}$,
   $x(t) = x(t+T) \ \forall t$.

$\text{frequency} = \frac{1}{T}$

$\text{angular frequency} = \frac{2\pi}{C}$

* Aperiodic → f^n/seq that is not periodic.

* non-periodic → $\omega = \frac{2\pi}{T}$

eg: $x(t) = \sin(2\pi\omega t)$
↳ if fractional aaya!!
↳ then non-periodic

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- Discrete time signal is periodic iff

$$x(n) = x(n+N) \ \forall n$$

$N > 0, N \in \mathbb{Z}$

↑ Period of signal measured in terms of no. of sample spacing (sample/cycle)

* No physical signal is Periodic, They are Aperiodic
  ↳ To bring periodicity, we've many method's

  non-stationary
  not stable
  anti-causal

Continuous time signal:

$$E_x = \text{power} = \int_{-\infty}^{+\infty} |x(t)|^2 (dt)$$

| | → to bring to real part, ignore imag/nay means Energy comes from real part only!

Discrete:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} |x(t)|^2 (dt)$$

(only if $T \to \infty$ ↳ we'll find P_x o/w E_x)

we'll use UNIT STEP f^n.
↳ Bring whole signal into 'and' causal
↳ means not give future value, will have past and present value only!

$$P_X = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

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* All signals are becoming ≥ 0, when we are squaring.

$x(n) \rightarrow$ complex discrete time signal

if $x(n) = x^*(-n)\ \forall n \rightarrow$ then conjugate sym.

even signal $\rightarrow$ real conjugate symmetric signal

$x(n) = -x^*(-n) \rightarrow x(n)$

conjugate anti-symmetric

if real

then called 'odd'

* Bounded, Absolutely Summable, and Square - summable signals :-

can't be (0) $\rightarrow E_x = P_x = 0$.

can't be -ve $\rightarrow E_x, P_x$ can't be < 0

so have to manage some Finite no.

$|x(n)| \leq B_x < \infty$

$\hookrightarrow x(n)$ is bounded

$\sum_{n=-\infty}^{\infty} |x(n)| < \infty \rightarrow x(n) \rightarrow$ absolutely summable

$\rightarrow$ same thing (have future value)

Causal, Anti-causal, Non-Causal :-

$x(t) = 0,\ t < 0$

$x(t) = 0,\ t \geq 0$

signal does not start before $t = 0$

if starts before $t=0 \Rightarrow$ non-causal signal

* multiplying signal by unit step ensures that resulting signal is causal.

Convolution → pt. to pt multiplication

W JMK

Unit step × non-causal = causal

$$x(t) = e^{-at}$$

$$x(t) = \underbrace{e^{-at}u(t)}_{\text{causal part of } x(t)} + \underbrace{e^{-at}u(-t)}_{\text{anti-causal part of } x(t)}$$

Systems :-

↳ entity that processes 1 or more signal to produce 1 or more output signals.

☆ no. of inputs :

↳ if 1 input → system with SI (single input) system is

↳ o/w → MI

Speech signal → gives only (time)

→ can be classified in terms of types of signals

① 1-D , M-D

② CT , DT

③ analog , digital

is mem (4) fns

① unit step ② ramp ③ ... ④ Parabolic

CT :→

(Excitation) Input $x(t)$ → [continuous-Time system] → output $y(t)$ (Response)

Input $x(n)$ → [Discrete Time system] → output $y(n)$

Impulse Response ⇒ $h(t)$ or $h(n)$

↳ response of system to an applied impulses

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Systems

- Static, Dynamic
- Causal, Non-causal
- Stable, Unstable
- system with or w/o memory
- Invertible and non-invertible
- Linear, non-
- Time variant, in

time-invariant

If behavior of system is Independent of time at which input is applied to system

If $\neq$, then TV

$$y(t,T) = y(t-T)$$

★ LTI (Linear Time Invariant)

↳ Linearity and Time Invariant ✓

$$y(t) = T\{x(t)\}$$ → multiplying signal with system response

↳ Transfer f^n of $x(t)$

↳ means Transferring input through system and $x(t)$ is input.

$$= x(t) \times h(t)$$

★ Linear and Non-Linear!

If satisfies superposition and homogenate principle

$x_1(t), x_2(t) \to 2$ ~~systems~~ inputs of systems

$y_1(t), y_2(t) \to$ — outputs

Then according to these principles ⇒

$$y_3(t) = a y_1(t) + b y_2(t)$$

$$y_3'(t) = a x_1(t) + b x_2(t)$$

If, $y_3(t) = y_3'(t) \to$ linear system o/w non-linear

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$\{ax_1(t) + bx_2(t)\}$ $T[\;\; \omega \;\;]$ $\downarrow$ $y(t) = ay_1(t) + by_2(t)$

$\{ax_1(t) + bx_2(t)\} \times h(t)$

$= \underbrace{ax_1(t) \times h(t)}_{y_1(t)} + \underbrace{bx_2(t) \times h(t)}_{y_2(t)}$

$= a(y_1(t)) + b(y_2(t))$

, system k through pass ho rha hai, output ~~same~~ utna hi kaise aayega? ↳ In linear system!!

for linear system, $h(t)$ is unit step ~~Basically~~ Basically
↳ $u(t)$

$$u(t) = \begin{cases} 1 ; & t \geq 0 \\ 0 ; & t < 0 \end{cases}$$

'1' ko kisse bhi multiply kro, ~~(t) hi~~ same hi rhega!!

⇓

linear system.

★ Static and Dynamic:

let $y(t) = \cancel{x(t)}\; T\{x(t)\}$ ← Static

then this is (static system)

↳ Jitna input, utna hi output.
↳ no deviation, no step deviation.
↳ pta nhi delay ho rha h, ya nhi
↳ pta nhi aage bdh rha h,

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Itna delay

$y(t) = T\{x(t-\tau)\}$ ← Dynamic

$y(t) = \frac{d}{dt}(x(t))$

$y(t) = \int_{-\infty}^{\infty} x(t)\,(dt)$ } → DYNAMIC

∫ → area, boundary region, sum of all values in accumulation form btaata !!

* Causal and Non-causal :-

$y(t) = x(t)$

$= x(t+4)$ } → causal

(won't analyze any value in future)

$= x(t-4)$ } → Non-causal

giving future value !!

$t-4=0$

$t=4$ → future prediction !

* Stable and unstable :-

$y(t) = \sin(t) \rightarrow T\{y(t)\}$

$\rightarrow T\{\sin(t)\}$

$= \sin(\pi t)$ } → Stable

Sinusodial gives sinusodial

Both are sinusodal only !

$y(t) = e^{at}$ → $a<0$, → $a>0$] Unstable

Kabhi nhi hoga everytime same trend output side, batta rhega!

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★ System with memory or w/o memory! –

output
If signal is either derivatives or $\int$ or delayed or advanced version of input signal ??, → then system with memory

$x(t) = \sin(t)$
output → $y(t) = -\sin(t)$ → system w/o memory

[sin (t) toh reh hi rha hai] means we are not carrying anything

we will study this only !!

★ If 'a' input → 'b' output
and if 'b' input → then 'a' output → Invertible system

$\sin(45^\circ) = \cos(45^\circ)$
$\sin(\pi/2 - 45^\circ) = \cos(45^\circ)$

★ Convolution of 2 signals! –
In system analysis, we'll try to determine the response to some specified input.

★ Unit Impulse Response :-

CT

$\delta(t) \rightarrow$ [LTI] $\rightarrow h(t)$
$x(t)$ $\qquad\qquad\qquad\qquad y(t)$

→ TV
→ Invertible

$\delta(t) \longrightarrow h(t)$
$x(t) \longrightarrow y(t) = T\{x(t)\}$

DT

$\delta(n) \longrightarrow h(n)$
$x(n) \longrightarrow y(n) = T\{x(n)\}$

multiplying 2 signals → one wrt same phase, other wrt delayed phase

w.Jack

CONVOLUTION ~~&~~ INTEGRALS :-

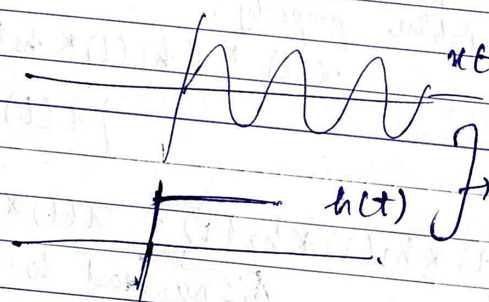
$\delta(t) \longrightarrow h(t)$

$\delta(t-\tau) \longrightarrow h(t-\tau)$

$x(\tau)\,\delta(t-\tau) \longrightarrow x(\tau)\,h(t-\tau)$

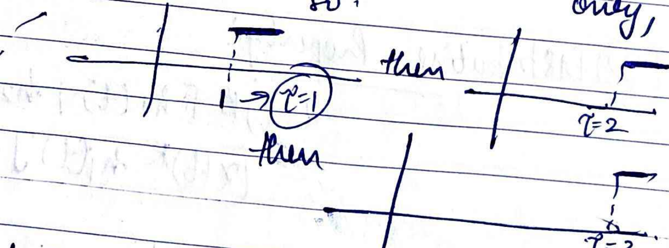
$$y(t) = T\{x(t)\}$$
$$= x(t) \otimes h(t)$$

→ when multiplying we must know at what time?



If not shift $h(t) \times x(t) = x(t)$ only,

so.



so shift the $h(t)$

$\Downarrow$

$h(t-\tau)$ → time instant at particular place

$$x(t) = \int_{\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,(d\tau) \longrightarrow$$

$$y(t) = \int_{\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,(d\tau)$$

$$y(t) = x(t) * h(t)$$

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$$x(t) * h(t) = h(t) * x(t)$$

$$x(t) * (h(t)) = \int_{-\infty}^{\infty} \cancel{\delta(t)} \; x(t)\delta(t-\tau)(d\tau)$$

let
$$t-\tau=\alpha$$
$$\tau = t-\alpha$$
$$d\alpha = -d\tau$$

$$\alpha \to \infty \quad \text{as } \tau \to \infty$$

& show
$$x(t) * h(t) = h(t) * x(t)$$

① Associative property:

$$x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$$

so, $x(t) * h_1(t) * h_2(t) = x(t) * h_2(t) * h_1(t) = h_1(t) * h_2(t) * x(t)$

no need to indicate which convolution to perform first

② Distributive Property:

$$x(t) * [h_1(t) + h_2(t)]$$
$$= [x(t) * h_1(t)] + [x(t) * h_2(t)]$$

• convolution with impulse

$$x(t) * \delta(t) = x(t)$$
$$= \int_{-\infty}^{\infty} x(\tau) \cdot \delta(t-\tau)(d\tau)$$

By sampling prop. ⇒ ??

$$x(\tau)\delta(t-\tau) = x(t)\,\delta(t-\tau)$$

$$= \int_{-\infty}^{\infty} \delta(t-\tau)(d\tau) = x(t)$$

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③ Width Property :-

duration (widths) of $x(t)$, $h(t) \to$ finite $\to$ W_x, W_h (resp)

then duration of $x(t) * h(t) \to W_x + W_h$

interval time b/w 1st and last non-zero value.

④ Differentiation Prop:

$$y(t) = x(t) * h(t)$$

$$\left(\frac{d}{dt}(x(t))\right) * h(t) = x(t) * \left(\frac{d}{dt}(h(t))\right)$$

$$= \frac{d}{dt}(y(t))$$

Dilation

⑤ Time-Scaling prop :

$$y(t) = x(t) * h(t)$$

then,

$$x(at) * h(at) = \frac{1}{|a|} y(at)$$

$a<0 \to$ expanding / enhancing signal

$a>1 \to$ comprehensing the signal

so based on req., bdha signal ko chota or chota —— bdha bna skte ho!!

Case 1
Case 2
Case 3

read from ppt

WJMK.

* $x(n) = \sum_{K=-\infty}^{\infty} x(k)\,\delta(n-k) \longrightarrow y(n) = \sum_{K=-\infty}^{\infty} x(k)\,h(n-k)$

$\delta(n-k)$: (location of pts) (k will vary)

$y(n) = x(n) * h(n)$

• Stability for LTI system.

ॐ WJMK

30/3/2023
Thursday

Tutorial-2

* Time variant and Invariant system :-
for CT:

$y(t,T) = y(t-T) \leftarrow$ Time invariant
$y(t,T) \neq y(t-T) \leftarrow$ Time variant

$y(t) = T\{x(t)\}$

eg: (1) $y(t) = \cancel{x} \; t\,x(t)$ → f^n variable will change

$y(t,T) = t\,x(t-T)$
↑
constant won't change

and $y(t-T) = (t-T)\,x(t-T)$

these (2) must be equal, but not equal!

$y(t,T) \neq y(t-T)$

↳ so system is Time Variant.

(2) $y(t) = x(t)\,\underbrace{\sin(10\pi t)}$ ↳ constant; not f^n; so do not shift it (in delayed or advanced version)

$y(t,T) = x(t-T)\sin(10\pi t)$

and $y(t-T) = x(t-T)\sin(10\pi(t-T))$

$y(t-T) \neq y(t,T)$

↳ so Time Variant system.

(3) $y(t) = e^{2x(t)}$ (Exp. f^n)

$y(t,T) = e^{2x(t-T)}$

$y(t-T) = e^{2x(t-T)}$ ⇒ $y(t,T) = y(t-T)$

↳ so Time Invariant

~~$\sin(\pi/n)$~~
~~$\sin(\pi(n/2k))$~~
~~$\sin(\pi n - 2\pi k)$~~
~~$-\sin(2\pi k/\pi n)$~~

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T → shifted version

(4) $y(t) = \sin\{x(t)\}$
$y(t,T) = \sin\{x(t-T)\}$
$y(t-T) = \sin\{x(t-T)\}$
so, $y(t,T) = y(t-T)$

In DT: (~~D~~ OR discrete domain)

→ $y(n,K) = y(n-K)$ → Time invariant
→ $y(n,K) \neq y(n-K)$ → ——— Variant

Eg: (1) $y(n) = x(n) - x(n-1)$
$y(n,k) = x(n-k) - x(n-k-1) = y(n-k)$
→ so Invariant.

means shifting 'k' ~~so~~; ~~of sig~~ (with) wherever 'n' is there.

(Just shifting (n))

(2) $y(n) = x(n)\cos(\omega_0 n)$
$y(n,k) = x(n-k)\cos(\omega_0 n)$
$y(n-k) = x(n-k)\cos(\omega_0(n-k))$
↳ means shifting the all values
→ wherever 'n', replace by (n-k).

so, $y(n,k) \neq y(n-k)$ ⇐ variant.

★ (3) Linear/Non-Linear Systems :-

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$ax_1(t) + bx_2(t) \rightarrow$ System $T\{\ \}$ $\rightarrow = T[a\,x_1(t) + b\,x_2(t)]$

$= y_3(t)$

$ay_1(t) + by_2(t)$

$= aT\{x_1(t)\} + bT\{x_2(t)\}$

$= y_3'(t)$

If $y_3(t) = y_3'(t)$ (Linear system)

If $y_3(t) \neq y_3'(t)$ (Non-Linear)

In DT, replace 't' by 'n'.

eg: ① $\boxed{y(t) = 2x^2(t)}$ → This is input!!! so here $x_1(t) \rightarrow 2x_1^2(t)$.

$y_1(t) = 2x_1^2(t)$

$y_2(t) = 2x_2^2(t)$

$y_3(t) = [2(x_1^2(t))]a + b[2(x_2^2(t))]$

$y_3(t) = 2a(x_1^2(t)) + 2b(x_2^2(t))$

and,

$y_3'(t) = T\{\ \} = a\,y_1(t) + b\,y_2(t)$

$\cancel{= 2a(x_1^2(t)) + 2b(x_2^2(t))}$

$= T\{2x^2(t)\}.$

$= T\{a x_1(t) + b(x_2(t))\}$

$= 2[a x_1(t) + b x_2(t)]^2$

$= 2[a^2 x_1^2(t) + b^2 x_2^2(t) + 2ab\,x_1(t)\,x_2(t)]$

as $y_3(t) \neq y_3'(t)$ (Non-Linear)

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eg: $y(t) = e^{x(t)}$

$\Rightarrow y_1(t) = e^{x_1(t)}$

$y_2(t) = e^{x_2(t)}$

$y_3(t) = y_3'(t)$ ← sum of weighted Linear Combination of the output response

sum of weighted Linear Combination of the input signals

$\underline{*}\ y_3(t) = a x_1(t) + b x_2(t)$ $(\Leftarrow a y_1(t) + b(y_2(t))$

$= a e^{x_1(t)} + b e^{x_2(t)}$

$y_3'(t) = e^{a x_1(t) + b x_2(t)}$

$= e^{a x_1(t)} \cdot e^{b x_2(t)}$

$y_3(t) \neq y_3'(t)$ ← Non-Linear!!

eg: $y(t) = x(t^2)$

$y_1(t) = x_1(t^2)$, $y_2(t) = x_2(t^2)$

$y_3(t) = a x_1(t^2) + b x_2(t^2)$

$y_3'(t) = a y_1(t) + b y_2(t)$

$= a x_1(t^2) + b x_2(t^2)$

$\Rightarrow y_3(t) = y_3'(t)$ ← Linear!!

WJMK m.

deals with ~~~~ initial value

at differentiation or integration part

Static or Dynamic System :-

If dealing with future or past values

If $\int$able or differentiable, then dynamics.

If dealing with ~~past or~~ present.

$y(t) = x^2(t)$
$y(n) = n(x(n))$

- $y(t) = \frac{d}{dt}(x(t))$
- $y(t) = x(t-1)$
- $y(t) = \int_{-\infty}^{\infty} x(t)(dt)$

eg : ① $y(t) = x(2t)$

$t=0 \Rightarrow y(0) = x(0)$
$t=1 \Rightarrow y(1) = x(2)$ ← future value !
$t=2 \Rightarrow y(2) = x(4)$

→ so, Dynamic systems !!

② $y(n) = x(n-2) + x(n)$ ← delayed version

$n=0$
$\Rightarrow y(0) = x(-2) + x(0)$
$n=1$
$\Rightarrow y(1) = x(-1) + x(1)$
$n=2$
$y(2) = x(0) + x(2)$

→ Dynamic !!

③ $y(t) = x^2(t) + x3(t) + x5(t)$ ← telling abt present value only

↳ Static !!

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(4) ~~xtt~~ $y(n) = \sum_{k=-\infty}^{(n+1)} x(k)$ ← Dynamic!!

$= \dots + x(-1) + x(0) + x(1) + \dots$

past ↑ ↑ future (for $n=0$)

$y(0) = \dots + x(-1) + x(0) + x(1)$

past ↑ ↑ Future

(5) $y(t) = \frac{d^2(x(t))}{dt^2} + 2x(t)$ ← Dynamic

↑ Derivable

(6) $y(n) = \log_{10}[x(n)]$ ← $\boxed{\text{static}}$

Causal or Non-Causal Systems:-

Causal ↓ Present and past state

Non-Causal ↓ deal with future values including past and present values. (Future must! past or present bhi ho sakta)

Eg: $y(t) = x^2(t) + x(t-4)$

$t=0$

$y(0) = x^2(0) + x(-4)$

↑ Present ↑ Present ↑ Past

WJMK

Eg : $y(t) = x(2-t) + x(-t-4)$

$t=0$

$y(0) = x(2) + x(-4)$ [Non- Causal]

Present Future Past

Eg : $y(t) = \int_{-\infty}^{3(t)} x(\cancel{t}\tau)(d\cancel{t}\tau) = x(t)\Big|_{-\infty}^{3t}$

$= x(3t) - x(-\infty)$

★ $y(t) = x(3t) - x(-\infty)$

$t=0$

$\Rightarrow y(0) = x(0) - x(-\infty)$

$t=1$

$\Rightarrow y(1) = x(3) - x(-\infty)$ [Non-causal]

Future Past

Eg: $y(t) = x(t/2) \rightarrow$ CAUSAL.

Eg : $y(t) = x[\sin(2t)]$ → Non-Causal

$t=0$

$\Rightarrow y(0) = 0$

$t = \pi/2 \Rightarrow y(\pi/2) = x(\sin(\cancel{2} \times \frac{\pi}{2}))$

$y(\pi/2) = x \sin(\pi)$

$\boxed{t = \pi, t = -\pi} \rightarrow y(-\pi) = \boxed{0}$ Future

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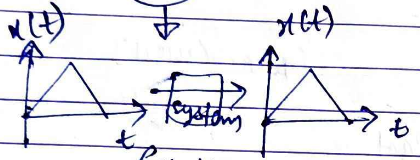
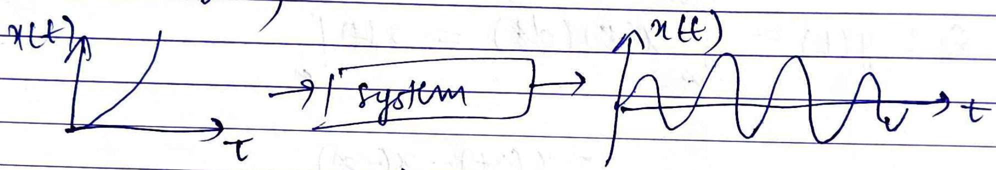
Bounded input - Bounded output ← superposition principle

Stable and Unstable System :-



(stable system)



UNSTABLE system.

Eg: (1) $y(t) = e^{x(t)}$; $|x(t)| \leq 8$

Bounded

x(t) ~~varies from~~

↑ Bounded input

$-8 \leq x(t) \leq 8$

x(t) varies from -8 to 8.

$y(t) = e^{x(t)}$

$e^{-8} \leq y(t) \leq e^{8}$

Bounded output. → stable system

(2) $h(t) = (2 + e^{-3t})(u(t))$

Dynamic ✓

$\int_{-\infty}^{\infty} |h(t)|\,dt \leq \infty$

(should be finite value!)

yeh hoga tabhi stable hoga

o/w unstable

dekhte hain hota h yaa nhi

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$$h(t) = \int_{-\infty}^{\infty} [2+e^{-3t}](u(t))\,(dt)$$

$$u(t) = \begin{cases} 1 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$

$$= \int_{0}^{\infty} (2+e^{-3t})(dt)$$

$$= 2t\Big|_0^{\infty} + \frac{e^{-3t}}{-3}\Big|_0^{\infty}$$

$$= \infty - 0 + \frac{1}{3}$$

$$= \infty + \frac{1}{3} = \infty \leftarrow \text{Not a finite value}$$

UNSTABLE SYSTEM.

output is not bounded.

Not causal

*. $h(t) = e^{2t} u(t) \leftarrow$ applying unit step f^n to make it a causal f^n.

apply from $-\infty$ to ∞

$$= \int_{-\infty}^{\infty} e^{2t}\, u(t)\,(dt)$$

$$= \int_{0}^{\infty} e^{2t}\,(dt) = \frac{e^{2t}}{2}\Big|_0^{\infty} = \frac{e^{2\infty}}{2} - \frac{e^0}{2}$$

$$= \infty - 1/2 = \infty$$

$\boxed{\text{Unstable}}$!!

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Eg: $h(t) = \frac{1}{RC}\left(e^{-t/RC}\right)(u(t))$

$$\Rightarrow h(t) = \int_{-\infty}^{\infty} \frac{1}{RC}\left(e^{-t/RC}\right)(u(t))(dt)$$

$$= \int_{0}^{\infty} \frac{1}{RC}\left(e^{-t/RC}\right)(dt)$$

$$= \frac{1}{RC}(-RC)\left[e^{-t/RC}\right]_0^{\infty}$$

$$= -1\,[0-1]$$

$$= 1 \leftarrow \text{stable system!}$$

Bounded output.

Eg: $y(n) = \cancel{a x(n-7}$ $a\,x(n-7)$

Inverse Impulse Response $h(n) = a\,\delta(n-7)$

only at one 'n' value will exist, o/w ZERO

$$\boxed{\sum_{k=-\infty}^{\infty} |h(k)| \leq \infty}$$

$\hookrightarrow$ Must!

$$h(n) = \sum_{n=-\infty}^{\infty} a\,\delta(n-7)$$

$\uparrow$ exist only at $n=7$

$$= \sum_{n=7} a\,\delta(n-7) = a\ (\text{for } n=7)$$

$\uparrow$ stable ✓

correlation within

~~Cycles~~ signal

WJMK

(seeing cor signal with ~~other~~ same signal)

Auto-correlation f^n ↳ correlating signal within signal

* Correlation of Signals :- (seeing how much similar or different they are)

Cross-correlation sequence of DT Energy signals:

$\to R_{xx}$

↳ so same signal is being shifted

x → 1st signal
y → 2nd signal

Doing cross-correlation (correlation b/w ~~sa~~ signals)

$$R_{xy}(m) = R_{yx}(m)$$

$$R_{xx}(m) = R_{xx}(-m)$$

(Its' up to u ki advanced version le rhe ho yaa delayed version)!!

(fourier series) ~~1 year~~

(fourier transf)

(Intro to Communication)

(Till this C1)

WJMK

[Derivations also imp]

5/4/2023
Wednesday

Lecture :- 3

Imp. (10 Marks) ***

(can't judge which part of signal have which info)

⊙ Fourier Series :→

↳ why need to apply this on signals ??

signals we get are in analog domain
↳ can be converted into discrete time signal.

↳ to bring it in some other domain, so that we can visualise the signal in different way,
→ to see what queues are present in signal

(to extract info & build model)

we need some transformations !
↳ to do analysis

Signal have diff-2 properties !!

→ we'll consider whole signal in 1 part, not as diff-2 parts (difficult)

In Frequency domain
↳ can visualise signals with periodicity etc.

($x(t)$) Signal in 3 parts :-

1) • constant term (amplitude)
2) • cosinusoidal part (cos) → real part
3) sinusoidal part (sin) → imaginary part

↳ If not periodic signal, then signal does not exist (assumption)

Fourier series of 2 types:
① Trigno
② Exponential

WOMK

① Fourier Series → mathematical tool that allows representation of any periodic signal as sum of harmonically related sinusoid.

Trigno : →
$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n w_0 t) + b_n \sin(n w_0 t)]$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,(dt)$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos(n w_0 t)\,(dt)$$

real part of signal

$$b_n = \frac{2}{T}\int_0^T x(t)\sin(n w_0 t)\,(dt)$$

Imaginary part of signal.

a_0, a_n, b_n → fourier series coefficients

will give weight or info about that signal at that particular instant of time

• Polar form Representation of Fourier Series :

Case I → To represent in terms of cos)

$a_n = C_n \cos(\theta_n)$

$b_n = -C_n \sin(\theta_n)$

$C_0 = a_0$ $\quad C_n = \sqrt{a_n^2 + b_n^2}$ $\quad \forall\ n \geq 1$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n w_0 t + \theta_n)$$

if dealing with even no → give C_n → given no. of ~~odd no.~~

WJMK

(To represent in terms of sin)

Case 2:

$$C_0 = a_0 \quad , \quad C_n = \sqrt{a_n^2 + b_n^2} \quad n \geq 1$$

$$\phi_n = \tan^{-1}\left(\frac{a_n}{b_n}\right)$$

$$a_n = C_n \sin(\phi_n)$$

$$b_n = C_n \cos(\phi_n)$$

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega_0 t + \phi_n)$$

* converting analog signal into frequency domain.

* $\int_0^T \sin(m\omega_0 t)(dt) = 0 \quad \forall m$

$$\int_0^T \cos(n\omega_0 t)(dt) = 0 \quad \forall n \neq 0$$

$$\int_0^T \sin(m\omega_0 t)\cos(n\omega_0 t)(dt) = 0 \quad \forall m, n$$

$$\int_0^T \sin(m\omega_0 t)\sin(n\omega_0 t)(dt) = \begin{cases} 0 ; & m \neq n \\ T/2 ; & m = n \end{cases}$$

$$\int_0^T \cos(m\omega_0 t)\cos(n\omega_0 t)(dt) = \begin{cases} 0 ; & m \neq n \\ T/2 ; & m = n \end{cases}$$

- Average value of sinusoid over m & n completes cycle in period T is ZERO.

(6) Evaluation of a_0, a_n, b_n :- Proofs:

(1) $a_0 = \frac{1}{T}\int_0^T x(t)(dt)$

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$$x(t) = a_0 + [a_1 \cos(w_0 t) + b_1 \sin(w_0 t)] + [a_2 \cos(2w_0 t) + b_2 \sin(2w_0 t)] + \dots + [a_n \cos(n w_0 t) + b_n \sin(n w_0 t)]$$

$$\Rightarrow \int_0^T x(t)(dt) = \int_0^T a_0 (dt) + \left[\int_0^T a_1 \cos(w_0 t)(dt) + \int_0^T b_1 \sin(w_0 t) dt\right] + \dots + \left[\int_0^T a_n \cos(n w_0 t)(dt) + \int_0^T b_n \sin(n w_0 t)(dt)\right]$$

$$= \int_0^T a_0 (dt) + 0 + 0 + \dots$$

$$= a_0 (T)$$

$$\Rightarrow a_0 = \frac{1}{T}\int_0^T x(t)(dt). \quad \text{H.P.}$$

got amplitude of the SIGNAL.

(2) for: $a_n = \frac{2}{T}\int_0^T x(t) \cos(n w_0 t)(dt)$

$\Rightarrow$ as

$$x(t) = a_0 + [a_1 \cos(w_0 t) + b_1 \sin(w_0 t)] + \dots + [a_n \cos(n w_0 t) + b_n \sin(n w_0 t)]$$

$$\Rightarrow \int_0^T x(t) \cos(n w_0 t)(dt) = \int_0^T a_0 \cos(n w_0 t)(dt) + \int_0^T a_1 \cos(n w_0 t) \cos(w_0 t)(dt) + \dots$$

$$\Rightarrow \int_0^T x(t) \cos(n w_0 t)(dt) = a_n(T/2)$$

WJMK

(3) $b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega_0 t)(dt)$

* In this way extracting coeff. values from given signal.

⊙ let $x(t) = 2 + t^2 + t^4$

as $x(t) = x(-t) \rightarrow$ Even signal.

$$\cancel{a_0 = \frac{1}{T}\int_0^T x(t)(dt)}$$

$$\cancel{= \frac{1}{T}\left(\int_0^{T/2} x(t)(dt) + \int_{T/2}^{T} x(t)(dt)\right)}$$

$$\boxed{a_0 = \frac{2}{T}\int_0^{T/2} x(t)(dt) \;,\; a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)(dt) \qquad b_n = 0}$$

as $a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)(dt)$

$$= \frac{1}{T}\left[\int_{-T/2}^{0} x(t)(dt) + \int_0^{T/2} x(t)(dt)\right]$$

$$= \frac{1}{T}\left[\int_0^{T/2} \underbrace{x(-t)}_{x(t)}(dt) + \int_0^{T/2} x(t)(dt)\right]$$

$$= \frac{2}{T}\int_0^{T/2} x(t)(dt)$$

WJMK.

- If imaginary signal → we'll consider odd part
- If (real) ———————— even —.

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos(n\omega_0 t)(dt)$$

$$= \frac{2}{T}\left[\int_{-T/2}^{0} x(t)\cos(n\omega_0 t)(dt) + \int_{0}^{T/2} x(t)\cos(n\omega_0 t)(dt)\right]$$

$$= \frac{2}{T}\left[\int_{0}^{T/2} (x(-t))\cos(-n\omega_0 t)(dt) + \int_{0}^{T/2} \text{''}\right]$$

$$= \frac{4}{T}\int_{0}^{T/2} x(t)\cos(n\omega_0 t)(dt)$$

If odd signal :- $a_0 = 0$, $a_n = 0$, $b_n = \frac{4}{T}\int_{0}^{T/2} x(t)\sin(n\omega_0 t)(dt)$

Proof:

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)(dt)$$

$$= \frac{1}{T}\left[\underbrace{\int_{-T/2}^{0} x(t)(dt)}_{\text{''}} + \int_{0}^{T/2} x(t)(dt)\right]$$

$$\underbrace{\int_{0}^{T/2} x(-t)(dt)}_{\text{''}}$$

$$-\int_{0}^{T/2} x(t)(dt)$$

(+) = 0

so, $\boxed{a_0 = 0}$.

@JMK

(i) Half-wave symmetry : (odd n only)

$$x(t) = -x\left(t \pm \frac{T}{2}\right)$$

$$a_0 = 0,$$

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)(dt)$$

$$b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin(n\omega_0 t)(dt)$$

• Even, odd, half-wave

$\underbrace{\text{Even, odd, half-wave}}$ are different names of symmetry

(i) Exponential Fourier Series :-

- • $x(t)$ is quarter-wave symmetry
- • has either even or odd symmetry
- • has half-wave symmetry

$$a_0 = 0,\ a_n = \frac{8}{T}\int_0^{T/2} x(t)\cos(n\omega_0 t)(dt)$$

$$b_n = \frac{8}{T}\int_0^{T/2} x(t)\sin(n\omega_0 t)(dt)$$

expressed as:-

$$x(t) = \sum_{n=(-\infty)}^{\infty} X_n e^{jn\omega_0 t}$$

$$X_n = \frac{1}{T}\int_0^{T} x(t)e^{-jn\omega_0 t}(dt)$$

WOMK

① R^n b/w Trigno and Exponential Fourier Series :—

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

$$= a_0 + \sum_{n=1}^{\infty} \left[a_n \left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right) + b_n \left(\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j} \right) \right]$$

$$= a_0 + \sum_{n=1}^{\infty} \left[a_n \left(\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2} \right) - j b_n \left(\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2} \right) \right]$$

$$= a_0 + \sum_{n=1}^{\infty} \left[\left(\frac{a_n - j b_n}{2} \right) e^{jn\omega_0 t} + \left(\frac{a_n + j b_n}{2} \right) e^{-jn\omega_0 t} \right]$$

$(j^2 = -1)$

as, $e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$e^{i\theta} + e^{-i\theta} = 2\cos\theta$

$\Rightarrow \cos\theta = \boxed{\dfrac{e^{i\theta} + e^{-i\theta}}{2}}$

substitution

$$X_n = \frac{a_n - j b_n}{2}$$

$$X_n = \frac{1}{2} \left(\frac{2}{T} \int_0^T x(t) \cos(n\omega_0 t)\,(dt) - j \frac{2}{T} \int_0^T x(t) \sin(n\omega_0 t)\,(dt) \right)$$

$$= \frac{1}{2} \times \frac{2}{T} \int_0^T [x(t) \cos(n\omega_0 t) - j \sin(n\omega_0 t))\,(dt)$$

$$= \frac{1}{T} \int_0^T x(t)\, e^{-jn\omega_0 t}\,(dt)$$

WOMK.

$$X_0 = \frac{1}{T}\int_0^T x(t)\,(dt) = a_0$$

$$x_n = \frac{a_n - j b_n}{2}.$$

$\boxed{a_{-n} = a_n}$ $\boxed{b_{-n} = -n}$

$$b_{(-n)} = -\frac{2}{T}\int_0^T (x(t))\sin(n\omega_0 t)\,(dt)$$

$$X_{-n} = \frac{a_n + j b_n}{2}.$$

$$x(t) = \sum_{n=-\infty}^{\infty} [X_n e^{jn\omega_0 t}]$$

$$a_n = 2\,\mathrm{Re}\{X_n\}$$
$$b_n = -2\,\mathrm{Im}\{X_n\}$$

$$a_n = X_n + X_{-n}$$
$$b_n = j(X_n - X_{-n})$$

Magnitude spectrum, $|X_n| = |X_{-n}| = \frac{\sqrt{a_n^2 + b_n^2}}{2} = \frac{C_n}{2}$ (from Polar form of Representation)

$$C_n = \begin{cases} 2|X_n| & ; n \geq 1 \\ X_0 & ; n = 0 \end{cases}$$

Magnitude spectrum is an even fn.

WDMK

Phase Spectrum $\angle X_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right) = \theta_n$

$$\angle X_{-n} = \tan^{-1}\left(\frac{-b_{-n}}{a_{-n}}\right)$$

$$\tan^{-1}\left(\frac{b_n}{a_n}\right) = -\tan^{-1}\left(-\frac{b_n}{a_n}\right) = -\theta_n = -\angle X_n$$

odd f^n.

DIRICHLET CONDNS: (If this satisfies, only then fourier series exist)

Over any period, $x(t)$ must be absolutely integrable :-

$$a_0 = X_0, \quad a_n = 2\,\mathrm{Re}\{X_n\}\frac{2}{T_0}, \quad b_n = 0$$

$$\int_0^T |x(t)|\,(dt) < \infty$$

$$|X_n| = \frac{1}{T}\left|\int_0^T x(t)\,e^{-jn\omega_0 t}\,(dt)\right| \leq \frac{1}{T}\int_0^T |x(t)\,e^{-jn\omega_0 t}|\,(dt)$$

$$= \frac{1}{T}\int_0^T |x(t)|\,(dt)$$

$$|X_n| < \infty.$$

$$x(t_0) = \frac{1}{2}\left[x(t_0^+) + x(t_0^-)\right]$$

DTFS: (Discrete Time fourier series) :-

$x(n) = x(n+N) \rightarrow$ discrete time signal is periodic with period (N).

WJMK

$$\phi_k(n) = e^{jk\omega_0 n} = e^{jk(2\pi/N)n}$$

↳ set of all discrete time complex exponential signals that are periodic with period 'N'.

$$\phi_k(n) = \phi_{(k+N)}(n)$$

$$X_k = \underbrace{|X_k|}_{\text{magnitude gives strength}} e^{(j\angle X_k)}$$ → phase part gives variations

① $$X(k+N) = \frac{1}{N}\sum_{n=\langle N\rangle} x(n)e^{-j(k+N)\omega_0 n}$$

$$= \frac{1}{N}\sum_{n=\langle N\rangle} x(n)e^{-jk\omega_0 n}e^{-j(2\pi/N)Nn}$$

$$= \frac{1}{N}\sum_{n=\langle N\rangle} x(n)e^{-jk\omega_0 n}e^{-j(2\pi)(n)}$$

$$= \frac{1}{N}\sum_{n=\langle N\rangle} x(n)e^{-jk\omega_0 n}$$

$$= X_k$$

so $X(k+N) = X_k$

↑ DTFS coefficients

② ↑ Periodic with period (N)

ωJMK

Properties of DTFS :

~~(1) Similarities b/w property~~

(1) **Linearity** : If $x(n)$ & $y(n) \rightarrow$ 2 periodic signals with period (N)

$$x(n) \longleftrightarrow X_K$$

$$y(n) \longleftrightarrow Y_K$$

$$z(n) = a\,x(n) + b\,y(n) \longleftrightarrow Z_K = aX_K + bY_K$$

~~$= aX_K$~~

(After Fourier series Transformation)

Proof:

$$Z_K = \frac{1}{N} \sum_{n=\langle N \rangle} z(n)\, e^{-jk\omega_0 n}$$

$$= \frac{1}{N} \sum_{n=\langle N \rangle} [a x(n) + b y(n)]\, e^{-jk\omega_0 n}$$

$$= a\left(\frac{1}{N}\right) \sum_{n=\langle N \rangle} x(n)\, e^{-jk\omega_0 n} + b\left(\frac{1}{N}\right) \sum_{n=\langle N \rangle} y(n)\, e^{-jk\omega_0 n}$$

$$\boxed{Z_K = aX_K + bY_K}$$

If condⁿ fulfilled → then Linear ✓
o/w ✗

(2) **Time shifting** : $x(n) \longleftrightarrow X_K$

$$y(n) = x(n - n_0) \longleftrightarrow Y_K = X_K e^{-jk\omega_0 n_0}$$

Input signal delayed by (n_0)

as may be delay in system

When shifted in time, then magnitude of fourier series coefficients remains unaltered.

$$|Y_K| = |X_K|$$

ωJMk

(3) Frequency shifting :- $x(n) \leftrightarrow X_k$

$$y(n) = e^{jMw_0 n}\, x(n) \leftrightarrow Y_k = X_{k-M}$$

Proof:

$$Y_k = \frac{1}{N} \sum_{n=N} y(n)\, e^{-jkw_0 n}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} y(n)\, e^{-jkw_0 n}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} e^{jMw_0 n}\, x(n)\, e^{-jkw_0 n}$$

$$= X_{k-M}.$$

(4) Time Reversal :- $x(n) \leftrightarrow X_k$

$$y(n) = x(-n) \leftrightarrow Y_k = X_{-k}$$

(5) Time Scaling :- (Dialation of SIGNAL)

Expansion & compression for the hair.

$X(2T) \rightarrow$ compression

$X(T/2) \rightarrow$ Expansion

(6) Periodic Convolution :-

$$x(n) \leftrightarrow X_k \; ; \; y(n) \leftrightarrow Y_k$$

then $$z(n) = x(n) \circledast y(n) = \sum_{r=\langle n \rangle}^{y(n) \leftrightarrow 1} x(r)\, y(n-r) \leftrightarrow$$

$$Z_k = N X_k Y_k$$

WJMK

* convolution in time transform to multiplication in frequency domain representation.

(7) multiplication :

(8) First difference :- $x(n) \leftrightarrow X_K$, $y(n) \leftrightarrow Y_K$

gives velocity

$$y(n) = x(n) - \underline{x(n-1)}$$

↑
1 delayed version.

$$\leftrightarrow Y_K = (1 - e^{-jk\omega}) X_K$$

Second difference → gives accn (or velocity)

(9) RUNNING Sum OR Accumulation :- (to see which part of signal very imp)
↳ or want to see signal at particular instant only!

$$y(n) = \sum_{K=-\infty}^{n} x(K)$$

(10) Conjugation & Conjugate Symmetry :-

$x(n) \leftrightarrow X_K$ $y(n) \leftrightarrow Y_K$

$$y(n) = x^*(n) \leftrightarrow Y_K = X^*_{-K}$$

WOMK

V. imp ☆☆

(11) Parseval's R^n or Th^m:

$$x(n) \leftrightarrow X_k$$

then, $$\underbrace{\frac{1}{N}\sum_{n=\langle N\rangle} |x(n)|^2}_{\text{Time Domain}} = \underbrace{\sum_{n=\langle N\rangle} |X_k|^2}_{\text{frequency domain}}$$

Time Domain → May be CT or DT

UJJMK

6 April, 2023
Thursday

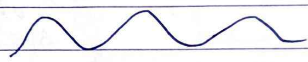
Tutorials :- 3

(i) Fourier Series :-

↳ Infinite

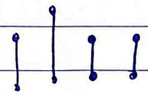
↳ mathematical tool to approximate any periodic function.

→ has value at any time



(1) CTFS (Continuous time Fourier series)

DTFS (Discrete " " ") → har time pe value nhi milegi



(2) (i) Exponential

(ii) Sinusoidal

$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$\boxed{j = i = \gamma}$

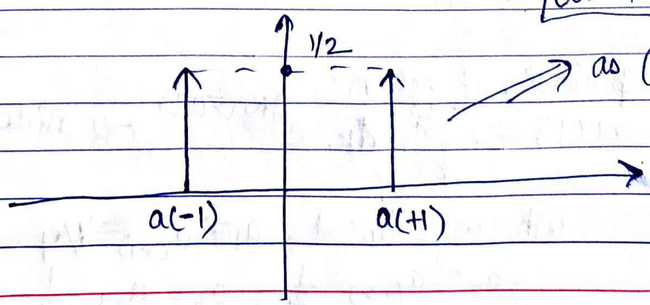
how to find components ?? of $\cos(\omega t)$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$$\cos(\omega t) = \underbrace{\frac{1}{2}}_{a(+1)} e^{j\omega t} + \underbrace{\frac{1}{2}}_{a(-1)} e^{-j\omega t}$$

$\boxed{\cos \to \text{even}}$

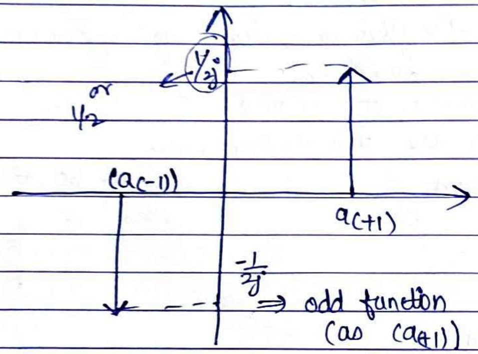


as (a(+1)) and (a(-1)) par same value aa hai!

WJMK

$$\sin(\omega t) = \frac{1}{2j} e^{j\omega t} + \left(-\frac{1}{2j}\right) e^{-j\omega t}$$

$a_{(+1)}$ $\qquad$ $a_{(-1)}$ $\qquad$ $\left|\frac{1}{2j}\right| = \frac{1}{2}$??



⇒ odd function (as $(a_{(+1)})$ and $(a_{(-1)})$ pair opposite values)

Q. $\sin(2x) + \sin(4x) + \sin(8x)$

↳ find its components in exponential form??

$$= \frac{e^{j(2x)} - e^{-j(2x)}}{2j} + \frac{e^{j(4x)} - e^{-j(4x)}}{2j} + \frac{e^{j(8x)} - e^{-j(8x)}}{2j}$$

$$= \cancel{\frac{1}{2j}\left(e^{j(2x)} + e^{j(4x)} + e^{j(8x)}\right) - \frac{1}{2j}\left(e^{-j(2x)} + e^{-j(4x)} + e^{-j(8x)}\right)}$$

don't write like this!

★ $x(t)$ → periodic function

$$x(t) = \sum_{k=-3}^{+3} a_k e^{jk(2\pi t)}$$ → Find out sinusoidal form?

Where, $a_0 = 1$; $a_1 = a_{(-1)} = 1/4$

$a_2 = a_{(-2)} = \frac{1}{2}$, $a_3 = a_{-3} = \frac{1}{3}$.

WJMK

$$\Rightarrow x(t) = a_0 + a_{-3} e^{-j(3)(2\pi t)} + a_{-2} e^{-j(2)(2\pi t)} + a_{-1} e^{-j(1)(2\pi t)} + a_3 e^{j(3)(2\pi t)} + a_2 e^{j(2)(2\pi t)} + a_1 e^{j(1)(2\pi t)}$$

$$= a_0 + (a_3)\left(e^{-j(3)(2\pi t)} + e^{j(3)(2\pi t)}\right) + a_2\left(e^{-j(2)(2\pi t)} + e^{j(2\pi)(2t)}\right) + a_1\left(e^{j(2\pi t)} + e^{-j(2\pi t)}\right)$$

$$= 1 + \frac{1}{3}\left(2\cos(6\pi t)\right) + \frac{1}{2}\left(2\cos(4\pi t)\right) + \frac{1}{4}\left(2\cos(2\pi t)\right)$$

#2 For the continuous time periodic signal;

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right).$$

Determine fundamental frequency & fourier coeff., such that $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$.

A. $x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$

$T_1 = \frac{2\pi}{(2\pi/3)} = 3$ sec $\qquad$ $T_2 = \frac{2\pi}{5\pi/3} = \frac{6}{5}$ sec

$\therefore \frac{T_1}{T_2} = \frac{3}{(6/5)} = \frac{15}{6} = \frac{5}{2}$ sec

$\Rightarrow 2T_1 = 5T_2 \Rightarrow T = 2T_1 = 5T_2 = 6$ sec.

Fundamental Period

fundamental frequency $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3}$

WOMK

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$$

$$= 2 + \frac{1}{2}\left[e^{j\left(\frac{2\pi}{3}\right)t} + e^{-j\left(\frac{2\pi}{3}\right)t}\right] +$$

$$4 \times \frac{1}{2j}\left[e^{j\left(\frac{5\pi}{3}\right)t} - e^{-j\left(\frac{5\pi}{3}\right)t}\right]$$

$$= -2j\, e^{j\left(\frac{5\pi}{3}\right)t} + 0.5\, e^{j\left(\frac{2\pi}{3}\right)t} + 2 + 0.5\left(e^{-j\left(\frac{2\pi}{3}\right)t}\right) +$$

$$2j\, e^{-j\left(\frac{5\pi}{3}\right)t}$$

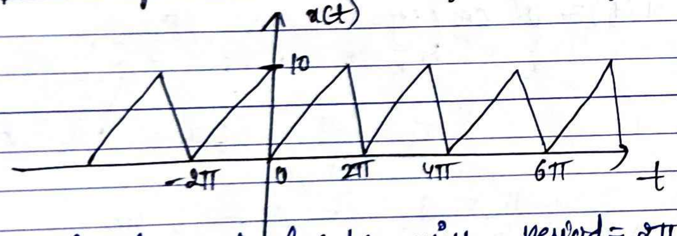
$$= -2j\, e^{-j(-5)\frac{\pi}{3}t} + 0.5\, e^{-j(-2)\left(\frac{\pi}{3}\right)t} + 2 +$$

$$0.5\, e^{-j(2)\left(\frac{\pi}{3}\right)t} + 2j\, e^{-5j\left(\frac{\pi}{3}t\right)}$$

$$\Rightarrow c_{-5} = -2j,\quad c_{-2} = 0.5,\quad c_0 = 2,\quad c_2 = 0.5,$$

$$c_5 = 2j$$

Q Find Trigno Fourier Series for waveform:



A Waveform is Periodic with period $= 2\pi$.

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$x(t) = \left(\frac{10}{2\pi}\right)(t);\quad 0 < t < 2\pi$$

WOMK

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right]$$

Where,

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi} \frac{10}{2\pi}t\,dt = \frac{10}{(2\pi)^2}\left[\frac{t^2}{2}\right]_0^{2\pi}$$

$$= 5$$

$$a_n = \frac{2}{T}\int_0^T (x(t))\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi} \frac{10}{2\pi}(t)\cos(nt)\,dt$$

$$= \frac{10 \times 2}{(2\pi)^2}\left[\frac{t}{n}\sin(nt) + \frac{1}{n^2}\cos(nt)\right]_0^{2\pi}$$

$$= \frac{10}{2\pi^2 n^2}\left[\cos(2n\pi) - \cos(0)\right] = 0$$

and $b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega_0 t)\,dt$

$$= \frac{2}{2\pi}\int_0^{2\pi} \frac{10t}{2\pi}\sin(nt)\,dt$$

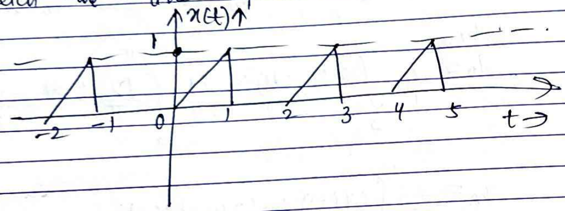
$$= \frac{10}{2\pi^2}\left[-\frac{t}{n}\cos(nt) + \frac{1}{n^2}\sin(nt)\right]_0^{2\pi} = -\frac{10}{n\pi}$$

$$\therefore x(t) = 5 - \sum_{n=1}^{\infty}\left[0 + \frac{10}{n\pi}\sin(nt)\right] = 5 - \sum_{n=1}^{\infty}\frac{10}{n\pi}\sin(nt)$$

$$= 5 - \frac{10}{\pi}\sin(t) - \frac{10}{2\pi}\sin(2t) - \frac{10}{3\pi}\sin(3t) - \dots$$

work

Q. Find Trigno fourier series for waveform and sketch the line spectrum:-



A: $T=2$, $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$$x(t) = \begin{cases} t ; & 0 \le t < 1 \\ 0 ; & 1 < t < 2 \end{cases}$$

Since wave is neither even nor odd, the series will contain both sine & cosine terms.

$$a_0 = \frac{1}{T}\int_0^T x(t)\,(dt) = \frac{1}{2}\left(\int_0^1 t\,(dt) + \int_1^2 0\,dt\right)$$

$$= \frac{1}{2}\left[\frac{t^2}{2}\right]_0^1 = \frac{1}{4}$$

$$a_n = \frac{2}{T}\int_0^T x(t)\cos(n\omega_0 t)\,(dt) = \frac{2}{2}\left[\int_0^1 t\cos(n\pi t)\,(dt) + \int_1^2 (0)\cos(n\pi t)\,(dt)\right]$$

$$= \int_0^1 t\cos(n\pi t)\,(dt)$$

$$\Rightarrow \underbrace{t\,\frac{\sin(n\pi t)}{n\pi}\Big|_0^1}_{0} + \frac{\cos(n\pi t)}{n\pi^2}\Big|_0^1 = \frac{1}{(n\pi)^2}\left[\cos(n\pi) - 1\right]$$

work

$$\therefore a_n = \begin{cases} \frac{-2}{(n\pi)^2} ; & n = 1, 3, 5, \dots \\ 0 ; & n = 2, 4, 6, \dots \end{cases}$$

$$b_n = \frac{2}{T}\int_0^T x(t)\sin(n\omega_0 t)\,(dt)$$

$$= \frac{2}{2}\left(\int_0^1 t\sin(n\pi t)\,(dt) + \int_1^2 0\sin(n\pi t)\,(dt)\right)$$

$$= \int_0^1 t\sin(n\pi t)\,(dt)$$

$$\Rightarrow \frac{-t\cos(n\pi t)}{n\pi}\Big|_0^1 + \frac{\sin(n\pi t)}{(n\pi)^2}\Big|_0^1 = \frac{-1}{n\pi}\cos(n\pi)$$

$$b_n = \begin{cases} \frac{1}{n\pi} ; & n = 1, 3, 5, \dots \\ \frac{-1}{n\pi} ; & n = 2, 4, 6, \dots \end{cases}$$

$$x(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)\right]$$

$$= \frac{1}{4} - \frac{2}{\pi^2}\cos(\pi t) - \frac{2}{(3\pi)^2}\cos(3\pi t) - \frac{2}{(5\pi)^2}\cos(5\pi t) + \dots$$

$$+ \frac{1}{\pi}\sin(\pi t) - \frac{1}{2\pi}\sin(2\pi t) + \frac{1}{3\pi}\sin(3\pi t) - \dots$$

∴ The even harmonic amplitudes are given directly by $|b_n|$. Since there are no harmonic cosine terms. ∴ However, odd harmonic amplitudes must be computed using $c_n = \sqrt{a_n^2 + b_n^2}$, $n \geq 1$ and $c_0 = a_0$.

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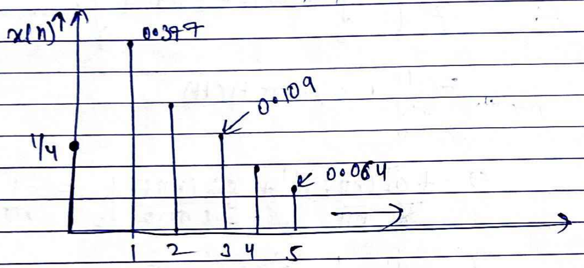
$$c_1 = \sqrt{\left(\frac{2}{\pi^2}\right)^2 + \left(\frac{1}{\pi}\right)^2}$$

$$= 0.377$$

$$c_3 = 0.109$$

$$c_5 = 0.064$$

$$c_0 = \frac{1}{4}$$



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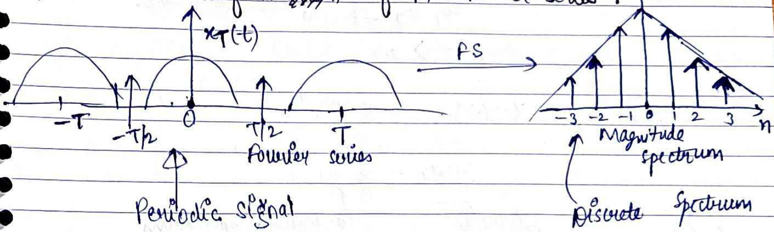
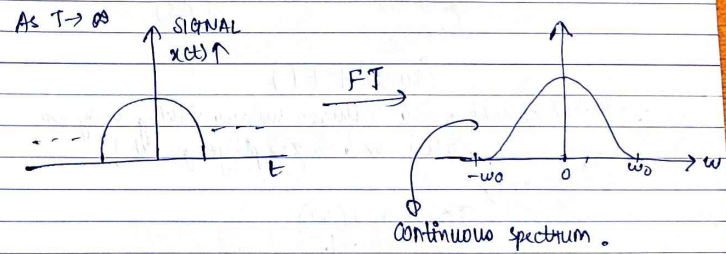
12/4/2023
wednesday

LECTURE:-4

* Fourier Transformation (FT) from Fourier series (FS) :-





(1) As we know, the exponential fourier series of periodic signals $x_T(t)$ is :

$$x_T(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad ; \quad t_0 < t < t_0+T$$

$$\boxed{T = \frac{2\pi}{\omega_0}}$$

Where,

$$C_n = \frac{1}{T}\int_{t_0}^{t_0+T} x_T(t)\, e^{-jn\omega_0 t}\,(dt)$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} x_T(t)\, e^{-jn\omega_0 t}\,(dt)$$

→ to get area under curve

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$\Rightarrow T C_n = \int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t} (dt)$

As $T \to \infty$

$\omega_0 = \frac{2\pi}{T}$

then $\omega_0 \to 0$

so, $\omega_0 \to d\omega \to$ very small value.

$n\omega_0 \to \omega$

$x_T(t) \to x(t)$

$n\omega_0 \to \omega$

Discrete Spectrum (DS) — Continuous Spectrum (CS)

(so Just FT)

→ Instead of taking all values, we are taking only one value and simplifying it!

and,

$TC_n \to x(\omega)$

$TC_n = \int_{-T/2}^{T/2} x_T(t)\, e^{-jn\omega_0 t} (dt)$

as $T \to \infty$, $TC_n \to x(\omega)$

$$\boxed{x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} (dt)} \quad \left\{ \text{as } x_T(t) \to x(t),\ n\omega_0 \to \omega \right\}$$

FT of $x(t)$

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so, $x(t) \xleftrightarrow{FT} x(\omega)$

so, $x(t)$ can be written as $x(\omega)$ if applying FT (fourier Transform)

$* \quad x_T(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \qquad t_0 < t < t_0 + T$

$\left.\begin{array}{l} TC_n \to x(\omega) \\ n\omega_0 \to \omega \end{array}\right\}$ as $T \to \infty$

$C_n = \frac{x(\omega)}{T}$ — (*).

using (*) $\Rightarrow$

$x_T(t) = \sum_{n=-\infty}^{\infty} \left(\frac{x(\omega)}{T}\right) (e^{jn\omega_0 t})$

and $\omega_0 = \frac{2\pi}{T}$

$\Rightarrow T = \frac{2\pi}{\omega_0}$ — (1)

use (1)

$x_T(t) = \sum_{n=-\infty}^{\infty} \left(\frac{x(\omega)}{2\pi}\right) (\omega_0) (e^{jn\omega_0 t})$

Now as $T \to \infty$, → Aperiodic or non-periodic signal

then $x_T(t) \to x(t)$ and $n\omega_0 \to \omega$ ↳ CS

Periodic signal — and $\omega_0 \to d\omega$ — DS

Summation becomes integration

so, $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega)\, e^{j\omega t} (d\omega)$ ← Inverse Fourier Transform

[aaj ki class tak Q1]

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☆ So →

$$\boxed{x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, (dt)}$$ → Forward Fourier Transform OR Analysis equations

and,

$$\boxed{x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega)\, e^{j\omega t}\, (d\omega)}$$

Backward (or Inverse) Fourier Transform OR synthesis equation

Inverse Fourier transform of $x(\omega)$ ← $\boxed{x(t) \xleftrightarrow{FT} x(\omega)}$ → Fourier Transform or Fourier integral of $x(t)$

Fourier Transform Pair

$$\boxed{\begin{aligned} x(\omega) &= F(x(t)) \\ x(t) &= F^{-1}(x(\omega)) \end{aligned}}$$

$x(\omega)$ complex f^n of 'ω'

$x(\omega) = X_R(\omega) + j X_I(\omega)$ — real part; imaginary part

(i) Continuous Time Fourier Transform:

Drawbacks of FS? why need FT?

only applicable to periodic signals.

- There are some naturally produced signal such as Aperiodic or non-periodic, which can't represent using Fourier Series. (mostly signals we get are not periodic)

To overcome this, Fourier developed mathematical model to transform signals b/w time (or spatial) domain to frequency domain & VICE-VERSA, which is called 'FT'.

FS → has to follow superpositional th^m, where bounded input & output are req.

(has to follow stability and causality)

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FT → realistic!

Just look into signal, and will analysis only 1 frame at a time

→ we ~~want~~ won't break into small parts to called $a_0, a_1, \ldots$ etc. and magnitude spectrum or phase spectrum dekhting

Jaise bhi in FS, FT mein abhi hum pura hi area dekh rahe hai !!

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$$X_n = \frac{1}{T} \int_{-T/2}^{T/2} \tilde{x}(t)\, e^{-jn\omega_0 t} (dt)$$

as, $T \to \infty$, $\omega_0 = \frac{2\pi}{T} \longrightarrow d\omega$,

$$\frac{1}{T} \to \frac{d\omega}{2\pi}$$

and, $x(\omega) = |x(\omega)|\, e^{j\angle x(\omega)}$

$$|x(\omega)| = \sqrt{X_R(\omega)^2 + X_I(\omega)^2}$$ ← magnitude

$$\angle x(\omega) = \tan^{-1}\left[\frac{X_I(\omega)}{X_R(\omega)}\right]$$ ← angle (or phase) of $x(\omega)$

(i) $|X(\omega)|$ plotted against ω → called magnitude spectrum of $x(t)$

$\angle X(\omega)$ ———— ω → phase

① FT → convert from time domain to frequency domain.

so helping only in change from 1 domain to another domain

→ same as Laplace transform.

* Properties of Continuous time Fourier Transform :- (CTFT)

useful to develop conceptual insights into transform and into r'ship b/w time-domain and frequency-domain descriptions of SIGNAL.

$x(t) \rightarrow$ time domain signal

① Linearity: $x_1(t) \rightarrow X_1(\omega)$ and $x_2(t) \rightarrow X_2(\omega)$

then,

$\underbrace{a x_1(t) + b x_2(t)}_{\text{linear combination of input signals}} \rightarrow a X_1(\omega) + b X_2(\omega)$

$$\mathcal{F}[a x_1(t) + b x_2(t)] = \int_{-\infty}^{\infty} [a x_1(t) + b x_2(t)] e^{-j\omega t} (dt)$$

applying Fourier transform

$$= \underbrace{a X_1(\omega) + b X_2(\omega)}_{\text{linear comb}^n \text{ of output signals}}$$

so amplitude neither ↓s nor ↑s

• Hmesha koi bhi signal khan bhi bhejna, pehle x ℓt by unit step signal (means convolve ℓt), then FT lgaayo, get signal in frequency domain

② Time shifting:

$$\mathcal{F}[x(t-t_0)] = \int_{-\infty}^{\infty} x(t-t_0) e^{-j\omega t} (dt)$$

$\tau = t - t_0$

$d\tau = dt$

$\tau \rightarrow -\infty$ as $t \rightarrow -\infty$

so, $\mathcal{F}(x(t-t_0)) = X(\omega) e^{-j\omega t_0}$

③ Frequency shifting: $x(t) \rightarrow X(\omega)$

$x(t) e^{j\omega_0 t} \leftrightarrow X(\omega - \omega_0)$

CORRECTION in slide, $\omega \rightarrow \omega_0$, $t_0 \rightarrow t$ ★

$$\mathcal{F}[x(t) e^{j\omega_0 t}] = \int_{-\infty}^{\infty} [x(t) e^{j\omega_0 t}] e^{-j\omega t} (dt)$$

$$\mathcal{F}[x(t) e^{j\omega_0 t}] = \int_{-\infty}^{\infty} x(t) e^{-j(\omega-\omega_0)t} (dt)$$

$$= X(\omega - \omega_0)$$

④ Time & Frequency shifting! —

is mein (X) means (÷) is mein

$x(t) \rightarrow X(\omega)$

$x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

$$\mathcal{F}[x(at)] = \int_{-\infty}^{\infty} x(at) e^{-j\omega t} (dt)$$

$\tau = at$

$$\Rightarrow \mathcal{F}[x(at)] = \frac{1}{a} \int_{-\infty}^{\infty} x(\tau) e^{-j\left(\frac{\omega}{a}\right)\tau} (d\tau)$$

$$= \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

WOMK

(5) Time Reversal :- → eql to conjugation in frequency domain

(6) Convolution Property :-

$$x_1(t) * x_2(t) \Leftrightarrow X_2(\omega) X_1(\omega)$$

convolution in time domain

multiplication in frequency domain

(7) multiplication Property

(8) Duality Property

(1) PALEY WELLEN criteria

(2) ?

→ If above criteria not followed, then signal won't exist!

$x(t) \leftrightarrow X(\omega)$

$X(t) \leftrightarrow 2\pi x(-\omega)$

as: $2\pi x(t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} (d\omega)$

replace t with '-t'

$\Rightarrow 2\pi x(-t) = \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} (d\omega)$

Interchange t and ω

$\Rightarrow 2\pi x(-\omega) = \int_{-\infty}^{\infty} X(t) e^{-j\omega t} (dt)$

★ Imp

(9) Parseval's relation :- $x(t) \leftrightarrow X(\omega)$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 (dt) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 (d\omega)$$

states that signal energies of an energy signal and its Fourier transform are equal

WOMK

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 (dt) = \int_{-\infty}^{\infty} x(t) \cdot x^*(t) (dt)$$

$$= \int_{-\infty}^{\infty} x(t) \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega \right) (dt)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(\omega) \left(\int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \right) (d\omega)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X^*(\omega) \cdot X(\omega) (d\omega)$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 (d\omega)$$

* DTFT (Discrete - time Fourier transform)

why we need?

Main objective : Angular Frequency!

$\boxed{K = \langle N \rangle \Downarrow \quad K \text{ varies from } -N \text{ to } N}$

* FT representation of aperiodic Discrete - Time Signals :-

$$x(n) = \sum_{K=k_0}^{k_0+N-1} \left(\sum_{m=-N/2}^{N/2-1} x(m) e^{-jk\omega_0 m} \right) e^{jk\omega_0 n} \left(\frac{\omega_0}{2\pi}\right)$$

$\forall$ N → even

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$$x(n) = \sum_{k=k_0}^{k_0+N-1} \left(\sum_{m=-\left(\frac{N-1}{2}\right)}^{\frac{N-1}{2}} x(m)\, e^{-jk\omega_0 m} \right) e^{jk\omega_0 n} \left(\frac{\omega_0}{2\pi}\right)$$

for N → odd

~~$X(e^{j\omega}) = \sum$~~

(*) FT representation of Aperiodic Discrete-Time Signals

(*)

$$X_R(e^{j\omega}) = \frac{1}{2}\left[X(e^{j\omega}) + X^*(e^{j\omega})\right]$$

$$X_I(e^{j\omega}) = \frac{1}{2j}\left[\ \text{''} - \text{''}\ \right]$$

complex conjugate of $X(e^{j\omega})$

$$\angle X e^{(-j\omega)} = -\angle X e^{(j\omega)}$$

ϕ

so phase spectrum ($\angle X(e^{j\omega})$) → odd fn.

~~(*) Properties of DTFT:-~~

① Periodicity of DTFT:-

$$X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$$

→ period fn in ω with period (2π).

Langregian's method or Range-kutta method for convergence

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$$X(e^{j(\omega+2\pi k)}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(\omega+2\pi k)n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(2\pi k)n}\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, (e^{-j\omega n})$$

$$= X(e^{j\omega})$$

$$= X(e^{j\omega})$$

→ so dusre cycle mein gye, then utna hi info paayenge, jitna pura 1st cycle mein paaya!

② Convergence of DTFT:

$$X_K(e^{j\omega}) = \sum_{n=-K}^{K}$$

(means 1 level mein jaane k baad, no much changes)

③ Gibb's Phenomenon:-

$$X(e^{j\omega}) = \begin{cases} 1 \ ; & |\omega| \le \omega_c \\ 0 \ ; & \omega_c < |\omega| \le \pi \end{cases}$$

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$$x(n) = \frac{1}{2\pi} \frac{e^{j\omega n}}{jn} \Bigg|_{-\omega c}^{\omega c}$$

$$= \frac{\sin(\omega_c n)}{n\pi}, \; n \neq 0$$

Properties of DTFT:-

1. Linearity property
2. Frequency shifting
3. Time Reversal
4. " Expansion
5. Differentiation in Time domain
6. " " frequency "
7. Convolution property
8. Accumulation "
9. Multiplication or Modulation or Windowing property.
10. Conjugation and Conjugate property
11. Parseval's Relation

* Signal Transmission through Linear Time - Invariant Systems

* Phase Delay and Group Delay

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13|4|2023

Thursday

(POC)

Tutorial :-

Q what is fourier transform?

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\,(e^{-j\omega t})\,(dt)$$ [Remember].!!

Q

$$\boxed{x(t) = e^{-at}(u(t))}$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-at}(u(t))\,e^{-j\omega t}\,\cancel{(d\omega)}\,(dt)$$

$$= \int_{0}^{\infty} e^{-at}\,e^{-j\omega t}\,(dt)$$

($u(t) \to$ unit step function)

$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\,(dt)$$

$$= \frac{1}{a+j\omega}$$

Q

$$\boxed{x(t) = e^{-at}(u(-t))}$$

$$x(-t) = \cancel{e^{at}\,u(t)} \quad X(-\omega)$$

$$\cancel{X(\omega) =}\ X(-\omega) = \frac{1}{a-j\omega}$$

Q

$$\boxed{x(t) = e^{-a|t|}}$$

$$= e^{-at} \quad ; \ t>0$$

$$= e^{at} \quad ; \ t<0$$

$$X(\omega) = \int_{-\infty}^{0} e^{at}\,e^{-j\omega t}\,(dt) + \int_{0}^{\infty} e^{-at}\,e^{-j\omega t}\,(dt)$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\,(dt) + \int_{0}^{\infty} e^{-(a+j\omega)t}\,(dt)$$

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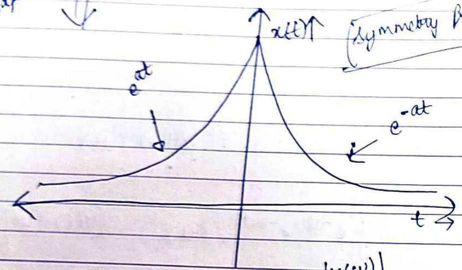
$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

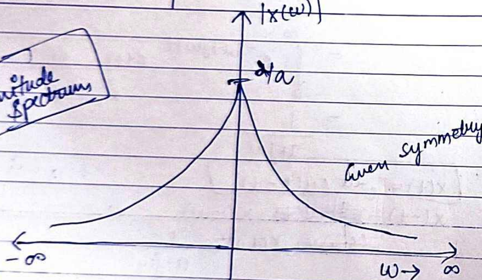
$$= \frac{2a}{a^2+\omega^2}$$

graphically ⇓

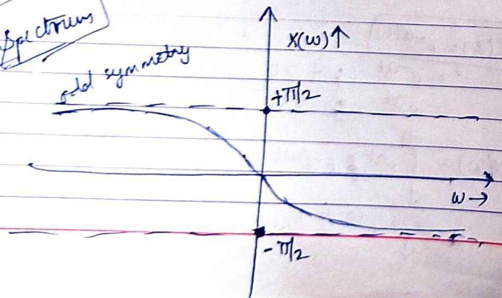
Symmetry Property



magnitude spectrum



Phase Spectrum



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As, $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} (dt)$

$$= \int_{-\infty}^{0} x(t)\, e^{-j\omega t} (dt) + \int_{0}^{\infty} x(t)\, e^{-j\omega t} (dt)$$

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t} (dt) + \int_{0}^{\infty} e^{-at} e^{-j\omega t} (dt)$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t} (dt) + \int_{0}^{\infty} e^{-(a+j\omega)t} (dt)$$

$$= \left. \frac{e^{(a-j\omega)t}}{a-j\omega} \right|_{-\infty}^{0} + \left. \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right|_{0}^{\infty}$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

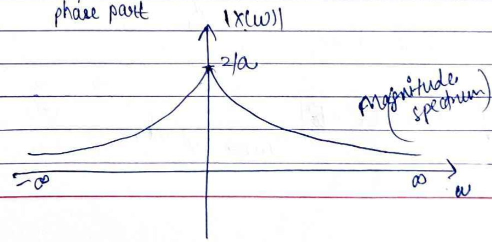
$$= \frac{2a}{a^2+\omega^2}$$

so, $|X(\omega)| = \frac{2a}{a^2+\omega^2} = \sqrt{(Re(X(\omega)))^2 + Im(X(\omega))^2}$

↑ magnitude part

$\angle X(\omega) = \tan^{-1}\left(\frac{Im(X(\omega))}{Real(X(\omega))}\right) = 0$ (as $Im(X(\omega)) = 0$)

↑ phase part



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Q

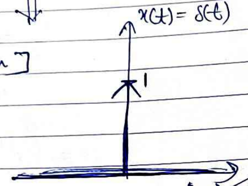
(Impulse Response)

$\boxed{x(t) = \delta(t)}$

as, $\delta(t) = \begin{cases} 1 ; \text{for } t = 0 \text{ or } \infty \\ 0 ; \text{for } t \neq 0 \end{cases}$

As, $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} (dt)$

[formula from fourier transform]



$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t} (dt)$

as $\delta(t) \neq 0$ only at $t = 0$

so don't say $\frac{e^{-j\omega t}}{-j\omega}$ at $t=0$

$= e^{-j\omega t} \cancel{\int} (\text{at } t = 0)$ [so koi $\int$ nhi ho rha hai]

[so $\delta(t)$ takes number as it is, no change hoga, no $\int$ required]

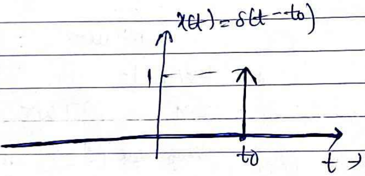
$= e^{-0}$

$= 1$

Q

$\boxed{x(t) = \delta(t - t_0)}$

$X(\omega) = \int_{-\infty}^{\infty} \delta(t - t_0)\, e^{-j\omega t} (dt)$



$= e^{-j\omega t}$ at $t = t_0$ $= \int_{t_0} e^{-j\omega t} (dt) = e^{-j\omega t}\Big|_{t = t_0}$

$= e^{-j\omega t_0}$

Q

$\boxed{x(t) = \delta(t + t_0)}$ $X(\omega) = e^{j\omega t_0}$

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ॐ

Q. Inverse Fourier Transform of $X(\omega) = \delta(\omega) \Rightarrow$

$x(t) \xleftrightarrow{FT} X(\omega)$

$X(\omega) = \mathcal{F}(x(t))$

Fourier transform

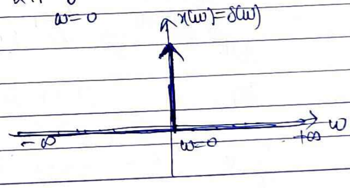
$x(t) = \mathcal{F}^{-1}(X(\omega))$

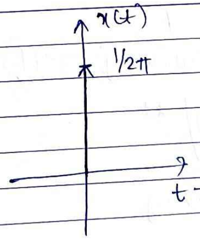
Synthesis Eqn $\Rightarrow$ $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t} (d\omega)$

as $X(\omega) = \delta(\omega)$

so, $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta(\omega)\, e^{j\omega t} (d\omega)$

$= \frac{1}{2\pi} \int_{\omega = 0} e^{j\omega t} (d\omega) = \frac{1}{2\pi} e^{j\omega t}\Big|_{\omega = 0} = \frac{1}{2\pi}$





$\frac{1}{2\pi} \longleftrightarrow \delta(\omega)$

$1 \longleftrightarrow 2\pi\,\delta(\omega)$ then, frequency Domain

if time domain

WJMK

Q. find the inverse FT (fourier transform) of $X(\omega) = \delta(\omega - \omega_0)$

(exist only at $\omega = \omega_0$)

A. $$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\,(d\omega)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega-\omega_0)\, e^{j\omega t}\,(d\omega)$$

$$= \frac{1}{2\pi}\int_{\omega=\omega_0} \delta(\omega-\omega_0)\, e^{j\omega t}\,(d\omega)$$

$$= \frac{1}{2\pi}\left[e^{j\omega t}\Big|_{\omega=\omega_0}\right]$$

$$= \frac{1}{2\pi}\left(e^{j\omega_0 t}\right)$$

so, $\mathcal{F}\left(\frac{1}{2\pi} e^{j\omega_0 t}\right) = \delta(\omega-\omega_0)$

$\frac{1}{2\pi} e^{j\omega_0 t} = \mathcal{F}^{-1}(\delta(\omega-\omega_0))$

so,

$$\frac{1}{2\pi} e^{j\omega_0 t} \xleftrightarrow{FT} \delta(\omega-\omega_0)$$

(Remember) imp ☆ $e^{j\omega_0 t} \xleftrightarrow{FT} 2\pi[\delta(\omega-\omega_0)]$ — ①

$$\boxed{\mathcal{F}[e^{j\omega_0 t}] = 2\pi\delta(\omega-\omega_0)} \;**$$

Similarly ⇒ $\mathcal{F}(e^{-j\omega_0 t}) = 2\pi\delta(\omega+\omega_0)$

so, ☆ imp $e^{-j\omega_0 t} \xleftrightarrow{FT} 2\pi\delta(\omega+\omega_0)$ — ②

(Remember)

WJMK

Q. find FT of $x(t) = \cos(\omega_0 t)$

→ don't write 'ω' here!! (WJMK) as ω → variable, ω_0 → constant

or, ω → continuous, ω_0 → discrete ???

$$= \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \quad \text{(Euler's formula)}$$

$$X(\omega) = \underbrace{\int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,(dt)}_{\mathcal{F}(x(t))}$$

$$= \int_{-\infty}^{\infty} \frac{1}{2} e^{j\omega_0 t}\cdot e^{-j\omega t}\,(dt) + \int_{-\infty}^{\infty} \frac{1}{2} e^{-j\omega_0 t}\cdot e^{-j\omega t}\,(dt)$$

$$= \frac{1}{2}\left[\mathcal{F}[e^{j\omega_0 t}] + \mathcal{F}[e^{-j\omega_0 t}]\right]$$

$$= \frac{1}{2}\left[2\pi\,\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right] \quad \text{(using ②)}$$

$$= \pi\left[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right]$$

Q. find FT of $x(t) = \sin(\omega_0 t)$

$$= \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,(dt)$$

$$= \frac{1}{2j}\left[\mathcal{F}[e^{j\omega_0 t}] - \mathcal{F}[e^{-j\omega_0 t}]\right]$$

$$= \frac{1}{2j}\left[2\pi\,\delta(\omega-\omega_0) - 2\pi\,\delta(\omega+\omega_0)\right] \quad \text{(using ① \& ②)}$$

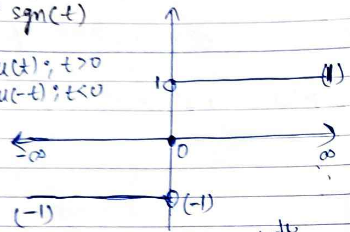
$$= \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

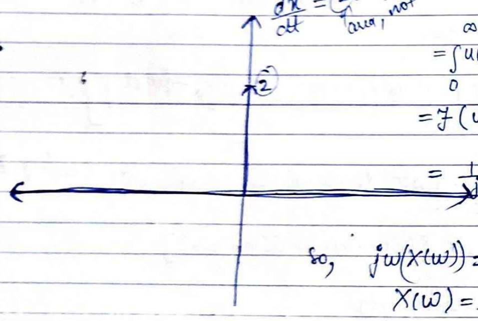
WJMK

Q $x(t) = \text{sgn}(t)$ → signum function of 't'

$x(t) = \begin{cases} u(t); & t>0 \\ u(-t); & t<0 \end{cases}$

(so sgn is unit step for $t \neq 0$)





$\frac{dx}{dt} = (2\delta(t))$ area, not magnitude

$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} (dt)$

$= \int_0^{\infty} u(t) e^{-j\omega t}(dt) + \int_{-\infty}^{0} u(-t) e^{-j\omega t}(dt)$

$= \mathcal{F}(u(t)) + \mathcal{F}(u(-t))$

$= \frac{1}{j\omega} + \delta\omega + \frac{1}{j\omega} - \delta(\omega)$

$= \frac{2}{j\omega}$

So, $j\omega(X(\omega)) = 2$

$X(\omega) = \frac{2}{j\omega}$

Q $H(\omega) = \frac{2\cos(\omega)\sin(2\omega)}{\omega}$

$h(0) = ?$

$= \frac{\sin(3\omega) + \sin(\omega)}{\omega}$

$= \frac{3\sin(3\omega)}{3\omega} + \frac{\sin(\omega)}{\omega}$

$\boxed{\mathcal{F}[u(t)] = \frac{1}{j\omega} + \delta(\omega)}$ (To remember) WJMK

$\boxed{\mathcal{F}[u(-t)] = \frac{1}{j\omega} - \delta(\omega)}$ ***

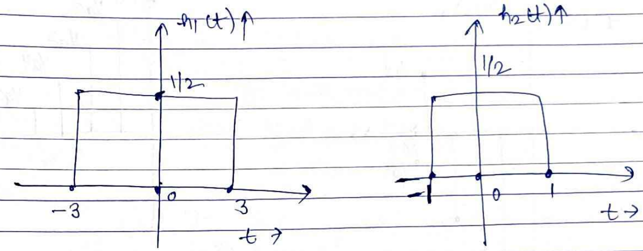
Ans. $= 3\,Sa(3\omega) + Sa(\omega)$ (Sa → sinc fn)

$= H_1(\omega) + H_2(\omega)$

$= h_1(0) + h_2(0)$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$ Ans



$\boxed{\text{Sinc} \to \text{function}, \quad \text{sinc} = \frac{\sin\theta}{\theta}}$

Sa → direction $\quad Sa(\theta) = \frac{\sin\theta}{\theta}$

Q Find FT of $x(n) = \delta(n)$ ← (Impulse function)

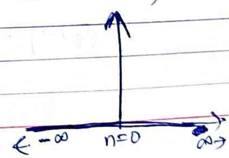
$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$ (Discrete)

as $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} (dt)$ (continuous)

$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \delta(n) e^{-j\omega n}$

$= e^{-j\omega n}\big|_{n=0}$

$= 1$

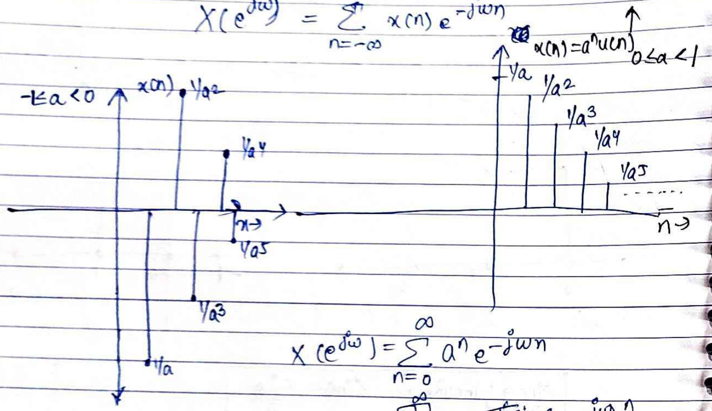


WJMK

So, $\delta(n) \xleftrightarrow{FT} 1$

Q. Find FT of $x(n) = a^n u(n)$; $|a|<1$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$



$$X(e^{j\omega}) = \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \cancel{e^{-j\omega}}\, (ae^{-j\omega})^n$$

$$= \frac{\text{First term}}{1-\text{difference term}} \quad [GP]$$

$$= \frac{1}{1-ae^{-j\omega}}$$

$$a^n u(n) \xleftrightarrow{FT} \frac{1}{1-ae^{-j\omega}} \; , |a|<1$$

$$X(e^{j\omega}) = \frac{1}{1-ae^{-j\omega}} * \frac{1-ae^{j\omega}}{1-ae^{j\omega}} = \frac{1-ae^{j\omega}}{(1-ae^{-j\omega})(1-ae^{j\omega})}$$

$$= \frac{1-ae^{j\omega}}{1-a(e^{j\omega}+e^{-j\omega})+a^2}$$

WJMK

$$= \frac{1-ae^{j\omega}}{1-2a\left[\frac{e^{j\omega}+e^{-j\omega}}{2}\right]+a^2}$$

$$= \frac{1-ae^{j\omega}}{1-2a\cos(\omega)+a^2}$$

$$= \frac{1-a(\cos\omega+j\sin\omega)}{1-2a\cos(\omega)+a^2}$$

$$= \frac{1-a\cos\omega-ja\sin\omega}{1-2a\cos\omega+a^2}$$

$$= \underbrace{\frac{1-a\cos\omega}{1-2a\cos\omega+a^2}}_{\text{Real part }(R)} + j\underbrace{\left(\frac{-a\sin\omega}{1-2a\cos\omega+a^2}\right)}_{\text{Imaginary part }(I)}$$

$R = Re(X(e^{j\omega}))$.

Magnitude spectrum of $X(e^{j\omega}) = |X(e^{j\omega})| = \sqrt{R^2+I^2}$

$$= \frac{1}{|1-2a\cos\omega+a^2|}\sqrt{(1-a\cos\omega)^2+(a\sin\omega)^2}$$

$$= \frac{\sqrt{1+a^2-2a\cos\omega}}{1-2a\cos\omega+a^2}$$

$$= \frac{1}{\sqrt{1+a^2-2a\cos\omega}}$$

Phase spectrum $= \angle X(e^{j\omega}) = \tan^{-1}\left(\frac{I}{R}\right)$

$$= \tan^{-1}\left(\frac{-a\sin\omega}{1-a\cos\omega}\right)$$

Draw their graphs as well!!