

Indian Institute of Information Technology Allahabad
Univariate and Multivariate Calculus
C1 Review Test

Program: B.Tech. 2nd Semester (IT+ECE)

Duration: **40 Minutes**

Date: May 22, 2022

Full Marks: 25

Time: 12:00 - 12:40 IST

Important Instructions:

1. Attempt all the questions. There is no credit for a solution if the appropriate work is not shown, even if the answer is correct. All the notations are standard and same as used in the lecture notes.
 2. Write down your name and enrolment number on a piece of paper. Write the solutions clearly with all the steps in details.
 3. Submit the solution in PDF format through Google Classroom. **Name the PDF as Enrolment number-UMC-22.pdf**. We will not accept the solution through emails.
 4. Extra 5 minutes is given for submission. Submission after 12:45 PM will attract penalty.
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Attempt all questions.

1. Let $f, g : [0, 1] \rightarrow [0, \infty)$ be continuous functions satisfying

$$\sup_{0 \leq x \leq 1} f(x) = \sup_{0 \leq x \leq 1} g(x).$$

Prove that there exists $x_0 \in [0, 1]$ such that $f(x_0) = g(x_0)$. [5]

2. Let $\alpha \neq \beta \in \mathbb{R} \setminus \mathbb{Q}$ and $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \alpha x, & \text{if } x \in \mathbb{Q} \\ \beta - \alpha x, & \text{otherwise} . \end{cases}$$

Find the points of continuity and discontinuity. [4]

3. Let $f : (0, \infty) \rightarrow \mathbb{R}$ be differentiable. Assume that there exist $\xi, \nu \in \mathbb{R}$ such that $\lim_{x \rightarrow \infty} f(x) = \xi$ and $\lim_{x \rightarrow \infty} f'(x) = \nu$. Using L'Hôpital's Rule, find the value ν . [3]

4. For $t > 0$, show that

$$t - \frac{t^2}{2} < \log(1+t) < t - \frac{t^2}{2(1+t)}. \quad [5]$$

5. Let $f(x) = x + \frac{1}{x}$ for $x \in \mathbb{R} \setminus \{0\}$. Find the points of local maxima/minima, points of inflection, domain of convexity/concavity, and horizontal/vertical/oblique asymptotes. Sketch the graph of f . [8]