

assumption for ML algo

- ↳ know that  $f$  is b/w  $x$ , and  $y$   
but  $x$  nahi pta
- ↳ and that we are trying  
to predict with help of ML algo.

Random Oracle model

- ↳ not a non-deterministic algo
- ↳ It's deterministic ~~algo~~ model.  
→ just mathematical fn has  $x$  and  $y$  ke  
bech mein but graphs will be  
look like random.
- ↳ then any ML algo won't ~~be~~ give  
any benefit.
- can't predict ' $y$ ' with ' $x$ '.

WJMK 3  
30  
2018

→ is it NP or not?

↳ ~~NP~~ Yes How?

→ can't write non-deterministic

poly time algo to solve this.

→ essential property of NP problem?

→ definition of NP problem?

↳ candidate soln must be

checked in poly time.

to give this soln, what to do??

what is soln domain here?

→ randomly taken from solution space

↳ binary string set.

so candidate soln

↳ any binary string.

→ as candidate soln can be checked in poly time.

→ what property to preserve so that problem is hard?

success probability of each iteration =

$$\frac{1}{2^k}$$

probability of soln → depends on size of output, not input.

assumption:-

→ each digit value is equally likely.

→ means every string is coming with equal probability

↳ only then this problem is

Hard!!

random oracle for



Name is RSA problem.

→ Identify attacker's problem:-

attacker is against encryption scheme

→ So attacker want to know  $m$ .

↳ this problem is hard or not?

\* factors of  $n$  again  $\text{PFA}$ , can solve in polynomial time

↳ as will repeat RSA algo 3 steps again.

→ have to perform integer factorization. (If)

\* RSA is not secured until integer "n" is hard → True.

\* RSA is secured bcoz → False.

Left assumption is difference assumption.

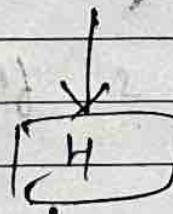
(RSA A) assumption.

but if AP related to RSA A assumption

or A assumption → OH A assumption

$$m \in \{0, 1\}^*$$

# Hash function:-



RSA → secure hash algorithm

↳ soundness req. of

hash fn ??

$$\{0, 1\}^*$$

It must be deterministic algo.

With given  $m \in \{0, 1\}^*$ , H.

find another  $m' \in \{0, 1\}^*$ .

$$\text{s.t. } H(m) = H(m')$$

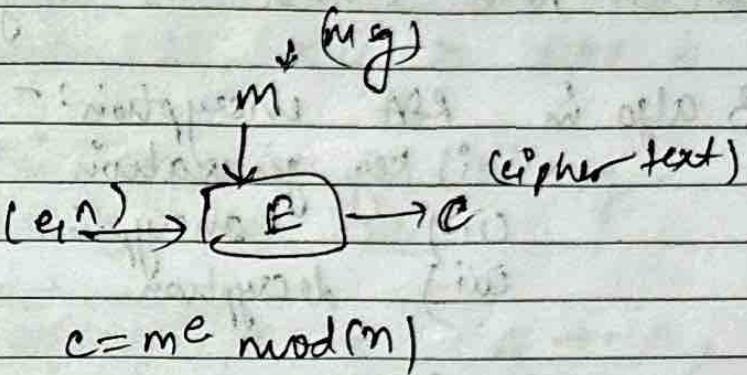
} what can ensure this problem is hard.

It must be deterministic

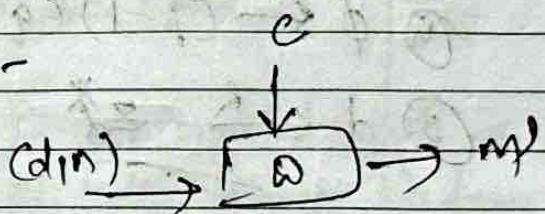
same input must give same output

- \* (1) inverse calculation is also polynomial time.  
 (2) gcd " " euclid algo.

## # ENCRYPTION:-



## # DECRYPTION:-



$$m' = (m^e)^d \pmod{n}$$

$$= m^{ed} \pmod{n}$$

$$m' = m^1 \pmod{n}$$

$$m' = m$$

Correctness proof

this can be written

$e, d$  are inverse of each other.

how??

as,  $e^{-1}$  is  $d^{-1}$   
 or  $e^{-1} \pmod{p}$ ,  
 but here is  $\pmod{n}$ .

$\phi \rightarrow$  euler's  $\phi$   
 & bcoz of euler's theorem

(C) Soundness of RSA:-

-they are related as :-



OH can never be hard as long as DL is not hard.

① 3 algo in RSA encryption:-

- (i) key generation
- (ii) encryption
- (iii) decryption

② keyGen algo :-

generate 2 large prime numbers

$$\begin{array}{l} \textcircled{1} p \leftarrow \text{large prime} \\ \textcircled{2} q \leftarrow \text{large prime} \end{array}$$

$$\textcircled{3} n \leftarrow (p * q)$$

$$\textcircled{4} \phi \leftarrow (p-1)(q-1)$$

$$\textcircled{5} d \leftarrow \mathbb{Z}_{\phi}^* \quad (d \text{ is random element of } \mathbb{Z}_{\phi}^*)$$

(d coming from random set  $\rightarrow \mathbb{Z}_{\phi}^*$ ).

b/w 1 &  $\phi - 1$ , but must be coprime with  $\phi$ .  
definitely nil paayega in polynomial time

$$\textcircled{6} e \leftarrow d^{-1} \bmod \phi$$

wrt  $\phi$ , multiplicative inverse of  $d$ .

saf yeh mod  $\phi$  se huya,  
baaki sare  $(\bmod(n))$

& honge.

$(e, n) \rightarrow$  public key

$(d, n) \rightarrow$  private key

So DHP's hardness depends on DLP,

→ But don't write this argument → as ~~DLP~~ DLP hard, can't calculate ' $a$ ', so DHP is hard.

This argument is wrong.

→ Inference rule  $\rightarrow$  if then sub

→ And

$A \rightarrow B$  is equivalent to  $\neg B \rightarrow \neg A$ ,  
but not equivalent to

$\neg A \rightarrow \neg B$ .

(Here)  $\rightarrow$

A : DLP is hard

DHP reduces to DLP.

B : DHP is hard,

(Show

↗ If

↳ VICE-VERSE  
also true?

→ Security of DH is ensured due to

security of DLP.

↳ True

→ DH is secure bcoz DLP is hard  $\rightarrow$  False.

→ \_\_\_\_\_ as long as \_\_\_\_\_  $\rightarrow$  False.

→ DL is hard  $\rightarrow$  DH is secure  $\rightarrow$  True.

# DH is hard bcoz till date no other algo has been found which can solve it in polynomial time.

→ can't say it will remain hard always.

# DH hardness assumption & DL hardness assumption.  $\rightarrow$  both are assumptions, but ...

they are related as :-

DH can never be hard as long as DL is not hard.

Q) 3 algo in RSA encryption:-

- (i) key generation
- (ii) encryption
- (iii) decryption

\* keyGen algo :-

generate 2 large prime numbers

$$\begin{array}{l} \textcircled{1} \quad p \leftarrow \text{large prime} \\ \textcircled{2} \quad q \leftarrow \text{" } \end{array}$$

$$\textcircled{3} \quad n \leftarrow (p * q)$$

$$\textcircled{4} \quad \phi \leftarrow (p-1)(q-1)$$

$$\textcircled{5} \quad d \leftarrow R \subset Z_{\phi}^* \quad (d \text{ is random element of } Z_{\phi}^*)$$

(d coming from random set  $\rightarrow Z_{\phi}^*$ ).

blw:  $1 \leq d < \phi$ , but must be coprime with  $\phi$ .  
definitely not payga in polynomial time.

$$\textcircled{6} \quad e \leftarrow d^{-1} \bmod \phi$$

wrt  $\phi$ , multiplicative inverse of  $d$ .

if yeh mod  $\phi$  se huya,  
baaki sare  $(\bmod(n))$

se hoga.

$(e, n) \rightarrow$  public key

$(d, n) \rightarrow$  private key

so DHP's hardness depends on DLP.

→ But don't write this argument → as ~~DLP~~ DLP hard, can't calculate 'a', so DHP is hard.

This argument is wrong.

→ Inference rule  $\rightarrow$  If  $\vdash A \rightarrow B$  then sub  $A \rightarrow B$

$A \rightarrow B$  is equivalent to  $\neg B \rightarrow \neg A$ ,

but not equivalent to

$\neg A \rightarrow \neg B$ .

Here  $\vdash$

A: DLP is hard

B: DHP is hard,

DHP reduces to DLP ✓

↳ show

↳ F/w ??

↳ CONVERSE VERSA  
also true?

→ security of DH  $\rightarrow$  depends on hardness of DL?

↳ True

↳ DH is secure bcoz DLP is hard  $\rightarrow$  false.

↳ \_\_\_\_\_ as long as \_\_\_\_\_  $\rightarrow$  False.

→ DL is hard  $\rightarrow$  DH is secure  $\rightarrow$  True.

# DH is hard bcoz till date no other algo has been found which can solve it in polynomial time.

→ can't say it will remain hard always.

# DH hardness assumption & DL hardness assumption.  $\rightarrow$  both are assumptions, but

WJMK  
50  
= = = = =

(1) RSA

→ OS first must be authentic  
 with OS → such web browser aad. to have  
b must be authentic.

② → If clock time change too → may be  
 browser naa chle → as per certificate validity  
 gone.

→ soundness:- diffie Hellman's protocol attack

attacker also want to calculate

$$(g^{ab}) \bmod p$$

Diffie Hellmann  
problem.

problem of  
 attacker against  
 Diffie Hellmann  
 protocol.

Now ques' comes whether this problem is Hard or  
 not?

If hard, ~~diffie~~ protocol is sound or not.

→ just need  $a, b$  from  $g^a \bmod p$  and  $g^b \bmod p$ .

→ Discrete log problem  $\rightarrow$  Hard problem.

→ can't calculate  $a, b$ .

so If discrete log problem (DLP) is hard

so diffie Hellmann protocol (DHP) is  
 hard.

$$\textcircled{3} \quad (k_B)^a \bmod p \quad \longleftrightarrow \quad \textcircled{3} \quad (k_A)^b \bmod p$$

$$\begin{cases} (k_B)^a \bmod p = (g^b)^a \bmod p = g^{ab} \bmod p \\ (k_A)^b \bmod p = (g^a)^b \bmod p = g^{ab} \bmod p \end{cases}$$

$$\therefore (k_A)^b \bmod p = (k_B)^a \bmod p = \text{kash (say)}$$

symmetric key.

behaves like

### soundness ??

→ Firstly identify objective of attacker in this protocol.  
 ↓  
 want to calculate kash.

assumption seq:-

exchanging public keys ( $k_A, k_B$ ) via channel → channel won't change.

→ A, B won't be able to detect (with help of packets) that hm ek dusre se interact nahi kar rahi.

③ anonymous diffie Hellmann

↳ without certificate, doing exchange.

→ Combo of both:- (??)

③ → Ephemeral diffie Hellmann

↳ cost higher as

certification Ghut vaar!

WORK

thus

$$(g^a \bmod p)$$

modular exponentiation is easy!!!  
 ↓ have library to solve it.  
 functions to  
 in polynomial time.

$g, p \rightarrow$  are constants

$$z_p^* \rightarrow z_p^* \rightarrow \text{BIJECTIVE}$$

FUNCTION!!

↑ inverse exists !!!

$$g^x \bmod p = \beta$$

$$\Rightarrow \text{discrete log } (\text{DLog } \beta)$$

$$x = \text{Dlog}_{g,p} \beta$$

stands for  
"discrete" log

base 'g'  
 wst  
 modulo 'p'

given  $\beta$ , can calculate  $x$  easily or not???

31 Aug 2024  
 saturday

(lec-8) :-

1) fixed

# Diffie - Hellman protocol:-

$p, q, g \rightarrow$  global parameters known to all (even, may be to attacker also).

public key of A

$$① a \leftarrow r z_q^*$$

$$② k_A \leftarrow g^a \bmod p$$

$$① b \leftarrow r z_q^*$$

$$② k_B \leftarrow g^b \bmod p$$

$a, b \rightarrow$  private keys of A, B.

lily subfield etc.

{ Super group  $\rightarrow$  "group extension" folle han-  
field  $\longrightarrow$  field }

$\Rightarrow$  field of  $\mathbb{C}$  is field extension over field of  $\mathbb{R}$ .

$$\mathbb{R} * \mathbb{R} \in \mathbb{R}$$

$$\mathbb{R} * \mathbb{C} \in \mathbb{C}$$

$$\mathbb{C} * \mathbb{C} \in \mathbb{C}$$

(\*) any element can generate subgroup.

(\*) Cyclic Subgroup?

let  $g$  is generator of  $\langle \mathbb{Z}_p^*, * \rangle$

$$\text{let } (g^\alpha) \text{ mod } (p) = \beta$$

$$\beta \in \mathbb{Z}_p^*$$

$$\text{as } \beta \neq 0$$

$$\alpha \beta \in \{1, \dots, p-1\}$$

ultimately

it repeats cyclically.

so, so not benefit of taking  $\alpha$  till  $\infty$ , as it will just repeat the things.

so, take  $\alpha \in \mathbb{Z}_p^*$  also.

$$\alpha \in \{1, \dots, p-1\}$$

but this is not order of  $\beta \rightarrow$  but some permutation of  $\mathbb{Z}_p^*$ .

thus

$$g^d \bmod p \leftarrow Z_p^* \rightarrow Z_p^* \rightarrow \text{BIJECTIVE FUNCTION!}$$

so inverse exists !!!

 $(g^d \bmod p)$ 

modular  
exponentiation is  
easy!!!  
have library to solve it.  
functions to polynomial  
in time.

$$g^d \bmod p = \beta$$

 $\text{DLog}(\beta)$ 

$$\alpha = \text{DLog}_{g, p} \beta$$

stands for  
"discrete" log

base 'g'  
wrt  
modulo 'p'

 $g, p \rightarrow$  are constantsgiven  $\beta$ , can calculate ' $\alpha$ ' easily or not???

31 Aug 2024  
Saturday

(lec-8) :-

① fixed.

# Diffie-Hellman protocol:-

$p, q, g \rightarrow$  global parameters  
known to all  
(even, may be to  
attacker also).

Diffie  
Hellman  
public  
key

A

KA

B

KB

$$① a \leftarrow Z_q^*$$

$$② K_A \leftarrow g^a \bmod p$$

$$① b \leftarrow Z_q^*$$

$$② K_B \leftarrow g^b \bmod p$$

 $a, b \rightarrow$  private keys of A | B.

liley subfield etc.

Super group  $\rightarrow$  "group extension" folle hain.  
 field  $\rightarrow$  "field"

$\Rightarrow$  field of  $C$  is field extension over field of  $R$ .

$$R * R \in R$$

$$R * C \in C$$

$$C * C \in C$$

(\*) any element can generate subgroup.

(\*\*) Cyclic Subgroup?

let  $g$  is generator of  $\langle Z_p^*, * \rangle$

$$\text{let } (g^\alpha) \text{ mod. } (p) = \beta \in Z_p^*$$

as  $\beta \neq 0$

$$\beta \in \{1, \dots, p-1\}$$

ultimately it repeats cyclically.

so not benefit of taking  $\alpha$  till  $\infty$ , as it will just repeat the things.

so, take  $\alpha \in Z_p^*$  also.

$$\alpha \in \{1, \dots, p-1\},$$

but this is not order of  $\beta \rightarrow$  but some permutation of  $Z_p^*$ .

this

$$(g^x \bmod p)$$

modular  
exponentiation is  
easy!!!

↓ have library to solve it  
functions to  
for polynomial

$g, p \rightarrow$  are constants

$$z_p^* \rightarrow z_p^{*x}$$

BIDJECTIVE

FUNCTION!!

so inverse exists !!!

$$g^x \bmod p = \beta$$

$$\Rightarrow \alpha \quad (\text{dlog } \beta)$$

$$\alpha = \text{dlog}_{g, p} \beta$$

stands for  
"discrete" log.

base 'g'  
wst  
modulo 'p'

given  $\beta$ , can calculate ' $\alpha$ ' easily or not???

31/ Aug/ 2024  
Saturday

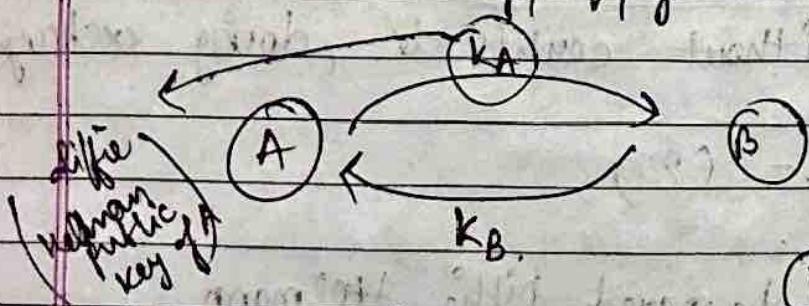
lec-8:-

(1) fixed.

# (2) Diffie - Hellman protocol:-

$p, q, g \rightarrow$  global parameters

known to all  
(even, may be to  
attacker also).



$$① a \leftarrow z_q^*$$

$$② KA \leftarrow g^a \bmod p$$

$$① b \leftarrow R z_q^*$$

$$② KB \leftarrow g^b \bmod p$$

$a, b \rightarrow$  private keys of A, B.

WJM

Ugly subfield etc.

[super group to "group extension" better name  
 field — field — field — ]

$\Rightarrow$  field of  $C$  is field extension over field of  $R$ .

$$R \neq R \in R$$

$$R \neq C \in C$$

$$C \neq C \in C$$

(\*) any element can generate subgroup.

(\*\*) Cyclic Subgroup?

let  $g$  is generator of  $\langle \mathbb{Z}_p^*, * \rangle$

$$\text{let } (g^\alpha) \bmod(p) = \beta \in \mathbb{Z}_p^*$$

as  $\beta \neq 0$

$$\alpha \beta \in \{1, \dots, p-1\}$$

unfortunately

it repeats cyclically,

$\therefore$  go not benefit of taking  $\alpha$  till  $\infty$ , as it will just repeat the things.

so, take  $\alpha \in \mathbb{Z}_p^*$  also.

$$\alpha \in \{1, \dots, p-1\},$$

but this is not order of  $\beta \rightarrow$  but some permutation of  $\mathbb{Z}_p^*$ .

this smallest ' $i$ '  $\rightarrow$  is 'order' of  $a$ .  
 In  $\mathbb{Z}_p^*$ , we'll have only one  $i$  s.t.  $a^i \text{ mod}(p) = 1$ .  
 $G = \langle \mathbb{Z}_p^*, * \rangle, p=11$

① let  $a = 2, p=11$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 2 \\ 4 \\ 8 \\ 5 \\ 10 \\ 9 \\ 7 \\ 3 \\ 6 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 4 \\ 8 \\ 3 \\ 5 \\ 10 \\ 9 \\ 7 \\ 6 \\ 2 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 8 \\ 3 \\ 5 \\ 10 \\ 9 \\ 7 \\ 6 \\ 2 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 6 \\ 5 \\ 10 \\ 9 \\ 7 \\ 3 \\ 2 \\ 8 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 5 \\ 10 \\ 9 \\ 7 \\ 3 \\ 2 \\ 8 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 10 \\ 9 \\ 7 \\ 3 \\ 2 \\ 8 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 9 \\ 7 \\ 3 \\ 2 \\ 8 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 7 \\ 3 \\ 2 \\ 8 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 3 \\ 2 \\ 8 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 2 \\ 8 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 8 \\ 1 \end{matrix}$$

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} \rightarrow \begin{matrix} 1 \end{matrix}$$

order of  $2 = 10$ .

subset,  $2$  in generating is  $\rightarrow \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$\mathbb{Z}_p^*$$

\* Primitive root ?

\* what is Generator ?

$\hookrightarrow$  element which generate entire group  
 like,  $2$  here is a generator.

known as  
 "SUBGROUP"

so we generated  
 a small group with  
 same operator  
 as that of  
 parent group

subset of main  
 group which in itself is a  
 group.

WJMK

\* Non-repudability can be achieved only by Digital signature  $\rightarrow$  public key cryptography, can't be achieved by symmetric key cryptography.

\* Elliptic Curve Cryptography. (Not in course)  
↳ Nowadays, mostly it is used.

# Hard Problem of Public key Cryptography:-

Discrete Log Problem

want  $i$  s.t.  $(a^i) \text{ mod}(p) = t$ ,

let,  $p = 11$ ,  $a = 5$

then  $(a^i) \text{ mod}(p)$

$$a^4 = a^3 \times a$$

$$a^3 = a^2 \times a \\ = 3 \times 5$$

$$= 15$$

$$\mod 11$$

for  $i \Rightarrow 0 \rightarrow 1$

$$1 \rightarrow 5$$

$$2 \rightarrow 3$$

$$3 \rightarrow 4$$

$$4 \rightarrow 9$$

$$5 \rightarrow 1$$

$$6 \rightarrow 5$$

$$7 \rightarrow 3$$

$$8 \rightarrow 4$$

whatever element u take take the inverse w.r.t 11.

OBSERVATION :-

① so  $a^i \text{ mod}(p) \in \{1, 5, 3, 4, 9\}$

② we'll have ' $\infty$ '. i.e. s.t.  $a^i \text{ mod}(p) = 1$ . But want smallest  $i$ .

$\rightarrow$  And we are looking in  $\mathbb{Z}_p^*$ .  $\rightarrow$  so that  $i = 0$  answer no. bcz  $a^0 \equiv 1 \pmod p$

WEEK 3  
MATERIAL

$$\mathbb{Z}_p = \{0, 1, \dots, p-1\}$$

$$\mathbb{Z}_p^* = \{1, \dots, p-1\}$$

①  $\langle \mathbb{Z}_p, +, * \rangle$  is a finite field.

↳ with identity of one operator, we did not require the inverse of another operator.

$\mathbb{Z}_p, + \rightarrow$  group ✓  
 $\mathbb{Z}_p, * \rightarrow$  group ✗

②  $\langle \{0, 1\}^n, +, * \rangle \rightarrow$  also a field.

so much → (almost 10-times)

③ Symmetric cryptography is faster than public key cryptography but both are equally secure.

↳ why is public key cryptography used then?



fact it takes little bit large key size to ensure security as much as of symmetric.

↓ that's why speed is.

To encrypt symmetric key !!

used to encrypt data!

costly!  
but only once!!

RSA  $\rightarrow$  128 bits of data can encrypt 1 by this costly algo.

so that symmetric key encrypt

no key paaya through open channel, and public key a liya, so jiske paas private key hogi, kisi no decrypt ke paayega  $\rightarrow$  so far mutual secure establishment  $\rightarrow$  public key is required.

Hybrid  
CRYPTO-  
GRAPHY

To establish  
secure  
communication  
between  
receiver and sender  
through open channel

WEEK 3

23 Aug | Friday

lec-7 :-

$$G = \langle S, \cdot \rangle$$

$\text{order}(G) = |S| = \text{cardinality of } S$

Let  $\Rightarrow$ 

$$G = \langle \mathbb{Z}_n, + \rangle, \quad \mathbb{Z}_n = \{0, 1, \dots, n-1\}$$

$\hookrightarrow$  this is  $\text{mod}(n)$

$\hookrightarrow$  is it group or not??

Closure property ✓

Associativity ✓

Additive inverse of 3 is 7.

Additive inverse of a is  $n-a$ .

so valid group.

Similarly  $\langle \mathbb{Z}_n^*, * \rangle$  is also group.

$\hookrightarrow$  NO!

$$\text{Let } n=10$$

2 ka inverse does not exist.

Numbers which are co-prime with 'n',

-  $\hookrightarrow$  if unk inverse milega.  $\hookrightarrow$  1, baakiyon ka nahi!

$\hookrightarrow \mathbb{Z}_n^*$  is that subset.

$\hookrightarrow \in \mathbb{Z}_n$ , but are also co-prime with n.

$\hookrightarrow$  satisfies closure property ✓

Associativity ✓

Identity exist ✓

Inverse exist ✓

$\therefore (\mathbb{Z}_n^*, *)$  is a valid group, (MG)

(multiplicative group)

to find another msg s.t. both have same hash value.

↳  $h \rightarrow \text{fixed}$  predetermined, 1-8 bit value.  
and it is possible as it is many-to one function.

↳ as  $n \gg l$ .

so, there does not exist only one such  $m'$ ,  
but saare honge s.t.  $H(m) = H(m')$ ,

then how can say 'Hash' is secure ??,

↳ so want this problem of finding such

$m'$  to be a hard problem.

also want to design hash f<sup>n</sup> s.t. above probm hard to.

↳ collision resistance property

Collision resistant hash fn

↳ collision exist but  
computationally infeasible to get,  
not impossible \*\*\*.

MAC

MAC takes 'K' as input while calculating  
hash.

Diffusion property ?

→ hashing is deterministic algorithm.

↳ as for same ' $m$ ',

$H(m)$  should be same.

→ Not a randomistic algo.

↳ hash chache it generates random  
string of bytes (esa lgta hai), but for  
same  $m$ ,  $H(m)$  must be same.

→ why attacker modifies file, can modify hash  
only if file detection fails na to  
paata ki attack hoga → but we assume  
file and hash ikathaa chitta hai.

Ans

Authenticity w/o Integrity has no meaning ???

→ SHA → secured version ???

① Encryption properties? (or Requirements)  
ensuring that

i) Correctness :- Encryption would work correctly when done.

ii) Soundness :- ??

→ Inv. encryption - Decryption :-

$C = E(m, k)$  → only  $k$  is unknown, want  $k$ !

↳ Hard problem.

② Soundness req. of hash :-

If msg changes, hash must also change.

Integrity :- If msg is modified; it must be detected.

→ In hashing, problem of attacker is computational problem.

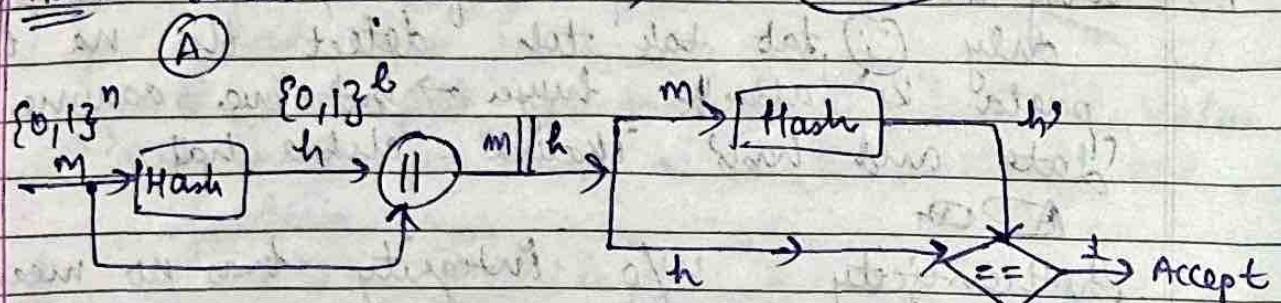
Given:  $m$ ,  $h$ , Hash algorithm also known

Compute: ? → Problem of attacker.

→  $m'$  (as  $m$  is passed over insecure channel)  
s.t.  $H(m) = H(m') = h$

① Hash :- sender . . . ( $n > > l$ )

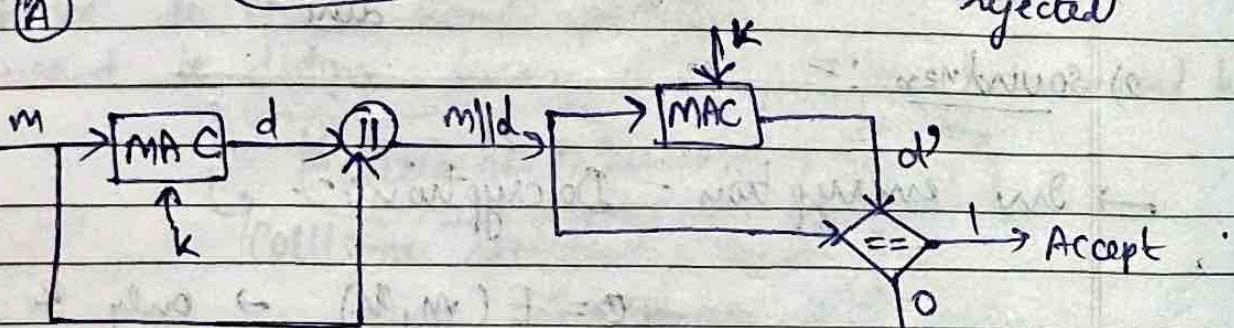
Integrity



② MAC :-  
sender

Authenticity &  
Integrity

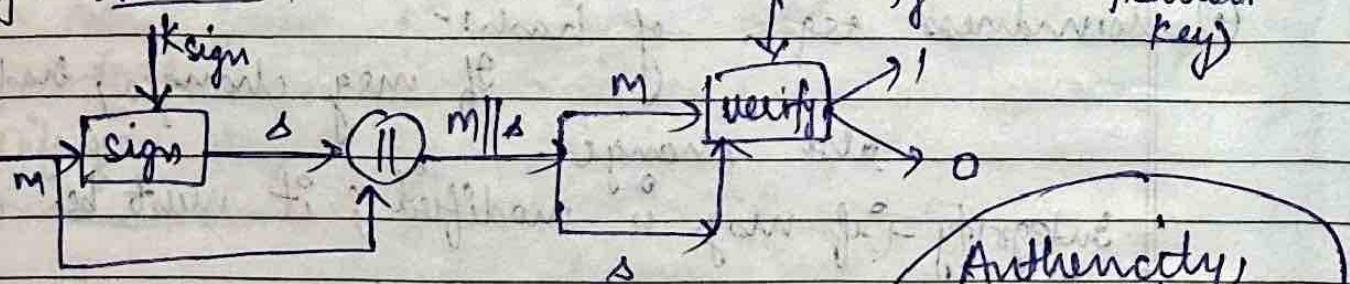
(Permission is  
rejected)



MAC is always symmetric cryptographic mechanisms.

③ Digital Signature Mechanism (DSM)

Kuverify → (Verification key)



$k_{sign}$  = Inverse ( $Kuverify$ )

$Kuverify$  = Public key A

$k_{sign}$  = Private key A

Authenticity,  
Integrity,  
non-repudiability

How?  
(H/W).

So, DSM is always asymmetric cryptographic mechanism.

to ensure in poly time, have to take constant numbers &  
 $W\leq \frac{c}{n^{\alpha}}$  Page No.:  
Date:

① f easy,  $f^{-1}$  hard, but if some key  
 is known,  $f^{-1}$  easy.

② Both functions are used in cryptography,

(NP solvable) from  $\mathbb{Z}^m$  space  $\rightarrow$  like random

③ Integer Factorization Problem :- no. what is the  
 to check divisibility  
 $36 = 1 \times 2^2 \times 3^2$ .  
 or  $H(\text{hard}) = H(\text{easy})$   
 or not  
 or vast  
 Chaining constants

Q:- If computation problem :- In Exam :- Write  
 given :-  $n$   
 find / compute :-  $p$  ]  $\rightarrow$  integer  
 such that :  $p | n$  ] factorization  
 cryptography  
 problem

Q:- If decision problem :- (number theory)  
 given :-  $n$  ] if known  
 whether  $n$  is prime  
 exactly product  
 of prime  
 numbers.  
 ans Hard / Easy ??

16 Aug 2024

Friday

[lec-6] :- (Block Diagrams:-)

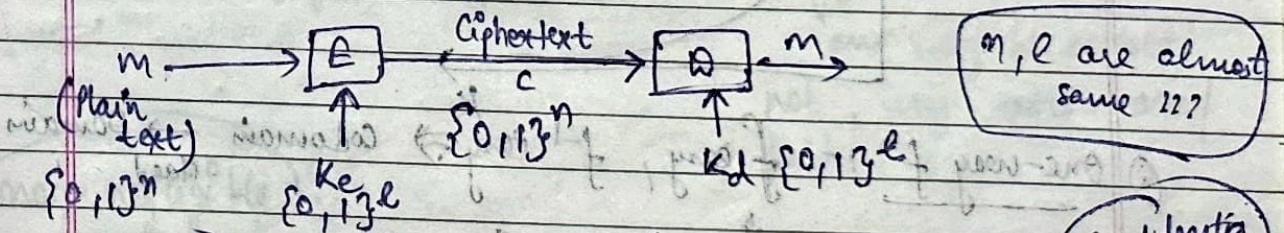
Encryption-decryption :-

Receiver

$K_e = \text{Inverse}$   
 $(K_d)$

(A)

(B)



for symmetric encryption :- Confidentiality

$C = E(M, K)$

$K_e = K_d$

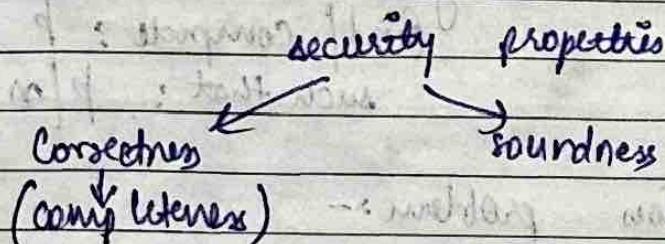
only unknown  
 $K$

$K_e$  : public key

$K_d$  : private key

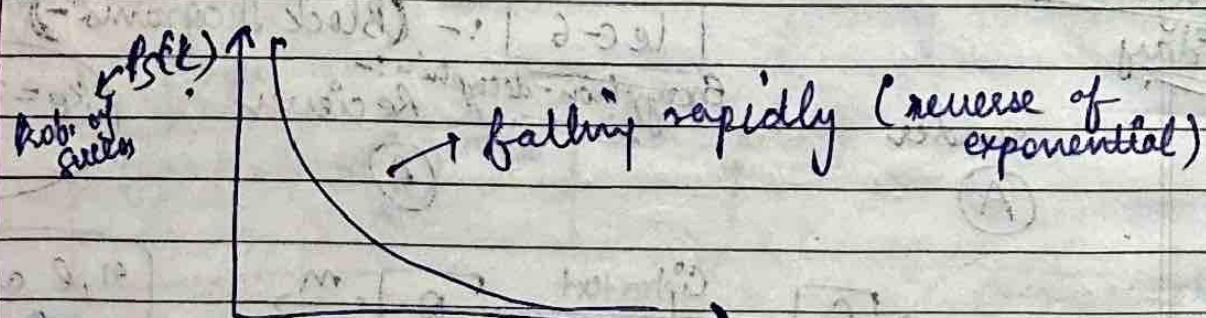
$K$  : shared secret key

- ① If growth rate of  $f^n$  is very high (exponential),  
 then  $f^n$  is non-negligible ( $\approx \infty$ )  
 ↗ power for attacker → opposite wall ka security protector etc.  
 → attacker is an algo to find private key etc.
- we want problem of attacker should be hard (for our safety) but want hashing, mapping, encryption etc to be hard (of w)  
 maybe problem, so cryptographic algo must be easy)



So that there should not exist deterministic poly. time complexity algo to solve its problem.

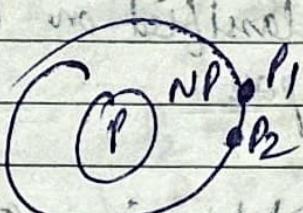
N Queen Hard only if  $n \approx \infty$ , not for  $n = 2, 3$  etc.



- ② One-way  $f^n$  i-  $f$  easy,  $f^{-1}$  easy  $\Rightarrow$  codomain  $\rightarrow$  domains Hard
- Domain  $\xrightarrow{\text{easy}} \text{Codomain}$

- ③ Trap door  $f^n$  i- rat can enter easily, and only if you want them rat can come out.  $\rightarrow$  easily, of w hard.

any NP problem can be reduced to NP complete (???)  
 one "complete" problem can be converted to  
 any other NP "problem".



P  $\rightarrow$  NP, P1, P2 mean reduce to  
 "adhega."

→ Hard problem which is NP  $\rightarrow$  may not be NP

Hard

any hard problem  $\in$  NP or NP-Hard or NP-  
 complete.

all have no problem with "poly. TC sol" algo.

Q. why not all hard problem are ~~NP~~ NP??

→ In CS we use NP type of problems.

NP  $\rightarrow$  have not deterministic Poly TC,

but must have non- " " algo to solve !!

NP-Hard  $\rightarrow$  This may not have non-deterministic poly TC  
 algo to solve.

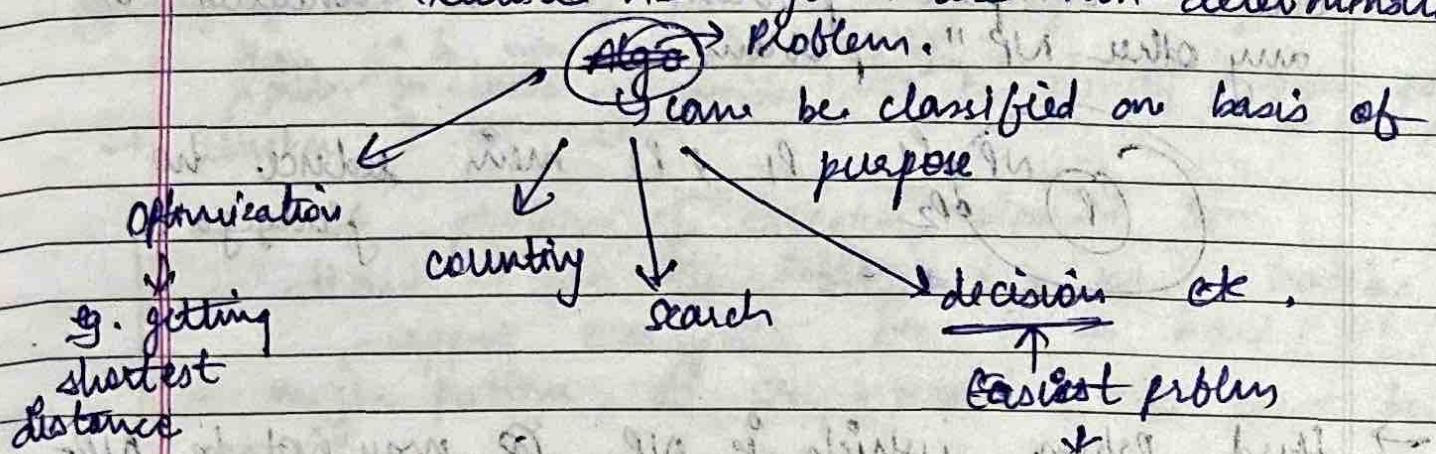
ans may be correct

not, may return ans/  
 failure.

non-negligible

get B not threshold, it is function  
 of input size ( $k$ ) (no. of bits)  
 → function is not negligible  $\rightarrow$  non-negligible  
 fn. !!

e.g. quick sort with random pivot everytime,  
Heuristic AI algo.  $\rightarrow$  are non-deterministic.



as any input but output is either "Yes"/1 or "No"/0.

$\hookrightarrow$  (0-1) problem.

also known as

$P_1$  is reducible to  $P_2 \Rightarrow$  if  $P_2$  is easy then  $P_1$  is easy.

$\hookrightarrow P_1$  is not more harder than  $P_2$ .

### Non-deterministic decision algo

LV  
 $\downarrow$   
Las Vegas

MC

Monte Carlo

It always gives answer, may  
may not be correct.

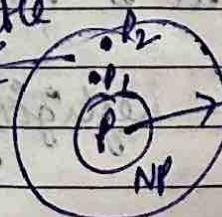
Yes

No biased.

Biased

$P_2$

reducible  
to  
 $P_1$



$\rightarrow$  as more fair, hardness  $P_2$

WDMK  
SUNDAY

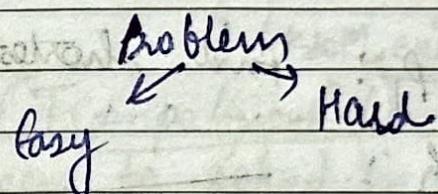
- ⇒ If  $P_2$  is easy, then  $P_1$  is easy.
  - ⇒ But if  $P_2$  is hard  $\Rightarrow P_1$  is hard. if  $P_1$  reduces to  $B$ .
  - ⇒  $P_1$  hard  $\Rightarrow B$  hard. (???)
- $\downarrow$   
 $(P_2 \text{ hard as long as } P_1 \text{ is hard.})$

9/8/24

sunday

WDMK  
SUNDAY

lec-5 :-



Algorithm

Deterministic      Non-deterministic / Probabilistic

① nature of algo is deterministic

i) means same input  $\rightarrow$  same output

ii) They ~~use~~ <sup>there is no</sup> source of randomness in algo/problm  $\rightarrow$  like generating random no.

iii) Flow chart nota hdi algo ka, so same input ke liye,

path from start  $\rightarrow$  end

is constant / same everytime, so

can determine path everytime, but

in non-deterministic we can not judge/

we guess the path as it is not fixed

$\rightarrow$  in AI mostly we use Deterministic algo.

↳ Non-deterministic

↳ diff output possible

e.g. quick sort with random pivot everywhere,  
Hanoi AI algo.  $\rightarrow$  are non-deterministic.

algo

Problem.

↳ can be classified on basis of purpose

optimization

counting

search

decision pk.

e.g. getting  
shortest  
distance

Easiest problem

as any input but output is

either "Yes"/1 or "No"/0.

$\hookrightarrow$  (0-1) problem.  
also known as

$P_1$  is reducible to  $P_2 \Rightarrow$  if  $P_2$  is easy, then  $P_1$  is easy.

$\hookrightarrow P_1$  is not more harder than  $P_2$ .

Non-deterministic decision algo

LV

Las Vegas

MC

Monte Carlo

↳ always gives answer, may  
may not be correct.

Yes

No biased.

$P_2$   
reducible

$P_1$   
to

$P_2$  is  
reducible  
to  $P_1$ .

$P_1$

$\rightarrow$  as more fair, hardness  $P_2$

NP

WDMK

⇒ If  $P_2$  is easy, then  $P_1$  is easy.

⇒ But if  $P_2$  is hard  $\Rightarrow P_1$  is hard. ] → If  $P_1$  reduces to  $P_2$ .

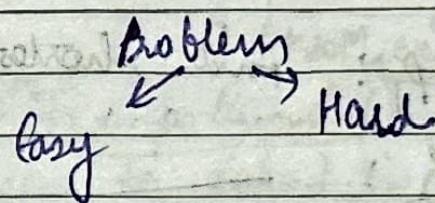
⇒  $P_1$  hard  $\Rightarrow P_2$  hard.

↓  
( $P_2$  hard as long as)  
 $P_1$  is hard.)

~~9/8/24~~  
Friday

WDMK

Lec-5 :-



Algorithm

Deterministic

Non-deterministic / Probabilistic

① nature of algo is deterministic

i) means same input  $\rightarrow$  same output

ii) They use ~~the~~ <sup>there are no</sup> sources of randomness in algo/problm(???)

iii) Flow chart nota hui algo ka, toh same input ke liye, path from start to end

iv) constant / same everytime, so

can determine path everytime, but

in non-deterministic can not judge /

guess the path as it is not fixed,

→ In AI, mostly we use deterministic algo.

Q. What do you mean by  $P_2$  is more harder than  $P_1$  or  $P_1$  is not as hard as  $P_2$ , both are hard. How you are comparing hardness???

A. It means that,  $P_1$  reduces to  $P_2$ .

↓  
↳ what do you mean by  
 $(P_1 \leq P_2)$  ??

using sol'g of  $P_2$ ,  $P_1$  can be solved.

Example:-

$P_1$ : find shortest route in unweighted graph

$P_2$ : \_\_\_\_\_ unweighted graph.

then  $P_2$  reduces to  $P_1$ .

(Polynomial time reduction)

(\*)  $P_1$  is reducible to  $P_2$  in polynomial time

means?

like unweighted to fully weighted  
main keinge; they fr. of (2) weighted  
wale ka use keinge; then again kya  
kuch keinge.

Joh jn ko use kene se phle kuch kiya, then  
jn call ke baad kuch kiya  $\rightarrow$  in  
dono mein jo time lgा poly. must  
(and jn call kiya  $\rightarrow$  may be ek se jyada  
vara ke she hours  $\rightarrow$  must call poly no.  
of times (P???)

WORK

in arr[ ].  $\rightarrow n = \text{no. of elements in array}$   
 ↳ as assuming every element  
 is taking constant space, so we take  
 ignore that.

But here input size  $\rightarrow$  not constant.  
 It's not like, & line ka code hai toh easy!!

\* what is easy and what is hard problem ???

↳ problem which  
 can not be solved  
 in polynomial  
 time complexity.  
 $\rightarrow$  PPT exists but does not

have. ( $Tc = \text{polynomial}$ ). (may)

↳ which can be solved in poly. time.

$O(1)$

$O(\log k)$

$O(k)$

$O(k^2)$

etc.

but in future  
 it may have!

$\alpha^{\log k} \rightarrow$  subexponential ( $>$  poly,  $<$  exp)

$\alpha^k \rightarrow$  exponential

↳ puzzle

↳ N queen

↳ graph coloring problem

Q. What is NP, NP Hard, NP complete problem?

↳ all are '∞', unbounded

→ centre se jitan aur jaayoge,  
 difficulty level extra bhega.  
 sets!

Q. What do you mean by  $P_2$  is more harder than  $P_1$  or  $P_1$  is not as hard as  $P_2$ . Both are hard. How you are comparing hardness???

A. It means that,  $P_1$  reduces to  $P_2$ .

↓ what do you mean by this??,  
 $(P_1 \leq P_2)$  ??

using sol' of  $P_2$ ,  $P_1$  can be solved.

Example:-

$P_1$ : find shortest route in weighted graph

$P_2$ :  unweighted graph.

then  $P_2$  reduces to  $P_1$ . (Polynomial time reduction)

K  $P_1$  is reducible to  $P_2$  in polynomial time

means?

like unweighted to fully weighted  
 mein kaise; then fr' of ~~fully~~ weighted  
 wale ke use kaise; then again kya naar  
 kuch kaise.

Jab jn ko use kese se phle kuch kya, then  
 jn call ke tsaad kuch kya → inn  
 dono mein jo time lgा poly. mud -  
 and jn call kya → may be ek se jyada  
 var ki the hain → must call poly no.  
 of times (????)

WORK  
STUDY

Time complexity  $\rightarrow n = \text{no. of elements in array}$

↳ as assuming every element

is taking constant space, so we take  
ignore that

But here input size  $\rightarrow$  not constant.

It's not like, a line ka code hai toh easy!!

\* what is easy and what is hard problem???

↳ problem which  
can not be solved  
in polynomial  
time complexity.

↳ exists but does not  
have. ( $TC = \text{polynomial}$ ), (may)

↳ which can be solved in poly. time.

$O(1)$

$O(\log k)$

$O(k)$

$O(k^2)$

etc.

but in future  
it may have!

$\alpha^{\log k} \rightarrow$  subexponential ( $> \text{poly}, < \text{exp}$ )

$\alpha^k \rightarrow$  exponential

↳ puzzle

↳ n queen

↳ graph coloring problems

Q. What is NP, NP Hard, NP complete problem?

↳ all are 'co', unbounded

→ centre se either 'dun jaafoge,  
difficulty level, itna bdaega'.

difficulty level, itna bdaega.

Public key  $\rightarrow$  key which is not supposed to be secret to somebody.

Symmetric key  $\rightarrow$  shared secret key (?)

$\hookrightarrow$  where no body can be sender, only one who knows your secret key can be sender.

But wait...

Private key = inverse (public key)

$\hookrightarrow$  tan private kaise milo wo?

where is secrecy of private key, then?

$\rightarrow$  Algos are public always.

$\hookrightarrow$  to generate keys

$\hookrightarrow$  to get  $k_2$  from  $k_1$ .

$\rightarrow$  2 options:- (1) Inverse does not exist.

(2) You can't calculate inverse!

$\hookrightarrow$  it's hard!

$\hookrightarrow$  it exists, but difficult.

$\rightarrow$  Not possible!

$\rightarrow$  as one-one hogi pkka!

$\hookrightarrow$  ek private key ek public key se  
li' map hogi,

$\rightarrow$  onto bhi hogi pkka!

So, Inverse exists.

$\&$  what do you mean by it?

$\rightarrow$  Hard can be problem, not algo.

$\rightarrow$  TC " " of algo, not of problem.

$\hookrightarrow$  size of input (in no. of bits)

$\hookrightarrow$  bits req. to hold input data.

Digital Sig nature

↳ <sup>ensures</sup> authentication, non-repudiability, integrity

⇒ Hash ↳ Receiver and Sender both uses same hash algorithms.

↳ does not require key, while all other classes req. key.

↳ so called keyless class.

⇒ symmetric key cryptography / symmetric cryptography →

↳ if  $\text{inverse}(\text{key}) = \text{key}$ , then

Asymmetric key cryptography / asymmetric cryptography.

↳ so Hash → is neither symmetric nor asymmetric.

Digital sig → always Asymmetric

MAC → " Symmetric

Encry-Decry → can be as well as Asymmetric

e.g. MD5, SHA are examples of 'hash' algo.

Asymmetric key cryptography → also known as Public ".

↳  $k_1, k_2$  should be secret to Receiver,

but  $k_1, k_2$  will be "the sender or someone else? → No!

↳ as sender can be anybody,  
so  $k_1$  should be announced to all, so  $k_1$  is public key and  $k_2$  is private key.

e.g.  $(\Sigma, +, *)$  is a ring, but not a field.

•  $\Rightarrow$  multiplicative identity exist,  
but " " inverse does not " ".

In ring, can perform  $+, -, *,$  not  $\div$ , as  
multiplicative inverse does not exist.

\* Group, ring, fields are 'as' as sets are 'as'.

→ Why we need closure property?

↳ as we need a fixed domain!!

Q. But if domain is very big,  $\infty$ , then what is use of  
closure property?

→ So in Cryptography, we need finite group/  
finite field.

Decryption key must only be known to receiver secretly.

$k_2 = \text{inverse}(k_1)$  ↗ Encryption key

→ want group where ~~all~~ inverse of no. is no. itself.

→ Let set  $\rightarrow \{0, 1\}^n$ , binary operator  $\rightarrow \oplus$ ,

↳ 4 binary strings of fixed length.  
Is it a group? "

Identity element =  $\underbrace{0000\dots 0}_{\text{on length}}$

Inverse (element) = element itself,

Cryptography

↳ set of algos to secure the fundamental  
req. of information security. (4 req.)

Q) what is inverse of an element?  
 →

- ↳ multiplicative inverse  $\rightarrow (1/3)$
- ↳ additive inverse  $\rightarrow (-3)$

5/8/24

Monday

Lec-4

WORK  
STATION

Group:  $\langle S, \text{ binary operator } (*) \rangle$

$a, b \in S$ .

$$f(a, b) = c$$

Group must satisfies:-

(1) closure i.e.  $c \in S$ .

(2) associativity : (3) identity (4) inverse.

Fees sit.  $a * e = e * a = a \forall a \in S$ .

$$\cancel{f(S \times S)} \rightarrow a * b = b * a$$

→ If group satisfies commutative property, then it is

abelian group.

field: → 2 operators  $\rightarrow$  usually  $+$ ,  $*$

↳ group w.r.t both operators

$+, *$  → fundamental operators.

$\div, \frac{1}{\cdot}$  are not fundamental operators, as  $\div$  is nothing but  $*$  with multiplicative inverse

→ Ring: - more than group, less than field.

↳ multiplicative inverse is missing in the ring.

\* key is req. for both encryption & decryption.

why for this, 'key' is mandatory & thus req.

to ensure that only 'B1' key 'key' in use decrypt the page!

→ so need to supply some secret which relates with secret on other side.

$$E : \{0,1\}^n \times \{0,1\}^l \rightarrow \{0,1\}^n$$

↑      ↗ plain text      ↗ key      ↗ cipher text  
 (Encryption function)

↳ binary f<sup>n</sup> (or input f<sup>n</sup>)

↳ 1<sup>st</sup> input: n bit binary string  
2<sup>nd</sup>      l

output: n

$$m \in \{0,1\}^n$$

$$k_1 \in \{0,1\}^l$$

$$c \in \{0,1\}^n$$

$k_2 \rightarrow$  receiver's private key

$k_1$  → somehow related to  $k_2$ .

so this is not random string of l bits,  
 $k_2$  can generate randomly, but  $k_1$  have to compute from  $k_2$

OR

$k_1$  can generate randomly, but then  $k_2$  have to compute from  $k_1$ .

so f relationship → 'inverse' of each other,  
as ek ka effect on plain text, dusse ne  
kaatna hai, means usko firse plain text  
baana hai!

Plain Text → Cipher Text

algo used is "Encryption".

Now, need plain text back from cipher text → for that algo used is called "Decryption".

Ciphertext → Plain Text

Decryption

Usually size of cipher and plain text is same.

- Q) Are these algorithms (Encryption / decryption) are secret or public?
- A) Public, obviously !!.

So attacker can also use them, and cipher text channel is like plain text.

This defn is not complete.

→ Decryption must req. some additional info

(Secret piece of info) to convert cipher text

to plain text. (It must not be known to attacker).

Known as "key"

different from Password.

also long

of bits

this must be random sequence of bytes

thus human can't remember such a sequence of almost 16 bytes.

→ Key is hash of Password.

must be  
human  
readable  
and  
rememberable

↳ seq. of  
characters

↳ go binary form  
↳ mem. store hots in memory.

↳ created/generated by computer via some algorithm.

↳ some

(algo / protocol, they may have standard) work ~~c/o.~~ ASCII → standard,  
not algo, not protocol.

## Cryptography :-

algo to ensure :-

→ I-A-NR.

eg: SHA, cipher, MD5, RSA, TLS

• Diff b/w algo & protocol.

can be  
executed in  
one system:

(part  
of  
protocol)

→ can't be executed on

one system,

at least 2 entities  
charlie, executes on top of  
computer n/w systems.

Inside TLS, SSL, we use cryptographic algo.

→ Cryptographic algo can be classified in diff-2 ways:-

(1) symmetric v/s asymmetric (from mathematical pt of view)

(2) from purpose pt of view, can classify as:-

Encryption,  
decryption  
to ensure  
"Confidentiality"

Hashing  
MAC  
media access control

Digital  
signature

msg → stream of bits

→ can't send data in raw form of text

→ plain text

↳ have to convert it into,

transformed text

transform it

↳ known as "Cipher text".

$$WOMK = \frac{2}{3} MCW$$

→ Can't ensure channel  
to physically safe  
from Date: attacker.

- ① what is 5th most important req.

3) Non-Repudability (NR)

↳ sender should not be able to deny that he has send the msg.

C-I-A-NR

↳ these 4 req. can be ensured by cryptography.

Availability → is req. in service, for eg. medium threat kde koi jisse msg bhejna tha !!  
DOS.

→ information service is one of the services.

→ not a req. in information security.

- ① Authorization and "access control"

↳ these are two very related requirements, cannot explain w/o another.

e.g. ID card milna, jisse can access.

→ purpose of authorization is to enable access control.

→ without defining an protocol by which we ensure that only legitimate entities / authorized entities can access that thing.

- ① Availability, authorization, access control

cryptography has important role in ensuring them, but not sufficient to ensure these three.

2/8/24

Friday

(lec-3)

Cyber Security :-

Book:-

Frozen

Information Security :-(A)  
sender  
node

m

(B)

Receiver  
node(sending single  
msg to B.)Requirements :-

→ apart from A &amp; B, no one should know the content of m.

1) Confidentiality

① msg should not be altered / tampered / modified

2) Integrity

② But this is not actual meaning of integrity

↳ practically can't avoid attacker from modifying the content.

Integrity → detection of modification.

↳ actual meaning

↳ agar attacker ne modify ki diya, toh

B detect ki ske, ki modified hai msg.

③ Availability

④ Authenticity (A)

↳ how B will know msg 'A' ne kya, this is authenticity of msg.

C-I - A tried aq.

3) Fundamental requirements of security