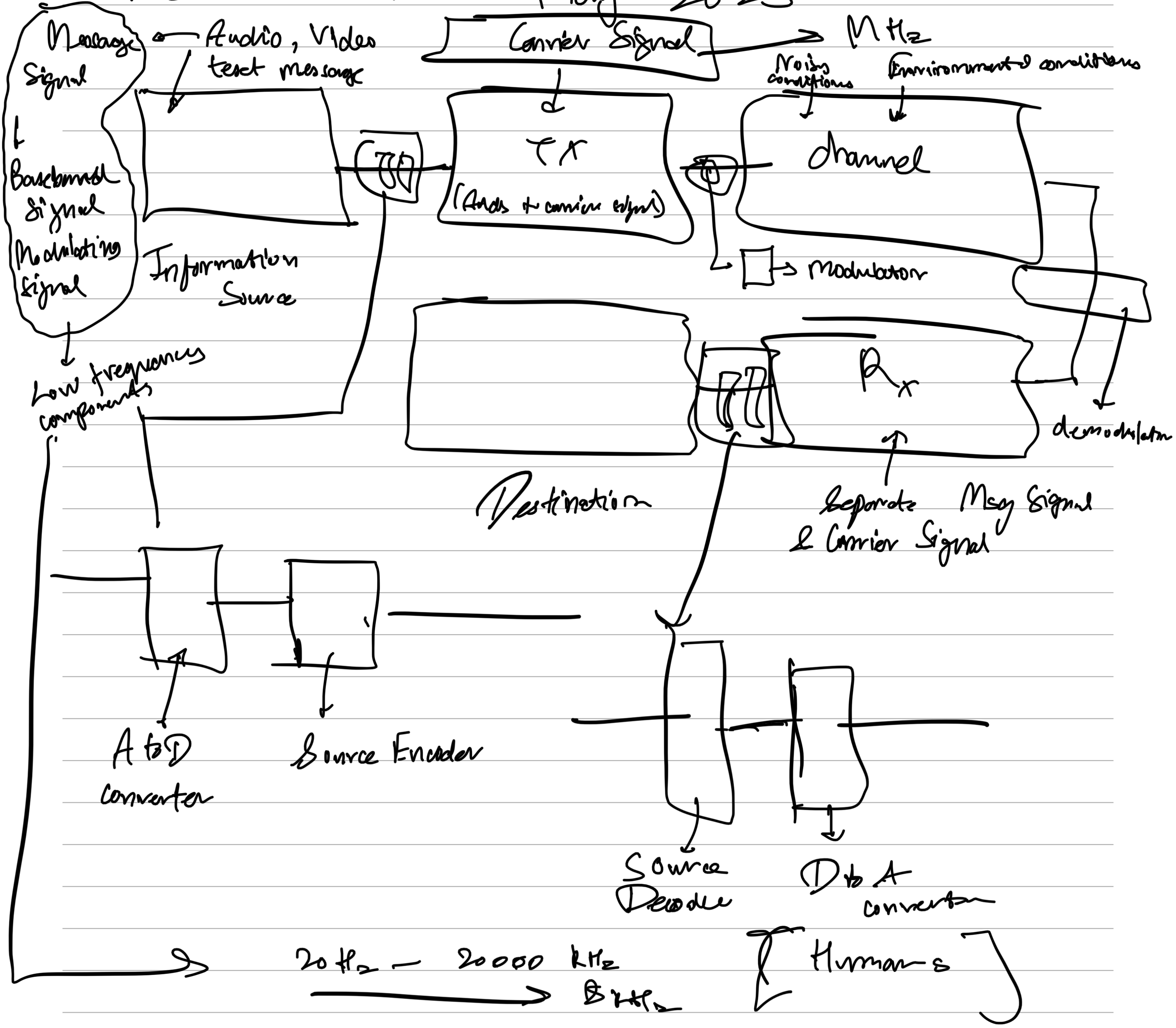
POC 10th May 2023 ca01



Antenna height $h = \frac{\lambda}{4}$ $\lambda = \frac{c}{f}$

$h = \frac{c}{4f}$

for 10 kHz

$\frac{3 \times 10^{8}}{4 \times 10^{4}}$

= 7500 metres

7.5 km

= 7.5 m → for 10 MHz

Modulator: Modulating & Carrier Mix

Demodulator: Separate Carrier & message signal separately

10 MHz + 10 KHz
10 MHz − 10 KHz
10 KHz.

Need for Modulation / Adv of Modulation Techniques:

→ To reduce the antenna height.

Eg:
Antenna height $h = \frac{\lambda}{4}$ $\lambda = \frac{c}{f}$

$h = \frac{c}{4f}$ for 10 KHz

$\frac{3 \times 10^8}{4 \times 10^4}$

$= 7500$ metres

$= 7.5$ Km

→ If we increase the frequency, then height of antenna will decrease.

$= 7.5\text{ m} \rightarrow$ for 10 MHz

→ To avoid mixing of signals

(Multiplexing the modulating and carrier signals)

→ To separate carrier signals and message signals at Tx and Rx

→ Commn channel requires shift of the range of Baseband frequencies into other frequency changes. Suitable for transmission and corresponding shift back to original frequency range after reception.

→ Modulation permits the use of multiplexing.

→ Increases the range of communications.

→ Allows adjustments in the bandwidth.

→ Improves the quality of reception.

Source → Increases SNR $\left(\frac{\text{Signal}}{\text{Noise}}\right)$

Destination → Removes unnecessary noises.

Modulation
- AM → Amplitude Modulation
- FM, PM → Angle modulation

$x(t) = A \sin(\omega t + \phi)$ — A: Amplitude, ω: frequency, ϕ: phase

$\omega = 2\pi f$

★ Amplitude Modulation (AM) – AM is defined as the modulation in which the Amplitude of the carrier signal varies in accordance with the amplitude of the modulating signal keeping its frequency and phase components constant.

Carrier Signal → $c(t) = A_c \cos(2\pi f_c t)$

Modulating Signal → $m(t) = A_m \cos(2\pi f_m t)$

The standard equation for AM

$$s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

$$s(t) = A_c \left[1 + k_a A_m \cos(2\pi f_m t) \right] \cos(2\pi f_c t)$$

$K_a A_m = \mu$

↳ Amplitude sensitivity of the AM

$\mu \rightarrow$ Modulation index

$$s(t) = A_c \left[1 + \mu \cos(2\pi f_m t) \right] \cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + A_c \mu \cos(2\pi f_c t) \cos(2\pi f_m t)$$

$\cos(A) \quad \cos(B)$

$\cos(A)\cos B = \frac{1}{2} \cos(A+B) + \cos(A-B)$

$$s(t) = A_c \cos(2\pi f_c t) + \frac{A_c \mu}{2} \cos(2\pi (f_c + f_m) t)$$

$$+ \frac{A_c \mu}{2} \cos(2\pi (f_c - f_m) t)$$

Apply Fourier Transform on $s(t)$

$$\mathcal{F}\left[\cos(2\pi f_c t) \right] = \frac{1}{2} \left[\delta(f + f_c) + \delta(f - f_c) \right]$$

$$\mathcal{F}\left[\frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2} \right] = \frac{1}{2} \left[\delta(f - f_c) + \delta(f + f_c) \right]$$

$$\mathcal{F}\left[\cos(2\pi (f_c - f_m) t) \right] = \frac{1}{2} \left[\delta(f + (f_c - f_m)) + \delta(f - (f_c - f_m)) \right]$$

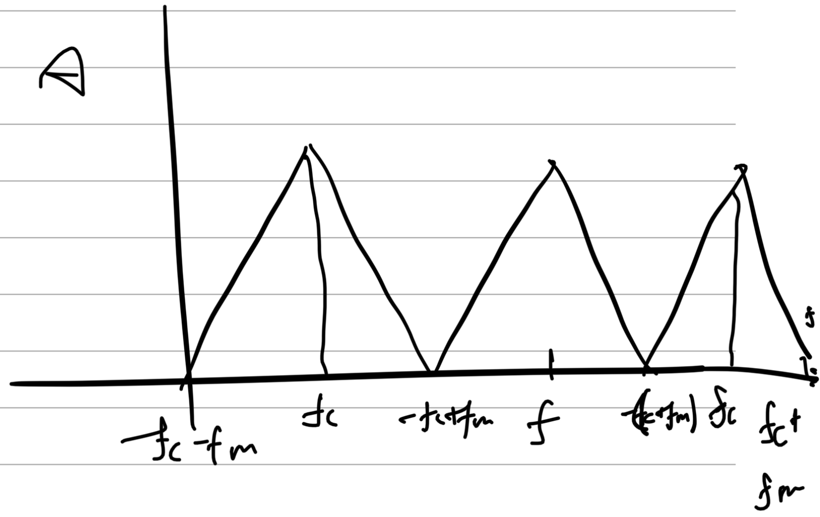
$$\mathcal{F}\left[\cos(2\pi (f_c + f_m) t) \right] = \frac{1}{2} \left[\delta(f + (f_c + f_m)) + \delta(f - (f_c + f_m)) \right]$$

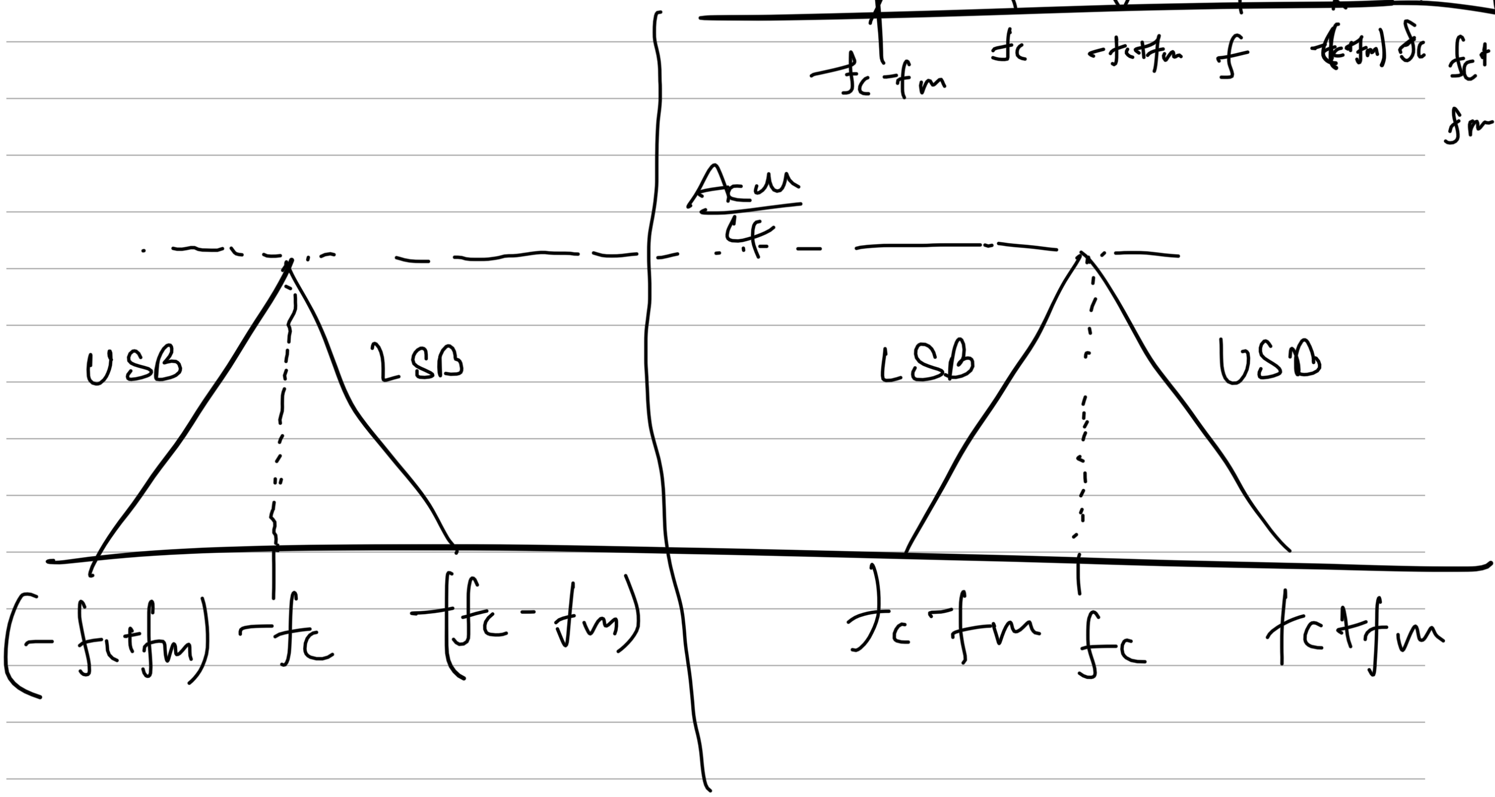
$$= \frac{A_c}{2}\left[\delta(f+(f_c)) + \delta(f-f_c)\right]$$

$$+ \frac{A_c \mu}{2}\left[\delta(f+(f_c-f_m)) + \delta(f-(f_c-f_m))\right]$$

$$+ \frac{A_c \mu}{2}\left[\delta(f+(f_c+f_m)) + \delta(f-(f_c+f_m))\right]$$

Spectrum of AM





$f_{LSB} : f_c - f_m$

$f_{USB} = f_c + f_m$

* Bandwidth: Difference b/w upper freq component & lower

Transmission

$$B_T = f_{USB} - f_{LSB}$$

$\hookrightarrow f_c + f_m \quad \Rightarrow f_c - f_m$

$$= f_c + f_m - (f_c - f_m)$$

$$\boxed{B_T = 2 f_m}$$

Bandwidth of the transmitted signal is twice the modulating signal frequency component.

Modulation Index < Low → Attenuated; High → Distorted.

$$\mu = K_a A_m \quad \text{or} \quad \mu = \frac{A_m}{A_c}$$

$$\eta = \frac{\text{Side band Power}}{\text{Total Power}}$$

[Efficiency of the AM]

$$= \frac{P_{LSB} + P_{USB}}{P_T}$$

$$= \frac{\frac{A_c^2 \mu^2}{8} + \frac{A_c^2 \mu^2}{8}}{\frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{4}}$$

$$\Rightarrow \frac{\frac{A_c^2 \mu^2}{4}}{\frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{4}}$$

$$P_c = \frac{A_c^2}{2R} \quad \left[\text{Ideally } R = 1\right]$$

$$P_{LSB} = \left(\frac{A_c \mu}{2\sqrt{2} R}\right)^2$$

$$P_{USB} = \left(\frac{A_c \mu}{2\sqrt{2} R}\right)^2$$

$$P_c = \frac{A_c^2}{2}$$

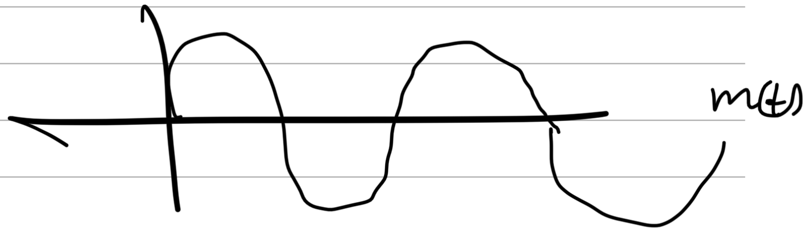
$$P_{USB} = P_{LSB} = \frac{A_c^2 \mu^2}{8}$$

$$\Rightarrow \quad \frac{\mu^2}{2+\mu^2}$$

$$\Rightarrow \quad \boxed{\eta = \frac{\mu^2}{2+\mu^2}}$$

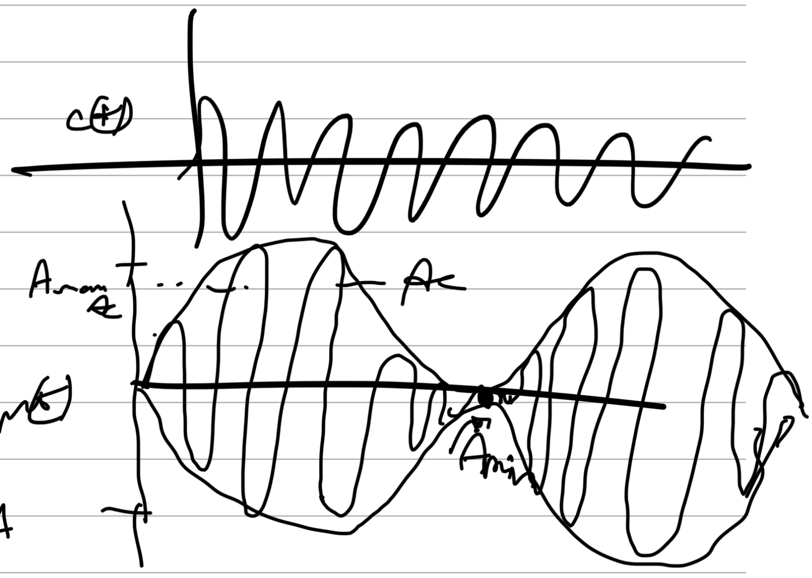
$$\boxed{\eta\% = \left(\frac{\mu^2}{2+\mu^2}\right) \times 100\ \%}$$

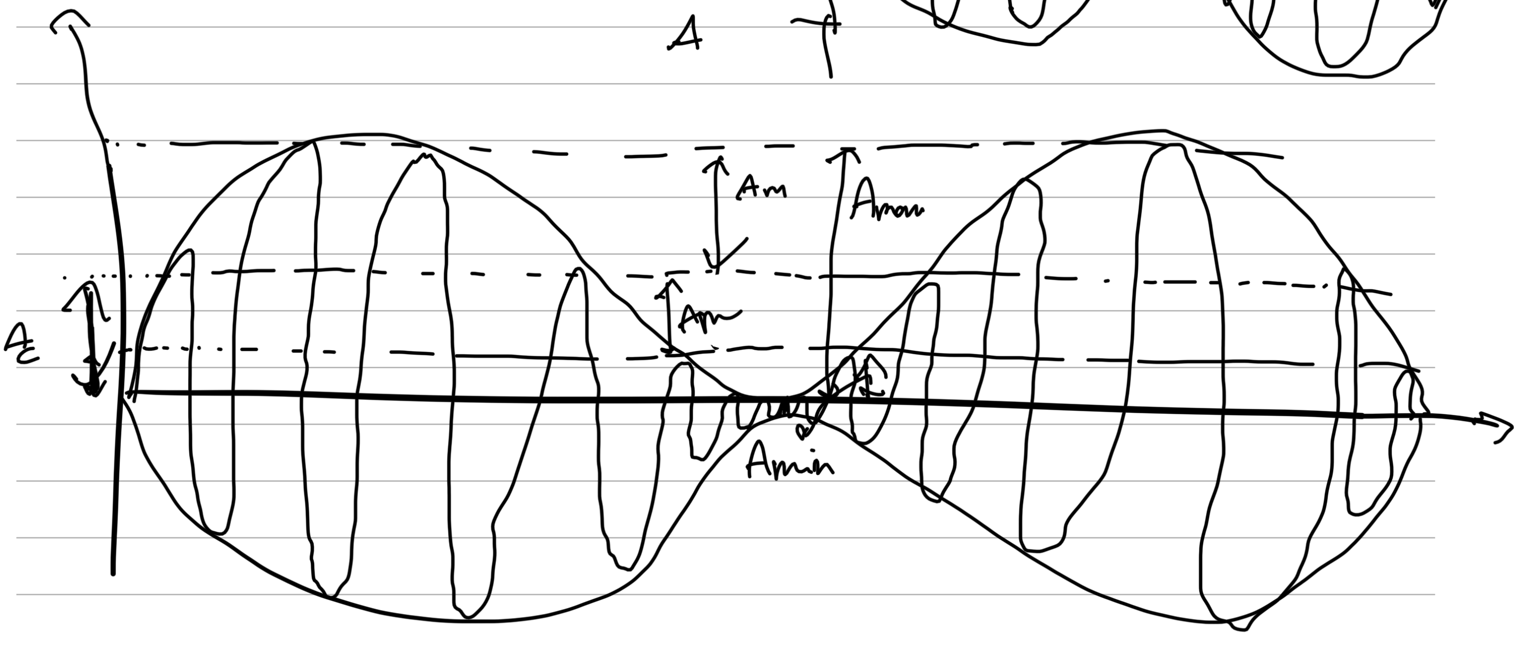
HW: $\mu = \frac{A_m}{A_c}$



Derive:

$\mu = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$





$$s(t) = A_c \left[1 + k_a m(t) \right] \cos(2\pi f_c t)$$

$$\left. \begin{matrix} c(t) \\ m(t) \end{matrix} \right\} \longrightarrow s(t) = c(t)\ m(t)$$

$$\mu = k_a A_m \quad \text{or} \quad \mu = \frac{A_m}{A_c}$$

$$A_m = \frac{A_{max} - A_{min}}{2}$$

$$A_c = A_{max} - A_m$$

$$A_m = \frac{A_{max} - A_{min}}{2}$$

$$A_c = A_{max} - A_m = A_{max} - \left(\frac{A_{max} - A_{min}}{2} \right)$$

$$A_c = \frac{A_{max} + A_{min}}{2}$$

$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

f_{LSB}

$$P_c = \frac{A_c^2}{2R}$$

$$P_{LSB} = P_{USB} = \frac{A_c^2}{2R} \frac{\mu}{4} = \frac{A_c \mu^2}{8R}$$

Ideally $\underline{\underline{R=1}}$

B_T

η

μ

✩ Multitone Amplitude Modulation :-

$$m(t) = m_1(t) + m_2(t) \longrightarrow A_{m_2} \cos(2\pi f_{m_2} t)$$

$$m_1(t) = A_{m_1} \cos(2\pi f_{m_1} t)$$

$$s(t) = A_c [1 + K_a m(t)] \cos(2\pi f_c t)$$

$$s(t) = A_c [1 + K_a (m_1(t) + m_2(t)] \cos(2\pi f_c t)$$

$$s(t) = A_c \left(1 + K_a \left(A_{m_1} \cos(2\pi f_{m_1} t) + A_{m_2} \cos(2\pi f_{m_2} t)\right)\right) \cos(2\pi f_c t)$$

$$\mu_1 = K_a A_{m_1} \qquad \mu_2 = K_a A_{m_2}$$

$$\boxed{s(t) \Rightarrow A_c \left(1 + \mu_1 \cos(2\pi f_{m_1} t) + \mu_2 \cos(2\pi f_{m_2} t)\right) \cos(2\pi f_c t)}$$

$$\Rightarrow A_c \cos(2\pi f_c t) + A_c \mu_1 \cos(2\pi f_{m_1} t) \cos(2\pi f_c t) + A_c \mu_2 \cos(2\pi f_{m_2} t) \cos(2\pi f_c t)$$

$$\cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\Rightarrow A_c \cos(2\pi f_c t) + \frac{A_c \mu_1}{2} \cos(2\pi (f_{m_1} + f_c)) + \frac{A_c \mu_1}{2} \cos 2\pi (f_c - f_{m_1}) t$$
$$+ \frac{A_c \mu_2}{2} \cos(2\pi (f_{m_2} + f_c)) + \frac{A_c \mu_2}{2} \cos(2\pi (f_c - f_{m_2}) t)$$

Apply Fourier Transform on $s(t)$

$$\boxed{\mathcal{F}[\cos(2\pi f_c t)] = \frac{1}{2}[\delta(f + f_c) + \delta(f - f_c)]}$$

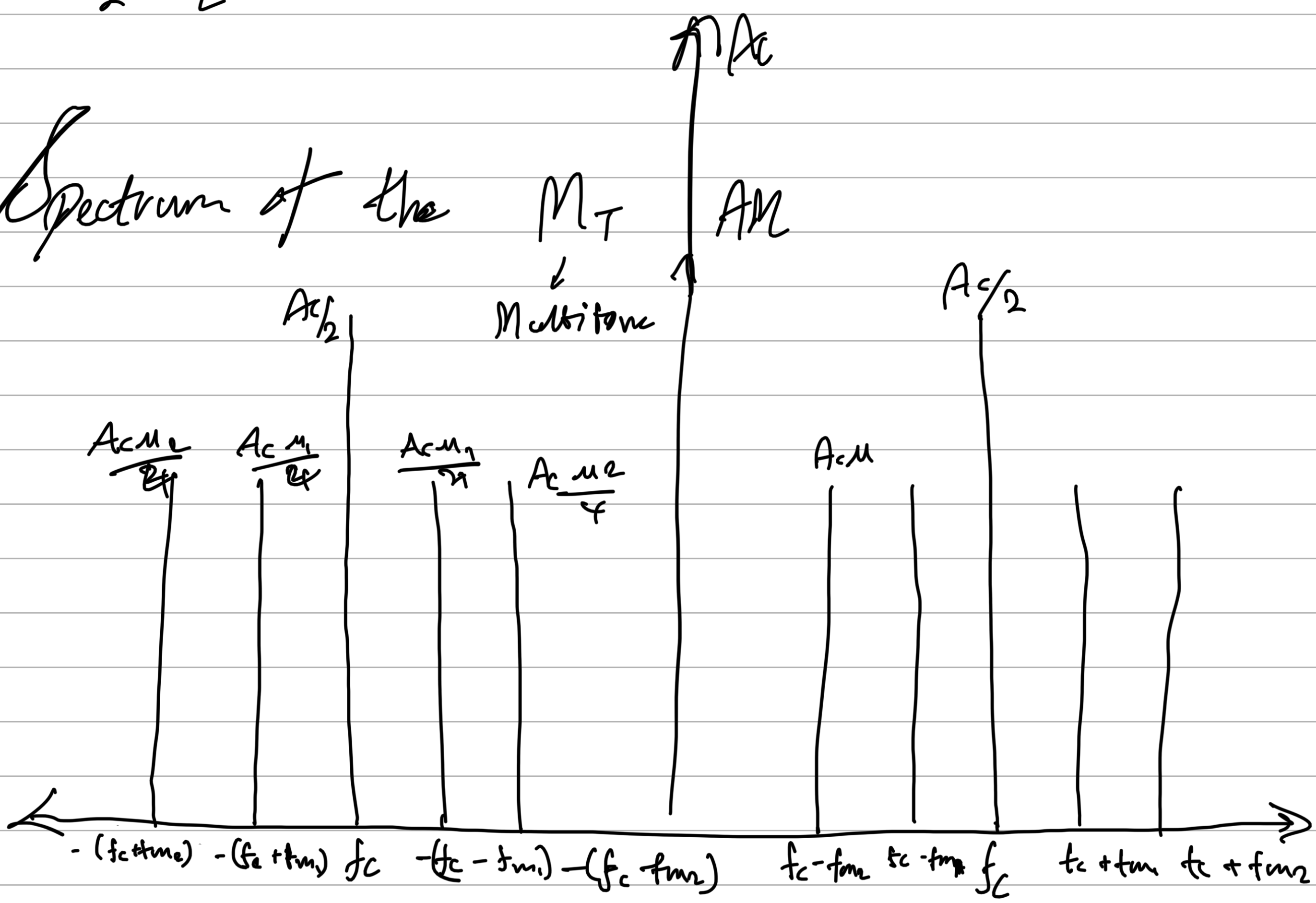
$$is\ s(f) = \frac{A_c}{2}\left[\delta(f+f_c) + \delta(f-f_c)\right] + \frac{A_c\mu}{2}\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right]$$

$$+ \frac{A_c\mu_1}{2}\left[\delta(f+(f_c-f_{m_1})) + \delta(f-f_c-f_{m_1})\right]$$

$$+ \frac{A_c\mu_2}{2}\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_{m_2}))\right]$$

$$+ \frac{A_c\mu_2}{2}\left[\delta(f+(f_c-f_{m_2})) + \delta(f-f_c-f_{m_2})\right]$$

Spectrum of the M_T AM



μ_T :

$$\eta = \frac{P_{SB}}{P_T} \qquad P_c = \frac{A_c^2}{2R}$$

$$P_{LSB_1} = \frac{A_c^2}{2R}\frac{\mu_1^2}{4}, \quad P_{LSB_2} = \frac{A_c^2}{2R}\frac{\mu_2^2}{4}$$

$$P_T = P_c + P_{LSB_1} + P_{LSB_2} + P_{USB_1} + P_{USB_2} \qquad P_{USB_2} = P_{LSB_2}$$

$$P_{SB} = P_{USB_1} + P_{USB_2} + P_{LSB_1} + P_{LSB_2}$$

$$\cancel{\mu_T^2 = \frac{2\mu_1^2 + 2\mu_2^2}{4 + 2\mu_1^2 + 2\mu_2^2}}$$

$$\cancel{\boxed{\mu_T = \frac{\mu_1^2 + \mu_2^2}{2 + \mu_1^2 + \mu_2^2}}}$$

$$P_{SB} = \frac{1}{2} P_c (\mu_1^2 + \mu_2^2)$$

$$P_T = P_c + P_{SB} = P_c + \frac{P_c}{2}(\mu_1^2 + \mu_2^2)$$

$$= P_c\left(1 + \frac{\mu_T^2}{2}\right)$$

$$\Rightarrow \mu_T^2 = \mu_1^2 + \mu_2^2$$

$$\boxed{\mu_T = \sqrt{\mu_1^2 + \mu_2^2}}$$

Q An Audio frequency signal $m(t) = 5\sin(2\pi(1000t)$ is used to AM a carrier of $c(t) = 100\sin(2\pi \times 10^6 t)$. Assume $\mu = 0.4$. Find side band frequency.
Find f_{LSB}, f_{USB}, Amplitude of each side
$B_T \rightarrow$ Transmission Bandwidth. P_T of $R = 100\ \Omega$

Solution:

$A_m = 5$ $\qquad$ $f_m = 1000$ Hz

$A_c = 100$ $\qquad$ $f_c = 10^6$ Hz

$((10^6 - 1000), 10^6 + 1000) \longrightarrow$

$$\boxed{B_T = 2f_m = 2000}$$

$f_{LSB} = 10^6 - 1000$

$f_{USB} = 10^6 + 1000$

$$A = \frac{A_c\ \mu}{4}$$

$$= \frac{100}{4} \times \frac{4}{10}$$

$A = 10$

$f_{LSB} = f_c - f_m$

$f_{USB} = f_c + f_m$

$B_T = 2f_m$

Carrier Amplitude $= \frac{A_c}{2} = \frac{100}{2} = 50V$

Amplitude of sideband $= \frac{A_c\mu}{4} = 10V$

$$P_T = P_c\left(1 + \frac{\mu^2}{2}\right)$$

$$= \frac{A_c^2}{2R}\left(1 + \frac{16}{100 \times 2}\right)$$

$$= \frac{10000}{2 \times 100}$$

$$= 50\left(1 + \frac{8}{100}\right)$$

$$= 50\,(1 + 0.08)$$

$$= 1.08 \times \frac{100}{2}$$

$$\boxed{P_T = 54\,W}$$

Q. Consider a message signal $m(t) = 20\cos(2\pi t)$

$c(t) = 50\cos(100\pi t)$ $\qquad f_c = 50\text{ Hz}$

Standard AM form $\qquad f_m = 1\text{ Hz}$

P_T of $R = 100\,\Omega$ $\qquad A_m = 20\,V$

$A_c = 50\,V$

$$P_T = \frac{A_c^2}{2R}\left(1 + \frac{\mu^2}{2}\right)$$

$$= \frac{(50)^2}{2\,\cancel{(100)}_2}\left(1 + \frac{9}{25\times 2}\right)$$

$$= \frac{\cancel{50}}{\cancel{4}} + \frac{\cancel{50}}{4} \times \frac{9}{\cancel{25\times 2}}$$

$$= 12.5 + \frac{9}{4}$$

$$\boxed{P_T = 14.75\,W}$$

$$\boxed{\begin{aligned} A_{max} &= A_c(1+\mu) \\ A_{min} &= A_c(1-\mu) \end{aligned}}$$

Calculate A_{max} & A_{min}

$$s(t) = 10\cos(2\pi 10^6 t) + 5\cos(2\pi 10^3 t)\cos(2\pi 10^6 t) + 2\cos(2\pi 10^6 t)\cos(2\pi 10^3 t) + \cos(4\pi 10^3 t)$$

holds.

then find Total power delivered & side band power & modulation index

$$s(t) = 10\cos(2\pi 10^6 t)\left[1 + \frac{\cos(2\pi 10^3 t)}{10} + \right.$$

$$s(t) = 10\left[1 + \frac{5}{10}\cos(2\pi 10^3 t) + \frac{2}{10}\cos(4\pi 10^3 t)\right]\cos(2\pi 10^6 t)$$

$A_c = 10$ $\quad$ $\mu_1 = 0.5$ $\quad$ $f_{m_1} = 10^3$ $\quad$ $\mu_2 = 0.2$ $\quad$ $f_{m_2} = 2\times 10^3$ $\quad$ $f_c = 10^6$

$$\mu = \sqrt{\mu_1^2 + \mu_2^2}$$

$$P_{SB} = P_{LSB_1} + P_{LSB_2} + P_{USB_1} + P_{USB_2}$$

$$P_T = P_c + P_{SB} \qquad P_{LSB_1} = \frac{A_c^2}{2R}\frac{\mu_1^2}{4}$$

$$= \frac{A_c^2}{2R}\frac{\mu_1^2}{4} + \frac{A_c^2}{R}\frac{\mu_2^2}{4}$$

$$= \frac{A_c^2}{4R}(\mu_1^2 + \mu_2^2)$$

$$= \frac{100}{4(1)}(0.25 + 0.04)$$

If R not given then 1Ω

$$= \frac{0.29}{4} \times 100$$

$$= \frac{29}{4}$$

$$P_{SB} = \boxed{7.25\ W}$$

$$P_T = 7.25 + \frac{A_c^2}{2R}$$

$$= 7.25 + \frac{100}{2}$$

$$\boxed{P_T = 57.25\ W}$$

$$\boxed{\mu_t = \sqrt{0.29}}$$

2 imp methods of AM generation for power applications :-

1) Switching modulator

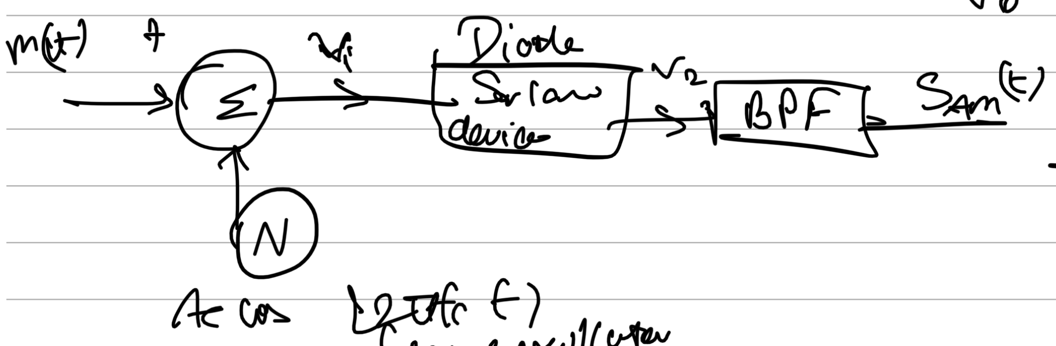
2) Square Law modulator.

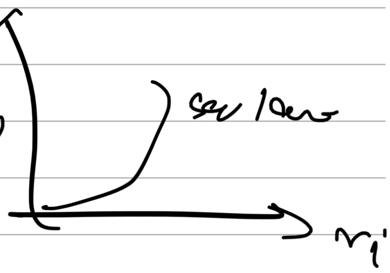
Square Law Modulator

Proportional to input through operation, the device is called non-linear

i/o relation of non lin device

$$V_0 = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \ldots$$





$V_0 = m(t) + c(t)$

$= m(t) + c_t \cos(2\pi f_c t)$

$$v_0 = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 \ldots$$

$= a_1 (m(t) + A_c \cos(2\pi f_c t)) + a_2 (m(t) + A_c \cos(2\pi f_c t))^2 + a_3 [\ldots]^3$

$= a_1 m(t) + a_1 A_c \cos(2\pi f_c t) + a_2 [m^2(t) + A_c^2 \cos^2(2\pi f_c t) + 2a_2 m(t) A_c \cos(2\pi f_c t) \ldots]$

[message signal can't be transmitted]

$\frac{1+\cos 2\theta}{2}$

AM = $s(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$

$\Rightarrow a_1 A_c \cos 2\pi f_c t + 2a_2 A_c \cos(2\pi f_c t)$

$$v_2 = a_1 A_c \cos(2\pi f_c t) + 2 a_2 A_c \cos(2\pi f_c t)$$

$$= a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos(2\pi f_c t)$$

→ Derived from Sq law modulator

$$s_{AM}(t) = A_c [1 + k_a m(t)] \cos(2\pi f_c t)$$

→ Standard eqⁿ of AM

$$= A_c' = a_1 A_c$$

$$k_a' = \frac{2a_2}{a_1} \quad \text{[Amplitude sensitivity]}$$

$$\mu = k_a m(t) = k_a m(t) \frac{2a_2}{a_1} A_m \cos(2\pi f_m t)$$

$$\boxed{\mu = \frac{2a_2}{a_1} A_m}$$ → Modulation index for Sq law modulator.

$$\boxed{f_c - f_m > 2 f_m \text{ or } f_c > 3 f_m}$$

Output AM signal is free from distortion and attenuation only when ↑

See diagram from TB [oscilloscope]

Switch modulator

$v_1 = m(t) + c(t)$

$= m(t) + A_c \cos(2\pi f_c t)$ — ①

$|m(t)| << A_c$

Output of diode is $v_2 = \begin{cases} v_1 & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$

the output of diode v/w

$T_0 = 1/f_c$ Output of diode varies b/w 0 and v_1 at a rate

When $c(t)$ is +ve, $v_2 = v_1$ and diode is forward biased.

When $c(t)$ is -ve, $V_2 = 0$ and diode is reverse biased.

Based upon above operation, switching response of diode is periodic rectangular wave with amplitude 1.

Mathematically output of diode is written as

$v_2 = v_1 \; g_p(t)$ —②

Is Fourier series transform

$$V_2 = [m(t) + A_c \cos(2\pi f_c t)]\, g_p(t)$$

$$T_0 = \frac{1}{f_c}$$

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c (2n-1) t)$$

Formula

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \frac{(-1)^{f-1}}{2n-1} \cos(2\pi f_c (1))$$

f odd harmonic

$$V_2 = [m(t) + A_c \cos(2\pi f_c t)]$$
$$\left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + \frac{2\cos(6\pi f_c t)}{3\pi} + \ldots\right.$$

$$V_2 = \frac{1}{2} m(t) + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t)$$
$$+ \frac{2A_c}{\pi}\cos^2(2\pi f_c t)$$

expand

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta$$

$$v_2 = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t)$$

$$v_1 = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t)$$

X → discard

$$+ \frac{A_c}{\pi} + A_c\cos(4\pi f_c t)$$

required AM is centered at f_c is obtained by passing v_2 through an ideal BPF having centre frequency f_c and $B_T = 2f_m$ Hz.

Output of BPF is

$$v_2' = \frac{2}{\pi} m(t)\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t)$$

$$= \frac{A_c}{2}\left[1 + \frac{2}{\pi} \times \frac{2}{A_c} m(t)\right]\cos(2\pi f_c t)$$

$$V_2' = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right]\cos(2\pi f_c t)$$

Comparing with standard AM wave

where $K_A = \frac{4}{\pi A_c}$ is called Amplitude sensitivity

$\therefore$ modulation index of output signal is given by

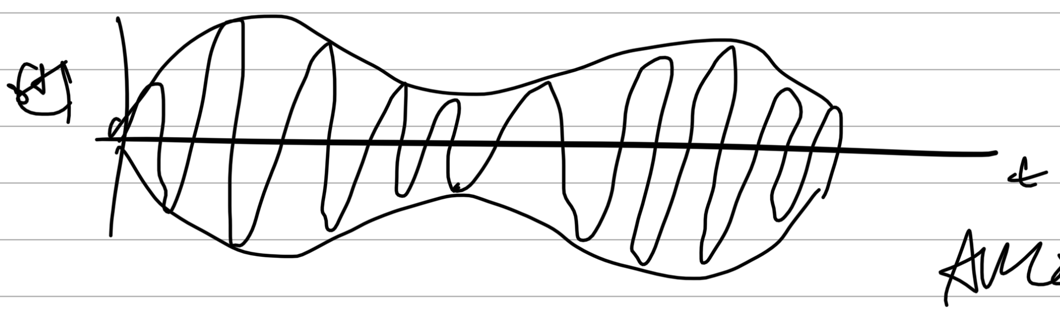
$$K_A = \frac{4}{\pi A_c}$$

$$\mu = A_c K_a$$

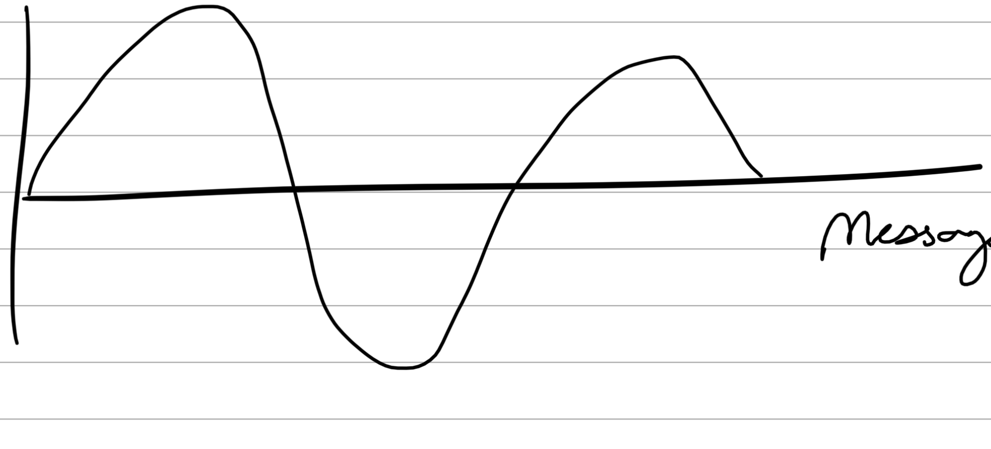
$$\boxed{\mu = \frac{4 A_m}{\pi A_c}}$$

✶ Demodulation or Detection

→ inverse of modulation process



AM signal



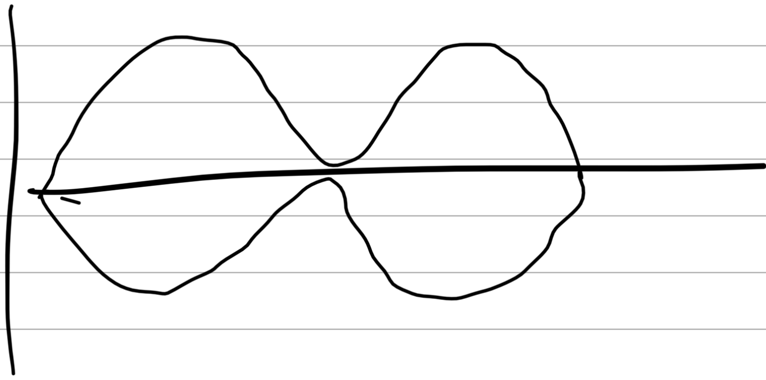
Message signal

$\mu > 1$ over modulation

$\mu = 1$ critical modulation

$\mu < 1$ under modulation

$\mu = 1$

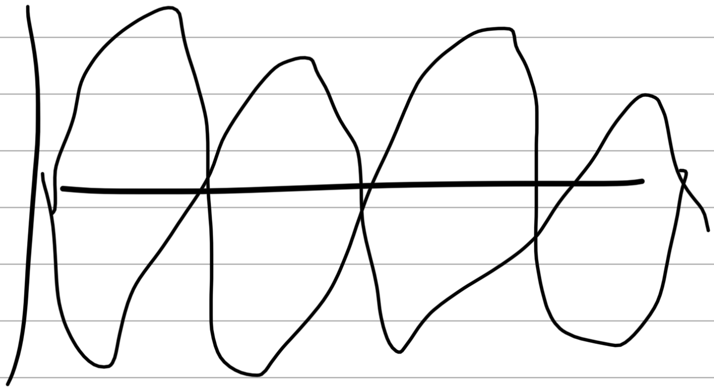


→ Sq law Demod

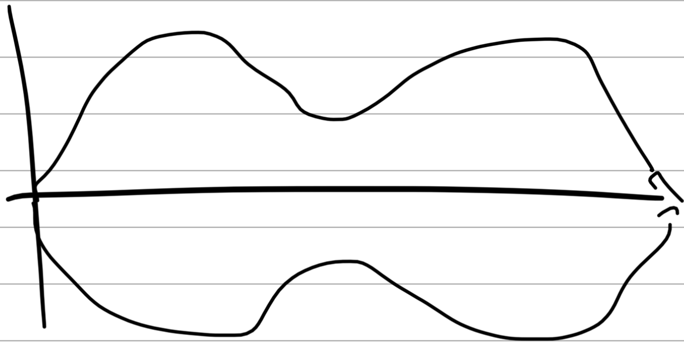
→ Envelope detector

→ synchronous detection

$\mu > 1$



$\mu < 1$



$$S_m(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

Sq Law Demodulator :

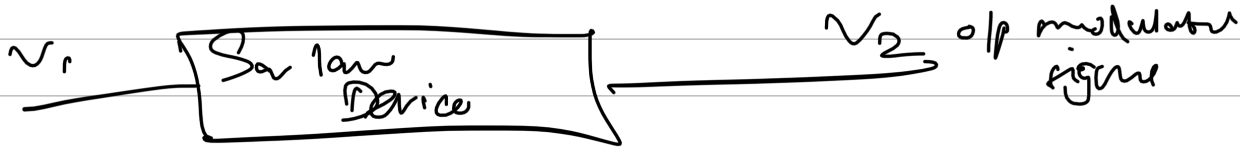
AM signal → Square Law mod → (SLP) o/p → LPF → Message signal

↳ Diode

{modulated signal ↳ non linear operation

As we know AM signal:

$$A_c\left(1+k_a m(t)\right)\cos(2\pi f_c t)$$



relation b/w input & output of a sq law device is given as

$$V_2 = a_1 V_1 + a_2 V_1^2 + a_3 V_1^3 + \dots + a_n V_1^n \rightarrow \text{nonlinear o/p.}$$

Where a_1, a_2, a_3 are sq law constants.

$$S_{AM}(t) = A_c\left(1+k_a m(t)\right)\cos(2\pi f_c t)$$

$$= A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t)$$

SLD o/p $= a_1 S_{AM}(t) + a_2 S_{AM}^2(t)$

$$= a_1\left(\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t)\right)$$

$$+ a_2\left(A_c^2\cos^2(2\pi f_c t) + A_c^2 k_a^2 m^2(t)\cos^2(2\pi f_c t) + 2A_c^2 k_a m(t)\cos^2(2\pi f_c t)\right)$$

SLD o/p $= a_1 A_c\cos(2\pi f_c t) + a_1 A_c k_a m(t)\cos(2\pi f_c t)$

$$+ a_2 \frac{A_c^2}{2} (1 + \cos(4\pi f_c t))$$

$$+ a_2 \frac{A_c^2 k_a^2 m^2(t)}{2} (1 + \cos(4\pi f_c t))$$ → eliminated by LPF

$$+ \frac{2 a_2 A_c^2 k_a m(t)}{2} (1 + \cos(4\pi f_c t))$$

After passing through LPF, we get

$$(SLD)_{o/p} = \frac{a_2 A_c^2 k_a^2 m^2(t)}{2} + a_2 A_c^2 k_a m(t)$$

unwanted signal / noise — Desired message signal

Signal to Noise Ratio:

⇒ Ratio of strength of signal carrying information (S) to that of unwanted interference (N).

Therefore, SNR: S/N

SNR >> 1, msg can be perfectly reconstructed by filter.

If SNR << 1, the message signal cannot be reconstructed back perfectly.

$$\frac{S}{N} = \frac{a_2 A_c^2 k_a m(t)}{\left(\frac{a_2 A_c^2 k_a^2 m^2(t)}{2}\right)} = \frac{2}{k_a m(t)}$$

Let the message signal, $m(t) = A_m \cos(2\pi f_m t)$

$$\frac{S}{N} = \frac{2}{K_a A_m \cos(2\pi f_m t)} = \frac{2}{\mu \cos(2\pi f_m t)}$$

$$\frac{S}{N} = \frac{\text{signal to noise}}{\text{ratio}} \propto \frac{1}{\mu}$$

Drawback of square law Demodulator:

→ The SNR at the input of a LPF is inversely proportional to the modulation index (μ).

→ Higher the modulation index, lower is the SNR.

→ To increase the SNR, modulation index has to be decreased which decreases the modulation efficiency and therefore, we get inefficient power distribution.

→ Practically not preferred for AM demodulation

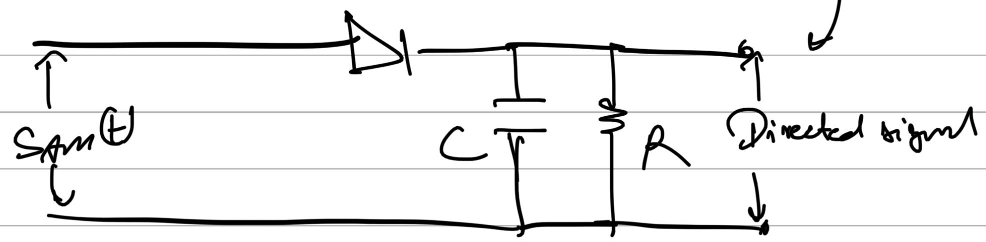
24 May 2023

Envelope Detector:

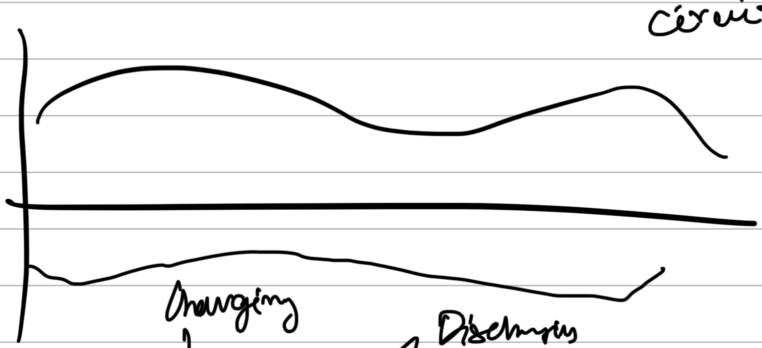
Limitation of sq law Demodulator can be overcome by simple and economical Envelope detector

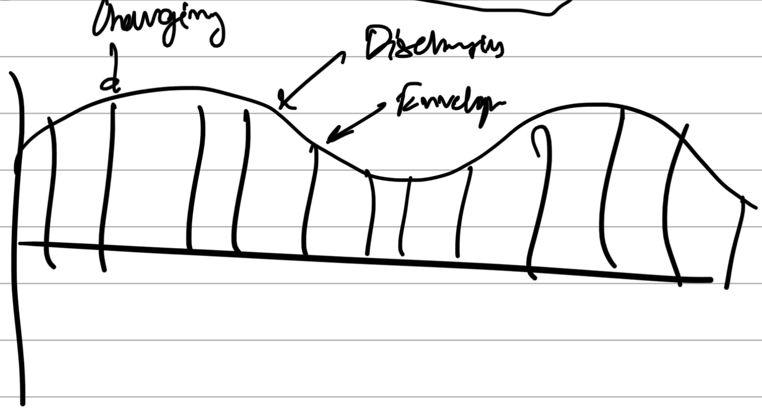
→ has diode & RC circuit

Add one more R_L resistor down



circuit w/ only +ve no -ve half cycles





→ In +ve half cycle of the AM signal, Diode conducts and current flows through 'R' through "R" whereas, in the -ve half cycle diode is reverse biased and no current flows through ~~R~~. As a result only +ve half of the AM wave appears across RC.

+ve half cycle → forward bias

'C' charges rapidly to peak value of input signal

When i/p signal falls below this val, diode becomes reverse biased & 'C' discharges slowly through load resistor R_L.

→ This discharging process continues until the next +ve half cycle when the input signal becomes greater than voltage across capacitor, the diode conducts again and process is repeated

* Selection of RC time constant:

The charging time constant '$R_s C$' should be [illegible] as compared to the carrier period $1/f_c$. Therefore, $R_s C << 1/f_c$

so capacitor 'C' charges rapidly

* On the other hand the discharging time constant "$R_L C$" should be long enough to ensure that capacitor discharges slowly through load resistance "R_L" between the positive peak of the carrier wave i.e.

$$\frac{1}{f_c} << R_L C << \frac{1}{f_m}$$

Where f_m is maximum modulating frequency

Envelope detector input.

$A_m \cos(2\pi f_m t)$

$A_c(1 + K_a m(t)) \cos(2\pi f_m t)$

$A \cos(2\pi f_c t) + B \sin(2\pi f_c t)$

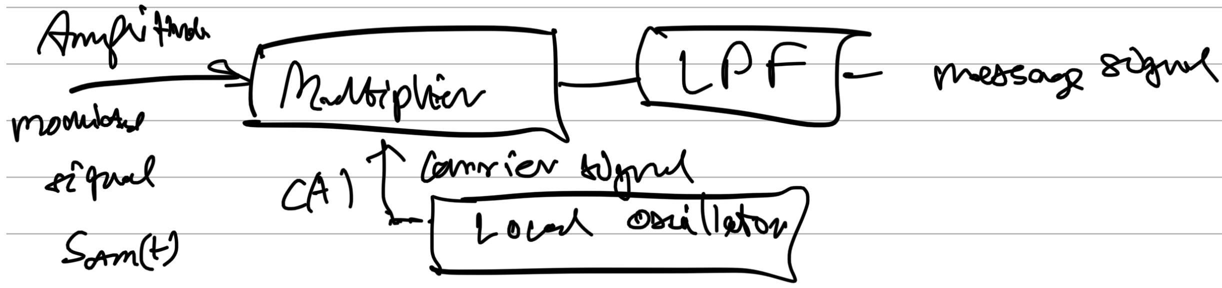
Envelope detector output.

$A_c(1 + K_a m(t))$

$\sqrt{A^2+B^2} \cos(2\pi f_c t + \tan^{-1}(B/A))$

Synchronous Detector.

* Unlike envelope detector, synchronous detector can be used for demodulation of under modulated as well as over-modulated signal



* It is known as Coherent Detector:

For perfect reconstruction of message signals.
the output of local oscillator should be perfectly synchronized in both frequency as well as phase with that of the carrier signal.

* Frequency synchronisation can be easily achieved However, achieving of phase synchronization is very complex and difficult to achieve.

* To achieve phase synchronisation, additional circuitry has to be added which makes synchronous detector very complex

AM signal

$$AM(t) = A_c (1 + K_a m(t)) \cos(2\pi f_c t)$$

Local oscillator output

$$c(t) = \cos(2\pi f_c t)$$

The frequency and phase of the local oscillator o/p and the amplitude modulated signal are both in synchronisation.

* The output of the multiplier can be written as:

$\Rightarrow A_c [1 + K_a m(t)] \cos(2\pi f_c t) * \cos(2\pi f_c t)$

$\Rightarrow A_c \cos^2 2\pi f_c t + A_c K_a m(t) \cos^2(2\pi f_c t)$

$= \frac{A_c}{2}(1 + \cos(4\pi f_c t)) + A_c K_a \frac{m(t)}{2}(1 + \cos(4\pi f_c t))$

$$= \frac{A_c}{2} + \frac{A_c}{2}\cos(4\pi f_c t) + \frac{A_c k_a}{2} m(t) + \frac{A_c k_a m(t)}{2} \cdot \cos(4\pi f_c t)$$

(Mul) $o/p = \frac{A_c}{2} + \frac{A_c}{2}\cos(4\pi f_c t) + \frac{A_c k_a}{2} m(t) + \frac{A_c k_a m(t)\cos(4\pi f_c t)}{2}$

DC term can be blocked by capacitor

Eliminated by LPF

* The output of synchronous detector is $\frac{A_c k_a m(t)}{2}$

The output of synchronous detector is the message signal with a different amplitude.
The desired amplitude can be achieved by passing the op through an amplifier.

Double Side band Suppressed Carrier (DSB SC)

Modulation :

Introduction :-

$s(t) = c(t)\, m(t)$ $\quad k_a\, m(t) \cos 2\pi f_c t$

Time Domain Representation & Frequency Domain of DSB SC wave :- OR single tone information :

Let $m(t)$, be the modulating signal having a bandwidth equal to "f_m" Hz and $c(t) = A_c \cos(2\pi f_c t)$ represents the carrier, then the time domain expression for DSB-SC wave is

$$s(t) = m(t)\, c(t)$$

$$s(t) = A_c \cos(2\pi f_c t) \cdot m(t)$$

Now substitute $m(t) = A_m \cos(2\pi f_m t)$, then

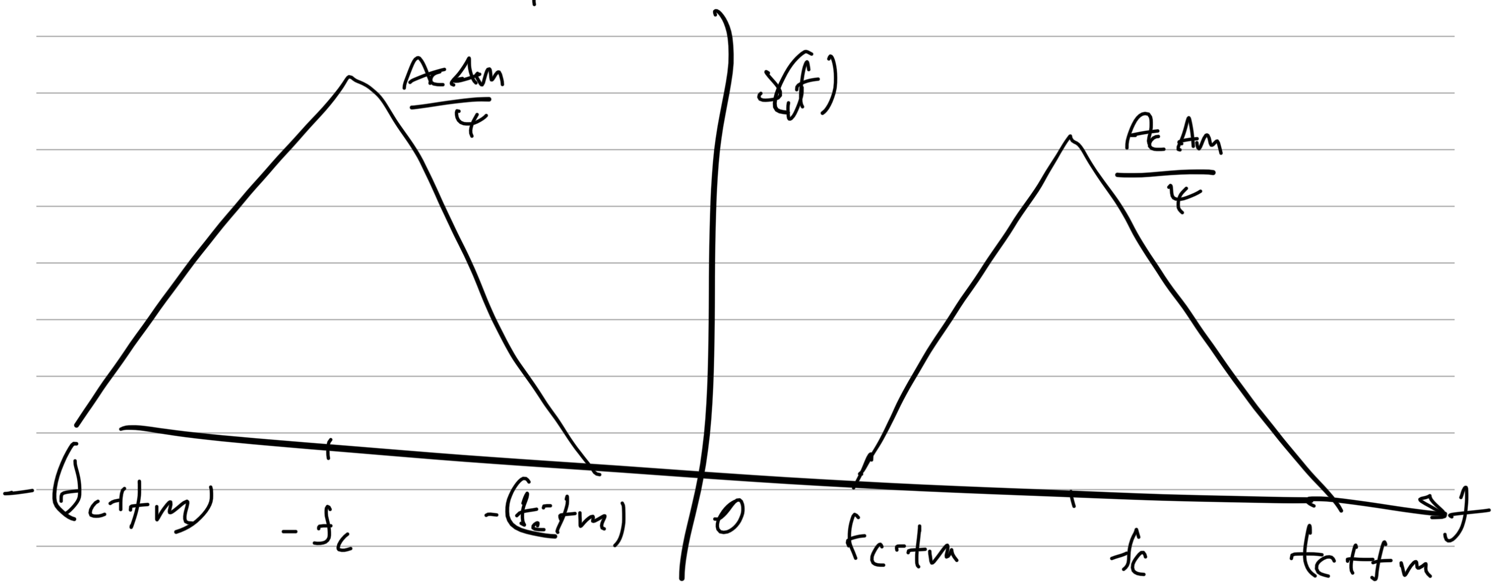
$s(t) = A_c \cos(2\pi f_c t) \cdot A_m \cos(2\pi f_m t)$

$$= \frac{A_c A_m}{2} \left(\cos(2\pi(f_c + f_m)t) + \cos(2\pi(f_c - f_m)t) \right)$$

Taking Fourier Transform on both sides, we have

$$\mathcal{F}\left[\cos(2\pi f_c t)\right] = \frac{1}{2}\left[\delta(f+f_c) + \delta(f-f_c)\right]$$

$$S(f) = \frac{A_c A_m}{4}\left\{\delta(f+(f_c-f_m)) + \delta(f-(f_c-f_m))\right\}$$
$$+ \frac{A_c A_m}{4}\left\{\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right\}$$



Spectrum of a DSB-SC wave

Fig shows amp spectrum of a DSB SC signal. We observe that either side of "f_c", we have lower and upper sideband also the carrier term is suppressed in the spectrum as there are no impulses at "f_c"

The minimum transmission bandwidth in DSB-SC wave is "$2f_m$".

Note : * The signal s(t) undergoes a phase reversal whenever the message signal crosses zero.

* A DSB_SC signal can be generated by a multiplier. A carrier signal can be suppressed by adding a carrier signal opposite in phase but equal in magnitude to the amplitude modulated wave, so the carrier gets cancelled.

Finally, Double side bands are available in the DSB-SC wave