IIT2021134

Indian Institute of Information Technology Allahabad
B.Tech. (IT) – III Sem (All Sections)
C3 Evaluation Exam
Subject: Introduction to Marketing

Date: 25.11.2022 Timings: 10:00 – 12:00
MM: 40
Note: Attempt all questions. All questions carry equal marks

Read the given case study and answer the questions given at the end

BMW is the ultimate driving machine. Manufactured by the German company Bayerische Motoren Werke AG, BMW stands for both performance and luxury. The company was founded in 1916 as an aircraft-engine manufacturer and produced engines during World Wars I and II. It evolved into a motorcycle and automobile maker by the mid-20th century, and today it is an internationally respected company and brand with $106 billion in sales in 2012.

BMW's logo is one of the most distinctive and globally recognized symbols ever created. The signature BMW roundel looks like a spinning propeller blade set against a blue sky background—originally thought to be a tribute to the company's founding days as an aircraft-engine manufacturer. Recently, however, a *New York Times* reporter revealed that the logo, which features the letters *BMW* at the top of the outer ring and a blue-and-white checkered design in the inner ring, was trademarked in 1917 and meant to show the colors of the Free State of Bavaria, where the company is headquartered.

BMW's growth exploded in the 1980s and 1990s, when it successfully targeted the growing market of baby boomers and professional yuppies who put work first and wanted a car that spoke of their success. BMW gave them sporty sedans with exceptional performance and a brand that stood for prestige and achievement.

The cars, which came in a 3, 5, or 7 Series, were basically the same design in three sizes. It was at this time that yuppies made Beemer and Bimmer the slang terms for BMW's cars and motorcycles, popular names still used today. At the turn of the century, consumers' attitudes toward cars changed. Research showed that they cared less about the bragging rights of the BMW brand and instead desired a variety of design, size, price, and style choices. As a result, the company took several steps to grow its product line by targeting specific market segments.

This resulted in unique premium-priced cars such as SUVs, convertibles, and roadsters, as well as less expensive compact cars like the 1 Series. In addition, BMW redesigned its 3, 5, and 7 Series cars, making them unique in appearance yet maintaining their exceptional performance. BMW's full range of cars now includes the 1 Series, 3 Series, 5 Series, 6 Series, 7 Series, X Series, Z4 Roadster, M Series, Hybrids, and BMWi.

BMW created the lower-priced 1 Series and X1 SUV to target the "modern mainstream," a group who are also family-focused and active but had previously avoided BMWs because of their premium cost. The 1 Series reached this group with its lower price point, sporty design, and luxury brand. The X1 and X3 also hit home with a smaller, less expensive SUV design.

The redesign of the 7 Series, BMW's most luxurious car, targeted a group called "upper conservatives." These wealthy, traditional consumers don't usually like sportier cars, so BMW added electronic components such as multiple options to control the windows, seats, airflow, and lights, a push-button ignition, and night vision, all controlled by a point-and-click system called iDrive. These enhancements added comfort and luxury, attracting drivers away from competitors like Jaguar and Mercedes.

BMW successfully launched the X Series by targeting "upper liberals" who had achieved success in the 1990s and gone on to have children and take up extracurricular activities such as biking, golf, and skiing. These consumers needed a bigger car for their active lifestyles and growing families, so BMW created a high-performance luxury SUV. BMW refers to its SUVs as sport *activity* vehicles in order to appeal even more to these active consumers.

BMW introduced convertibles and roadsters to target "post-moderns," a high-income group that continues to attract attention with more showy, flamboyant cars BMW's 6 Series, a flashier version of the high-end 7 Series, also targeted this group. BMW uses a wide range of advertising tactics to reach each of its target markets. However, the company's U.S. tagline, "The Ultimate Driving Machine," has remained consistent since it first launched there in 1974. During that time, sales have grown to more than 300,000 units in the United States in 2013. In recent years, BMW has returned to emphasizing performance over status, stating, "We only make one thing, the ultimate driving machine."

BMW owners are very loyal to the brand, and enthusiasts host an annual Bimmerfest each year to celebrate their cars. The company nurtures these loyal consumers and continues to research, innovate, and reach out to specific segment groups year after year.

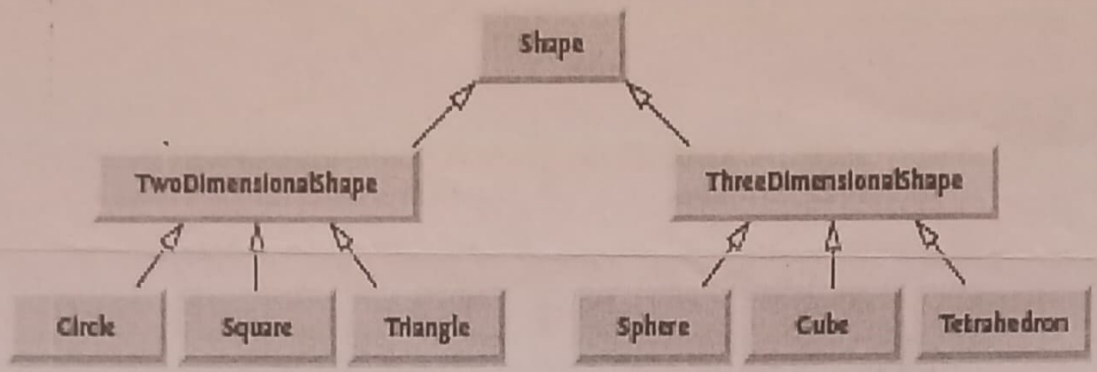
Questions

1. How does BMW segment its consumers? Why does this work for BMW?
2. What does BMW do well to market to each segment group? Where could it improve its marketing strategy?
3. Should BMW ever change its tagline, "The Ultimate Driving Machine"? Why or why not

Object Oriented Methodologies
C3 Review Test
Time: 2 hours

Question. 1 **[05 Marks]**

The world of shapes is much richer than the shapes included in the inheritance hierarchy of Fig. below. Write down all the shapes you can think of both two-dimensional and three-dimensional and form them into a more complete Shape hierarchy with as many levels as possible. Your hierarchy should have class Shape at the top. Classes TwoDimensionalShape and ThreeDimensionalShape should extend Shape. Add additional subclasses, such as Quadrilateral and Sphere, at their correct locations in the hierarchy as necessary. Write the attributes and methods of parent and child classes.



Question 2: A programmer writes the following code in order to be able to completely clone an object of type Car. Answer the following with suitable reason based on the below code snippet:

```
public class Tyre {
                private int treadRemaining;
                public void SetTread(int t) { treadRemaining=t; }
                public int GetTread() { return treadRemaining; }
}
public class Car extends Vehicle implements Cloneable {
                private Tyre tyres[] = new Tyre[4];
                public Car() {
                        for (int i=0; i<4; i++) tyres[i] = new Tyre();
                            }
public Object clone() throws CloneNotSupportedException {
                        Car c = new Car();
                        c.tyres = this.tyres;
                        return c;
   }
}
```

A. Explain what it means for the treadRemaining field to be private. Explain why it is good programming practice for such fields to be private. **[1.5 marks]**
B. Identify and explain two reasons why this code may not function as intended. **[1.5 marks]**
C. Rewrite the code to address the problems you have identified and allow Car objects to be fully cloned. **[2 marks]**

Question 3:

A. Explain the use of access modifiers in java. Write different access modifiers in java with examples. **[2 Marks].**
B. Differentiate between the usage of Class inheritance and Object composition in Java with examples and relative merits-demerits **[2 Marks]**
C. What is polymorphism? With examples explain how polymorphism is implemented in Java. **[1 Mark]**

Question 4:

Explain the role of a Layout Manager in the GUI design. Explain the Card and Grid Layout Managers. **[3 Mark]**

Question 5:

What do you understand by cohesion and coupling? Explain if the metrics need to be generally high or low for a good object-oriented design. **[2 Mark]**

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Department of Information Technology

Theory of Computation, C3 Review Test

B.Tech. IT & IB (III semester)

Duration: 90 minutes (10:00 AM – 11:30 AM) **Maximum Marks: 30**

Instructions:

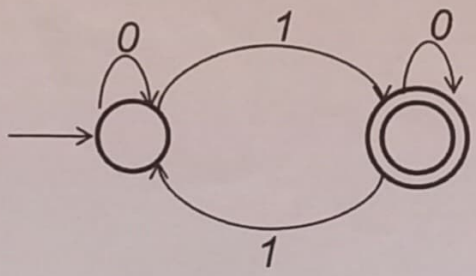
1. *All questions are compulsory. All the subparts of a question are to be attempted together.*

1. A commonly used operation in Functional Programming languages is zip, which takes two lists of equal size as arguments and combines them by interleaving their elements. Languages like LISP, Haskell, R, python, scala etc. implement this operation in some form. An example of its use in Haskell is as below: **[5 marks]**

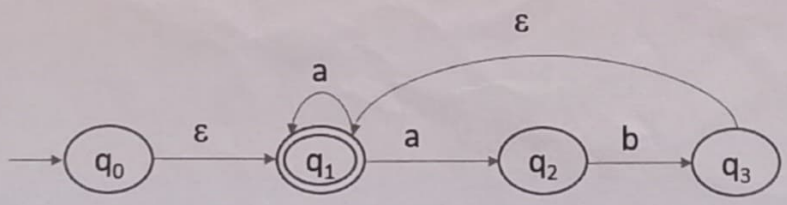
   zip [a, b, c] [d, e, f] = [a, d, b, e, c, f]

   Now, for the following DFA M, show that L = {w=w_1 zip w_2 : w_1, $w_2 \in$ L(M) and $|w_1| = |w_2|$} is regular by constructing a DFA M' with L(M') = L.

   [eg. w_1 =abc, w_2=xyz, w_1 zip w_2 = axbycz]



2. Convert the following ε-NFA to DFA (directly, without converting it to NFA). **[2 marks]**



3. Simplify the grammar and convert it to CNF: **[3 marks]**

$S \rightarrow aAa \mid bBb \mid \varepsilon$

$A \rightarrow C \mid a$

$B \rightarrow C \mid b$

$C \rightarrow CDE \mid \varepsilon$

$D \rightarrow A \mid B \mid ab$

4. Design a Turing machine to obtain the reverse of a string from the given input string over $\Sigma = \{a, b\}$. **[5 marks]**

5. Design a Turing Machine to compute proper subtraction. **[5 marks]**

i.e., m-n for $m \geq n$

0 for $m < n$

6. State true or false with proper justification:

a) If L is a recursive language, its complement is also recursive. **[2.5 marks]**
b) Intersection of two Context-Free Languages is also Context-Free. **[2.5 marks]**

7. Find an NPDA with two states for the language $L = \{a^n b^{n+1} : n \geq 0\}$ by first finding the CFG for L and then converting it to NPDA. **[5 marks]**

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INDIAN INSTITUTE OF INFORMATION TECHNOLOGY ALLAHABAD
Department of Information Technology
Operating Systems (IOPS)
B.Tech. (IT) -3rd Semester, Session: JULY-DEC 2022

Full marks: 80 Marks **Duration: 2 hrs.**

Group-A: All questions are compulsory (20 marks):-

A.1	What is the purpose of paging the page table?	2
A.2	A computer has **six** tape drives, with *n* processes competing for them. Each process may need **two** drives. What is the maximum value of *n* for the system to be deadlock free, Explain?	2
A.3	What is the disadvantage of using total resource ordering for deadlock prevention in distributed systems?	2
A.4	When performing a context switch, the old process information is saved. Where does such information get saved?	2
A.5	What can be the consequences if the memory management unit handling a segmented memory does not check the segment limit register?	2
A.6	State True/False. If false give the correct answer a. Linked allocation of files prevents internal disk fragmentations b. Swap space exists in main memory c. mounting a file system removes portion of file system into a directory structure d. In UNIX, open system call returns pointer to the entry in the system wide table.	4
A.7	A compare-and-swap instruction (CAS, or CMPXCHG on Intel systems) allows you to: (a) Modify a memory location only if its contents match a given value. (b) Exchange the contents of two memory locations if their values are different. (c) Exchange the contents of two memory locations if a lock is not set. (d) Exchange the contents of two memory locations if a lock is set.	1
A.8	A thread that is blocked on a semaphore is awakened when another thread: (a) Tries to decrement a semaphore's value below 0. (b) Tries to increment the semaphore. (c) Causes the semaphore's value to reach a specific number. (d) Tries to block on the same semaphore	1
A.9	What is race condition?	2
A.10	State the role of short term scheduler and long term scheduler.	2

Group-B: Each of the following question contain 12 Marks. (12*5=60 marks):-

Answer any FIVE

B.1	a	What is the difference between mutex_trylock and mutex_lock? Write a code in C for summing numbers from an array using thread.	4
	b	What are the advantages of Thread over Process? Describe different kinds of Multithreading models. What is Hyper Threading or Simultaneous Multithreading (SMT)? Describe different threading issues.	4
	c	What do you mean by Message Passing Systems? Discuss briefly about different types of Message Passing Systems. What are the issues related to Message Passing Systems?	4
B.2	a	Can a system be in a state that is neither deadlocked nor safe? If so, give an example. If not, prove that all states are either deadlocked or safe.	3
	b	Consider a main memory with five-page frames and the following sequence of page references: 3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3. Compare the number of page faults with respect to page replacement policies First In First Out (FIFO) and Least Recently Used (LRU)?	4
	c	Consider a system with 4 types of resources R1 (3 units), R2 (2 units), R3 (3 units), R4 (2 units). A non-preemptive resource allocation policy is used. At any given instance, a request is not entertained if it cannot be completely satisfied. Three processes P1, P2, P3 request the sources as follows if executed independently.	5

		<table><tr><td>**Process P1:** t=0: requests 2 units of R2 t=1: requests 1 unit of R3 t=3: requests 2 units of R1 t=5: releases 1 unit of R2 and 1 unit of R1. t=7: releases 1 unit of R3 t=8: requests 2 units of R4 t=10: Finishes</td><td>**Process P2:** t=0: requests 2 units of R3 t=2: requests 1 unit of R4 t=4: requests 1 unit of R1 t=6: releases 1 unit of R3 t=8: Finishes</td><td>**Process P3:** t=0: requests 1 unit of R4 t=2: requests 2 units of R1 t=5: releases 2 units of R1 t=7: requests 1 unit of R2 t=8: requests 1 unit of R3 t=9: Finishes</td></tr></table> If all three processes run concurrently starting at time t=0, Is there any chance of occurrence of Deadlock? (Justify your answer with proper explanation)	
B.3	a	Recall that Belady's anomaly is that the page-fault rate may increase as the number of allocated frames increases. Now, consider the following statements: S1: *Random page replacement algorithm (where a page chosen at random is replaced) suffers from Belady's anomaly.* S2: *LRU page replacement algorithm suffers from Belady's anomaly.* Justify whether the above statements are correct or not.	**3**
	b	A computer whose processes have 1024 pages in their address spaces keeps its page tables in memory. The overhead required for reading a word from the page table is 500 nsec. To reduce this overhead, the computer has an associative memory which holds 32 (virtual page, physical page frame) pairs and can do look up in 100 nsec. What hit rate is needed to reduce the mean overhead to 200 nsec?	**4**
	c	A computer system uses 32-bit virtual address, and 32-bit physical address. The physical memory is byte addressable and the page size is 4Kbytes. It is decided to use two level page tables to translate from virtual address to physical address. Equal number of bits should be used for indexing first level and second level page table, and the size of each table entry is 4 bytes. i) Give a diagram showing how a virtual address would be translated to a physical address. ii) What is the number of page table entries that can be contained in each page?	**5**
B.4	a	Consider three CPU-intensive processes, which require 10, 20, and 30 time units and arrive at times 0, 2 and 6, respectively. How many context switches are needed if the operating system implements a shortest remaining time first scheduling algorithm? Also compute the average turnaround time.	**4**
	b	Consider three processes, all arriving at time zero, with total execution time of 10, 20, and 30 units, respectively. Each process spends the first 20% of execution time doing I/O, the next 70% of time doing computation, and the last 10% of time doing I/O again. The operating system uses a shortest remaining compute time first scheduling algorithm and schedules a new process either when the running process gets blocked on I/O or when the running process finishes its compute burst. Assume that all I/O operations can be overlapped as much as possible. For what percentage of time does the CPU remain idle?	**4**
	c	What is the cause of thrashing? How does the system detect thrashing? Once it detects thrashing, what can the system do to eliminate this problem?	**4**
B.5	a	A 160GB SATA HDD has 720rpm (revolutions per minute) and 3MB/s data transfer rate. Suppose the sector size is 512 bytes and 200 sectors per track. i. What is the average rotational latency and transfer time to read one sector from the disk? ii. How much time it requires to transfer 1MB data from contiguous sectors from a track/cylinder? iii. If the disk has 500 cylinders (numbered 0-499), and the read-write head is positioned at cylinder no. 499 (the innermost cylinder), calculate the total distance moved by the read-write head if the I/O	**6**

		requests (in the form of cylinder nos.) arrive as : 19 304 281 480 22 192 144. The disk scheduling algorithms to be considered are : FCFS, Scan and Elevator.	
	b	Consider two file systems that keep the inodes of files in two different ways : Strategy I - inodes are kept at the start of the disk. Strategy II - inodes get allocated during the time a file is created and the i-node is put at the start of the first block of the file i. What is the advantage and (or) disadvantage of adopting strategy II over strategy I. [Hint : Try to address the problem in terms of file type and file size] ii. In order to access a file using Strategy II mentioned in the question above, the file path (provided as input to a file operations - open/read/write) is parsed and inode of the required file is fetched. How many disk operations will be needed to fetch the inode of the file **C3_review_test.pdf** whose path is provide as /home/iiita/Documents/OS/C3_review_test.pdf ? *[Assume, i-node for the root directory is in the memory, but nothing else along the path is in the memory. Also assume that each directory fits in one disk block. Thus, it takes one disk operation to retrieve the i-node, and one more operation for the directory content.]*	3
	c	Suppose we are using UNIX system. A process opens a file myFile, reads *100 bytes*, from the file and then forks a child process. The child makes a read of one byte on the same file descriptor. What byte does it read?	1
	d	Let us assume a File system has 1K byte blocks and its file inode holds the following : pointer to X direct blocks of data, pointer to Y indirect blocks. The indirect block holds pointer to Z other direct blocks of data. Find the maximum file size supported by the inodes?	2
B.6		Consider a simple reader-writer implementation. int counter = 0; Readers : return counter; Writers: counter = counter +1;	
	a	i. Does this implementation cause race hazard? Explain. ii. Is there a race condition when there is only one dedicated writer? Justify your answers.	3
	b	Suppose we have n>1 concurrent processes, P1.....Pn. At some point of time Pi will execute its code section Si. We however, want the code sections are executed in the sequence S1,.....Sn. How do you solve the synchronization problem using semaphores? Can you solve it using mutexes?	2
	c	Consider a traffic crossing between roads, one in the East-West direction and the other in the North-South direction. Suppose the crossing is modelled as a shared data structure and cars are modelled as processes that access the crossing in order to pass through it. Two or more cars are allowed to pass through the crossing if they are headed in the same or direction (e.g. North bound or South bound car). But East-west traffic and Noth-South traffic must not cross the crossing simultaneously. Construct a synchronization solution for the cars using semaphores. Write two routines **east_west()** and **north_south()** to show how cars travelling in the respective direction should behave.	4
	d	Fill the blanks. Provide an example if needed to explain the statement : 1. Priority inversion occurs when..... and can be circumvented by ..... 2. Lamport's Bakery algorithm is a solution to ..... 3. Lost wakeup problem occurs when	3

END

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, ALLAHABAD

C3 Review Test, B-Tech-3rd Sem (IT & BI)

Paper Title: Introduction to Finance

Max Marks: 40 Duration: 2.5 hours

Instructions: Answer all the questions. Except calculator, no other materials are allowed.

Marks: 16 X 1=16

1. A firm has a cost of capital of 10%. The firm should accept any project that has an internal rate of return (IRR)
   a) less than 10%
   b) greater than 10%
   c) cannot be decided based on the information

2. Which of the following would be consistent with an aggressive approach to financing working capital?
   a) Financing short-term needs with short-term funds.
   b) Financing permanent inventory build-up with long-term debt.
   c) Financing some long-term needs with short-term funds.
   d) Financing seasonal needs with short-term funds.

3. What is the beta of the risk-free rate?
   a) Zero
   b) Greater than 1
   c) Equal to 1
   d) Can be any nonnegative number

4. The _____ form of Efficient Market Hypothesis suggests that there is little to be gained from studying past market data, or any other public information about firms.
   a. Weak
   b) Strong
   c) Semi strong

5. In capital budgeting, a profitability index of 0.79 for a project means that _____.
   a) The project's costs (cash outlay) are (is) less than the present value of the project's benefits
   b) The project's NPV is greater than zero
   c) The project's NPV is greater than 1
   d) The project returns 79 cents in present value for each current dollar invested (cost)

6. Which of the following statement is true related to the management of working capital?
   a) Profitability moves together with risk.
   b) Liquidity moves together with risk.
   c) Profitability varies inversely with risk.
   d) Profitability moves together with liquidity.

7. Which of the following is NOT a cash outflow for the firm?
   a) dividends.
   b) interest payments.

c) depreciation.
d) taxes.

8. As the number of stocks on a portfolio increases:
   a) The expected return decreases
   b) The expected return increases
   c) The non-systematic risk of the portfolio decreases
   d) The systematic risk of the portfolio increases

9. The curved segment that envelopes the set of optimal combinations of stocks is known as the
   a) characteristic line.
   b) new frontier.
   c) efficient frontier.
   d) security market line.

10. A U.S.-based company is considering an aggressive expansion into Asia. If management is very uncertain about the forecasted cashflows, then the cost of capital for the high risks project should be
   a) low
   b) high
   c) independent of risk

11. Which of the following statements regarding the payback period, is false?
   a) The payback period is simple to calculate and understand
   b) The payback period measures the time that a project will take to generate enough cash flows to cover the initial investment
   c) It takes account of the time value of money
   d) The payback period ignores cash flows after the payback point has been reached

12. When _____ is greater than zero the project should be accepted.
   a) Internal rate of return
   b) Profitability index
   c) Net present value
   d) Modified internal rate of return

13. In an efficient market, assume that a firm that is perceived to have great growth opportunities announces that it will start paying dividends for the first time. What effect will this announcement have on stock prices?
   a) To fall
   b) To rise
   c) To stay unchanged.

14. Measuring the length of the cash cycle, which of the following is not used as a metric?
   a) Accounts receivable days.
   b) Accounts payable days.
   c) Inventory days.
   d) Acid test days.

15. To evaluate projects, a single, overall cost of capital is often used because:
   a) it is the only way to measure a firm's required return.
   b) it acknowledges that most new investment projects have about the same degree of risk.

c) it avoids the problem of computing the required rate of return for each investment proposal
d) it acknowledges that most new investment projects offer about the same expected return.

16. As a diversified investor in Facebook, which of the following types of risk would you consider in your discount rate calculation?
a) The risk that a global economic slowdown will affect how much companies spend on advertising
b) The risk that new privacy laws will restrict data gathering and access
c) The risk that users will find a different social media platform to spend their time on
d) The risk that Mark Zuckerberg will leave as CEO

Marks: 6 X 4 =24

17. Explain systematic and idiosyncratic risk with some examples.

18. Explain three different forms of market efficiency.

19. What does it mean "Positive covariances increase portfolio variance; negative covariances decrease portfolio variance (diversification)"? Explain.

20. What are the problems of using internal rate of return (IRR) in project evaluation?

21. You are required to form a portfolio of two assets, A and B. The assets A and B have the following characteristics:

Asset A: Expected return: 0.08, Standard deviation: 0.20
Asset B: Expected return: 0.13, Standard deviation: 0.36
Correlation coefficient between A and B =0.4

a) What proportion of your wealth should you invest in A and B, if you demand an expected return of 10%?
b) What is the portfolio's standard deviation?

22. Suppose a firm has to decide between two capital investment projects A and B. The table below gives the NPVs, IRRs and the first three years' cashflow for the two capital investment projects. Each project's cash flows continue for several more years. The cost of capital is 12% for both projects.

Table: (Figures in millions)

Project	Initial Investment	C1	C2	C3	NPV	IRR
A	100	20	20	20	57	17.8%
B	200	0	20	40	64	14.5%

a) Which project A or B would the firm choose, based on the above table data and why?
b) Suppose the firm rejects all projects with payback periods greater than 3 years. Then what will be decision following the payback period policy?

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Indian Institute of Information Technology, Allahabad
C3 Review Test
Probability and Statistics (PAS)
B. Tech. (3rd Semester)

Date: November 22, 2022 (05:30 PM - 07:00 PM) Total Marks:42

Important Instructions

1. Question paper consists of 7 questions. Answer all questions. Writing on question paper is not allowed.
2. Attempt all the parts of question 4 at the same place. Parts done separately will not be graded.
3. Number the pages of your answer booklet. On the first page of your answer booklet, make a table (as shown below) to indicate the page number in which respective questions have been answered. If you did not attempt a particular question, write down NA.

Question No.	1	2	3	4	5	6	7
Page No.							

4. Use of any electronic gadgets is not allowed.

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1. Three factories produce the same tool and supply it to the market. Factory A produces 30% of the tools for the market and the remaining 70% of the tools are produced in factories B and C. Given that 98% of the tools produced in factory A, 95% of the tools produced in factory B and 97% of the tools produced in factory C are not defective. What percent of tools should be produced by factories B and C so that a tool picked at random in the market will have a probability of being non defective equal to 0.96? [6]
2. Show that the necessary and sufficient conditions for two given real numbers a and b to be respectively the mean and the variance of some binomial distribution are that $a > b > 0$ and $\frac{a^2}{a-b}$ is an integer. [5]
3. You are allowed to take a certain test three times, and your final score will be the maximum of the test scores. Assume that your score in each test takes one of the values from 1 to 10 with equal probability 1/10, independently of the scores in other tests. What is the probability mass function of the final score? [5]
4. Let $(\mathcal{S}, \mathcal{F}, P)$ be a probability space and $A, B \in \mathcal{F}$ with $0 < P(A) < 1, 0 < P(B) < 1$. [8]
   (a) Find the correlation coefficient ρ between the random variables I_A and I_B, where I_A and I_B are the indicator functions of A and B, respectively.
   (b) Show that $\rho = 0$ if and only if A and B are independent.
   (c) What happens if $A = B$ or if $A = B^c$.
   (d) Show that $\rho > 0 \Leftrightarrow P(A|B) > P(A)$.
5. Let X and Y be independently and identically distributed exponential random variables with $\lambda = 1$. Find the msrginsl probability density functions of $U = X_1 + X_2$ and $V = X_1 - X_2$ using transformation of variables technique. [10]

6. Let (X_n) be a sequence of random variables with probability mass functions

$$f_{X_n}(x) = \begin{cases} \frac{1}{n}, & \text{if } x = 1 \\ 1 - \frac{1}{n}, & \text{if } x = 0 \\ 0, & \text{otherwise} \end{cases}$$

Find a real number a such that (X_n) converges to a in probability. [4]

7. Let $X_1, \ldots, X_n$ be independently and identically distributed random variables with probability density function

$$f(x, \theta) = \begin{cases} (1+\theta)x^{\theta}, & 0 < x < 1 \\ 0, & \textit{otherwise.} \end{cases}$$

Find the maximum likelihood estimator of θ. [4]