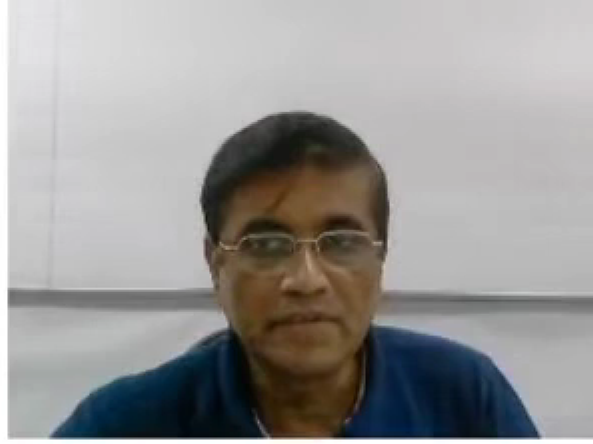


GDA Vs Logistic Regression

GDA

$P(X|Y=0) \sim N(\mu_0, \Sigma)$

$P(X|Y=1) \sim N(\mu_1, \Sigma)$

$P(Y) \sim \text{Bernoulli}(\phi)$

From MLE $\Rightarrow$

$$\mu_0 = \frac{\sum_{i=1}^{m} 1\{Y^{(i)}=0\} X^{(i)}}{\sum_{i=1}^{m} 1\{Y^{(i)}=0\}}$$

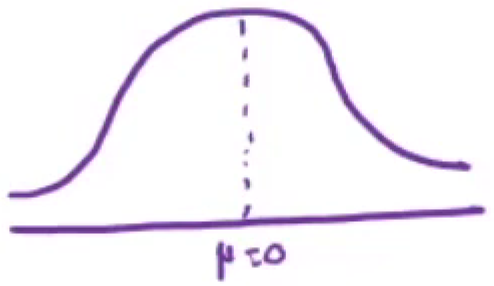
$$\mu_1 = \frac{\sum_{i=1}^{m} 1\{Y^{(i)}=1\} X^{(i)}}{\sum_{i=1}^{m} 1\{Y^{(i)}=1\}}$$

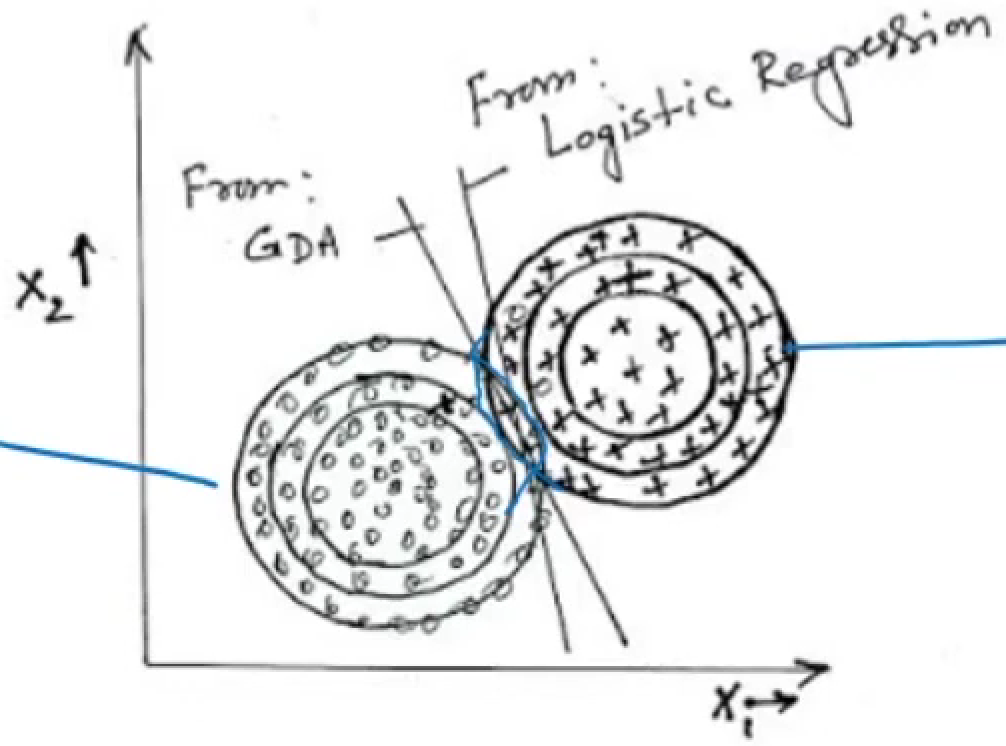
$$\Sigma = \frac{1}{m}\sum_{i=1}^{m} (x^{(i)} - \mu_{Y^{(i)}})(x^{(i)} - \mu_{Y^{(i)}})^T$$

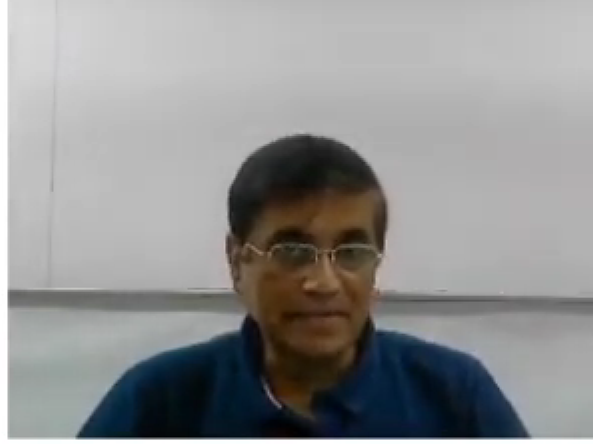
$$\phi = \frac{1}{m}\sum_{i=1}^{m} 1\{Y^{(i)}=1\}$$

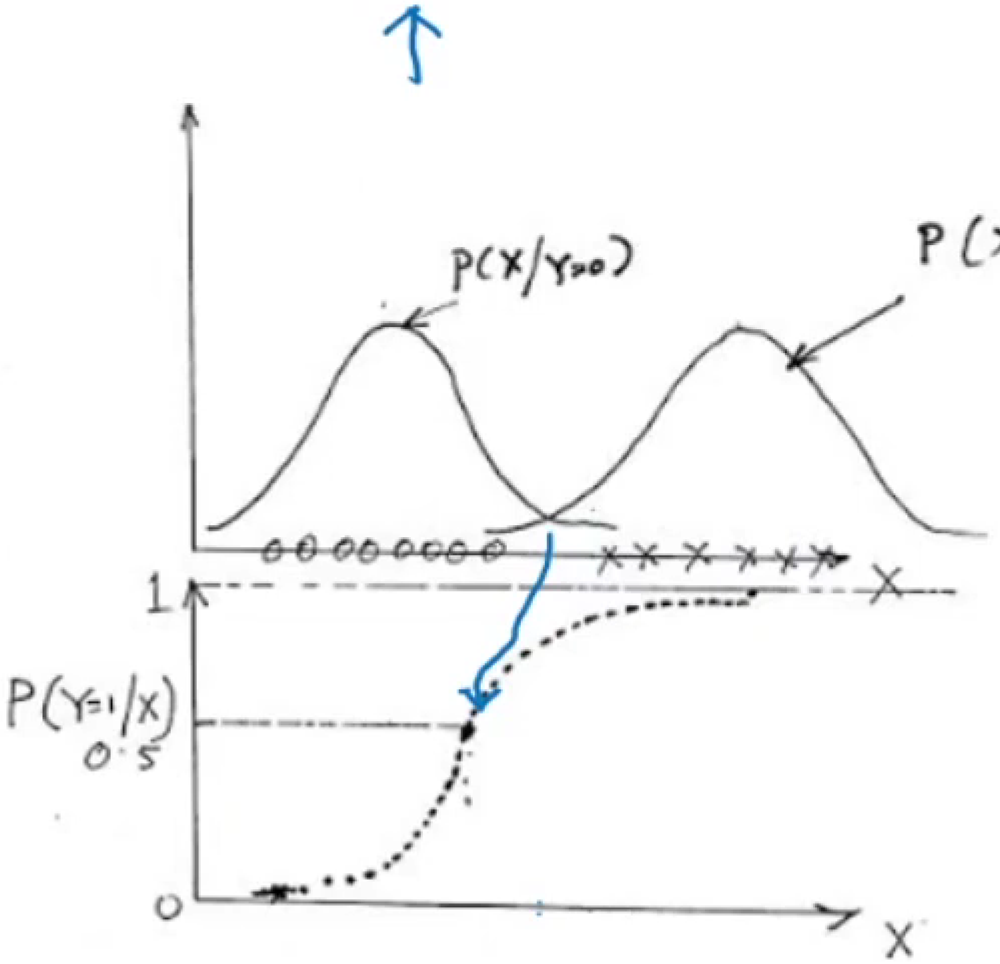
Logistic Regression

$$P(Y=1|X) = \frac{1}{1+e^{-W^T X}}$$

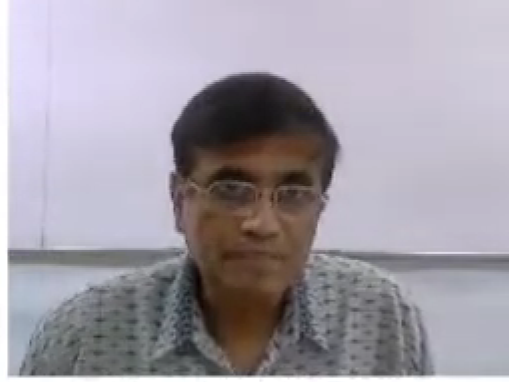








$P(Y=0) = P(Y=1)$
$= 0.5$ say

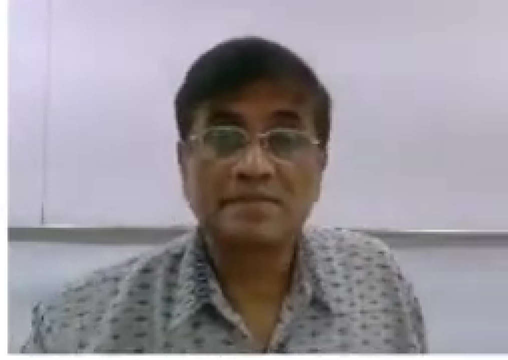


Learning Naïve Bayes Classifier for developing Spam mail Filter

- We want our Classifier will tell us whether an email received is a Spam or non –Spam through text classification.
- We will train this using labelled data in line with generative Model principle.
- We will then use it as mail reader which will automatically filter out Spam messages and possibly will keep it in another folder.
- Depending on how Feature is represented we will present two Naïve Bayes models:
  - Multi-variate Bernoulli event model
  - Multinomial event model
- Multi-variate Bernoulli event model

$$X = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{array}{l} \text{a} \\ \text{abacus} \\ \text{absence} \\ \vdots \\ \text{Lottery} \\ \text{million} \\ \vdots \\ \text{zero} \end{array} \quad \text{create a dictionary.}$$

$10{,}000 \times 1$



Learning Naïve Bayes Classifier for developing Spam mail Filter

$P(X_1, X_2, \ldots X_{10,000} \mid Y)$

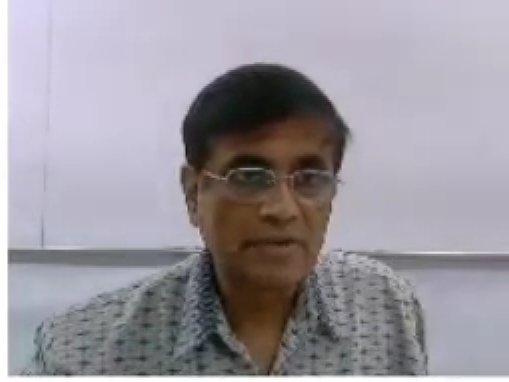
$= P(X_1 \mid Y)\, P(X_2 \mid Y, X_1)\, P(X_3 \mid Y, X_1, X_2) \cdots P(X_{10,000} \mid Y, X_1 X_2 \cdots X_{10,000})$

Assumptions :

The words are occurring in the mail independently.

$= \underbrace{P(X_1 \mid Y)\, P(X_2 \mid Y)\, P(X_3 \mid Y) \cdots P(X_{10,000} \mid Y)}$ – Naive Bayes.

$= \prod_{j=1}^{n} P(X_j \mid Y)$



Naïve Bayes Classifier

In GDA , the feature vectors were continuous, real-valued vectors.
In Naïve Bayes, another generative model, we assume the features are discrete valued.

Model:

$P(X/Y)\ P(Y)$

$$P(X/Y) = \prod_{j=1}^{n} P(x_j / Y)$$

Parameters:

$P(Y=1) = \phi_Y \rightarrow$ Bernoulli distribution

$P(x_j = 1 / Y=0) = \phi_{j/Y=0}$

$P(x_j = 1 \mid Y=1) = \phi_{j/Y=1}$

Using MLE the parameters are evaluated as follows:

$$\phi_Y = \frac{\sum_{i=1}^{m} 1\{Y^{(i)} = 1\}}{m} \leftarrow \text{indicator function}$$

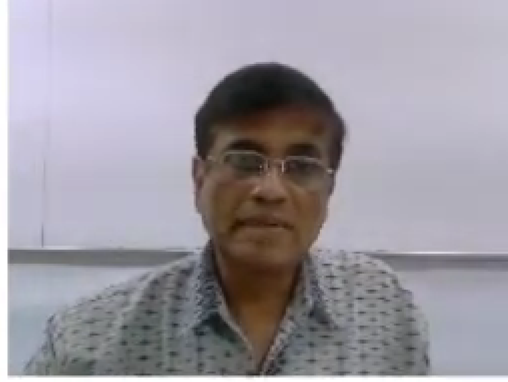
AND

$$\phi_{j/Y=0} = \frac{\sum_{i=1}^{m} 1\{x_j^{(i)} = 1 \wedge Y^{(i)} = 0\}}{\sum_{i=1}^{m} 1\{Y^{(i)} = 0\}}$$

$$\phi_{j/Y=1} = \frac{\sum_{i=1}^{m} 1\{x_j^{(i)} = 1 \wedge Y^{(i)} = 1\}}{\sum_{i=1}^{m} 1\{Y^{(i)} = 1\}}$$

This simply consider the entire training set and count the number of times the word X_j has appeared when the mail is spam.

Naïve Bayes Classifier



Testing the model:

$$P(Y/x_j) = \frac{P(x_j/Y).P(Y)}{P(x_j^{(i)})}$$

specifically,

$$P(Y=1/x_j) = \frac{P(x_j/Y=1)\cdot P(Y=1)}{P(x_j/Y=1).P(Y=1) + P(x_j/Y=0).P(Y=0)}$$

$$= \frac{\prod_{j=1}^{n} P(x_j/Y=1)\, P(Y=1)}{P(x_j/Y=1)\, P(Y=1) + P(x_j/Y=0)\, P(Y=0)}$$

Naïve Bayes Classifier

Testing the model:

$$P(Y=1/x_j) = \frac{P(x_j/Y)\cdot P(Y)}{P(x_j^{(i)})}$$

specifically,

$$\underbrace{P(Y=1/x_j)} = \frac{P(x_j/Y=1)\cdot P(Y=1)}{P(x_j/Y=1)\cdot P(Y=1) + P(x_j/Y=0)\cdot P(Y=0)}$$

$$= \frac{\prod_{j=1}^{n} P(x_j/Y=1)\, P(Y=1) \qquad = 0}{P(x_j|Y=1)\, P(Y=1) + P(x_j/Y=0)\, P(Y=0)}$$