WOMK 3

24/09/2024
Tuesday

(Lec-8) :-

* Polygon Clipping :-

Rendering Pipeline :-

Model and Camera Parameters → (Basically coordinates) ⇒

- Transform → model → world (so that saare k saare models same coordinate system mein aa jaayein)
- Illuminate
- Transform → world → camera → projection
- Clip → kitna part object ka screen pe display krna chahte ho.
- Project
- Rasterize

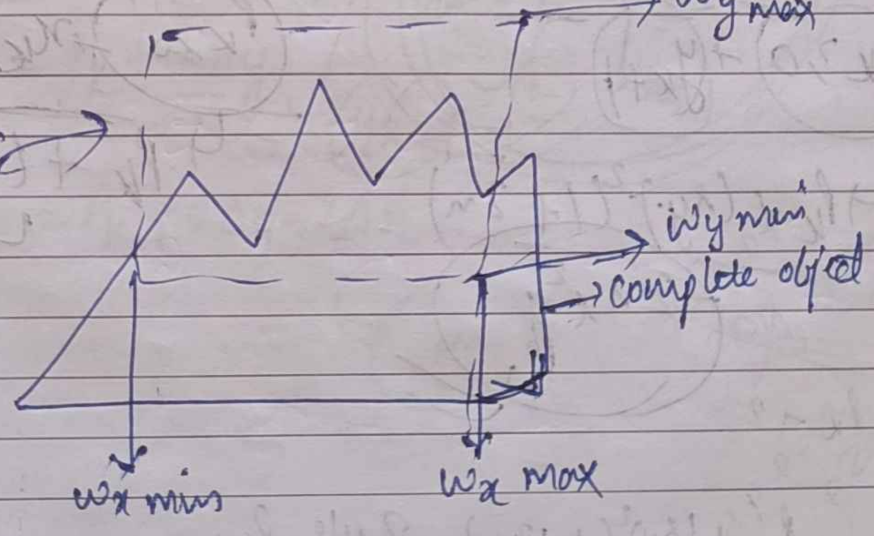
Rendering Pipeline

↓

Frame Buffer → DISPLAY

WINDOW :-

(sirf itna object screen pe dikhega)



WONK:

* CLIPPING POLYGONS :-

↳ complex than clipping individual lines

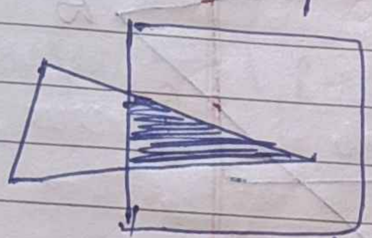
Input : Polygon

Output : Original ", new polygon or nothing.

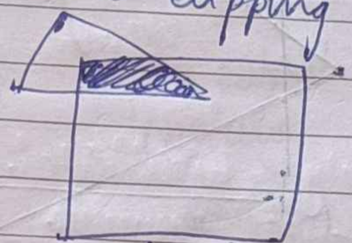
→ tabhi jab △ banatte hain, sirf 3 lines nahi dikhti, usmein colour bhi dikhta

↓

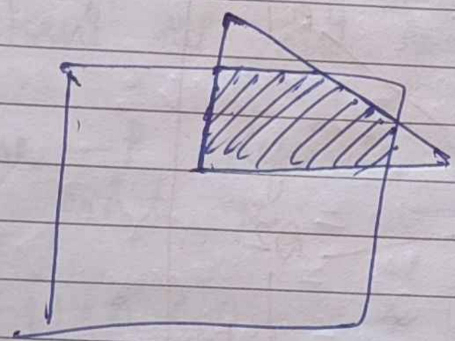
for this clipping polygon is ~~more~~ important than clipping lines.



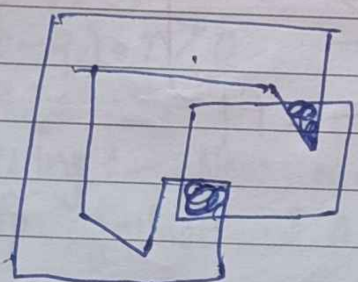
triangle ⇒ triangle.



triangle ⇒ quad.



triangle ⇒ 5-gon.



→ that's why clipping Hard ⇒ so have some algo.

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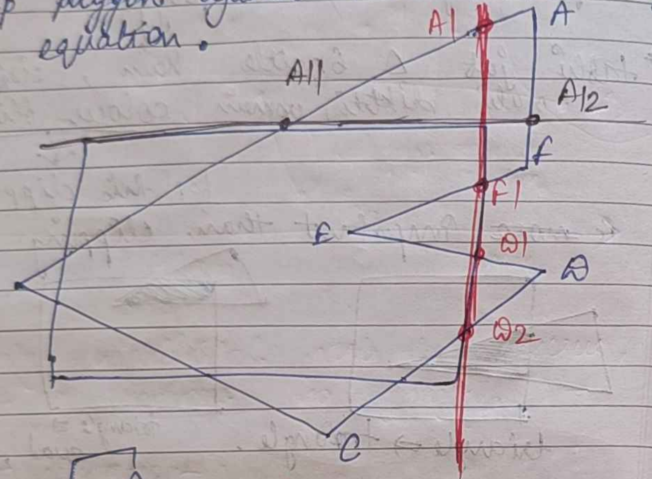
SUTHERLAND - HODGMAN CLIPPING :-

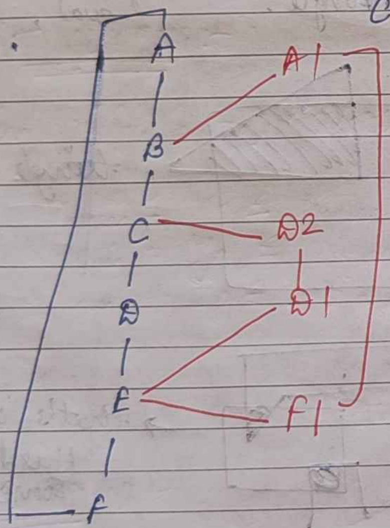
step ① → Consider each edge of the view window individually.

step ② → clip polygon against view window edge's equation.

eg :-



Step I ⇒



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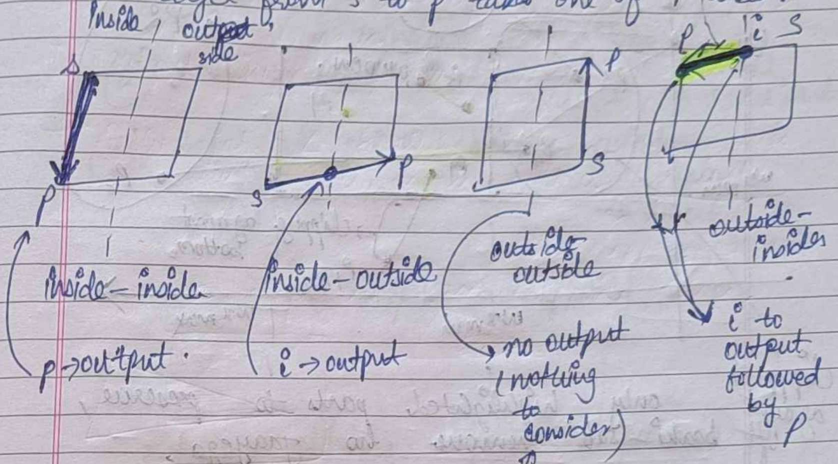
※ 4 cases possible :-

yeh pura highlighted output hoga.

Edge from s to p takes one of 4 case :-



as if intersection consider ~~kari~~ kari thi and here no intersection.

① Plane - P defined by q and n then point p.

$(p-q)\cdot n < 0$ → then p inside

$(p-q)\cdot n == 0$ → then p on plane

$(p-q)\cdot n > 0$ → then p outside.

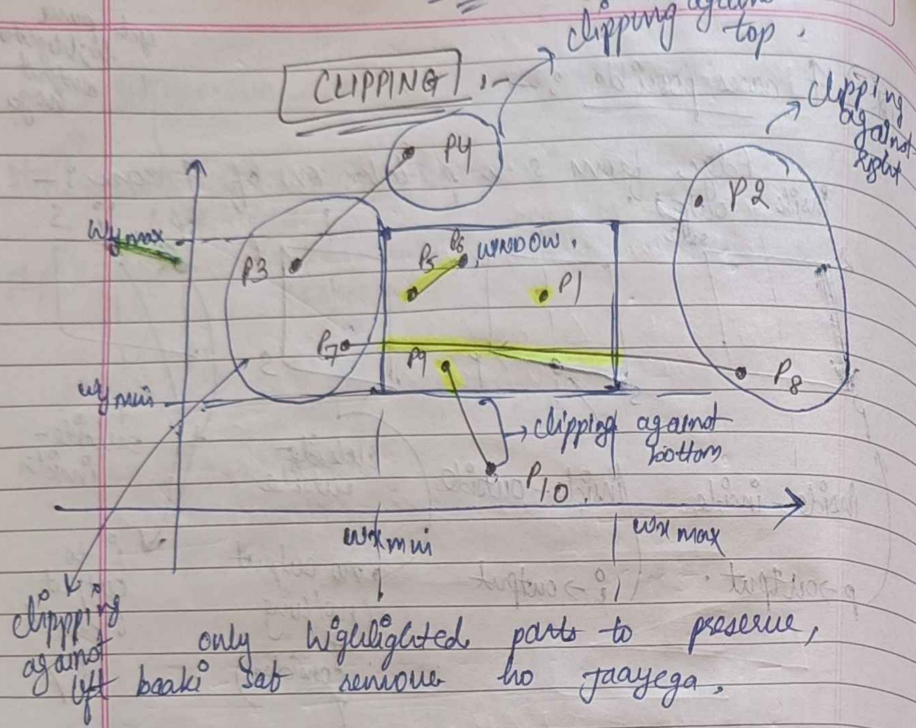
3D viewing :- Now need 6 diff. planes to clip against 3-D view

top, down, front, back, left, right.

{baaki same, algo saari}

WOMK 3 om.

CLIPPING :-



only highlighted parts to preserve, baaki sab remove ho jaayega.

POINT CLIPPING :-

given (x_p, y_p, z_p) check:

$$wx_{min} \le x_p \le wx_{max}$$
$$wy_{min} \le y_p \le wy_{max}$$
$$wz_{min} \le z_p \le wz_{max}$$

→ if satisfied, take that point o/w, not clip them

2-D clipping mein, no cond^n for 'z-coordinate' obviously.

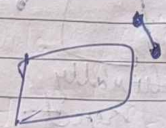
LINE CLIPPING :-

WOMK 3 om.

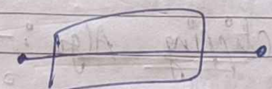
* Cases :-

1. Both end points inside window → Don't clip.
2. One end point inside and one outside → must clip some part.
3. Both end points outside window → Don't know whether to clip or not.

eg:



clip completely.

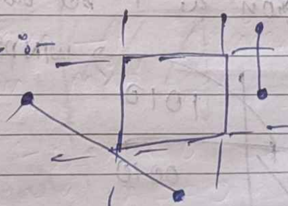


clip some part.

↳ clip

ohh!!

Consider :-



check against each and every boundary.

Brute force line clipping :-

1) Don't clip lines with both end points within window.
2) For lines with one end pt. inside the window and one end-pt. outside, calculate the intersection pt. (using the line eq^n) & clip from this pt. out.

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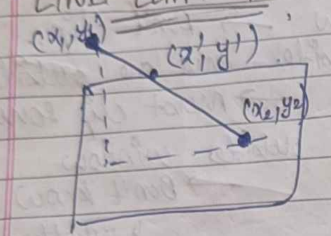
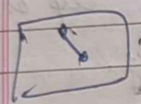
3) for lines with both end pt outside . . .
have to think!

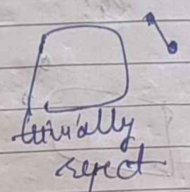
LINE CLIPPING (II) :-



Expensive !



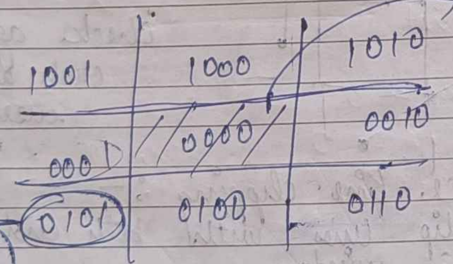
→ trivially accept



trivially reject

(*) Cohen – Sutherland Clipping Algo :-

1) Divide view – plane into region defined by view – port edges.
2) Assign each region, a 4 – bit outcodes:–

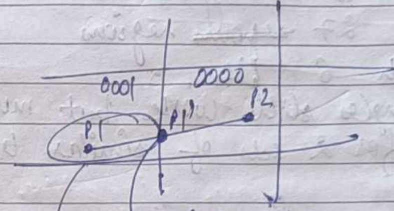


(above, below, right, left). → bit
✓ 1, X 0, X 0

→ Assign vertices of line 4 bit value.

steps :-

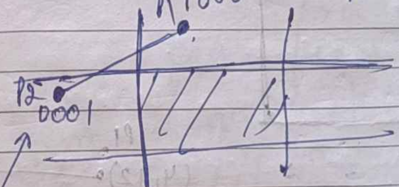
1 * If both 0000, trivial accept –
2 * If (v1 & v2 != 0), trivial reject –
3 * clip against one side (where one is non-zero)
4 * Assign new vertex 4 bit
5 * Go to step 1.



→ clipping pt

→ itna clip and $\boxed{P1' \to 0000}$
(clip against left boundary)

eg :-

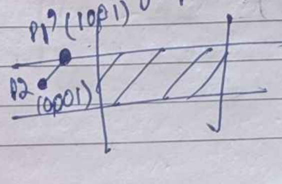


⇒ perform clipping, as can't do trivial reject

clip against left boundary

M-1

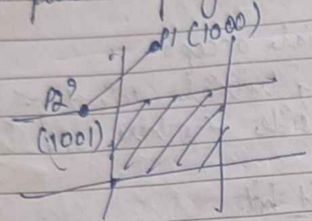
say pehle clip against top :-



→ 1001
& 0001
0001 → Now, trivial reject !!

WOMK 3 Step 2

(M2) while clip against left :-



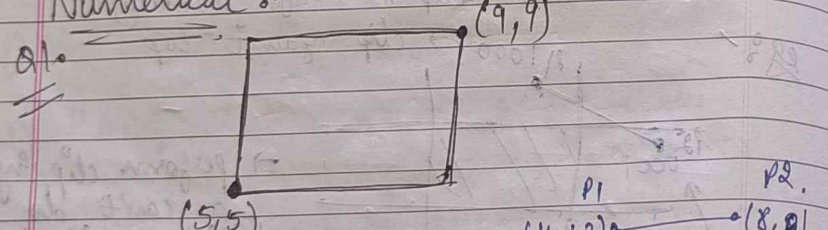
$P2 \& P1 = 1000 \neq 0$

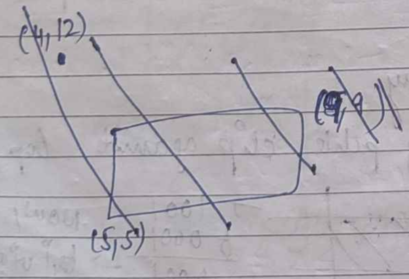
so trivial reject,

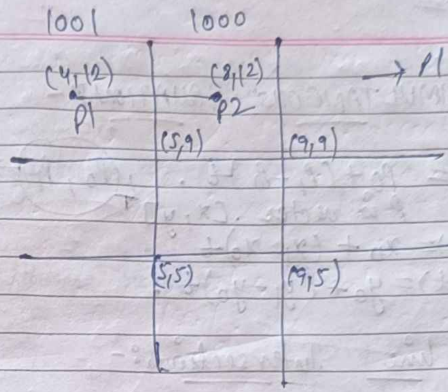
→ Extends easily to 3D region :-
  → with 27 ~~regions~~ regions
  → with 6 bits.
→ similar triangles still work but must be done for 2 sets of similar triangles.

Numerical :-

Q1.

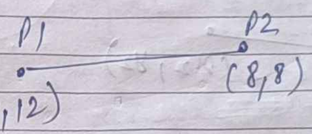


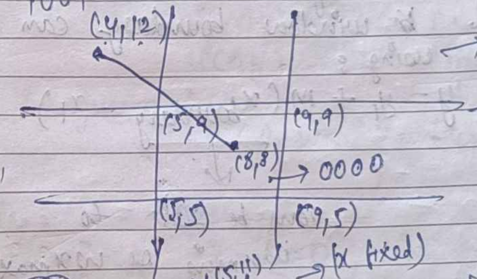




→ P1 & P2 →
1001
1000
1000
→ trivially reject

Now





→ 1001
& 0000
0000
→ can't trivially reject & accept.

$m = \frac{4}{-4} = -1$

$y = mx + c$

$y = -x + c$

$y + x = c$

$c = 16$

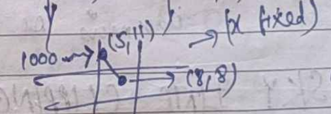
$y = -x + 16$

$9 = -x + 16$

$9 - 16 = -x$

7

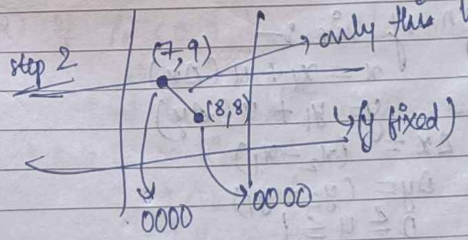
Step 1



Clip from ~~top~~ left, then from top.

→ as (1001) → 2 one → done bataane hain, ek-ek karke kataao, → so pehle '1' → the step.

step 2



WOMK ☺ ☺mm.

SOLVING SIMULTANEOUS EQUATIONS :-

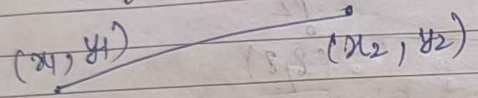
$P(t) = P_0 + (P_1 - P_0)t$. $(P_0, P_1) \to 2$ points

P is vector . [x, y]

$x(t) = x_0 + (x_1 - x_0)t$

$y(t) = y_0 + (y_1 - y_0)t$.

Calculating line Intersections :-



y-coordinate of an intersection with a vertical window boundary can be calculated using :

$$y = y_1 + m(x_{boundary} - x_1).$$

can be set to wx_{min} or wx_{max}

LIANG-BARSKY LINE CLIPPING (II) :-

Parametric defn of line :-

$x = x_1 + u(\Delta x)$

$y = y_1 + u(\Delta y)$

$\Delta x = (x_2 - x_1)$

$\Delta y = (y_2 - y_1)$

$0 \leq u \leq 1$

WOMK ☺ ♡.

Goal :- Find range of u for which x and y both inside the viewing window.

→ Drawback batata hai pichle algo ki !
↳ konsi Drawback ??

Mathematically :-

$\Rightarrow x_{min} \leq x_1 + u(\Delta x) \leq x_{max}$

$\Rightarrow y_{min} \leq y_1 + u(\Delta y) \leq y_{max}$.

Rearranged :-

$u*(-\Delta x) \leq (x_1 - x_{min})$

$u*(\Delta x) \leq (x_{max} - x_1)$

$u*(-\Delta y) \leq (y_1 - y_{min})$

$u*(\Delta y) \leq (y_{max} - y_1)$

In general :- $u*(p_k) \leq (q_k)$

Rules :-

1. If $p_k = 0$, line parallel to boundaries
   - → if for that same k, $q_k < 0$, it's outside
   - → otherwise it's inside.
2. If $p_k < 0$ → line starts outside this boundary
   - ⇒ $r_k = q_k / p_k$
   - ⇒ $u_1 = \max(0, r_k, u_1)$

→ $p_k > 0$ → line starts inside boundary

⇒ $r_k = q_k / p_k$

⇒ $u_2 = \min(1, r_k, u_2)$

→ If $u_1 > u_2$, line is completely outside.

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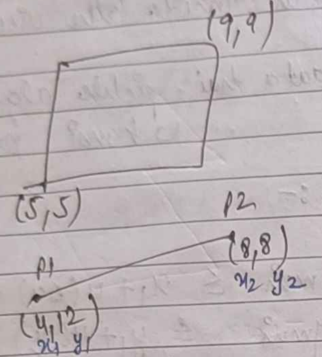
$u_1 = 0$ (initially)

Numerical :-



$P1 = -\Delta x$
$P2 = \Delta x$
$P3 = -\Delta y$
$P4 = \Delta y$

$q_1 = x_1 - x_{min}$
$q_2 = x_{max} - x_1$
$q_3 = y_1 - y_{min}$
$q_4 = y_{max} - y_1$

$\Delta x = 8-4 = 4$
$\Delta y = 8-12 = -4$
$x = 4 + \cancel{4(4)}\ 4u$
$y = 12 - 4u.$

so, $P1 = -4$
$P2 = 4$
$P3 = 4$
$P4 = -4$

as $P1 = -4 < 0 \Rightarrow r_k = r_1 = q_1/p_1 = (-1)/(-4) = 1/4$
↳ find value of u_1

~~so, $u_1 = \max(0, \frac{1}{4}, 0) = 1/4$~~ → as $P1, P_4 < 0$.

$u_1 = \max(0, q_1/p_1, q_4/p_4)$

$u_2 =$ ~~max (1,~~ $\min(1, q_2/p_2, q_3/p_3)$

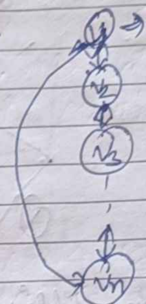
$u_1 = 3/4$
$u_2 = 1$
↳ as $P2, P3 > 0$

WJMK ☺ WJMK ☺ WJMK ☺

1/9/24 Tuesday

Lec-9 :-

Fill algorithms :- polygon :-

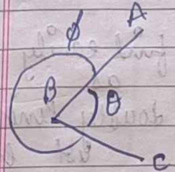


→ Doubly linked list.

→ How to know whether convex or concave?

① all inner angles are < 180°
↳ How to measure? difficult.

② $0 < \theta < 180° \Rightarrow \sin(\theta) > 0$
$180° < \theta < 360° \Rightarrow \sin(\theta) < 0.$
How sin ?
↳ By cross product.



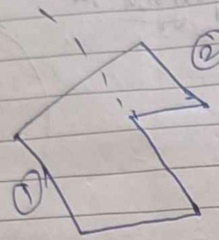
$\vec{BA} \times \vec{BC} \Rightarrow \theta$ dega
$\vec{BC} \times \vec{BA} \Rightarrow \phi$ dega.
ek mein +ve, dusre mein -ve!
dikkat !!

There should not be a flip.
ek vaar $\vec{BC}$ wali direction se start kiya, throughout usi direction mein jaao → same sign aana chahihe ⇒ +ve hai toh +ve rahe, -ve hai toh -ve rahe.

WJMK MM

vertex pe line break ho jaayega

③ concavity :- when ek edge across jaayein, and 2 edges different sign dein.

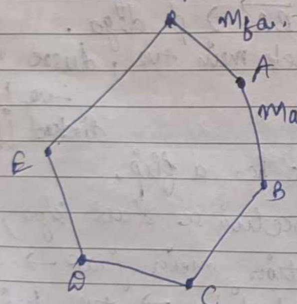


line ① and ② will have different signs.

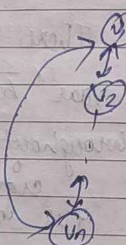
Polygon filling Algorithms :- (How antialiasing can be implemented).

* A triangle is always convex!
↳ so in algo always try to use Δ as usko colour krna very easy.

⇒ Trapezoid and Quadrilaterals are also always convex !!!!



→ can find easily as doubly linked list hoti

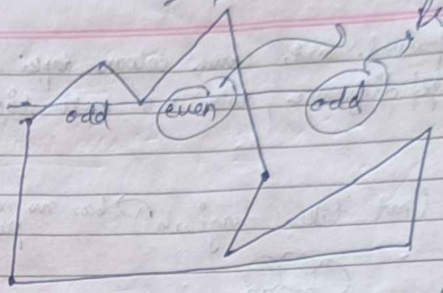


\# flag = odd or 'ON' ⇒ color, flag = even or 'OFF' ⇒ don't color.
⇓
but ∃ problem with this

WJMK MM

Problem!

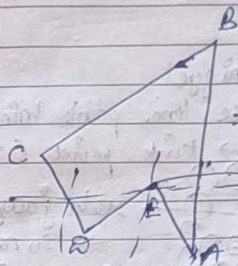
as :-



soln :- at extrema, print 2 points, not ①.

when there is flip in sign of slope

eg :-



→ my first pt → (A) is lowest pt.

→ splitting
↳ as concave problem as E pe slope sign flips

Doubly linked list ⇒

A ⟷ $1/m_{AB}$ ⟷ $1/m_{EA}$
⇕
B ⟷ $1/m_{AB}$ ⟷ $1/m_{BC}$
⇕
C ⟷ $1/m_{BC}$ ⟷ $1/m_{CD}$
⇕
D ⟷ $1/m_{CD}$ ⟷ $1/m_{DE}$
⇕
E ⟷ $1/m_{DE}$. ⟷ $1/m_{EA}$.

⇓
Now sort it!
⇓
(top - bottom sorting)

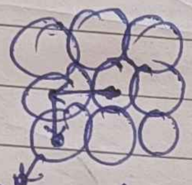
\# any convex shape can be decomposed into triangles.

Boundary - Fill Algorithm :- let say blue colour krna hai, and blue hai, stop algo else colour it blue and see its neighbours → if all blue then stop else colour uncoloured ... so on! → stop at boundary pixel ~~tak~~ or recursion continues.

WJMK ☺

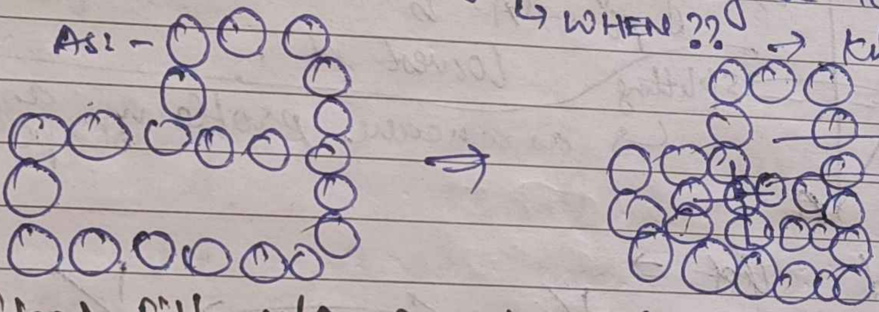
[removing background and same colour aapki shirt ka, as no edge/ difference, toh wo aapki shirt bhi remove krdega.]

→ 4 connectivity (not taken diagonals) → bcoz no need to → udhar pauch hi jaana hai



Diagonal tak paunch hi gye!
But kyi ↓ vaar alag se lene pdhte hain!
↳ WHEN ??

As :-



→ kink point (such edge pt) reh jaayega.
→ so need to go diagonally!
↳ Now!

① Flood fill v/s Boundary fill → H/w
② Inside - Outside Test :- ① odd even rule
② Non zero winding number rule

CGV :-

WOMK

8 oct 2024
Tuesday

lec-

Q. How to project 3D scene into 2D image?

Note :-

→ Orthographic projection does not retain info of depth, while projective projection does.

(*) By taking photo from camera having known focal length and pixel value, then can height of object (eg. Burj Khalifa) jiski photo lekhi hai, with help of orthographic and projective projections concept.

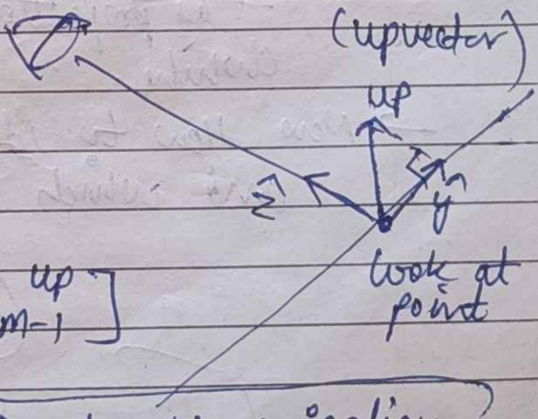
→ we need to define camera position say x_c which will be dependent on $[x_0^c, x_1^c, \ldots, x_{m-1}^c]$. (m-dimensional positional vector).

→ ~~looking~~ look at vector

look at point
↓
$l_c\,[x_0^l, x_1^l, \ldots, x_{m-1}^l]$.

$$\hat{z} = \frac{\vec{l_c} - \vec{x_c}}{\|\vec{l_c} - \vec{x_c}\|}$$

(Z camera cap)



$$\vec{up}\,[x_0^{up}, x_1^{up}, \ldots, x_{m-1}^{up}]$$

$$\vec{u} = \vec{up} - \vec{l_c}$$

$$\hat{x} = \frac{\hat{y} \times \vec{up}}{\|\hat{y} \times \vec{up}\|}$$

$\vec{up}$ vector ki projection on place ki direction $= \hat{y}$

WORK from.

$\hat{y} = \hat{x} \times \hat{z}$

$$d\,[x_0^d, x_1^d, \ldots, x_{n-1}^d]$$
$$d_x = \vec{d}\cdot\hat{x}$$
$$d_y = \vec{d}\cdot\hat{y}$$
$$d_z = \vec{d}\cdot\hat{z}$$

* Moon's rotation is face locked around earth's orbit, isiliye moon ka sirf ek part hi dikhta hai.
- ↳ but more than 180° steradian dikhta hai, not saara
- → the pehli photo 'apollo' ne li hai!

↳ Example of 'looking at data'.

→ why removed back surface?
- ↳ as useless!
- ↳ so why to have burden with you.
- → remove it!

→ as projection mein toh front side hi chahiye.

→ Now, How to identify which is front side, and which is back side?

CLIPPING :-
- ↳ world very big
- ↳ and when projecting → limited area dikhaayenge
- ↳ baaki ka clip krna padhega.

reason for clipping

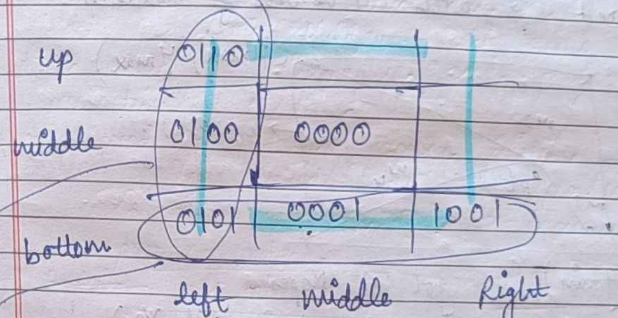
ALICE IN WONDERLAND

WORK from.

→ clipping can be in 2D, as well as 3D
- → like when superimposing one object on another object.

→ only showing inside of house, baath baahar jaa raha → chop it out. (3D clipping)

Sutherland Algo :- (bits for coding)



⇒ 9 sections.
- ↳ need 4 bits to represent. (redundancy, as can go upto 16)
  - ↳ so 7 binary codes ka use nahi hoga
  - ↓ so ∃ flexibility in selecting regions.

Adv ?
- inka and = 1.
- inka and = 1.

so,
→ any line which falls in ▭ part, can easily be cut out / clipped with help of boolean expression / operation like AND.

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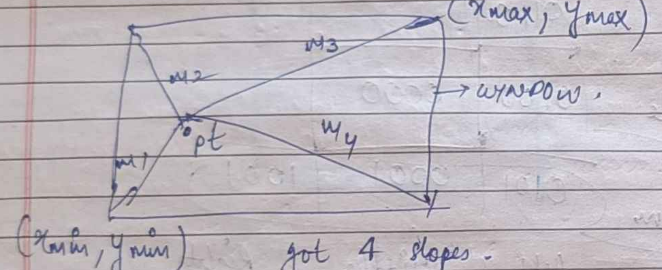
How to use linked list to get new polygon after clipping it!

NLN algo :-

↳ designing action dependent on 1st pt of edge.

connect pt with 4 vertices of window.



got 4 slopes.

edge → $E_{12}(V_1, V_2)$

↳ say slope → m_{12}.

$m_1 < m_{12} < m_2$

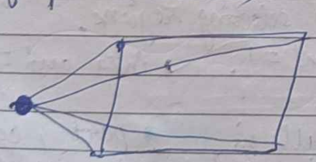
$m_2 < m_{12} < m_3$

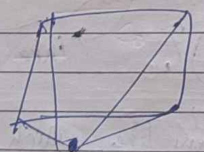
$m_3 < m_{12} < m_4$

$m_4 < m_{12} < m_1$

→ from these cond^{n}s, we will get to know, kis edge se chop krna hai.

If pt is outside →





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→ triangle clipping mein max 7 sides ka ban skta.

3 + 4

triangle ki 3 sides

window ki 4 sides

clipping

↳ difficult when we have concave polygon.

→ can't forget / work on actual linked list, uski copy banao and usmein changes kro!

H/W

→ implicit form of line eq^n?

→ Explicit form of line eq^n?

→ Parametric form of line eq^n ⇒

$V(u) = V_1 + (V_2 - V_1)(u)$

↳ $u = 0 \rightarrow V(u) = V_1$

$u = 1 \rightarrow V(u) = V_2$

$0 < u < 1 \rightarrow$ within window ✓

HIDDEN SURFACE REMOVAL :-

→ agar dekhna konsa front side aur konsa back side → pehle toh normal nikaalo to the plane

↳ $(Ax + By + Cz + D) = 0$

↳ normal = (A, B, C)

⇒ 'dot product' help kregа.

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→ hm utna hi paint krenge na jitna dikhta hai, so fill algo etc. se only front side paint hoti hai not back side.

Painter's Algo

Z-buffering v/s Ray tracing

Ray Casting (addition to painter's algo)

→ interlocked, can't make decision which is on top
↳ so then split polygon into two, so that decision can be taken.

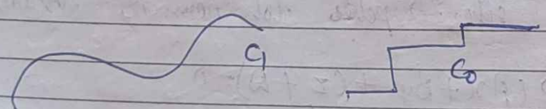
(*) BSP Tree algo
↳ Binary Space Partitioning

WARNOCK'S ALGORITHM:- ??

→ different levels of continuity:-

$C_0, C_1, \ldots, C_n$.

C^ith continuity means ith derivative also continuous.



read about them and their conditions

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→ How to create higher level of continuity.

Special functions:-

eg: Gamma function etc.

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SPLINE:-

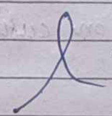
Har image → line, circle, ellipse se toh nahi bn skti; so "spline" chahiye.

Spline History → ducks (wooden strips)

↓ Drawbacks (??)

$y = mx + c$ (explicit)
$Ax + By + C = 0$ (implicit)

$x = x_0 + (x_1 - x_0)u$
$y = y_0 + (y_1 - y_0)u$ → (parametric)



→ this is not even f^n
↳ How to draw this?

SPECIFYING CURVES:-

1. control points
2. knots
3. Interpolating spline
4. Approximating spline.

(i) If figure has k splines, then it always follow C_{k-1} continuity. (???)

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② cubic spline has C_2 continuity at joints.

CUBIC CURVE:-

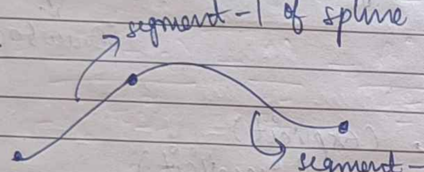
Eq^n for single parametric cubic spline segment →

$$P(t)=\sum_{i=1}^{4} B_i t^{i-1} \; ; \; t_1 \le t \le t_2$$

t_1, t_2 → parameter values at beginning and end of segment.

$$f(x) = Ax^3+Bx^2+Cx+D$$

spline:

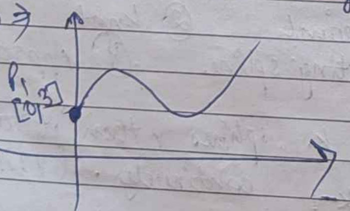


→ means 2 curves in same spline.

$P(t)$ → position vector of any pt. on cubic spline segment.

If 3D → $P[t] = [x(t) \; y(t) \; z(t)]$
→ we'll work in 2D mostly.

In 2D ⇒



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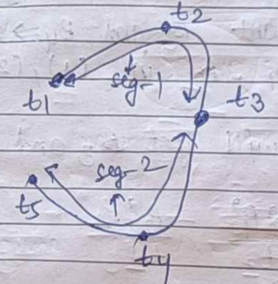
$$x(t)=\sum_{i=1}^{4} B_{ix} t^{i-1} \; ; \; t_1 \le t \le t_2$$

$$y(t)=\sum_{i=1}^{4} B_{iy} t^{i-1} \; ; \; t_1 \le t \le t_2$$

$$z(t)=\sum_{i=1}^{4} B_{iz} t^{i-1} \; ; \; t_1 \le t \le t_2$$



If considering t_1 to t_3 → one segment, then t_2 is intermediate value of t.

Usually we start from '0', thus $t_1 = 0$.

for seg-1 → $0 \le t \le t_3$

for seg-2 $t_3 \le t \le t_5$
as start for '0', so
$0 \le t \le t_5$

→ We want to find 2nd derivative of curve eq^n!!

$$P(t) = B_1 + B_2 t + B_3 t^2 + B_4 t^3 \quad \text{— (1)}$$

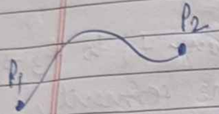
$$P'(t) = \sum_{i=1}^{4} B_i (i-1) t^{i-2}$$

tangent to curve.

$$= B_2 + 2B_3 t + 3B_4 t^2 \quad \text{— (2)}$$

$$P'(t) = [x'(t) \; y'(t) \; z'(t)]$$

so, $P(0) = P_1$ ← starting pt $(= B_1)$
$P(t_2) = P_2$ ← last point
$P'(0) = P_1'$ $(= B_2)$
$P'(t_2) = P_2'$



Now want B_3 and B_4 ⇒ ?
Put B_1, B_2 in (1), (2)
and
Solve, u will get B_3 and B_4.

$$B_3 = \frac{3(P_2 - P_1)}{t_2^2} - \frac{2P_1'}{t_2} - \frac{P_2'}{t_2}$$

$$B_4 = \frac{2(P_1 - P_2)}{t_2^3} + \frac{P_1'}{t_2^2} + \frac{P_2'}{t_2^2}$$

$$B_1 = P_1$$

$$B_2 = P_1'$$

↳ so to know B_i's want:-
P_1, P_2, P_1', P_2'

↳ Formula for 1 spline only (with 2 seg).
Similarly dusre ka dhundo

Now to check
whether connected or not?
→ whether smoothness hai yaa nahi?

→ for that find 2nd derivative of $P(t)$.

$$P'(t) = B_2 + 2B_3 t + 3B_4 t^2$$

$$P''(t) = 2B_3 + 6B_4(t)$$

Segment-1 :- $2B_3 + 6B_4(t_2)$ —— (*)
2nd derivative at last pt. of 1st segment.

Segment-2 :-

but at start $t = 0$
2nd derivative at 1st pt. of 2nd segment
$= 2B_3 + 6B_4(0)$
$= 2B_3$ —— (#)

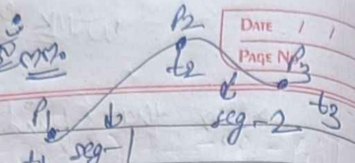
for continuity, (*) = (#) (must).

$$2B_3 = 2B_3 + 6B_4(t_2)$$

↳ these B_3 will be different.

so, $2B_3^{2} = 2B_3^{1} + 6B_4^{1}(t_2)$ ← seg-2, seg-1

$$t_2 = \frac{2B_3^2 - 2B_3^1}{6B_4^1} = \frac{B_3^2 - B_3^1}{(3)B_4^1}$$



Final equation :-

$$\underbrace{6t_2\left[\frac{2(P_1-P_2)}{t_2^3}+\frac{P_1'}{t_2^2}+\frac{P_2'}{t_2^2}\right]}_{B_4'} + \underbrace{2\left[\frac{3\cancel{(P_1-P_2)}\,(P_2-P_1)}{t_2^2} - \frac{2P_1'}{t_2} - \frac{P_2'}{t_2}\right]}_{B_3'} = \underbrace{2\left[\frac{3\cancel{(P_2-P_3)}\,(P_3-P_2)}{t_3^2} - \frac{2P_2'}{t_3} - \frac{P_3'}{t_3}\right]}_{B_3^2}$$

Final eq^n :-

$$(t_3)(P_1') + 2(t_3+t_2)P_2' + t_2P_3' = \frac{3}{t_2t_2}\left[t_2^2(P_3-P_2)+t_3^2(P_2-P_1)\right]$$

agar 2 ki jgaah --> 9 segments hote

↓

P_{10} tak jaata

How to generalise!

As can have many segments.

So in general ⇒

$$t_{(k+2)}P_k' + 2(t_{k+1}+t_{k+2})P_{k+1}' + t_{k+1}P_{k+2}' = \frac{3}{(t_{k+1})(t_{k+2})}\left[t_{k+1}^2(P_{k+2}-P_{k+1}) + t_{k+2}^2(P_{k+1}-P_k)\right]$$

(*) for (n) points, u will have (n-2) equations.

↳ $P_1, P_2, \ldots, P_n$.

like here P_1, P_2, P_3 ↳ and $(3-2)=1$ eq^n.

⇒ So computer mein toh esse nahi de skte, so 'matrix' form mein denge.

$$\begin{bmatrix}
1 & 0 & 0 & \cdots & \cdots & \cdots & 0\\
t_3 & 2(t_2+t_3) & t_2 & 0 & 0 & 0 & 0\\
0 & t_4 & 2(t_3+t_4) & t_3 & 0 & 0 & 0\\
0 & 0 & t_5 & 2(t_4+t_5) & t_4 & \vdots & \vdots\\
\vdots & \vdots & 0 & \vdots & \vdots & \vdots & \vdots\\
\vdots & \vdots & \vdots & \vdots & t_n & 2(t_{n-1}+t_n) & t_{n-1}\\
0 & 0 & \cdots & \cdots & \cdots & \cdots & 1
\end{bmatrix}$$

|| X

not diagonal matrix.

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$$X \begin{bmatrix} P_1' \\ P_2' \\ \vdots \\ P_n' \end{bmatrix} = \begin{bmatrix} \frac{3}{t_2 t_3}\left[t_2^2(P_3-P_2)+t_3^2(P_2-P_1)\right] \\ \vdots \\ \frac{3}{(t_{n-1})(t_n)}\left[(t_{n-1})^2(P_n-P_{n-1})+(t_n)^2(P_{n-1}-P_{n-2})\right] \end{bmatrix}$$

we need values of P_1', P_2' for seg-1,
P_2', P_3' for seg-2

$\vdots$

P_n', P_{n+1}' for seg-n

so, ~~[M] [P'] = [R]~~ $[M][P'] = [R]$

→ want to make (X) as diagonal matrix.

by adding row-1 ⇒
1 0 0 0 0 0 0 . . 0

and last row
⇒ 0 0 0 0 . . 0 1

at diagonal matrix bnjaayegi

as show by blue on previous page

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(*) then 'n' rows ho jaayengi!

$$[P'] = [M]^{-1}[R]$$
$$[M][P'] = [R]$$

$$P_k(t) = [1 \; t \; t^2 \; t^3] \begin{bmatrix} B_{1k} \\ B_{2k} \\ B_{3k} \\ B_{4k} \end{bmatrix}$$

Now what will be generalized form (?)

$$B_{1k} = P_k$$

$$B_{2k} = P_k'$$

$$B_{3k} = \frac{3(P_{k+1}-P_k)}{t_{k+1}^2} - \frac{2P_k'}{t_{k+1}} - \frac{P_{k+1}'}{t_{k+1}}$$

$$B_{4k} = \frac{2(P_k - P_{k+1})}{t_{k+1}^3} + \frac{P_k'}{t_{k+1}^2} + \frac{P_{k+1}'}{t_{k+1}^2}$$

So, final matrix:-

$$\begin{bmatrix} B_{1k} \\ B_{2k} \\ B_{3k} \\ B_{4k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{-3}{t_{k+1}^2} & \frac{-2}{t_{k+1}} & \frac{3}{t_{k+1}^2} & \frac{-1}{t_{k+1}} \\ \frac{2}{t_{k+1}^3} & \frac{1}{t_{k+1}^2} & \frac{-2}{t_{k+1}^3} & \frac{1}{t_{k+1}^2} \end{bmatrix} \begin{bmatrix} P_k \\ P_k' \\ P_{k+1} \\ P_{k+1}' \end{bmatrix}$$

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So, final equation is :-

$$= \left[\left(1 - \frac{3t^2}{t_{k+1}^2} + \frac{2t^3}{t_{k+1}^3}\right) \left(t - \frac{2t^2}{t_{k+1}} + \frac{t^3}{t_{k+1}^2}\right) \left(\frac{-2t^3}{t_{k+1}^3} + \frac{3t^2}{t_{k+1}^2}\right) \left(\frac{-t^2}{t_{k+1}} + \frac{t^3}{t_{k+1}^2}\right) \right] \begin{bmatrix} P_k \\ P'_k \\ P_{k+1} \\ P'_{k+1} \end{bmatrix}$$

After rearranging →

$$= [f_1(\tau) \ \ f_2(\tau) \ \ f_3(\tau) \ \ f_4(\tau)] \begin{bmatrix} P_k \\ P'_k \\ P_{k+1} \\ P'_{k+1} \end{bmatrix}$$

where, $\tau = \frac{t}{t_{k+1}}$

where,

$$f_1(\tau) = 1 - \frac{3t^2}{t^2_{k+1}} + \frac{2t^3}{t^3_{k+1}} = 1 - 3\tau^2 + 2\tau^3 = 2\tau^3 - 3\tau^2 + 1$$

$$f_2(\tau) = -2\tau^3 + 3\tau^2$$

$$f_3(\tau) = t_{k+1}(\tau(\tau^2 - 2\tau + 1))$$

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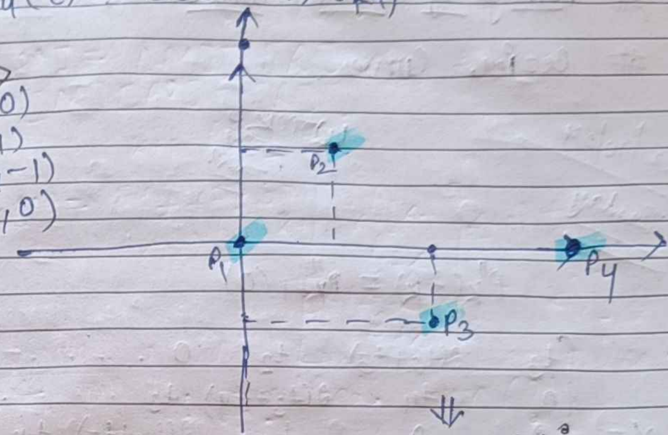
$$f_4(\tau) = \tau(\tau^2 - \tau)\, t_{k+1}$$

Question →

$P_1(0,0)$
$P_2(1,1)$
$P_3(2,-1)$
$P_4(3,0)$



⇒ 4 control points.

⇓

Draw Spline passing through these 4 interpolating points.

Tangent vectors for P_1 and P_4 are (1,1) and (1,1). find value for $\boxed{\tau = \frac{2}{3}}$.

as, $\tau = \frac{t}{t_{k+1}} = \frac{2}{3}$

Hermite Basis (Blending) functions :-

$$x(u) = [u^3 \ u^2 \ u \ 1] \begin{bmatrix} a_x \\ b_x \\ c_x \\ d_x \end{bmatrix}$$

$$= U \cdot C$$

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family of Hermite Curves :- ?

Bezier Curves

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$$P'_K(t_K) = P''_{K+1}(0)$$

$$X = \begin{bmatrix} t_3 & 2(t_2+t_3) & t_2 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & t_4 & 2(t_3+t_4) & t_3 & \cdots & & 0 & \cdots \\ 0 & 0 & t_5 & 2(t_4+t_5) & t_4 & 0 \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & & & & \vdots \\ & & & & 0 & t_n & 2(t_n+t_{n-1}) & t_{n-1} \end{bmatrix}$$

$$P = \begin{bmatrix} P_1' \\ P_2' \\ \vdots \\ P_n' \end{bmatrix} \qquad A = \begin{bmatrix} \frac{3}{t_2 t_3}\{t_2^2(P_3-P_2) + t_3^2(P_2-P_1)\} \\ \vdots \\ \frac{3}{(t_{n-1})(t_n)}\{t_{n-1}^2(P_n-P_{n-1}) + t_n^2(P_{n-1}-P_{n-2})\} \end{bmatrix}$$

then,

$$\boxed{XP = A}$$

Q1. Cubic Spline ← Nothing but interpolating spline
↳ means always pass through control points

P_1 [0 0]
P_2 [1 1]

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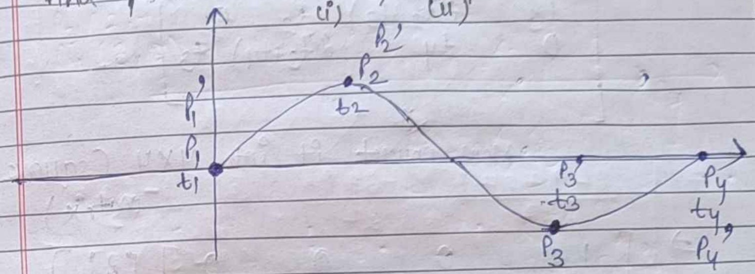
P_3 [2 -1]
P_4 [3 0]
P_1' [1 1] } → tangent vectors
P_4' [1 1]

find points on $\tau = 1/3$ (i) and $\tau = 2/3$ (ii).

A1.



(i) $\tau = \frac{t}{t_{k+1}}$ want t.
initially $t = 0$

so, $t_1 = 0$
want t_2, t_3 and t_4.

here $\boxed{n=4}$.

P_1 (0, 0)
P_2 (1, 1)

distance b/w P_2 and P_1 $= \sqrt{(1-0)^2+(1-0)^2}$
$= \sqrt{2}$

and, t_2 will be equal to this distance only.

$t_2 = \sqrt{2}$

and, distance b/w P_3 and P_2 $= \sqrt{(2-1)^2+(-1-1)^2}$
$= \sqrt{1^2+2^2}$
$= \sqrt{5}$

thus, $t_3 = \sqrt{5}$

distance b/w P_4 and P_3 = $\sqrt{(3-2)^2+(0+1)^2}$
$= \sqrt{1^2+1^2}$
$= \sqrt{2}$

thus, $t_4 = \sqrt{2}$.

Now

$$\begin{bmatrix} t_3 & 2(t_2+t_3) & t_2 & 0 \\ 0 & t_4 & 2(t_3+t_4) & t_3 \end{bmatrix}$$

⇓

Now convert it into 4×4 (square matrix)

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ t_3 & 2(t_2+t_3) & t_2 & 0 \\ 0 & t_4 & 2(t_3+t_4) & t_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_1' \\ P_2' \\ P_3' \\ P_4' \end{bmatrix} = \begin{bmatrix} \\ \frac{3}{t_2 t_3}\{t_2^2(P_3-P_2)+t_3^2(P_2-P_1)\} \\ \frac{3}{t_3 t_4}\{t_3^2(P_4-P_3)+t_4^2(P_3-P_2)\} \\ \\ \end{bmatrix}$$

$P_3 P_2$ distance b/w P_3, P_2

$\frac{3}{\sqrt{2}\sqrt{5}}\{(2)(\sqrt{5}) + (5)(\sqrt{2})\}$

(11.54)

$= \frac{3}{\sqrt{10}} \times 34.62$

$= \frac{34.62}{\sqrt{10}}$

$= 6.64$

Now convert into 4×1

$$= \begin{bmatrix} P_1' \\ \frac{3}{t_2 t_3}\{\dots\} \\ \frac{3}{t_3 t_4}\{\dots\} \\ P_4' \end{bmatrix}$$

Now put values ⇒

idhar jaake inverse hogya.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \sqrt{5} & 2(\sqrt{2}+\sqrt{5}) & \sqrt{2} & 0 \\ 0 & \sqrt{2} & 2(\sqrt{2}+\sqrt{5}) & \sqrt{5} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} [1 \quad 1] \\ P_2' \\ P_3' \\ [1 \quad 1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 6.64 & \\ 6.64 & \\ 1 & 1 \end{bmatrix}$$

0.949

0.949

finally ⇒

$$\begin{bmatrix} [1 \quad 1] \\ P_2' \\ P_3' \\ [1 \quad 1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0.525 & -0.148 \\ 0.525 & -0.148 \\ 1 & 1 \end{bmatrix}$$

inverse leke multiply kiya

Now,

$P_K(\tau) = [F][G]$

$= [(2\tau^3 - 3\tau^2 + 1) \quad (-2\tau^3 + 3\tau^2) \quad t_{k+1}(\tau(\tau^2 - 2\tau + 1)) \quad \tau(\tau^2 - \tau)t_{k+1}]$

$$\begin{bmatrix} P_K \\ P_{K+1} \\ P'_K \\ P'_{K+1} \end{bmatrix} \Rightarrow [G]$$

$$\boxed{P(t) = \sum_{i=1}^{4} B_i t^{i-1}}$$

There are 3 segments in spline

for 1st segment, $k=1$ so $P_K = P_1$

as $\tau = 1/3$ → will lie in 1st segment, 2nd seg, or 3rd seg,

for 1st seg ⇒ thus, $t_{k+1} = t_2$

$\tau = 1/3$,

WJMK 3rd sem.

$P_k(\tau) =$

$$\left[\left(\frac{2}{27} - \frac{3}{9} + 1\right) \quad \left(-\frac{2}{27} + \frac{3}{9}\right) \quad (\sqrt{2})\left[\frac{1}{3}\left(\frac{1}{9} - \frac{2}{3} + 1\right)\right] \quad \left(\frac{1}{3}\right)\left(\frac{1}{9} - \frac{1}{3}\right)(\sqrt{2})\right] \begin{bmatrix} P_1 \\ P_2 \\ P_1' \\ P_2' \end{bmatrix}$$

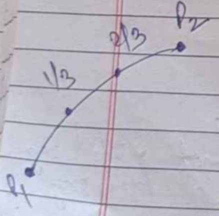
$$= \left[\frac{2-9+27}{27} \quad \frac{-2+9}{27} \quad (\sqrt{2})\left[\frac{1-6+9}{27}\right] \quad (\sqrt{2})\left(\frac{1-3}{27}\right)\right]$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 1 & 1 \\ 0.525 & -0.148 \end{bmatrix}$$

→ Similarly hm 2nd and 3rd segment mein nikaalenge (solve it) ↳ (3) values dayenge $t = 1/3$ ki

(ii) Now for $\tau = 2/3 \leftarrow <1$

↳ so ~~also in~~ for segment 1.



thus $\tau = 2/3$ and $k = 1$

to in eq^n of $P_k(\tau) = [F][G]$.

→ similarly hm 2nd and 3rd segment mein nikaalenge !!!!

WJMK 3rd sem.

Bezier Curve :- approximating spline curve

$$P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t)$$

$J_{n,i}(t) \rightarrow$ blending function

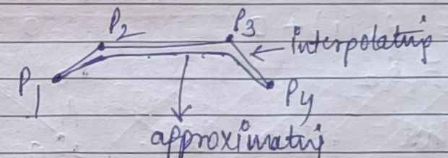
↳ tells nature of curve.

$$J_{n,i}(t) = \binom{n}{i} t^i (1-t)^{n-i}$$

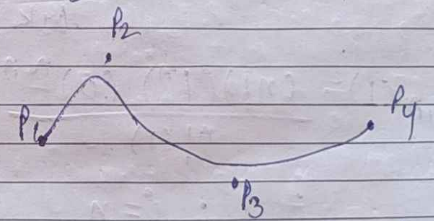
where,

$$\binom{n}{i} = \frac{n!}{(n-i)!\,i!}$$

say:



↳ P_1 se move towards P_2, but P_2 touch nhi krega, then move towards P_3 but P_3 — — —, then comes to P_4.



Now any spline start ~~for~~ from $t = 0 \Rightarrow$

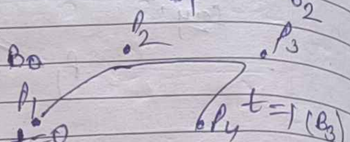
$$J_{n,0}(0) = \binom{n}{0}(0)^0(1-0)^{n-0}$$
$$= (1)(1)(1)$$
$$= 1.$$

(as $0^0 = 1$ in computer)
↓
koan chahe theoretically undefined.

and,

$$J_{n,i}(0) = \binom{n}{i}(0)^i(1-0)^{n-i}$$
$$= 0$$

Now,
$$P(0) = \sum_{i=0}^{n} B_i \cdot (J_{n,i}(0))$$
$$\cancel{= \sum_{i=0}^{n} B_i (0) = 0}$$
$$= B_0(J_{n,0}(0)) + \sum_{i=1}^{n} B_i(J_{n,i}(0))$$
$$= B_0(1) + 0$$
$$= B_0$$



at last, here P_4

$\boxed{t=1}$ (as ~~segment~~ spline t=0 se t=1 tak jaata hai)

Now,
at t=1
$$\Rightarrow J_{n,n}(1) = \frac{(n!)(1^n)(0)^{n-n}}{n!(1)}$$
$$= 1;\ i=n$$

$$\Rightarrow J_{n,i}(1) = \begin{cases} \frac{n!}{i!(n-i)!}(1)^i(1-1)^{n-i} = 0; & i \neq n \\ 1; & i = n \end{cases}$$

$$P(1) = B_n J_{n,n}(1) = \sum_{i=0}^{n} B_i(J_{n,i}(1))$$

$\boxed{P(1) = B_n,}$
$$= \sum_{i=0}^{n-1} B_i(0) + B_n(J_{n,n}(1))$$
$$= 0 + B_n(1) = B_n$$

Bezier curve

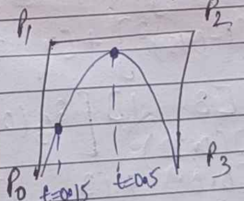
↳ always convex hull
↳ iski saari properties, it will have.

and also ⇒

$\boxed{\sum_{i=0}^{n} J_{n,i}(t) = 1}$ → this is for 'basis function'.

Q. Bezier curve
$B_0 = [1\ \ 1]$
$B_1 = [2\ \ 3]$
$B_2 = [4\ \ 3]$
$B_3 = [3\ \ 1]$

We know:-
at t=0 → B_0
at t=1 → B_3,
find value B' at t=0.15,
t=0.5



$P(0) = P_0 = B_0 = [1\ \ 1]$

~~Now, $P(1) = B_n$~~

n=3, as starting from 0.

⇓

but in computer:- i=0 to n, nahi, i=1 to n+1 leke chlo, as 0° problem nahi aayegi tab.

NOTE: WOMK

① Beizer curve can be given in matrix form:-

$$P(t)=[F][G]$$

$$F=[J_{n,0} \quad J_{n,1} \quad J_{n,2} \quad \dots \quad J_{n,n}]$$

$$G^T=[B_0 \quad B_1 \quad \dots \quad B_n]$$

$$P(t)=[(1-t)^3 \quad 3t(1-t)^2 \quad 3t^2(1-t) \quad t^3]\begin{bmatrix}B_0\\B_1\\B_2\\B_3\end{bmatrix}$$

$$P(t)=[T][N][G]$$

$$=\underbrace{[t^3 \quad t^2 \quad t \quad 1]}_{[T]}\underbrace{\begin{bmatrix}-1 & 3 & -3 & 1\\3 & -6 & 3 & 0\\-3 & 3 & 0 & 0\\1 & 0 & 0 & 0\end{bmatrix}}_{[N]}\underbrace{\begin{bmatrix}B_0\\B_1\\B_2\\B_3\end{bmatrix}}_{[G]}$$

A. ~~$P(0.15)=[1.5 \quad 1.0765]$~~

~~$P(0.5)=[2.75 \quad 2.5]$.~~

as $P(t)=\sum_{i=0}^{3} B_i J_{3,i}(t)$

$$P(0.15)=B_0 J_{3,0}(0.15)+B_1 J_{3,1}(0.15)+B_2 J_{3,2}(0.15)+B_3 J_{3,3}(0.15)$$

$$=[1 \quad 1]\left[\binom{3}{0}(0.15)^0(1-0.15)^3\right]+[2 \quad 2]\left[\binom{3}{1}(0.15)^1(1-0.15)^2\right]+[4 \quad 3]\left[\binom{3}{2}(0.15)^2(1-0.15)^1\right]+[3 \quad 1]\left[\binom{3}{3}(0.15)^3(1-0.15)^0\right]$$

$$=[1 \quad 1](0.61)+[2 \quad 2](0.32)+[4 \quad 3](0.05)+[3 \quad 1](0.003)$$

$$=[0.61 \quad 0.61]+[0.64 \quad 0.64]+[0.20 \quad 0.15]+[0.009 \quad 0.003]$$

WOMK

$$=[1.459 \quad 1.403]$$

Similarly can find $P(0.5)$

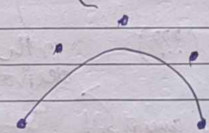
(By matrix, it can be easily found).

① What is 1st / 2nd derivative of $J_{n,i}$? → for connectivity

In general

$$[T]=[t^n \quad t^{n-1} \quad \dots \quad t \quad 1]$$

$$[N]=\begin{bmatrix}\binom{n}{0}\binom{n}{n}(-1)^n & \binom{n}{1}\binom{n-1}{n-1}(-1)^{n-1} & \dots & \binom{n}{n}\binom{n-n}{n-n}(-1)^0\\ \binom{n}{0}\binom{n}{n-1}(-1)^{n-1} & \binom{n}{1}\binom{n-1}{n-2}(-1)^{n-2} & \dots & 0\\ \vdots & & & \\ \binom{n}{0}\binom{n}{1}(-1)^1 & \binom{n}{1}\binom{n-1}{0}(-1)^0 & \dots & 0\\ \binom{n}{0}\binom{n}{0}(-1)^0 & 0 & \dots & 0\end{bmatrix}$$



$$[G]=\begin{bmatrix}B_0\\B_1\\ \vdots\\ B_n\end{bmatrix}$$

(i) 1st derivative of $J_{n,i}$ ⇒

$$J'_{n,i}(t) = \binom{n}{i}\left\{ i\,(t^{i-1})(1-t)^{n-i} - (n-i)\, t^{i}(1-t)^{n-i-1} \right\}$$

$$= \binom{n}{i} t^{i}(1-t)^{n-i}\left\{ \frac{i}{t} - \frac{(n-i)}{(1-t)} \right\}$$

$$= \frac{(i-nt)}{t(1-t)} J_{n,i}(t)$$

(ii) Second derivative of $J_{n,i}$ ⇒

$$J''_{n,i}(t) = \left\{ \frac{(i-nt)^2 - nt^2 - i(1-2t)}{t^2(1-t)^2} \right\} J_{n,i}(t)$$

→ will be used to check smoothness.

↳ (Just equate both) (????)

$$P^{r}(0) = \frac{n!}{(n-r)!} \sum_{i=0}^{r} (-1)^{n-i} \binom{r}{i} (B_i)$$

→ and will get smoothness eqⁿ (???)

rth derivative of P

generalized form.

$$P'(0) = n(B_1 - B_0)$$

tangent → $P'(t) = \sum_{i=0}^{n} B_i \, J'_{n,i}(t)$.

$$P''(t) = \sum_{i=0}^{n} B_i \, J''_{n,i}(t)$$

$$P'(1) = n(B_n - B_{n-1})$$

$$P''(0) = n(n-1)(B_0 - 2B_1 + B_2)$$

$$P''(1) = n(n-1)(B_n - 2B_{n-1} + B_{n-2})$$

lec-

5/11.
Tuesday

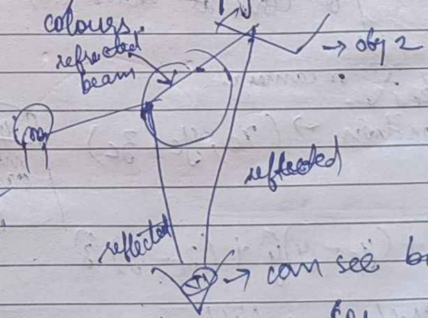
Blending fⁿ → 4 types (??)

→ In convex polygon, no problem, once u get one pt. inside polygon, you can completely fill the whole ", problem comes in concave polygon.

instead of this,
take multiple convex polygons, whose sum = concave polygon.

→ Illumination ?

Illumination of light on everyone is same, but we see diff colours on objects.
↳ property of objects like texture, colour, pigment etc. that show different colours.



→ can see both objects.

(If multiple bouncing, then there is problem)

WOMK

H/W n-dimension? → illumination where reg? → influence, diffuse data

(*) Ambient Illumination :-

light band kadi room ki, still can see booz "coming from window etc which is not direct sunlight.

$$I(x,y,z,\theta,\phi,\lambda) = K_E(\lambda) + K_a(\lambda)\cdot I_a(\lambda)$$

→ wavelength

$$+ (K(\lambda))\, I_0(\tau,\lambda) \cos\theta$$

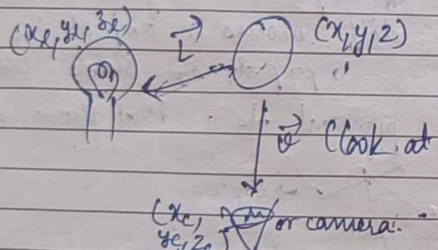
position in world coordinates as generating whole frame in world coordinates, not eye coordinates

as can have multiple light sources so coordinate of ith light source

$(x_\ell^i, y_\ell^i, z_\ell)$ (position coordinates of light source)

Now, define:

$$\vec{L} = (x_\ell - x)\hat{x} + (y_\ell - y)\hat{y} + (z_\ell - z)\hat{z}$$



Camera position → (x_c, y_c, z_c)

H/W multidimensional camera position

$$\vec{v} = (x_c - x)\hat{x} + (y_c - y)\hat{y} + (z_c - z)\hat{z}$$

→ If my pt, itself is a light source (as some objects have some amt of light source), but has very little lights, like WINDOW → $K_E(\lambda)$

like stock market advisors

WOMK

→ property of object.

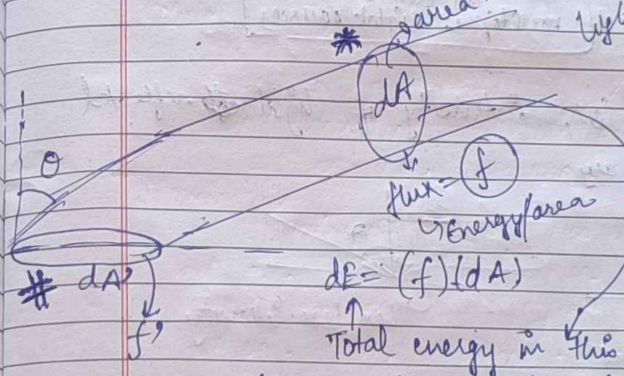
$K_a(\lambda)$?

$I_a(\lambda)$?

→ property of light source.

⇒ 'K' component gives diff. colours of object, that differentiate object

can change with time, like in day diff, in night diff, but ka always same!

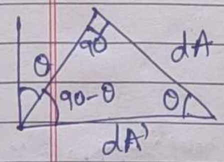


$dE = (f)(dA)$

Total energy in this area.

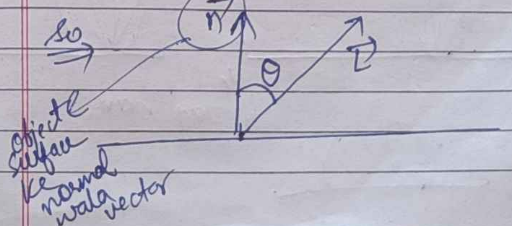
$(f)(dA) = (f')(dA')$ (as * light falling on #)

$$f' = f\left(\frac{dA}{dA'}\right)$$



$dA = (dA')\cos\theta$

so, $(f') = f\cos\theta$



$$\cos\theta = \hat{n}\cdot\hat{L} = \frac{\vec{n}\cdot\vec{L}}{||\vec{n}||\,||\vec{L}||}$$

So

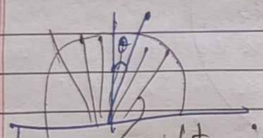
$$I(x,y,z,\theta,\phi,\lambda) = K_E(\lambda) + (K_a(\lambda))(I_a(\lambda)) + (K(\lambda))\, I_0(r,\lambda)\cos\theta + (k_\lambda)(I_0(r,\lambda))(\cos\phi)$$

$\hat{n}\cdot\hat{L}$

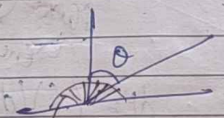
$\hat{n}\cdot\hat{L}_i$

as can have multiple light sources

depending on θ, intensity of light ~~the~~ reflected will vary.

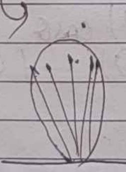


light reflected of high intensity

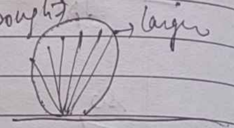


here less intensity

what if roughness on surface?

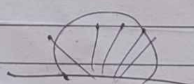


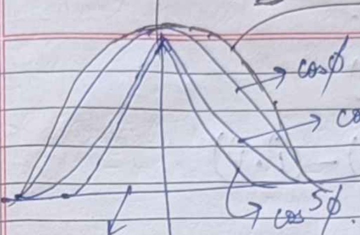
→ if more rough



larger

if very rough



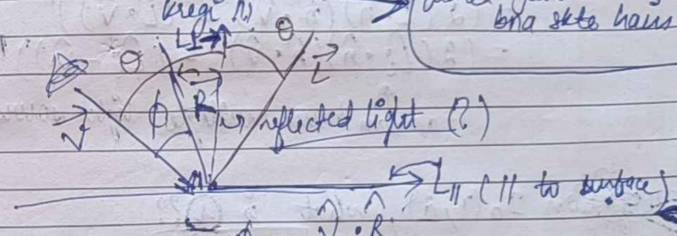


→ expanding flux by ↓ing power.

so concentrating flux to narrow pencil like beam by ↑ing power.

so roughness is par dependent kregi n

dheere-2 square jaisa bna skte hain



so, $\cos\phi = \hat{V}\cdot\hat{R}$

How to know $\hat{R}$?

$$\vec{L} = \vec{L_\perp} + \vec{L_\parallel}$$

$$\vec{R} = \vec{L_\perp} - \vec{L_\parallel}$$

what is $L_\perp$?

$$\vec{L_\perp} = (\vec{L}\cdot\hat{n})\hat{n}$$

$$L_\parallel = \vec{L} - \vec{L_\perp}$$

$$= \vec{L} - (\vec{L}\cdot\hat{n})\hat{n}$$

so,

$$\vec{R} = \vec{L_\perp} - \vec{L_\parallel}$$

$$= (\vec{L}\cdot\hat{n})\hat{n} - (\vec{L} - (\vec{L}\cdot\hat{n})\hat{n})$$

$$= 2(\vec{L}\cdot\hat{n})\hat{n} - \vec{L}$$

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so, $\hat{R} = 2(\hat{L}\cdot\hat{n})\hat{n} - \hat{L}$

so, $\hat{V}\cdot\hat{R} = 2(\hat{L}\cdot\hat{n})(\hat{n}\cdot\hat{V}) - (\hat{L}\cdot\hat{V})$

so, $\cos\phi = \hat{V}\cdot\hat{R}$

so, $(\cos\phi)^{\mu} = (\hat{V}\cdot\hat{R})^{\mu}$

$= (2(\hat{L}\cdot\hat{n})(\hat{n}\cdot\hat{V}) - (\hat{L}\cdot\hat{V}))^{\mu}$

$= (2(\hat{L_i}\cdot\hat{n})(\hat{n}\cdot\hat{V}) - (\hat{L_i}-\hat{V}))^{\mu}$

as can have multiple light sources.

In $I_0(r,\lambda)$, what is r??

$\boxed{r = \|\vec{L}\|}$

as intensity will depend on it!!!

$\boxed{\frac{I_0}{(r^2)}(e^{-\sigma r})}$????