Theory of Computation

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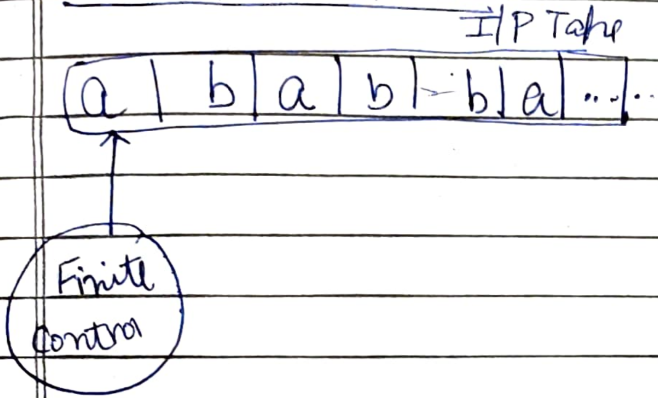
Finite Automata :-



Deterministic Finite Automata (DFA) :-

$$DFA = \langle Q, \Sigma, \delta, q_0, F \rangle$$

Q = Finite set of states.
Σ = Finite set of alphabets.
δ = Transition function
q_0 = initial state $\quad q_0 \in Q$
F = finite set of final states. $\quad F \subseteq Q$

$$\delta : Q \times \Sigma \rightarrow Q$$

$$q_0 \xrightarrow{a} q_1$$

$$\delta(q_0, a) = a_1$$

aaa
baaa

ab

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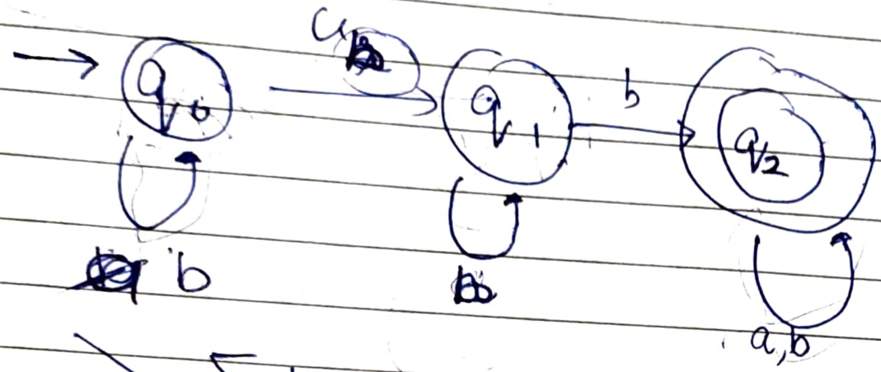
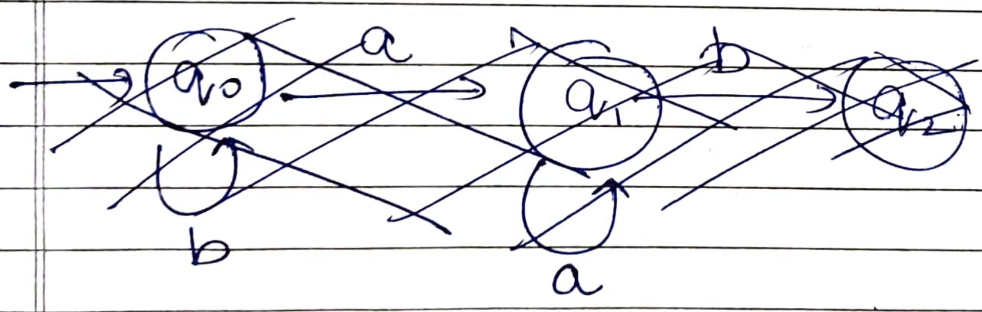
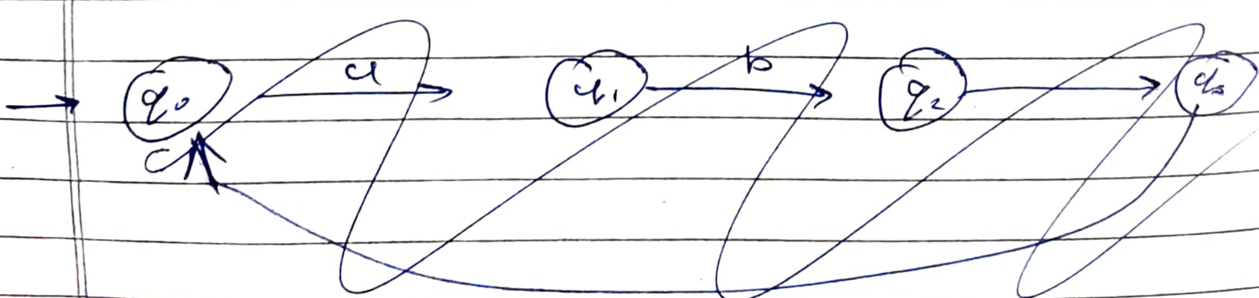
$L = \langle w \mid w \in (a,b)^*,$ ab anywhere $\rangle$

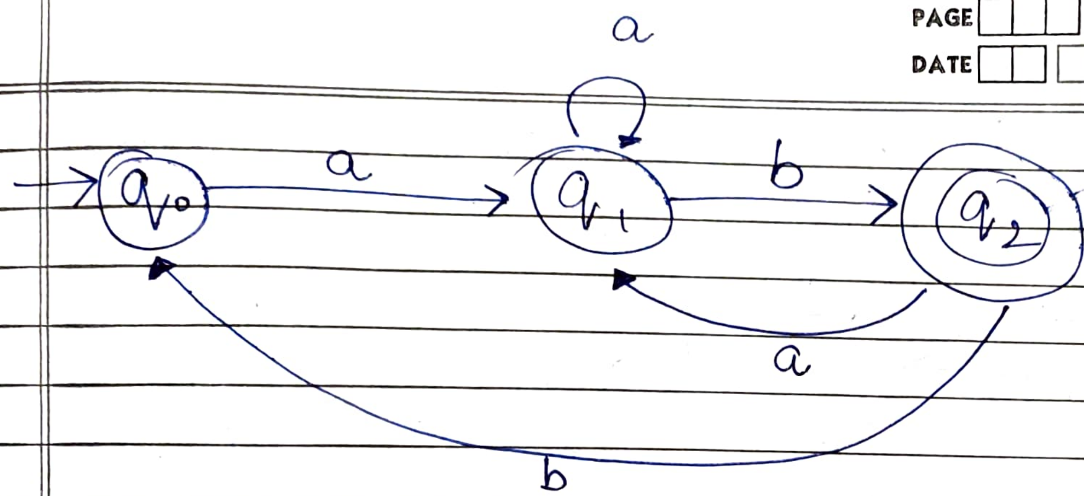


$Q \setminus \Sigma$	a	b
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_2	q_2

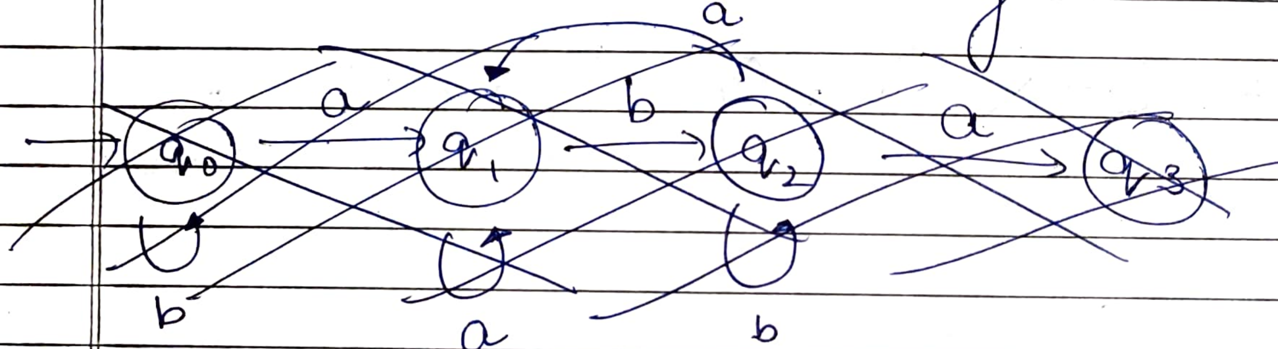
$L = \langle w \mid w \in (a,b)^*,$ ends with ab $\rangle$

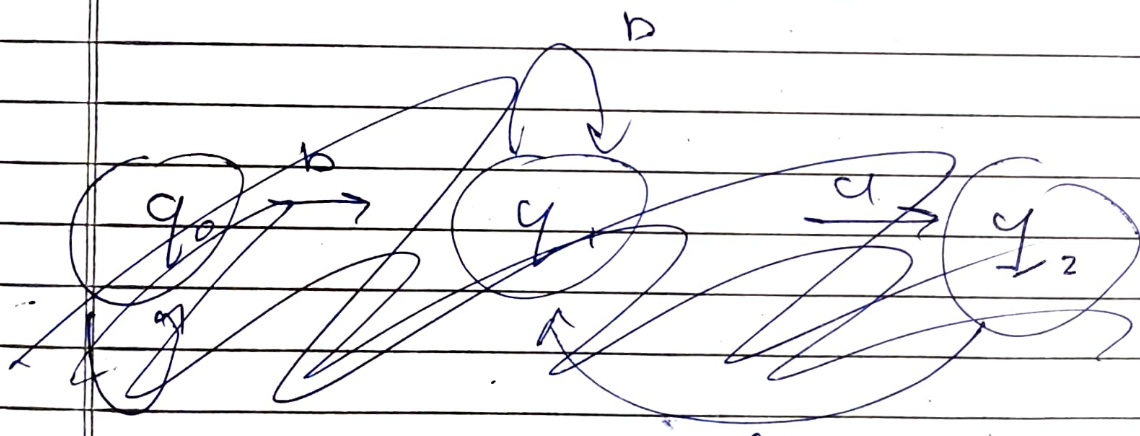


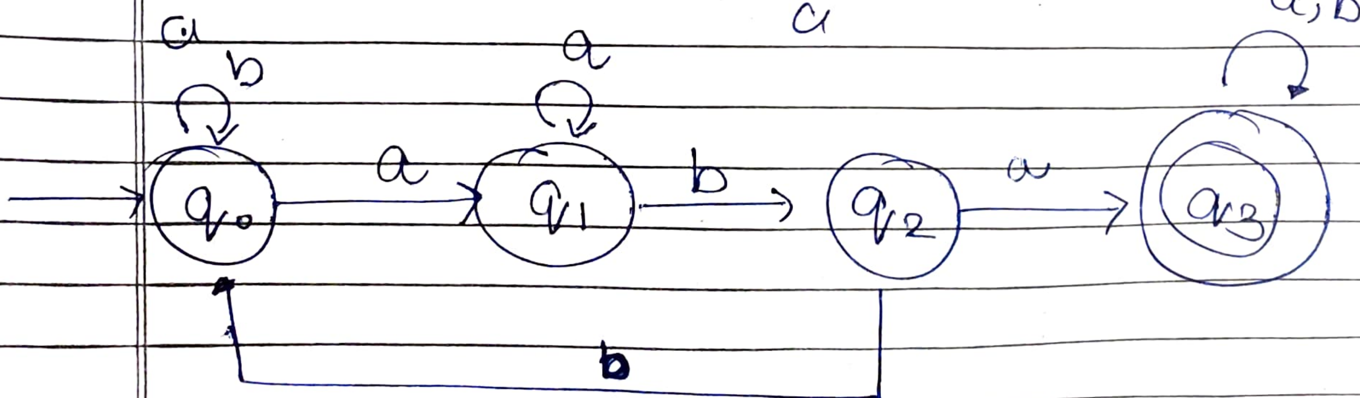




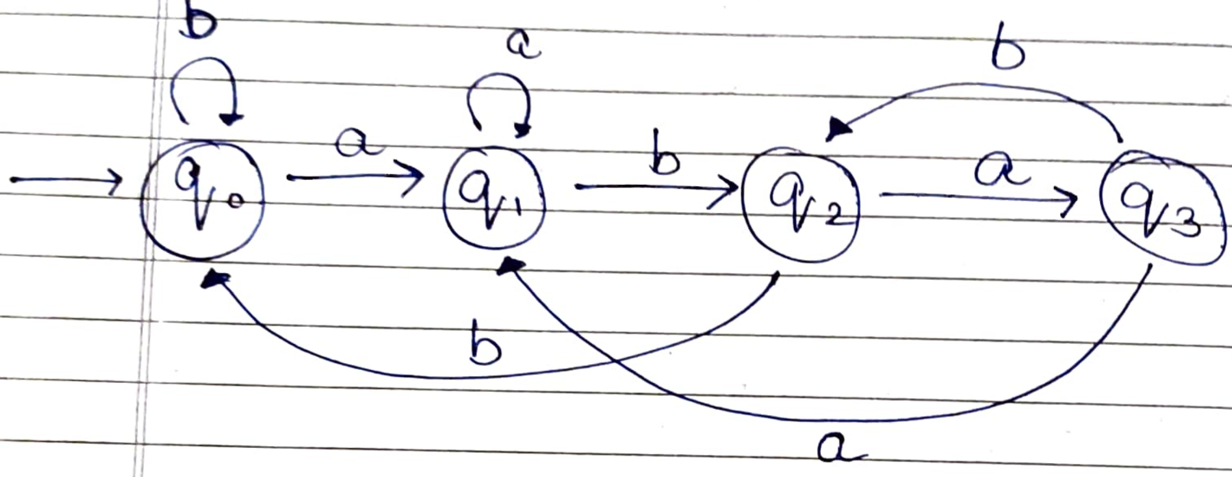
* $L = \langle w / w \in (a,b)^*$, aba anywhere $\rangle$



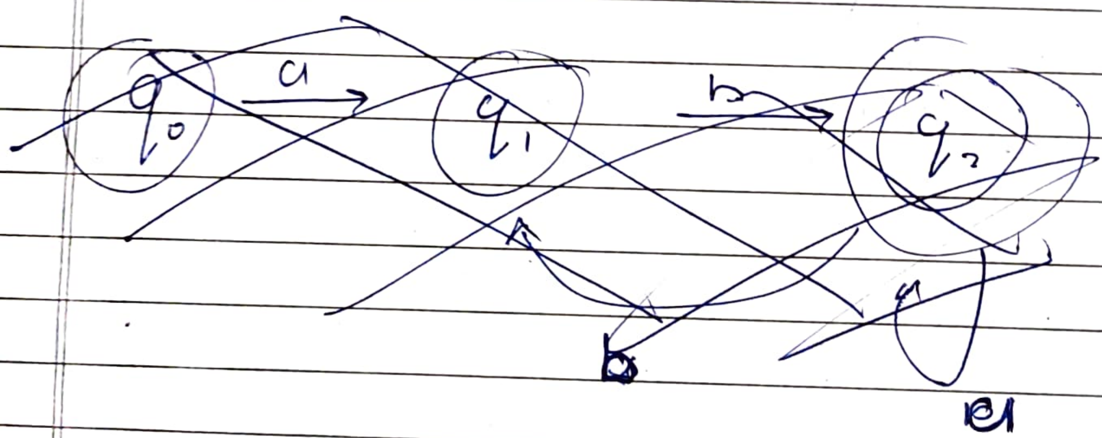


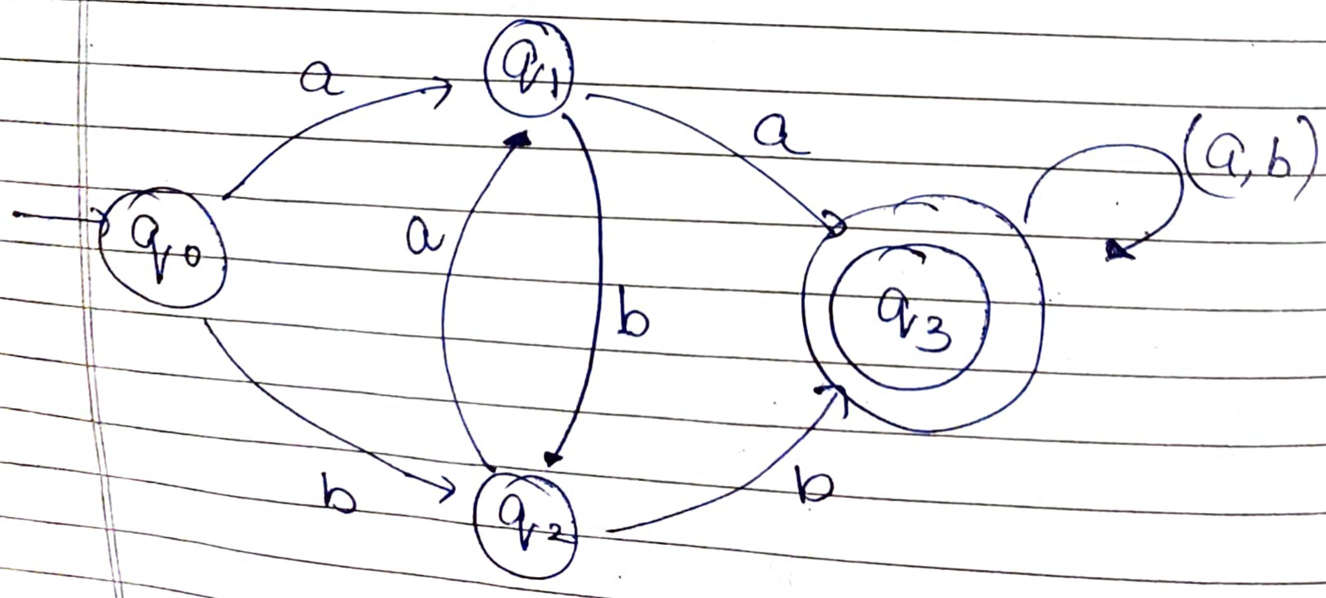


* $L = \langle w / w \in (a,b)^*$, ends with aba $\rangle$



* $L = \langle w / w \in (a,b)^*$, such that aa or bb are present anywhere $\rangle$





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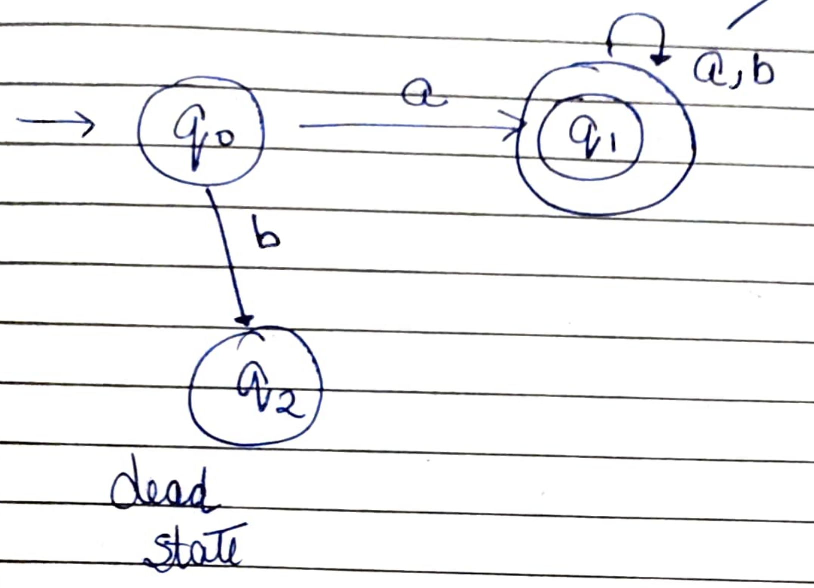
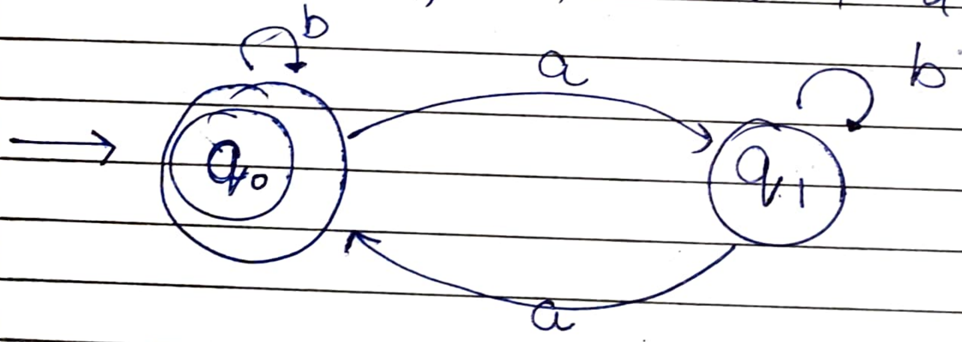
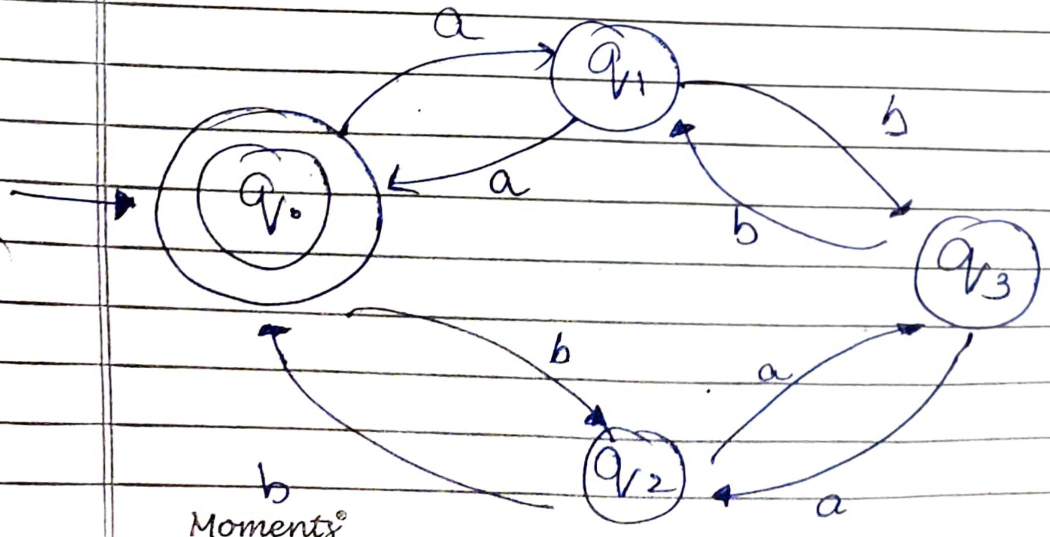
* $L = \{ aw \mid w \in (a,b)^* \}$



* $L = \{ w \mid w \in (a,b)^*$, such that a should be even$\}$



* $L = \{ w / w \in (a,b)^*$, such that a, b both even$\}$



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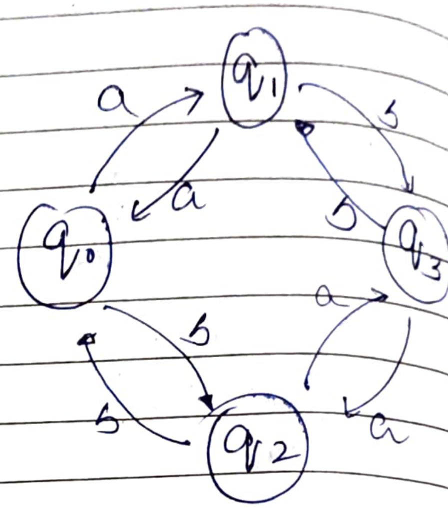
* $L = \langle w, w \in (a,b)^*$ such that.

(i) ~~ab~~ a even, b odd

q_2 is final

(ii) a odd, b even

q_1 is final.

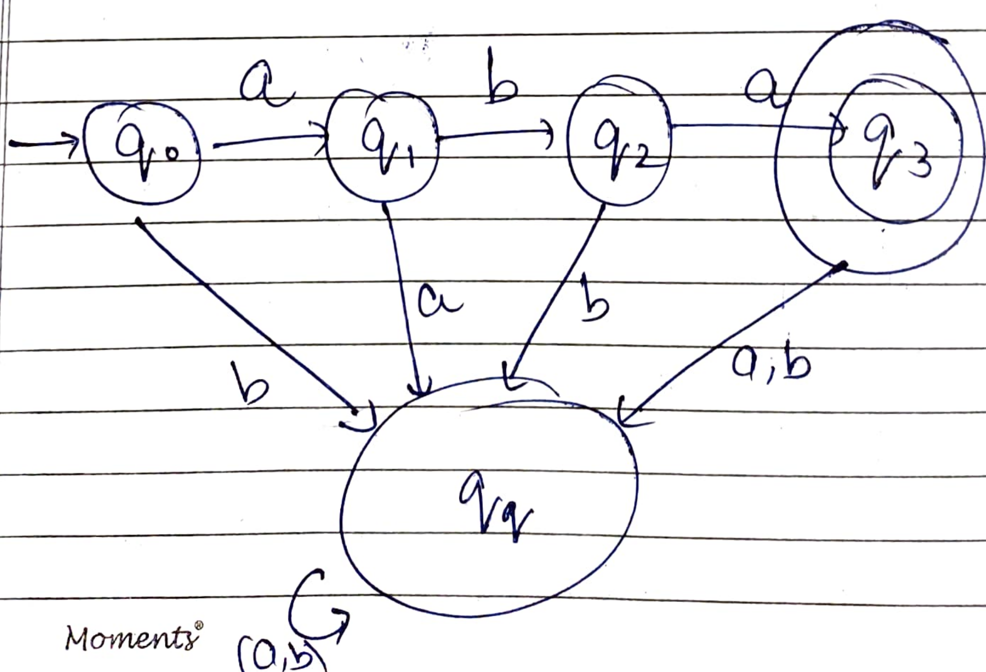


(iii) a odd, b odd

q_3 is final state

~~$L = \langle a, b, a \rangle$~~

$L = \langle aba \rangle$



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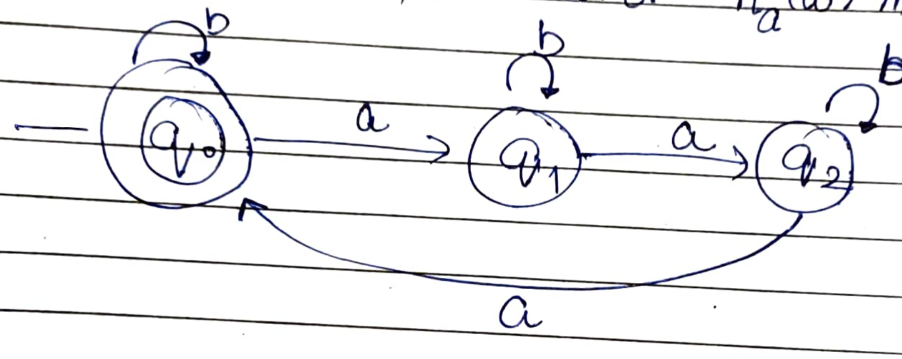
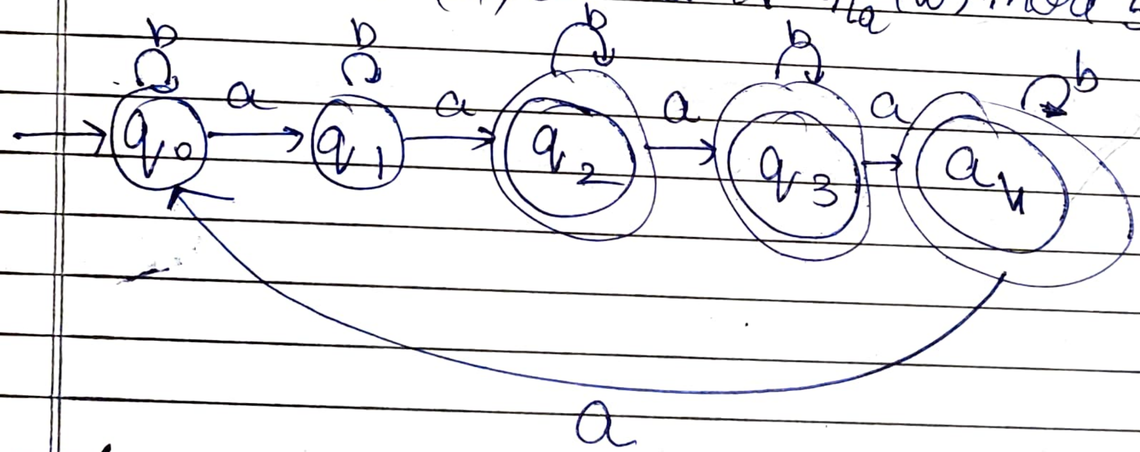
To make the complement of DFA interchange final states with other states.

$L = \langle w / w \in (a,b)^* \text{ and } n_a(w) \bmod 3 = 0 \rangle$

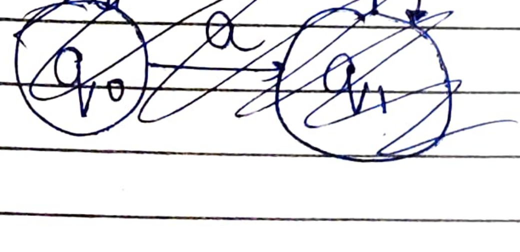


$L = \langle w / w \in (a,b)^* \text{ and } n_a(w) \bmod 5 > 1 \rangle$



$L = \langle w / w \in (a,b)^*$ and a should be third last

$w\, a\, w_1 w_2$



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$$L = \{ w \,/\, w \bmod 5 = 0, \; w \in (0,1)^* \}$$

w is a binary string

⑤

0	00000
5	00101
10	01010
15	01111
20	10100
25	11001
30	11100

$Q \backslash \Sigma$	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

Non-Deterministic Finite Automata (NDFA or NFA)

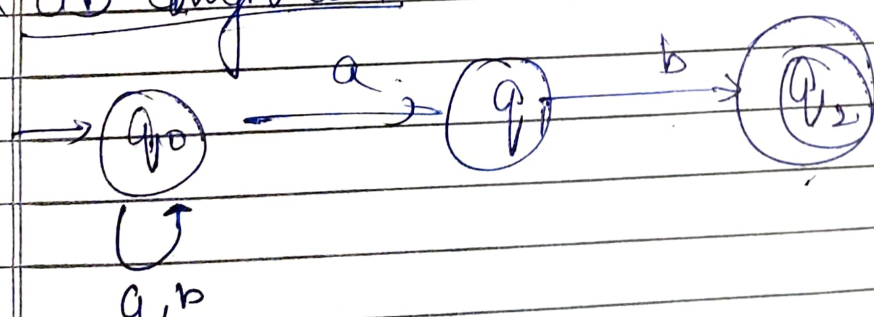
$$NFA = \langle Q, \Sigma, \delta_N, q_0, F \rangle$$

$$\delta_D : Q \times \Sigma \longrightarrow Q$$

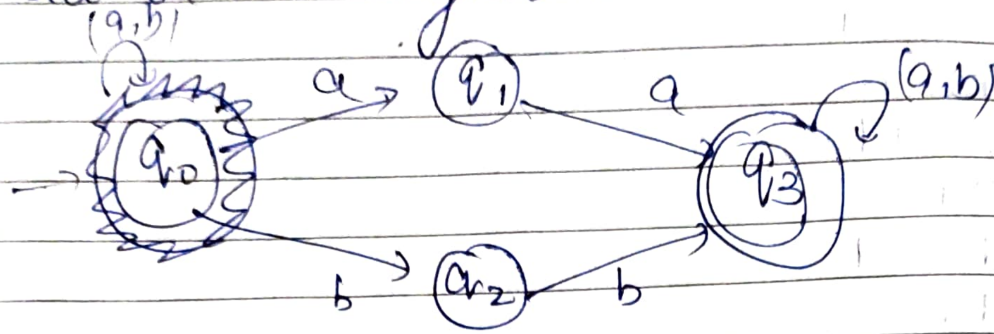
$$\delta_N : Q \times \Sigma \longrightarrow P(Q)$$

$$DFA \subseteq NFA$$

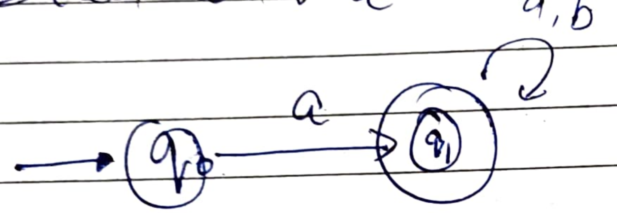
* ab anywhere



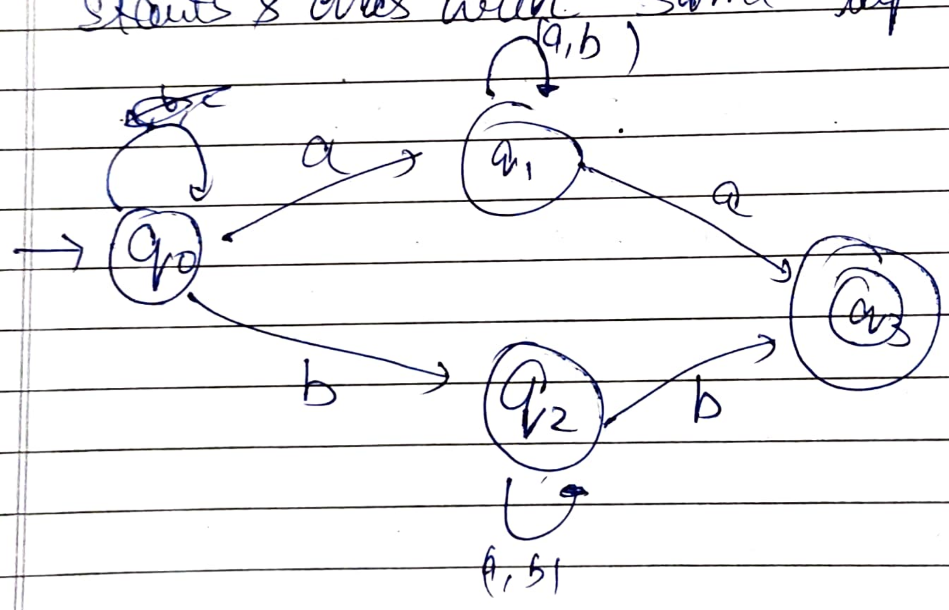
* aa or bb anywhere



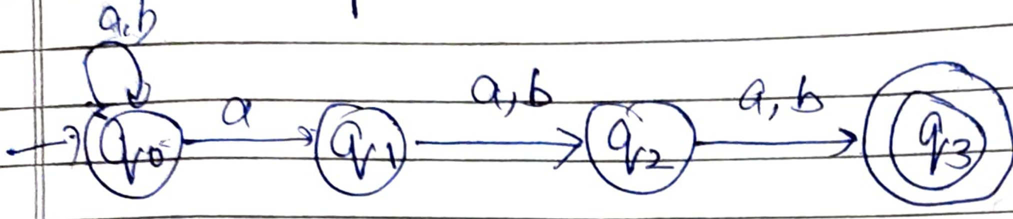
* Start with a



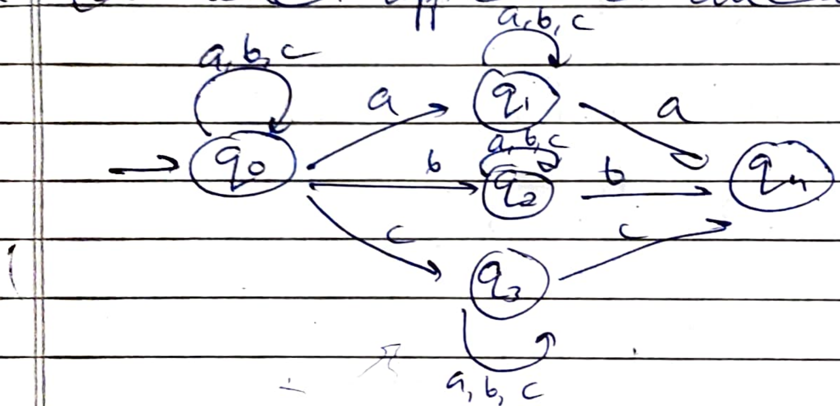
* Starts & ends with same alphabet/character



* 3rd last alphabet should be an a.



* last letter appear atleast twice



NFA to DFA :→

$$NFA = \langle Q_N, \Sigma, \delta_N, q_0, F_N \rangle$$

$$DFA = \langle Q_D, \Sigma, \delta_D, q_0, F_D \rangle$$

$$\delta_N \longrightarrow \delta_D$$

Steps

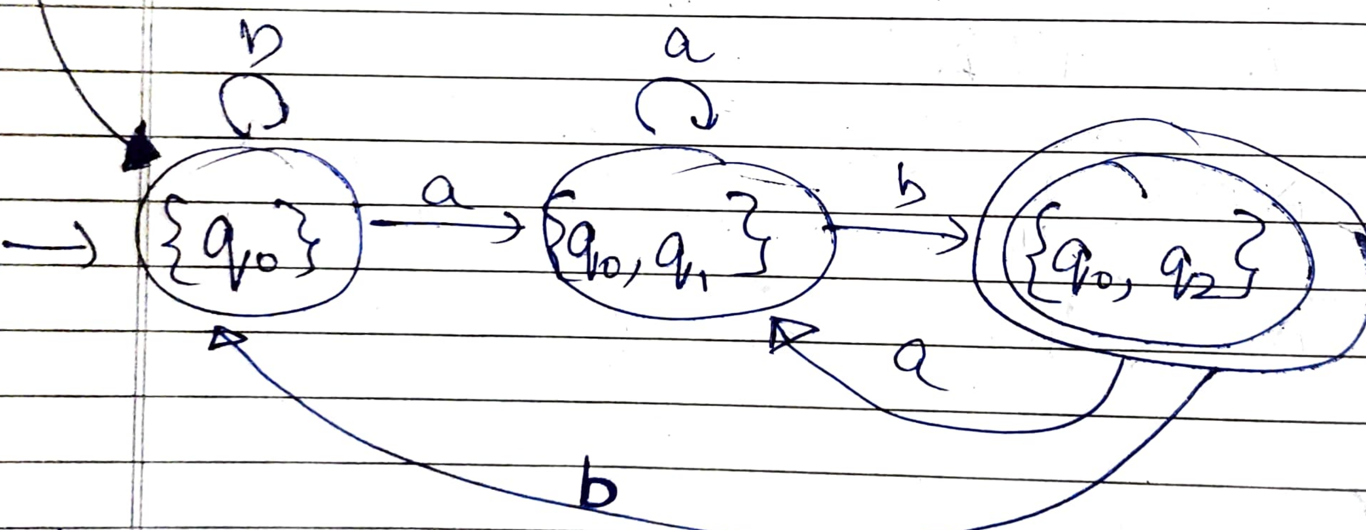
1.) $Q_D = \{q_0\}$

2.) Read all the symbols from Σ over Q_N and generate newer states $q_D \leftarrow q_D \cup q$

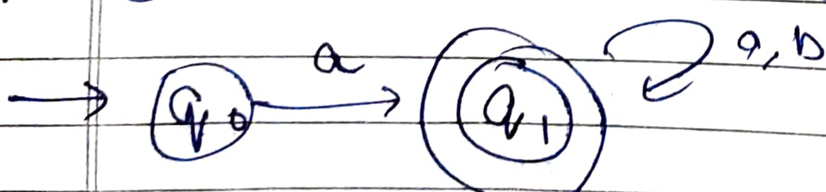
3.) Repeat step-2 until all the transitions over Q_D are not covered.

(i) F_D is all those states where F_N is present in it.

Q_0 \ Σ	a	b
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0\}$



★ Start with an a

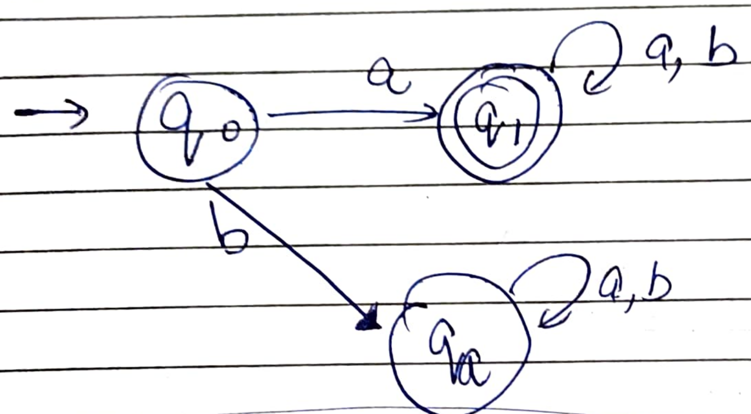


Convert to DFA.

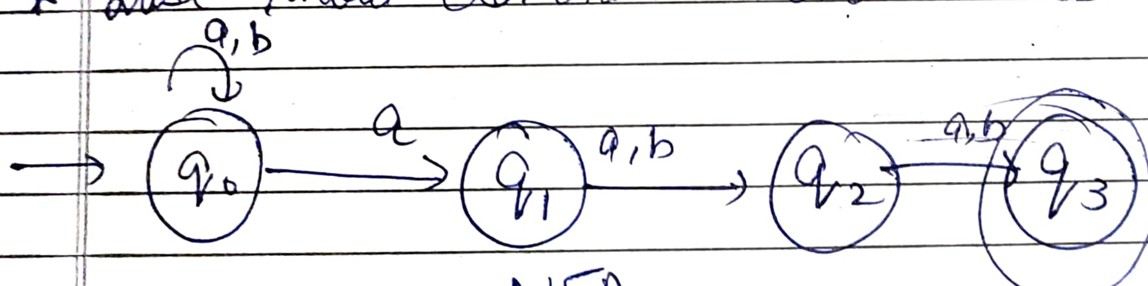
$\boxed{\text{NFA}}$

Q_0 \ w	a	b
$\rightarrow \{q_0\}$	$\{q_1\}$	$\emptyset$ $\{q_d\}$
$\{q_1\}$	$\{q_1\}$	$\{q_1\}$.
$\{q_d\}$	$\{q_d\}$	$\{q_d\}$

replace $\emptyset$ by q_d (dead state)



* Last third element should be a.



NFA

* Convert it to DFA

$Q \backslash \Sigma$	a	b
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_2, q_3\}$
$\{q_0, q_2\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_3\}$
$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_2, q_3\}$
$\{q_0, q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$	$\{q_0, q_3\}$
$\{q_0, q_1, q_3\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1\}$	$\{q_0\}$

Extended Transition Function

$\delta : Q \times \Sigma \longrightarrow Q$ for DFA

$\delta : Q \times \Sigma \longrightarrow P(Q)$ for NFA

$\hat{\delta} : Q \times \Sigma^* \longrightarrow Q$ for DFA

$\hat{\delta} : Q \times \Sigma^* \longrightarrow P(Q)$ for NFA

Transition function

$\delta(q_0, a) = q_1$

$\hat{\delta}(q_0, abaab) = q_3$

$\hat{\delta}(q, a) = P$

$\hat{\delta}(q, aw) = \hat{\delta}(\delta(q, a), w)$

$$\hat{\delta}(q, wa) = \delta(\hat{\delta}(q, w), a)$$

$$\hat{\delta}(q, \epsilon) = q$$

$$\hat{\delta}(q, w) = \bigcup_{P \text{ in } q} \hat{\delta}(P, w)$$

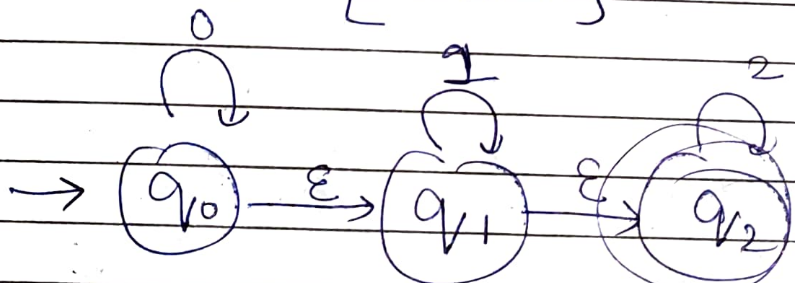
Set of States

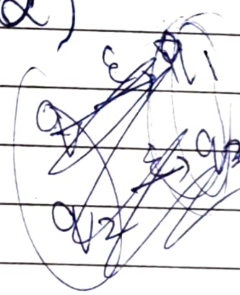
$\cancel{\epsilon = \phi}$

NFA with ϵ transitions (ϵ-NFA)

$$\epsilon\text{-NFA} = \langle Q, \Sigma, \delta, q_0, F \rangle$$

$$\delta : Q \times \{\Sigma \cup \epsilon\} \rightarrow P(Q)$$





1. ϵ-closure (q) denotes all the states P such that there is a path from q to P labelled by ϵ.

2. $$\epsilon\text{-closure}(P) = \bigcup_{q \text{ in } P} \epsilon\text{-closure}(q)$$

3. $$\hat{\delta}(q, wa) = \epsilon\text{-closure}(\delta(\hat{\delta}(q, w), a))$$

0 1 2

| | | | |

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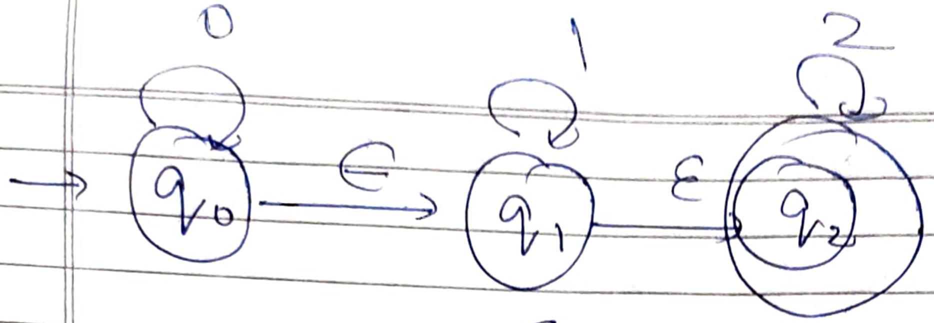


ϵ-closure $(q_0) = \{q_0, q_1, q_2\}$

ϵ-closure $(q_1) = \{q_1, q_2\}$

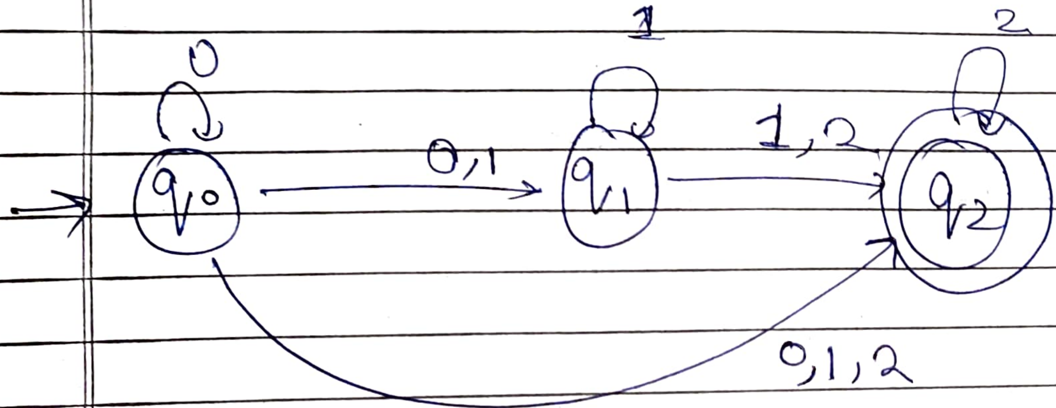
ϵ-closure $(q_2) = \{q_2\}$

$Q \backslash \Sigma \cup \epsilon$	0	1	2	ϵ
$\rightarrow q_0$	q_0	$\emptyset$	$\emptyset$	q_1
q_1	$\emptyset$	q_1	$\emptyset$	q_2
q_2	$\emptyset$	$\emptyset$	q_2	$\emptyset$

NFA $= \langle Q, \Sigma, \delta_N, q_0, F_N \rangle$

ϵ-NFA $= \langle Q, \Sigma, \delta_\epsilon, q_0, F_\epsilon \rangle$

$Q \setminus \Sigma$	0	1	2
$\rightarrow q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	$\emptyset$	$\{q_1, q_2\}$	$\{q_2\}$
q_2	$\emptyset$	$\emptyset$	$\{q_2\}$



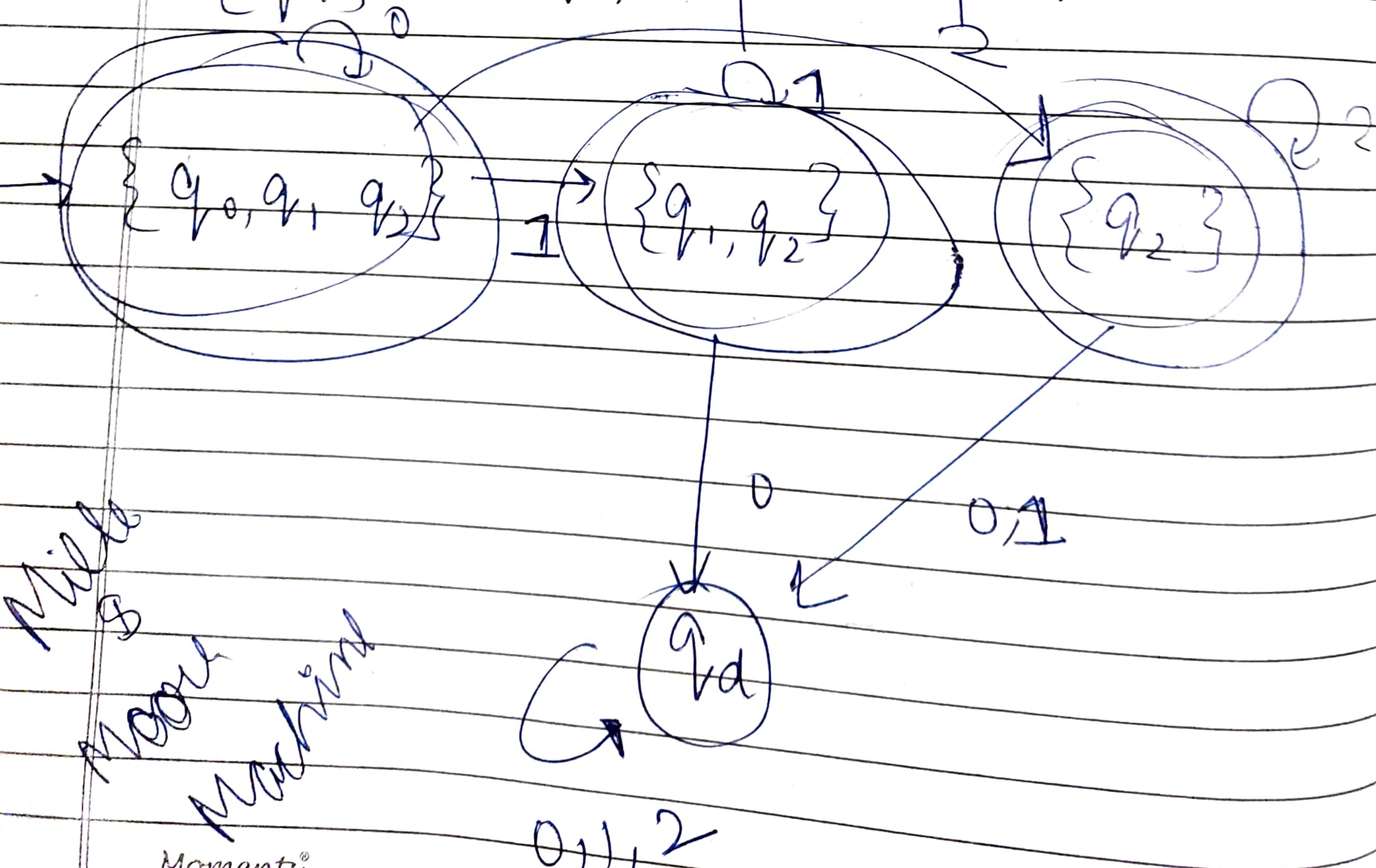
$$F_N = \begin{cases} F_\epsilon \cup \{q_0\} ; & \text{if } \epsilon\text{-closure}(q_0) \text{ includes states from } F_\epsilon \\ F_\epsilon & \end{cases}$$

$\epsilon\text{-NFA} = \langle Q_\epsilon, \Sigma, \delta_\epsilon, q_0, F_\epsilon \rangle$

$DFA = \langle Q_D, \Sigma, \delta_D, q_0', F_D \rangle$

$q_0' = \epsilon\text{-closure}(q_0)$

$Q \backslash \Sigma$	0	1	2
$\rightarrow \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_2\}$
$\{q_1, q_2\}$	q_d	$\{q_1, q_2\}$	$\{q_2\}$
$\{q_2\}$	q_d	q_d	$\{q_2\}$
$\{q_d\}$	q_d	q_d	q_d



Miele & Moore Machine

Minimization of Finite Automata

1) Two states q_1 & q_2 are equivalent

$q_1 \equiv q_2$ if

$\hat{\delta}(q_1, x)$ & $\hat{\delta}(q_2, x)$ are final states or both of them are non-final states $\forall x \in \Sigma^*$

2) Two states q_1 & q_2 are k-equivalent if both $\hat{\delta}(q_1, x)$ & $\hat{\delta}(q_2, x)$ are final states or non-final states $\forall$ strings x of length atmost k.

$|x| \leq k$.

$Q = \{Q_1, Q_2, \ldots Q_n\}$

π_k

$\downarrow$

$\pi_{k+1} = \pi_k$

Regular Expressions

$\Sigma \quad r_1 + r_2$

$\epsilon \quad r_1 r_2$

$(\quad) \quad r_1^*$

$+ \quad$ aa ~~aa~~ bb

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$r_1 = a(a+b)^*$ starts with a

$r_2 = (a+b)^* a$ ends with a

$r_3 = (a+b)^* aba (a+b)^*$ aba anywhere

$r_4 = (a+b)^* aba$ aba at end

$r_5 = (a+b)^* (aa+bb) (a+b)^*$ aa or bb anywhere

$r_6 = a(a+b)^* a + b(a+b)^* b$ starts & ends with same symbol

$r_7 = a(a+b)^* b + b(a+b)^* a$ starts & ends with different symbol

$r_8 = (a+b)^* a (a+b)(a+b)$

$r_9 = (ab^*a + b)^*$ no. of a's are even.

$r_{10} = aba$

$r_{11} = [(a+b)(a+b)(a+b)]^*$ Length 3 string

$r_{12} = (ab^*ab^*a + b)^*$ no of a's are divisible by 3.

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$(aa)^* b^*$

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Language which should accept atleas 1 a

$r_{13} = (a+b)^* a (a+b)^*$

$r_{14} = a^* b a^* b a^*$ Exactly two b's

$L = \langle a^n b^m \mid n \geq 4 \ \& \ m \leq 3 \rangle$

$aaa\,a^*(\varepsilon + b + bb + bbb)$

$L = \{$ atmost three a $\rangle$

$[b^* + b^* a b^* + b^* a b^* a b^* + b^* a b^* a b^* a b^*]$

$\Rightarrow b^*(\varepsilon + a)\, b^*(\varepsilon + a)\, b^*(\varepsilon + a)\, b^*$

$L = \langle a^n b^m \mid n = \text{even},\ m \geq 0 \rangle$

$(aa)^* b^*$

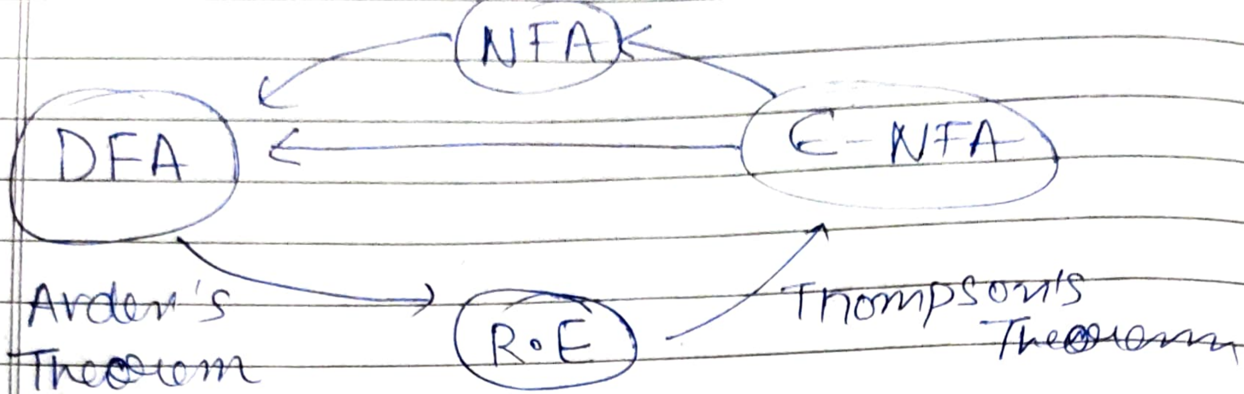
$L = \langle a^n b^m \mid n + m = \text{even} \rangle$

$(aa)^*(\varepsilon + ab)(bb)^*$

$L = ($ a,b & atleast one occurrence of two b should be there

$\Rightarrow (a+b)^* a (a+b)^* b (a+b)^* + (a+b)^* b \cdot (a+b)^* a (a+b)^*$

Moment



Let r be a regular expression then there exists an ε-NFA that accepts L(r)

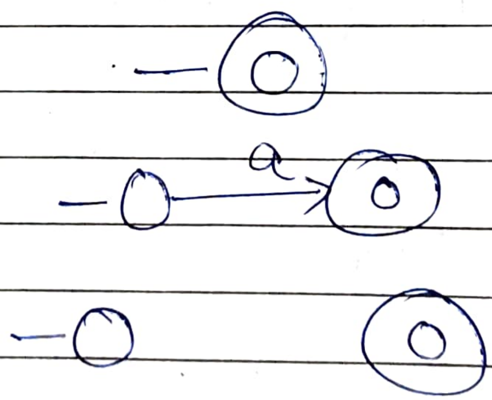
Proof → Mathematical operation on the number of operators in the regular expression. So that there is ε-NFA has one final state & no transitions go out of the final state.

Basis

$r_1 = \epsilon$

$r_2 = a$

$r_3 = \phi$



Induction Assume that the theorem is true for the regular expression with fewer than i operator ($i \geq 1$). Let r has i operators then there exists three cases.

Case-1: $r = r_1 + r_2$

Case-2: $r = r_1 r_2$

Case-3: $r = r_1^*$

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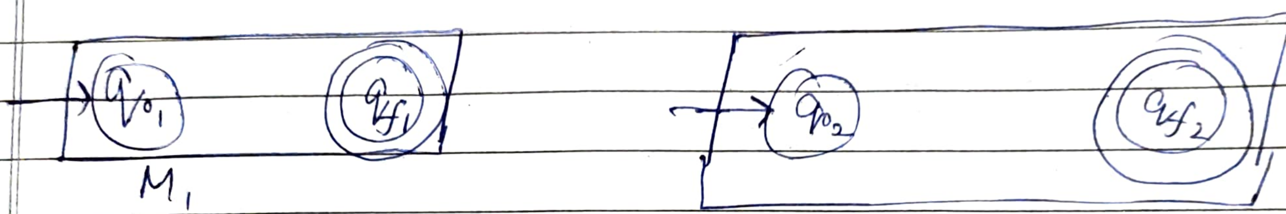
r_1 $L(r_1)$ M_1

r_2 $L(r_2)$ M_2

$M_1 = \langle Q_1, \Sigma_1, \delta_1, q_{0_1}, F_1 \rangle$

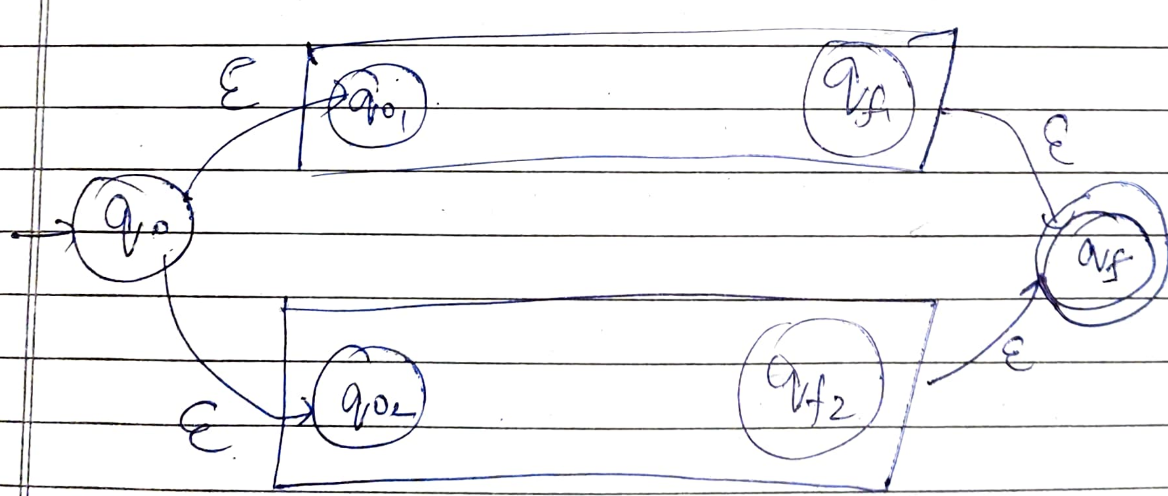
$M_2 = \langle Q_2, \Sigma_2, \delta_2, q_{0_2}, F_2 \rangle$



M_1

Case I: $r = r_1 + r_2$

$M_{union} = \langle Q_{union}, \Sigma_{union}, \delta_{union}, q_{0_{union}}, F_{union} \rangle$



$Q_{union} = Q_1 \cup Q_2 \cup \{q_0, q_f\}$

$\Sigma_{union} = \Sigma_1 \cup \Sigma_2 \cup \{\varepsilon\}$

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$F_{union} = \{q_f\}$

1) $\delta_{union}(q_0, \epsilon) = \{q_{01}, q_{02}\}$

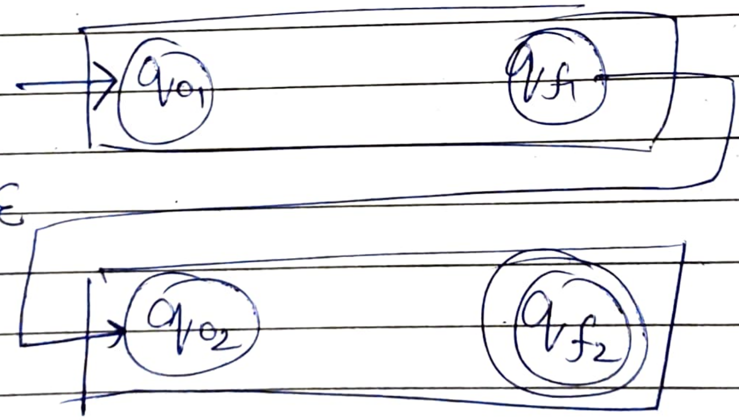
2) $\delta_{union}(p, a) = \delta_1(p, a) \; \forall p \in Q_1 \; \forall a \in \Sigma$

3) $\delta_{union}(p, a) = \delta_2(p, a) \; \forall p \in Q_2$ & $\forall a \in \Sigma_2$

4.) $\delta_{union}(q_{f_1}, \epsilon) = q_f \quad \forall q_{f_1} \in F_1$

5) $\delta_{union}(q_{f_2}, \epsilon) = q_f \quad \forall q_{f_2} \in F_2$

Case 2: $r = r_1 r_2$



$Q_{concat} = Q_1 \cup Q_2$

$\Sigma_{concat} = \Sigma_1 \cup \Sigma_2 \cup \{\epsilon\}$

~~F union~~ $F_{concat} = F_2$

Moments

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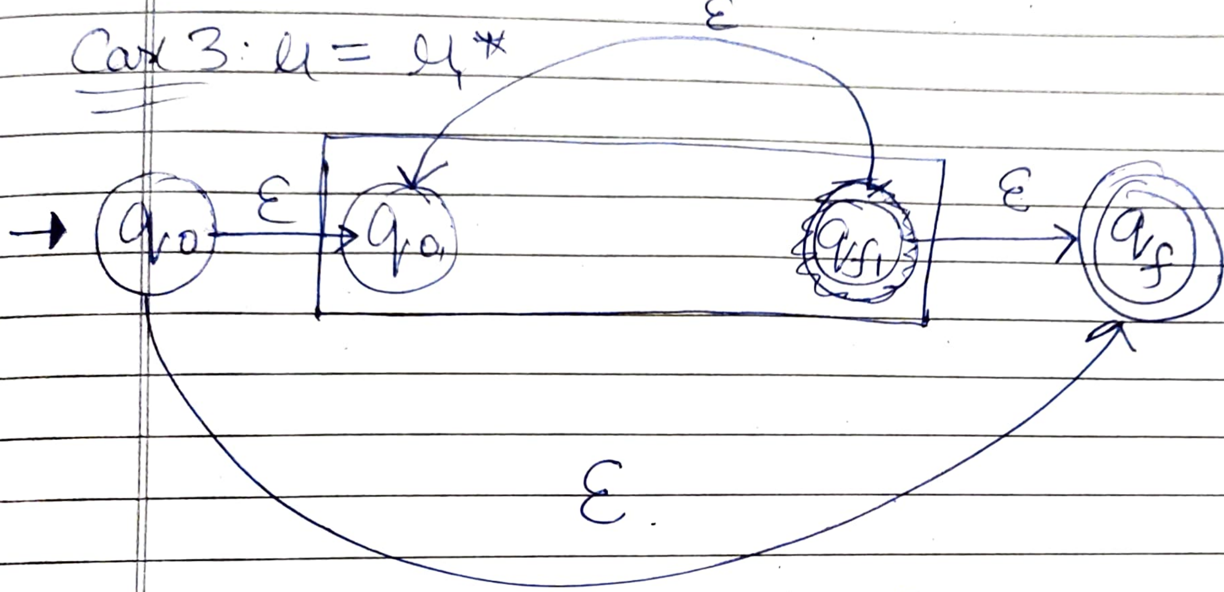
1) $\delta_{concat}(q_f, \epsilon) = q_{02} \quad \forall q_{f_1} \in F_1$

2) $\delta_{concat}(P, a) = \delta_1(P, a) \quad \forall P \in Q_1 \ \& \ \forall a \in \Sigma_1$

3) $\delta_{concat}(P, a) = \delta_2(P, a) \quad \forall P \in Q_2 \ \forall a \in \Sigma_2$

Case 3: $L = L_1^*$



$Q_{closure} = Q_1 \cup \{q_0, q_f\}$

$\Sigma_{cl} = \Sigma_1 \cup \{\varepsilon\}$

$F_{cl} = \{q_f\}$

1) $\delta_{cl}(q_0, \epsilon) = \{q_{01}, q_f\}$

2) $\delta_{cl}(P, a) = \delta_1(P, a) \quad \forall P \in Q_1 \ \& \ \forall a \in \Sigma$

3) $\delta_{cl}(q_f, \epsilon) = \{q_f, q_{01}\}$