INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, ALLAHABAD

CI Assessment, 15 September 2023,

Graphics & Visual Computing IGVC-5211

B.Tech - IT: V-Semester

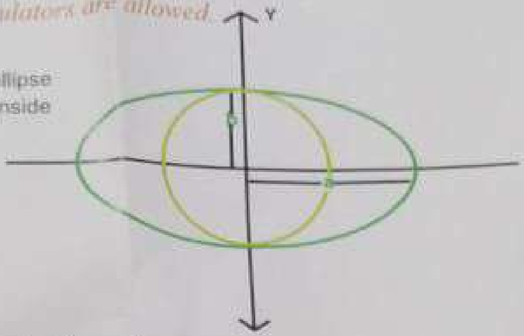
Full Marks - 20 Which will be Scaled to 10. Time – 2.0 hrs.

Answers should be brief and to the point. Marks will be deducted for unnecessary writing. Calculators are allowed.

1. Write the midpoint line drawing algorithm of ellipse and draw the following figure where circle is inside the ellipse

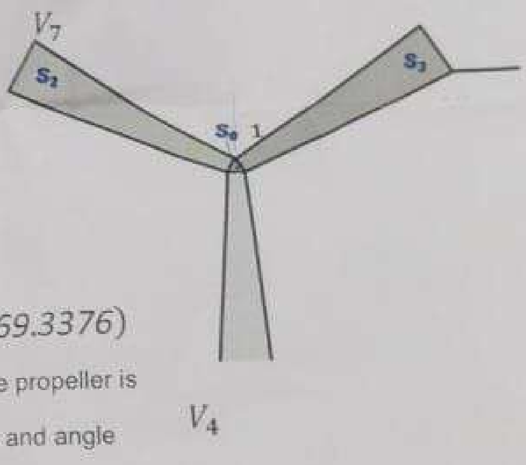
Consider the radius of the circle as

6 i.e. **b** and **a** is 8.



[5]

2. A Helicopter Propeller needs to be designed using **3 equal isosceles trapezoids** as shown in the figure. The dimensions of each **trapezoid** are ***height=100*** and the ***bases*** **are *10* and *30*.**



A vector $\vec{R}$ formed from the vertex

$(x_a = 20.0, y_a = 30.0, z_a = 25.0)$ and

$(x_b = 164.3376, y_b = 174.3376, z_b = 169.3376)$

is the axis of rotation of the propeller. The centre of the propeller is placed at (x_b, y_b, z_b). The propeller is rotated by and angle ωt around $\bar{R}$.

a) Describe and generate all the vertices of the propeller and then make the data structure of the vertices, the edges and the surfaces $S_0(V_1, V_2, V_3)$, $S_1(V_2, V_3, V_5, V_4)$, $S_2(V_1, V_2, V_6, V_7)$ and $S_3(V_1, V_3, V_8, V_9)$ and in a hierarchical manner. Where $V_i[x_i, y_i, z_i]$. Assume $z_i = 0.0$ in the **Object Frame**. Compute and Tabulate the coordinates of the vertex $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9$. The Origin of the **Object Frame** is the center of S_0.

Algorithm :-

1. Input r_x, r_y, and ellipse center (x_c, y_c) and obtain the first point on an ellipse centered on the origin as

$$(x_0, y_0) = (0, r_y)$$

2. Calculate the initial value of the decision parameter in region 1 as

$$P1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2$$

3. At each x_k position in region 1, starting at $k=0$, perform the following: if $P1_k < 0$, the next point along the ellipse centered on $(0,0)$ is (x_{k+1}, y_k) and

$$P1_{k+1} = P1_k + 2r_y^2 x_{k+1} + r_y^2$$

otherwise, the next point along the ellipse is (x_k+1, y_k-1) and

$$P1_{k+1} = P1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$$

with

$$2r_y^2 x_{k+1} = 2r_y^2 x_k + 2r_y^2$$

$$2r_x^2 y_{k+1} = 2r_x^2 y_k - 2r_x^2$$

and continued until $2r_y^2 x \geq 2r_x^2 y$

4. Calculate the initial value of the decision parameter in region 2 as

$$P2_0 = r_y^2 \left(x_0 + \frac{1}{2}\right)^2 + r_x^2 (y_0 - 1)^2 - r_x^2 r_y^2$$

where (x_0, y_0) is the last position calculated in region 1

5. At each y_k position in region 2, starting at $k=0$, perform the following test: If $P2_k > 0$, the next point along the ellipse centered on $(0,0)$ is $(x_k, y_k - 1)$ and

$$P2_{k+1} = P2_k - 2r_x^2 y_{k+1} + r_x^2$$

Otherwise, the next point along the ellipse is (x_k+1, y_k-1) and

$$P2_{k+1} = P2_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

using the same incremental calculations for x and y as in region 1.
Continue until $y = 0$.

6. For both regions, determine symmetry points in the other three quadrants.

7. Move each calculated pixel position (x, y) onto the elliptical path centered on (x_c, y_c) and plot these coordinate values:

$$x = x + x_c, \quad y = y + y_c$$

Q. $r_x = 8$, $r_y = 6$

The initial values and increments for the decision parameter calculations are

$$2r_y^2 x = 0$$

$$2r_x^2 y = 2r_x^2 r_y$$

For region 1, $(x_0, y_0) = (0, 6)$

$$P1_0 = r_y^2 - r_x^2 r_y + \frac{1}{4} r_x^2 = -332$$

$$P1_{k+1} = P1_k + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_y^2$$

Successive midpoint decision-parameter values and pixel positions along the ellipse are listed in the following table:

K	$P1_K$	(x_{k+1}, y_{k+1})	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-332	(1,6)	72	768
1	-224	(2,6)	144	768
2	-44	(3,6)	216	768
3	208	(4,5)	288	640
4	-108	(5,5)	360	640
5	288	(6,4)	432	512
6	244	(7,3)	504	384

$[2r_y^2 x_{k+1} = 2 \times 6^2 \times 1 = 72]$
$[2r_x^2 y_{k+1} = 2 \times 8^2 \times 6 = 768]$

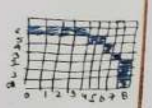
Now move out of region 1 because $2r_y^2 x > 2r_x^2 y$

For region 2, the initial point is $(x_0, y_0) = (7, 3)$ and the initial decision parameter is

$$P2_0 = f_{ellipse}\left(7 + \frac{1}{2}, 2\right) = -151$$

The remaining positions along the ellipse path in the first quadrant are then calculated as

K	$P2_K$	(x_{k+1}, y_{k+1})	$2r_y^2 x_{k+1}$	$2r_x^2 y_{k+1}$
0	-151	(8,2)	576	256
1	233	(8,1)	576	128
2	754	(8,0)	–	–



Pixels in first Quadrant of ellipse

e Total figure will not change. Technically it is said the rotation of the Propellor is Aliased.

C) Notce that

$\Delta x = x_c - x_b = 1463.3758 - 164.3376 = 1299.0382$

$\Delta y = y_c - y_b = 1473.3758 - 174.3376$

$= 1299.0382$

$\Delta z = z_c - z_b = 1468.3758 - 169.3376$

$= 1299.0382$

$\Delta x = \Delta y = \Delta z$ The Lookat vector is along $\overrightarrow{R}$ therefore $x_{cam} = x_{obj}, y_{cam} = y_{obj}, z_{cam} = z_{obj}$ and

$V_{Cam}[i] = V_{Obj}[i] = V_i(x_i, y_i, -2000)$

D) $z_{Cam} = -2000$ and $f = 1000$

$$x_{PProj} = \frac{f}{|z_{Cam}|} x_{Cam} = \frac{1}{2} x_i;\ y_{PProj} = \frac{f}{|z_{Cam}|} y_{Cam} = \frac{1}{2} y_i$$

$$V_{PProj}[i] = \frac{f}{|z_{Cam}|} V_{Cam} = \frac{1}{2} V_i \text{ where } \{i = 1,2,3,4,5,6,7,8,9\}$$

E) Center pixel $X_0 = INT\left(\frac{1920}{2} + 0.5\right) = 960pix$ $Y_0 = INT\left(\frac{1080}{2} + 0.5\right) = 540pix.$

$x_{Screen}[i] = X_i = INT\left(10 \times \frac{x_i}{2} + 935 + 0.5\right)$ *pixels*

$y_{Screen}[i] = Y_i = INT\left(10 \times \frac{y_i}{2} + 540 + 0.5\right)$ *pixels*

F) Point within Polygon:

$S_0(V_1, V_2, V_3) \longrightarrow Average(V_1, V_2, V_3)$

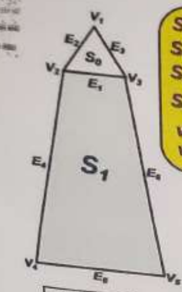
$S_1(V_2, V_3, V_4, V_5) \longrightarrow Average(V_2, V_3, V_4, V_5)$

$S_2(V_1, V_2, V_6, V_7) \longrightarrow Average(V_1, V_2, V_6, V_7)$

$S_3(V_1, V_3, V_8, V_9) \longrightarrow Average(V_1, V_3, V_8, V_9)$

POLIGONE	$x_{in(pixel)}$	$y_{in(pixel)}$
$S_0(V_1, V_2, V_3)$	960	540
$S_1(V_2, V_3, V_4, V_5)$	960	276
$S_2(V_1, V_2, V_6, V_7)$	731	672
$S_3(V_1, V_3, V_8, V_9)$	1189	672

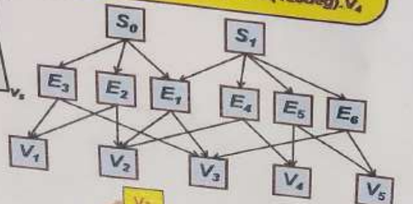
$V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9$. The Origin of the **Object Frame** is the center of S_0

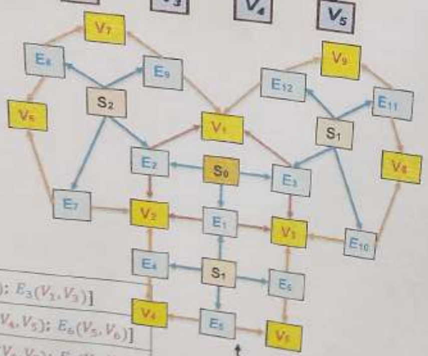


Similar for S_2 & S_3

$S_0\ \{E_1(V_2,V_3),\ E_2(V_1,V_2),\ E_3(V_1,V_3)\}$
$S_1\ \{E_1(V_2,V_3),\ E_4(V_2,V_4),\ E_5(V_4,V_5),\ E_6(V_5,V_3)\}$
$S_2\ \{E_2(V_2,V_1),\ E_7(V_2,V_6),\ E_8(V_6,V_7),\ E_9(V_7,V_1)\}$
$S_3\ \{E_3(V_1,V_3),\ E_{10}(V_3,V_8),\ E_{11}(V_8,V_9),\ E_{12}(V_9,V_1)\}$

V_6= Rot(-120deg).V_5 V_7= Rot(-120deg).V_4
V_8= Rot(120deg).V_5 V_9= Rot(120deg).V_4

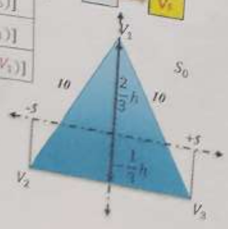




$S_0[E_1(V_2,V_3);\ E_2(V_2,V_1);\ E_3(V_1,V_3)]$
$S_1[E_1(V_2,V_3);\ E_4(V_2,V_4);\ E_5(V_4,V_5);\ E_6(V_5,V_6)]$
$S_2[E_2(V_2,V_1);\ E_7(V_2,V_6);\ E_8(V_6,V_7);\ E_9(V_7,V_1)]$
$S_3[E_3(V_1,V_3);\ E_{10}(V_3,V_8);\ E_{11}(V_8,V_9);\ E_{12}(V_9,V_1)]$

$h = \sqrt{10^2 - 5^2} = 5\sqrt{3}$

$\frac{2}{3}h = \frac{10}{\sqrt{3}} = 5.7735 \qquad -\frac{1}{3}h = \frac{-5}{\sqrt{3}} = -2.8868$



$V_1\left(0, 10/\sqrt{3}\right)$ $V_2\left(-5, -5/\sqrt{3}\right)$ $V_3\left(+5, -5/\sqrt{3}\right)$
$V_4\left(-15, -\left(100 + 5/\sqrt{3}\right)\right)$ $V_5\left(+15, -\left(100 + 5/\sqrt{3}\right)\right)$

$V_6 = R_z\left(-\frac{2\pi}{3}\right)V_5$ $V_7 = R_z\left(-\frac{2\pi}{3}\right)V_4$ $V_8 = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}V_6$ $V_9 = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}V_7$

	Answer of (a)			For (C along with x, y of a)	Answer of (d)		Answer of (e)	
	x	y	z	z_{Camera}	x_{PProj}	y_{PProj}	x_{Screen} (pixels)	x_{Screen} (pixels)
V_1	0	5.7735	0.0	-2000.000	0.000	2.887	960	569
V_2	-5	-2.8868	0.0	-2000.000	-2.500	-1.443	935	526
V_3	5	-2.8868	0.0	-2000.000	2.500	-1.443	985	526
V_4	-15	-102.8868	0.0	-2000.000	-7.500	-51.443	885	26
V_5	15	-102.8868	0.000	-2000.000	7.500	-51.443	1035	26
V_6	-96.6025	38.4530	0.000	-2000.000	-48.301	19.226	477	732
V_7	-81.6025	64.4338	0.000	-2000.000	-40.801	32.217	552	862
V_8	96.6025	38.4530	0.000	-2000.000	48.301	19.226	1443	732
V_9	81.6025	64.4338	0.000	-2000.000	40.801	32.217	1368	862

B)

$$T_r(x_b, y_b, z_b)R_y\left(\frac{\pi}{4}\right)R_x\left(\frac{\pi}{4}\right)V_i \qquad \{i = 1,2,3,4,5,6,7,8,9\}$$

$$(x_b = 164.3376, y_b = 174.3376, z_b = 169.3376)$$

If $\omega = \frac{2\pi}{3}$ $Radiance/second$, and if the animation frames are buffered at every $t = 1\ second$, then the Propeller will not appear to move. Since all 3 blades of the Propeller are identical, a rotation of $\omega = \frac{2\pi}{3}$ $Radiance/second$ S_1 will replace the position of S_2:

$S_1 \longrightarrow S_2,\quad S_2 \longrightarrow S_3\quad$ and $\quad S_3 \longrightarrow S_1$

... coordinates of the vertex $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9$. The ... the **Object Frame** is the center of S_0

$S_0\{E_1(V_2,V_3), E_2(V_1,V_2), E_3(V_1,V_3)\}$
$S_1\{E_1(V_2,V_3), E_4(V_2,V_4), E_5(V_4,V_5), E_6(V_5,V_3)\}$
$S_2\{E_2(V_2,V_1), E_7(V_2,V_6), E_8(V_6,V_7), E_9(V_7,V_1)\}$
$S_3\{E_3(V_1,V_3), E_{10}(V_1,V_8), E_{11}(V_8,V_9), E_{12}(V_9,V_3)\}$
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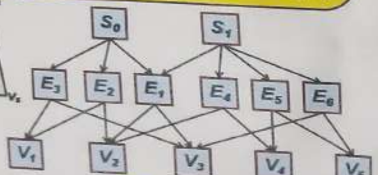
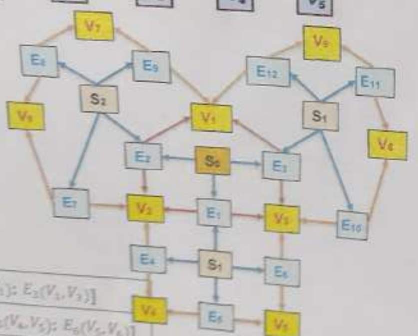
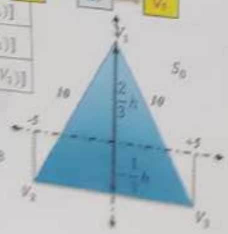




$S_0[E_1(V_2,V_3); E_2(V_2,V_1); E_3(V_1,V_3)]$
$(V_2,V_3); E_4(V_2,V_4); E_5(V_4,V_5); E_6(V_5,V_4)]$
$(V_2,V_1); E_7(V_2,V_6); E_8(V_6,V_7); E_9(V_7,V_1)]$
$(V_1,V_3); E_{10}(V_3,V_8); E_{11}(V_8,V_9); E_{12}(V_9,V_1)]$

$\sqrt{10^2-5^2} = 5\sqrt{3}$

$= \frac{10}{\sqrt{3}} = 5.7735 \quad -\frac{1}{3}h = \frac{-5}{\sqrt{3}} = -2.8868$



$V_1(0, 10/\sqrt{3})$ $V_2(-5, -5/\sqrt{3})$ $V_3(+5, -5/\sqrt{3})$
$V_4(-15, -(100+5/\sqrt{3}))$ $V_5(+15, -(100+5/\sqrt{3}))$

$V_6 = R_z\left(-\frac{2\pi}{3}\right)V_5$ $V_7 = R_z\left(-\frac{2\pi}{3}\right)V_4$ $V_8 = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}V_6$ $V_9 = \begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix}V_7$

	Answer of (a)			For (C along with x, y of a)	Answer of (d)		Answer of (e)	
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B)

$$T_T(x_b, y_b, z_b)R_y\left(\frac{\pi}{4}\right)R_x\left(\frac{\pi}{4}\right)V_i \qquad \{i = 1,2,3,4,5,6,7,8,9\}$$

$$(x_b = 164.3376, y_b = 174.3376, z_b = 169.3376)$$

If $\omega = \frac{2\pi}{3}$ $Radiance/second$, and if the animation frames are buffered at every $t = 1\ second$, then the Propeller will not appear to move. Since all 3 blades of the Propeller are identical, a rotation of $\omega = \frac{2\pi}{3}$ $Radiance/second$ S_1 will replace the position of S_2:

$S_1 \longrightarrow S_2$, $S_2 \longrightarrow S_3$ and $S_3 \longrightarrow S_1$

b) Derive the transformations to rotate the propeller. Define the Transformation matrix and then use short notation only. (Hint: It is easier to represent this transformation as a product of multiple transformations). If $\omega = \frac{2\pi}{3}$ $Radiance/second$, and if the animation frames are buffered at every $t = 1\ second$, then what is observed and why? What is the technical word for the observation?

c) The Camera is located at $(x_c = 1463.3758, y_c = 1473.3758, z_c = 1468.3758)$ The ***Lookat* point** is the **center** of surface $S_0\ (V_1, V_2, V_3)$ and the ***UP Vector*** is towards the vertex V_1 from the ***Lookat* point**. Compute and Tabulate the coordinates of the vertex $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9$ in the ***Camera Frame***.

d) If the Focal length of the camera $f = 1000$, Compute and Tabulate the coordinates of the vertex $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9$ in the ***Perspective Projection***.

e) In the case of a monitor with an industry-standard Full HD 1080p resolution, this display has a resolution of 1920 x 1080. This means that the screen will have a width of 1,920 pixels while the height of the screen will be 1,080 pixels. The $ScalingFactor = 10.0$. find the pixel coordinates of the vertex $V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9$. Please Note: the ***Lookat point*** is the **center of the Screen.**

f) To colour the polygons/ surfaces $S_1(V_2, V_3, V_4, V_5)$, $S_2\ (V_1, V_2, V_6, V_7)$, $S_3(V_1, V_3, V_8, V_9)$ and $S_0(V_1, V_2, V_3)$ using **Boundary-Fill / Flood-Fill Algorithm A** point within each polygons/ surfaces is needed. Compute and Tabulate:

POLIGONE	$x_{in(pixel)}$	$y_{in(pixel)}$
$S_0(V_1, V_2, V_3)$		
$S_1(V_2, V_3, V_5, V_4)$		
$S_2(V_1, V_2, V_6, V_7)$		
$S_3(V_1, V_3, V_8, V_9)$		

[(1+2)+(2+2)+2+2+2+2=15]