Tutorial (PS1).

Random exp
- → All outcomes known in advance
- → Not predictable, a particular outcome of trial.
- → can be repeated.

Sample space ⇒ collection of all outcomes

Event ⇒ subset of sample space.

For uncountable :-

S - uncountable sample

$P(\{x\}) = 0$

$U \subseteq S$

$U \subseteq V$

not found such that $\boxed{P(V) = 0}$

Proof :-

U - maximal.

$P(U) = 0$

$x \in S - U$

$P(U \cup \{x\}) = 0.$

is contradictory

—x—

Events :- E, F, G

ans. 1)

a) only F occurs.

ans:- $F \cap E^c \cap G^c$

b) both E and F occurs but NOT G.

ans:- $E \cap F \cap G^c$

c) at least one event occurs

ans:- $E \cup F \cup G$

d) at least two event occurs.

ans:- $(E \cap F) \cup (F \cap G) \cup (E \cap G)$

e) all three events occur.

ans:- $E \cap F \cap G$

f) none occurs.

ans:- $E^c \cap F^c \cap G^c$

g) at most one event occurs.

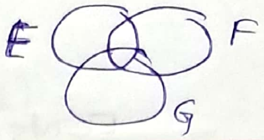
ans:- $(\cancel{E \cap F})^c$ means → ϕ, 1 occurs

→ (3, 2) atleast 2 event not occurs

$$(E \cap F^c \cap G^c) \cup (F \cap G^c \cap E^c) \cup (G \cap E^c \cap F^c)$$
$$\cup (E^c \cap F^c \cap G^c)$$

~~(and)~~ (or)

$$(E \cap F)^c \cap (F \cap G)^c \cap (E \cap G)^c$$



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h) at most 2 event occur.

ans:- $(E\cap F\cap G)^c$

ans.2) $S=\{0,1,2,3,\ldots\}$ and $E \overset{\text{event}}{\subseteq} S$

$\hookrightarrow$ countable non-finite set

verify P is probability on S.

a) • It should follow $\rightarrow$ $P(S)=1$

$0 \leq P(E) \leq 1$

$P\left(\bigcup_i E_i\right)=\sum_i P(E_i)$

Probability function:- $P: P(S) \longrightarrow \mathbb{R} \hookrightarrow [0,1]$

$$P(E)=\sum_{x\in E}\frac{e^{-\lambda}\lambda^x}{x!}$$ } convergent series to 1

all terms positive

$\therefore$ sum ≥ 0

$$P(S)=\frac{e^{-\lambda}\cdot\lambda^0}{0!}+\frac{e^{-\lambda}\cdot\lambda^1}{1!}+\frac{e^{-\lambda}\cdot\lambda^2}{2!}+\frac{e^{-\lambda}\cdot\lambda^3}{3!}+\cdots$$

$$=e^{-\lambda}\left[\frac{\lambda^0}{0!}+\frac{\lambda^1}{1!}+\frac{\lambda^2}{2!}+\cdots\right]$$

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$$\therefore \boxed{P(S) = e^{-\lambda} \times e^{\lambda} = 1}$$

---x---

Why converging?

$$\frac{r_{t+1}}{r_t} = \frac{\left(\frac{e^{-\lambda}\cdot\lambda^{t+1}}{(t+1)!}\right)}{\left(\frac{e^{-\lambda}\cdot\lambda^{t}}{t!}\right)} = \frac{\lambda}{(t+1)!} \times t! = \boxed{\frac{\lambda}{t+1}}$$

as $t \to \infty$, $1 > \frac{\lambda}{t+1} > 0$

$\therefore$ converging

$P(E) \geq 0$

$P(S) = 1$

$P(E) \leq P(S)$

$P(E)$ as "i" increases

$P(E)$ increases

but $P(E)$ bounded

$\therefore$ $0 \leq P(E) \leq 1$

---x---

(iii) $P\left(\bigcup_i E_i\right) = \sum_i P(E_i)$

$E_i \Rightarrow$ M.E

Reason:- since $P(S)$ is converging series.

Then $E \subseteq S$, $\therefore P(E)$ will also converging

Also by rearrangement of terms E_i will also Lead to converging series.

ex:- $E_1 =$ even , $E_2 =$ odd.

$$P(E_1 \cup E_2) = \sum_{x \in E_1 \cup E_2} \frac{e^{-\lambda}\cdot\lambda^{x}}{x!}, \quad \lambda > 0.$$

$$P\left(\bigcup_i E_i\right) = \sum_{x \in (\bigcup_i E_i)} \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \quad \lambda > 0$$

$$= \sum_i \left(\sum_{x \in E_i} \frac{e^{-\lambda} \cdot \lambda^x}{x!} \right)$$

$$= \sum_{x \in E_1} \frac{e^{-\lambda} \cdot \lambda^x}{x!} + \sum_{x \in E_2} \frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

$$+ \sum_{x \in E_3} \frac{e^{-\lambda} \cdot \lambda^x}{x!} + \cdots$$

$$= P(E_1) + P(E_2) + \cdots$$

$$= \sum_i P(E_i)$$

Yes, P.F ✓

b) $P(E) = \sum_{x \in E} p(1-p)^x, \quad 0<p<1$

same as (a)

c) $\because P(E) \geq 0$ and $P(S) = 1$

Hence $0 \leq P(E) \leq 1$

Lets take $E_1 = $ odd $E_2 = $ even.

$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2)$

$P(S) = 1+1$

$\boxed{1 = 2}$ absurd.

also $P(E_1 \cup E_2) \geq 1$, impossible.

$\therefore \boxed{NO}$

— x —

Q. $\boxed{2}$

(a) $S = \mathbb{R}, \quad \Sigma = \mathcal{B}\mathbb{R}$

- $\Sigma =$ all collection of open intervals, unions, (countable) countable intersection, all closed intervals.

Let $A \subseteq \mathcal{B}\mathbb{R}$

$$P(A) = \frac{1}{\pi} \int_A \frac{1}{1+x^2} \cdot dx.$$

since it is collection of intervals, so breakable at integra

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$$P(\mathbb{R}) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{1}{1+x^2} \cdot dx = 1$$

$P(E)$ has all positive terms

$\therefore$ absolutely convergent, so rearrangement possible.

ans. 3 (b) $\mathbb{R}$ is infinite interval.

$\therefore P(\mathbb{R}) = 0$ instead of $P(\mathbb{R}) = 1$

ans. 3 (c)

$$P(I) = \int_I \frac{1}{2} \cdot dx \quad , \quad I_1 \subseteq [1,\infty)$$

$$= 1 \quad , \text{ if } \quad I_2 \subset (-\infty, 1]$$

ex.

$\because (2,6) \in [1,\infty)$

$$P(I_{2-6}) = \boxed{\int_2^6 \frac{1}{2} \cdot dx = 2}$$ impossible.

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4) $P\left(\bigcup_{i=1}^{n} E_i\right) \leq \sum_{i=1}^{n} P(E_i)$

a)

ans:- $n=2$; $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$\therefore \quad P(E_1 \cup E_2) < P(E_1) + P(E_2)$

if $P(E_1 \cap E_2) \neq 0$

a

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$

if $P(E_1 \cap E_2) = 0$

$n=3$; $P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3)$

$- P(E_1 \cap E_2)$ $\quad + P(E_1 \cap E_2 \cap E_3)$

Tough way :- $\quad - P(E_2 \cap E_3)$

$- P(E_1 \cap E_3)$

Easy way:-

$P(E_1 \cup E_2 \cup E_3) \leq P(E_1 \cup E_2) + P(E_3)$

$\leq P(E_1) + P(E_2) + P(E_3)$

b

b) $$P\left(\bigcap_{i=1}^{n} E_i\right) \geq \sum_{i=1}^{n} P(E_i) - (n-1)$$

$n=2$ $\quad P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2)$

$$\geq P(E_1) + P(E_2) - (2-1)$$

$$\therefore \boxed{P(E_1 \cup E_2) \leq 1}$$

$n=k$ Let

$$P(E_1 \cap E_2 \cap E_3 \cdots E_k) \geq P(E_1) + P(E_2) + \cdots - (k-1)$$

then for $n = k+1$

$$P(E_1 \cap E_2 \cap E_3 \cdots E_{k+1}) = P\left((E_1 \cap E_2 \cdots E_k) \cap E_{k+1}\right)$$

$$= P(E_1 \cap E_2 \cdots E_k) + P(E_{k+1}) - P\left(E_{k+1} \cap (E_1 \cap E_2 \cdots E_k)\right)$$

$$\geq P(E_1) + P(E_2) + \cdots - (k-1) - 1 + P(E_{k+1})$$

$$\geq P(E_1) + P(E_2) + \cdots + P(E_{k+1}) - (k+1-1)$$

x——x——x

(5) Independent Events

→ If E and F are independent

$\therefore P(E \cap F) = P(E) \times P(F)$ —— (1)

To prove E^c and F are independent

$$P(E^c \cap F) = P(F) - P(E \cap F)$$
$$= P(F) - P(E).P(F) \quad [\text{From } (1)]$$
$$= P(F)[1 - P(E)]$$
$$= P(F) \times P(E^c)$$
$$\therefore \boxed{P(E^c \cap F) = P(F) \times P(E^c)}$$

b) similar as a)

c)
$$P(E^c \cap F^c) = 1 - P[E \cup F]$$
$$= 1 - [P(E) + P(F) - P(E \cap F)]$$
$$= 1 - [P(E) + P(F) - P(E).P(F)]$$
$$= 1 - [P(E)[1 - P(F)] + P(F)]$$
$$= 1 - [P(E).P(F^c) + P(F)]$$
$$= 1 - P(E).P(F^c) - P(F)$$
$$= 1 - P(F) - P(E).P(F^c)$$
$$= P(F^c) - P(E).P(F^c)$$
$$= P(F^c)[1 - P(E)]$$
$$\boxed{P(E^c \cap F^c) = P(F^c).P(E^c)}$$

Tutorial 1)

Q.8] Probability $= \frac{\pi (2)^2}{\pi (10)^2} = \boxed{\frac{4}{100}}$

Q.9) $\frac{366}{7} = 52$ ~~sundays~~ weeks. in a Leap year

& 2 days remaining.

2 days = { MT, TW, W Th, Th F, FS, S Sun, Sun M }

7 possibilities.

Favour :- { S Sun , Sun M } 2

$\therefore$ Probability $= \boxed{\frac{2}{7}}$

Q.10) E :- both are boys

F :- one is boy.

~~$P(E \cap F) = P(\text{boy}) P(\text{girl})$~~
~~$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$~~

~~$P(F) = \frac{1}{2} \times 1 = \frac{1}{2}$~~

~~$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$~~

Baye theorem.

$$\text{Prob} = \frac{P(\text{one is boy})}{\text{Total prob}} = \frac{\frac{1}{2} \cdot \frac{1}{2}}{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot (\frac{1}{1})}$$

$$= \boxed{\frac{1}{3}}$$

Prob (Both boys) $= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

Prob (1 boy) $= \frac{1}{2} \cdot 1 = \frac{1}{2}$.

Q.11) non-favour $\Rightarrow$ 18 $\{(1\,2\,3),(2,3,4),\ldots(18,19,20)\}$

Total $\Rightarrow$ $^{20}C_3 = \frac{20!}{17!\,3!}$

$= \frac{20\times19\times18}{6}$

$= \boxed{60\times19}$

Prob $= 1 - \frac{\text{non-favour}}{\text{Total}}$

$= 1 - \frac{18}{60\times19}$

$$\begin{array}{r} 190 \\ -\ 3 \\ \hline 187 \end{array}$$

$= 1 - \frac{3}{190}$

$= \boxed{\frac{187}{190}}$

— x —

Q.13)

a) greater than 5 :- 6, 7, 8

fav $^3C_2 \rightarrow 3$

Total ways :- $^8C_2$

Probability $\Rightarrow \frac{^3C_2}{^8C_2} = \frac{\frac{3!}{2!\,1!}}{\frac{8!}{2!\,6!}} = \frac{3\times2}{8\times7}$

$^8C_2 \Rightarrow 28$

$= \boxed{\frac{3}{28}}$

b) $P(\text{Sum}=5) = \frac{2}{28} = \frac{1}{14}$

ans. 12) $P(A) = 0.4$, $P(B) = 0.7$.

a) Min $P(A \cap B) = 0.1$
Max $P(A \cap B) = 0.4$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

This will be min when $P(A \cup B)$ is max.

$\Rightarrow P(A \cup B)_{max} = 1$

$$P(A \cap B)_{min} = 0.4 + 0.7 - 1$$
$$= \boxed{0.1}.$$

Condition :- A & B are exhaustive

b) ~~similarly $P(A \cap B)$ max when $P(A \cup B)$ min.~~

$(A \cap B) \subset A$
$(A \cap B) \subset B$.

$P(A \cap B) \leq P(A) = 0.4$

$P(A \cap B) \leq P(B) = 0.7$.

$\therefore P(A \cap B) \leq \min P(A)$ or $P(B)$

$\Rightarrow \boxed{P(A \cap B)_{max} = 0.4}$

JQK

14) ~~ace of spade ⇒ 2×4 ⇒ 8~~ ace ⇒ 3 } except spade

even ⇒ 2, 4, 6, 8, 10 → 5

4 groups ⇒ $5 \times 3 = \boxed{15}$.

spade ⇒ 13.

∴ Total fav ⇒ $3 + 13 + 15 = 31$

$$\therefore \boxed{\text{prob} = \frac{31}{52}}$$

15) 2, 3, 4, 5, 6 ⇒ x

1 ⇒ $2x$.

Total students ⇒ $5x + 2x = 7x$.

grade 3 ⇒ $\frac{x}{7x} = \boxed{\frac{1}{7}}$.

16) infected + positive → 99.99%. ~~infect / neg → 0.01%~~

not infect + positive ⇒ 0.02%. ~~not inf / neg ⇒ 99.98%.~~

nice

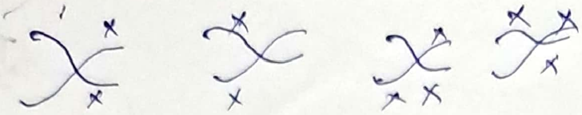
generally 1 infected ⇒ $\frac{1}{10,000}$ ⇒ 0.0001%

P(~~Random~~ is positive) = ? $\frac{\text{P(infect \& ~~positive~~)} + \text{P(not infect \& positive)}}{\text{P(inf \& pos)} + \text{P(inf \& neg)} + \text{P(not inf \& pos)} + \text{P(not inf \& neg)}}$

P(inf)

$$= \frac{\frac{99.99}{100} + \frac{0.02}{100}}{\frac{99.99}{100} + \frac{0.01}{100} + \frac{0.02}{100} + \frac{99.98}{100}}$$

$$= \frac{100.01}{2} = 50$$



16) I = people infected

T = test +ve

$$P(T \mid I) = \frac{9999}{10{,}000}$$

$$P(T \mid I^c) = \frac{2}{10000}$$

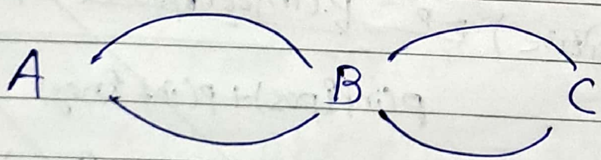
$$P(I) = \frac{1}{10000}$$

$$P(T) = P(T \cap I) + P(T \cap I^c)$$

$$= P(T \mid I) \cdot P(I) + P(T \mid I^c) \cdot P(I^c)$$

H.W solve yourself

(18)



P(open road A to c) = ??

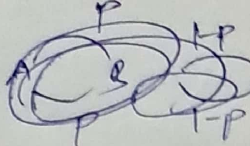
P (snow - block) = p

P (snow not block) = 1 − p

E = R_i is blocked.

$P(E) = p$

$P(E^c) = 1 - p$.



probability $= P(E_1^c \cup E_2^c) \cdot P(E_3^c \cup E_4^c)$

(independent)

$= P(E_1 \cap E_2)^c \cdot P(E_3 \cap E_4)^c$

$= (1 - p \cdot p)(1 - p \cdot p)$ ($\because$ independent)

$= \boxed{(1-p^2)^2}$.

19) Dearrangement of n

$$D_n = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^{n+2} \frac{1}{n!}$$

Given :- E_i = Event that ith letter entered in ith envelope.

E_i^c = ith letter doesn't enter ith envelope.

$P(E_1^c \cap E_2^c \cap E_3^c \cdots \cap E_n^c)$

$= 1 - P(E_1 \cup E_2 \cup \cdots E_n)$

$$= 1 - \left\{ \sum_{i=1}^{n} P(E_i) - \sum_{i=1}^{^nC_2} P(E_i \cap E_j) + \sum_{i=1}^{^nC_3} P(E_i \cap E_j \cap E_k) \cdots + (-1) \sum_{i=1}^{^nC_k} P(E_i \cap E_j \cdots E_k) \right.$$

$$= 1 - \left\{ n \times \left[\frac{(n-1)!}{n!}\right] - {^nC_2} \times \left[\frac{(n-2)!}{n!}\right] + {^nC_3}\left[\frac{(n-3)!}{n!}\right] + \cdots \right.$$

$P(E_i) = P(\text{one goes into its envelope})$

$= \frac{1}{n}$.

$P(E_i \cap E_j) = P(\text{two goes into their envelope})$

$= P(\text{1 goes correctly}) \cdot P(\text{2 goes correctly} \mid \text{1 has gone correctly})$

$= \frac{1}{n} \times \frac{1}{n-1}$

Reason :- $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2 | E_1)$

$= \frac{1}{n} \cdot \frac{1}{n-1}$

$\because P(E_1 \cap E_2 \cap E_3) = P(E_1) \cdot P(E_2|E_1) \cdot P(E_3 | E_1 \cap E_2)$

$= \frac{1}{n} \times \frac{1}{n-1} \times \frac{1}{n-2}$

$\vdots$

$P(E_1 \cap E_2 \cdots \cap E_n) = \frac{1}{n!}$

$$\Rightarrow 1 - \left\{ n \times \frac{1}{n} - {}^{n}C_2 \times \frac{1}{n} \times \frac{1}{n-1} + \cdots \right\}$$

$$\Rightarrow 1 - \left\{ 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} - \cdots \right\}$$

$$= \left\{ \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \cdots\cdots (-1)^{n+2} \left(\frac{1}{n!}\right) \right.$$

(20)

R, W, B, Y, G

$P(\text{Red}) = \frac{1}{5}$

$P(W) = 2/5$

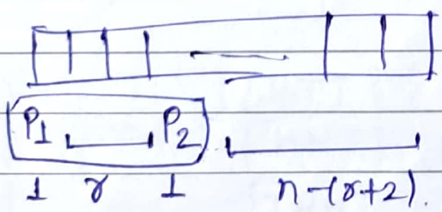
$P(B) = ?\quad P(Y) = ?\quad P(G) = ?$

$$P(G \cup Y \cup B) = 1 - P(R) - P(W)$$

$$= 1 - \frac{1}{5} - \frac{2}{5}$$

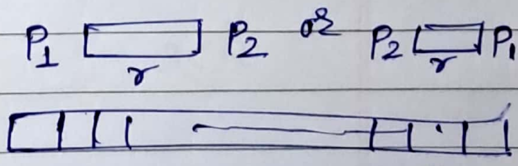
$$= \boxed{\frac{2}{5}}$$

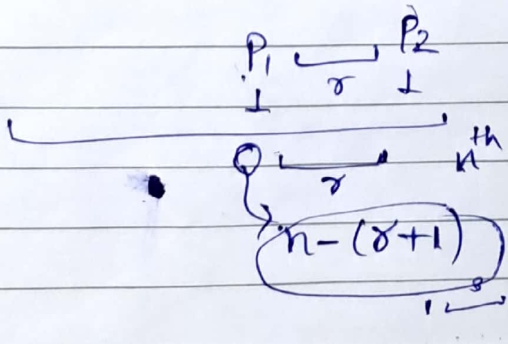
(21)



$n - (r+2)$.

Total seatings P_1 & P_2 pack





$\boxed{\text{Total} \Rightarrow n!}$

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shifting

P_1, P_2 interchange ... places ... people randomize

$$\text{Prob} \Rightarrow \frac{\text{favour}}{\text{Total}} = \frac{2 \times [n-(r+1)] \times [n-2)]!}{n!}$$

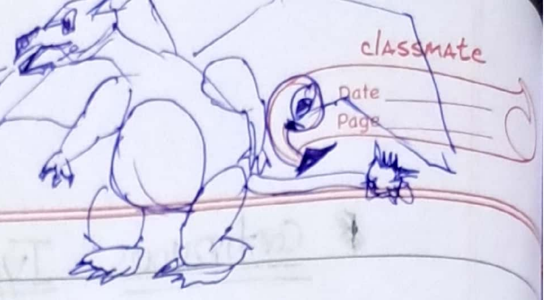
$$\frac{2 \times [n-(r+1)]!}{n!}$$

$$= \frac{}{n \times (n-1) \times (n-2) \ldots \times (n-(r+2)) \times (n-(r+3)) \times \ldots 3 \times 2 \times 1}$$

$$= \boxed{\frac{2 \times (n-(r+1))}{n \times (n-1)}}$$

—X—

Charizard



Tutorial 2

Problem set 2

Random variable :-

$X : S \longrightarrow \mathbb{R}$ such that

$\forall B \in B_{\mathbb{R}}$

$X^{-1}(B) \in \Sigma$ = event space

also $X^{-1}\left((-\infty, x]\right) \in \Sigma$

Distribution function :-

$F_X : \mathbb{R} \to \mathbb{R}$ codomain $\mathbb{R}$, Range :- $[0,1]$

such that
- F_X is increasing
- Right continuous
- $F_X(-\infty) = 0$, $F_X(\infty) = 1$

Q.1 a) $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x} & \text{if } x \geq 0. \end{cases}$

i) increasing :- $x_1 < x_2 \Rightarrow F(x_1) \leq F(x_2)$

if $x_1, x_2 < 0$

$f(x_1) = f(x_2)$.

if $x_1, x_2 \geq 0$

$x_1 < x_2$

$-x_1 > -x_2$

$e^{-x_1} > e^{-x_2}$ (e is increasing)

$-e^{-x_1} < -e^{-x_2}$

$\boxed{1 - e^{-x_1} < 1 - e^{-x_2}}$

- Function is right continuous.
  (It is continuous everywhere).
  By checking continuity at 0

$\lim_{x \to 0^-} F(x) \Rightarrow 0$ Left continuous ✓ $\quad \lim_{x \to 0^+} F(x)\ (1-e^{-x}) = 1-1 = \boxed{0}$

Right continuous ✓

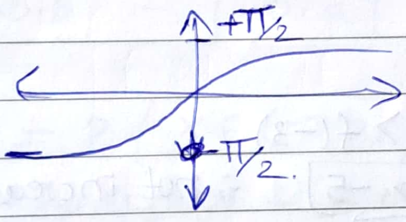
$F_X(-\infty) = \lim_{x \to -\infty} F(x) = 0$

$F_X(+\infty) = \lim_{x \to +\infty} F(x) = 1 - e^{-\infty} = 1 - 0 = \boxed{1}$

—— x ——

b) $F(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1} x$, $-\infty < x < \infty$.

ans- $\tan^{-1} x$



$\tan^{-1} x$

i) Since $\tan^{-1} x$ is increasing function $\left(\because \frac{d(\tan^{-1} x)}{dx} = \frac{1}{1+x^2} > 0 \right.$ so increasing.$\left. \right)$

$\therefore$ $F(x)$ is increasing.

ii). $\tan^{-1} x$ is completely continuous
Hence $F(x)$ is also continuous.
($\because \tan^{-1} x$ is differentiable).

iii) $F_X(+\infty) = \lim_{x \to +\infty} F(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}(+\infty)$

$= \frac{1}{2} + \frac{1}{\pi} \times \frac{\pi}{2}$

$= \frac{1}{2} + \frac{1}{2} = \boxed{1}$.

Similarly $\boxed{F_X(-\infty) = 0}$

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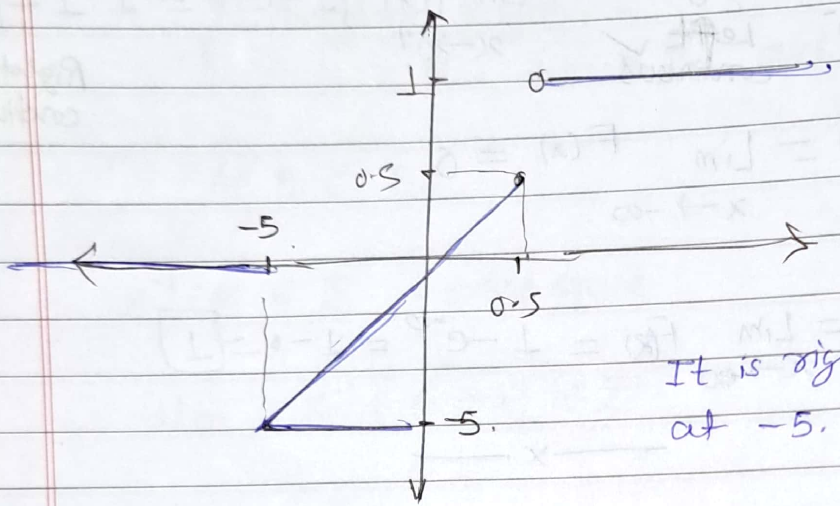
c) $F(x) = \begin{cases} 0 & x < -5 \\ x & -5 \leq x \leq 0.5 \\ 1 & x > 0.5 \end{cases}$



It is right continuous at -5.

disprove :- not right-continuous at 0.5.

$-7 < -3$

but $f(-7) > f(-3)$

$\boxed{0 > -5}$ $\therefore$ not increasing.

x ——— x ——— x

ans.2

$$F(x) = \begin{cases} 0 & \text{if } x<0 \\ \frac{2}{3} & 0\le x<1 \\ \frac{7-6c}{6} & 1\le x<2 \\ \frac{4c^2-9c+6}{4} & 2\le x\le 3. \\ 1 & x>3 \end{cases}$$

Revision

Note:- $\{X\le x\}$ means $\{\omega\in S : X(\omega)\le x\}$

$$\{X\le x\} = X^{-1}((-\infty, x])$$

$$P(a<X\le b) = P(\{a<X\le b\}).$$

$$(a,b] = (-\infty,b] - (-\infty,a]$$

$$P(X\in(a,b]) = P(X\in(-\infty,b]) - P(X\in(-\infty,a])$$
$$= F_X(b) - F_X(a)$$

X — discrete Random variable if $\exists\, E_X \subset \mathbb{R}$

$$P(X=x)>0 \quad \forall x\in E_X.$$

$$\& \; P(X\in E_X) = \sum_{x\in E_X} P(X=x) = 1.$$

p.m.f $\quad f_X(x) = P(X=x) = F_X(x) - F_X(x^-)$ c.d.f

$$= \begin{cases} P(X=x & \text{if } x\in E_X \\ 0 & \text{otherwise} \end{cases}$$

$= P(X \le x)$

$$F_X(x) = \sum f_X(y)$$

$$y \in (-\infty, x] \cap E_X$$

$P(E_X^c) = 0$

$$P(X \in A) = P(X \in A \cap E_X) + P(X \in A \cap E_X^c)$$

$$= P(X \in A \cap E_X)$$

$$= \sum_{x \in A \cap E_x} P(X = x)$$

$$= \sum_{x \in A \cap E_x} f_X(x)$$

Q.2) continue:-

$F(x)$ is right continuous ——— ($\because$ CDF)

Checking continuity at 1

common mistake

$$\frac{2}{3} = \frac{7-6c}{6}$$

$$\frac{12}{3} = 7-6c$$

$$4 = 7-6c$$

$$6c = 3$$

$$\boxed{c = \frac{1}{2}}$$

It is not right continuous at 1.

fn is also not increasing at $c = \frac{1}{2}$

since $F(x)$ is right continuous at 3.

$$\lim_{x \to 3^+} F(x) = \lim_{h \to 0} F(3+h) = 1.$$

now, $F(3) = \frac{4c^2 - 9c + 6}{4}$

$$1 = \frac{4c^2 - 9c + 6}{4}$$

$$4 = 4c^2 - 9c + 6$$

$$4c^2 - 9c + 2 = 0$$

$$4c^2 - 8c - c + 2 = 0$$

$$4c(c-2) - 1(c-2) = 0$$

$$(c-2)(4c-1) = 0$$

$$\boxed{c=2} \quad \boxed{c=\tfrac{1}{4}}$$

if $c = 2$, f^n will not be creasing

if $\boxed{c=\frac{1}{4}}$, f^n is increasing

$$\boxed{F(x) = \begin{cases} 0 & \text{if } x<0 \\ 2/3 & 0 \le x < 1 \\ 11/12 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases}}$$

Yes, $f(x)$ is increasing

$\frac{14-3}{2} = \frac{11}{12}$ $\quad 4\left(\frac{1}{16}\right) \quad \frac{1}{4} - \frac{9}{4} + 6 = \frac{-8}{4} + 6 = \frac{-4}{4} = 1$

$7 - \frac{8}{4}2$

c.d.f $F(x)$ •) is not continuous everywhere
•) is Right continuous everywhere
•) continuous at E_x^c
•) discontinuous at E_x.

$$P(X=0) = F_X(0) - F_X(0^-)$$
$$= \frac{2}{3} - 0$$

$$\boxed{P(X=0) = \frac{2}{3}}$$

similarly $P(X=1) = F_X(1) - F_X(1^-)$
$$= \frac{11}{12} - \frac{2}{3}$$
$$= \frac{3}{12} = \boxed{\frac{1}{4}}.$$

$$P(X=2) = F_X(2) - F_X(2^-)$$
$$= 1 - \frac{11}{12}$$
$$= \boxed{\frac{1}{12}}.$$

$$\sum_{x \in (0,1,2)} P(X=x) = \frac{2}{3} + \frac{1}{4} + \frac{1}{12} = \boxed{1}$$

$\underset{x \in (0,1,2)}{P(X=x)} > 0$ Hence $E_x = \{0\}, \{1\}, \{2\}$

ii) pmf is function from $\mathbb{R} \to \mathbb{R}$

$$f_X(x) = p(x) = \begin{cases} \frac{2}{3} & x=0 \\ \frac{1}{4} & x=1 \\ \frac{1}{12} & x=2 \\ 0 & \text{otherwise} \end{cases}$$

use small p for p.m.f.

or small f

c) $p(1 < X < 2) = P(X<2) - P(X \leq 1)$

$= F_X(2^-) - F_X(1)$

$= \frac{11}{12} - \frac{11}{12}$

$= \boxed{0}$.

$p(2 \leq X < 3) = p(X<3) - P(X<2)$

$= F_X(3^-) - F_X(2^-)$

$= 1 - \frac{11}{12}$

$= \boxed{\frac{1}{12}}$.

$P(0 < X \leq 1) = P(X \leq 1) - P(X \leq 0)$

$= F_X(1) - F_X(0)$

$= \frac{11}{12} - \frac{2}{3}$

$= \frac{3}{12} = \boxed{\frac{1}{4}}$.

$$P(1 \le X \le 2) = \boxed{\frac{1}{3}}$$

$$P(X \geq 3) = 1 - 1 = 0.$$

$$P(X = 2.5) = P(X = 2.5) - P(X = 2.5^-)$$
$$= 1 - 1$$
$$= \boxed{0}.$$

ans. d) $P(\{X=1\} \mid \{1 \le X \le 2\})$

$$= P(\{A\} \mid \{B\})$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$P(1 \le X \le 2)$$
$$= P(X=2) + P(X=1) + P(1 < X < 2)$$
$$0$$

$$A \cap B = (X=1) \cap (1 \le X \le 2)$$
$$= \boxed{(X=1)}$$

$$B = \{1 \le X \le 2\}$$

Homework

$\frac{a}{r}, a, ar^2$ $\quad {}^nP_r = \frac{n!}{(n-r)!}$ $\quad {}^nC_r = \frac{n!}{(n-r)!\, r!}$

$n=3$
$r=1$

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$$\frac{2 \times (0)! \times \cancel{(0)!}\; 01! \times {}^{3-2}C_1}{3!} = \frac{1}{3}$$

ans 22

Exam Question 1–100

Q.22 & Q.28

$(1,2,3), (2,3,4) \ldots (48,49,50) \Rightarrow 48$
$(1,3,5), (5,7,9) \ldots (46,48,50) \Rightarrow 46$

$(1,25,49)\ (2,26,50) \Rightarrow 2$

$\therefore$ summation $\Rightarrow 2+4+ \ldots + 46+48$
$= 2[1+2+ \ldots +24]$
$= 2 \times \frac{24 \times 25}{2}$
$= \boxed{600}$

$\therefore$ Probability $= \dfrac{600 \text{ A.P ways}}{\text{total ways of selecting } 3} = \boxed{\dfrac{600}{{}^{50}C_3}}$

ans 23

If $r>1 \Rightarrow$ Yes G.P.
If $r=1$, $a,a,a \Rightarrow$ No G.P. $\Rightarrow$ hence Rejected.
If $r<1 \Rightarrow$ Reverse order G.P.

Case 1) $r>1$
$a_{min} = 1$
$\therefore ar^2 < 50$
ie $r^2 < 50$
$\therefore \boxed{r \le 7}$

$\therefore r = \{2,3,4,5,6,7\}$

If $r=2$,
$1 \le a \le 12 \;] \therefore 12$ triplets

$ar^2 < 50$
$a < \frac{50}{r^2} = \frac{50}{4} = 12.5$

If $r=3$ $a < \frac{50}{r^2}$, $\frac{50}{9} \Rightarrow 5$ triplets

If $r=4$, $a < \frac{50}{16} \Rightarrow 3$ triplets

If $r=5$, $a < \frac{50}{25} \Rightarrow 2$

If $r=6$, $a < \frac{50}{36} \Rightarrow 1$

If $r=7 \Rightarrow a < \frac{50}{49} \Rightarrow 1$ triplet

$\therefore$ Total numbers $\Rightarrow 12+5+3+2+1+1 = \boxed{24}$

<u>case 2)</u> $r \leq 1$

$r = \frac{m}{n}$, $(m,n)=1$ $\therefore$ coprime m & n.

$ar^2 = \frac{am^2}{n^2}$ is an integer

$\Rightarrow \therefore$ a must be divided by n^2.

ie $n^2 | a$

Now, $ar^2 \leq 50$

$a \leq \left[\frac{50n^2}{m^2}\right]$.

(m,n)	$\left[1, \left[\frac{50n^2}{m^2}\right]\right)$	
(3,2)	[1,22]	4, 8, 12, 16, 20.
(5,2)	[1,8]	4,8.
(7,2)	[1,4]	4
(4,3)	[1,28]	9, 18, 27.
(5,3)	[1,18]	9, 18.
(7,3)	[1,9]	9
(5,4)	[1,32]	16, 32.
[7,4]	[1,16]	16
[6,5]	[1,34]	25.
[7,5]	[1,25]	25.
[7,6]	[1,36]	1

Total favourable cases

$= 24 + 5 + 2 + 1 + 3 + 2 + 1 + 2 + 1 + 1 + 1 + 1$

$= 44$

$\therefore$ answer $= \boxed{\frac{44}{{}^{50}C_3}}$

ans. 28)

Total favourable cases $\Rightarrow$ Take 3 defective fuses. $\Rightarrow {}^5C_3$.

Total cases $\Rightarrow {}^{20}C_3$

ans:- $\frac{{}^5C_3}{{}^{20}C_3}$

Reason:- Taking balls without replacement = Taking balls together

ans. 30. answer is $\frac{r}{100}$. Let red balls be 'r'.

a) First ball drawn will be red.

$$\Rightarrow \frac{r}{100} \times \frac{r-1}{99} + \frac{100-r}{100} \times \frac{r}{99}$$

$$= \boxed{\frac{r}{100}}$$

b). ~~50th~~ 3rd ball will be red.

$$\Rightarrow \frac{r}{100} \times \frac{r-1}{99} + \frac{r-2}{98}$$

$$+ \frac{r}{100} \times \frac{100-r}{99} \times \frac{r-1}{98}$$

$$+ \frac{100-r}{100} \times \frac{99-r}{100} \times \frac{r}{98}$$

$$+ \frac{100-r}{100} \times \frac{r}{99} \times \frac{r-1}{98}$$

$$\Rightarrow \boxed{\frac{r}{100}}$$

Hence 50^{th} ball will also be red.

(c) Probability that Last ball is red $= \boxed{\frac{1}{100}}$.

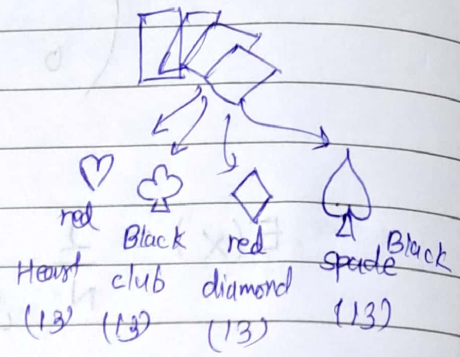
Remember this important result

(TUTORIAL)

Problem set 2)

ans. 3) 52 $\longrightarrow$ 5 at random without replacement

ways of choosing (sample space) $= {}^{52}C_5 = \binom{52}{5}$



Random variable

$X =$ No. of Hearts

$\therefore X(\omega) = \{0, 1, 2, 3, 4, 5\}$

E_X (support) $= \{0, 1, 2, 3, 4, 5\}$

a) pmf of X.

$$P(X=0) = \frac{\binom{13}{0} \cdot \binom{39}{5}}{\binom{52}{5}}$$

$$P(X=1) = \frac{\binom{13}{1}\binom{39}{4}}{\binom{52}{5}}$$

$\vdots$

$$P(X=5) = \frac{\binom{13}{5}\binom{39}{0}}{\binom{52}{5}}$$

$$\therefore \quad P(X=x) = \frac{\binom{13}{x}\binom{39}{5-x}}{\binom{52}{5}}$$

So, pmf will be

$$p(x) = \begin{cases} \dfrac{\binom{13}{x}\binom{39}{5-x}}{\binom{52}{5}} & \text{when } x \in \{0,1,2,3,4,5\} \\ 0 & \text{otherwise} \end{cases}$$

(b) $\quad p(X \leq 1) = p(X=0) + p(X=1)$

[Q.6 is same as Q.3]

Q.4 $\quad X$ – RV with pmf.

$$p(x) = \begin{cases} \dfrac{c}{(2x-1)(2x+1)} & , \text{ if } x \in \{1,2,3,\dots\} \\ 0 & , \text{ otherwise} \end{cases}$$

c is a constant.

ans:

a) $$\sum_{x \in E_X} p(x) = 1.$$

$$\Rightarrow \sum_{x=1}^{\infty} \frac{c}{(2x-1)(2x+1)} = 1.$$

$$c \sum_{x=1}^{\infty} \frac{1}{(2x-1)(2x+1)} = 1$$

$$\frac{c}{2} \sum_{x=1}^{\infty} \left(\frac{1}{2x-1} - \frac{1}{2x+1} \right) = 1$$

$$\frac{c}{2} \left(\frac{1}{1} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{5}} + \cdots + \cancel{\frac{1}{2x-1}} - \frac{1}{2x+1} \right) = 1$$

$$\frac{c}{2} \left(1 - \frac{1}{2x+1} \right) = 1$$

$$\boxed{\because \lim_{x \to \infty} \left(\frac{1}{2x+1} \right) = 0}$$

$$\therefore \quad \frac{c}{2}(1-0) = 1$$

$$\boxed{c = 2}$$

b) c.d.f of X.

$$p(x) = \begin{cases} \frac{2}{(2x-1)(2x+1)} & x \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

For c.df, find $P(X \leq x)$, $x \in \mathbb{R}$

$$\boxed{F_X(x) = P(X \leq x), \quad x \in \mathbb{R}}$$

$2 \quad \frac{4}{5}$ ——— $13.$ $\quad \frac{2}{3}+\frac{2}{15}=\frac{12}{15}$

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$$F_X(x) = \begin{cases} 0 & , \text{ if } x<1 \\ \frac{2}{1\cdot 3} & , \text{ if } x=1 \\ \frac{2}{1\cdot 3} & , 1<x<2 \\ \frac{2}{1\cdot 3}+\frac{2}{3\cdot 5} & , 2\leq x<3 \\ \frac{2}{1\cdot 3}+\frac{2}{3\cdot 5}+\frac{2}{5\cdot 7} & , 3\leq x<4 \end{cases}$$

$$F_X(x) = \begin{cases} 0 & , \text{ if } x<1 \\ \frac{2[x]}{2[x]+1} & , \text{ if } x\geq 1. \end{cases}$$

——— x ———

$p(X=a) :\to \{\omega : X(\omega)=a\}$

(c) $m, n \in \mathbb{N}, \quad m<n$

$P(X<m+1) = ?$

ans:-

$P(X<m+1)$

$= P(X \leq m+1) - P(X=m+1)$

$= \underbrace{F_X(m+1)}_{\text{c.d.f}} - \underbrace{p(m+1)}_{\text{pmf}}$

$= \dfrac{2[m+1]}{2[m+1]+1} - \dfrac{2}{(2(m+1)-1)(2(m+1)+1)}$

$= \dfrac{2}{2(m+1)+1}\left[(m+1) - \dfrac{1}{2(m+1)-1}\right]$

$= \dfrac{2}{2(m+1)+1}\left[\dfrac{(m+1)(2m+1)-1}{2m+1}\right]$

H.W

(d) $P\left(\underbrace{(X>1)}_{A} \mid \underbrace{(1 \leq X < 4)}_{B}\right)$

$= \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{P(1<X<4)}{P(1 \leq X<4)} = \dfrac{P(X \leq 4) - P(X=4) - P(X \leq 1)}{P(X \leq 4) - P(X=4) - P(X<1)}$

(5) X — RV , c.d.f

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{x^2}{2} & \text{if } 0 \leq x < 1 \\ \frac{x}{2} & \text{if } 1 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$

a) X is of continuous type and then pdf of x??

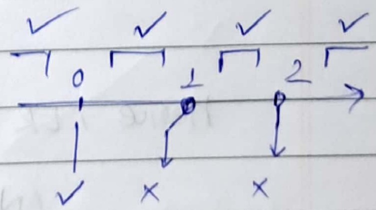
ans:- X is C.T.R.V if cdf is differentiable at critical points. { ie 0, 1, and 2 }
↳ where definition of function changes

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \quad \text{RHL}$$

$$\lim_{h \to 0} \frac{f(a) - f(a-h)}{h} \quad \text{LHL}$$

RHL & LHL should be same.

Observation :- differentiability



Finite no. of non-differentiable points, hence it is continuous Type R.V. (piece-wise continuous)

$$F'(x) = \begin{cases} 0 & , x < 0 \\ x & , 0 \leq x < 1 \\ \frac{1}{2} & , 1 < x < 2 \\ 0 & , x \geq 2 \end{cases}$$

(Note:- not-differentiable at 1 & 2)

$$= \begin{cases} x & , 0 \leq x < 1 \\ \frac{1}{2} & , 1 < x < 2 \\ 0 & , x \notin \{1, 2\} \\ \text{Any arbitrary constant} & , x \in \{1, 2\} \end{cases}$$

Hence, Ek R.V का pdf unique हो जरूरी नहीं
(Not needed that a R.V, should have unique p.d.f)

ans. (b) $P(1 < X < 2)$

$= P(X \leq 2) - P(X = 2) - P(X \leq 1)$

$= P(X < 2) - P(X \leq 1) = F_X(2^-) - F_X(1)$

Q.6 HW.

Q.7 X - R.V.

$$F(x) = \begin{cases} c(4x - 2x^2), & \text{if } 0 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

a) Find c?

ans:- use properties

$$\int_{-\infty}^{\infty} f(t).dt = 1$$

$$\Rightarrow \int_0^2 c(4x - 2x^2).dx = 1$$

$$c \left[\frac{4x^2}{2} - \frac{2x^3}{3}\right]_0^2 = 1$$

$$c \left[\left(\frac{4\times4}{2} - \frac{2\times8}{3}\right) - (0)\right] = 1$$

$$c\left(\frac{16}{2} - \frac{16}{3}\right) = 1$$

$$c\left(\frac{3-2}{6}\right) = \frac{1}{16}$$

$$\boxed{c = \frac{6}{16} = \frac{3}{8}}$$

b) cdf of X

$$F_X(x) = P\{X \leq x\}$$

$$= \int_{-\infty}^{x} f(t)\,dt$$

$$= \begin{cases} 0 & \text{if } x < 0 \\ \int_0^x \frac{3}{8}\cdot(4t - 2t^2)\,dt & \text{if } 0 \leq x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$$

Question type covered till now:-

$X \to$ discrete $\longrightarrow$ cdf $\rightleftarrows$ pmf

$X \to$ continuous $\longrightarrow$ cdf $\rightleftarrows$ pdf

(8) $P(X=r) = \binom{n}{r} p^r (1-p)^{n-r}$ $\Rightarrow \boxed{P(X \neq r) = 0}$

$r = 0, 1, 2, \dots n$ $0 \leq p \leq 1.$

Find pmf

a) $Y = aX + b$ b) $Y = X^2$

continuous continuous

Y are R.V.

ans:- $P(Y=y)$

$P(aX+b = y) = ?$

$Y(w) = b \quad \forall w$

Note:- $P(X=-2) = 0$
$P(X=-7) = 0$
$\because P(X=\text{negative})$ not defined

case I :- when $a = 0$

$$P(Y=y) = \begin{cases} 1, & y = b \\ 0, & y \neq b \end{cases}$$

case II :- when $a \neq 0$.

$P(Y=y) = P(aX+b = y)$
$= P\left(X = \frac{y-b}{a}\right)$

We already have $P(X=r)$, use the definition.

$$P\left(X = \frac{y-b}{a}\right) = \begin{cases} \binom{n}{\frac{y-b}{a}} p^{\left(\frac{y-b}{a}\right)} (1-p)^{n-\left(\frac{y-b}{a}\right)} ; \\ \frac{y-b}{a} = 0, 1, 2, \dots n. \\ \therefore y = b, a+b, 2a+b, \dots na+b \end{cases}$$

$\qquad 0 \quad ; \text{ otherwise}$

$\therefore$ PMF of $Y = aX + b$.

$$P(Y=y) = P\left(X = \frac{y-b}{a}\right)$$

$$= \begin{cases} \binom{n}{\left(\frac{y-b}{a}\right)} p^{\left(\frac{y-b}{a}\right)} (1-p)^{n-\left(\frac{y-b}{a}\right)} ; & \text{where } y = b, a+b, 2a+b \dots na+b \\ 0 ; & \text{otherwise.} \end{cases}$$

b) $P(Y = X^2) = P(X = \sqrt{Y}) = P(X = \sqrt{y})$

$$\therefore \quad = \begin{cases} \binom{n}{\sqrt{y}} p^{(\sqrt{y})} (1-p)^{n-\sqrt{y}} ; & \text{where } \sqrt{y} = 0, 1, 2 \dots n \\ & \therefore y = 0, 1, 4, 9, \dots n^2. \\ 0 ; & \text{otherwise} \end{cases}$$

$$P(Y = X^2) = P(X = \pm\sqrt{Y}) = P(X = \pm\sqrt{y}) = P(X = \sqrt{y}) + \underbrace{P(X = -\sqrt{y})}_{\text{value is zero}}$$

$$= P(X = \sqrt{y}) \text{ only}$$

$$= \begin{cases} \binom{n}{\sqrt{y}} p^{\sqrt{y}} (1-p)^{n-\sqrt{y}} ; & \sqrt{y} = 0, 1, 2, \dots n \\ & y = 0, 1, 4, 9, \dots n^2 \\ 0 ; & \text{otherwise} \end{cases}$$

(9) X has p.d.f $f(x) = \begin{cases} c \cdot e^{-x} ; & 0 < x < \infty \\ 0 ; & \text{elsewhere} \end{cases}$

continuous

Find c & $P(1 < X \leq 2) = ?$

ans:-

$$\int_{-\infty}^{+\infty} f(x) = 1$$

$$\int_{0}^{\infty} c \cdot e^{-x} = 1$$

$$-c \left[e^{-x} \right]_0^{\infty} = 1$$

$$-c \left[\frac{1}{e^{\infty}} - \frac{1}{e^{0}} \right] = 1$$

$$\boxed{c = 1}$$

$$P(1 < X \leq 2) = P(X \leq 2) - P(X \leq 1)$$

$$= F_X(2) - F_X(1)$$

$$= \int_{-\infty}^{2} f(x) \cdot dx - \int_{-\infty}^{1} f(x) \cdot dx = \int_{1}^{2} f(x) \cdot dx$$

$$= \boxed{e^{-1} - e^{-2}}$$

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ans.10) Let Range $(X) = [0,3]$. $\rightarrow$ support of X
E_X.

pdf $f_X(x) = kx^2$

Let $Y = X^3$.

find

(a) Find k and c.d.f of X??

ans:-

$$f(x) = \begin{cases} kx^2 & 0 \leq x \leq 3 \\ 0 & \text{else} \end{cases}$$

Now
$$\int_{-\infty}^{+\infty} f(x) = \int_0^3 kx^2 = 1.$$

$$\boxed{k = \frac{1}{9}}$$

$$F_X(x) = \text{cdf of } X = \begin{cases} P(X \leq x) & , x \in E_x \\ 0 & , \text{otherwise} \end{cases} = \begin{cases} \int_{-\infty}^{x} f_X(t)\,dt, & x \in E_x \\ 0, & \text{otherwise.} \end{cases}$$

$$= \begin{cases} P(X < 0) = 0 & ; \quad \underline{x < 0} \\ P(0 \leq X \leq 3) = \int_0^x \frac{x^2}{9} & ; \\ P(X > 3) = 1 & ; \end{cases}$$

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(b) $E(Y) = \int_{-\infty}^{+\infty} y \cdot f_Y(y) \cdot dx$

expectation

$$E(h(x)) = \int_{-\infty}^{+\infty} h(x) \cdot f_X(x) \cdot dx$$

Roadmap:- pdf of X

$\Downarrow$

cdf of ~~pdf~~ $Y = X^3$ $\Rightarrow$ pdf of $Y = X^3$.

X is R.V

$h: \rightarrow R \longrightarrow R$, $h(x) = x^3$.

If h is strictly monotone ✓

+

differentiable ✓.

then

pdf of $h(X)$

$$= \begin{cases} f_X(h^{-1}(y)) \left| \frac{d(h^{-1}(y))}{dy} \right|, & y = h(x). \\ 0, & \text{otherwise} \end{cases}$$

$$E(h(x)) = \int_{-\infty}^{+\infty} h(x) \cdot f_X(x) \cdot dx.$$

$$= \int_{0}^{3} x^3 \cdot f_X(x) \cdot dx.$$

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(c) $Var(Y) = E\left[(y-\mu_y)^2\right]$.

where $\mu_y = E(y)$

now

$$Var(Y) = E\Big[\big(\underbrace{x^3 - \tfrac{27}{2}}_{h(x)}\big)^2\Big]$$

—— x ——

ans. 11) X — RV

$$p(x) = \begin{cases} 1/7 & x \in \{-2,-1,0,1\} \\ 3/14 & x \in \{2,3\} \\ 0 & \text{otherwise} \end{cases}$$

a) Find mgf of x??

ans. mgf of $x = E(e^{tx})$

$$E(e^{tx}) = \sum_{x \in E_x} e^{tx} \cdot p(x), \quad t \in \mathbb{R}$$

$$= \frac{1}{7}\left(e^{-2t} + e^{-t} + e^0 + e^t\right)$$

$$+ \frac{3}{14}\left(e^{2t} + e^{3t}\right)$$

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(b) pmf of $Z = X^2$

$= P_Z(z)$

$$= \begin{cases} P(Z = z), & z \in E_Z \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} P(X^2 = z), & z \in \{0, 1, 4, 9\} \\ 0, & \text{otherwise} \end{cases}$$

$P(X^2 = 0) = P(X = 0) = \frac{1}{7}$

$P(X^2 = 1) = P(X = 1) + P(X = -1)$

$= \frac{1}{7} + \frac{1}{7}$

$= \boxed{\frac{2}{7}}$

$P(X^2 = 4) = P(X = +2) + P(X = -2)$

$= \frac{3}{14} + \frac{1}{7}$

$= \boxed{\frac{5}{14}}$

$P(X^2 = 9) = P(X = 3) + P(X = -3)$

$= \frac{3}{14} + 0 \hookrightarrow$ ($\because$ not given)

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$\therefore$ pmf $Z = X^2 = \begin{cases} P(Z=0) = \frac{1}{7}, \\ P(Z=1) = \frac{2}{7} \\ P(Z=4) = \frac{5}{14} \\ P(Z=9) = \frac{3}{14} \end{cases}$

c.d.f $Z = X^2 = \begin{cases} 0 & Z < 0 \\ \left(\frac{1}{7}\right) & \text{if } 0 \leq Z < 1 \\ \frac{3}{7} & 1 \leq Z < 4 \\ \frac{11}{14} & 4 \leq Z < 9 \\ 1 & Z \geq 9. \end{cases}$

14) X – R.V with pdf.

★★★ Exam

$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

Find cdf of $Z = X^2(3-2X)$
find its pdf.

ans:- We cannot find inverse of $X^2(3-2X)$

Reason :- $Y = 3X^2 - 2X^3$

$X = f(\sim)$??

$x(3-2x) \Rightarrow -2x^2 - 2x^2$
$+ x^2(-2)$ $(-4x^2)$

Lets go through cdf of Z

$\Downarrow$

$$F_Z(z) = \begin{cases} P(Z \leq z) & , z \in E_Z \\ 0 & , \text{otherwise.} \end{cases}$$

$h(x) \in E_Z$

$h([0,1]) = [0,1]$

cdf of Z

$$\hookrightarrow = \begin{cases} 0 & z < 0 \\ \int_{-\infty}^{h^{-1}(z)} h'(x) & 0 < z < 1 \end{cases}$$

$$= \begin{cases} 0 & ; z < 0 \\ \int_{-\infty}^{h^{-1}(z)} 6(x) \cdot (1-x) & ; 0 \leq h(x) < 1 \\ 1 & h(x) \geq 1 \end{cases}$$

continued on next page.

$Z = X^2 (3-2X) = h(X)$

c.d.f of Z

$$F_Z(z) = \begin{cases} P(Z \leq z), & z \in E_Z \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} P(h(X) \leq z), & -\!\!-\!\!\text{II}\!\!-\!\!- \\ 0, & -\!\!-\!\!\text{II}\!\!-\!\!- \end{cases}$$

$$= \begin{cases} P\left(X \in \frac{h^{-1}(z)}{x}\right), & -\!\!-\!\!\text{II}\!\!-\!\!- \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \int_{-\infty}^{h^{-1}(z)} f_X(x)\,dx & z \in E_Z \\ 0, & \text{otherwise} \end{cases}$$

$p(Z \leq z) = p(Z < 1) + p(Z \geq 1)$

$\Rightarrow p(Z < 1)$

$= \boxed{1}$

moment generating function.

ans) [15] m.g.f $= \dfrac{e^{t} - e^{-2t}}{3t}$, $t \neq 0$.

$$E(e^{tx}) = \frac{e^{t} - e^{-2t}}{3t}$$

$$\int_{x \in E_x} e^{tx} \cdot f_X(x) \cdot dx = \frac{e^{t} - e^{-2t}}{3t}$$

~~Replay~~ $M_X(t) = E(e^{xt})$.

continuous

discrete (not our question)

$$M_X(t) = \sum_{x} e^{xt} \cdot p(x)$$

$$M_X(t) = E(e^{xt})$$

$$= \int_{-\infty}^{+\infty} e^{xt} \cdot f_X(x) \cdot dx$$

ans:- $M_X(t) = \int_a^b e^{xt} \cdot f_X(x) \cdot dx = \dfrac{e^{t} - e^{-2t}}{3t}$.

∴ It should be :-

$$\int_{-2}^{1} e^{xt} \cdot \underbrace{f_X(x)}_{\left(\frac{1}{3}\right)} \cdot dx = \boxed{\frac{e^{t} - e^{-2t}}{3t}}$$

so, pdf of X.

$$f_X(x) = \begin{cases} 1/3 , & -2 < x < 1 \\ 0 , & \text{otherwise.} \end{cases}$$

17) series is not convergent

18) integration convergent.

19) RV – X.

Mean – μ , Finite second moment.

Show:- $E((X-\mu)^2) \leq E((X-c)^2) \quad \forall c \in \mathbb{R}$

Soln:- $E((X-c)^2) = E(X^2 - 2cX + c^2)$

$= E(X^2) + c^2 - 2c\,E(X)$

let $h(c) = E(X^2) + c^2 - 2c\,E(X)$

To check nature $\quad h'(c) = 2c - 2E(X)$

$= 2(c-\mu)$.

If $c > \mu$, then $\quad h'(c) > 0$

$c < \mu$, then. $\quad h'(c) < 0$.

So, h is increasing $c \in (\mu, \infty) \rightarrow \mu < c \Rightarrow h(\mu) \leq h(c)$

h is decreasing $c \in (-\infty, \mu) \rightarrow \mu > c \Rightarrow h(\mu) \leq h(c)$

$$h(\mu) \leq h(c) \quad \forall c \in \mathbb{R}$$

$$E\left((X-\mu)^2\right) \leq E\left((X-c)^2\right) \quad \forall c \in \mathbb{R}$$

Q2b) Question is:- $E(X) = \mu$

gives hint $E(X-\mu) = 0$.

ie $E(X) - \mu = 0$

$$E(X-\mu) = \int_{-\infty}^{+\infty} (x-\mu) f_X(x) \cdot dx.$$

$$= \int_{-\infty}^{0} (x-\mu) f_X(x) \cdot dx \quad \mu - x$$

$$+ \int_{0}^{\infty} (x-\mu) \cdot f_X(x) \cdot dx \quad \mu + x.$$

using change of variable

let's first replace x by t

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$$= \int_{-\infty}^{0} (t-\mu) f_X(t) \cdot dt + \int_{0}^{\infty} (t-\mu) f_X(t) \cdot dt$$

Now, Replace t by $(\mu - x)$ Replace t by $(\mu + x)$

$\therefore dt =$

$\mu - x = t$

$x = \mu - t$

$dx = -dt$

$$= \cancel{\int_{-\infty}^{0} (-x) \cdot f_X(\mu - x) \cdot}$$

$$= \int_{-\infty}^{\mu} (x-\mu)\, f_X(x) \cdot dx + \int_{\mu}^{\infty} (x-\mu)\, f_X(x) \cdot dx$$

$\mu - t$ $\mu + t$

$$= \int_{+\infty}^{0} (-t) \cdot f_X(\mu - t) \cdot dt + \int t \cdot f_X(\mu + t) \cdot dt$$

— x —

Q.21 ~~(28)~~

$$\boxed{\begin{array}{l} \text{Markov's Inequality} \\ X \text{ is R.V, neither discrete nor continuous} \\ \text{where } X \text{ takes non-negative values.} \\ X(\omega) \in [0, \infty) \\ P(X \geq 0) = 1 \\ \text{For all } t > 0 \\ P(X \geq t) \leq \dfrac{E(X)}{t} \end{array}}$$

ans. 21) Given $P(X \leq 0) = 0$, μ is finite

$$P(X \geq 2\mu) \leq 0.5$$

ans:- Replace t by (2μ) in Markov's Inequality.

$$P(X \geq 2\mu) \leq \frac{E(x)}{2\mu}$$

we know, $E(x) = \mu$.

$$\therefore P(X \geq 2\mu) \leq \frac{\mu}{2\mu} = \boxed{0.5}$$

but all this is true when $2\mu > 0 \Rightarrow \mu > 0$.

Let's prove:-

case I):- X discrete $(\mu > 0)$.

$$\mu = E(X) = \sum x\, p(x)$$

$$= \cancel{\sum_{x \leq 0} x . p(x)} + \underbrace{\sum_{x > 0} x . p(x)}_{\boxed{> 0}} \quad \because \mu \text{ finite}$$

case II):- X is continuous R.v

$$\mu = E(X) = \int_{-\infty}^{+\infty} x . f_X(x) . dx \quad \because \mu \text{ finite}$$

$$= \boxed{\int_{-\infty}^{0} x f_X(x) . dx} + \underbrace{\int_{0}^{\infty} x . f_X(x) . dx}_{\boxed{> 0}}$$

$\times$

ans-22) Chebyshev Inequality

If $\varepsilon > 0$

$$P\{|X-c| \geq \varepsilon\} \leq \frac{E((X-c)^2)}{\varepsilon^2}$$

$\Downarrow$ same as

$$1 - P\{|X-c| < \varepsilon\} \leq \frac{E((X-c)^2)}{\varepsilon^2}$$

$$1 - P\{c-\varepsilon < X < c+\varepsilon\} \leq \frac{E((X-c)^2)}{\varepsilon^2}$$

$\therefore$ $c = 3$ $\quad$ $\cancel{E(X)=3}$

$\varepsilon = 5.$ $\quad$ $\cancel{E(X^2)=13}$

$E(X)=3 \qquad E(X^2)=13$

$$\text{var}(X) = E(X^2) - (E(X))^2 = 4$$

Problem Set-3

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The Bernoulli Distribution

X	0	1
$P(X=x)$	$1/2$	$1/2$

There is single trial.

$P(\text{success}) = p$ — $X=1$ is success.
$p(\text{failure}) = 1-p$ — $X=0$ is failure.

Then X has Bernoulli distribution:

$P(X=x) = p^x (1-p)^{1-x}$

Binomial Trials

There are n trials.

Each single Trial $\to$ failure, success

$P(S) = p$ (success)
$P(F) = 1-p$ (failure)

$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$

X — # of success in n trials.

Q. 60 times Head, in 100 trials.

ans:- $P(X=60) = \binom{100}{60} \cdot \left(\frac{1}{2}\right)^{60} \left(\frac{1}{2}\right)^{40}$

$= \boxed{\binom{100}{60}\left(\frac{1}{2}\right)^{100}}$

Q] What is probability in 3 rolls of dice, that 5 comes 2 times?

ans] $\binom{3}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 = \cancel{3}\times\frac{1}{\cancel{36}_{12}}\times\frac{5}{6} = \boxed{\frac{5}{72}}$

Problem. set - 3

ans. 3)

$P(x \geq 1, \; \underbrace{n-x \geq 1}_{\downarrow}) \geq 0.95$

because atleast a girl also.

$P(1 \leq x \leq n-1) \geq 0.95$

$\Downarrow$ (complement it)

$1 - P(x=0) - P(x=n) \geq 0.95$

$1 - \binom{n}{0}\cdot\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^0 - \binom{n}{n}\cdot\left(\frac{1}{2}\right)^n\cdot\left(\frac{1}{2}\right)^0 \geq 0.95$

$1 - \left(\frac{1}{2}\right)^{n-1} \geq 0.95$

$1 - 0.95 \geq \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{32}$ $\frac{5}{100}$

$0.05 \geq \left(\frac{1}{2}\right)^{n-1}$

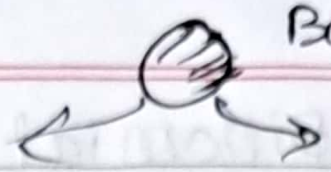
$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

$\frac{\cancel{5}^{1}}{\cancel{100}_{20}} \geq \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{20} \geq \left(\frac{1}{2}\right)^{n-1}$

$\boxed{\frac{1}{20} \geq \left(\frac{1}{32}\right)}$ $\therefore \left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{n-1}$

$\therefore \boxed{n=6}$



Ball.

(6) 18 balls.

$$\underbrace{\boxed{B_1}\quad\boxed{B_2}\quad\boxed{B_3}}_{p=\frac{3}{7}}\quad\underbrace{\boxed{B_4}\quad\boxed{B_5}\quad\boxed{B_6}\quad\boxed{B_7}}_{1-p=\left(\frac{4}{7}\right)}$$

Trials 18 ball

find probab B_1, B_2, B_3 has 6 balls in total.

$$P(X=6)=\boxed{\left({}^{18}C_6\right)\times\left(\frac{3}{7}\right)^6\times\left(\frac{4}{7}\right)^{12}}$$

X