$\boxed{PS-5}$

Q2: Poisson $\to$ discrete $\to$ continuous correction (must!).

$P(X_1 + \dots + X_{10} \geq 15) \simeq ??$

By CLT $\Rightarrow$

$$\frac{(X_1 + \dots + X_{10}) - 10 \times 1}{\sqrt{10} \times \sqrt{1}} \sim N(0,1).$$

$P(X_1 + \dots + X_{10} > 14.5)$

$$\simeq P\left(\frac{X_1 + \dots + X_{10} - 10}{\sqrt{10}} > \frac{14.5 - 10}{\sqrt{10}}\right)$$

$$\simeq P\left(Z > \frac{4.5}{\sqrt{10}}\right) \simeq 1 - P\left(Z \leq \frac{4.5}{\sqrt{10}}\right)$$

$$\simeq 1 - \Phi\left(\frac{4.5}{\sqrt{10}}\right)$$

Problem- Set V

(2) $E(X_i)=1$ & $Var(X_i)=1$

By CLT $\dfrac{(X_1+X_2+\cdots+X_{10})-10\times 1}{\sqrt{10}\times\sqrt{1}} \sim N(0,1)$

i.e. $\dfrac{(X_1+\cdots+X_{10})-10}{\sqrt{10}} \sim N(0,1)$

So

$$P\left((X_1+\cdots+X_{10})\geq 15\right) = P\left(\frac{X_1+\cdots+X_{10}-10}{\sqrt{10}} \geq \frac{5}{\sqrt{10}}\right)$$

$$\simeq P\left(Z\geq \frac{5}{\sqrt{10}}\right)$$

$$= 1-P\left(Z\leq \frac{5}{\sqrt{10}}\right)$$

$$= 1-\Phi\left(\frac{5}{\sqrt{10}}\right)$$

3) $Var(X_i)=\frac{1}{12}$ & $E(X_i)=\frac{1}{2}$

By CLT

$$P\left(\sum_{i=1}^{10} X_i > 7\right) = P\left(\frac{\sum_{i=1}^{10} X_i - 10\times\frac{1}{2}}{\sqrt{10\left(\frac{1}{12}\right)}} > \frac{7-5}{\sqrt{10\times\frac{1}{12}}}\right)$$

$$\simeq P\left(Z > \frac{2}{\sqrt{\frac{10}{12}}}\right)$$

$$= P(Z>2.2) = 1-P(Z\leq 2.2)$$

$$= 1-\Phi(2.2)$$

$$\simeq 0.0139$$

Let X_i — the lifetime of the i-th battery to be put in use.

$p = P(X_1+\cdots+X_{25}>1100)$?

$$p = P\left(\frac{\sum_{i=1}^{25} X_i - 25\times 40}{\sqrt{25}\times 20} > \frac{1100-1000}{20\sqrt{25}}\right)$$

$$\simeq P(Z>1) = 1-\Phi(1)$$

$$\simeq 0.1587$$

(5) $P(X=20) = \binom{40}{20}\left(\frac{1}{2}\right)^{20}\left(\frac{1}{2}\right)^{20}$ X - no. of success $X \sim Bin(40, \frac{1}{2})$

$= \binom{40}{20}\left(\frac{1}{2}\right)^{40} \simeq 0.1268$

Using Normal Approximation

$P(X=20) = P(19.5 < X < 20.5)$ (Continuity correc

$= P\left(\frac{19.5-20}{\sqrt{40\times\frac{1}{2}\times\frac{1}{2}}} < \frac{X-20}{\sqrt{40\times\frac{1}{4}}} < \frac{20.5-20}{\sqrt{40\times\frac{1}{4}}}\right)$

$= P\left(\frac{-0.5}{\sqrt{10}} < \frac{X-20}{\sqrt{10}} < \frac{0.5}{\sqrt{10}}\right)$

$= P\left(-0.16 < \frac{X-20}{\sqrt{10}} < 0.16\right)$

$\simeq P(-0.16 < Z < 0.16)$

$= \Phi(0.16) - \Phi(-0.16)$

$= 2\Phi(0.16) - 1$ $(\because \Phi(-2) = 1 - \Phi(2))$

$\simeq 0.1272$

(6) Let X denote the no. of students that each accepted applicant will independently attend, Then $X \simeq Bin(450, 0.3)$

Our aim is to find $P(X > 150)$.

Using normal approximation, $X \sim N(450\times0.3, 450\times0.3\times0.7)$

$P(X>150) \simeq P(X > 150.5)$ (Continuity correc

$= P\left(\frac{X-135}{\sqrt{94.5}} > \frac{150.5-135}{\sqrt{94.5}}\right)$

$= P\left(Z > \frac{15.5}{9.72}\right)$

$= P(Z > 1.59)$

$= 1 - \Phi(1.59)$

$= 1 - 0.9441$

$= 0.0559 = 0.06$

(7) $X \sim Bin(1700, 0.6)$ Then $E(X) = ...$

$$P(X > 1060) \doteq P\left(\frac{X-1020}{20} > \frac{1060.5-1020}{20}\right) \quad \text{(by continuity correction)}$$

$$= P\left(Z > \frac{40.5}{20}\right)$$

$$= P(Z \geq 2.025)$$

$$= 1 - \Phi(2.025)$$

$$= 1 - \Phi(2.03)$$

$$= 1 - 0.97831 = 0.02169$$

(8) X_i = upper face of die in ith roll

Then the sum is a r.v.

$$X = X_1 + \dots + X_{420}$$

The p.m.f of X_i is

$$p_X(x) = \begin{cases} 1/6 & \text{if } x \in \{1, 2, \dots, 6\} \\ 0 & \text{o.w.} \end{cases} \quad \forall 1 \leq i \leq 420$$

The expectation

$$E(X_i) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = 7/2 \quad \forall 1 \leq i \leq 420$$

$$E(X_i^2) = \frac{1^2+2^2+3^2+4^2+5^2+6^2}{6} = \frac{1}{\cancel{6}} \times \frac{\cancel{6} \times 7 \times 13}{6} = \frac{91}{6} \quad \forall 1 \leq i \leq 420$$

$$Var(X_i) = \frac{91}{6} - \frac{49}{4} = \frac{182-147}{12} = \frac{35}{12}$$

$$E(X) = 420 \times 7/2 = 1470$$

$$\& \ Var(X) = 420 \times \frac{35}{12} = 35 \times 35 = 1225$$

$$\Rightarrow \sqrt{Var(X)} = 35$$

Hence

$$P(1400 \leq X \leq 1550) \simeq P(1399.5 \leq X \leq 1550.5) \quad \text{(by conti. corr.)}$$

$$= P\left(\frac{1399.5-1470}{35} \leq \frac{X-1470}{35} \leq \frac{1550.5-1470}{35}\right)$$

$$= P(-2.01 \leq Z \leq 2.30) = \Phi(2.30) - \Phi(-$$

$$= \Phi(2.30) + \Phi(2.01) - 1$$

$$= 0.98928 + 0.97778 - 1 = 0.96706$$

a 100(1-α) percent confidence interval for ...

(9) X – sum of the weights of 100 packages.

The mean and the variance of the weight of a ~~big~~ single package are

$$\mu = \frac{5+50}{2} = 27.5, \quad \sigma^2 = \frac{(50-5)^2}{12} = 168.75$$

$$P(X > 3000) = P\left(\frac{X - 100\times 27.5}{\sqrt{168.75 \times 100}} > \frac{3000 - 2750}{129.9} = 1.92\right)$$

$$\simeq P(Z > 1.92) \quad \text{(by CLT)}$$

$$= 1 - \Phi(1.92)$$

$$= 1 - 0.9726 = 0.0274$$

(11) (a) The expected value and standard deviation of the proportion that favor the candidate are

$$E(X) = 200 \times (0.45) = 90, \quad \sigma = \sqrt{200(0.45)(0.55)} = 7.0356$$

(b) $X \sim Bin(200, 0.45)$.

$$P(X \geq 101) = P(X \geq 100.5) \quad \text{(by conti. corr.)}$$

$$= P\left(\frac{X-90}{7.0356} \geq \frac{100.5-90}{7.0356}\right) \equiv P(Z \geq 1.4924)$$

$$= 1 - \Phi(1.4924) = 0.0678$$

Scanned with OKEN Scanner