Amplitude Modulation.

Introduction: There are many instances when the baseband signals are incompatible for direct transmission over the medium.

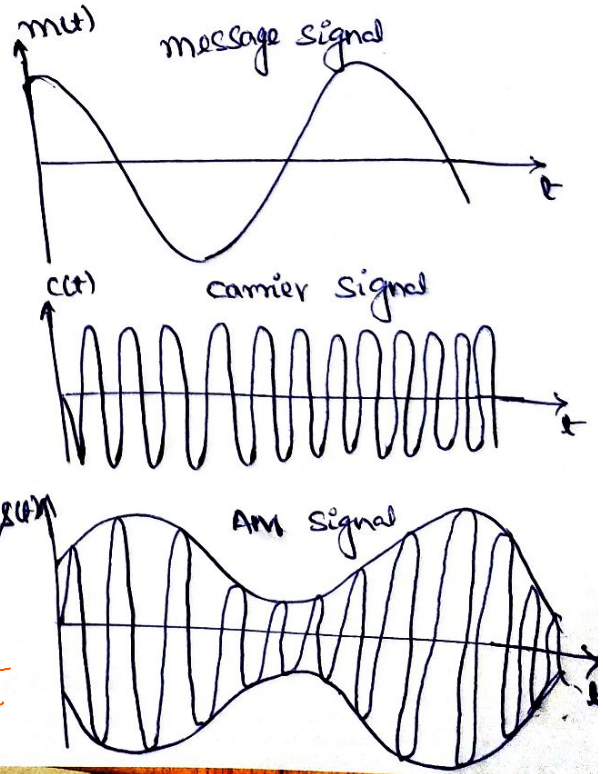
For example; voice signals cannot travel longer distances in air, the signals gets attenuated rapidly. Hence for transmission of baseband signals by radio, modulation technique has to be used.

Modulation Techniques:

* In the modulation process, the baseband signal (such as voice, video etc) modifies another higher frequency signal called the carrier.
  The carrier is usually a sine wave that is higher in frequency than the highest baseband signal frequency.

* Modulation is the process of changing some characteristics (amplitude, frequency and phase) of a carrier wave in accordance with the instantaneous value of the modulating signal.

* There are three types of modulations.
  - i) Amplitude Modulation (AM)
  - ii) Frequency Modulation (FM)
  - iii) Phase Modulation. (PM)



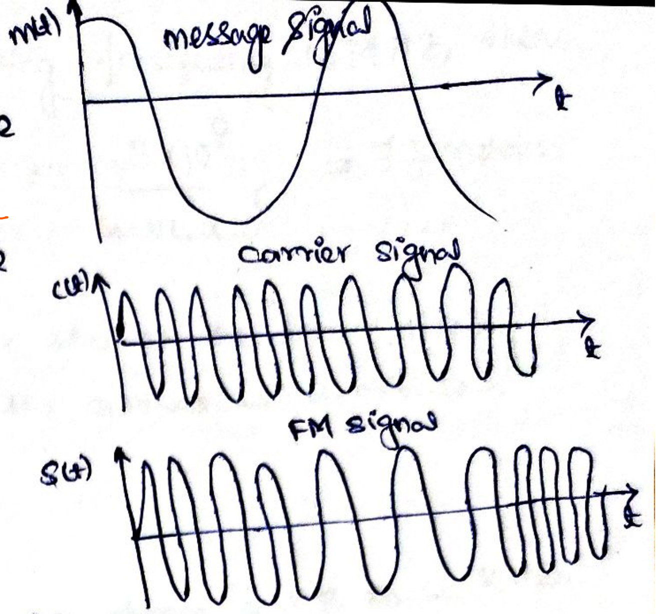
i) **Amplitude Modulation:**
Amplitude Modulation is defined as the modulation in which the amplitude of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its carrier frequency and phase constant.

W_c & ϕ_c → Constant

ii> Frequency Modulation:

Frequency modulation is defined as the modulation in which the frequency of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its carrier amplitude and phase constant.

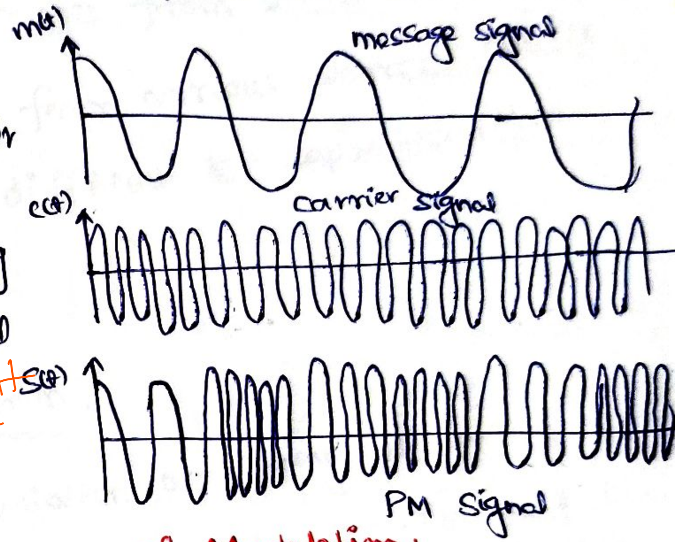
A_c & ϕ_c → Constant



iii> Phase Modulation:

Phase Modulation is defined as the modulation in which the phase of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal, keeping its carrier amplitude and frequency constant.

A_c & W_c → Constant



Need For Modulation or Advantages of Modulation:

The advantages of Modulation are:

i) Reduces the height of antenna:

The minimum height of antenna is a function of wavelength "λ".

$\frac{\lambda}{4}$ = height of antenna

height of antenna is given by $\lambda/4$.

$\therefore \boxed{\lambda = \frac{c}{f}}$

i.e.; height of antenna $= \boxed{\lambda/4 = \frac{c}{4f}}$

where, $\lambda = \frac{c}{f}$

$c = 3\times10^8$ m/sec, velocity of light

f = transmitted frequency

For example; if we choose transmitted frequency $f = 10$ kHz then,

Height of antenna $= \lambda/4 = \frac{c}{4f} = \frac{3\times10^8}{4\times10\times10^3} = 7500 = 7.5$ km

Height of antenna = 7.5 km = 7500 meters

Similarly, if we choose transmitting frequency 10 MHz, then,

$$\text{Height of antenna} = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3\times10^8}{4\times10\times10^6} = 7.5 \text{ meters}$$

From the above two examples, it is clear that as the transmitting frequency is increased, height of the antenna is decreased.

ii) **Avoid mixing of signals:**

All audio (message) signals ranges from 20 Hz to 20 kHz. 20 Hz to 20 kHz

The transmission of message signals from various sources causes mixing of signals and then it is difficult to separate these signals at the receiver end.

iii) **Increases the range of communication:**

* Low frequency signals have poor radiation retention and they get highly attenuated. Therefore baseband signals cannot be transmitted directly over long distances.

* Modulation increases the frequency of the signal and thus they can be transmitted over long distances.

iv) **Allows multiplexing of signals:**

Modulation allows multiplexing of signals.

For example, number of TV channels operating simultaneously.

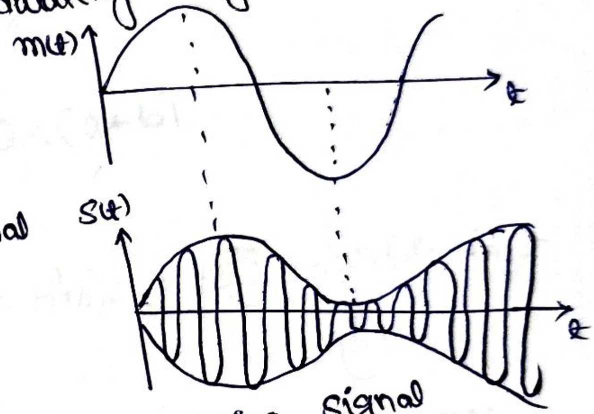
v) **Allows adjustments in the bandwidth:**

Bandwidth of a modulated signal may be made smaller or larger vi> Improves quality of reception :-

Modulation techniques like frequency modulation, pulse code Modulation reduces the effect of noise to great extent. Therefore, reduction of noise, improves the quality of reception.

* Amplitude Modulation : Time & Frequency Domain Approach:

Amplitude Modulation is defined as the modulation in which the amplitude of the carrier wave is varied in accordance with the instantaneous amplitude of the modulating signal keeping its carrier frequency and phase constant.



* The instantaneous value of modulating signal is given by.

$m(t) = A_m \cos(2\pi f_m t)$ — ①

$m(t) = A_m \cos(2\pi f_m t)$

Where, $A_m \rightarrow$ maximum amplitude of modulating signal
$f_m \rightarrow$ frequency of modulating signal.

* The instantaneous value of carrier signal is given by,

$c(t) = A_c \cos(2\pi f_c t)$ — ②

Where, $A_c \rightarrow$ Maximum amplitude of carrier signal
$f_c \rightarrow$ frequency of carrier signal

The standard equation for AM wave is given by

$s(t) = A_c[1 + K_a m(t)] \cos(2\pi f_c t)$ — ③

Where, K_a is a constant called the amplitude sensitivity of the modulator.

$K_a \rightarrow$ Amplitude sensitivity

Substituting eq ① in eq ③, we get

$$s(t) = A_c\,(1 + k_a A_m \cos(2\pi f_m t))\cos(2\pi f_c t)$$

$$= A_c\,(1 + \mu \cos(2\pi f_m t))\cos(2\pi f_c t)$$

$\mu = k_a A_m = \frac{A_m}{A_c}$ → modulation index

Where, $\mu = k_a A_m$ is called the modulation index.

$$\therefore\ s(t) = A_c \cos(2\pi f_c t) + A_c \mu \cos(2\pi f_m t)\cos(2\pi f_c t) \quad -④$$

Eq → ④ can be further expanded, by means of the trignometrical equation.

** $$\boxed{\cos a \cdot \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)}$$

$$\therefore\ s(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2}\cos 2\pi(f_c - f_m)t + \frac{\mu A_c}{2}\cos 2\pi(f_c + f_m)t \quad -⑤$$

Eq → ⑤ is the amplitude modulated signal, consists of three frequency components

f_c → center frequency or carrier frequency

$f_c - f_m$ → Lower Side band (LSB)

$f_c + f_m$ → Upper Side Band (USB)

$f_c - f_m$ → LSB

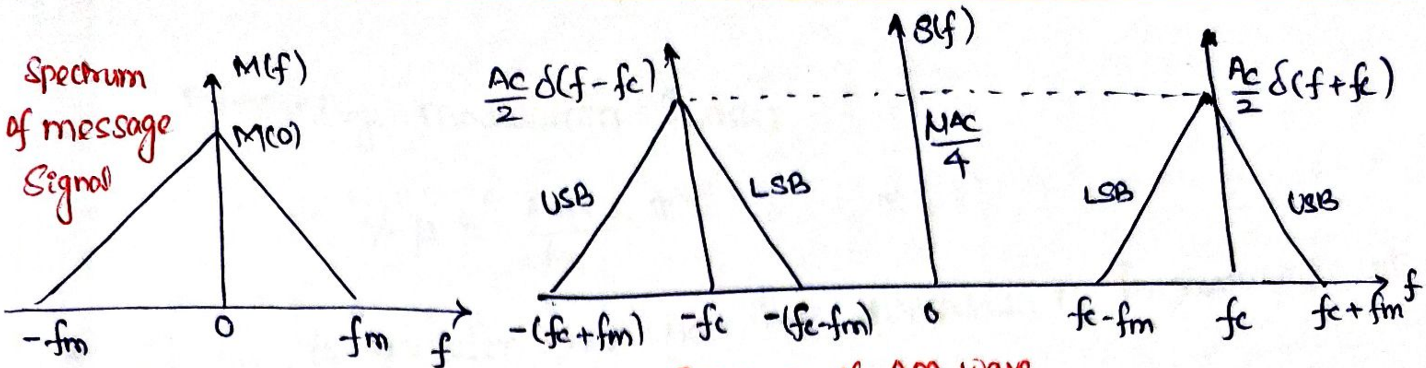
$f_c + f_m$ = USB

Taking Fourier Transforms on both sides of Eq^n → ⑤, we get,

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{\mu A_c}{4}\left[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))\right] + \frac{\mu A_c}{4}\left[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))\right]$$

** $$A_c \cos(2\pi f_c t) \xrightarrow{F.T} \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

f → Sampling frequency



Spectrum of AM wave

* The amplitude spectrum of the AM wave has two sidebands on either sides of $\pm f_c$

* For +ve frequencies, the highest frequency component of the AM wave equals $(f_c + f_m)$, called "Upper Side band f_{USB}" and the lowest frequency component equals $(f_c - f_m)$, called "Lower Sideband f_{LSB}".

Transmission Bandwidth : (B_T) :

The difference between Upper Sideband and Lower Sideband frequencies defines the transmission band width "B_T".

$$B_T = f_{USB} - f_{LSB} = (f_c + f_m) - (f_c - f_m) = 2f_m$$

$$= (f_c + f_m) - (f_c - f_m)$$

$$= f_c + f_m - f_c + f_m = 2f_m$$

***

$$\boxed{B_T = 2f_m}$$

$\therefore$ Bandwidth required for transmission of AM wave is twice the modulating signal frequencies i.e. $2f_m$

Modulation Index And Percentage Modulation Index : (μ) :

The ratio of change in amplitude of modulating signal to the amplitude of carrier wave is known as Modulation Index or Modulation factor.

$$\boxed{\mu = K_a A_m} \quad \text{or} \quad \boxed{\mu = \frac{A_m}{A_c}}$$

modulation index

Percentage modulation Index;

$A_m > A_c \Rightarrow$ Distortion happen

** $$\boxed{\% \mu = \frac{A_m}{A_c} \times 100}$$

* If A_m is greater than A_c then ** distortion is introduced into the system.

* The modulating signal voltage "A_m" must be less than carrier signal voltage "A_c" for proper amplitude modulation.

Transmission Efficiency of an AM wave : (η):

Transmission Efficiency is defined as the ratio of the total transmitted power power carried by the sidebands to the total transmitted power is called transmission efficiency 'η' and is given by,

** $$P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$$

** $$\boxed{\eta = \frac{P_{USB} + P_{LSB}}{P_T}}$$

Let $R = 1\Omega$

$$P_T = P_c\left(1+\frac{\mu^2}{2}\right)$$

$$P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}$$

$$\therefore \boxed{P_T = P_c\left(1+\frac{\mu^2}{2}\right)}$$

** $$P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8R}$$

R=1

$$\eta = \frac{\frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}}{P_c(1+\mu^2/2)} = \frac{\frac{\cancel{2}\,\mu^2 A_c^2}{8R\,4}}{P_c(1+\mu^2/2)} \qquad P_c = \frac{A_c^2}{2R}$$

$$= \frac{\frac{\mu^2}{2}\left(\frac{A_c^2}{2R}\right)}{P_c(1+\mu^2/2)} = \frac{\frac{\mu^2}{2}\cancel{P_c}}{\cancel{P_c}\left(1+\frac{\mu^2}{2}\right)} = \frac{\mu^2/2}{1+\frac{\mu^2}{2}} = \frac{\mu^2/2}{\frac{2+\mu^2}{2}}$$

*** $$\boxed{\therefore \eta = \frac{\mu^2}{2+\mu^2}}$$

Transmission efficiency

*** $$\boxed{\eta\% = \frac{\mu^2}{2+\mu^2} \times 100}$$

Modulation Index Expression Using AM wave :

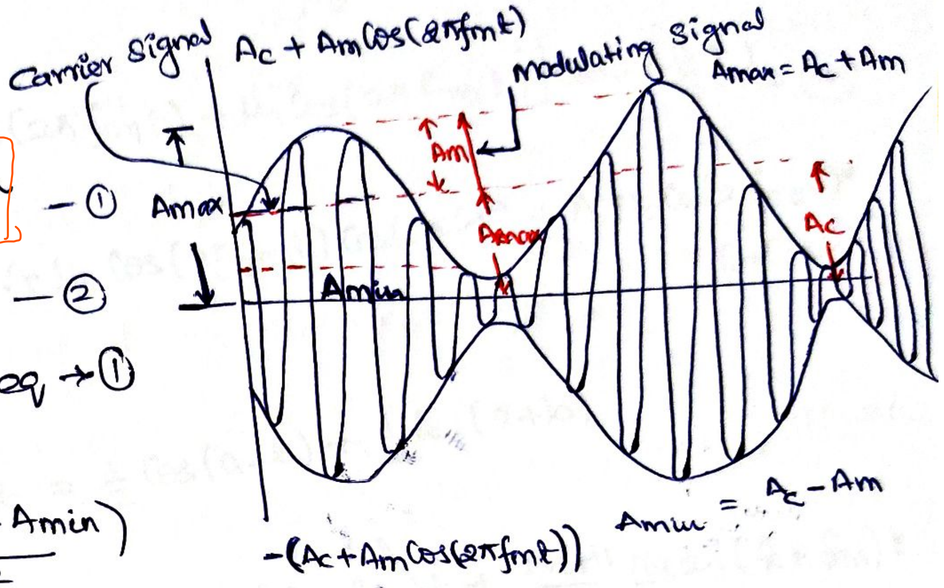
We can calculate the modulation index from the amplitude modulated wave,

$$\mu = \frac{A_m}{A_c}$$

from figure,

$$\boxed{A_m = \frac{A_{max} - A_{min}}{2}} \quad \text{—} ①$$

and $A_c = A_{max} - A_m$ — ②



Amplitude Modulated Wave

Substituting eq → ② in eq → ①

we get,

$$A_c = A_{max} - \left(\frac{A_{max} - A_{min}}{2}\right)$$

$$= \frac{2A_{max} - A_{max} + A_{min}}{2}$$

$$\therefore \; A_c = \frac{A_{max} + A_{min}}{2}$$

This implies, $\mu = \frac{A_m}{A_c} = \frac{\left(\frac{A_{max} - A_{min}}{2}\right)}{\left(\frac{A_{max} + A_{min}}{2}\right)} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}}$$

Expression for Multitone Amplitude Modulation:-

Consider two modulating signals

$m_1(t) = A_{m1} \cos 2\pi f_{m1} t$ — ①

$m_2(t) = A_{m2} \cos 2\pi f_{m2} t$ — ②

In General, an amplitude modulated wave is expressed as,

$$\boxed{s(t) = A_c\left(1 + k_a m(t)\right) \cos(2\pi f_c t)} \quad \text{—} ③$$

$m(t) = m_1(t) + m_2(t)$

Now substitute eq → ① and eq → ② in eq → ③, we get,

$$\therefore\ s(t) = A_c\left(1 + k_a\left(m_1(t) + m_2(t)\right)\right)\cos(2\pi f_c t)$$

$$= A_c\left(1 + k_a\left(A_{m1}\cos(2\pi f_{m1} t) + A_{m2}\cos(2\pi f_{m2} t)\right)\right)\cos(2\pi f_c t)$$

$$= A_c\left[1 + \underbrace{k_a A_{m1}}_{\mu_1}\cos(2\pi f_{m1} t) + \underbrace{k_a A_{m2}}_{\mu_2}\cos(2\pi f_{m2} t)\right]\cos(2\pi f_c t)$$

$$\therefore\ s(t) = A_c\left[1 + \mu_1\cos(2\pi f_{m1} t) + \mu_2\cos(2\pi f_{m2} t)\right]\cos(2\pi f_c t) \quad \text{--- (4)}$$

$$s(t) = A_c\cos(2\pi f_c t) + A_c\mu_1\cos(2\pi f_{m1} t)\cos(2\pi f_c t) + A_c\mu_2\cos(2\pi f_{m2} t)\cos(2\pi f_c t)$$

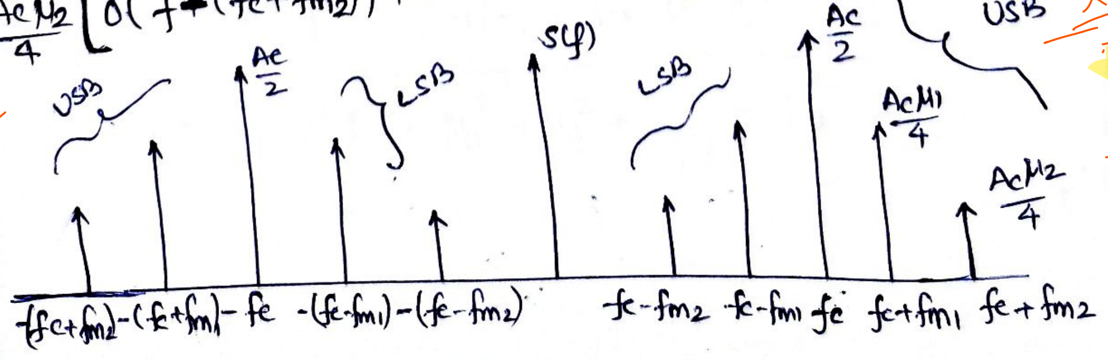
As we know, $\cos a \cdot \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)$

$$\therefore\ \boxed{s(t) = A_c\cos(2\pi f_c t) + \frac{A_c\mu_1}{2}\cos 2\pi(f_c - f_{m1})t + \frac{A_c\mu_1}{2}\cos 2\pi(f_c + f_{m1})t + \frac{A_c\mu_2}{2}\cos 2\pi(f_c - f_{m2})t + \frac{A_c\mu_2}{2}\cos(2\pi(f_c + f_{m2})t)} \quad \text{--- (5)}$$

From eq → (5), it is clear that, when we have two modulating frequencies, we get four additional frequencies, two upper side bands (USB), $f_c + f_{m1}$, $f_c + f_{m2}$. and two lower sidebands (LSB), $f_c - f_{m1}$, $f_c - f_{m2}$.

Applying Fourier Transform to eq → (5) we get,

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{A_c\mu_1}{4}\left[\delta(f - (f_c - f_{m1})) + \delta(f + (f_c - f_{m1}))\right] + \frac{A_c\mu_1}{4}\left[\delta(f - (f_c + f_{m1})) + \delta(f + (f_c + f_{m1}))\right] + \frac{\mu_2 A_c}{4}\left[\delta(f - (f_c - f_{m2})) + \delta(f + (f_c - f_{m2}))\right] + \frac{A_c\mu_2}{4}\left[\delta(f - (f_c + f_{m2})) + \delta(f + (f_c + f_{m2}))\right] \quad \text{--- (6)}$$



** Spectrum for Multitone Amplitude Modulation

Expression for Total transmitted Power and total Modulation Index for Multitone Amplitude Modulation :-

The total power in the amplitude modulated wave is calculated as follows:

**
$$\boxed{P_T = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}}$$

we know that, ** $\boxed{P_c = \frac{(A_c/\sqrt{2})^2}{R} = \frac{A_c^2}{2R}}$

$$\left(\frac{\mu_1}{2}\right)^2 P_c = P_{USB1} = P_{USB2} = P_{LSB1} = P_{LSB2} \Rightarrow \frac{A_c^2\mu_1^2}{8R} = \frac{A_c^2\mu_1^2}{8R} = \frac{A_c^2\mu_2^2}{8R} = \frac{A_c^2\mu_2^2}{8R}$$

$$\Rightarrow \frac{\mu_1^2}{4}(P_c)$$

$$\therefore P_T = \frac{A_c^2}{2R} + \frac{A_c^2\mu_1^2}{8R} + \frac{A_c^2\mu_2^2}{8R} + \frac{A_c^2\mu_1^2}{8R} + \frac{A_c^2\mu_2^2}{8R}$$

$$= \frac{A_c^2}{2R} + \cancel{2}\frac{A_c^2\mu_1^2}{\cancel{8}R} + \cancel{2}\frac{A_c^2\mu_2^2}{\cancel{8}R} \qquad \because P_c = \frac{A_c^2}{2R}$$

$$= \frac{A_c^2}{2R} + \frac{A_c^2\mu_1^2}{4R} + \frac{A_c^2\mu_2^2}{4R} = \frac{A_c^2}{2R}\left[1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right]$$

$$P_T = P_c\left(1 + \frac{\mu_t^2}{2}\right)$$

**
$$\boxed{P_T = P_c\left(1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right)} = P_c\left(1 + \frac{\mu_t^2}{2}\right)$$

where $\frac{\mu_t^2}{2} = \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2} \Rightarrow \boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2}}$

In general, Total modulation index is given by

***
$$\boxed{\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2 + \cdots + \mu_n^2}}$$

Generation of Amplitude Modulation wave :

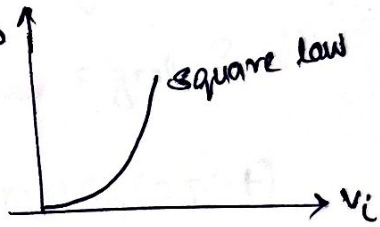
The device which is used to generate an amplitude modulated wave is known as Amplitude modulator

There are two important methods of AM generation for low power applications:

i) Switching Modulator
ii) Square Law Modulator

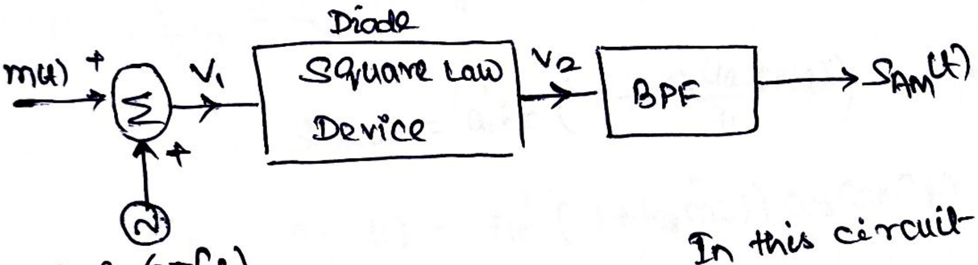
Square Law Modulator : When the output of a device is not directly proportional to input throughout the operation, the device is said to be non-linear.

The input-output relation of a non-linear device can be expressed as



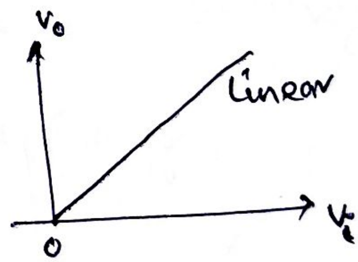
$$V_o = a_1 V_i + a_2 V_i^2 + a_3 V_i^3 + \cdots$$

$$* \; V_o = a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \cdots$$



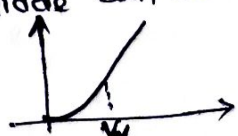
$A_c \cos(2\pi f_c t)$
↳ Local oscillator

In this circuit a square law device like diode is used therefore it is known as square law modulator



Diode characteristics
At low voltages : Square law characteristics
At high voltages : Linear characteristics

The strength of practical message signal is much small and if we generate carrier with little strength the diode can be operate in square law characteristic region.



Now consider the sum of message signal and carrier signal is represented by V_1 and its peak voltage is near to V_γ

$\boxed{V_1 = m(t) + c(t)}$

$= m(t) + A_c \cos(2\pi f_c t)$ — ①

The output of square law device is given by

$V_2 = a_1 v_1 + a_2 v_1^2 + a_3 v_1^3 \dots$

$$V_2 = a_1 V_1 + a_2 V_1^2 + a_3 V_1^3 + \dots$$

consider only first two terms

$$V_2 = a_1 V_1 + a_2 V_1^2$$

$$= a_1 (m(t) + A_c \cos(2\pi f_c t)) + a_2 (m(t) + A_c \cos(2\pi f_c t))^2$$

To avoid overlapping of undesired signals

$$= a_1 (m(t) + A_c \cos(2\pi f_c t)) + a_2 (m^2(t) + A_c^2 \cos^2(2\pi f_c t) + 2 A_c m(t) \cos(2\pi f_c t)) \quad \text{— ②}$$

× ✓ × ×

can't travell alone

The output of diode (i.e. V_2) is applied through BPF

The BPF output will be the final modulated signal $S_{AM}(t)$

$$(BPF)_{o/p} = S_{AM}(t) = a_1 A_c \cos(2\pi f_c t) + 2 a_2 A_c m(t) \cos(2\pi f_c t)$$

$$S_{AM}(t) = a_1 A_c \left(1 + \frac{2 a_2}{a_1} m(t)\right) \cos(2\pi f_c t)$$

comparing with standard AM signal

i.e, $S_{AM}(t) = A_c' (1 + k_a m(t)) \cos(2\pi f_c t)$

where $A_c' = a_1 A_c$

$K_a = \frac{2 a_2}{a_1}$

$k_a = \frac{2 a_2}{a_1}$ Amplitude sensitivity

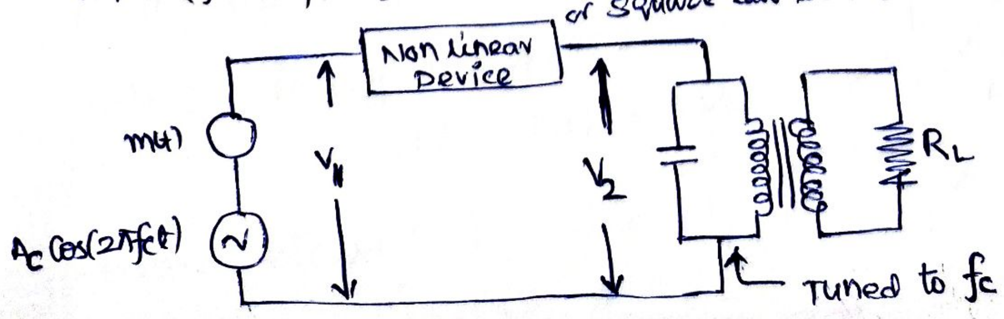
Therefore, modulation index of the output signal is given by

**

$$\boxed{\mu = 2 \frac{a_2}{a_1} A_m}$$

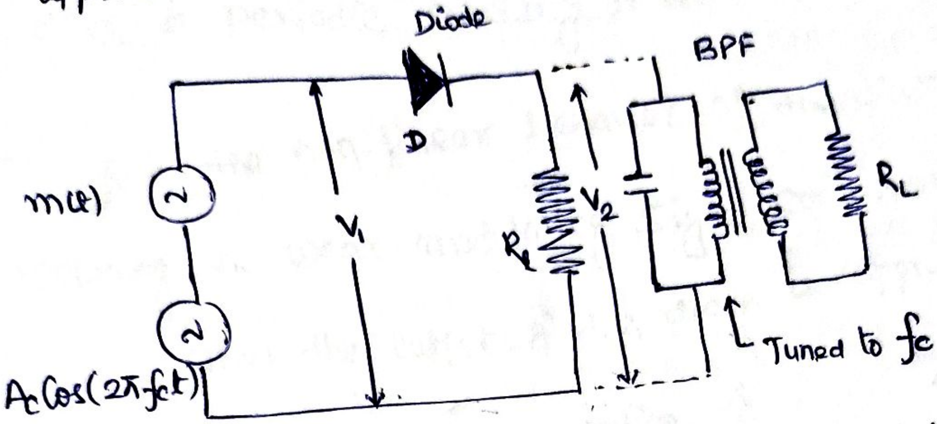
The output AM signal is free from distortion and attenuation only when $(f_c - f_m) > 2 f_m$ or $f_c \gg 3 f_m$

$f_c \gg 3 f_m$



Switching Modulator:

Consider a semiconductor diode used as an ideal switch to which a carrier wave c(t) and an message signal m(t) are simultaneously applied.



It is assumed that the carrier wave c(t) applied to the diode is Large in amplitude.

The total input V_1 for the diode at any instant is given by

$$V_1 = m(t) + c(t)$$
$$= m(t) + A_c Cos(2\pi f_c t) \quad — ①$$

Where $|m(t)| << A_c$

The output of the diode is $V_2 = \begin{cases} V_1, & c(t) > 0 \\ 0, & c(t) < 0 \end{cases}$

i.e., the output of the diode varies between 0 and V_1 at a rate equal to carrier frequency $T_0 = \frac{1}{f_c}$

When the peak amplitude of c(t) is maintained more than that of information signal, the operation is assumed to be dependent on only c(t) irrespective of m(t).

When c(t) is +ve, $V_2 = V_1$ Since the diode is forward biased.

Similarly, when $c(t)$ is $-ve$, $V_2 = 0$ Since diode is reverse biased.

Based upon above operation, switching response of the diode is periodic rectangular wave with an amplitude unity.

Therefore, the non-linear behavior of the diode can be replaced by assuming the weak modulating signal compared with the carrier wave.

Thus the output of the diode is approximately equivalent to linear-time varying operation.

Mathematically, the output of the diode can be written as,

$$V_2 = V_1 \cdot g_p(t) \quad \text{—} ②$$

Substitute eq → ① in eq → ② we get,

$$V_2 = [m(t) + A_c \cos(2\pi f_c t)]\, g_p(t). \quad \text{—} ③$$

Where $g_p(t)$ is a rectangular pulse train with a period equal to $T_0 = \frac{1}{f_c}$

Representing $g_p(t)$ by its Fourier series, we have

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c (2n-1) t) \quad \text{—} ④$$

For $n=1$, then eq → ④ becomes,

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \frac{(-1)^{1-1}}{2(1)-1} \cos(2\pi f_c (2(1)-1)) + \text{odd harmonic} \quad \text{—} ⑤$$

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos(2\pi f_c (2n-1) t)$$

Substituting eq→ 5 in eq→ 3, we get,

$$V_2 = [m(t) + A_c \cos(2\pi f_c t)]\left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) - \frac{2}{3\pi}\cos(6\pi f_c t) + \cdots\right]$$

$$V_2 = \frac{1}{2}m(t) + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t) + \frac{2A_c}{\pi}\cos^2(2\pi f_c t) + \cdots$$

As we know that, $\cos^2\theta = \frac{1+\cos 2\theta}{2}$ $\therefore \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos 2\theta.$

$$V_2 = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t) + \frac{2A_c}{\pi}\left[\frac{1}{2} + \frac{\cos(2\pi f_c t)}{2}\right] + \cdots$$

$$V_2 = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t) + \frac{2A_c}{2\pi} + \frac{2A_c}{\pi}\cdot\frac{\cos(2\pi f_c t)}{2} + \cdots$$

$$\therefore V_2 = \frac{m(t)}{2} + \frac{2m(t)}{\pi}\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t) + \frac{A_c}{\pi} + A_c\cos(4\pi f_c t) + \cdots \quad \text{—} (6)$$

The required AM wave centered at "f_c" is obtained by passing V_2 through an ideal BPF having centre frequency f_c and $B_T = 2f_m$ Hz.

$\therefore$ The output of BPF is

$$V_2' = \frac{2}{\pi}m(t)\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t)$$

$$= \frac{A_c}{2}\left[1 + \frac{2}{\pi}\cdot\frac{2}{A_c}\cdot m(t)\right]\cos(2\pi f_c t)$$

$$V_2' = \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c}m(t)\right]\cos(2\pi f_c t)$$

Comparing with Standard AM wave

Where $k_a = \frac{4}{\pi A_c}$ is called Amplitude Sensitivity

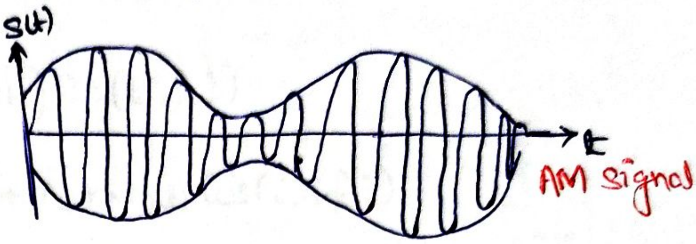
$\therefore$ Therefore, the modulation index of the output signal is given by

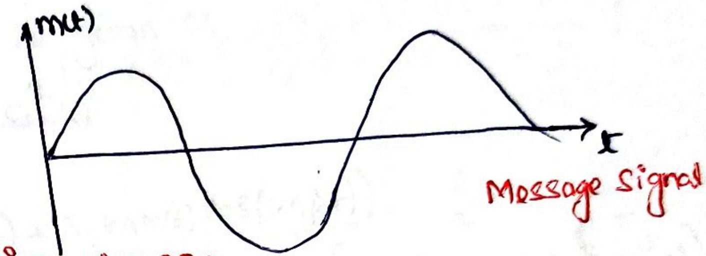
$$\boxed{k_a = \frac{4 A_m}{\pi A_c} = \mu}$$

Demodulation or Detection :

Demodulation or Detection is the process of recovering the original message signal from the modulated wave at the receiver.

Demodulation is the inverse of the modulation process.





Detection of Amplitude Modulation Waves:

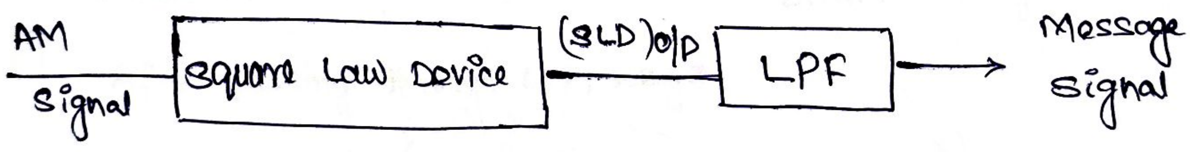
There are three methods of detection of Amplitude Modulation.

i) Square Law demodulator
ii) Envelope detector
iii) Synchronous detector

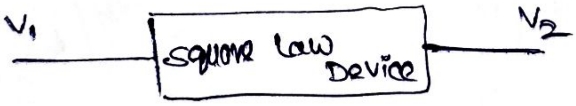
} The most commonly used AM Demodulators

Square Law Demodulator :

low pass filter- filter out carrier frequency



As we know that AM signal : $A_c(1+k_a m(t))\cos(2\pi f_c t)$



The relation between input and output of a square law device is given as

$$V_2 = a_1 V_1 + a_2 V_1^2 + a_3 V_1^3 + \dots + a_n V_1^n$$

where a_1, a_2, a_3 are square law constants.

The characteristics of LPF should be such that it allows the message signal to pass through it.

f_m is the maximum frequency present in the message signal.

Output of square law device

$$S_{AM}(t) = A_c(1+k_a m(t))\cos(2\pi f_c t)$$

$$= A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t)$$

Output of square law device may be given as

$$(SLD)o/p = a_1 S_{AM}(t) + a_2 S_{AM}^2(t)$$

$$= a_1(A_c\cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t))$$

$$+ a_2(A_c^2\cos^2(2\pi f_c t) + A_c^2 k_a^2 m^2(t)\cos^2(2\pi f_c t) + 2A_c^2 k_a m(t)\cos^2(2\pi f_c t))$$

$$(SLD)o/p = a_1 A_c\cos(2\pi f_c t) + a_1 A_c k_a m(t)\cos(2\pi f_c t) + \frac{a_2 A_c^2}{2}(1+\cos(4\pi f_c t))$$

$$+ \frac{a_2 A_c^2 k_a^2 m^2(t)}{2}(1+\cos(4\pi f_c t)) + \frac{2a_2 A_c^2 k_a m(t)}{2}(1+\cos(4\pi f_c t))$$

Eliminated by LPF

After passing through LPF, we get

$$(SLD)o/p = \frac{a_2 A_c^2 k_a^2 m^2(t)}{2} + a_2 A_c^2 k_a m(t)$$

Unwanted signal / Noise

Desired message signal

Signal to Noise Ratio (SNR):

SNR is defined as the ratio of the strength of the signal carrying information (S) to that of unwanted interference (N). Therefore,

SNR = S/N.

* If SNR >> 1, the message signal can be perfectly reconstructed by the filter.

* If SNR << 1, the message signal cannot be reconstructed back perfectly.

$$\frac{S}{N} = \frac{a_2 \cdot A_c^2 \cdot k_a \cdot m(t)}{\left(\frac{a_2 A_c^2 k_a^2 \cdot m^2(t)}{2}\right)} = \frac{2}{k_a m(t)}$$

Let the message signal, $m(t) = A_m \cos(2\pi f_m t)$

$$\frac{S}{N} = \frac{2}{k_a A_m \cos(2\pi f_m t)} = \frac{2}{\mu \cos(2\pi f_m t)}$$

** $\frac{S}{N}$ = Signal to Noise Ratio $\propto \frac{1}{\mu}$

Drawback of Square Law Demodulator :

* As we have seen, the signal to Noise Ratio at the input of a LPF is inversely proportional to the modulation Index (μ):

* Higher the modulation Index, lower is the Signal to Noise Ratio

* To increase the SNR, Modulation Index has to be decreased, which decreases the modulation efficiency and therefore, we get unefficient power distribution

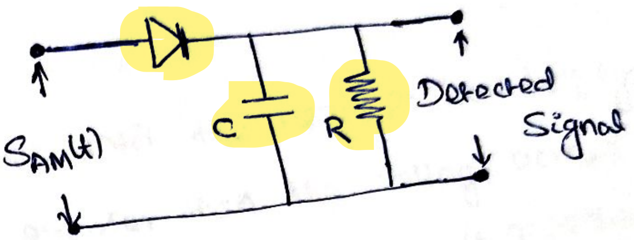
* This method is practically not preffered for AM demodulation.

Envelope Detector

* The limitation of the Square Law Demodulator can be overcome by using simple and economical Envelope Detector.

* It consists of a diode and an RC circuit

An envelope detector is a simple and highly effective device that is well suited for the demodulation of narrowband AM wave.

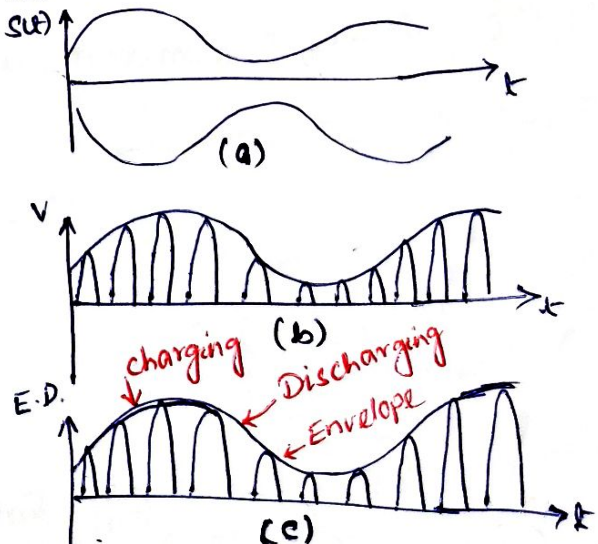


* In the envelope detector, the output of the detector follows the envelope of the modulated signal, hence the name.

In the Figure, the envelope detector consists of a diode and a resistor-capacitor filter. This circuit is also known as diode detector.

* In the positive half cycle of the AM signal, Diode conducts and current flows through "R" whereas, in the Negative half cycle, diode is reverse biased and no current flows through R. As a result only positive half of the AM wave appears across RC. as shown in Fig (b)

* Let us see, how RC filter responses to this positive half of AM Wave.



On the positive half cycle, the diode is forward biased and the capacitor "C" charges up rapidly to the peak value of the input signal.

When the input signal falls below this value, the diode becomes reverse biased and the capacitor "C" discharges slowly through the load resistor R_L.

* This discharging process continues until the next positive half cycle. When the input signal becomes greater than the voltage across capacitor, the diode conducts again and the process is repeated.

Selection of RC time constant:

The charging time constant "$R_S C$" should be short as compared to the carrier period $1/f_c$. therefore, $R_S C << \frac{1}{f_c}$. so capacitor 'C' charges rapidly.

$1/f_c$

* On the otherhand, the discharging time constant "$R_L C$" should be long enough to ensure that the capacitor discharges slowly through the load resistance "R_L" between the positive peak of the carrier wave i.e., $\frac{1}{f_c} << R_L C << \frac{1}{f_m}$

where f_m is the maximum modulating frequency.

Envelope Detector input

$A_m \cos(2\pi f_m t)$

$A_c(1+k_a m(t)) \cos(2\pi f_c t)$

$A\cos(2\pi f_c t) + B\sin(2\pi f_c t)$

$\sqrt{A^2+B^2}\cos(2\pi f_c t + \tan^{-1}(B/A))$

Envelope Detector output

output-amplitude of input

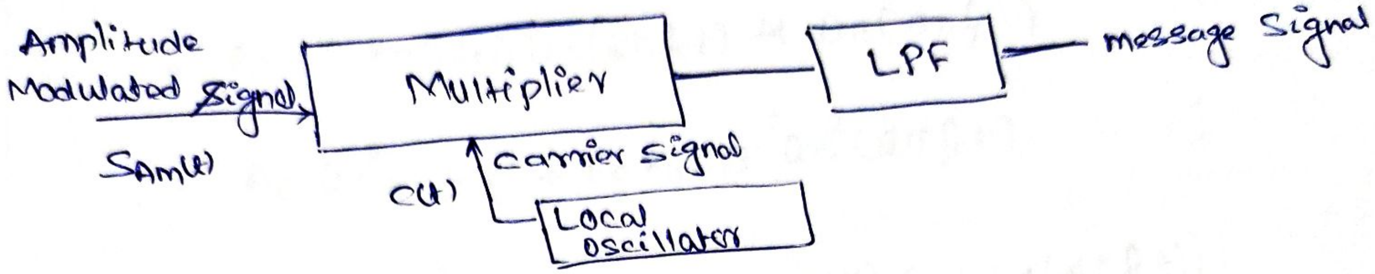
A_m

$A_c(1+k_a m(t))$

$\sqrt{A^2+B^2}$

Synchronous Detector

* Unlike envelope detector, Synchronous detector can be used for demodulation of under-modulated as well as over-modulated signal.



* It is also known as Coherent detector:

For the perfect reconstruction of message signal, the output of local oscillator should be perfectly synchronized in both frequency as well as phase with that of the carrier signal.

* Frequency synchronization can be easily achieved. However, achieving of phase synchronization is very complex and difficult to achieve.

* To achieve Phase Synchronization, additional circuitry has to be added which makes the synchronous detector very complex

AM signal $S_{AM}(t) = A_c(1+k_a m(t))\cos(2\pi f_c t)$

Local Oscillator output $c(t) = \cos(2\pi f_c t)$

The frequency and phase of the local oscillator output and the amplitude modulated signal are both in synchronization.

* The output of the multiplier can be written as:

$$= A_c\,(1+k_a m(t))\cos(2\pi f_c t) * \cos(2\pi f_c t)$$

$$= A_c \cos^2 2\pi f_c t + A_c k_a m(t) \cos^2(2\pi f_c t)$$

$$= \frac{A_c}{2}(1+\cos(4\pi f_c t)) + \frac{A_c k_a m(t)}{2}(1+\cos(4\pi f_c t))$$

$$= \frac{A_c}{2} + \frac{A_c}{2}\cos(4\pi f_c t) + \frac{A_c k_a m(t)}{2} + \frac{A_c k_a m(t)}{2}\cdot\cos(4\pi f_c t)$$

$$(\text{Mul})\,o/p = \frac{A_c}{2} + \frac{A_c}{2}\cos(4\pi f_c t) + \frac{A_c k_a m(t)}{2} + \frac{A_c k_a m(t)}{2}\cos(4\pi f_c t)$$

DC Term: can be blocked by a capacitor

Eliminated by LPF

* The output of synchronous detector is $\frac{A_c k_a m(t)}{2}$

The output of synchronous detector is the message signal with a different amplitude.

The desired amplitude can be achieved by passing the output through an amplifier.

Double Sideband Suppressed Carrier (DSB-SC) Modulation:

Introduction:

* To overcome the drawback of power wastage in AM wave an DSB-SC method is used.

* DSB-SC is a method of transmission where only the two sidebands are transmitted without the carrier

or

The conventional AM wave in which the carrier is suppressed is called DSB-SC modulation.

Time Domain Representation & Frequency Domain approach of DSB-SC wave : OR Single tone information :

Let $m(t)$ be the modulating signal having a bandwidth equal to "f_m" Hz and $c(t) = A_c \cos(2\pi f_c t)$ represents the carrier, then the time domain expression for DSB-SC wave is,

$$s(t) = m(t) \cdot c(t).$$

$$s(t) = A_c \cos(2\pi f_c t) \cdot m(t)$$

Now Substitute $m(t) = A_m \cos(2\pi f_m t)$, then

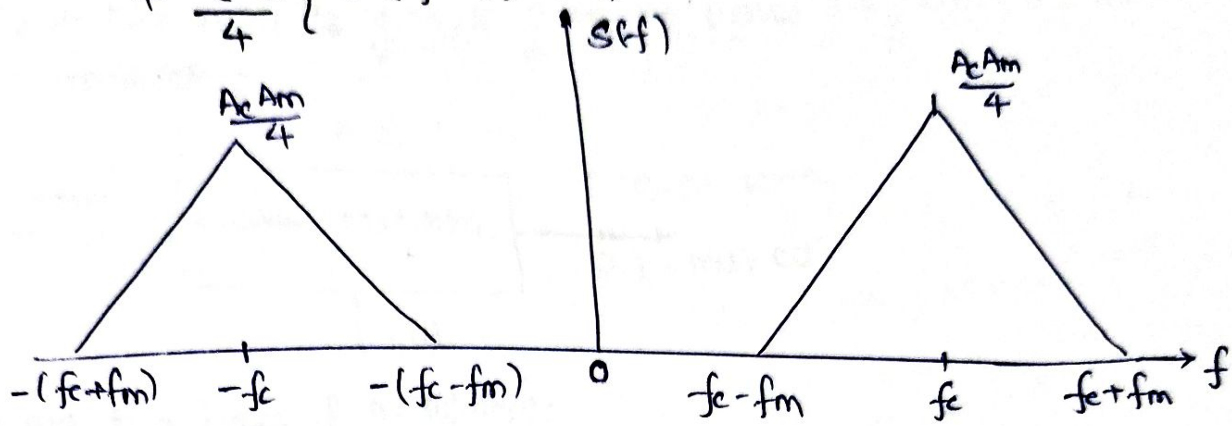
$$s(t) = A_c \cos(2\pi f_c t) \cdot A_m \cos(2\pi f_m t)$$

$$\therefore s(t) = \frac{A_c A_m}{2} \cos(2\pi (f_c - f_m)t) + \frac{A_c A_m}{2} \cos(2\pi (f_c + f_m) t)$$

$$\because \cos a \cdot \cos b = \frac{\cos(a-b) + \cos(a+b)}{2}$$

Taking Fourier Transform on both Sides, we have

$$S(f) = \frac{A_c A_m}{4}\left\{\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))\right\} + \frac{A_c A_m}{4}\left\{\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right\}$$



Spectrum of a DSB-SC wave

The figure shows amplitude spectrum of a DSB-SC signal. We observe that either side of "f_c", we have lower and upper sideband also the carrier term is suppressed in the spectrum as there are no impulses at "f_c"

The minimum transmission bandwidth in DSB-SC wave is "$2f_m$".

Note: * The signal $s(t)$ undergoes a phase reversal whenever the message signal crosses zero.

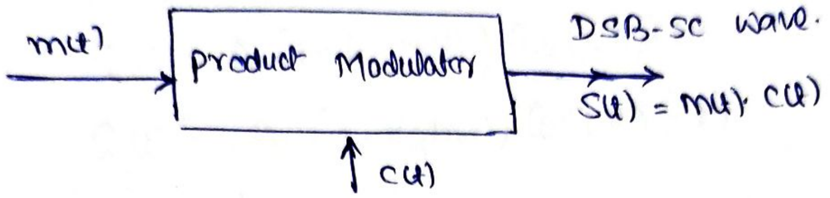
* A DSB-SC signal can be generated by a multiplier. A carrier signal can be suppressed by adding a carrier signal opposite in phase but equal in magnitude to the amplitude modulated wave, so the carrier gets cancelled.

Finally, Double side bands are available in the DSB-SC wave.

Generation of DSB-SC wave:

* A DSB-SC wave simply consists of the product of the modulating signal and the carrier signal.

* The devices used to generate DSB-SC waves are known as the product modulators.



There are two types of modulators:

1) Balanced modulator

2) Ring modulator

Balanced Modulator:

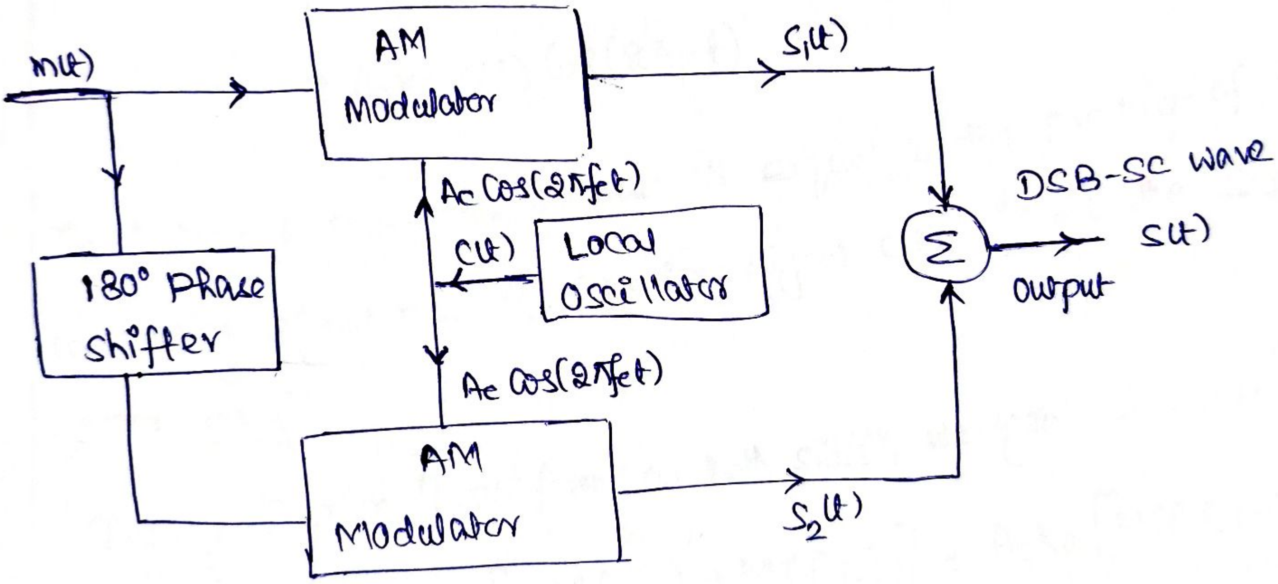


Figure shows the block diagram of a balanced modulator used for generating a DSB-SC signal

* It consists of two amplitude modulators that are interconnected in such a way as to suppress the carrier.

* One I/p to the amplitude modulator is from an oscillator that generates a carrier wave. The second I/p to the amplitude modulator in the top path is the modulating signal $m(t)$ while in the bottom path is $-m(t)$.

* The o/p of the two modulators are as follows:

$$S_1(t) = A_c(1+k_a m(t))\cos(2\pi f_c t)$$

$$S_2(t) = A_c(1-k_a m(t))\cos(2\pi f_c t)$$

The output of the summer is $s(t) = S_1(t) - S_2(t)$

$$s(t) = A_c(1+k_a m(t))\cos(2\pi f_c t) - A_c(1-k_a m(t))\cos(2\pi f_c t)$$

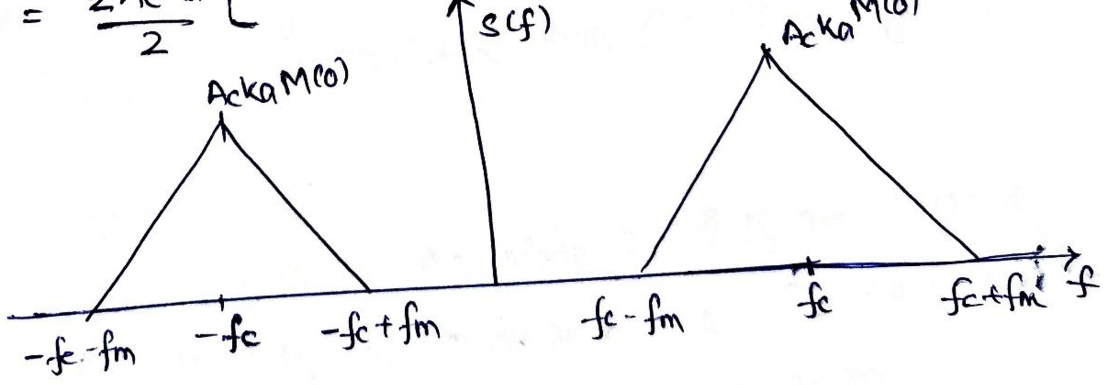
$$= \cancel{A_c\cos(2\pi f_c t)} + A_c k_a m(t)\cos(2\pi f_c t) - \cancel{A_c\cos(2\pi f_c t)} + A_c k_a m(t)\cos(2\pi f_c t)$$

$$= 2A_c k_a m(t)\cos(2\pi f_c t)$$

The balanced modulator output is equal to the product of the modulating signal $m(t)$ & carrier signal $c(t)$ except the scaling factor "$2k_a$"

Taking Fourier Transform on both sides, we get

$$S(f) = \frac{2A_c k_a}{2}\left[M(f-f_c) + M(f+f_c)\right] = A_c k_a\left[M(f-f_c) + M(f+f_c)\right]$$

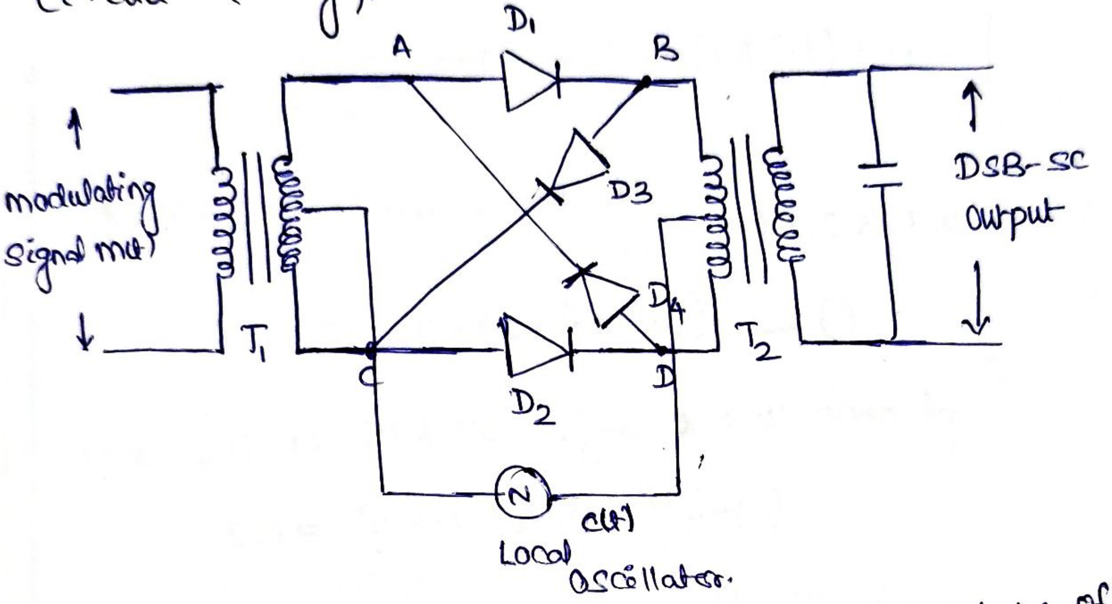


Since the carrier component is eliminated, the output is called DSB-SC signal.

Ring Modulator:

Ring Modulator is a product modulator used for generating DSB-SC modulated wave.

The Ring modulator consists of input transformer "T_1" output Transformer "T_2" and four diodes connected in a bridge circuit. (Ring).



Here, the carrier signal is applied to the center taps of the input and output transformers, and modulating signal is applied to the input transformer "T_1". The output appears across the secondary of the transformer "T_2".

* The diodes connected in the bridge act like switches, and their switching is controlled by the carrier signal as it is usually higher in frequency and amplitude than the modulating signal.

Working principle:

* When the carrier is positive, the diode D_1 & D_2 are forward biased and diodes D_3 & D_4 are reverse biased. Hence the modulator multiplies the message signal m(t) by +1 i.e., $y(t) = m(t)$.

* When the carrier is negative, the diodes D_3 & D_4 are forward biased and diodes D_1 & D_2 are reverse biased. Hence the modulator multiplies the message signal $m(t)$ by -1. i.e., $v_0(t) = -m(t)$

* The square wave carrier $c(t)$ can be represented by a Fourier series as:

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[2\pi(2n-1)f_c t]$$

$$c(t) = \frac{4}{\pi}\left[\cos(2\pi f_c t) - \frac{1}{3}\cos(6\pi f_c t) + \ldots\right]$$

The Ring modulator output is $\boxed{s(t) = c(t) \cdot m(t)}$

$$v(t) = s(t) \cdot \cos(2\pi f_c t + \phi) \quad \text{—} ①$$

Where $s(t)$ is DSB-SC signal and is given by

$$s(t) = A_c \cos(2\pi f_c t) \cdot m(t) \quad \text{—} ②$$

Substituting eq → ② in eq → ① we get

$$v(t) = A_c \cos(2\pi f_c t) \cdot \cos(2\pi f_c t + \phi) \cdot m(t)$$

As we know that, $\cos a \cdot \cos b = \frac{1}{2}\cos(a-b) + \frac{1}{2}\cos(a+b)$

$$v(t) = \frac{A_c m(t)}{2}\left[\cos(2\pi f_c t + \phi - 2\pi f_c t)\right] + \frac{A_c m(t)}{2}\left[\cos(2\pi f_c t + \phi + 2\pi f_c t)\right]$$

$$\therefore v(t) = \frac{A_c m(t)}{2}\cos\phi + \frac{A_c m(t)}{2}\cos(4\pi f_c t + \phi)$$

* The desired output is $v(t)$. The desired message signal is obtained by passing $v(t)$ through a Lowpass filter having a bandwidth greater than "f_m" Hz.

The output of the LPF is

$$V_o(t) = \frac{A_c}{2} \cos\phi \; m(t)$$

The demodulated signal $V_o(t)$ is therefore proportional to $m(t)$

Where, $\phi \rightarrow$ Phase error

* When $\phi = 0$, then amplitude of $V_o(t)$ is maximum
* When $\phi = \pi/2$, then amplitude of $V_o(t)$ is minimum (represents the quadrature null effect of the coherent detector).

Substituting eqn (1) in eqn $\rightarrow$ (2), we get

$$S(t) = \left[\frac{4}{\pi} \cos(2\pi f_c t) - \frac{4}{3\pi} \cos 6\pi f_c t + \ldots \right] m(t)$$

$$= \frac{4}{\pi} m(t) \cos(2\pi f_c t) - \frac{4}{3\pi} m(t) \cos 6\pi f_c t + \ldots \quad \text{—— (3)}$$

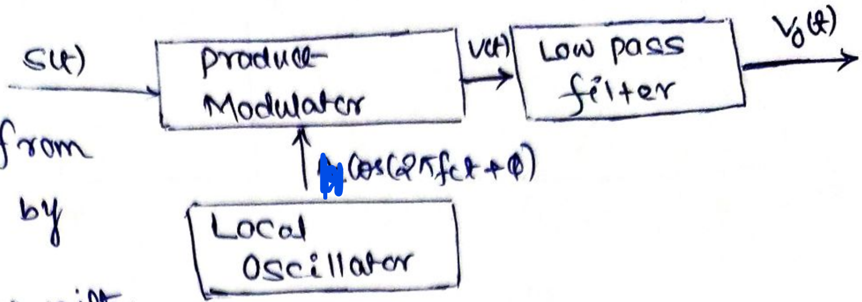
* The DSB-SC wave is extracted from $S(t)$ by passing information from eqn $\rightarrow$ (3) through the ideal BPF having center frequency "f_c" and Bandwidth equal to "$2f_m$" Hz

$\therefore$ The output of the BPF is

$$\boxed{S(t) = \frac{4}{\pi} m(t) \cos(2\pi f_c t).}$$

Coherent Detection of DSB-SC wave:

* The modulating signal $m(t)$ is recovered from a DSB-SC wave $s(t)$ by first multiplying $s(t)$ with a locally generated carrier wave and then low pass filtering the product as shown in the figure.



* For faithful recovery of modulating signal $m(t)$, the local oscillator output should be exactly coherent or synchronized in both frequency and phase with the carrier wave $c(t)$ used in the product modulator to generate $V_0(t)$ with the local oscillator output equal to $\cos(2\pi f_c t+\phi)$

The product modulator output can be given as:

$$v(t) = s(t) \cdot \cos(2\pi f_c t+\phi) \longrightarrow ①$$

We know that $$s(t) = A_c \cos(2\pi f_c t) \cdot m(t) \longrightarrow ②$$

Substituting eqn → ② in eqn → ①, we get

$$v(t) = A_c \cos(2\pi f_c t+\phi) \cdot \cos(2\pi f_c t) \cdot m(t)$$

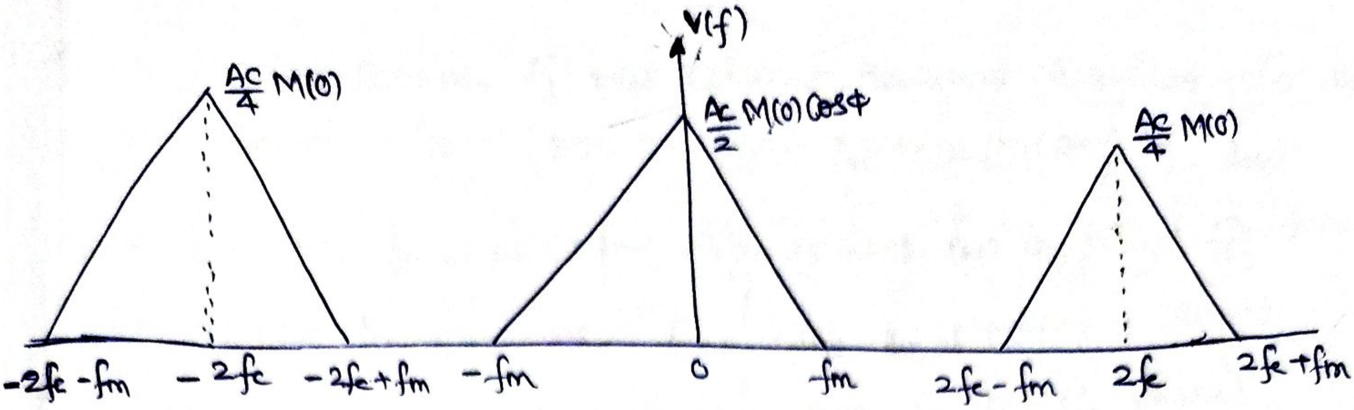
$$\because \cos A \cdot \cos B = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$$

$$v(t) = \frac{A_c m(t)}{2}\left[\cos(2\pi f_c t+\phi-2\pi f_c t)\right] + \frac{A_c m(t)}{2}\left[\cos(2\pi f_c t+\phi+2\pi f_c t)\right]$$

$$v(t) = \frac{A_c m(t)}{2}\cos\phi + \frac{A_c m(t)}{2}\cos(4\pi f_c t+\phi) \longrightarrow ③$$

Taking Fourier Transform on both sides of eqn → ③, we get

$$V(f) = \frac{A_c}{2} M(f)\cos\phi + \frac{A_c}{4}\left[M(f-2f_c)+M(f+2f_c)\right]$$



Amplitude Spectrum of V(f)

* The desired message signal is obtained by passing v(t) through a LPF having the bandwidth greater than "fm" Hz but less than $2f_c-f_m$

* The output of the LPF is

$$V_0(t) = \frac{A_c}{2} m(t) \cos\phi.$$

The demodulated signal $V_0(t)$ is therefore proportional to m(t)

Where $\phi \rightarrow$ phase error

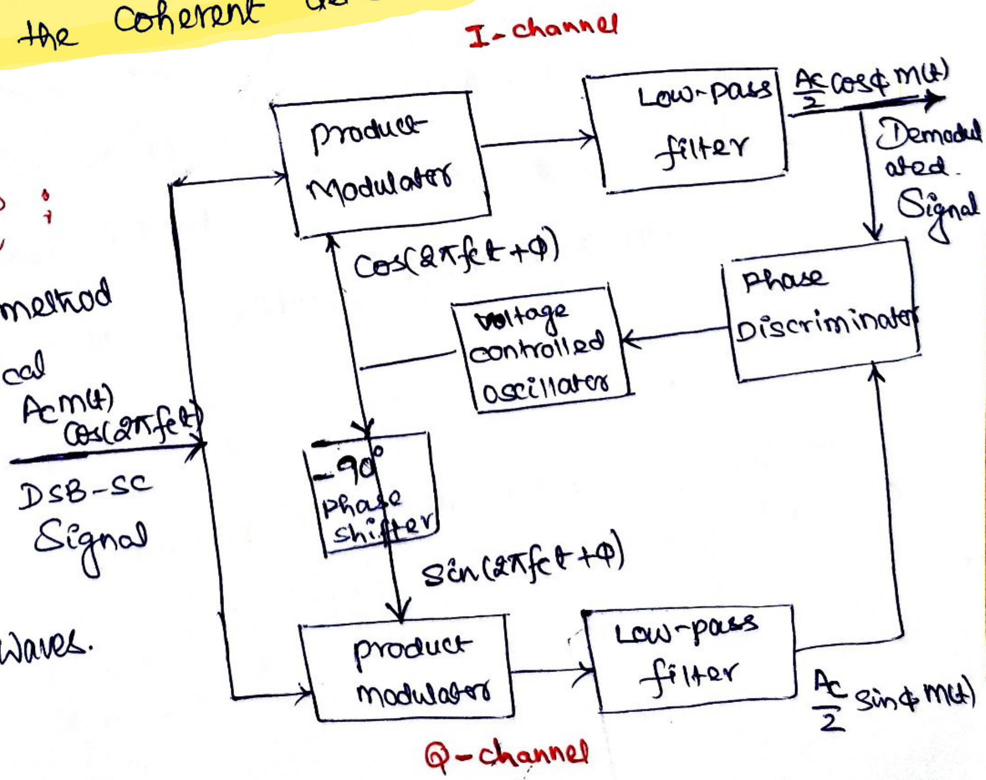
When ϕ = constant, $V_0(t)$ is proportional to m(t)

When $\phi = 0$, Amplitude of $V_0(t)$ is maximum

When $\phi = \pi/2$, Amplitude of $V_0(t)$ is minimum(represents the quadrature null effect of the coherent detection).

COSTAS LOOP :

* The costas loop is a method of obtaining a practical synchronous receiver system, suitable for Demodulating DSB-SC waves.



* The receiver consists of two Coherent detectors Supplied with the Same input Signal (DSB-SC wave) $A_c\, m(t) \cos(2\pi f_c t)$, but with individual local oscillator Signals that are in phase Quadrature with respect to each other (i.e., the local oscillator signal Supplied to the product modulators are 90° out of phase).

* The frequency of the local oscillator is adjusted to be the same as the carrier frequency "f_c".

* The detector in the upper path is referred to as the In-phase Coherent detector or I-channel and that in the lower path is referred to as the Quadrature-phase Coherent detector or Q-channel

These two detectors are coupled together to form a Negative Feedback System defined in such a way as to maintain the local oscillator Synchronous with the carrier wave.

<u>Operation:</u>

When the local oscillator signal is of the same phase as the carrier wave $A_c \cos(2\pi f_c t)$ used to generate the incoming DSB-SC wave under these conditions.

* The I-channel output contains the desired demodulated signal $m(t)$, whereas Q-channel output is zero.

$$V_{oI} = \frac{A_c\, m(t)}{2} \cos\phi$$

i.e., whenever the carrier is Synchronized. $\phi = 0$ and $\cos\phi = \cos(0) = 1$ and $\sin\phi = \sin(0) = 0$

$$\therefore\ V_{oQ} = 0$$

when phi is very small -> cos(phi)~1 and sin (phi)~phi

* When local oscillator phase changes by a small angle 'ϕ' radians. The I-channel output will remain unchanged, but Q-channel produces same output which is proportional to $\sin\phi$.

multiplier- m(t)^2

?

The output of I and Q-channels are combined in phase discriminator (which consists of a multiplier followed by a LPF), a dc control signal is obtained that automatically corrects for local phase errors in the voltage controlled oscillator.

Disadvantages of DSB-SC Coherent detection:-

Amplitude of the demodulated signal is maximum when $\phi = 0$ & minimum when $\phi = \pm\pi/2$. So, perfect synchronization has to be achieved for detection which inturn increases the cost of the receiver.

The received multiplexed signal $S(t)$ is applied to the two product modulators. The output of the Top product modulator is given by $\quad s_1(t) = S(t)\cos(2\pi f_c t)$

The Top LPF removes the high frequency term and allows only $\frac{A_c m_1(t)}{2}$

$$\therefore \; s_1(t) = \frac{A_c m_1(t)}{2}$$

The bottom LPF removes the high frequency term and allows only $\frac{A_c m_2(t)}{2}$. Thus the output of LPF is

$$s_2(t) = \frac{A_c m_2(t)}{2}$$

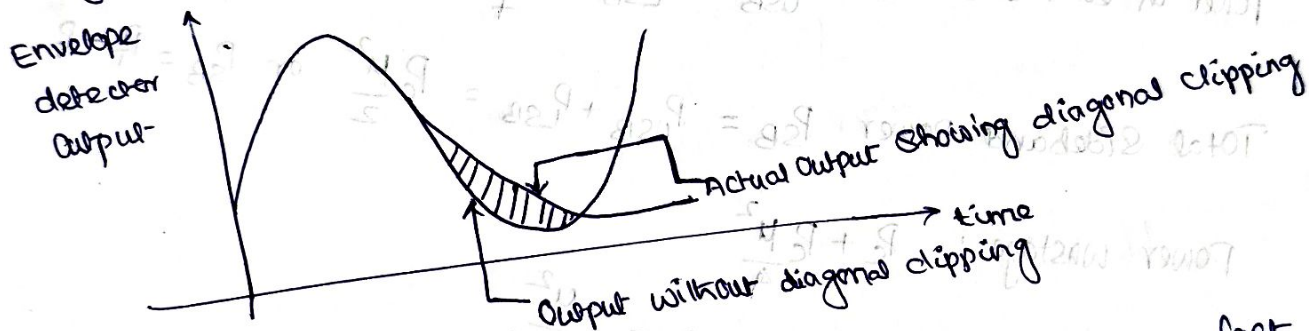
Distortion in Envelope Detector :-

There are two types of distortions which can occur in the detector output. They are :

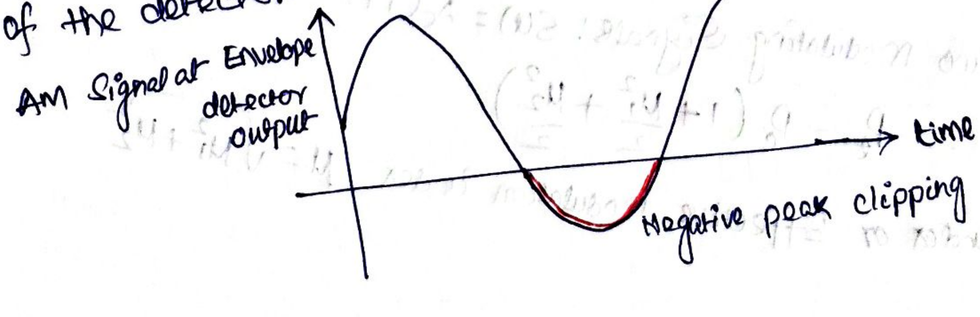
* Diagonal clipping
* Negative peak clipping

Diagonal clipping :-

This type of distortion occurs when the RC time constant of the load current is too long. Due to this the RC circuit cannot follow the fast change in the modulating envelope and is as shown in figure



Negative peak clipping : This distortion occurs due to a fact that the modulation index on the output side of the detector is higher than that on its input side. So at higher depths of modulation of the transmitted signal, the over modulation may takes place at the output of the detector.



AM wave : $S(t) = A_c(1+\mu \cos(2\pi f_m t)) \cos(2\pi f_c t)$

Modulation Index(μ) :

$\mu = \frac{A_m}{A_c}$

Amplitude of each Side band $= \frac{A_c \mu}{2}$

Upper Sideband frequency : $f_{USB} = (f_c + f_m)$

Lower Sideband frequency : $f_{LSB} = (f_c - f_m)$

Transmission Bandwidth / Bandwidth of AM : $BW = 2f_m$

Total Transmitted power : $P_t = P_c + P_{USB} + P_{LSB}$

$= P_c\left(1 + \frac{\mu^2}{2}\right)$ $\quad P_t = I_t^2 R_L$

power in each sideband : $P_{USB} = P_{LSB} = \frac{P_c \mu^2}{4}$

Total Sideband Power $P_{SB} = P_{USB} + P_{LSB} = \frac{P_c \mu^2}{2}$ or $P_{SB} = P_T - P_c$

Power wastage : $P_c + \frac{P_c \mu^2}{4}$

Transmission efficiency : $\eta = \frac{\mu^2}{2+\mu^2}$

Carrier power : $P_c = \frac{A_c^2}{2R_L}$ or $P_c = \frac{I_c^2}{R_L}$

Maximum frequency in AM wave : $f_{max} = f_c + f_m$

Minimum frequency in AM wave : $f_{min} = f_c - f_m$

Modulation index from AM wave $\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

AM wave with two modulating Signals : $S(t) = A_c(1 + \mu_1 \cos 2\pi f_{m1} t + \mu_2 \cos 2\pi f_{m2} t)$

Transmitted power : $P_t = P_c\left(1 + \frac{\mu_1^2}{2} + \frac{\mu_2^2}{2}\right)$

Total Modulation index or Effective modulation index $\mu = \sqrt{\mu_1^2 + \mu_2^2}$

Advantages

* low power consumption or power saving
* The modulation system is simple
* Efficiency is more than AM
* Carrier wave is suppressed
* Linear modulation type is required
* It can be used for point to point communication.

Disadvantages

* Design of receiver is complex
* Bandwidth required is same as that of AM

Applications:

* Analogue TV system to transmit color information.

* Total Transmitted power: $P_t = P_c\left(1+\frac{\mu_t^2}{2}\right)$ $\because \mu_t^2 = \mu_1^2 + \mu_2^2$
* Amplitudes of AM wave: $A_{max} = A_c(1+\mu^2)$ and $A_{min} = A_c(1-\mu^2)$
* Peak amplitude of carrier $A_c = (A_{max} + A_{min})/2$
* Peak amplitude of message signal : $A_m = \frac{A_{max} - A_{min}}{2}$
* Modulation index from AM wave : $\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

Single Sideband Modulation (SSB Modulation)

* Standard Amplitude modulation & DSB-SC modulation are wasteful of bandwidth because they both require a transmission bandwidth equal to twice the message bandwidth

i.e. $BW = 2f_m$.

* In both case (AM & DSB-SC) half of the transmission bandwidth is occupied by the upper sideband of the modulated wave, whereas the other half of the transmission bandwidth is occupied by the lower sideband of the modulated wave.

* The Upper & Lower sidebands are uniquely related to each other by virtue of their symmetry about the carrier frequency "f_c".

Thus only one sideband is necessary for transmission of information & if both the carrier and the other sideband are suppressed at the transmitter, no information is lost.

∴ The channel required the same bandwidth as the message signal f_m

* When only one sideband is transmitted, the modulation referred to as Single Sideband Modulation.

Single tone Modulation

Only Lower Side band (LSB) frequency of SSB Modulation

* Let the modulating signal m(t) is

$$m(t) = A_m \cos(2\pi f_m t) \quad \text{--- (1)}$$

* The Hilbert transform of the modulating signal m(t) is obtained by passing it through a -90° Phase shifter. So the Hilbert transform is given by

$$\hat{m}(t) = A_m \sin(2\pi f_m t) \quad \text{--- (2)}$$

We know that the SSB wave with only LSB is given by

$$S_L(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) + \hat{m}(t)\sin(2\pi f_c t)\right] \quad \text{--- (3)}$$

Substituting eq (1) & eq (2) in eq (3), we get

$$S_L(t) = \frac{A_c}{2}\left[A_m \underbrace{\cos(2\pi f_m t)}_{\cos A}\,\underbrace{\cos(2\pi f_c t)}_{\cos B} + A_m \underbrace{\sin(2\pi f_m t)}_{\sin A}\,\underbrace{\sin(2\pi f_c t)}_{\sin B}\right]$$

W.K.T $\cos(A-B) = \cos A.\cos B + \sin A.\sin B$.

$$S_L(t) = \frac{A_c A_m}{2}\left[\cos(2\pi f_c t - 2\pi f_m t)\right] = \frac{A_c A_m}{2}\cos 2\pi(f_c - f_m)t$$

Eq (4) shows that the SSB wave consists of only the Lower Sideband frequency $(f_c - f_m)$. This is exactly same as the result obtained by suppressing the upper sideband frequency $(f_c + f_m)$ of the corresponding DSB-SC wave.

The SSB wave with only USB is given by

$$S_U(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t)\right] \quad \text{--- (4)}$$

Substituting eq (1) & eq (2) in eq (4), we get

$$S_U(t) = \frac{A_c}{2}\left[\underbrace{A_m\cos(2\pi f_m t)}_{\cos A}\underbrace{\cos(2\pi f_c t)}_{\cos B} - A_m \underbrace{\sin(2\pi f_m t)}_{\sin A}\underbrace{\sin(2\pi f_c t)}_{\sin B}\right]$$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B.$

$$S_U(t) = \frac{A_c A_m}{2}\left[\cos(2\pi f_c t + 2\pi f_m t)\right] = \frac{A_c A_m}{2}\cos 2\pi(f_c + f_m)t \quad \text{--- (5)}$$

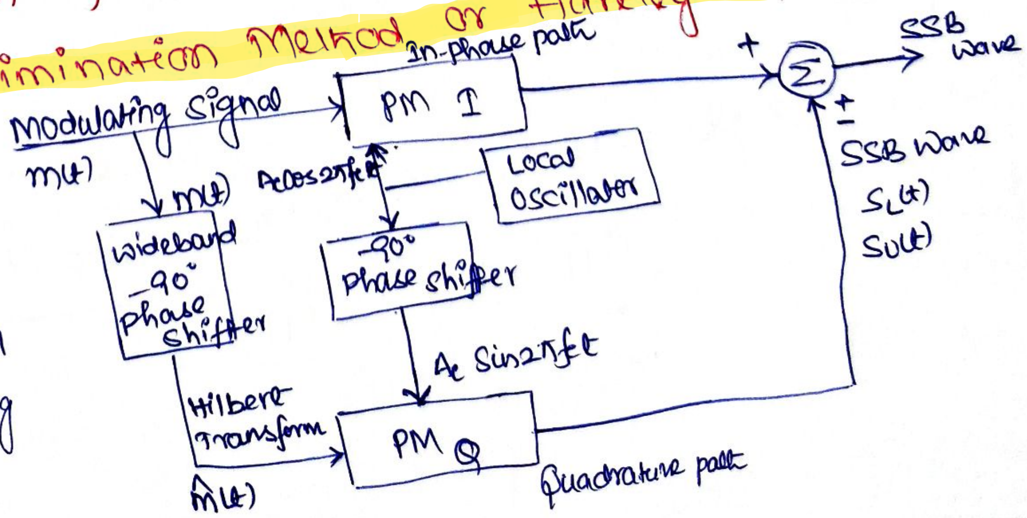
Eq → (5) Shows that the SSB wave consists of only the upper sidebands of frequency $(f_c + f_m)$.

This is exactly same as the result obtained by suppressing the lower sideband frequency $(f_c - f_m)$ of the corresponding DSB-SC wave.

Generation of SSB wave:

Phase Discrimination Method or Hartley Modulator

The figure shows the block diagram of phase discrimination method for generating SSB wave



* The SSB modulator uses two product modulators I & Q, Supplied with carrier waves in phase quadrature to each other.

* The message signal $m(t)$ & a carrier signal $A_c \cos(2\pi f_c t)$ is directly applied to the product modulator I, producing a DSB-SC wave.

* The Hilbert transform $\hat{m}(t)$ (-90° phase shifter) of $m(t)$ & carrier signal shifted by 90° are applied to the product modulator Q, producing DSB-SC wave.

* The output of product modulator "I" is

$$\boxed{S_I(t) = A_c m(t) \cos(2\pi f_c t)}$$

* The output of product modulator "Q" is

$$\boxed{S_Q(t) = A_c \cdot \hat{m}(t) \cdot \sin(2\pi f_c t).}$$

These signals $S_I(t)$ & $S_Q(t)$ are fed to a summer.

The output of the summer is $S(t) = S_I(t) + S_Q(t)$.

$$S(t) = A_c m(t) \cos(2\pi f_c t) + A_c \hat{m}(t) \sin(2\pi f_c t)$$

The plus sign at the summing junction yields an SSB with only the LSB i.e.,

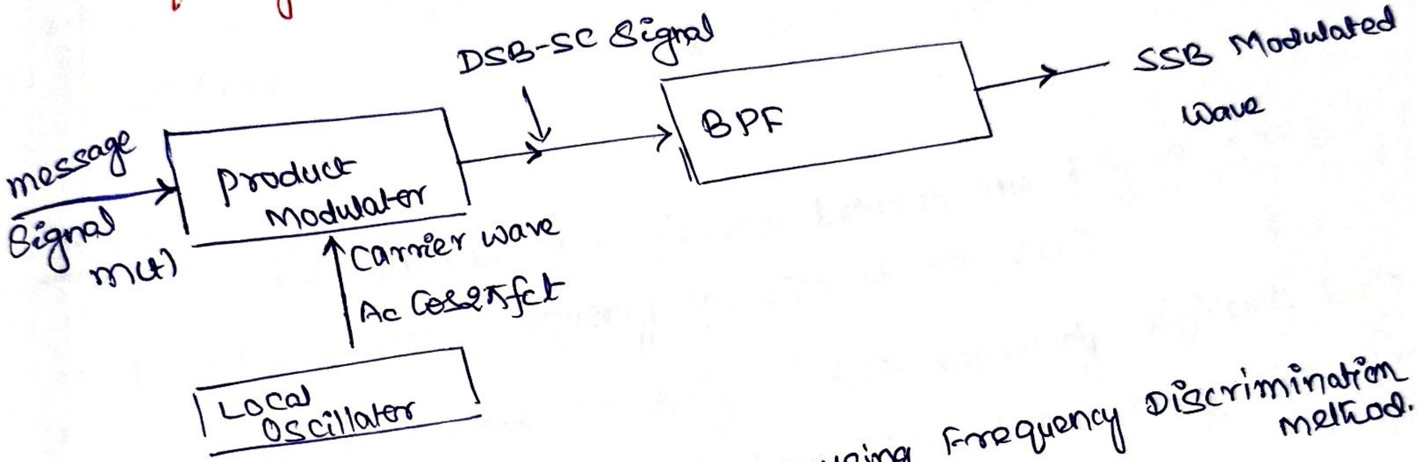
$$\boxed{S_L(t) = A_c m(t) \cos(2\pi f_c t) + A_c \hat{m}(t) \sin(2\pi f_c t)}$$

* Similarly the minus sign at the summing junction yields an SSB with only the SSB i.e.,

$$\boxed{S_u(t) = A_c m(t) \cos(2\pi f_c t) - A_c \hat{m}(t) \sin(2\pi f_c t)}$$

This SSB modulator is also known as the Hartley modulator.

Frequency Discrimination Method or Filtering method.



* Block diagram of SSB modulator using Frequency Discrimination method.

* Frequency discrimination method can be used for generating the SSB modulated wave if the message signal satisfies the following conditions:

*→ The message signal should not have any low frequency content (i.e., the message spectrum M(f) has "holes" at zero frequency).

The audio signals passes this property

* The telephone signals will have a frequency range extending from 300Hz to 3.4KHz. The frequencies in the range 0-300 Hz are absent, thereby creating an energy gap from 0 to 300 Hz.

if we want USB->fc+fm
- signals of freq range from fc to fc+fm should pass through BPF - centre freq=fc+fm/2 and bandwidth=fm
if we want LSB->fc-fm
-signals of freq range from fc-fm to fc should pass through BPF-centre freq=fc-fm/2 and bandwidih=fm

* The highest frequency component "fm" of the message signal m(t) is much less than the carrier frequency "fc".

* This modulator consists of a product modulator, carrier oscillator and BPF designed to pass the desired sideband.

* At the output of the product modulator, we get the DSB-SC modulated wave which contains only two sidebands

* The BPF will pass only one sideband and produce the SSB modulated wave and its output

Demerits: The frequency difference between the highest frequency in LSB & lowest frequency in USB is too small

This makes the design of BPF extremely difficult, because its frequency response need to have very sharp change over from attenuation to pass band & vice-versa.

Design of BPF:

The design of BPF must satisfy two basic requirements

* The passband of the filter occupies the same frequency range as the spectrum of the desired SSB modulated wave.

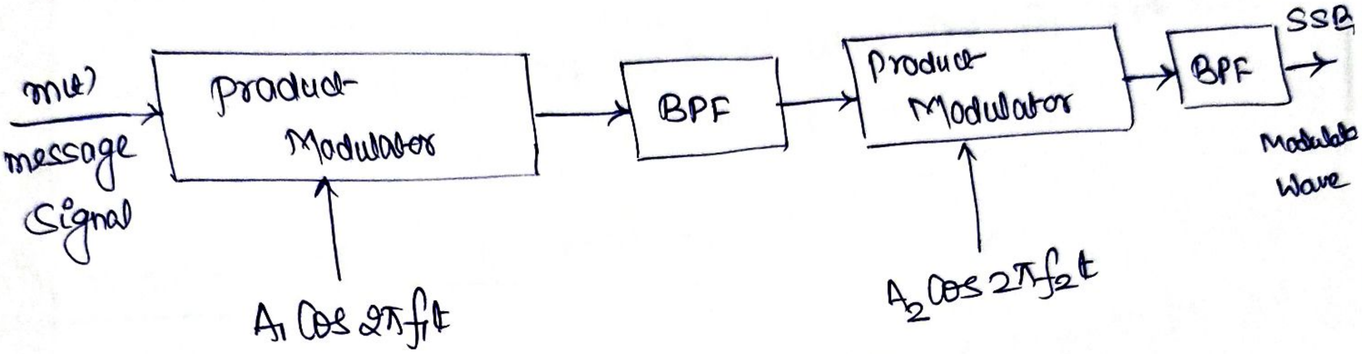
* The width of the guard band which separates the passband from stop band be twice the lowest frequency component of the message signal

i.e., $\boxed{\text{Guard band} = 2f_1 \text{ Hz.}}$?

f_m

The conditions mentioned above are satisfied only by the highly selective filters using crystal resonators with a high Q-factor typically in the range 1000 to 2000Hz.

Two stage SSB Modulator :



A Two stage SSB modulator

* The message signal m(t) modulates the carrier f_1 to produce a DSB-SC signal. This signal is passed through the 1st BPF to produce an SSB modulated signal.

* The output of the first BPF is then used to modulate another carrier "f_2" which is higher than "f_1" then the output of the 2nd product modulator we get another DSB-SC signal.

* Thus increases the Guard band between sideband frequency which will make the filter design easy.

Advantages :

* The filter method gives the adequate sideband suppression

* The sideband filters also helps to attenuate carrier if present in the output of balanced modulator.

learn

* The bandwidth is sufficiently flat & wide.

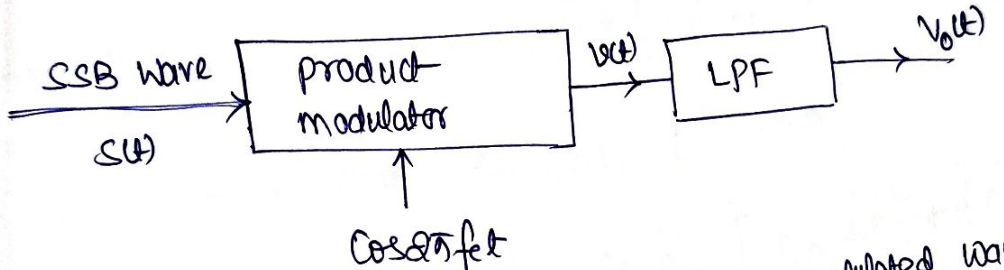
(i) Disadvantages:

* They are bulky

* Due to the inability of the spectrum/system to generate SSB at high radio frequencies, the frequency up conversion is necessary.

* TWO experience filters are to be used one for each sideband.

Demodulation of SSB wave:-



* Coherent detection of an SSB modulated wave.

* The baseband signal m(t) can be recovered from the SSB wave s(t) by using coherent detection.

* The product modulator is having two inputs. One input is the SSB modulated wave s(t) & another input is the locally generated carrier $\cos(2\pi f_c t)$ then low pass filtering the modulator output as shown in the above figure.

Thus product modulator output is given by

$$v(t) = s(t)\cos(2\pi f_c t) \quad \text{—— (1)}$$

We know that, $s(t) = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) \pm \hat{m}(t)\sin(2\pi f_c t)\right]$ —(2)

Substituting eq→(2) in eq→(1), we get

$$v(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin(2\pi f_c t)\right]\cos(2\pi f_c t)$$

$$v(t) = \frac{A_c}{2} m(t)\cos 2\pi f_c t \cdot \cos 2\pi f_c t + \frac{A_c}{2}\hat{m}(t)\sin 2\pi f_c t \cdot \cos 2\pi f_c t$$

We know that $\cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$\cos A \cdot \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$

$$v(t) = \frac{A_c}{4} m(t)\left[\cos(2\pi f_c + 2\pi f_c)t + \cos(2\pi f_c - 2\pi f_c)t\right] \pm \frac{A_c}{4}\hat{m}(t)$$
$$\left[\sin(2\pi f_c + 2\pi f_c)t + \sin(2\pi f_c - 2\pi f_c)t\right]$$

$$v(t) = \frac{A_c}{4} m(t)\left[\cos 4\pi f_c t + \cos(0)\right] \pm \frac{A_c}{4}\hat{m}(t)\left[\sin(4\pi f_c t) + \sin(0)\right]$$

We know that $\cos(0) = 1$, $\sin(0) = 0$

$$v(t) = \frac{A_c}{4} m(t)\left[1 + \cos 4\pi f_c t\right] \pm \frac{A_c}{4}\hat{m}(t)\left[\sin 4\pi f_c t + 0\right]$$

$$v(t) = \frac{A_c}{4} m(t) + \frac{A_c}{4} m(t)\cos(4\pi f_c t) \pm \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)$$

$$= \underbrace{\frac{A_c}{4} m(t)}_{\text{Scaled message signal}} + \underbrace{\frac{A_c}{4} m(t)\cos(4\pi f_c t) \pm \frac{A_c}{4}\hat{m}(t)\sin(4\pi f_c t)}_{\text{unwanted terms}}$$

* When $v(t)$ is passed through the filter, it will allow only the 1st term to pass through & will reject all other unwanted components/terms.

* Thus at the output of the filter we get the scaled message signal & the coherent SSB demodulation is achieved.

$$\therefore V_0(t) = \frac{A_c}{4} m(t)$$

The detection of SSB modulated waves is based on the assumption that there is perfect synchronization between local carrier & that in the transmitter both in frequency and phase

* But in practice a phase error ϕ may arise in the locally generated carrier wave. Thus the detector output is modified due to phase error as follows.

$$V_0(t) = \frac{A_c}{4} m(t) \cos\phi + \frac{A_c}{4} \hat{m}(t) \sin\phi$$

Note: The phase distortion is not serious with voice communication because the human ear is relatively insensitivity to phase distortion. The presence of phase distortion gives rise to what is called the Donald Duck voice effect

* The phase distortion cannot be tolerable in the transmission of music and video signals.

Advantages :-

* SSB required half the bandwidth required of AM & DSB-SC
* Due to suppression of carrier and one sideband, power is saved.
* Reduced Interference of noise. This is due to the reduced bandwidth. As the bandwidth increases, the amount of noise added to the signal will increase.
* Fading does not occur in SSB transmission.

Disadvantages :

* The generation & reception of SSB signal is a complex process.
* Since carrier is absent, the SSB transmitter & receiver need to have an excellent frequency stability
* The SSB modulation is expensive & highly complex to implement.

Applications:

* SSB transmission is used in the applications where the power saving is required in mobile systems.

* SSB is also used in applications in which bandwidth requirements are low.

* point to point communication, Land, air, maritime mobile communications, TV, Telemetry, military communications, Radio Navigation, amature Radio

Vestigial Sideband (VSB) Modulation:

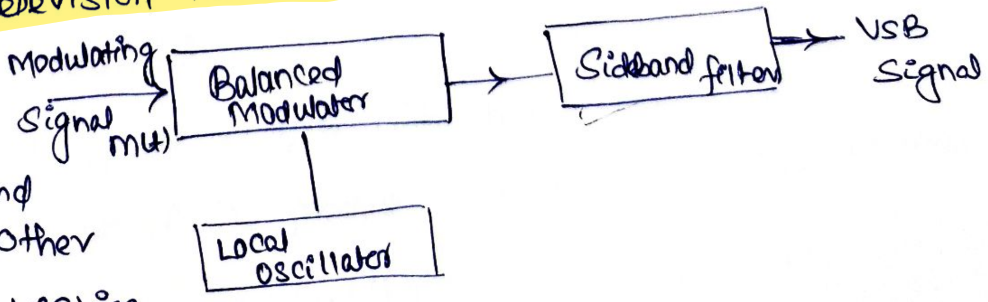
The Stringent (very strict condition) frequency response requirements on the Sideband filter is SSB-SC modulation can be relaxed by allowing a part of the unwanted Sideband (Called as vestige) to appear in the output of the modulator.

Due to this, the design of the sideband filter is simplified to a great extent. But the bandwidth of the system is increased slightly.

* The SSB modulation is not appropriate way of modulation when the message signal contains significant components at extremely low frequencies. Because in such cases the upper and low sidebands meet at the carrier frequency and it is difficult to isolate one sideband. To overcome this difficulty, the modulation technique known as vestigial sideband modulation (VSB) is used.

In this technique, one sideband is passed almost completely whereas just a trace, or vestige of the other sideband is retained.

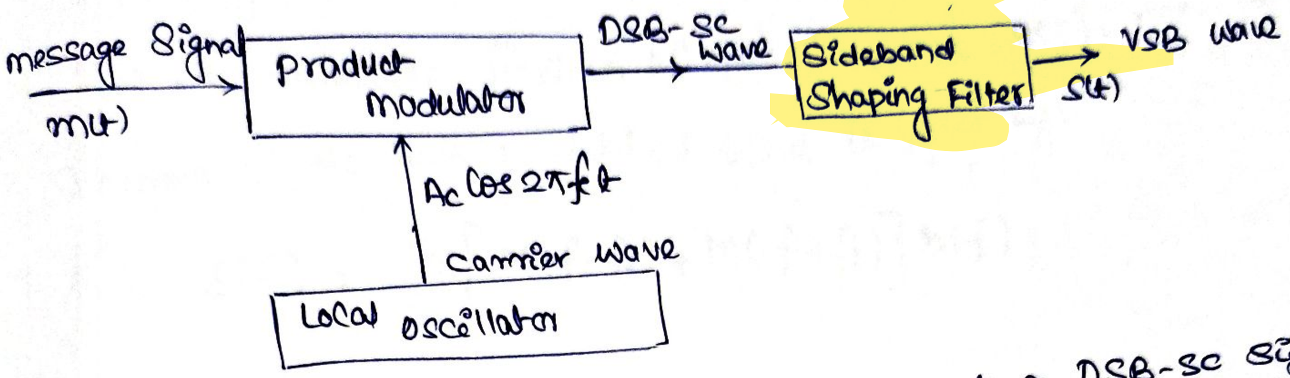
The television signals contain significant components at extremely low frequencies and hence vestigial sideband modulation is used in television transmission.



In VSB, one sideband and a part of the other sideband called as vestige is transmitted.

So, the bandwidth required for VSB transmission is somewhat higher than that of SSB modulation.

Generation of VSB Modulated wave:

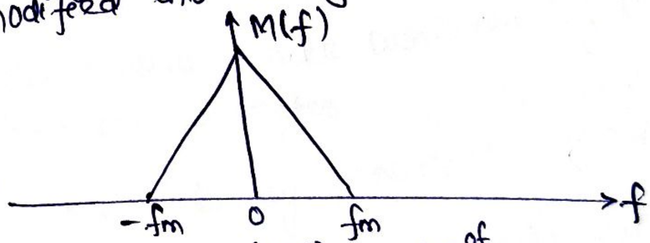


To generate a VSB signal, first we have to generate a DSB-SC signal and then pass it through a sideband shaping filter as shown in the figure.

This filter will pass the wanted sideband as it is along with a part of unwanted sideband.

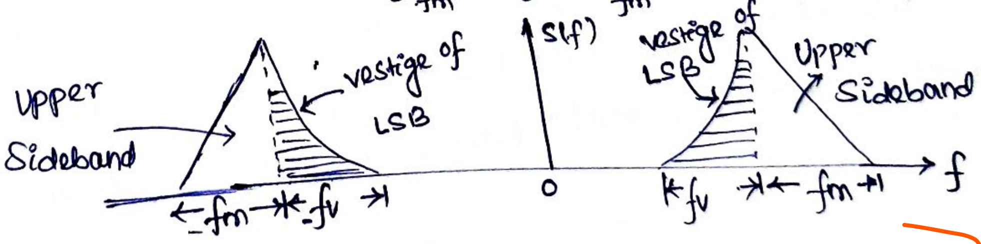
The spectrum of a VSB modulated wave s(t) along with the message signal m(t)

Here lower sideband is modified into vestigial sideband.



Transmission bandwidth is given by

$BW = f_m + f_v$



where

f_m is message bandwidth

f_v is the width of the vestigial sideband.

* The output of the product modulator is the DSB-SC wave and is given by $S(t) = m(t) \cdot c(t)$.

$\therefore\ s(t) = m(t) \cdot A_c \cos(2\pi f_c t)$

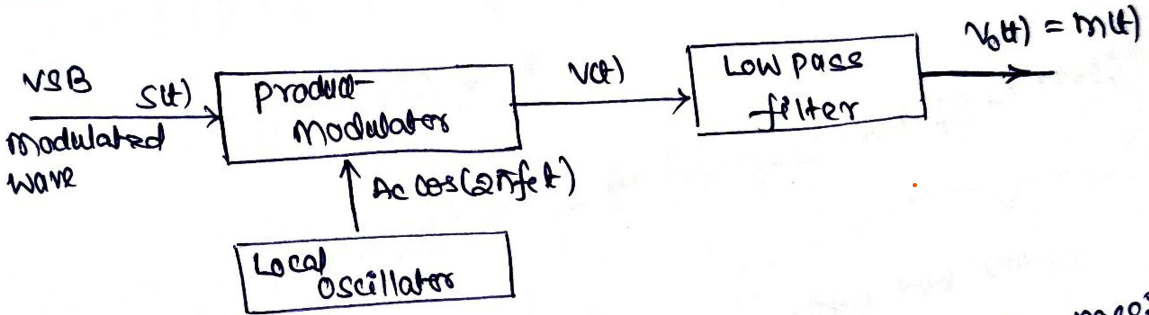
This DSB-SC signal is then applied to a sideband shaping filter. The filter will pass the wanted sideband as it is & the vestige of the unwanted sideband.

Let the transfer function of the filter be $H(f)$. Hence the Spectrum of the VSB modulated wave is given by:

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f)$$

Where $M(f)$ is the message spectrum.

Demodulation of VSB Modulated Wave:



The demodulation of VSB modulated wave can be achieved by passing VSB wave $s(t)$ through a coherent detector.

* Thus, multiplying $m(t)$ by a locally generated carrier wave $A_c \cos(2\pi f_c t)$, which is synchronous with the carrier wave $A_c \cos(2\pi f_c t)$ in both frequency and phase as shown in figure,

$$v(t) = s(t) \cdot A_c \cos(2\pi f_c t) \quad \text{——} \ (1)$$

Taking Fourier Transform on both side of eqⁿ → (1), we get

$$V(f) = \frac{A_c}{2}\left[S(f-f_c) + S(f+f_c)\right] \quad \text{——} \ (2)$$

We know that

$$S(f) = \frac{A_c}{2}\left[M(f-f_c)+M(f+f_c)\right] H(f)$$

$$S(f+f_c) = \frac{A_c}{2}\left[M(f-f_c+f_c)+M(f+f_c+f_c)\right] H(f+f_c) \quad — ③$$

$$= \frac{A_c}{2}\left[M(f)+M(f+2f_c)\right] H(f+f_c) \quad — ③$$

III'ly

$$S(f-f_c) = \frac{A_c}{2}\left[M(f-2f_c)+M(f)\right] H(f-f_c) \quad — ④$$

Substituting eq ③ & ④ in eq ②, we get

$$V(f) = \frac{A_c}{2}\left\{\frac{A_c}{2}\left[M(f)+M(f+2f_c)\right] H(f+f_c) + \frac{A_c}{2}\left[M(f)+M(f-2f_c)\right] H(f-f_c)\right\}$$

$$= \frac{A_c}{4} M(f-2f_c) H(f-f_c) + \frac{A_c}{4} M(f) H(f-f_c) + \frac{A_c}{4} M(f) \cdot H(f+f_c)$$

$$+ \frac{A_c}{4} M(f+2f_c) H(f+f_c)$$

$$V(f) = \underbrace{\frac{A_c}{4} M(f)\left[H(f-f_c)+H(f+f_c)\right]}_{\text{wanted VSB wave}} + \frac{A_c}{4}\underbrace{\left[M(f-2f_c) H(f-f_c) + M(f+2f_c) H(f+f_c)\right]}_{\text{unwanted VSB wave}} \quad ⑤$$

Eqⁿ ⑤ is passed through a LPF, which eliminates unwanted VSB wave and passes only wanted VSB wave i.e., VSB wave given by

$$V_0(f) = \frac{A_c}{4} M(f)\left[H(f-f_c)+H(f+f_c)\right].$$

* To obtain the undistorted message signal $m(t)$ at the output of the demodulator, the transfer function $H(f)$ should satisfy the condition as follows

$$H(f-f_c)+H(f+f_c) = 2H(f_c)$$

Hence $H(f_c)$ is constant. Thus design of VSB filter is therefore less complicated to an SSB filter.

Envelope Detection of a VSB Wave plus Carrier

* VSB modulation is used in the commercial TV broadcasting in which along with VSB transmission a carrier signal of substantial size is transmitted.

$\therefore$ The modulated wave can be demodulated by using Envelope detector.

We know that, the VSB modulated wave with full USB and a vestige of LSB is given by

$$s(t)_{VSB} = \frac{A_c}{2}\left[m(t)\cos(2\pi f_c t) - m_Q(t)\sin(2\pi f_c t)\right] \quad \text{--- (1)}$$

Adding carrier component $A_c\cos(2\pi f_c t)$ to eqn → (1) scaled by a factor k_a, modifies the modulated wave applied to the envelope detector input as

$$s(t) = A_c\cos(2\pi f_c t) + k_a s_{VSB}(t)$$

$$s(t) = \frac{A_c}{2}k_a\left[m(t)\cos(2\pi f_c t) - m_Q(t)\sin(2\pi f_c t)\right] + A_c\cos(2\pi f_c t)$$

$$s(t) = \frac{A_c}{2}k_a m(t)\cos(2\pi f_c t) - \frac{A_c}{2}k_a m_Q(t)\sin 2\pi f_c t + A_c\cos(2\pi f_c t)$$

$$= A_c\cos(2\pi f_c t)\underbrace{\left[1 + \frac{k_a}{2}m(t)\right]}_{\text{In-phase}} - \frac{k_a A_c}{2}\underbrace{m_Q(t)\sin(2\pi f_c t)}_{\text{Quadrature}}$$

Where the constant "k_a" determines the percentage modulation.

The envelope detector output, denoted by $a(t)$ is therefore,

$$a(t) = \sqrt{(\text{In-phase Component})^2 + (\text{Quadrature component})^2}$$

$$= \sqrt{A_c^2\left[1 + \frac{k_a}{2}m(t)\right]^2 + A_c^2\left[\frac{k_a}{2}m_Q(t)\right]^2}$$

$$= \sqrt{A_c^2\left[1+\frac{k_a m(t)}{2}\right]^2\left\{1+\left[\frac{k_a/2\, m_Q(t)}{1+\frac{k_a}{2} m(t)}\right]^2\right\}}$$

Taking as common

$$= A_c\left[1+\frac{k_a m(t)}{2}\right]\sqrt{1+\left[\frac{\frac{k_a m_Q(t)}{2}}{1+\frac{k_a m(t)}{2}}\right]} \quad \text{—— (2)}$$

Eqⁿ → (2) indicates that the distortion is contributed by the quadrature component $m_Q(t)$.

This distortion can be reduced using two methods.

* 1> Reducing the percentage modulation to reduce k_a.
* 2> Increasing the width of the vestigial sideband to reduce $m_Q(t)$.

Advantages:

* The reduction in bandwidth. It is almost as efficient as the SSB.
* Easy to design the filter.

Applications:

VSB modulation has become standard for the transmission of TV signals. Because the video signals need a larger transmission bandwidth, if transmitted using DSB FC (AM modulation) or DSB-SC techniques.