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[B.Tech IT] * Linear Algebra C2 Assignment *

Q1 $A = \{n, i, j, v, a, l\}$

$B = \{c, h, a, v, d, r, i\}$

$n = 2+0+3+3$

$= 8$

$C = A \cup B$

$= \{n, i, j, v, a, l, c, h, u, d, r\}$

$D = A \times B$

$\boxed{|A| = 6}$

$\boxed{|B| = 7}$

$\boxed{|C| = 11}$

$\boxed{|D| = |6|\,|7| = 42}$

$\theta = |C| + |D| + n$

$= 11 + 42 + 8$

$\boxed{\theta = 61^\circ}$

$r^* = |C| + 1 = 12$

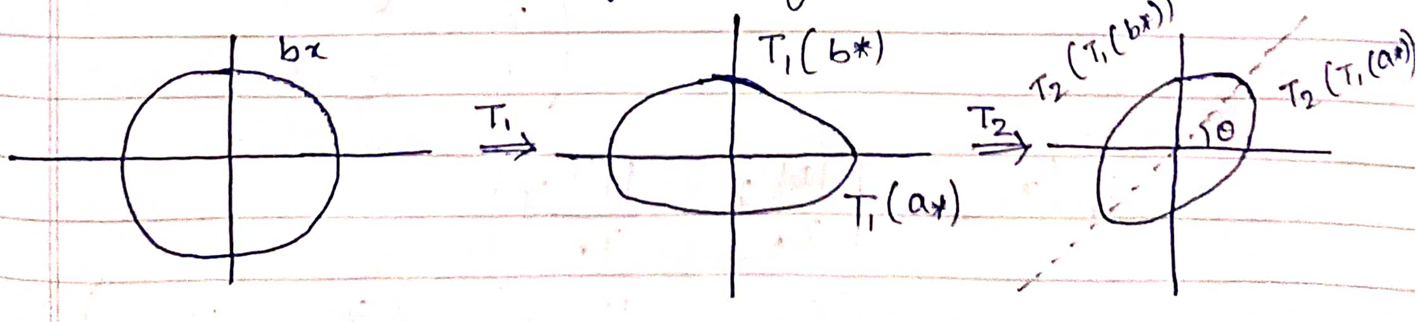
$a^* = |D| = 42$

$b^* = |C| = 11$

Linear Transformation would be composition of two Transformation given by:-

$T_1 \rightarrow$ Tranfers circle to ellipse of given dimensions

$T_2 \rightarrow$ Rotates ellipse by θ angle in +ive x direction.



Let $B = \{(1,0), (0,1)\}$ be the basis for $\mathbb{R}^2$

We need a linear transformation from,

$$(a_*, 0) \longrightarrow (|D|, 0)$$

and

$$(0, b_*) \longrightarrow (0, |C|)$$

$$a_x = b_x = r_x = |C| + 1 = 12$$

Let T_1 be a linear transformation from $x^2 + y^2 = r_*^2$ to

$$\frac{x^2}{|D|^2} + \frac{y^2}{|C|^2} = 1$$

$T_1 = \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

From ①

$$T_1(1,0) = \left(\frac{|D|}{r_*}, 0\right)$$

$$T_1(0,1) = \left(0, \frac{|C|}{r_*}\right)$$

$$T_1 = \begin{pmatrix} \frac{|D|}{r_*} & 0 \\ 0 & \frac{|C|}{r_*} \end{pmatrix} = \begin{pmatrix} \frac{42}{12} & 0 \\ 0 & \frac{11}{12} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{7}{2} & 0 \\ 0 & \frac{11}{12} \end{pmatrix}$$

$$T_1(x,y) = \left(\frac{|D|}{r_*}x, \frac{|C|}{r_*}y\right)$$

$$= \left(\frac{42}{12}x, \frac{11}{12}y\right) = \left(\frac{7}{2}x, \frac{11}{12}y\right)$$

For T_2

We need a transformation from,

$$(T_1(a_x), 0) \longrightarrow (T_1(a_*)\cos\theta, T_1(a_x)\sin\theta)$$

$$(0, T_1(b_x)) \longrightarrow (T_1(b_*)(-\sin\theta), T_1(b_*)\cos\theta)$$

$T_2 : \mathbb{R}^2 : \mathbb{R}^2$

$$T_2\begin{pmatrix} \frac{7}{2}x \\ \frac{11}{12}y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{\text{Matrix for rotation [anticlockwise]}} \begin{pmatrix} \frac{7}{2}x \\ \frac{11}{12}y \end{pmatrix}$$

$$\therefore T_2\left(\frac{7}{2}x, \frac{11}{12}y\right) = \left(\frac{7}{2}x\cos\theta - \frac{11}{12}y\sin\theta, \frac{7}{2}x\sin\theta + \frac{11}{12}y\cos\theta\right)$$

$\boxed{\theta \longrightarrow 61^\circ}$

$T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$

$T = T_2 \circ T_1(x,y)$

$$T = \begin{pmatrix} \frac{7}{2} & 0 \\ 0 & \frac{11}{12} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$T = \begin{pmatrix} \frac{7}{2}\cos(61) & -\frac{11}{12}\sin(61) \\ \frac{7}{2}\sin(61) & \frac{11}{12}\cos(61) \end{pmatrix}$$

$T(x,y) =$

$$= \left(\frac{|D| \cos x}{r_*} - \frac{|C| \sin\theta\, y}{r_*}, \right.$$

$$\left. \frac{|D| \sin\theta\, x}{r_*} + \frac{|C| \cos\theta\, y}{r_*} \right)$$

$$\boxed{T(x,y) = \left(\frac{7}{2}\cos(61)x - \frac{11}{12}\sin(61)y,\ \frac{7}{2}\sin(61)x + \frac{11}{12}\cos(61)y \right)}$$

Q2 $K = |A| = 6$

First, we find eigen values of T as $\begin{pmatrix} Tx = \lambda x \\ T^5 x = \lambda^5 x \end{pmatrix}$

Transition matrix of T with respect to standard basis of $\mathbb{R}^2$:-

$$T = \begin{pmatrix} \frac{7}{2}\cos(61) & -\frac{11}{12}\sin(61) \\ \frac{7}{2}\cos(61) & \frac{11}{12}\cos(61) \end{pmatrix}$$

$$|T - \lambda I| = 0$$

Let λ_1 and λ_2 be eigenvalues of T:-

We know, $Tr(T) =$ sum of eigen values $= \lambda_1 + \lambda_2$

$$= \frac{53}{12}\cos(61)$$

$|T| =$ product of eigenvalues $= \lambda_1 \lambda_2$

$$= \frac{77}{24}$$

Equation with eigen values as roots:

$$\lambda^2 - \left(\frac{53}{12}\cos(61)\right)\lambda + \frac{77}{24} = 0$$

$D = b^2 - 4ac$

$= \left(\frac{53}{12}\cos(61)\right)^2 - 4(1)\left(\frac{77}{24}\right)$

$= 4.58 - 12.83$

$= -8.24$

$D < 0$

$\therefore$ Eigen values will be complex.

Eigen values of T^5 = (Eigen value of $T)^5$
↳ again complex for complex eigen value.

Eigen values can't be complex as $\mathbb{R}^2$ is defined over the field $\mathbb{R}$ and eigen values belong to the field. We have defined $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ so field cannot be complex.

So, there will be no eigen vectors.

Note: If λ is complex
λ^5 will also be complex