$$\boxed{f''(x) > 0}$$

7] b]

$$f(x) \geq f(x_0) + f'(x_0)[x - x_0].$$

let $f(x) = \cos x$

$f'(x) = -\sin x$ convex $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$f''(x) = -\cos x$

~~if $f''(x) \leq 0$~~

* see the domain

$x \to y$

$x_0 \to x$

$$\cos y \geq \cos(x) - \sin x[x - x_0].$$

$$\cos y - \cos x \geq \sin x[x - y]$$

8] a]

Assume $z = \lambda x + (1-\lambda)y$

$z \in [a, b]$ as $x, y\ [a, b]$.

$$f(x) = f(x_0) + f'(x_0)[x - x_0] + \underbrace{\frac{f''(c)[x - x_0]^2}{2!}}_{+\text{ve term}}$$

let

$x = x$ & $x_0 = z$

$$f(x) \geq f(z) + f'(z)[x - z]$$

$||^{\text{ly}}$

$$f(y) \geq f(z) + f'(z)[x - y]$$

multiply by λ

multiply by $1-\lambda$ and add both

$\therefore$ $\lambda f(x) + (1-\lambda) f(y) \geq f(z) + f'(z)[\lambda x - \lambda z - z + y]$

$f'(z)[\underbrace{\lambda x + y(1-\lambda) - z}_{z}]$

$\therefore$ $\lambda f(x) + (1-\lambda) f(y)$
$\geq f(z)$

only when $f''(x) \geq 0$

when $f''(x) \geq 0$

(b) In above qu put $y = 0$

as $\sin x$ is concave in $[0, \pi]$

$\therefore$ $f(\lambda x) \geq \lambda f(x)$

$\therefore$ $\boxed{\sin(\lambda x) \geq \lambda \sin x}$

9]

$\rightarrow$ $f''(x) \geq 0$

From qu. 7 we get

if $f''(x) \geq 0$

$\forall$ $f(x) \geq f(x_0) + f'(x_0)(x - x_0)$ $\forall x$ — ①

also as f is bounded

$$|f(x)| \le M$$

$$-M \le f(x) \le M$$

$$\boxed{|f(x)| \le M}$$

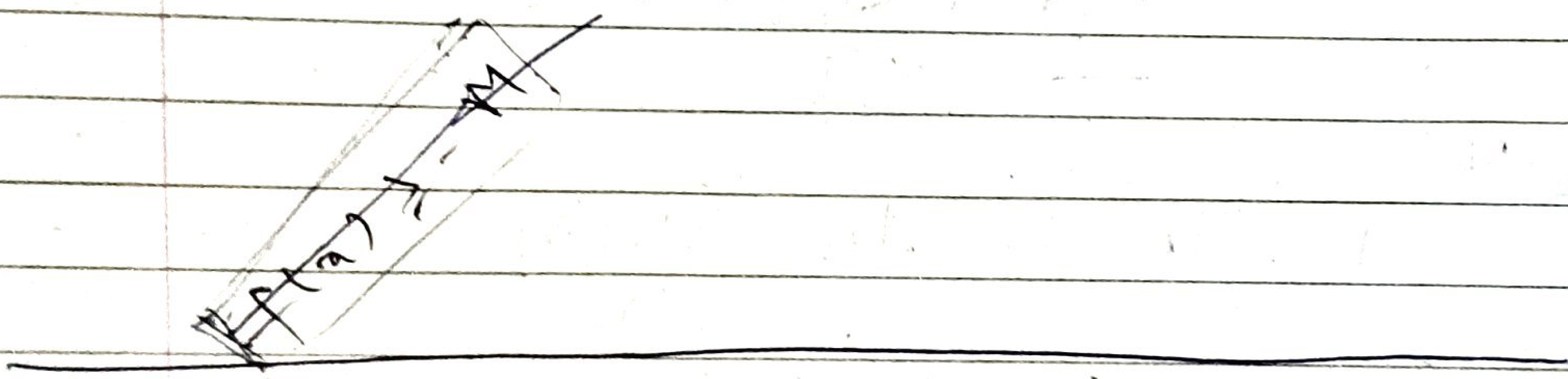
$$f(x_0) + f'(x_0)(x - x_0) \le M$$

$$A + Bx \le M \qquad \forall\, x \in (-\infty, \infty).$$

$$\boxed{Bx \le M - a}$$

$$c \qquad x \in (-\infty, \infty)$$

$$\boxed{Bx \le c} \qquad \forall\, x \in (-\infty, \infty)$$



$$f(x) \le M$$

$$f(x_0) + f'(x_0)[x - x_0] \le M$$

$$\boxed{Bx < M - A}$$ should be true $x \in (-\infty, \infty)$.

$$\boxed{B = 0}$$

$$f'(x_0) = 0$$

But $B = f'(x_0)$

So $f(x)$ is const. function.

Page No.
Date

10] $\rightarrow$ $x_1 < \frac{x_1+x_2}{2} < x_2$

Firstly taylor theorem on

x_1 & $\frac{x_1+x_2}{2}$

$$f(x) = f(x_0) + f'(x_0)[x-x_0] + \frac{f''(x_0)[x-x_0]^2}{2!} + \frac{f'''(c)[x-x_0]^3}{3!}$$

Let $x = x_1$ & $x_0 = \frac{x_1+x_2}{2}$

$$f(x_1) = f\left(\frac{x_1+x_2}{2}\right) + f'\left(\frac{x_1+x_2}{2}\right)\left[\frac{x_1-x_2}{2}\right] + K + f'''(c)\underbrace{\left[x_1 - \left(\frac{x_1+x_2}{2}\right)\right]}_{-ve}$$

$$\therefore f(x_1) \leq f\left(\frac{x_1+x_2}{2}\right) + f'\left(\frac{x_1+x_2}{2}\right)\left[\frac{x_1-x_2}{2}\right] + K. \quad \text{— (1)}$$

||ly

Taylor for

$\frac{x_1+x_2}{2}$ & x_2

$\downarrow$ $\downarrow$

x_0 x

we get

$$f(x_2) \geq f\left(\frac{x_1+x_2}{2}\right) + f'\left(\frac{x_1+x_2}{2}\right)\left[\frac{x_2-x_1}{2}\right] + t$$

$$\therefore f\left(\frac{x_1+x_2}{2}\right) + f'\left(\frac{x_1+x_2}{2}\right)\left[\frac{x_1-x_2}{2}\right] + t \leq f(x_2) \quad \text{--- (2)}$$

Add (1) & (2)

$$f(x_1) + \cancel{f\left(\frac{x_1+x_2}{2}\right)} + f'\left(\frac{x_1+x_2}{2}\right)\left(\frac{x_2-x_1}{2}\right) + \cancel{t}$$

$$\leq \cancel{f\left(\frac{x_1+x_2}{2}\right)} + f'\left(\frac{x_1+x_2}{2}\right)\left[\frac{x_1-x_2}{2}\right] + \cancel{t}$$

$f(x_2)$

$$\therefore f(x_1) - f(x_2) \leq f'\left(\frac{x_1+x_2}{2}\right)[x_2-x_1].$$