Indian Institute of Information Technology Allahabad
Linear Algebra (LAL)
Quiz 2

Program: B.Tech. 1^{st} Semester
Duration: **25 Minutes** Full Marks: 20
Date: March 04, 2022 Time: : 10 AM- 10:25 AM

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1. Let A be a 3×3 matrix such that $\text{rank}(A - I) = 1$ and $\text{rank}(A - 2I) = 2$. Then

   (a) A has three distinct eigenvalues.
   (b) The determinant of A is 2.
   (c) The rank of A is 3.
   (d) The trace of A is 4.

   Solution: (b), (c), (d)

2. Let $A \in M_n(\mathbb{R})$ be a non-zero matrix such that $A^k = 0$ for some $k \in \mathbb{N}$. Then

   (a) $\text{rank}(A)$ is at most $(n - 1)$.
   (b) the minimal polynomial of A is x.
   (c) A has only one eigenvalue with multiplicity n.
   (d) A is similar to a diagonal matrix.

   Solution: (a), (c)

3. Let A be an $n \times n$ matrix over $\mathbb{C}$ such that every non-zero vector of $\mathbb{C}^n$ is an eigenvector of A. Then which of the following is(are) correct.

   (a) All the eigenvalues of A are same.
   (b) A need not be diagonalizable.
   (c) If $\lambda(\neq 0)$ is a non-zero eigenvalue of A then $\text{rank}(A) = n$.
   (d) If λ is an eigenvalue of A, then the minimal polynomial of A is $(x - \lambda)^n$.

   Solution: (a), (c),

4. Which of the following can be the minimal polynomial of a diagonalizable matrix?

   (a) $3x^2 + 5x + 6$.
   (b) $x^3 + 3x^2 + 2x$.
   (c) $x^2 + 8x + 16$.
   (d) $x^3 - 6x^2 + 11x - 6$.

   Solution: (b), (d).

5. Let λ be an eigenvalue of a matrix A. Then which of the following will be (possible) eigenvalues of B^T and B^{-1}, where $B = A^2 + 2A + I$, respectively?

(a) $\lambda^2 + 2\lambda + 1$ and $\lambda^{-2} + 2\lambda^{-1} + 1$.
(b) $\lambda^2 + 1$ and $\lambda^{-2} + 1$.
(c) $\lambda^2 + 2\lambda + 1$ and $(\lambda^2 + 2\lambda + 1)^{-1}$.
(d) $\lambda^2 + 1$ and $(\lambda + 2\lambda + 1)^{-1}$.

Solution: (c).

6. If A=$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ and matrix B $= 10A^4 - 8A^2 + 2I_2$ then which of the following is/are **false**?

(a) Trace$(B) >$ det(B).
(b) Trace$(B) =$ det(B)
(c) B is diagonalizable.
(d) B is invertible.

Solution: (a),(b)

7. If $X_n = \{x \in \mathbb{R} \mid x = \text{trace}(A), A \in M_n(\mathbb{R}), A^2 = I_n\}$, then which of the following is/are true?

(a) $1 \in X_{10}$
(b) $1 \notin X_{20}$
(c) $0 \in X_{73}$
(d) $0 \notin X_{1001}$

Solution: (b),(d)

8. If A is a 2×2 matrix over $\mathbb{C}$ with $\det(A + I) = 1 + \det(A)$, then which of the following is/are true?

(a) trace$(A) = 0$.
(b) $A = 0$.
(c) A is non singular
(d) $\det(A) = -\lambda^2$, where λ is an eigenvalue of A.

Solution: (a),(d)

9. Let $A, B \in M_n(\mathbb{R})$. Suppose λ is an eigenvalue of A and μ is an eigenvalue of B. Then

(a) $\lambda\mu$ is an eigenvalue of AB.
(b) $\lambda + \mu$ is an eigenvalue of $A + B$.
(c) If zero is an eigenvalue of B, then zero is an eigenvalue of AB.
(d) If eigenvalues of A and B are non-zero, then eigenvalues of AB are non-zero.

Solution: (c), (d)

10. Let $A \in M_{m \times n}(\mathbb{R})$ and $m < n$. Then

(a) $A^T X = 0$ has a non-zero solution.

(b) If $AX = b$ is solvable for every $b \in \mathbb{R}^n$, then $A^T X = b$ is also solvable for every $b \in \mathbb{R}^m$.

(c) $A^T AX = 0$ has a non-zero solution.

(d) $AX = 0$ has a non-zero solution.

Solution: (c), (d)