## Indian Institute of Information Technology Allahabad Discrete Mathematical Structures C1 Review test

Program: B.Tech. 2<sup>nd</sup> Semester (IT+IB)

Duration: **60+ 10 minutes** Full Marks: 16

Date: May 22, 2022 Time:: 5:00 PM - 6:10 PM

## **Important Instructions:**

1. Attempt all the questions.

- 2. Write down your name and enrolment number. Write the solutions clearly with all the steps in details.
- 3. Submit the solution in PDF format through Google Classroom. Name the PDF as DMS-your enrolment number. We will not accept the solution through emails.
- 1. Let us define sets A and B as follows:

 $A = \{ Your first name \}$  and  $B = \{ Your last name \}$ 

In the absense of last name, take  $B = \{l, a, s, t, n, m, e\}$ 

For example: if your name is Peter Massopust, then  $A = \{p, e, t, r\}$  and  $B = \{m, a, s, o, p, u, t\}$ .

Now, define  $C = A \cup B$  and  $D = A \cap B$ . Then

- (I) Construct two distinct partial order relations (name these relations  $R_1$  and  $R_2$  respectively) on C.
- (II) Find all maximal and minimal elements of the constructed partial ordered sets  $(C, R_1)$  and  $(C, R_2)$ . [4]
- (III) Find the supremum and infimum ( if they exist) of the constructed partial ordered sets  $(C, R_1)$  and  $(C, R_2)$ . [2]
- (IV) Determine whether the following sets are finite, countably infinite (countable) or uncountable: [3]
  - (a) X = the collection of all functions from C to D.
  - (b) Y = the collection of all functions from C to  $\mathbb{N}$ , where  $\mathbb{N}$  denotes the set of natural numbers.
  - (c) Z = the collection of all functions from  $\mathbb{N}$  to C.
- 2. Let  $n \in \mathbb{N}$  and suppose we are given real numbers  $a_1 \geq a_2 \geq \ldots \geq a_n \geq 0$ . Show that Arithmetic mean  $(AM) = \frac{a_1 + a_2 + \ldots a_n}{2} \geq (a_1 a_2 \ldots a_n)^{\frac{1}{n}} = GM$  (Geometric mean). [3]