Program: B.Tech. 2nd Semester (IT+IB)
Duration: **30+ 7 minutes**   Full Marks: 10
Date: June 10, 2022   Time:: 12:15 PM - 12:52 PM

Important Instructions:

1. Attempt all the questions.

2. Write down your name and enrolment number. Write the solutions clearly with all the steps in details.

3. Submit the solution in PDF format through Google Classroom. **Name the PDF as DMS-your enrolment number**. We will not accept the solution through emails.

Let us define sets A, B and E as follows:

$$A = \{\textbf{Your first name}\}, \quad B = \{ \textbf{Your last name}\}$$

$$\text{and } E = \{ \textbf{Your state/union territory name}\}$$

In the absense of last name, take $B = \{l, a, s, t, n, m, e\}$
For example: if your name is **Peter Massopust** and you are from **Kerala**, then $A = \{p, e, t, r\}$, $B = \{m, a, s, o, p, u, t\}$ and $E = \{k, e, r, a, l\}$.

Now, define $C = A \cup B \cup E$ and $D = A \times B \times E$ (the Cartesian product of three sets A, B and E)
and **Num** = 600 - last three digits of your enrolment no.

We denote the number of elements in the set C by $|\textbf{C}|$.

(1) If $|\textbf{C}| \geq |\textbf{D}|$ then find the number of onto functions from C to D. Otherwise, find the number of onto functions from D to C. [4]

(2) How many solutions does the equation

$$x_1 + x_2 + x_3 + x_4 = \textbf{Num}$$

have, where x_1, x_2, x_3 and x_4 are non-negative integers such that $x_1 \geq 2, \; x_2 \geq 3, \; x_3 \geq 6$ and $x_4 \geq 7$? [2]

(3) Find the solution to the recurrence relation

$$x_n = \textbf{Num } x_{n-1} - |\textbf{C}| \; x_{n-2} + |\textbf{D}| \; x_{n-3}$$

with the initial conditions $x_0 = 2, \; x_1 = 3$, and $x_2 = 5$. [4]