Satellite Communication

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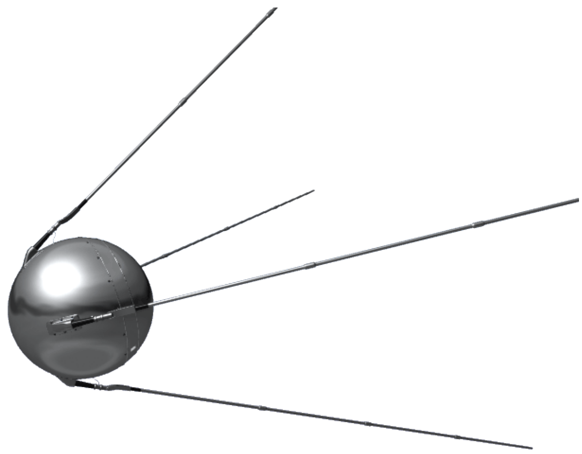

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Nomenclature

ACTS Advanced Communication Technology Satellite

ADC Analog Digital Converter

AIT Assembly Integration test

AOCS Attitude and Orbital Control System

ASIC Application Specific Integrated Circuit

AWGN Additive White Gaussian Noise

BCR Battery Charge Regulator

BDR Battery Discharge Regulator

BER Bit Error Rate

BPSK Binary Phase Shift Keying

CCITT Comitee Consultatif International du Telegraphe et du Telephone (The International Telegraph and Telephone Consultative Committee)

CCSDS Consultative Committee for Space Data Systems

CTU Central Terminal Unit

dB Decibel

DBS Digital Broadcasting Satellite

DSCS Defence Satellite Communication System

EIRP Equivalent Radiated Isotropic Power of an Antenna

ESA European Space Agency

FCC The Federal Communications Commission (FCC) is an independent agency of the United States government regulating frequency use in space among other regulating tasks.

FSK Frequency Shift Keying

FSS Fixed Satellite Service

G/S Ground Station

GEO Gesostationary Earth Orbit

GPS Global Positioning System

GTO Deostationary Transfer Orbit

LEO Low Earth Orbit

LO Local Oscillator

MEO Medium Earth Orbit

MMDS Multipoint Multichannel Distribution System

MSS Mobile Satellite Service

NASA National Aeronautics and Space Administration (NASA) is the agency of the United States government that is responsible for the nation's civilian space program and for aeronautics and aerospace research.

OFDM Othogonal Frequency Domain Multiplex

OOK On Off Keying

PCS Personal Communication Service

PM Project Management

PO Polar Orbit

QAM Quadrature Amplitude Modulation

QPSK Quadrature Phase Shift Keying

Rx Reception or Receiver

S/C Space Craft

SEU Single Event Upset

SNR Signal to Noise Ratio

SSO Sun Syncronous Orbit

SSPA Solid State Power Amplifier

TC/TM Tele Command / Telemetry

TLE Two Line Element of a satellite orbit

TT& C Telemetry, Tracking and Control

TT& R Telemetry, Telecontrol and Ranging

TWT Travelling Wave Tube

TWTA Travelling Wave Tube Amplifier

Tx Transmitter or Transmission

UHF Ultra High Frequency

VHF Very High Frequency

VSAT Very Small Aperture Antenna Terminal

1 Introduction

1.1 Overview

Earth is a sphere and radio waves propagate in straight lines.

A satellite can act as a relay station for radio signals

Or can be used as a broadcast station or a switchboard

satellites can be placed in orbits

The Orbit determines speed and movement of satellite, orbital mechanics are different from our normal understanding of movements

Geostationary orbit (35786 km) are special, by Arthur C. Clarke suggested in 1945

Disadvantage, Signals very weak and expensive to launch ($25000-$50000 per kg)

Costs are a significant factor. The cost must be recovered in the 10 to 15 years of operation

Low earth orbits are easier to handle, but satellite moves, tracking needed, constellations are needed.

Power generation only by solar cells. Hence power is limited in the Satellite

Efforts are made in the Ground stations. Up to 30 m Dishes with cryocooled receivers

In the other side now TV-Direct Broadcasting Satellites with 30 cm Dishes available

Frequencies used are 1 to 40 GHz. Rain attenuation problem at high frequencies above 10 GHz

FM is replaced now by digital modulation schemes. Higher Data rates and more flexibility

Space is a business. Planning is complicated.

1.2 History

[1] Arthur C. Clark wrote the first well-known article on communication satellites. "Extra-Terrestrial Relays" was published in Wireless World in 1945. In the article, Clark discussed geosynchronous earth orbit and the possibility of covering the earth with three satellites.

The actual journey into space began October 4, 1957, when the Soviet Union launched Sputnik 1, the world's first orbital spacecraft, which orbited the world for three months. A month later the Soviets launched Sputnik 2 and its passenger Laika, a dog who has the distinction of being the first known living creature to escape earth and enter outer space. The space race was on, and in February of 1958, the United States launched Explorer 1.

The first communication satellite was launched on December 18, 1958. Signal Communication by Orbital Relay (SCORE), which broadcasted a Christmas message from President Eisenhower - "Peace on Earth, Good will toward men" - orbited the earth for 12 days until the batteries failed. The main purpose of the SCORE project was to prove that an atlas missile could be put into orbit.

Combined, the U.S. and U.S.S.R. launched six satellites in 1958, 14 satellites in 1959, 19 in 1960 and 35 in 1961. In 1962, the United Kingdom and Canada launched satellites of their own, along with the 70 satellites launched by the U.S. and U.S.S.R.

On August 12, 1960, the United States launched Echo 1, a passive reflector satellite with no amplification possibilities. Echo 1 could only reflect the radiation back to earth. At the time of its launch, it was thought that passive reflector satellites could serve a purpose in communications, but the technology was soon abandoned.

Bell Telephone Laboratories assisted in the Echo 1 project. Knowledge gained working on Echo 1 helped Bell to develop Telstar, an experimental satellite that relayed television signals. Telstar was launched into medium earth orbit in 1962. In the six months following the launch, stations in the United States, Britain and France conducted about 400 transmissions with multi-channel telephone, telegraph, facsimile and television signals, and they performed over 250 technical tests and measurements.

Near complete Earth coverage (excluding polar areas) was achieved with the development of Intelsat and the launching of satellites into geosynchronous earth orbit over the Atlantic (1965), Pacific (1967), and Indian oceans (1969). A combination of more than 130 governments and international organization control Intelsat. Intelsat, along with Inmarsat, which is used in international shipping, is open to use by all nations. The Intelsat consortium owns the satellites, but each nation owns their own earth stations. In 1997 Intelsat had 19 satellites in geostationary orbit.

NASA led the new wave of communication satellite technology with the launch of Advanced Communications Technology Satellites (ACTS) in 1993. ACTS pioneered the use of spot beams, on-board storage and processing, and all digital transmission, which combined made a successful communication satellite constellation more feasible. Each of these innovations serve a certain technological purpose that makes and internet in the sky more likely.

Spot beams subdivide a satellite's footprint which allows the satellite to use its portion of the spectrum more efficiently On-board storage and processing allows for inter-satellite communication and the caching of information until a spot beam finds its target All-digital transmission allows a satellite to incorporate error codes into its signal which helps to overcome rain fade.

Following the breakthrough, several corporations decided to get invest in broadband satellites. In 1997 the FCC gave permission to 13 companies to use a portion of the sky and a portion of the electromagnetic for their satellites systems and their signals. Among the companies was Hughes, Loral, Motorola, EchoStar and Teledesic, a company with the backing of Bill Gates and Craig McCaw.

Not all of these companies will be able to make it in space, and Motorola has already failed with their 66 satellite system, Iridium, which was supposed to provide mobile telephone service.

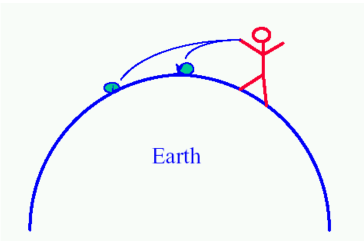
Since Sputnik more than 4500 Satellites have been launched.

References

[1] Coffey. [Online]. Available: http://iml.jou.ufl.edu/projects/Fall99/Coffey

Orbital Mechanics 2

2.1 Principles of Orbital Mechanics



2.2 Kepler's Laws

Newton's law of gravity is

$$F = \frac{G\,m_1 m_2}{r^2} \tag{2.1}$$

where G =the universal gravitational constant $6.6730 \cdot 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}$

In case of $m_1$being the earth with $m_1 = M = 5.9742 \cdot 10^{24}$kg we get the gravitational force on a body with mass m as

$$F = \frac{G\,M\,m}{r^2} \tag{2.2}$$

since G and M cannot be determined very accurately , but the product

$$GM = \mu = 3.986 \cdot 10^{14}\,\frac{\mathrm{m}^3}{\mathrm{s}^2} \tag{2.3}$$

is called standard gravitational parameter and is known very accurately. We have

$$F = \frac{\mu\,m}{r^2} \tag{2.4}$$

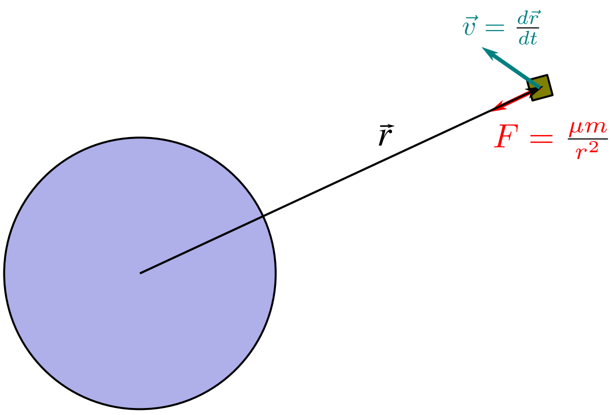


Figure 2.1: Equilibrium of Forces

We can interpret this equation as Newton's law of acceleration $F = ma$. Where $a = \mu/r^2$. At the surface of the earth ($r = R_{earth} = 6,378.135$ km) this acceleration has the value 9.80665ms^{-2}.

Vectorial Description

Using a vector description we have for Newton's law of acceleration

$$\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2} \tag{2.5}$$

where r is the vector leading to the small mass m in space. The big mass M is in the origin of the coordinate system.

When $M \gg m$ the acceleration is in the opposite direction of $\vec{r}$, hence we can write

$$\begin{aligned} \vec{F}_{grav} &= \vec{F}_{acc} && (2.6)\\ -m\frac{\mu}{r^2}\frac{\vec{r}}{r} &= m\frac{d^2\vec{r}}{dt^2} && (2.7)\\ m\frac{d^2\vec{r}}{dt^2} + m\frac{\mu}{r^2}\frac{\vec{r}}{r} &= 0 && (2.8) \end{aligned}$$

This is the differential equation for the movement in space. Using the dot-notation for derivatives with respect to time t we get the form

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = 0 \tag{2.9}$$

The solution of this differential equation are tracks in the shape of a conic section (circle, ellipse, parabola, hyperbola), where the big mass M is in the focus or centre. The major semi-axis is called a. The eccentricity is called e.

A conic section , or just conic , is a curve formed by passing a plane through a right circular cone. As shown in the figure to the right, the angular orientation of the plane relative to the cone determines whether the conic section is a circle, ellipse, parabola, or hyperbola . The circle and the ellipse arise when the intersection of cone and plane is a bounded curve.

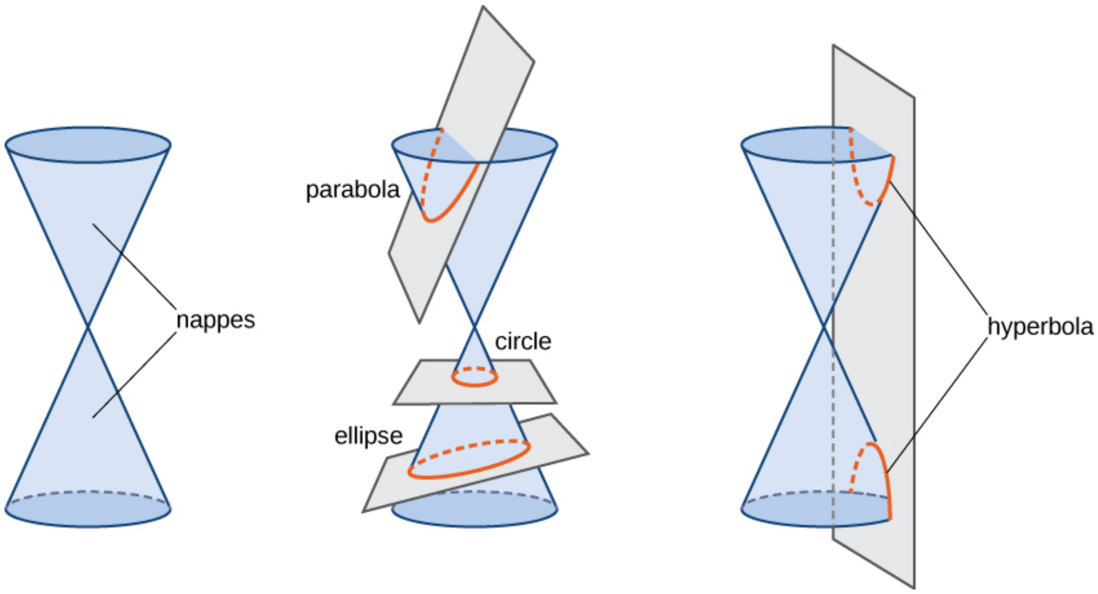


Figure 2.2: Conic Sections, from https://philschatz.com/calculus-book/contents/m53846.html

Conic Section	Eccentricity e	Semi-major axis a
Circle	0	$= r$ (Radius)
Ellipse	$0 < e < 1$	> 0
Parabola	1	∞
Hyperbola	> 1	< 0

Table 2.1: Types of Conic Section Curves

- The **circle** is a special case of the **ellipse** in which the plane is perpendicular to the axis of the cone.
- If the plane is parallel to a generator line of the cone, the conic is called a **parabola**.
- Finally, if the intersection is an unbounded curve and the plane is not parallel to a generator line of the cone, the figure is a **hyperbola**.

In the latter case the plane will intersect both halves of the cone, producing two separate curves. We can define all conic sections in terms of the eccentricity. The type of conic section is also related to the semi-major axis and the energy. The table below shows the relationships between eccentricity, semi-major axis, and energy and the type of conic section.

In polar coordinates the satellite follows the path

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \tag{2.10}$$

in the plane. The eccentricity describes the deviation from the exact circle as seen in Table 2.1.

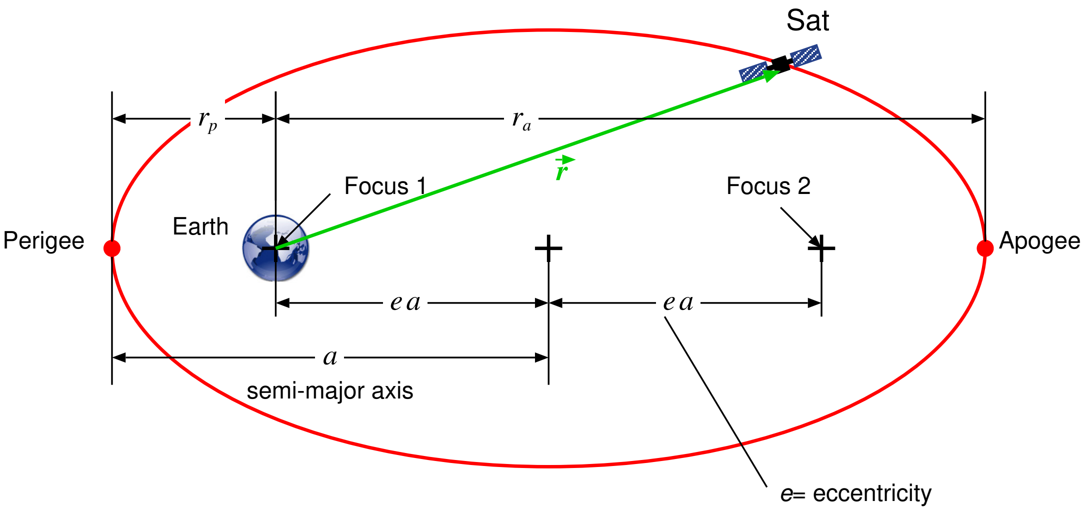


Figure 2.3: Path of Satellite as an Ellipse, Perigee Distance $r_p = a(1-e)$ and Apogee Distance $r_a = a(1+e)$

```
a=1.
t=arange(0,2*pi+0.02,0.02)
for e in [0,0.1,0.5,0.8,0.9]:
        r=a*(1-e**2)/(1+e*cos(t))
        polar(t,r,label='e='+str(e))
        legend()
savefig('/home/speik/soridat/vorlesung/asc/linkbudget/ellipseplot.svg')
close()
```

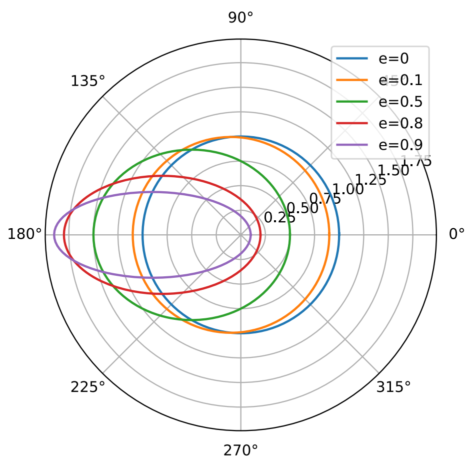


Figure 2.4: Plots of Elliptical Paths following Eqn. 2.10

2.2.1 Numerical calculation and Animation

We can solve the diff. eqn of eqn. 2.9 using Eulers method for 2nd order differential equations.

The sat moves following the diff-eqn

$$\frac{d^2\vec{r}}{dt^2} + \frac{\mu}{r^3}\vec{r} = 0$$

we can rearrange this in several eqations

$$\frac{d^2\vec{r}}{dt^2} = -\frac{\mu}{r^3}\vec{r}$$

$$\frac{d\vec{r}}{dt} = \vec{v}$$

Now we can move in small steps Δt and iterate

$t_0 = 0, \vec{r}_0 = \text{Startingpoint}, v_0 = \text{startvelocity}$

$$t_{i+1} = t_i + \Delta t$$

$$\vec{r}_{i+1} = \vec{r_i} + \Delta t\, \vec{v}_i$$

$$\vec{v}_{i+1} = \vec{v_i} + \Delta t \frac{-\mu}{r^3}\vec{r}$$

```python
import matplotlib.pyplot as plt
from numpy import array, arange, sqrt
from time import sleep
### constants ####
mu = 3.986e14
Rearth = 6378e3
dt = 5.0 # time steps in sec
tdeci = 40  # decimation for sector markings

### defining initial conditions
### Apogee dist, Perigee Dist., initial velocity, arg. of periapsis, zoom value
scene = {}

scene['iss']     = (Rearth,Rearth+640e3, 182,0,5)
scene['example'] = (Rearth,30000e3,5e-2,60,2) # point example
scene['geo']     = (42164e3,42164e3,0,0,1) # GEO Orbit
scene['molniya'] = (Rearth+500e3,40000e3,0,78,1) #
scene['hohmann'] = (Rearth,42164e3,3074.66-1576.14,40,1) # GTo to GEO
scene['extreme'] = (Rearth+300,70000e3,0,45,1) # GTo to GEO

setscene = 'hohmann' ### <<=== SELECT A SCENE HERE
drawspokes = True    ### Select drawing of spokes for equal time traces


#################################################
rp,ra,deltav, arg_periapsis, zoom = scene[setscene]
omega = arg_periapsis*pi/180
vcir=sqrt(mu/ra)
print("vcir=",vcir)
va = sqrt(2*mu/ra-2*mu/(ra+rp))
vp = sqrt(2*mu/rp-2*mu/(ra+rp))
print("vp=",vp)
print("va=",va)
a = (ra+rp)/2
## Start in Apogee
```

```
r0 = array([-rp*cos(omega),-rp*sin(omega)])
v0 = array([vp*sin(omega),-vp*cos(omega)])

## Orbital Period
T = 2*pi*sqrt(a**3/mu)
print('Orbital Period=',T/60/60,'h')
points_per_orbit = 60
dt = T/points_per_orbit/ 250
r = r0
v = v0
fig = plt.figure(1,figsize=(10,10))
ax2 = plt.subplot()
ax2.add_patch(plt.Circle((0, 0), radius=Rearth/1e3, ec='black', fc='blue'))
ax2.set_ylim(-60000/zoom,60000/zoom)
ax2.set_xlim(-60000/zoom,60000/zoom)
ax2.spines['left'].set_position('center')
ax2.spines['right'].set_color('none')
ax2.spines['bottom'].set_position('center')
ax2.spines['top'].set_color('none')
ax2.text(40000/zoom,-10000/zoom,"Dist. in km")
ax2.grid()
satloc, = ax2.plot(r[0]/1e3,r[1]/1e3,'or',ms=10)
trace, =  ax2.plot(0,0,'r')
rlist = [r]
tlist = [0]
count = 0
boost = False
tracex = []
tracey = []
for t in arange(0.,100*T,dt):
    tlist.append(t)
    rnew = r + dt * v
    rmag = sqrt(rnew[0]**2+rnew[1]**2)
    vnew = v + dt * (-mu/rmag**3 * rnew)
    r = rnew
    v = vnew
    tracex.append(r[0]/1e3)
    tracey.append(r[1]/1e3)
    count += 1
    ## Add a boost after half a period
    if t>T/2 and not boost and deltav>0:
        vboost = deltav*v/sqrt(v[0]**2+v[1]**2)
        v+= vboost
        ax2.plot(r[0]/1e3,r[1]/1e3,"*y",ms=15)
        boost = True
    if count%200==0:
        satloc.set_xdata([r[0]/1e3])
        satloc.set_ydata([r[1]/1e3])
        if t<=T and drawspokes:
            ax2.plot([0,r[0]/1e3],[0,r[1]/1e3],'y')
            pass
        trace.set_xdata(tracex)
        trace.set_ydata(tracey)
        fig.canvas.draw()
        #print(t,rnew)
        #sleep(0.001)
```

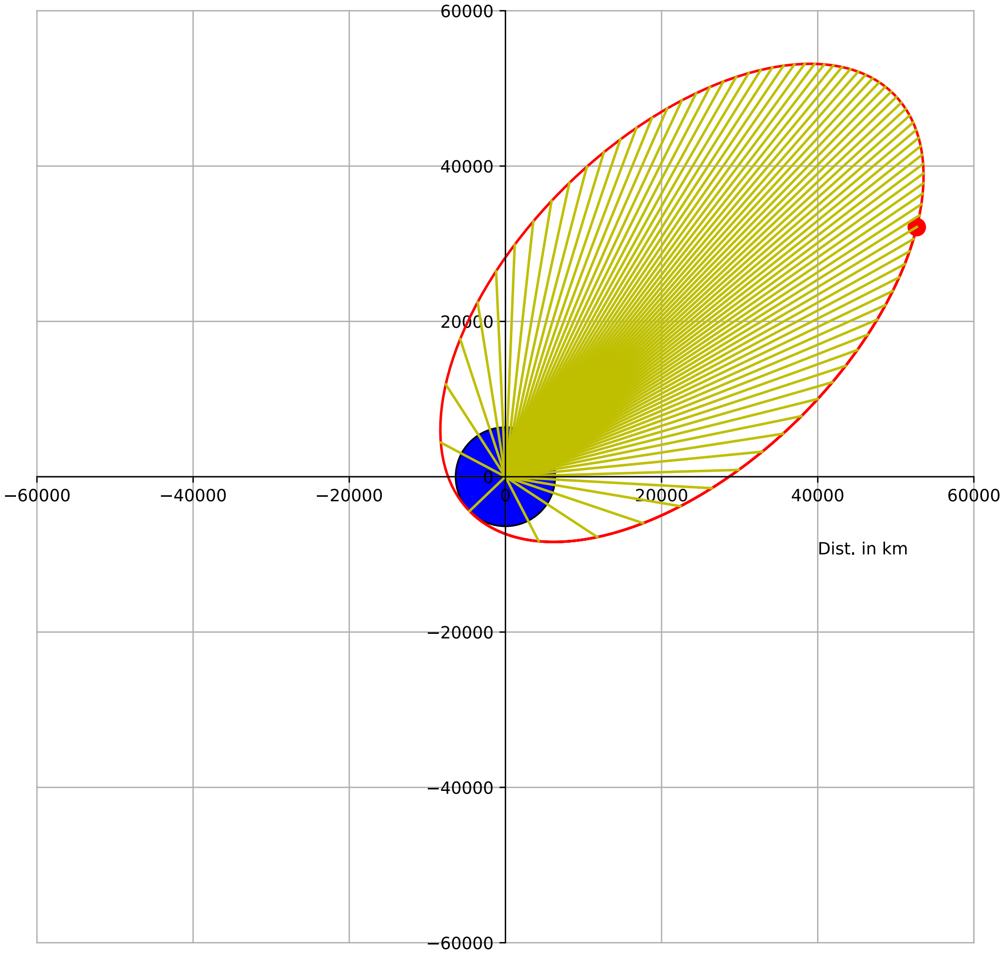


Figure 2.5: Orbit path generated by the python differential eqn. solver above

2.3 Kepler's Laws of planetary motion

German mathematician and astronomer Johannes Kepler (1571–1630) discovered the above relationships empirically around 1605 by analyzing the detailed astronomical observations of Tycho Brahe.

In their classic formulation they are:

1. The orbit of every planet is an ellipse with the Sun at a focus.

2. A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.

3. The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.

Isaac Newton was able to derive Kepler's relationships from Newton's own laws of motion and law of universal gravitation, using classical Euclidean geometry.

2.4 Orbit velocity

Let us reconsider eqn. 2.11 and multiply by $\dot{\vec{r}}$

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = 0 \tag{2.11}$$

$$\dot{\vec{r}}\ddot{\vec{r}} + \frac{\mu}{r^3}\dot{\vec{r}}\vec{r} = 0 \tag{2.12}$$

Since the velocity v is the first derivative of the distance r, i.e. $v = \dot{\vec{r}}$ we get

$$\vec{v}\,\dot{\vec{v}} + \frac{\mu}{r^3}\vec{r}\,\dot{\vec{r}} = 0 \tag{2.13}$$

$$v\,\dot{v} + \frac{\mu}{r^3}r\,\dot{r} = 0 \tag{2.14}$$

$$\frac{d}{dt}\left(\frac{v^2}{2}\right) + \frac{d}{dt}\left(-\frac{\mu}{r}\right) = 0 \tag{2.15}$$

$$\frac{d}{dt}\left(\frac{v^2}{2} - \frac{\mu}{r}\right) = 0 \tag{2.16}$$

Now we can see that $\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} =$ const. $= -\frac{\mu}{2a}$, where ε describes the specific energy of the movement. Hence

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \tag{2.17}$$

Where a is the semi-major axis, r is the distance of the satellite from the centre of the earth, and v is the velocity at that point. Following for the instantaneous velocity of the satellite in the orbit being

$$v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} \tag{2.18}$$

2.4.1 Uniform Circular Motion

for a uniform circular movement we have $a = r$ and hence

$$v_{circ} = \sqrt{\frac{\mu}{r}} \tag{2.19}$$

Orbit period

$$\boxed{T = 2\pi\sqrt{\frac{r^3}{\mu}} \tag{2.20}}$$

or

$$T^2 = 4\pi^2\frac{r^3}{\mu} \tag{2.21}$$

which is Kepler's third law. This is a basic equation of planetary and satellite motion. It also holds for elliptical orbits if we define r to be the semi-major axis (a) of the orbit.

For the circular motion we can also define an angular velocity called the mean motion

$$n = \frac{2\pi}{T} \tag{2.22}$$

Example 1:

International Space Station $h = 640$km above ground

With the Earth radius 6378 km follows

$v = \sqrt{\frac{\mu}{R_{earth}+h}} = 7531\,\frac{\text{m}}{\text{s}}$

$T = 2\pi\sqrt{\frac{r^3}{\mu}} = 98\,\text{min}$

python code:

```
mu = 3.98e14
r = 6378e3 + 640e3
v = sqrt(mu/r)
T = 2*pi*sqrt(r**3/mu)
print("v=",round(v)," m/s    ")
print("T=",round(T/60)," min")
```

```
v= 7531 m/s T= 98 min
```

Example 2:

Geostationary Earth Orbit GEO requires a Period time of one sidereal day[1] 23 hours, 56 minutes, 4.1 seconds (T =86,164.1 seconds).

Hence the radius follows from eqn. 2.21

$$r^3 = \frac{\mu T^2}{4\pi^2} \tag{2.23}$$

$$r = \sqrt[3]{\frac{\mu T^2}{4\pi^2}} = 4.2164 \cdot 10^7 \mathrm{m} = 42164\mathrm{km} \tag{2.24}$$

The velocity in orbit is

$$v = \sqrt{\frac{\mu}{r}} = 3074.7 \frac{\mathrm{m}}{\mathrm{s}} \tag{2.25}$$

The angular velocity of the GEO satellite is

$n = \frac{2\pi}{T} = 7.292 \cdot 10^{-5} \frac{\mathrm{rad}}{\mathrm{s}}$

2.5 Escape Velocity

Remember

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \tag{2.26}$$

now for $a \to \infty$ the ellipse becomes a parabola. and

$$\frac{v^2}{2} - \frac{\mu}{r} = 0 \tag{2.27}$$

$$v^2 = \frac{2\mu}{r} \tag{2.28}$$

This is the so called escape velocity

$$v = \sqrt{\frac{2\mu}{r}} \tag{2.29}$$

Example 3:

for the **Earth** we get $v_{esc} = 11.2$km/s on the ground (when $r = R_{earth}$)

In **9,000 km altitude** in "space," it is already less than 7.1 km/s.

The escape velocity on the **surface of the Moon** is 2.4 km/s

2.6 Elliptical Orbits

In an elliptical orbit the orbital period can be found equal to the circular orbit assuming a as the radius:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{2.30}$$

2.6.0.1 Calculating Perigee and Apogee Velocities

Since $a = (r_p + r_a)/2$ we get for the apogee

$$v_{apo} = \sqrt{\frac{2\mu}{r_a} - \frac{2\mu}{r_p + r_a}} \tag{2.31}$$

and for the perigee

$$v_{per} = \sqrt{\frac{2\mu}{r_p} - \frac{2\mu}{r_p + r_a}} \tag{2.32}$$

where r_p and r_a are the distances to the centre of the Earth in perigee and apogee, respectively.

2.7 Orbit Changes

2.7.1 Launch in Orbit

In order to launch something into a circular orbit with radius r_2, we use an elliptical transfer orbit as shown in Figure 2.6.

First we accelerate the object on the ground to the required perigee velocity

$$v_{per} = \sqrt{\frac{2\mu}{r_p} - \frac{2\mu}{r_p + r_a}} \tag{2.33}$$

with $r_p = R_{earth} = 6360$km

When the object is in the apogee point of the transfer orbit we accelerate a second time to “circlerise” the orbit

We find the apogee velocity first

$$v_{apo} = \sqrt{\frac{2\mu}{r_a} - \frac{2\mu}{r_p + r_a}} \tag{2.34}$$

and the required circular orbit velocity

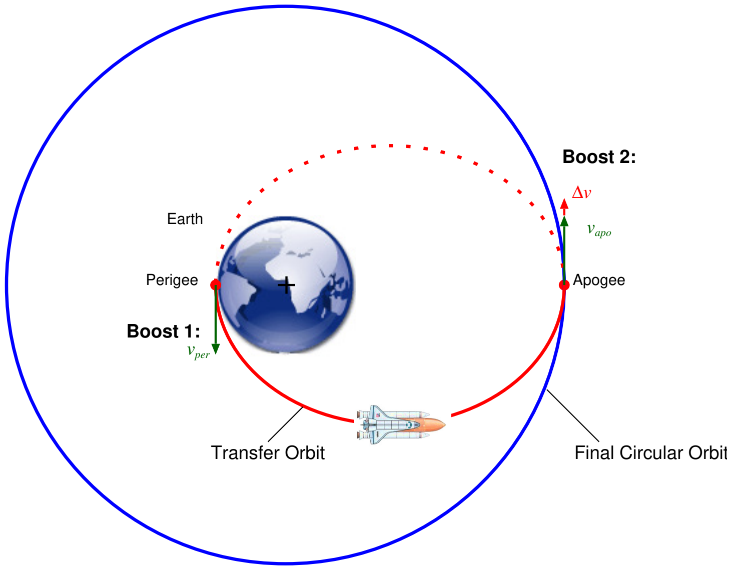


Figure 2.6: Hohmann Transfer from Ground to Circular Orbit

$$v_{circ} = \sqrt{\frac{\mu}{r_a}} \tag{2.35}$$

the velocity difference gives us the required velocity change

$$\Delta v = v_{circII} - v_{apo} = \sqrt{\frac{\mu}{r_a}} - \sqrt{\frac{2\mu}{r_a} - \frac{2\mu}{r_p + r_a}} \tag{2.36}$$

In real situations the launch is not parallel to the ground. In order to escape the friction losses of the atmosphere (so called atmospheric drag), we launch vertically and manoeuvre into horizontal movement outside the atmosphere around 30 km height.

Example 4:

We launch the Space Shuttle from the ground $r_p = R_e = 6378\text{km}$ into the ISS orbit $r_a = R_e + 640\text{km}$

Required launch velocity: $v_{per} = \sqrt{\frac{2\mu}{r_p} - \frac{2\mu}{r_p+r_a}} = 8092.1\,\frac{\text{m}}{\text{s}}$

Resulting velocity in apogee point: $v_{apo} = \sqrt{\frac{2\mu}{r_2} - \frac{2\mu}{r_p+r_a}} = 7354.1\,\frac{\text{m}}{\text{s}}$

Required velocity for ISS circular orbit: $v_{circ} = \sqrt{\frac{\mu}{r_a}} = 7536.4\,\frac{\text{m}}{\text{s}}$

Additional velocity for changing from transfer orbit into circular orbit: $\Delta v = V_{circ} - v_{apo} = 182.23\,\frac{\text{m}}{\text{s}}$

```
mu=3.986e14
rp=6378000
ra=7018000
vper=sqrt(2*mu/rp-2*mu/(rp+ra))
vapo=sqrt(2*mu/ra-2*mu/(rp+ra))
vcir=sqrt(mu/ra)
delv=vcir-vapo
print(vper,vapo,vcir,delv)
```

2.7.2 Hohmann Transfer

The transfer is similar to the launch from the ground. We accelerate in the lower orbit with Δv_1 and again in the higher orbit with Δv_2.

2.7.3 Orbital Plane Change

To change the orientation of a satellite's orbital plane, typically the inclination, we must change the direction of the velocity vector. This manoeuvre requires a component of V to be perpendicular to the orbital plane and, therefore, perpendicular to the initial velocity vector. If the size of the orbit remains constant, the manoeuvre is called a simple plane change . We can find the required change in velocity by using the law of cosines. For the case in which V_f is equal to V_i , this expression reduces to

$$\Delta v = 2v_i \sin\frac{\theta}{2} \tag{2.37}$$

where Vi is the velocity before and after the burn, and is the angle change required.

From equation (3.64) we see that if the angular change is equal to 60 degrees, the required change in velocity is equal to the current velocity. Plane changes are very expensive in terms of the required change in velocity and resulting propellant consumption. To minimize this, we should change the plane at a point where the velocity of the satellite is a minimum: at apogee for an elliptical orbit. In some cases, it may even be cheaper to boost the satellite into a higher orbit, change the orbit plane at apogee, and return the satellite to its original orbit.

Typically, orbital transfers require changes in both the size and the plane of the orbit, such as transferring from an inclined parking orbit at low altitude to a zero-inclination orbit at geosynchronous altitude. We can do this transfer in two steps: a Hohmann transfer to change the size of the orbit and a simple plane change to make the orbit equatorial. A more efficient method (less total change in velocity) would be to combine the plane change with the tangential burn at apogee of the transfer orbit. As we must change both the magnitude and direction of the velocity vector, we can find the required change in velocity using the law of cosines,

$$\Delta v = \sqrt{v_i^2 + v_f^2 - 2v_i v_f \cos\theta} \tag{2.38}$$

where V_i is the initial velocity, V_f is the final velocity, and is the angle change required. Note that equation (3.65) is in the same form as equation (3.60). Click here for example problem #3.20 As can be seen from equation (3.65), a small plane change can be combined with an altitude change for almost no cost in V or propellant. Consequently, in practice, geosynchronous transfer is done with a small plane change at perigee and most of the plane change at apogee. Another option is to complete the manoeuvre using three burns. The first burn is a co-planar manoeuvre placing the satellite into a transfer orbit with an apogee much higher than the final orbit. When the satellite reaches apogee of the transfer orbit, a combined plane change manoeuvre is done. This places the satellite in a second transfer orbit that is coplanar with the final orbit and has a perigee altitude equal to the altitude of the final orbit. Finally, when the satellite reaches perigee of the second transfer orbit, another coplanar manoeuvre places the satellite into the final orbit. This three-burn maneuver may save propellant, but the propellant savings comes at the expense of the total time required to complete the maneuver.

2.8 Orbital Elements

To mathematically describe an orbit one must define five quantities, called orbital elements . They are

- **Eccentricity e:**
  shape of the ellipse, describing how much it is elongated compared to a circle
- **Semi-major axis a:**
  the sum of the periapsis and apoapsis distances divided by two.
- **Inclination i:**
  vertical tilt of the ellipse with respect to the reference plane, measured at the ascending node (where the orbit passes upward through the reference plane)
- **Argument of periapsis ω:**
  defines the orientation of the ellipse in the orbital plane, as an angle measured from the ascending node to the periapsis
- **Right Ascension of ascending node Ω:**
  horizontally orients the ascending node of the ellipse (where the orbit passes upward through the reference plane) with respect to the reference frame's vernal point

A sixth element is necessary for spotting the satellite with respect to time:

- **Mean anomaly at epoch** M_0:
  defines the position of the orbiting body along the ellipse at a specific time (the "epoch").

Celestial longitude is analogous to longitude on Earth and is measured in degrees counterclockwise from zero with zero longitude being in the direction of the *vernal equinox*[2]. In general, three observations of an object in orbit are required to calculate the six orbital elements. Two other quantities often used to describe orbits are *period* and *true anomaly*. Period , P , is the length of time required for a satellite to complete one orbit. True anomaly ,v , is the angular distance of a point in an orbit past the point of periapsis, measured in degrees.

2.8.1 Two-Line Element Sets

A **Two-Line Element set** (**TLE**) is a set orbital elements that describe the orbit of an earth satellite. A computer program called a *model* can use the TLE to compute the precise position of a satellite at a particular time. The TLE is in a format specified by NORAD and used by NORAD and NASA. Orbital elements are determined for many thousands of space objects by NORAD and are freely distributed on the Internet in the form of TLEs. A TLE consists of a title line followed by two lines of formatted text.

[2]At equinox the Sun will cross directly over the Earth's equator. This moment is known as the vernal equinox (=spring point) in the Northern Hemisphere. For the Southern Hemisphere, this is the moment of the autumnal equinox. Equinox literally means "equal night". On the vernal (spring) and autumnal (fall) equinoxes, day and night are the same length.
The Direction of the line sun-vernal equinox is the direction of vernal equinox.

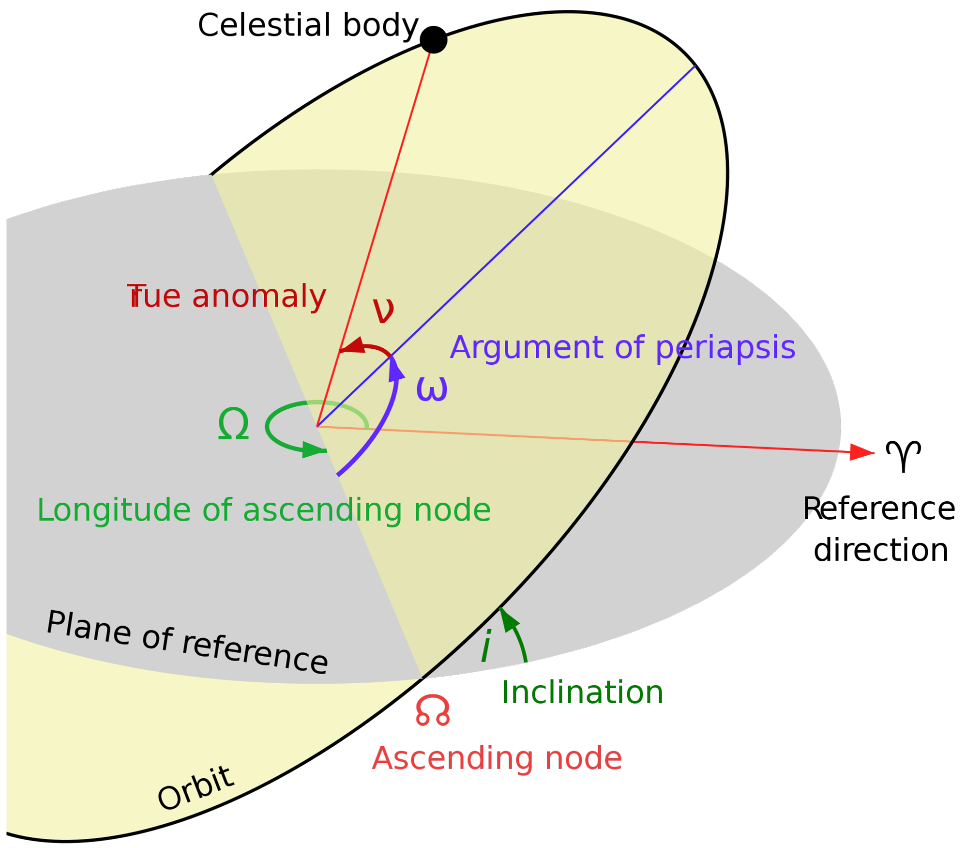
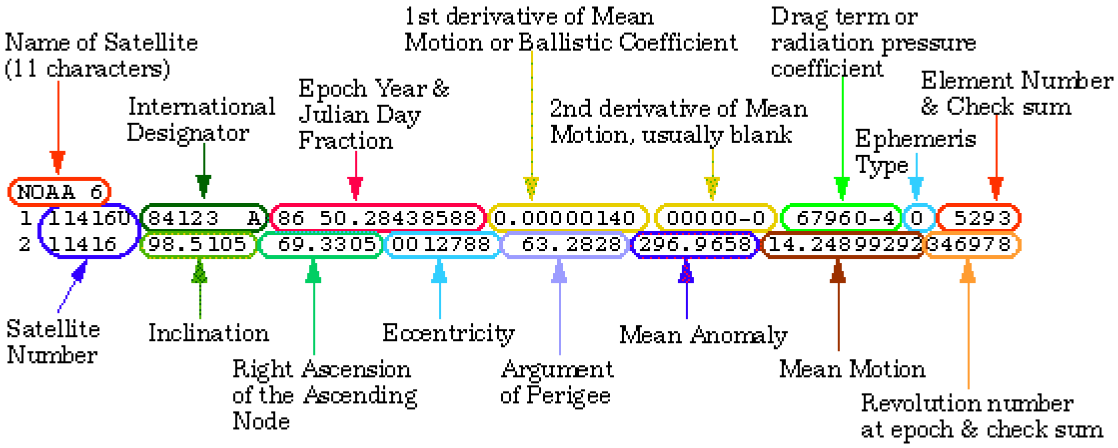


Figure 2.7: The Orbital Parameters, source: wikipedia

Format

The following is an example of a TLE (for a NOAA Satellite)

From NASA's Definition of Two-line Element Set Coordinate System



The meaning of this data is as follows:

Title:

Field	Columns	Content	Example
1	01-24	Satellite name	ISS (ZARYA)

LINE 1:

Field	Columns	Content	Example
1	01-01	Line number	1
2	03-07	Satellite number	25544
3	08-08	Classification (U=Unclassified)	U
4	10-11	International Designator (Last two digits of launch year)	98
5	12-14	International Designator (Launch number of the year)	067
6	15-17	International Designator (Piece of the launch)	A
7	19-20	Epoch Year (Last two digits of year)	08
8	21-32	Epoch (Day of the year and fractional portion of the day)	264.51782528
9	34-43	First Time Derivative of the Mean Motion divided by two	-.00002182
10	45-52	Second Time Derivative of Mean Motion divided by six	00000-0
11	54-61	BSTAR drag term (decimal point assumed)	-11606-4
12	63-63	The number 0 (Originally "Ephemeris type")	0
13	65-68	Element number	292
14	69-69	Checksum (Modulo 10)	7

LINE 2:

Field	Columns	Content	Example
1	01-01	Line number	2
2	03-07	Satellite number	25544
3	09-16	Inclination [Degrees]	51.6416
4	18-25	Right Ascension of the Ascending Node [Degrees]	247.4627
5	27-33	Eccentricity (decimal point assumed)	0006703
6	35-42	Argument of Perigee [Degrees]	130.5360
7	44-51	Mean Anomaly [Degrees]	325.0288
8	53-63	Mean Motion [Revs per day]	15.72125391
9	64-68	Revolution number at epoch [Revs]	56353
10	69-69	Checksum (Modulo 10)	7

The checksums for each are calculated by adding the numerical digits on that line. One is added to the checksum for each negative sign (-) on that line. All other characters are ignored.

2.9 Types Of Orbits

(taken from [1])

For a spacecraft to achieve Earth orbit, it must be launched to an elevation above the Earth's atmosphere and accelerated to orbital velocity. The most energy efficient orbit, that is one that requires the least amount of propellant, is a direct low inclination orbit. To achieve such an orbit, a spacecraft is launched in an eastward direction from a site near the Earth's equator. The advantage being that the rotational speed of the Earth contributes to the spacecraft's final orbital speed. At the United States launch site in Cape Canaveral (28.5 degrees north latitude) a due east launch results in a "free ride" of 1,471 km/h (914 mph).

Launching a spacecraft in a direction other than east, or from a site far from the equator, results in an orbit of higher inclination. High inclination orbits are less able to take advantage of the initial speed provided by the Earth's rotation, thus the launch vehicle must provide a greater part, or all, of the energy required to attain orbital velocity. Although high inclination orbits are less energy efficient, they do have advantages over equatorial orbits for certain applications. Below we describe several types of orbits and the advantages of each:

Geosynchronous orbits (GEO) are circular orbits around the Earth having a period of 24 hours. A geosynchronous orbit with an inclination of zero degrees is called a geostationary orbit . A spacecraft in a geostationary orbit appears to hang motionless above one position on the Earth's equator. For this reason, they are ideal for some types of communication and meteorological satellites. A spacecraft in an inclined geosynchronous orbit will appear to follow a regular figure-8 pattern in the sky once every orbit. To attain geosynchronous orbit, a spacecraft is first launched into an elliptical orbit with an apogee of 35,786 km (22,236 miles) called a *geosynchronous transfer orbit (GTO)*. The orbit is then circularized by firing the spacecraft's engine at apogee.

Polar orbits (PO) are orbits with an inclination of 90 degrees. Polar orbits are useful for satellites that carry out mapping and/or surveillance operations because as the planet rotates the spacecraft has access to virtually every point on the planet's surface.

Walking orbits : An orbiting satellite is subjected to a great many gravitational influences. First, planets are not perfectly spherical and they have slightly uneven mass distribution. These fluctuations have an effect on a spacecraft's trajectory. Also, the sun, moon, and planets contribute a gravitational influence on an orbiting satellite. With proper planning it is possible to design an orbit which takes advantage of these influences to induce a precession in the satellite's orbital plane. The resulting orbit is called a walking orbit , or precessing orbit.

Sun synchronous orbits (SSO) are walking orbits whose orbital plane precesses with the same period as the planet's solar orbit period. In such an orbit, a satellite crosses periapsis at about the same local time every orbit. This is useful if a satellite is carrying instruments which depend on a certain angle of solar illumination on the planet's surface. In order to maintain an exact synchronous timing, it may be necessary to conduct occasional propulsive manoeuvres to adjust the orbit.

Molniya orbits are highly eccentric Earth orbits with periods of approximately 12 hours (2 revolutions per day). The orbital inclination is chosen so the rate of change of perigee is zero, thus both apogee and perigee can be maintained over fixed latitudes. This condition occurs at inclinations of 63.4 degrees and 116.6 degrees. For these orbits the argument of perigee is typically placed in the southern hemisphere, so the satellite remains above the northern hemisphere near apogee for approximately 11 hours per orbit. This orientation can provide good ground coverage at high northern latitudes.

Hohmann transfer orbits are interplanetary trajectories whose advantage is that they consume the least possible amount of propellant. A Hohmann transfer orbit to an outer planet, such as Mars, is achieved by launching a spacecraft and accelerating it in the direction of Earth's revolution around the sun until it breaks free of the Earth's gravity and reaches a velocity which places it in a sun orbit with an aphelion equal to the orbit of the outer planet. Upon reaching its destination, the spacecraft must decelerate so that the planet's gravity can capture it into a planetary orbit. To send a spacecraft to an inner planet, such as Venus, the spacecraft is launched and accelerated in the direction opposite of Earth's revolution around the sun (i.e. decelerated) until it achieves a sun orbit with a perihelion equal to the orbit of the inner planet. It should be noted that the spacecraft continues to move in the same direction as Earth, only more slowly. To reach a planet requires that the spacecraft be inserted into an interplanetary trajectory at the correct time so that the spacecraft arrives at the planet's orbit when the planet will be at the point where the spacecraft will intercept it. This task is comparable to a quarterback "leading" his receiver so that the football and receiver arrive at the same point at the same time. The interval of time in which a spacecraft must be launched in order to complete its mission is called a launch window .

2.10 Kepler's Problem

Kepler's problem is to find the position of the satellite for a given orbit and time.

Using the orbital parameters

- Semi-Major Axis, a
- Eccentricity, e
- Inclination, i
- Argument of Periapsis (=angle of perigee) ω

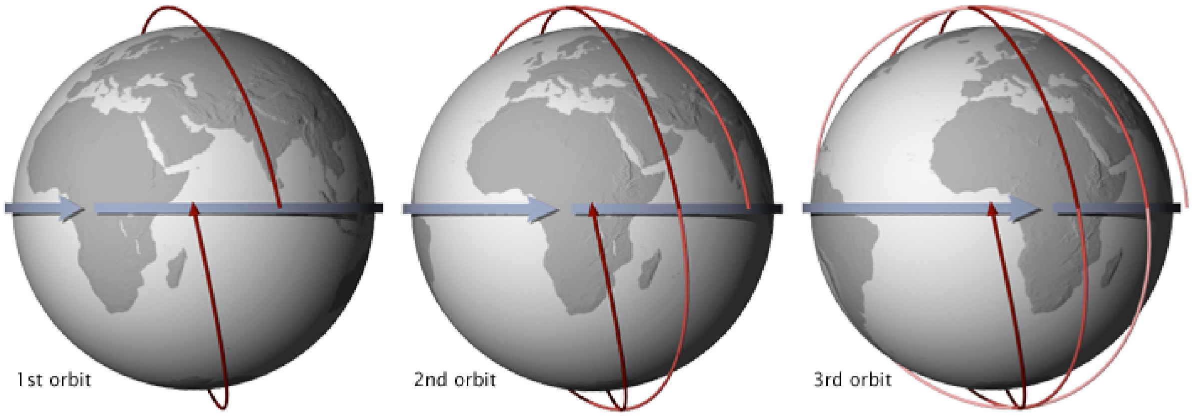


Figure 2.8: Sunsyncronous Orbit, from: `http://earthobservatory.nasa.gov`

- Right Ascension of Ascending Node Ω
- time of perigee passage

Details can be found in [2].

2.10.1 Anomalies

In an ellipse we can define different angles called anomalies

The eccentric anomaly E is the angle between the direction of periapsis and the point p and locating its intercept p with the auxiliary circle, a circle of radius a (the semi-major axis of the ellipse) that inscribes the entire ellipse.

The mean anomaly M of an orbiting body is the angle the body would have travelled about the centre of the orbit's auxiliary circle. Unlike other measures of anomaly, the mean anomaly grows linearly with time.

The true anomaly ν is the angle between the direction of periapsis and the current position p of an object on its orbit, measured at the focus s of the ellipse (i.e. centre of Earth). This is the angle usually of interest, because it can be used to locate the satellite in the sky

using the angle ν (true anomaly) between the direction of the perigee and the direction of the satellite we can find the Eccentric anomaly using eqn. (2.20d) from [4]

$$\cos E = \frac{(\cos\nu + e)}{1 + e\cos\nu} \tag{2.39}$$

We can define a *mean movement*

$$n = \frac{2\pi}{T} \tag{2.40}$$

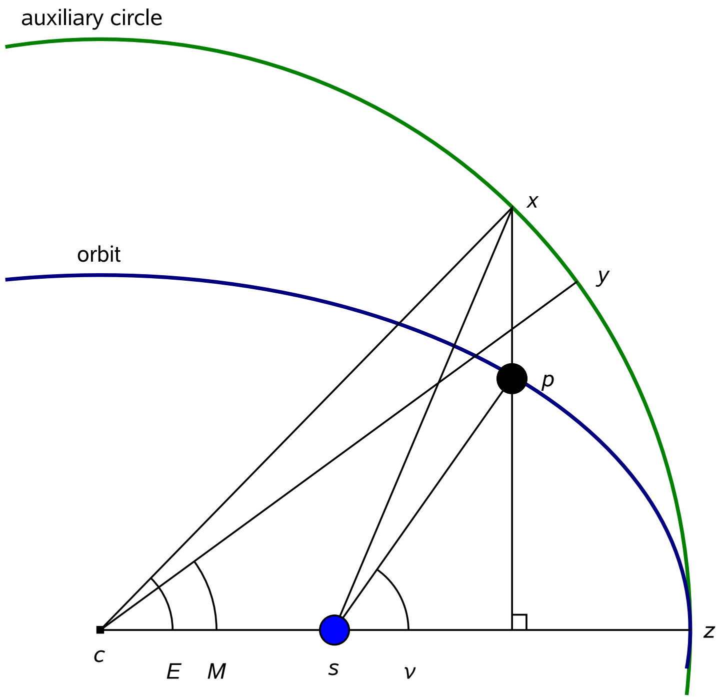


Figure 2.9: Definition of Anomalies

and a mean anomaly

$$M = \frac{2\pi}{T}(t - t_p) = n(t - t_p) \tag{2.41}$$

where t_p is the instant of passing through the perigee. The mean anomaly is related to the eccentric anomaly by Kepler's equation

$$M = E - e \sin E \tag{2.42}$$

We will see later, that we have to find E from M. There is no analytic solution to this problem. We have to use numerical methods , i.e. root finding algorithms, to find $E(M)$. In Figure 2.10 is the function for $E(M)$ and $\nu(M)$ for an eccentricity of $e = 0.7$.

The true anomaly is related to the eccentric anomaly by

$$\tan \frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}} \cdot \tan \frac{E}{2} \tag{2.43}$$

Conversely, the eccentric anomaly E is related to the true anomaly ν by:

$$\tan \frac{E}{2} = \sqrt{\frac{1-e}{1+e}} \cdot \tan \frac{\nu}{2} \tag{2.44}$$

The distance r of the satellite from the centre of the Earth can now be written as using eqn. (2.21) from [4]

$$r = a(1 - e \cos E) \tag{2.45}$$

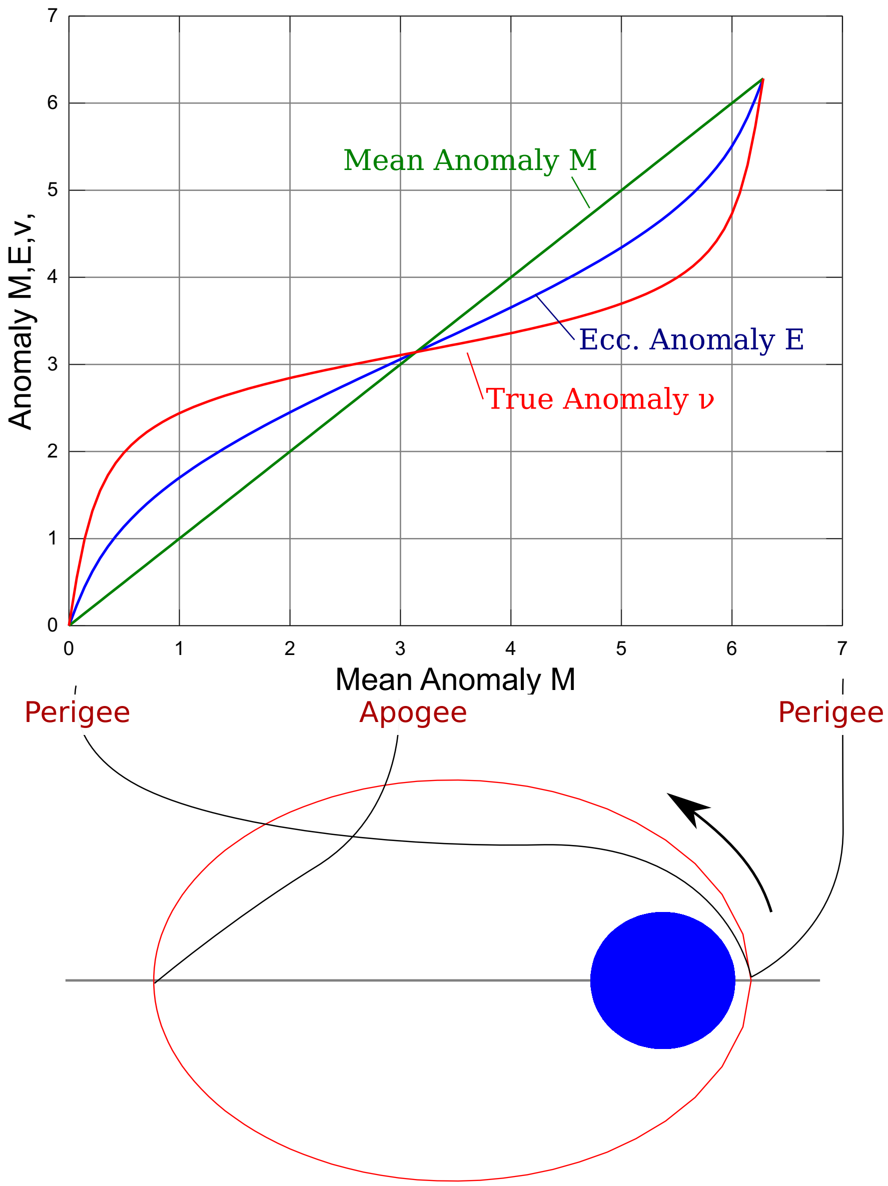


Figure 2.10: Trace of the Anomalies for a Molniya orbit with an eccentricity of $e = 0.7$

2.10.2 Finding the Movement of the Sat

We can now apply an algorithm to calculate the movement of the satellite in the ellipse over time

1. From the instantaneous time t we can find the mean movement n and the mean anomaly M using eqn. 2.41
2. From M we can find the eccentric anomaly E. Note, E must be found by iterating the eqn. 2.42.
3. From E we can find the true anomaly ν directly from eqn. 2.43.
4. The distance from the centre of the earth is eqn. 2.45
5. We can now plot in polar coordinates the ellipse in polar coordinates with $\phi = \nu$ and $r = r$. An example is given in Figure 2.11.

These steps can be be performed in Python by:

```
### Timing ################################################
T=2*pi*sqrt(a**3/mu) # Orbital Period
n=2*pi/T            # Mean Velocity
M=arange(0,endangle,2*pi/(steps-1)) # Mean Movement
t=M*T/2/pi; # timeline in seconds

### Solve M ###############################################
E=[]
for Mrun in M:
   func= lambda E:  E-e*sin(E)-Mrun
   Erun = fsolve(func,2)
   E.append(Erun[0])
E=asarray(E)
### Find Anomalies ########################################
v=2*arctan((((1+e)/(1-e))**0.5)*tan(E/2));
v=unwrap(v);
print("found True Anomaly Vector")
phi= arcsin(sin(i)*sin(omega+v));                  ##### Maral 2.39;
lamda=arctan(tan(omega+v)*cos(i))-((OmegaE/n)*E*rotate);  ##### Maral 2.38b
print("found SubSat Vector ")
```

2.11 Time Reference

The sidereal day is the time for one complete rotation of the earth, i.e. period of earth rotation

$$T_{sid} = 23\text{h}\,56\,\text{min}\,4.1\,\text{s} = 86164.1\text{s} \tag{2.46}$$

In contrast the solar day is the commonly known

$$T_{solar} = 24\text{h} = 86400\text{s} \tag{2.47}$$

Sidereal day an solar day differ, because the earth moves around the sun with a mean value of 0.9856^o per day.

For more details see [4] Section 2.1.5.5 .

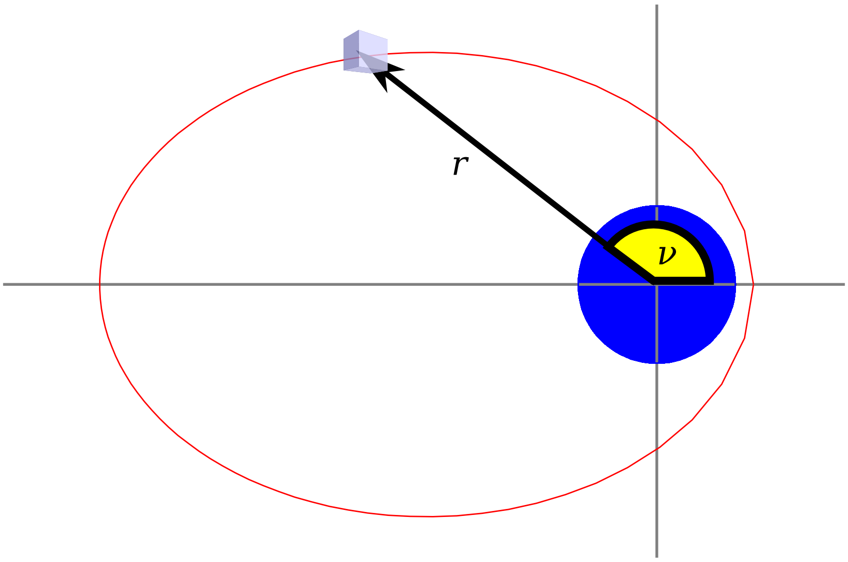


Figure 2.11: Polar Coordinate Plot of Satellite Track

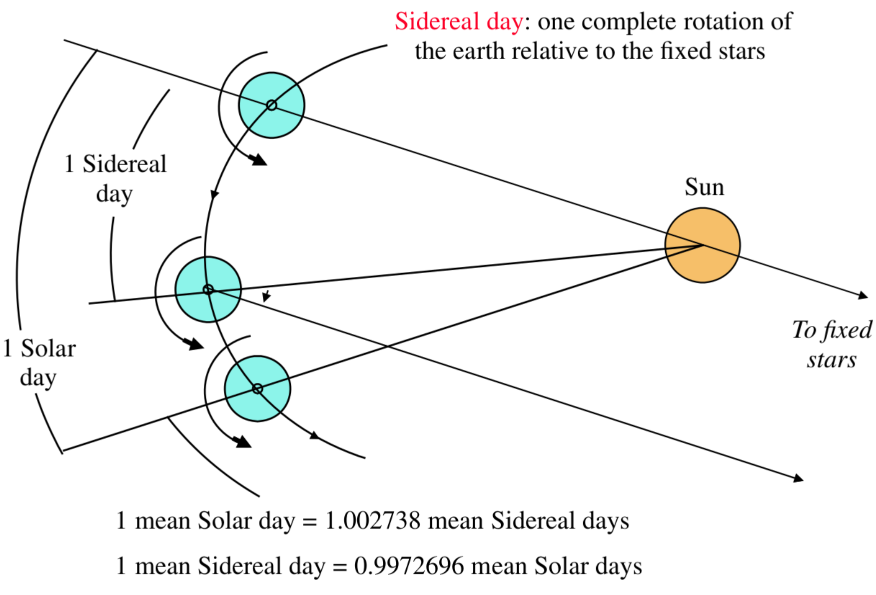


Figure 2.12: Sideral time and sideral day, from

2.12 Sub Satellite Point

The sat period is using eqn (2.19) from [4]

$$T = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{2.48}$$

with the *mean movement* from (2.22) from [4]

$$n = \frac{2\pi}{T} \tag{2.49}$$

and the velocity using (2.18b) from [4]

$$v = \sqrt{\frac{\mu}{a}} \tag{2.50}$$

Latitude of Sat is using (2.39) from [4]

$$\boxed{\varphi = \arcsin\left[\sin(i) \cdot \sin(\omega + \nu)\right] \qquad (2.51)}$$

and the Longitude is (2.38b) from [4]

$$\boxed{\lambda = \arctan\left[\tan(\omega + \nu) \cdot \cos(i)\right] - \left[\frac{\Omega_E}{n}(E - e \cdot \sin E) \quad \underbrace{-\frac{\Omega_E}{n} \cdot (E_N - e \cdot sinE_N)}_{\text{Offset due to launch time}}\right] \qquad (2.52)}$$

where

- i is the inclination of the orbit
- ω is the argument of perigee of the orbit
- ν is the true anomaly
- Ω_e is the angular velocity of the earth ($\Omega_e = \frac{2\pi}{T_{sideral}} = 7.2921 \cdot 10^{-5} \frac{\text{rad}}{\text{s}} = 6.300388097 \frac{\text{rad}}{\text{day}}$)
- E is the eccentric anomaly of the satellite at time t and
- E_N is the eccentric anomaly on passing through the ascending node
- $n = \frac{2\pi}{T}$ is the mean movement with the orbital period T

λ and φ can be expressed as a function of only one of the anomalies E or ν using one of equation. the time t associated with this anomaly E can be found from eqn. 2.41 and 2.42.

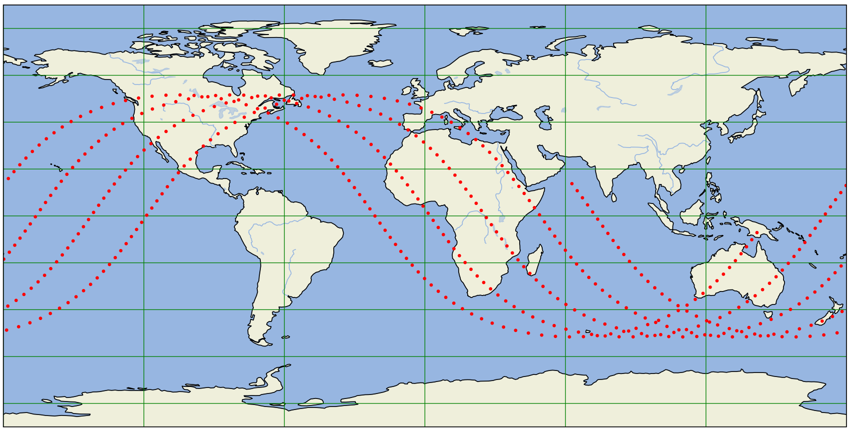


Figure 2.13: Track of ISS using Python, Cartopy and Ephem

2.12.1 Sub Satellite Point for Circular Orbits

For circular orbits the eccentricity is zero and $E = \nu = M$ and the eccentricity is $e = 0$ and ω is irrelevant. Hence

$E = \nu = M = \frac{2\pi}{T}(t - t_p)$

Further $E_N$is not relevant, as there is no defined angle between ascending node an perigee.

and the latitude of Sat is

$$\varphi = \arcsin\left[\sin(i) \cdot \sin(M)\right] \tag{2.53}$$

and the Longitude is (2.38b) from [4]

$$\lambda = \arctan\left[\tan(M) \cdot \cos i\right] - \frac{\Omega_E}{n} M \tag{2.54}$$

where

$$M = \frac{2\pi}{T}(t - t_p) \tag{2.55}$$

2.12.2 Distance and Azimuth and Elevation from an Observation Point

When we observe the satellite in the sky the can determine the direction in terms of an azimuth angle A and an elevation angle E and a distance R.

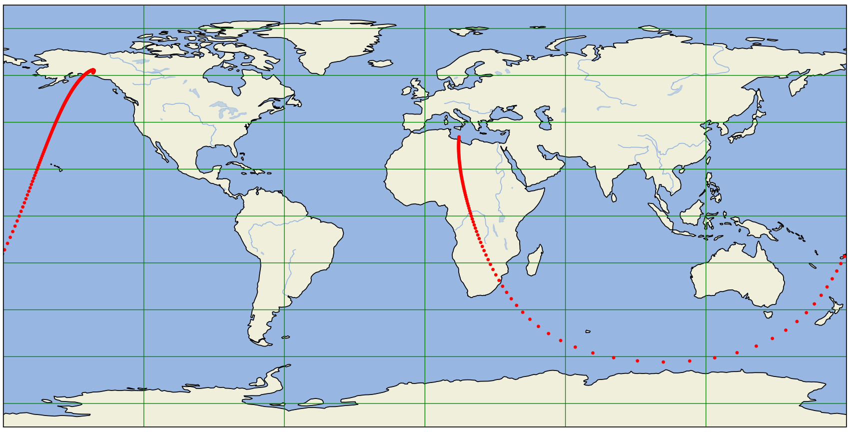


Figure 2.14: Track of a Molniya Satellite using Python, Cartopy and Ephem

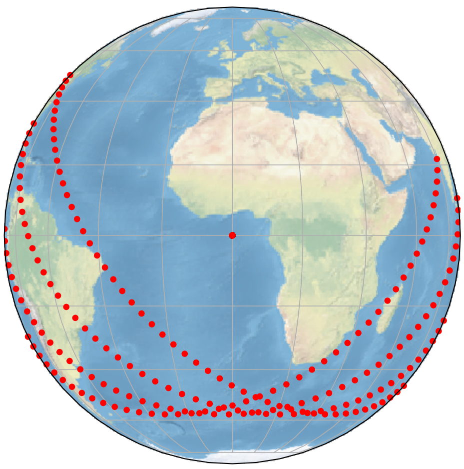


Figure 2.15: Track of ISS using Python, Cartopy and Ephem

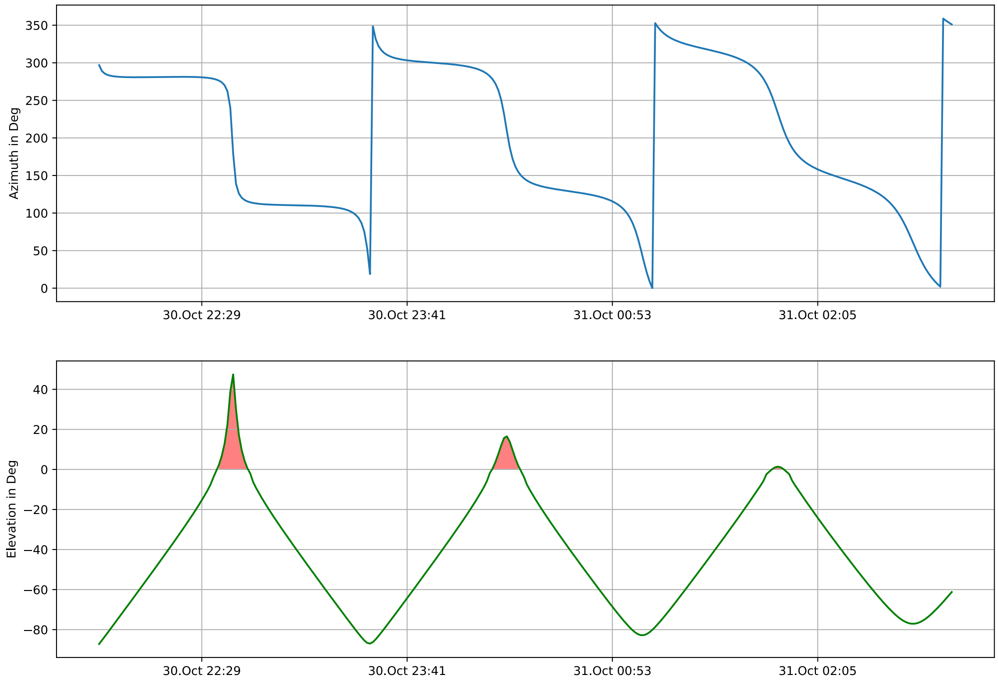


Figure 2.16: Azimuth and Elevation of a Satellite using Python with Ephem, Visible Sections marked red

Let us assume the observation point at longitude ψ and latitude l and a satellite with a sub-satellite point at longitude λ and latitude φ. Further we define the difference of longitude as

$$L = |\psi - \lambda| \tag{2.56}$$

Distance

Now, the distance observer-satellite is using cosine law as shown in eqn.(2.40) of [4]

$$R = \sqrt{R_E^2 + r^2 - 2R_E\, r\, \cos\phi} \tag{2.57}$$

with R_E being the radius of the Earth and

$$\cos\phi = \cos L \cdot cos\varphi \cos l + \sin\varphi \cdot \sin l \tag{2.58}$$

Example 5:

We observe the satellite Hotbird 6 at Bremen.

The coordinates of Bremen are ψ =8.8^{o}East and l =53.0^o North

Hotbird is geostationary ($i = 0$ and $\varphi = 0$) with $\lambda = 13^o$East, $a = 42164$km

We have $L = \psi - \lambda = |8.8 - 13| = 3.2^o$

and:$\cos\phi = \cos L \cdot cos\varphi \cos l + \sin\varphi \cdot \sin l = \cos(-3.2) \cdot 1 \cdot \cos(53^o) + 0 = 0.60088$

Hence the distance is

$$\begin{aligned} R &= \sqrt{R_E^2 + r^2 - 2R_E\, r\, \cos\phi} && (2.59)\\ &= \sqrt{6,378^2 + 42164^2 - 2 \cdot 6,378 \cdot 42164 \cdot 0.60088} && (2.60)\\ &= 38,670\text{km} && (2.61) \end{aligned}$$

The satellite is 38,670 km away from Bremen

Elevation

The angle between horizon and the satellite in the sky is called the *elevation* E of the satellite. Using (2.43a) of [4] we get for the elevation

$$E = \arccos(\frac{r}{R}\sin\phi) \tag{2.62}$$

where $\sin\phi = \sqrt{1 - \cos^2\phi}$. Another form is

$$\tan E = \frac{\cos\phi - \frac{R_E}{r}}{\sin\phi} \tag{2.63}$$

where R_E is the radius of the Earth.

Position of the Sub-Sat Point with Respect to observation point	Azimuth A
South-East	$A = 180^o - a$
North-East	$A = a$
South-West	$A = 180^o + a$
North-West	$A = 360^o - a$

Table 2.2: Determination of Azimuth Angles

Azimuth

The horizontal angle between the North direction on the ground and the satellite position is called the Azimuth direction A. Using (2.44) of [4] we get

$$\sin a = \frac{\sin L \cos \varphi}{\sin \phi} \tag{2.64}$$

and hence

$$a = \arcsin\left(\frac{\sin L \cos \varphi}{\sin \phi}\right) \quad \text{when } \phi > 0 \text{ and } L > 0 \tag{2.65}$$

For other angles of ϕ and L with have to add extra constant angles as shown in Table

Example 6:

We observe the satellite Hotbird 6 at Bremen, again.

With the intermediate result $\sin \phi = \sqrt{1 - \cos^2 \phi} = \sqrt{1 - 0.60088^2} = 0.79934$.

Elevation is

$$\begin{aligned} E &= \arccos(\frac{r}{R} \sin \phi) && (2.66) \\ &= \arccos(\frac{42,164}{38,670} 0.79934) && (2.67) \\ &= 29.359^0 && (2.68) \end{aligned}$$

Double Check the Result by googleing Hotbird, Elevation for Bremen!

Azimuth:

$$\begin{aligned} \sin a &= \frac{\sin L \cos \varphi}{\sin \phi} && (2.69) \\ \sin a &= \frac{\sin(3.2)1}{0.79934} = 0.069834 && (2.70) \\ a &= 4^o && (2.71) \end{aligned}$$

Since the satellite is south-east

$$A = 180 - (+4) = 176^o \tag{2.72}$$

Nadir Direction

see [4] eqn 2.45c

$$\sin\theta = (\cos E)R_E/r \tag{2.73}$$

2.12.3 Doppler Effect

The Doppler shift in frequency is

$$\Delta f = \frac{V_r f}{c} \tag{2.74}$$

see Maral 2.1.6.5 for details

2.13 Perturbations

The orbital elements discussed provide an excellent reference for describing orbits. However there are other forces acting on a satellite that perturb it away from the nominal orbit. These perturbations, or variations in the orbital elements, can be classified based on how they affect the Keplerian elements.

The main perturbations are

- Third-Body Perturbations due to the Moon and the Sun
- Non Spherical Earth

Third-Body Perturbations

(taken from [1])

The gravitational forces of the Sun and the Moon cause periodic variations in all of the orbital elements, but only the longitude of the ascending node, argument of perigee, and mean anomaly experience secular variations. These variations arise from a gyroscopic precession of the orbit about the ecliptic pole. The variation in mean anomaly is much smaller than the mean motion and has little effect on the orbit, however the variations in longitude of the ascending node and argument of perigee are important, especially for high-altitude orbits.

For nearly circular orbits the equations for the secular rates of change resulting from the Sun and Moon are

Drift of Longitude of the ascending node:

$$\Delta\Omega_{moon} = -0.00338\cos(i)/n \tag{2.75}$$

$$\Delta\Omega_{sun} = -0.00154\cos(i)/n \tag{2.76}$$

Drift of Argument of perigee:

$$\Delta\omega_{moon} = 0.00169(4 - 5\sin^2 i)/n \quad (2.77)$$
$$\Delta\omega_{sun} = 0.00077(4 - 5\sin^2 i)/n \quad (2.78)$$

where i is the orbit inclination, n is the number of orbit revolutions per day, and $\Delta\Omega$ and $\Delta\omega$ are in degrees per day. These equations are only approximate; they neglect the variation caused by the changing orientation of the orbital plane with respect to both the Moon's orbital plane and the ecliptic plane.

2.13.1 Perturbations due to Non-spherical Earth

(taken from [**?**])

When developing the two-body equations of motion, we assumed the Earth was a spherically symmetrical, homogeneous mass. In fact, the Earth is neither homogeneous nor spherical. The most dominant features are a bulge at the equator, a slight pear shape, and flattening at the poles. For a potential function of the Earth, we can find a satellite's acceleration by taking the gradient of the potential function. The most widely used form of the geopotential function depends on latitude and geopotential coefficients, J_n, called the zonal coefficients.

The potential generated by the non-spherical Earth causes periodic variations in all the orbital elements. The dominant effects, however, are secular variations in longitude of the ascending node and argument of perigee because of the Earth's oblateness, represented by the J_2 term in the geopotential expansion. The rates of change of Ω and ω due to J_2 are

$$\frac{d\Omega}{dt} = -\frac{3}{2}J_2\left(\frac{R_e}{a(1-e^2)}\right)^2 n\cos(i) \quad (2.79)$$

$$\frac{d\omega}{dt} = \frac{3}{4}J_2\left(\frac{R_e}{a(1-e^2)}\right)^2 n(4 - 5\sin^2(i)) \quad (2.80)$$

where n is the mean motion in degrees/day, J_2 has the value 0.00108263, R_E is the Earth's equatorial radius, a is the semi-major axis in kilometres, i is the inclination, e is the eccentricity, and Ω and ω are in degrees/day.

For satellites in GEO and below, the J_2 perturbations dominate; for satellites above GEO the Sun and Moon perturbations dominate.

Molniya orbits are designed so that the perturbations in argument of perigee are zero. This conditions occurs when the term 4-5sin2i is equal to zero or, that is, when the inclination is either 63.4 or 116.6 degrees.

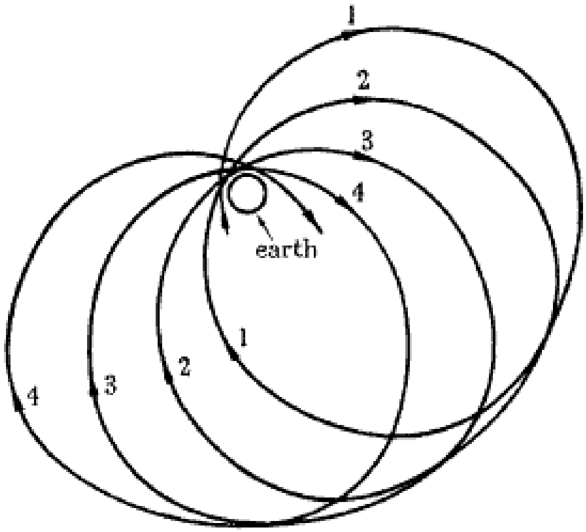


FIGURE 45. Sketch of an orbit in the equatorial plane. The rate of rotation of the major axis is exaggerated (about fifty times) for clarity.

Perturbations

The fact that the Earth is not a sphere but an ellipsoid causes the orbit of a satellite to be perturbed

Geopotential function:

$\Phi(r,L) = (\mu/r) \bullet [1 - \sum J_n(R_E/r)^n P_n(\sin L)]$

where:

L is latitude

R_E is the Earth radius at Equator

P_n is the Legendre polynomials

J_n are dimensionless coefficients:

$J_2 = 0.00108263$

$J_3 = -0.00000254$

$J_4 = -0.00000161$

...

Orbit	Effect of J_2 (Eqs. 6-19, 6-20) (deg/day)	Effect of Moon (Eqs. 6-14, 6-16) (deg/day)	Effect of Sun (Eqs. 6-15, 6-17) (deg/day)
Shuttle	a = 6700 km, e = 0.0, i = 28 deg		
$\Delta\Omega$	−7.35	−0.000 19	−0.000 08
$\Delta\omega$	12.05	0.002 42	0.001 10
GPS	a = 26,600 km, e = 0.0, i = 60.0 deg		
$\Delta\Omega$	−0.033	−0.000 85	−0.000 38
$\Delta\omega$	0.008	0.000 21	0.000 10
Molniya	a = 26,600 km, e = 0.75, i = 63.4 deg		
$\Delta\Omega$	−0.30	−0.000 76	−0.000 34
$\Delta\omega$	0.00	0.000 00	0.000 00
Geosynchronous	a = 42,160 km, e = 0, i = 0 deg		
$\Delta\Omega$	−0.013	−0.003 38	−0.001 54
$\Delta\omega$	0.025	0.006 76	0.003 07

$\Delta\Omega$ is the drift of the ascending node
$\Delta\omega$ is the drift of periapsis / perigee

2.14 Atmospheric Drag

(taken from [1])

Drag is the resistance offered by a gas or liquid to a body moving through it. A spacecraft is subjected to drag forces when moving through a planet's atmosphere. This drag is greatest

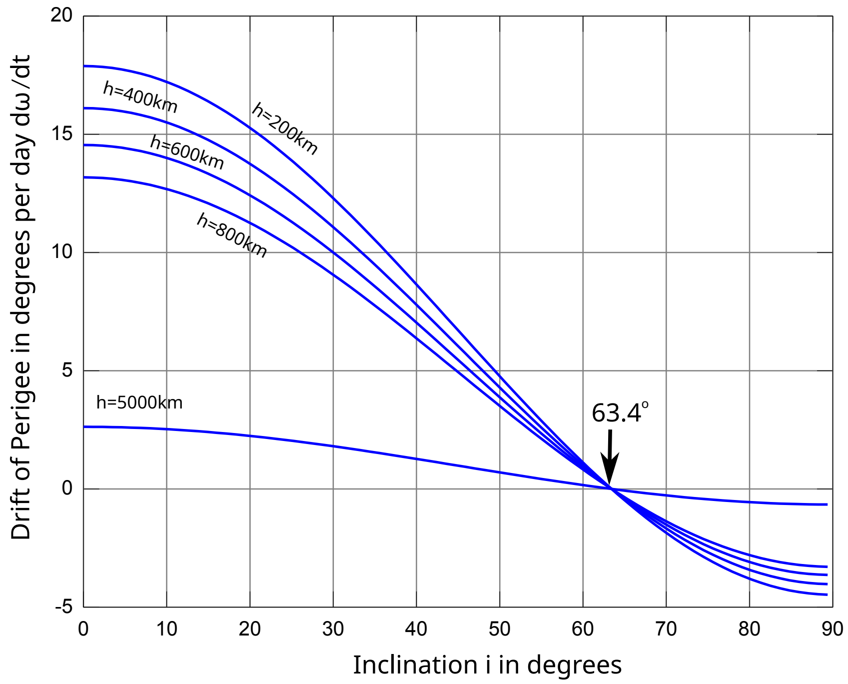


Figure 2.17: Drift of Perigee due to the not-spherical shape of the earth

during launch and reentry, however, even a space vehicle in low Earth orbit experiences some drag as it moves through the Earth's thin upper atmosphere. In time, the action of drag on a space vehicle will cause it to spiral back into the atmosphere, eventually to disintegrate or burn up. If a space vehicle comes within 120 to 160 km of the Earth's surface, atmospheric drag will bring it down in a few days, with final disintegration occurring at an altitude of about 80 km. Above approximately 600 km, on the other hand, drag is so weak that orbits usually last more than 10 years - beyond a satellite's operational lifetime. The deterioration of a spacecraft's orbit due to drag is called decay.

The drag force F_D on a body acts in the opposite direction of the velocity vector and is given by the equation

$$F_D = \frac{1}{2} C_D \rho v^2 A \tag{2.81}$$

where C_D is the drag coefficient,ρ is the air density,v is the body's velocity, and$A$ is the area of the body normal to the flow. The drag coefficient is dependent on the geometric form of the body and is generally determined by experiment. Earth orbiting satellites typically have very high drag coefficients in the range of about 2 to 4.

The density, pressure and temperature of air changes with height by a model created by the NASA[3]

$$T_{celcius} = \begin{cases} 15.04° - 0.00649 \cdot h & \text{for } h < 11\text{km} \\ -56.46° & \text{for } 11\text{km} < h < 25\text{km} \\ -131.21° + 0.00299 \cdot h & \text{for } h > 25\text{km} \end{cases} \tag{2.82}$$

[3] https://www.grc.nasa.gov/www/k-12/airplane/atmosmet.html

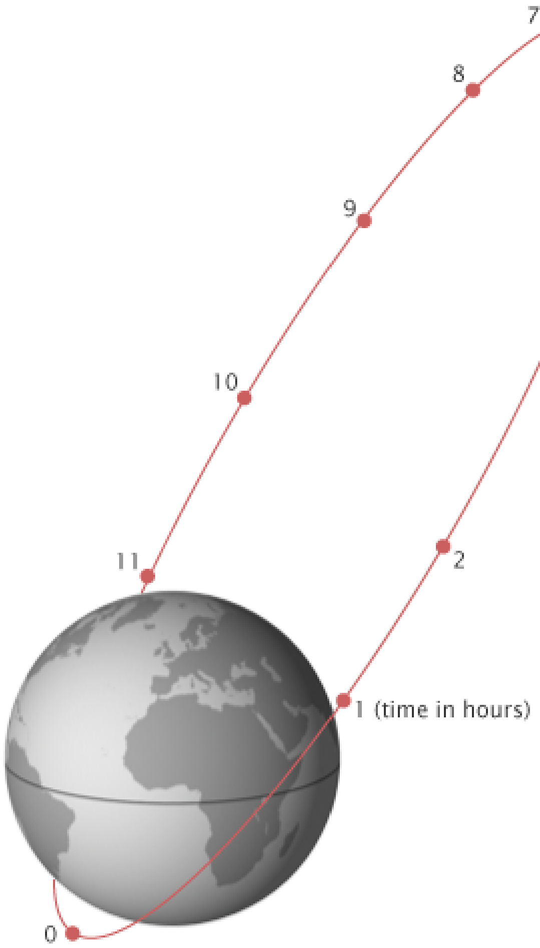


Figure 2.18: Molniya Orbit

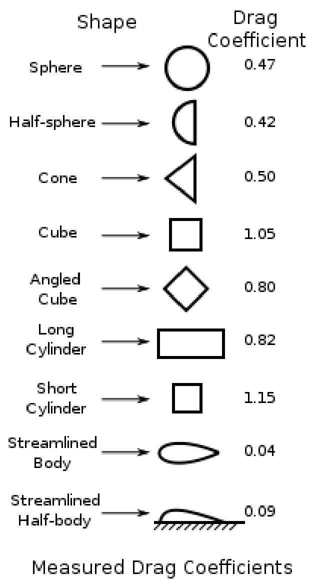


Figure 2.19: Drag Coefficient of Common Shapes, from wikipedia.com

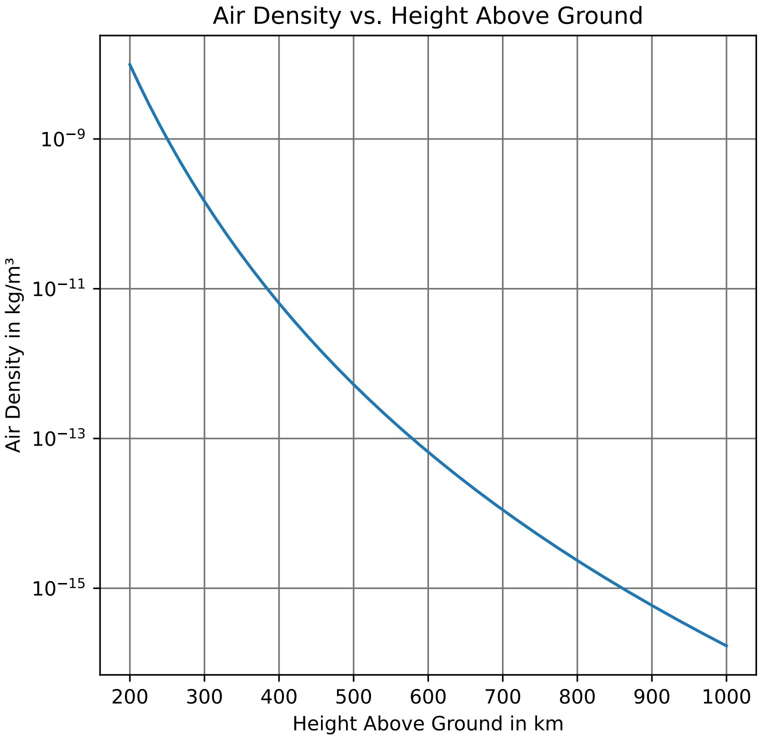


Figure 2.20: Earth Atmospheric Model, Density change over height

$$p = \begin{cases} 101.29 \cdot ((T + 273.1)/288.08)^{5.256} & \text{for } h < 11\text{km} \\ 22.65 \cdot \exp(1.73 - .000157 \cdot h) & \text{for } 11\text{km} < h < 25\text{km} \\ 2.488((T + 273.1)/216.6)^{-11.388} & \text{for } h > 25\text{km} \end{cases} \tag{2.83}$$

$$\rho = p/(0.2869(T + 273.1)) \tag{2.84}$$

Example 7:

A satellite is in a circular polar orbit, with $h = 587$km over ground. The satellite has a spherical shape with a diameter of 2m and is gold-coated. Weight of the satellite is 800 kg.

Find the velocity and orbital period of the satellite.

Find the drag force due to the atmosphere, assume an air density of of $2 \cdot 10^{-13} \frac{\text{kg}}{\text{m}^3}$.

Find the required velocity change after one period to keep the satellite in orbit.

Solution:

$v = \sqrt{\frac{\mu}{R_E+h}} = 7565.0$m/s

with a period $T = \frac{2\pi(R_E+h)}{v} = 5784.9$s

$\rho = 2 \cdot 10^{-6} \frac{kg}{m^3}$ and $C_d = 0.47$ from table and $A_n = \frac{\pi D^2}{4} = 3.14\mathrm{m}^2$

following: $F = \frac{1}{2}\rho v^2 C_d A_n = 8.45 \cdot 10^{-6}\mathrm{N}$,

now $a = \frac{F}{m} = 1.056 \cdot 10^{-8}\frac{\mathrm{m}}{\mathrm{s}^2}$

hence for one orbit $\Delta v = a \cdot T = 6.11 \cdot 10^{-5}\frac{\mathrm{m}}{\mathrm{s}}$

it takes 16365 seconds (4.5 hours) to reduce the velocity by 1 m/s.

It is a good idea to boost every 34 hours by about 10 m/s

Example 8:

Similar a cube Sat with 10 kg and 10cm x 1cm size:

Solution:

80 hours for 10 m/s

The following table provides a very rough guide to the lifetime of an object in a circular or near circular orbit at various altitudes. Satellite from https://www.spaceacademy.net.au/watch/debris/or

Altitude	Lifetime
200 km	1 day
300 km	1 month
400 km	1 year
500 km	10 years
700 km	100 years
900 km	1000 years

References

[1] Robert A. Braeunig. Orbital mechanics.

[2] Michel Capderou. *Satellites: orbits and missions*. Springer, 2005.

[3] Michel Capderou. *Handbook of satellite orbits: From kepler to GPS*. Springer Science & Business, 2014.

[4] Gerard Maral and Michel Bousquet. *Satellite Communications Systems*. Wiley-Blackwell, 6 edition, 4 2020.

[5] Dennis Roddy. *Satellite Communications*. McGraw-Hill, 4. edition edition, 2006. Available as E-Book at Library.

3 Propulsion Systems

3.1 Rocket Propulsion

During powered flight the propellants of the propulsion system are constantly being exhausted from the nozzle. As a result, the weight of the rocket is constantly changing. In the following derivation, we are going to neglect the effects of aerodynamic lift and drag.

3.2 Thrust

Thrust is the force that is pushing a rocket forward, due to the repulsion of mass in the back of the rocket. In Figure 3.1we observe the following dynamic system: In a short time interval dt a rocket of mass M travelling at velocity v ejects fuel mass dm with the exhaust velocity v_{ex}. Due to the ejection, the rocket gains velocity of Δv. From conservation of momentum we get

$$\sum \text{Momentum}(t=0) = \sum \text{Momentum}(t=\Delta t) \tag{3.1}$$
$$(M+\Delta m)\,v = M\,(v+\Delta v)+\Delta m\,(v-v_{ex}) \tag{3.2}$$
$$M\,v+\Delta m\,v = M\,v+M\Delta v-\Delta m\,v_{ex}+\Delta m\,v \tag{3.3}$$
$$0 = M\Delta v-\Delta m\,v_{ex} \tag{3.4}$$
$$\Delta v = \frac{1}{M}v_{ex}\Delta m \tag{3.5}$$

Note, the Δv is in opposite direction of the exhaust velocity.

or using infinite small time steps:

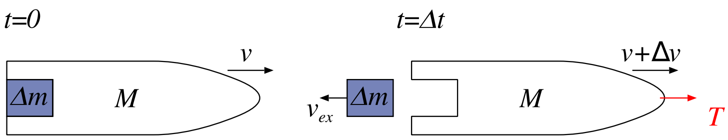


Figure 3.1: Mass Repulsion System

$$\begin{aligned} dv &= \frac{v_{ex}}{M} dm && (3.6) \\ \frac{dv}{dm} &= \frac{v_{ex}}{M} && (3.7) \end{aligned}$$

The thrust T is the force, that is accelerating the rocket. Using newtons first law of force

$$T = M\,a \tag{3.8}$$

where a is the acceleration, which is the first derivative of the velocity, i.e. $a = \frac{dv}{dt}$, we get

$$\begin{aligned} \text{Thrust} &= \text{Mass} \times \text{Acceleration} && (3.9) \\ T &= M\,a = M\frac{dv}{dt} && (3.10) \\ &= M\frac{dv}{dm}\frac{dm}{dt} && (3.11) \\ T &= M\frac{v_{ex}}{M}\frac{dm}{dt} && (3.12) \\ T &= v_{ex}\frac{dm}{dt} && (3.13) \end{aligned}$$

The Thrust is in opposite direction of the acceleration, hence the minus sign. Or as scalar quantities:

$$\boxed{T = v_{ex}\frac{dm}{dt} \qquad (3.14)}$$

The trust if equal the exhaust velocity times the rate of fuel ejection (=Fuel mass per second).

3.2.1 Specific Impulse

The total impulse of a rocket engine is defined as the fuel mass times the exhaust velocity

$$p = m_{fuel} \cdot v_{ex} \tag{3.15}$$

From this we can define a **mass specific impulse** as total impulse per fuel mass

$$p_{sp} = \frac{p}{m_{fuel}} = v_{ex} \tag{3.16}$$

with the unit $\frac{\text{m}}{\text{s}}$. This specific impulse is identical with the exhaust velocity

Alternative we can define a **weight specific impulse as** the total impulse per fuel weight $(= mass \cdot g_0)$

$$I_{sp} = \frac{p}{m_{fuel} g_0} \tag{3.17}$$

or

$$I_{sp} = \frac{v_{ex}}{g_0} \tag{3.18}$$

The specific impulse is a way to describe the efficiency of rocket and jet engines. It represents the impulse (change in momentum) per unit of propellant or in other words

Specific Impulse is the ratio of Thrust T to rate of fuel ejection

$$I_{sp} = \frac{\text{Thrust}}{\text{Rate of Fuel Ejection}} = \frac{T}{Rate} \tag{3.19}$$

The higher the specific impulse, the less propellant is needed to gain a given amount of momentum. I_{sp} is a useful value to compare engines, much like "liter per km" is used for cars. A propulsion method with a higher specific impulse is more propellant-efficient. Propellant is normally measured either in units of mass, or in units of weight at sea level on the Earth.

Substituting in eqn 3.14 yields

$$T = I_{\text{sp}} \cdot \frac{dm}{dt} \cdot g_0 \tag{3.20}$$

where:

- T is the thrust obtained from the engine, in newtons.
- $I_{sp}$is the specific impulse measured in seconds.
- $\frac{dm}{dt}$ is the propellant mass flow rate in kg/s , which is minus the time-rate of change of the vehicle's mass since propellant is being expelled.
- g_0 is the acceleration at the Earth's surface, in m/s² , =9.81m/s²

3.2.2 Tsiolkovsky Equation

Rocket mass ratios versus final velocity calculated from the rocket equation Tsiolkovsky's rocket equation, or ideal rocket equation is named after Konstantin Tsiolkovsky, who independently derived it and published in his 1903 work, considers the principle of a rocket: a device that can apply an acceleration to itself (a thrust) by expelling part of its mass with high speed in the opposite direction, due to the conservation of momentum. The equation relates the delta-v with the effective exhaust velocity and the initial mass and the end mass of a rocket.

Starting from the thrust equation with a total Mass $m(t)$ that changes over time due to the burned propellant:

Engine Type	used in	Propellant	sp. Impulse	Thrust	v_{ex}
NK-33 (Russian rocket engine)	Rockets	Kerosine+ LOX	330 s		$3240\frac{\text{m}}{\text{s}}$
Space Shuttle main engines (SSMEs)	Space Shuttle	Liq. Oxygen (LOX)	452 s	1.78 MN	$4423\frac{\text{m}}{\text{s}}$
		+liq. Hydrogen	363 s		
CF6-80C2B1F turbofan	Boing 747	Kerosine + Air	5950 s		$58400\frac{\text{m}}{\text{s}}$
Magnetoplasma Thruster	experimental	Plama acc.	14600 s		$144000\frac{\text{m}}{\text{s}}$

Table 3.1: Engine Specific Impulses

Characteristics	PAM-D	IUS	Centaur
Length (m)	2.04	5.2	9.0
Diameter (m)	1.25	2.9	4.3
Mass (kg)	2180	14,865	18,800
Thrust (N)	66,440	200,000	147,000
Isp	292.6	292.9	442
Structure mass	180	1255	2100
Propellant mass	2000	9710	16,700
Airborne support equipment mass	1140	3350	4310

Table 3.2: Characteristics of Some Commercial Rocket Transfer Vehicle

Figure 3.2: A Titian IV Centaur rocket launches at Vandergerg Air Force Base, California. The rocket is the largest unmanned space booster used by the Air Force. The vehicle carries payloads equivalent to the size and weight of those carried on the space shuttle. The Centaur is the third Stage of the rocket. (U.S. Air Force photo)

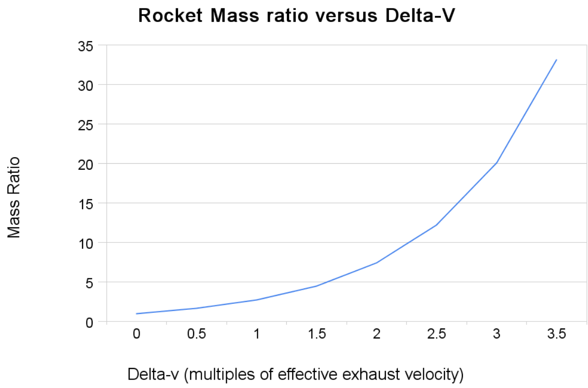


Figure 3.3: Rocket mass ratios versus final velocity calculated from the rocket equation

$$m(t)\frac{dv}{dt} = -v_{ex}\frac{dm}{dt} \tag{3.21}$$

$$\frac{dv}{dt} = -\frac{v_{ex}}{m(t)}\frac{dm}{dt} \tag{3.22}$$

$$dv = -\frac{v_{ex}}{m}\frac{dm}{dt}dt = -\frac{v_{ex}}{m}dm \tag{3.23}$$

$$v = \int dv = -v_e\int_{M_0}^{M_1}\frac{1}{m}dm \tag{3.24}$$

$$v = -v_{ex}[\ln m]_{M_0}^{M_1} \tag{3.25}$$

$$v = -v_{ex}\ln\frac{M_1}{M_0} \tag{3.26}$$

This is the velocity gained when starting with weight M_0 and ending with weight M_1.

$$\boxed{\Delta v = v_{\text{ex}}\ln\frac{M_0}{M_1}} \tag{3.27}$$

where:

M_o is the initial total mass, including propellant, in kg (or lb)

M_1 is the final total mass in kg (or lb)

v_{ex} is the effective exhaust velocity in m/s or (ft/s) or $v_{\text{e}} = I_{\text{sp}} \cdot g_0$

Δv is the delta-v in m/s (or ft/s)

Example 9:

Assume an exhaust velocity of 4.5 km/s and a Δv of 9.7 km/s (Earth to LEO).

Single stage to orbit rocket:

$$1 - e^{-\frac{9.7}{4.5}} = 0.884 \tag{3.28}$$

, therefore 88.4 % of the initial total mass has to be propellant. The remaining 11.6 % is for the engines, the tank, and the payload.

In the case of a **space shuttle**, it would also include the orbiter. Two stage to orbit: suppose that the first stage should provide a Δv of 5.0 km/s;

$$1 - e^{-5.0/4.5} = 0.671 \tag{3.29}$$

therefore 67.1% of the initial total mass has to be propellant. The remaining mass is 32.9 %. After disposing of the first stage, a mass remains equal to this 32.9 %, minus the mass of the tank and engines of the first stage. Assume that this is 8 % of the initial total mass, then 24.9 % remains. The second stage should provide a Δv of 4.7 km/s;

$$1 - e^{-4.7/4.5} = 0.648 \tag{3.30}$$

therefore 64.8% of the remaining mass has to be propellant, which is 16.2 %, and 8.7 % remains for the tank and engines of the second stage, the payload, and in the case of a space shuttle, also the orbiter. Thus together 16.7 % is available for all engines, the tanks, the payload, and the possible orbiter.

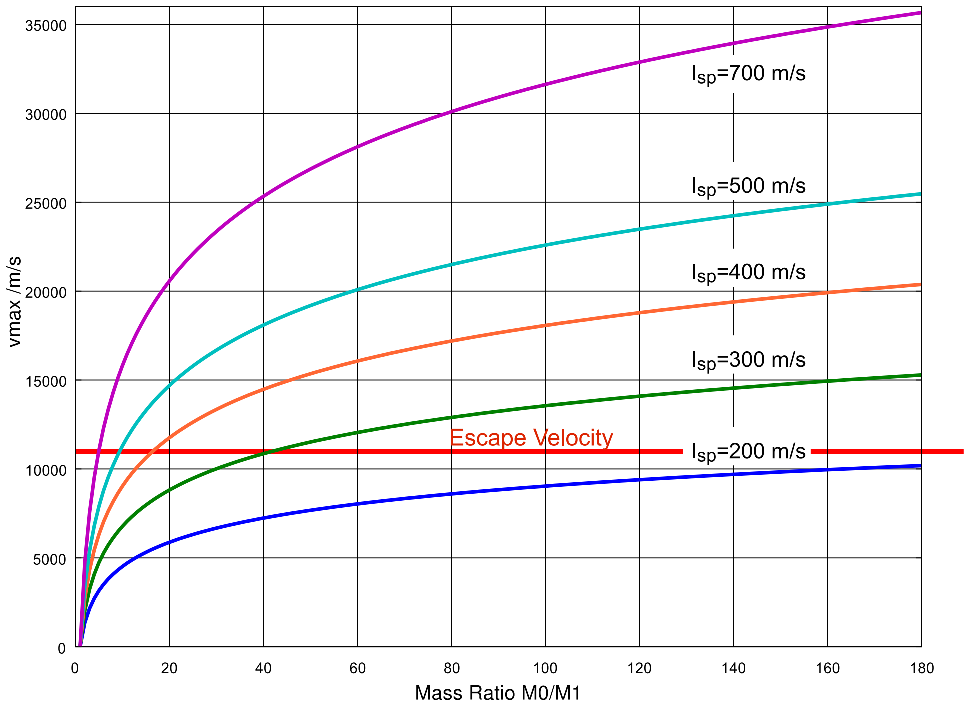


Figure 3.4: Velocities possible

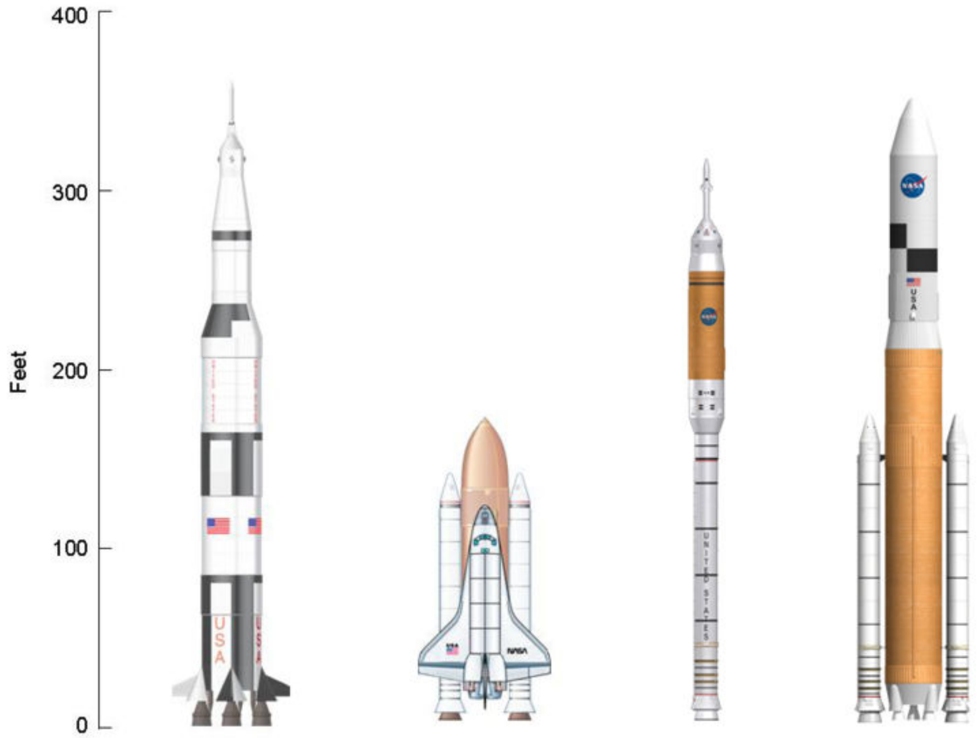


Figure 3.5: Some typical large Launchers: Saturn V of the Apollo Program, Space Transportation System (STS, a.k.a. as Space Shuttle), Ares I, Ares V; source: `http://www.nasa.gov/images/content/125170main_comparison_full.jpg`

3.3 Vehicles

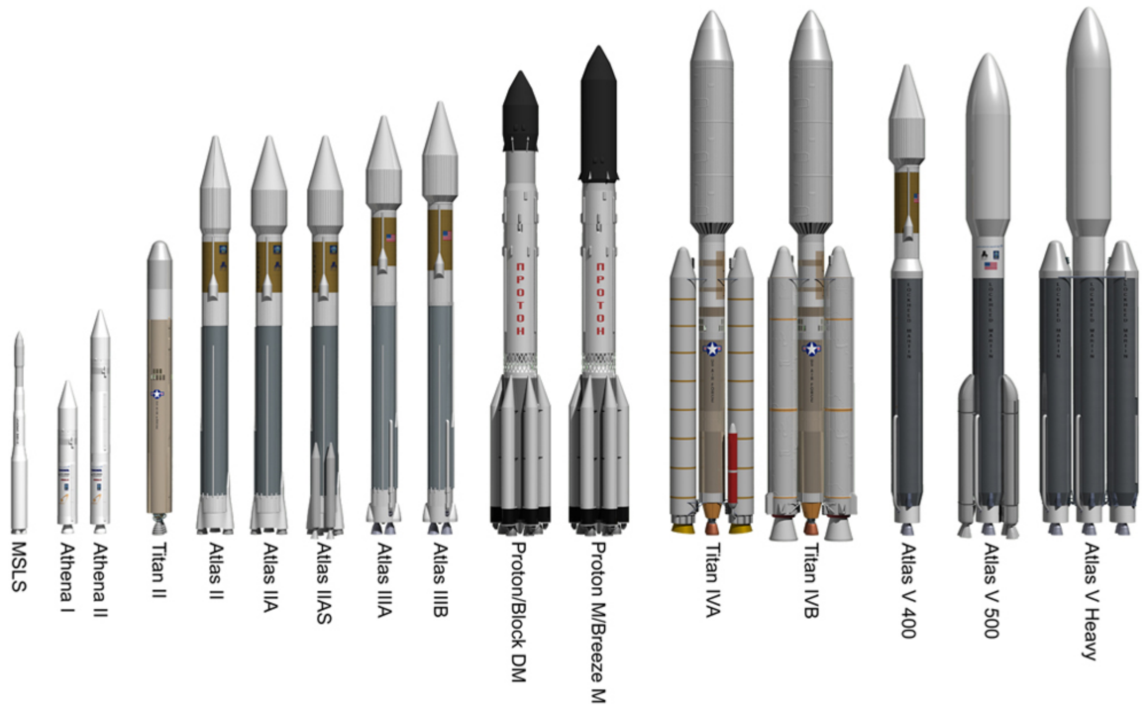
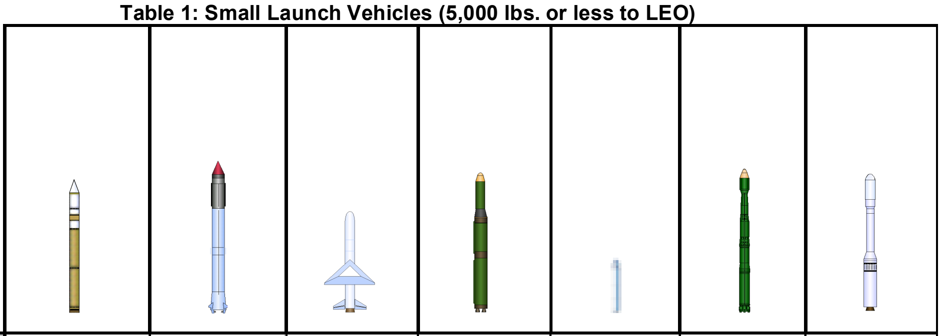


Figure 3.6: The Lockheed Family of Launch Vehicles

3.4 Launch Costs

from [?]

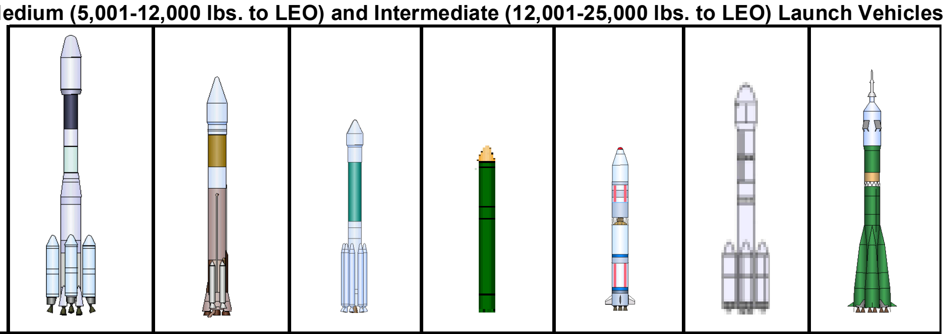
Table 1: Small Launch Vehicles (5,000 lbs. or less to LEO)



Vehicle name	Athena 2	Cosmos	Pegasus XL	Rockot	Shtil	START	Taurus
Country/Region of origin	USA	Russia	USA	Russia	Russia	Russia	USA
LEO capacity lb (kg)	4,520 (2,065)	3,300 (1,500)	976 (443)	4,075 (1,850)	947 (430)	1,392 (632)	3,036 (1,380)
Reference LEO altitude mi (km)	115 (185)	249 (400)	115 (185)	186 (300)	124 (200)	124 (200)	115 (185)
GTO capacity lb (kg)	1,301 (590)	0	0	0	0	0	988 (448)
Reference site and inclination	CCAFS 28.5 deg.	Plesetsk 62.7 deg.	CCAFS 28.5 deg.	Plesetsk 62.7 deg.	Barents Sea 77-88 deg.	Svobodny 51.8 deg.	CCAFS 28.5 deg.
Estimated launch price (2000 US$)	$24,000,000	$13,000,000	$13,500,000	$13,500,000	$200,000*	$7,500,000	$19,000,000
Estimated LEO payload cost per lb (kg)	$5,310 ($11,622)	$3,939 ($8,667)	$13,832 ($30,474)	$3,313 ($7,297)	$211 ($465)	$5,388 ($11,687)	$6,258 ($13,768)
Estimated GTO payload cost per lb (kg)	$18,448 ($40,678)	N/A	N/A	N/A	N/A	N/A	$19,234 ($42,411)

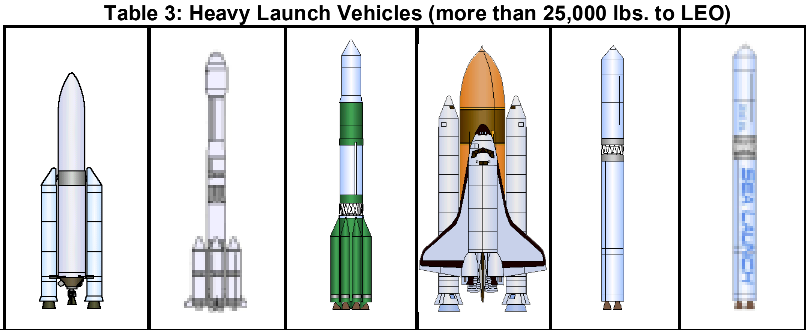
* Shtil launch costs partially subsidized by the Russian Navy as part of missile launch exercises

Table 2: Medium (5,001-12,000 lbs. to LEO) and Intermediate (12,001-25,000 lbs. to LEO) Launch Vehicles



Vehicle name	Ariane 44L	Atlas 2AS	Delta 2 (7920/5)	Dnepr	Long March 2C	Long March 2E	Soyuz
Country/Region of origin	Europe	USA	USA	Russia	China	China	Russia
LEO capacity lb (kg)	22,467 (10,200)	18,982 (8,618)	11,330 (5,144)	9,692 (4,400)	7,048 (3,200)	20,264 (9,200)	15,418 (7,000)
Reference LEO altitude mi (km)	124 (200)	115 (185)	115 (185)	124 (200)	124 (200)	124 (200)	124 (200)
GTO capacity lb (kg)	10,562 (4,790)	8,200 (3,719)	3,969 (1,800)	0	2,205 (1,000)	7,431 (3,370)	2,977 (1,350)
Reference site and inclination	Kourou 5.2 deg.	CCAFS 28.5 deg.	CCAFS 28.5 deg.	Baikonur 46.1 deg.	Taiyuan 37.8 deg.	Taiyuan 37.8 deg.	Baikonur 51.8 deg.
Estimated launch price (2000 US$)	$112,500,000	$97,500,000	$55,000,000	$15,000,000	$22,500,000	$50,000,000	$37,500,000
Estimated LEO payload cost per lb (kg)	$5,007 ($11,029)	$5,136 ($11,314)	$4,854 ($10,692)	$1,548 ($3,409)	$3,192 ($7,031)	$2,467 ($5,435)	$2,432 ($5,357)
Estimated GTO payload cost per lb (kg)	$10,651 ($23,486)	$11,890 ($26,217)	$13,857 ($30,556)	N/A	$10,204 ($22,500)	$6,729 ($14,837)	$12,598 ($27,778)

Table 3: Heavy Launch Vehicles (more than 25,000 lbs. to LEO)



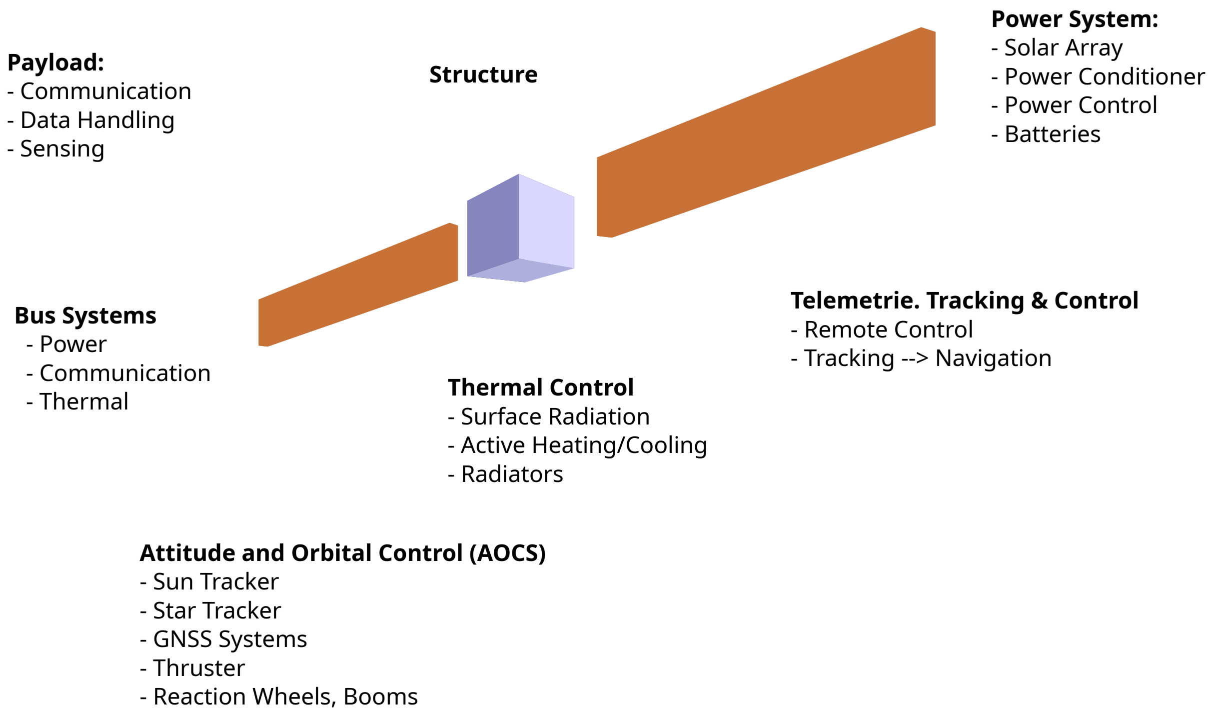
Vehicle name	Ariane 5G	Long March 3B	Proton	Space Shuttle	Zenit 2	Zenit 3SL
Country/Region of origin	Europe	China	Russia	USA	Ukraine	Multinational
LEO capacity lb (kg)	39,648 (18,000)	29,956 (13,600)	43,524 (19,760)	63,443 (28,803)	30,264 (13,740)	34,969 (15,876)
Reference LEO altitude km (mi)	342 (550)	124 (200)	124 (200)	127 (204)	124 (200)	124 (200)
GTO capacity lb (kg)	14,994 (6,800)	11,466 (5,200)	10,209 (4,630)	13,010 (5,900)	0	11,576 (5,250)
Reference site and inclination	Kourou 5.2 deg.	Xichang 28.5 deg.	Baikonur 51.6 deg.	KSC 28.5 deg.	Baikonur 51.4 deg.	Odyssey Launch Platform 0 deg.
Estimated launch price (2000 US$)	$165,000,000	$60,000,000	$85,000,000	$300,000,000	$42,500,000	$85,000,000
Estimated LEO payload cost per lb (kg)	$4,162 ($9,167)	$2,003 ($4,412)	$1,953 ($4,302)	$4,729 ($10,416)	$1,404 ($3,093)	$2,431 ($5,354)
Estimated GTO payload cost per lb (kg)	$11,004 ($24,265)	$5,233 ($11,538)	$8,326 ($18,359)	$23,060 ($50,847)	N/A	$7,343 ($16,190)

References

Space Segment

4

4.1 Subsystems



4.2 Thermal Control

see also [**?**, Chap. 10.6]

In Space we do not have any heat transfer due to convection or conduction. The heat transfer is limited to radiation.

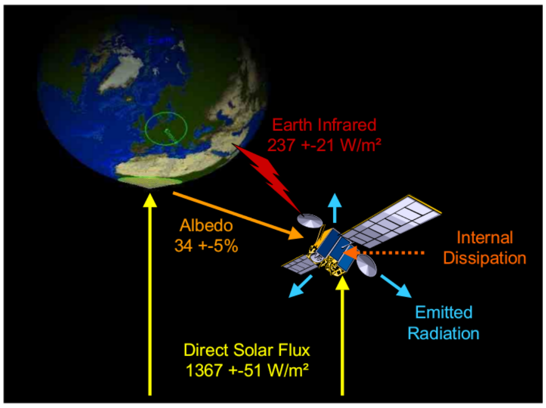


Figure 4.1: Radiation into and out of a Satellite

4.2.1 Equilibrium Temperature

In order to calculate the equilibrium temperature we have to sum up the incident and outgoing heat power and set them equal.

The temperature of a satellite depends on a several factors:

- Heat incoming:
  - Incident radiation from the sun Q_{sun}
  - Incident radiation from the earth Q_{earth}
  - The albedo effect causing a fraction of light to be reflected by the earth Q_{albedo}
  - Energy dissipated internally Q_{diss}
- Heat outgoing
  - Radiated power into space Q_{rad}

The incoming heat radiation is absorbed by

$$Q_{in} = A_{ill} \cdot G \cdot \alpha \tag{4.1}$$

where A is the illuminated area of the object, mathematically the projected area of the direction of the incoming radiation (shadow area). The G is the power flux density of the radiation, usually given in W per m^2 and α is the absorptivity of the illuminated surface. A table of various absorptivities for common surfaces are given in 4.1.

The radiated power of the satellite follows

$$Q_{out} \quad = \quad Q_{rad} = \epsilon\sigma A_{sur} T^4 \tag{4.2}$$

where $A_{sur}$is the surface of the satellite, $\sigma = 56,7E-9$W/K^{4}m^2 is the Stefan-Boltzmann Constant, ϵ is the emissivity of the surface as given in Table 4.1. The temperature T is the temperature of the satellite and determines the radiated power by the power of four. Hence, with increasing temperature the radiated power increases by power of four.

Material	Measurement Temperature T (K)	Surface Condition	Solar Absorptivity (VIS/NIR) alpha	Infrared Emissivity (TIR) epsilon	Absorptivity/ Emissivity Ratio a/e	Eqilibrium Temp. T (K)
Gold	294	As Received	0,299	0,023	13,00	749
Alu (6061-T6 @22°C)	294	As Received	0,379	0,0346	10,95	717
Alu (6061-T6 @150°C)	422	As Received	0,379	0,0393	9,64	695
Alu polisched (6061-T6 @22°C)	294	Polished	0,2	0,031	6,45	628
Alu polisched (6061-T6 @150°C)	422	Polished	0,2	0,034	5,88	614
Black Paint	295	Al. Substrate	0,975	0,874	1,12	405
Solar Cell-Fused Silica Cover	295		0,805	0,825	0,98	392
White Epoxy (@150°C)	422	Al. Substrate	0,248	0,888	0,28	287
White Epoxy (@22°C)	294	Al. Substrate	0,248	0,924	0,27	284
Aluminized Teflon	295		0,163	0,8	0,20	265
Silvered Teflon	295		0,08	0,66	0,12	233
OSR	295		0,077	0,79	0,10	220

Table 4.1: Absorptivity and Emissivity of Some Materials

The equilibrium temperature is reached when the incident heat plus internal generated heat is equal to the radiated heat

$$Q_{in} = Q_{out} \tag{4.3}$$

$$\begin{aligned}
Q_{in} &= Q_{sun} + Q_{albedo} + Q_{earth} + Q_{diss} && (4.4)\\
\texttt{where:} & && (4.5)\\
Q_{sun} &= A_{il} G_s \alpha && (4.6)\\
Q_{albedo} &= A_{il} G_a \alpha && (4.7)\\
Q_{earth} &= A_{il} G_{earth} \alpha && (4.8)\\
Q_{diss} &= \texttt{Power dissipated inside Sat.} && (4.9)
\end{aligned}$$

Example 10:

Calculate the Equilibrium Temperature of a spherical satellite with a diameter of 80 cm. The satellite has a black painted aluminum substrate shell. Internal power dissipation is 100 W. Include sun, albedo and IR earth incident radiation into your calculation.

Stefan-Boltzmann: $Q_{rad} = \epsilon \sigma A T^4$

$A_{illuminated} = \pi r^2 = 0.5\text{m}^2$

$A_{surface} = 4\pi r^2 = 2\text{m}^2$

$G_s = 1367 \frac{\text{W}}{\text{m}^2}$ sun incident power density

$G_a = G_s \cdot 0,34$ Albedo incident power density

$G_{earth} = 237 \frac{\text{W}}{\text{m}^2}$ earth IR incident power density

$\alpha = 0,975$ (black paint) absorptivity

$\epsilon = 0,874$ (black paint) emissivity

$\sigma = 56,7E-9W/K^4m^2$ Stefan-Boltzmann Constant

see Slide 6 to 9 Thermal:

$Q_{in} = Q_{out}$

$Q_{sun} + Q_{albedo} + Q_{earth} + Q_{diss} = Q_{radiated}$

$\underbrace{A_{il} G_s \alpha}_{Q_{sun}} + \underbrace{A_{il} G_a \alpha}_{Q_{albedo}} + \underbrace{A_{il} G_{earth} \alpha}_{Q_{earth}} + \underbrace{100\text{W}}_{Q_{diss}} = \underbrace{\epsilon \sigma A_{sur} T^4}_{Q_{radiated}}$

$Q_{in} = 666.41\text{W} + 226.58\text{W} + 115.54\text{W} + 100\text{W} = 1108.5\text{W}$

$T = \sqrt[4]{\frac{Q_{in}}{\epsilon \sigma A_{surface}}} == 324\text{K} = 50°\text{C}$

[6.70000000e+02 2.28000000e+02 1.16000000e+02 1.00000000e+02 1.17127224e+08]
[1.11400000e+03 1.17127224e+08]

4.3 Attitude and Orbit Control

4.4 Telemetry, Tracking, Command Monitoring

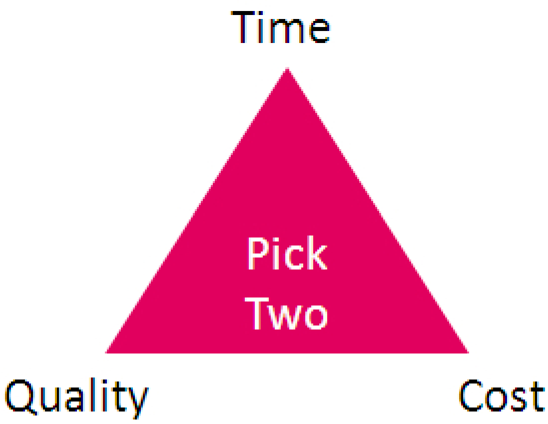
5 Space System Project Management

6 Space System Engineering

6.1 Designing for the Space Environment

6.1.1 Design Constraints

Engineering triangle



6.1.2 Design for Space

Weight

Size

Gravitational Effects

Thermal Issues, see Section 4.2

Reliability

6.1.3 Reliability

Redundancy

Testing

Heritage

Documentation

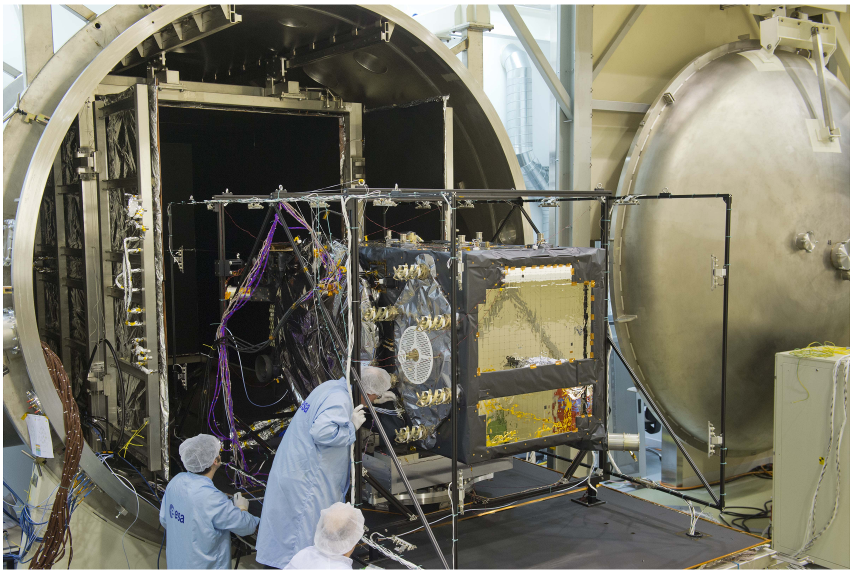
6.1.4 Radiation Hardening

Latch-Up Control

6.1.5 Vibration

6.2 Testing

6.2.1 Vacuum



The first Galileo Full Operational Capability (FOC) satellite beside the Phenix test chamber in the ESTEC Test Centre in October 2013, being readied for its five-week-long thermal vacuum test. Note the thermal tent visible inside the chamber, used to reproduce the temperature extremes of Earth orbit. ESTEC Test Centre engineers examine the Galileo Full Operational Capability satellite after it has been slid out of the Phenix test chamber on 29 November 2013, following a five-week thermal–vacuum test. The box within the Phenix is the 'thermal tent', used to reproduce the extreme temperatures of space. (source: http://www.esa.int/ESA_Multimedia/Images) esa united space in europe

European Space Agency

CUBESATS TAKE A BUMPY RIDE BEFORE GOING TO SPACE


Final preparations before installation on the shaker

27 November 2014

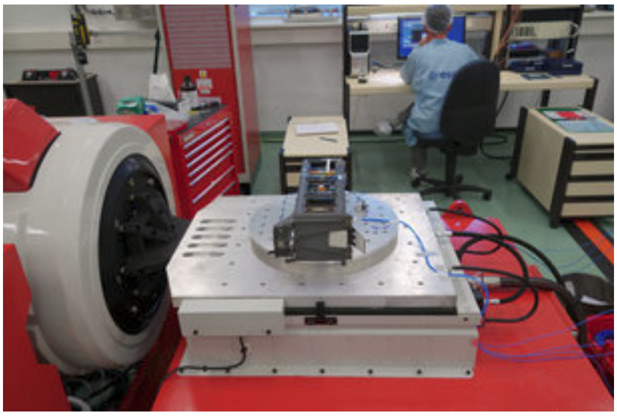
Two satellites chosen by the Fly Your Satellite! programme have undergone vibration testing at the European Space Research and Technology Centre (ESTEC) in Noordwijk, the Netherlands. The tests have provided important information that the teams can now use to help them obtain their 'ticket to orbit'.

The tests took place this month at the ESTEC Mechanical Systems Laboratory (MSL). The vibration tests showed how the satellites will respond to the harsh mechanical jarring that accompanies launch.

The two satellites are small CubeSats designed and built by university students. These satellites were chosen to progress to this part of the programme by a board of ESA experts in February 2014.

The vibration tests are part of Phase 2 of the Fly Your Satellite! programme. This phase incorporates tests to make sure that the CubeSats are capable of operating in the harsh space environment, and that they can survive the launch.

Before going through the vibration tests, both satellites had already passed thermal vacuum and thermal cycling tests, demonstrating that they are able of operating in space-like conditions.


Vibration test set-up

The value of the test was proven when one of the satellites underwent an unscheduled antenna deployment on the vibration table.

Specialists from the ESTEC Materials Technology Section laboratory performed a 3D X-ray inspection, and helped the students interpret the three-dimensional radiographic pictures of their satellite. This showed that the antenna deployment was caused by a nut, which had unscrewed itself because of the vibrations.

The students got to plan how to retrieve the nut and repair the satellite with minimum disruption to the rest of its systems. Corrective action was taken to ensure that this does not happen again, and the team is now preparing the CubeSat for a new campaign of united space in europe

European Space Agency

BELGIAN STUDENTS CLOSER TO THEIR 'TICKET TO ORBIT'

Inspection of the OUFTI-1 satellite

22 October 2014

A team of Belgian students have taken steps towards gaining their 'Ticket to Orbit!', not for themselves, but for the CubeSat they are developing.

OUFTI-1 is designed and developed, and now being tested by a team of students from the University of Liège, Belgium. It has a mass of approximately 1kg and dimensions of approximately 10x10x10cm. It is designed to demonstrate the D-STAR digital communication protocol and validate high-efficiency solar cells.

D-STAR is an amateur radio digital communication protocol. Once OUFTI-1 is in orbit, it will allow radio operators worldwide to communicate through the CubeSat.

The students brought OUFTI-1 to ESTEC, the European Space Research and Technology Centre, in Noordwijk, The Netherlands. This is where ESA tests many important satellites. Prior to arriving, OUFTI-1 had completed Phase 1 of the Fly Your Satellite! Programme, which included tests performed in ambient conditions in the university's laboratories to check the good functionality of the satellite.

During Phase 2 of the programme the CubeSat is tested to demonstrate that it can survive the environmental conditions to which it will be exposed during launch and while in orbit. To simulate the cold and hot temperature cycles that the satellite will undergo during its lifetime in space, a thermal vacuum chamber is used to provide the expected temperatures in vacuum.

Preparing the test set-up

First the chamber was pumped down to reach vacuum conditions equal to approximately 10^{-6} mbar. Then four hot-cold cycles were performed. At predefined moments of the thermal cycling sequence a so-called Functional Performance Check was executed to determine if the CubeSat was working as expected.

The thermal vacuum/thermal cycling test campaign is now finished and the team is currently performing additional tests back at the University of Liège to confirm the CubeSat's readiness to proceed to the second part of Phase 2: the vibration test to verify OUFTI-1's capability to survive the launch.

References

7 The Free Space Link

A satellite communication system consists of a transmitter channel and receiver. There are three different types of links:

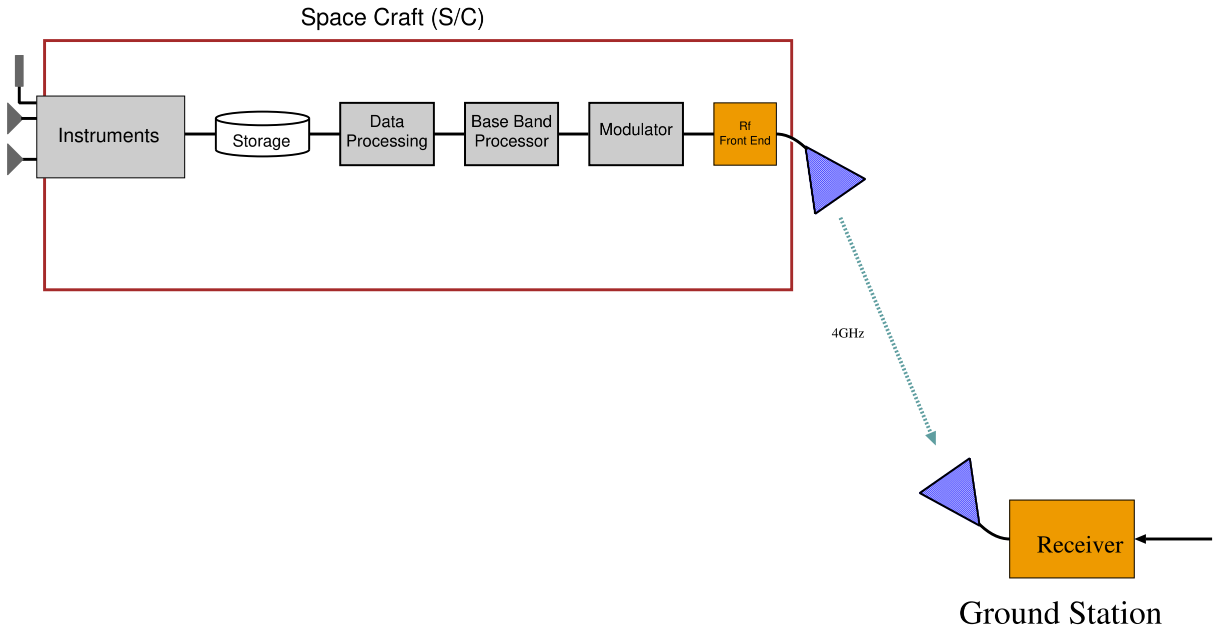
- The up-link from the earth station to the satellite
- The down-link from the satellite to the ground station
- The inter-satellite link between satellites

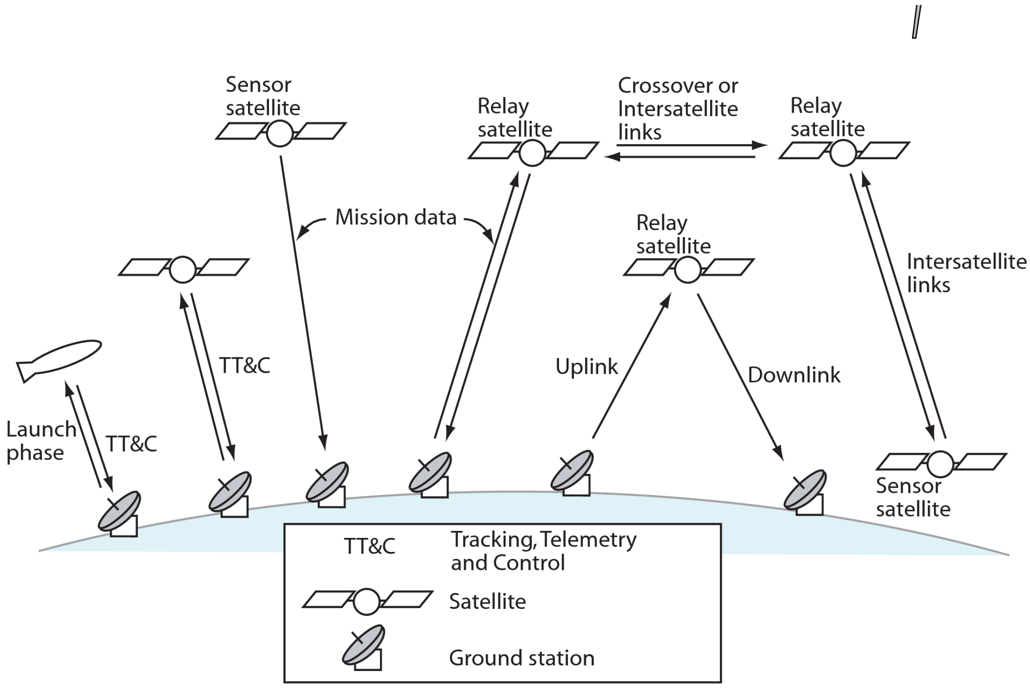
as seen in Figure 7.1

The link is established either through a microwave link, or by optical methods, such as laser beams.

7.1 Satellite Types

Store Forward Sat:

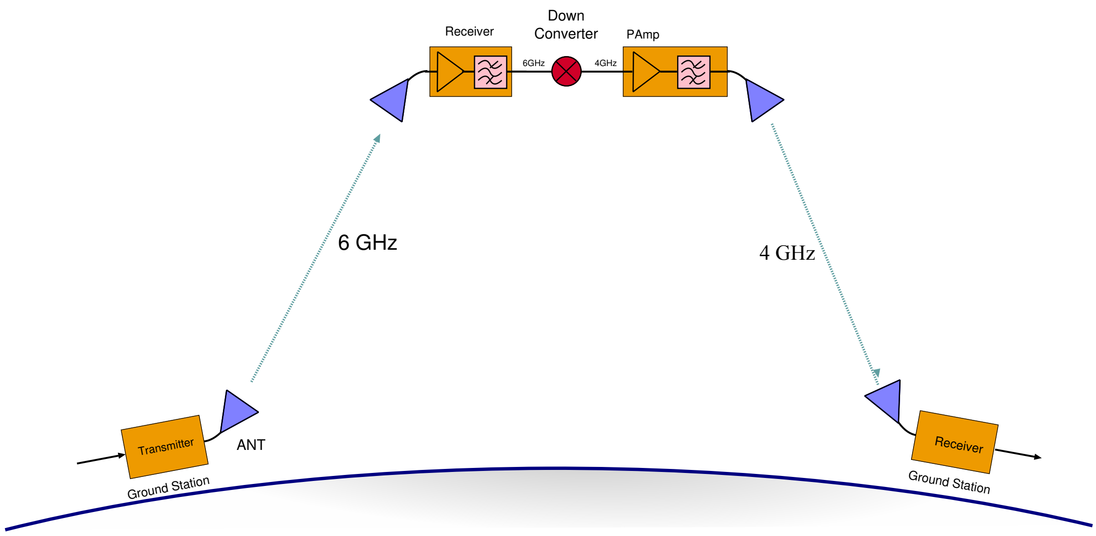




The communications architecture consists of satellites and ground stations interconnected with communications links. (Adapted from SMAD.)

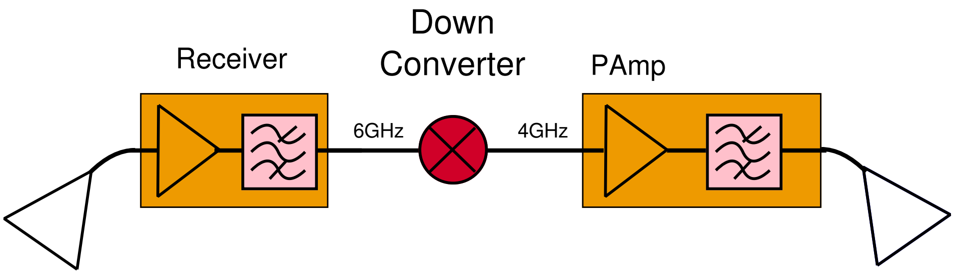
Figure 7.1: Types of Satellite Links, from [?]

Bend-Pipe:



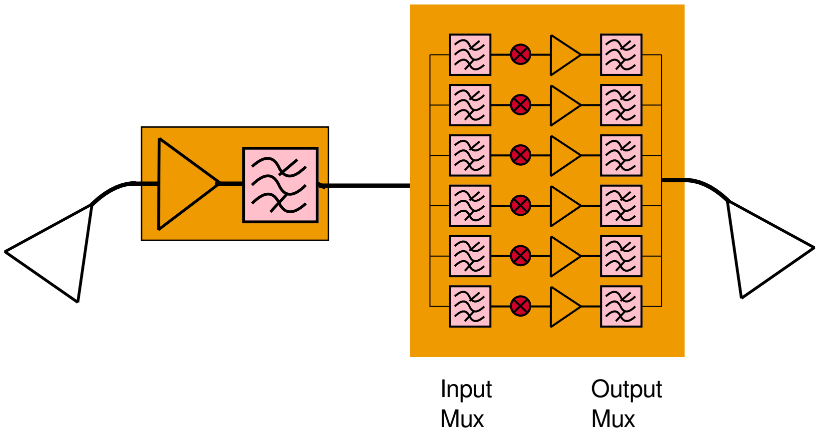
Bend-Pipe

A bend pipe satellite receives a signal from ground and amplifies it and transmits it back to the ground at a different location. To avoid interference between received and transmitted signal the down-link signal is converted to a frequency different from the up-link. Typically the down-link frequency is about $\frac{2}{3}$ of the up-link frequency.

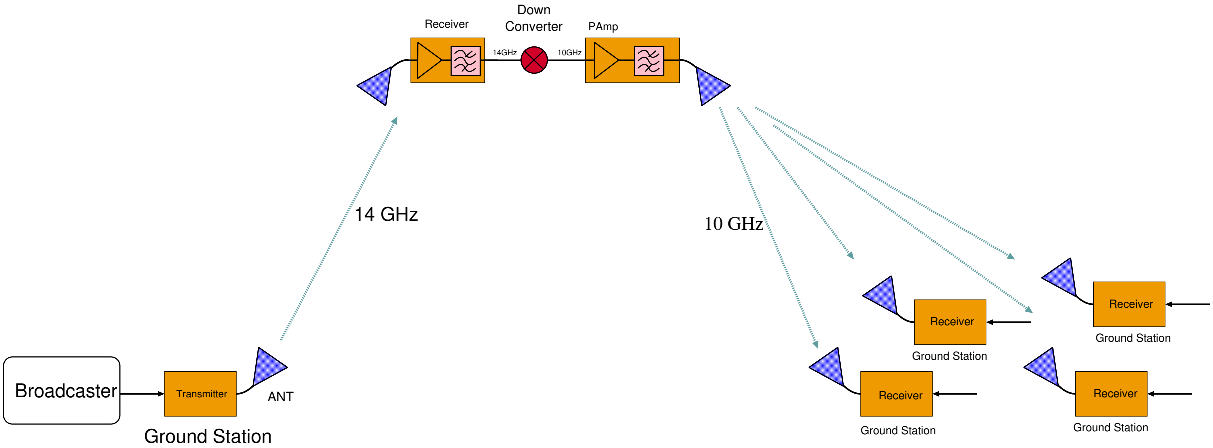


Multiplexer

Multiplexer split a wide-band signal in several small-band signals. This is necessary due to the limitations of linear amplifiers.



Broadcast Satellite



7.2 Building Blocks of a Link

The physical link takes the base-band signal (analog or digital) and modulates it onto a carrier. This RF signal is amplified and transmitted by an antenna. The receiver receives the signal by an antenna and amplifies demodulates it back into the base-band.

The quality of the link is characterized by

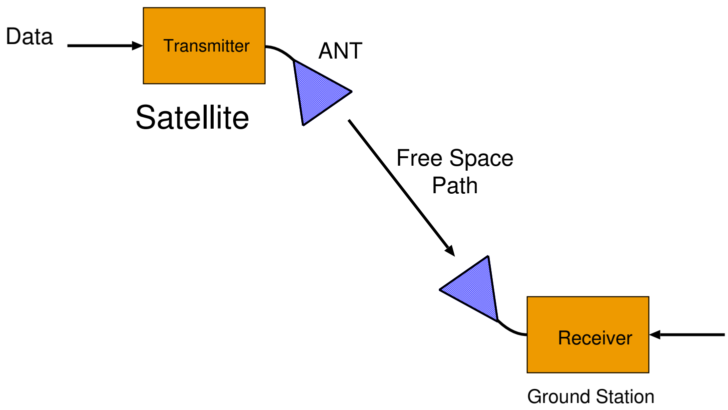


Figure 7.2: Block Diagram of a Typical Free Space Down Link

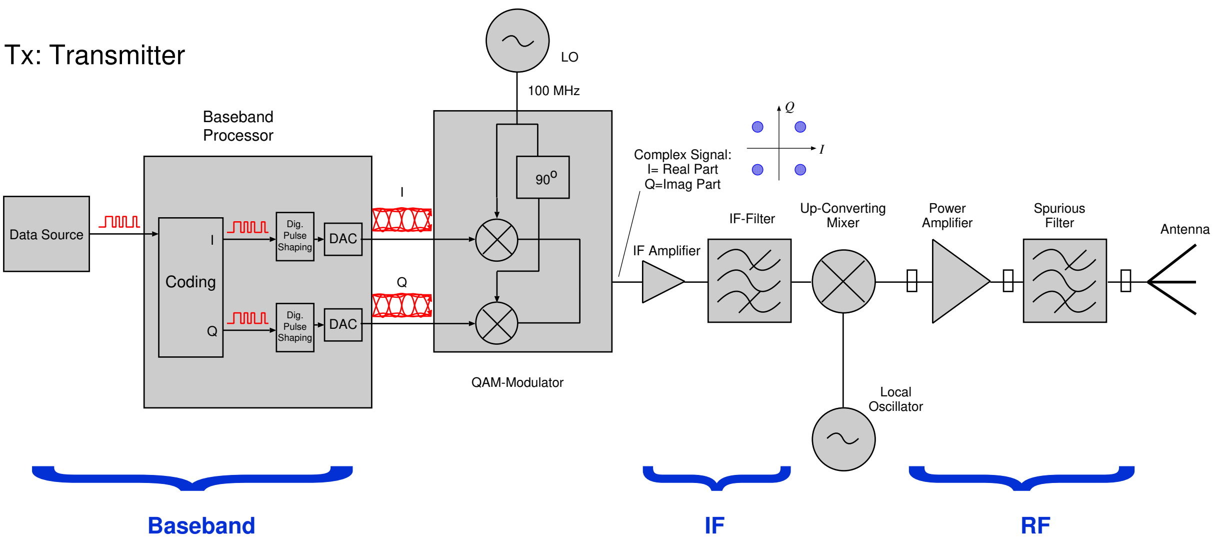


Figure 7.3: Block Diagram of a Typical Transmitter

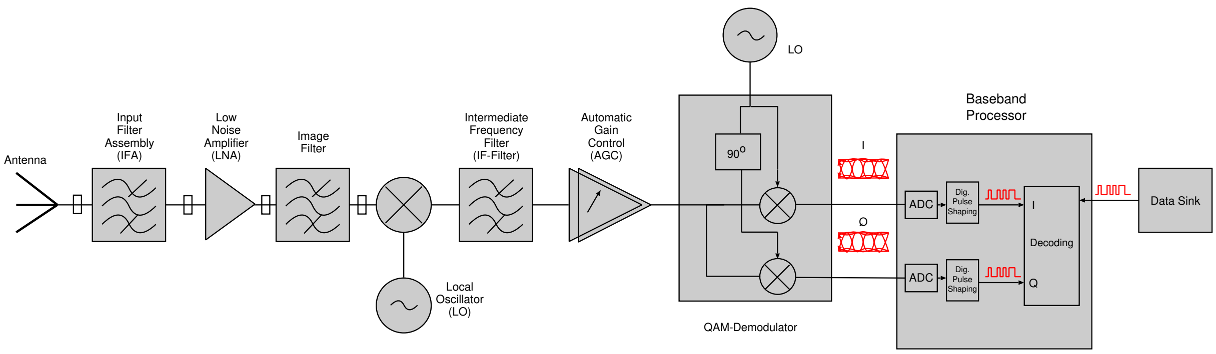


Figure 7.4: Block Diagram of a Typical Receiver

- the signal-to-noise ration S/N in analog systems or
- the bit error rate BER in digital systems.

Typical values for SNR are 20dB for television and 10 dB in speech channels. A typical BER is 10^{-6}in digital systems. When designing a satellite link we have to ensure that these values are met at all times or at a specified guaranteed on-time, e.g. 99.9% of all time.

7.3 Frequency Designations

Frequencies **above about 30 MHz** can pass through the ionosphere and so are available for communicating with satellites and other extra-terrestrial sources. Frequencies below 30MHz are liable to be reflected by the ionosphere at certain stages of the sunspot cycle. The ionosphere consists of several layers of ionized gas which alter in height during the 24 hour daylight cycle. The ionosphere has an effect on satellite communications even if it does not completely prevent them.

Frequencies from about 100MHz to 2 GHz are used for communicating with low earth orbit satellites (LEOs). Since the range from ground station to satellite is only a few hundreds of km, it is not necessary to use high gain ground based antennas. Of course, there is a direct link between the beam divergence angle of an antenna and its directivity and its gain.

Very high frequencies of more than 40 GHz are highly attenuated by the atmosphere. For that reason they cannot be used for links with ground stations. Inter-satellite links in free space (vacuum) are possible. The same applies to optical links using laser beams.

7.3.1 Frequency Spectrum

Waves can be characterized by its wave length or frequency. They are related by

$$\lambda \cdot f = c \tag{7.1}$$

Art	Frequency	Wellenlänge
P-Band	220-300MHz	115cm
L-Band	1-2 GHz	20cm
S-Band	2-4GHz	10cm
C-Band	4-8 GHz	5cm
X-Band	8-12,5 GHz	3cm
Ku-Band	12,5-18GHz	2cm
K-Band	18-26,5GHz	1,35cm
Ka-Band	26,5-40GHz	1cm

Table 7.1: Radar Bands

The selected frequency used in Satellite communication depend on

- Propagation effects

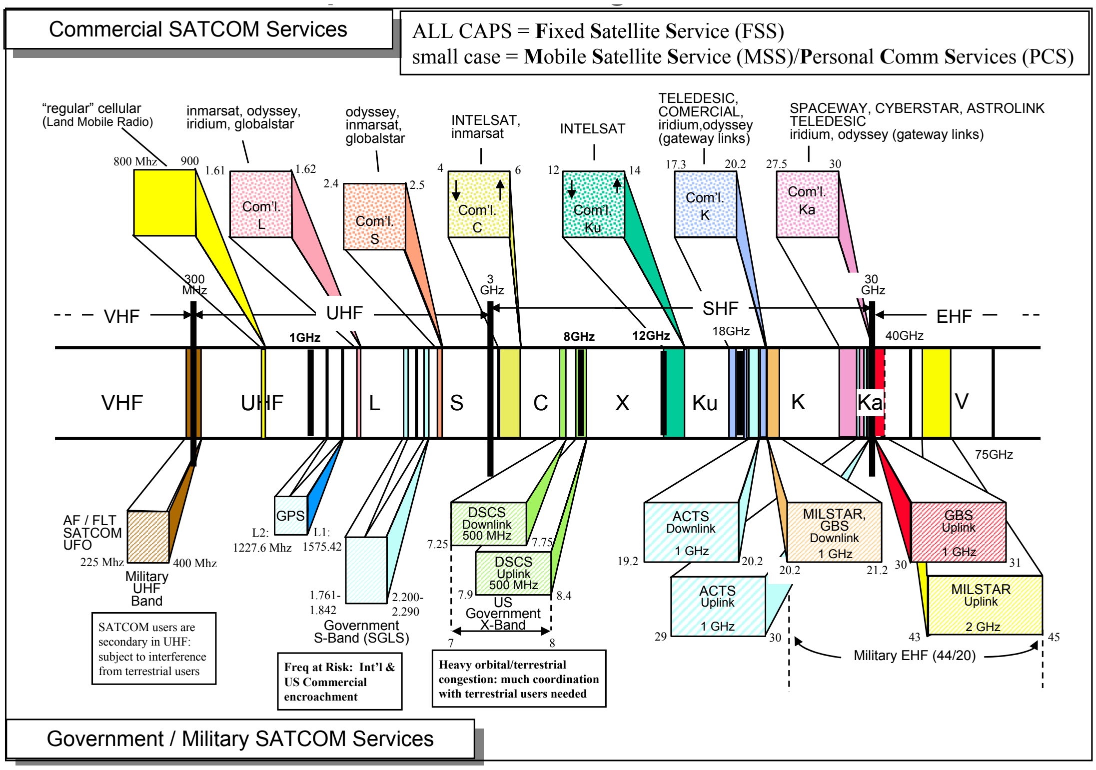


Figure 7.5: Frequency Allocation, from [1]

- Antenna size
- regulations
- required fractional bandwidth
- Cost of equipment

For satellite services are the frequency ranges allocated as follows:

- LEO satellites can operate low RF frequencies because of the short distance only low antenna gain (small) is needed. We have only small bandwidth available
- GEO satellites need high frequencies as more gain is needed and low frequencies are occupied. At high frequencies we can achieve lots of antenna gain with small antennas.

7.3.2 Regulation

Satellites share bands in the UHF, C and X-Band. The L,S and K-Bands are almost exclusively used for satellite communication. As mobile terrestrial services become more popular more and more bandwidth at higher frequencies is also allocated to terrestrial systems.

For more information take a look at `http://www.ntia.doc.gov/osmhome/osmhome.html`

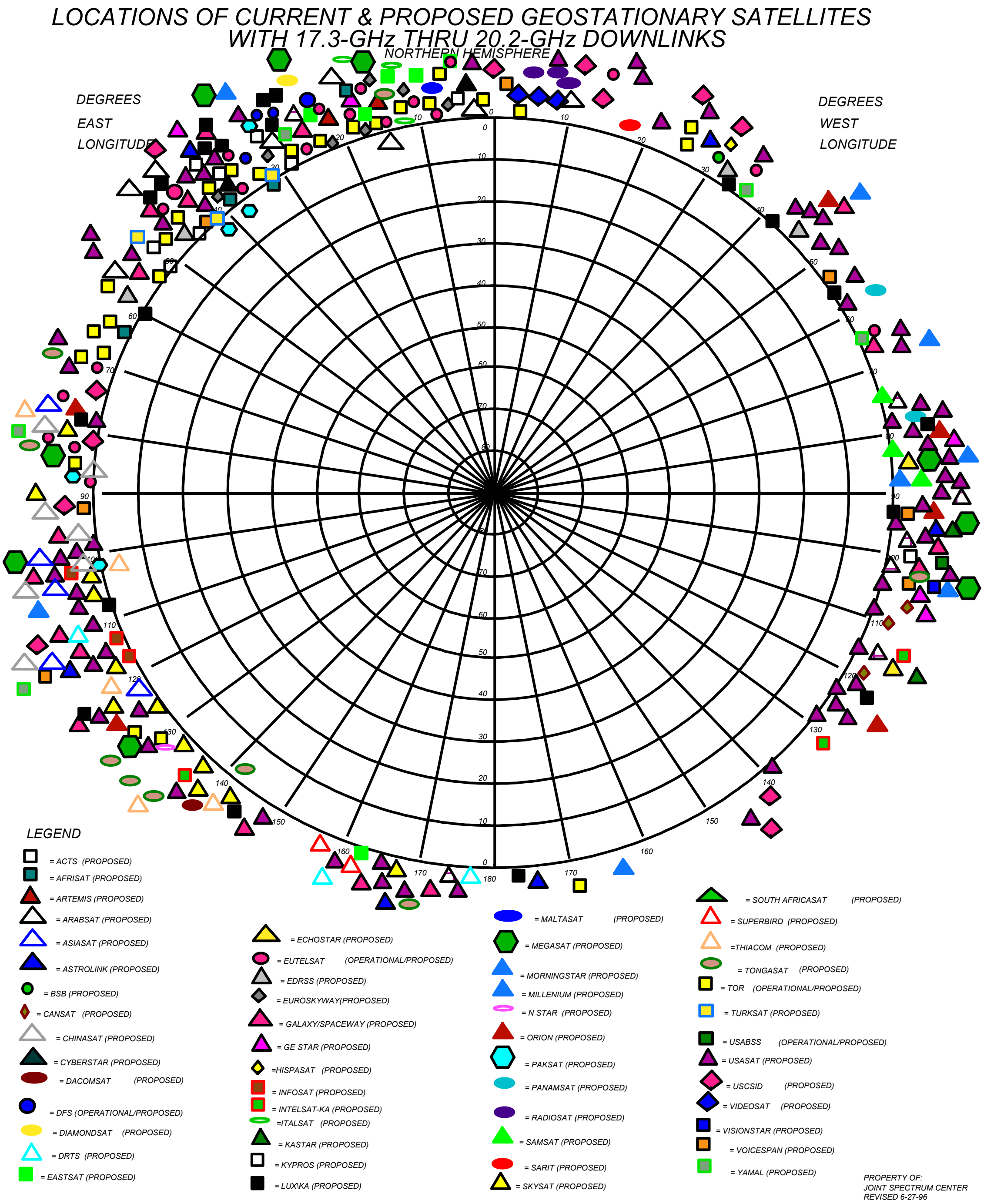


Figure 7.6: Frequency Allocation, from [1]

7.3.3 Up- and Down-link Bands

Satellite services cannot use the same frequency for up-link and down-link. The transmitted down-link signal from a satellite would interfere with the very sensitive receiver front end of the up-link. Due to possible harmonics, the up-link frequency is ideally selected between the down-link frequency and the doubled down-link frequency (it's first harmonic). In C-Band, for example, the down-link is at 4 GHz whereas the up-link is at 6 GHz.

The frequencies are different for up- and down-link in satellite communication. The allocation is as follows:

Band	Up-link [MHz]	Downlink [MHz]	Short [GHz]
C	5,925-6,425	3,700-4,200	6/4
X	7,900-8,400	7,250-7,750	8/7
K_u	14,000-14,500	10,950-11,200 11,450-11,700	14/11
K_u	14,000-14,500 17,300-18,100	11,700-12,200 12,200-12,700	14/11
K_a	27,500-30,000	17,700-20,200	30/20

The typical bandwidth of one satellite channel is 36 to 112 MHz.

7.3.4 Fractional Bandwidth.

Fractional bandwidth is defined as (Δf) divided by fo, where fo is the band centre and (Δf) the bandwidth in Hz which is occupied by the signal or signals.

At 10 GHz, 1% fractional bandwidth gives us 100 MHz of numerical bandwidth, which is sufficient for 10 TV channels each requiring 10MHz bandwidth. At 5 GHz, 1% fractional bandwidth only gives us 5 TV channels of 10 MHz each.

The concept of fractional bandwidth is central to satcoms. Many microwave transmission systems can only work over a limited fractional bandwidth. The widest band systems, like dual directional Lange couplers, Travelling Wave Tubes (TWTs), and circulators or isolators, have only 50% to 100% fractional bandwidth. It is possible to make distributed amplifiers with wider fractional bandwidth than this, but they tend to have rather poor group-delay characteristics across the band and have rather specialized applications.

References

[1] Wiley Larson. *Space Mission Analysis and Design*. K1uwer Academic Publishers, 2005.

8 Link Budget

8.1 The Link as a System

Example 11:

Given is a satellite in GEO orbit (40000km to earth station), transmits with 2 W, Tx-Antenna-Gain is 17 dB at 11 GHz

Calculate:

a) Flux density on earth surface

b) Power received by antenna with effective aperture of 10m²

c) Received power when Earth station antenna has gain of 52.3 dB

Solution:

a) Flux density is

$$S = \frac{P_t}{4\pi r^2} = \frac{2\mathrm{W}\cdot 10^{-1.7}}{4\pi(4\cdot 10^7\mathrm{m})^2} = 4.97\cdot 10^{-15}\,\frac{\mathrm{W}}{m^2} \frown -143\mathrm{dBW} \tag{8.1}$$

Or EIRP in dB by $EIRP = (P_t + G_t)\mathrm{dBW} = 20\mathrm{dBW}$

with Path Loss (with $\lambda = \frac{c}{f} = 0.027\mathrm{m}$)

$$L_P = 20\log\left(\frac{4\pi r}{\lambda}\right) = 20(\frac{4\pi 40000\cdot 10^3}{\lambda}) = 205.4\mathrm{dB} \tag{8.2}$$

translating transmit power in dB: $P_r = 10\log_{10}(\frac{P_r}{1\mathrm{mW}}) = 33\mathrm{dBm}$

and calculating the gain of the antenna $G = \frac{4\pi A}{\lambda^2} = 169181 = 52.3\mathrm{dB}$, we get

$$P_r = P_t + G_t + G_r - L_p = 33\mathrm{dBm} + 17\mathrm{dB} + 52.3\mathrm{dB} - 205.4\mathrm{dB} \tag{8.3}$$

$$= -103.1\mathrm{dBm} = 4.89\cdot 10^{-14}\mathrm{W} \tag{8.4}$$

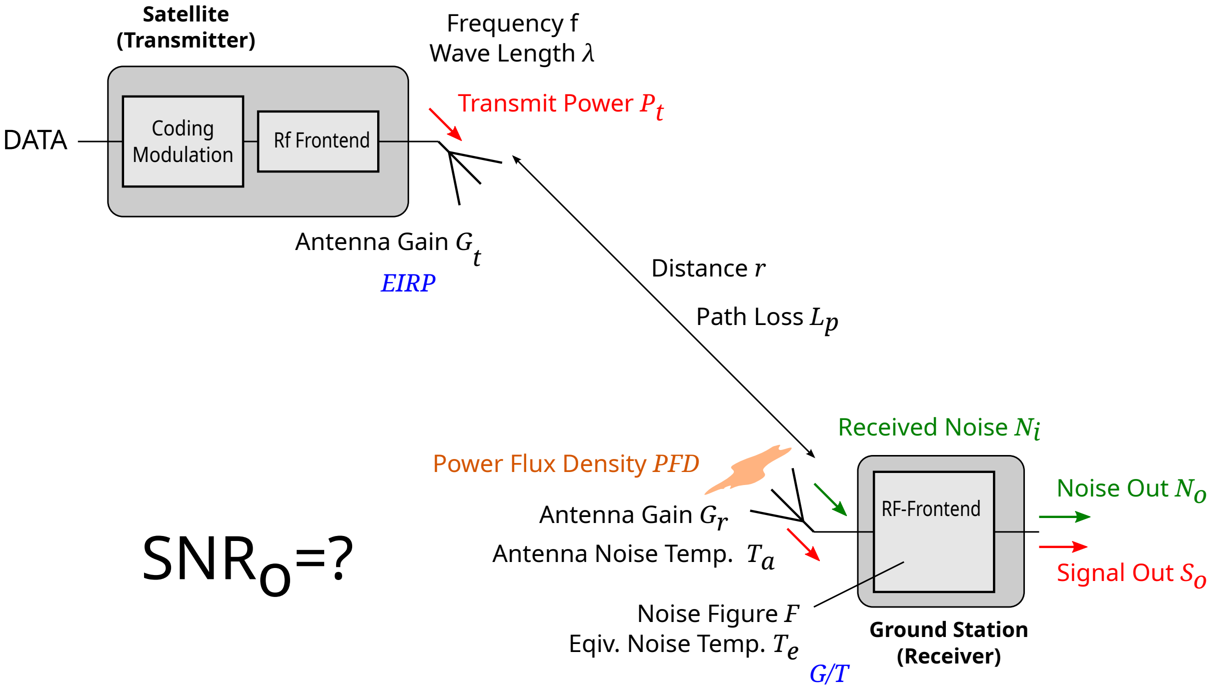


Figure 8.1: Overview over Link Parameters

8.2 The Link Budget

The Link Budget calculates the Power losses and gains through the complete RF link, it includes:

- Transmitter Power P_t
- Transmission losses in the Transmitter through lines etc L_{tl}
- Tx Antenna Gain G_t
- Antenna Pointing Loss $L_{point,t}$
- Free Space Loss L_p
- Atmospheric Loss (Clouds, Vapour etc.) L_{atm}
- Rx Antenna Pointing loss $L_{point,r}$
- Polarization Loss L_{pol}
- Reception Losses in cables etc. L_{rl}
- Noise Temperature Contribution L_N
- Other Losses L_{other}

8.3 A Simple Link Budget

A simple link budget includes the main power levels of the link.

- Frequency, Wavelength
- Transmit Power
- Transmit Antenna Gain, EIRP
- Distance with Path Loss

- Receiving Antenna Gain, Antenna effective Aperture
- Received Antenna Noise, Receiver Noise
- Signal-to-Noise Ratio

Example 12:

for a Hotbird-consumer-receiver-link in 20,000 km Distance. see also example above.

The satellite is specified with: 250W transmit power, and an Antenna gain of 26 dB. The transmitted Signal has a centre frequency of $f_0 = 11.5$GHzand a bandwidth of 33 MHz.

The Receiver is a consumer device with a 60cm dish and an LNA with a noise figure $F = 0.8$dB and a receiver gain of 60dB. We assume an antenna noise temperature of 15K. The antenna has 60% antenna aperture efficiency.

Figure 8.2 shows a simple Link budget for this link.

Simple Link Budget

	Boltzmann Const	k	1.38E-023	J/K
			:input fields	
Transmitter	Input power		1	mW
			-30	dBW
	Power Amplifier	Gpow	53.98	dB
			250000	
	Transmit Power	Pt	250	W
			23.98	dBW
	Frequency	f	1.15E+010	Hz
	Wave Length	lambda	0.0261	m
	Bandwidth	B	3.30E+007	Hz
	Sat Antenna Gain	G	400	
			26.02	dBi
	EIRP	EIRP	100000	
			80	dBm
			50	dBW
Path	Distance to Ground	R	20000000	m
	Path Loss	Lp	9.28E+019	
			199.68	dB
	Powerfluxdensity Ground	S	1.99E-011	W/m²
			-107.01	dBW/m²
	Received P of Isotropic Antenna		1.08E-015	W
			-149.68	dBW
Ground Station	Antenna Diameter	D	0.6	
	Antenna physical Size		0.28	m²
	Antenna Efficiency	eta	0.6	
	Eff Antenna Area	Aeff	0.170	m²
	Antenna Gain	Gr	3131.02	
			34.96	dB
	Received Power	Pr	3.37E-012	W
			-114.72	dBW
	Receiver Gain	G	60	dB
			1000000	
	Power at Receiver out	So	3.37E-006	W
			-54.72	**dBW**
	Noise			
	Antenna Noise	Ta	15	K
	Receiver Noise Figure	f	0.8	dB
			1.2	
	Equiv Noise Temp	Te	58.66	K
	Noise Output	No	3.35E-008	W
			-74.74	**dBW**
System	SNR	**SNR**	**100.57**	
			20.02	**dB**
	G/T	G/T	42.51	1/K
			16.28	dB/K
System in dB	EIRP	EIRP	50	dBW
	G over T	G/T	16.28	dB/K
	Path Loss	-Lp	-199.68	dB
	Boltzmann Const	1/k0	228.6	dB
	S/N per Hz	**S/N**	**95.21**	**dB/Hz**
	S/N per Band	**S/N**	**20.02**	**dB**

Figure 8.2: Link budget Simple

8.4 Power Diagram

In the diagram shown below, the power levels at various points of the transmission chain can be visualized.

```
P0 = -30
stages = {'Power Amp': 54, 'Tx-Ant.Gain': 26, 'Path Loss': -199.6, 'Rx-Ant.Gain': 35, 'Rec. Gain':60,'Amp.': 24.7}

Plevel = cumsum([P0]+list(stages.values()))
fig,ax = subplots(figsize=(8,5))
plot(Plevel,'o-')
for vv in range(len(stages)):
    y = (Plevel[vv]+Plevel[vv+1])/2
    txt = str(Plevel[vv+1]-Plevel[vv])+"dB"
    text(vv+0.5,y-10,txt,fontsize=9)
grid()
xticks(arange(7)-0.5,['Pin']+list(stages.keys()),rotation='vertical',fontsize = 12)
ylabel('Power Level in dBW')
tight_layout()
savefig('/home/speik/soridat/vorlesung/asc/linkbudget/powerdigram.svg')
```

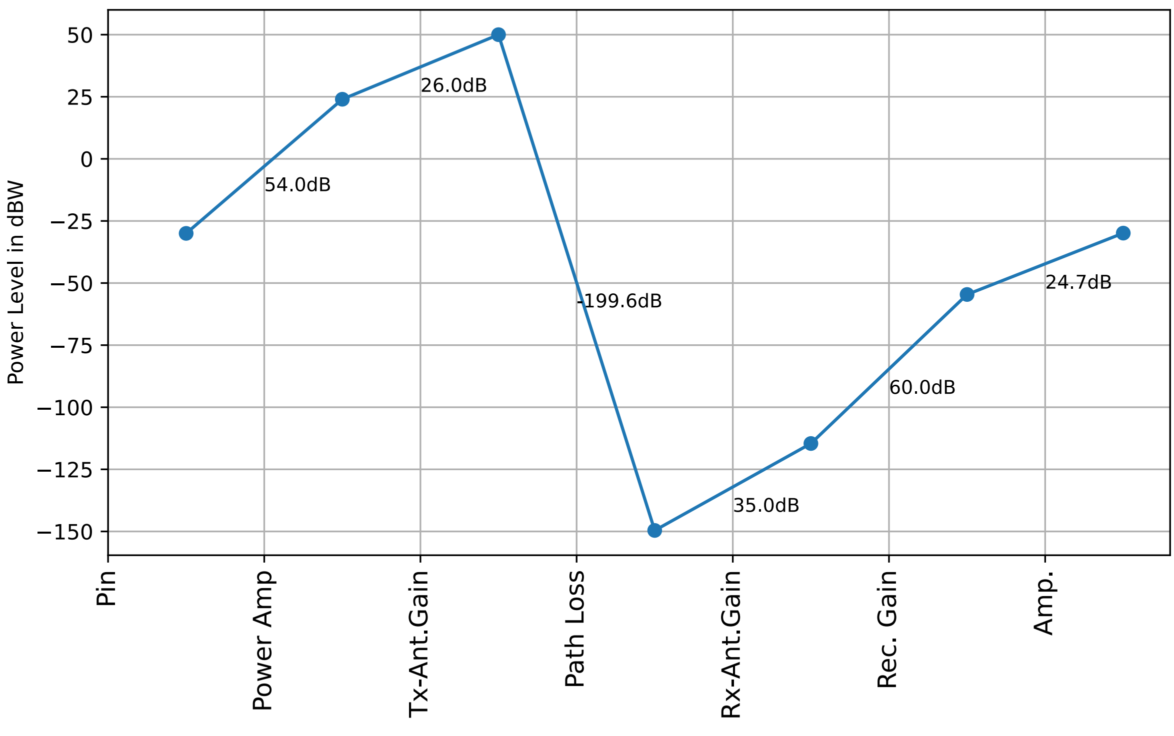


Figure 8.3: Power Diagram from Python generated

8.5 Detailed Link Budget

The following parameters contribute to the link budget or are used as figure of merits for the link budget. The quantities are interacting with each other

Operation Frequency f

The frequency is related to the wave length over the speed of light c by

$$f = \frac{c}{\lambda} \tag{8.5}$$

Transmit Power P_t

The transmit power is the physical available power at the power amplifier (PA) output of the satellite

Transmitter Losses L_{Tx}

Transmitter losses include losses in cables, connectors and a possible inserted power filter

Antenna Gain G

The antenna gain is the gain of power flux density in main radiation direction compared to the power flux density generated by an isotropic antenna

$$G = \frac{S}{S_{iso}} = \frac{S}{\frac{P_s}{4\pi R^2}} \tag{8.6}$$

The gain is often given in log-scale as dBi

The Gain is related to the antenna aperture by

$$G = \frac{4\pi A_e}{\lambda^2} \tag{8.7}$$

Antenna Aperture

The antenna aperture represents the collecting area of an antenna in the receiving case. the total received power is

$$P_{rec} = A_e \cdot S \tag{8.8}$$

where S is the power flux density at the antenna location. See S definition below.

Antenna Aperture Efficiency η_{ant}

The antenna efficiency captures the ratio between physical antenna area and effective antenna aperture

$$\eta_{ant} = \frac{A_e}{A_{phy}} \tag{8.9}$$

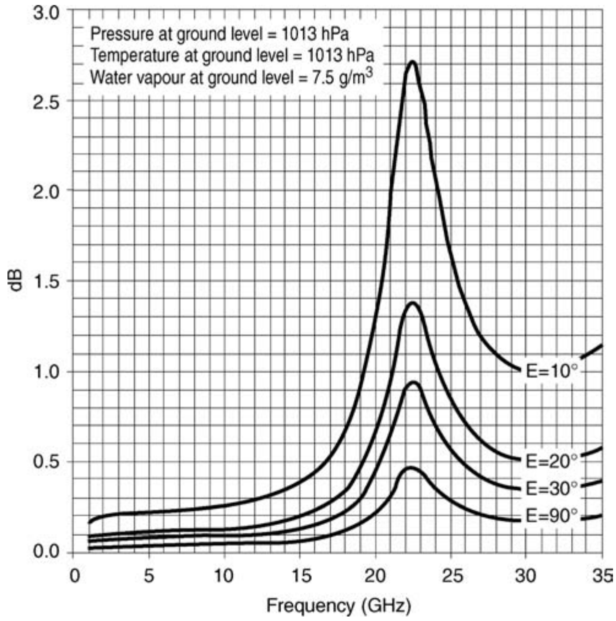


Figure 8.4: Attenuation due to Atmosphere, from [?]

Antenna Beam Width Θ_{BW}, Φ_{BW}

See details in Section Antennas

$$G = \frac{4\pi}{\Theta_{BW}\Phi_{BW}} \tag{8.10}$$

Distance R

The distance between Satellite and ground station (receiver)

Elevation Angle h

Elevation angle of the ground station antenna with respect to the horizon, depends on the satellite position and may change over time (for non GEO)

Atmospheric Attenuation L_{atm}

The atmospheric attenuation includes all losses due to the molecules, Water and dust in the atmosphere. This does not include the path loss. The atmospheric losses strongly depend on the elevation angle. See Figure 8.4.

Receiver Noise Figure F

See details Chapter Noise

Figure 8.5: Link Budget of the Terra SAR X Satellite

Receiver Noise Temperature T_e

Equivalent Noise temperature T_e of the receiver excluding antenna receives noise

Receiver Losses L_{Rx}

Transmitter losses include losses in cables, connectors and a possible inserted power filter

Polarization Losses

Losses due to polarization mismatch, typically up to 3 dB

Pointing Error, Depointing Loss

Losses due to pointing errors of the antenna. The bigger the gain, the more likely we have depointing errors

Antenna Noise Temperature T_a

The Antenna noise temperature is a combination of the ground noise temperature T_g and the sky noise temperature T_{sky}. See a chart at Figure **??**

Contour Attenuation

System Noise Temperature

G/T

Antenna Gain versus system noise temperature. The G/T is a figure of merit for receiver.

ENTR or SNR

$SNR = \frac{P_r}{N} = \frac{EIRP \cdot L_{path} \cdot G}{k \cdot T \cdot B} = EIRP \cdot L_{path} \cdot \frac{G}{T} \cdot \frac{1}{k} \frac{1}{B}$

Description	Quantity	Value	Unit
Frequency Band:			
Frequency	f=	8.20E+009	Hz
Wave Length	lambda=	3.66E-002	m
Transmitter			
Transmit Power	Ptx=	8	W
		9	dBW
Transmitter Losses	Ltx=	0	dB
Antenna Efficiency	Eta=	55	%
Gain	G=	5.6	
Gain		7.5	dBi
Link			
Distance	R=	1731.7	km
Elev.Angle	H=	10	deg
Atm.Dämpfung	Latm=	1	dB
Receiver			
Location	Neustrelitz		
Receiver Noise Figure	F=	1	dB
Receiver Noise Temp	Trx=	75.1	K
Receiver Losses	Lrx=	0.5	dB
Physical Temperature	T=	290	K
Antenna Diameter	D=	7.3	m
Antenna Efficiency	Eta=	60	%
Antenna Gain	GantRx=	236081.1	
		53.7	dB
Beam Width	Theta3db=	0.35	deg
Polaristion Loss	Lpol=	0	dB
Ground Noise Temp	Tboden=	45	K
Raumrauschtemp.	Tsky=	20	K
Antenna Noise Temp	Tant=	65	K
Pointing Error	Thetar=	0.1	deg
Contour Attenuation	Lcont=	3	dB
System			
EIRP		13.5	dBW
Path Loss		175.5	dB
Total Link Loss		176.5	dB
Depointing Atten.	Lp=	1.0	dB
System Noise Temp	Trx=	164.6	K
G/T Empfänger	G/T=	30.1	dB/K
Carrier-to-Noise-Ratio	C/No=	95.7	dBHz
Receiver Bandwidth	BW=	250.00	Mhz
C/N	C/N=	11.74	dB

Table 8.1: Link Budget Terra SAR

Digital Transmission Systems 9

9.1 Channel Capacity

The most important question associated with a communication channel is the maximum rate at which it can transfer information. Information can only be transferred by a signal if the signal is permitted to change. Analogue signals passing through physical channels may not change arbitrarily fast. The rate at which a signal may change is determined by the bandwidth. In fact it is governed by the same Nyquist-Shannon law as governs sampling; a signal of bandwidth B may change at a maximum rate of $2B$. If each change is used to signify a bit, the maximum information rate is $2B$.

The Nyquist-Shannon theorem makes no observation concerning the magnitude of the change. If changes of differing magnitude are each associated with a separate bit, the information rate may be increased. Thus, if each time the signal changes it can take one of n levels, the information rate is increased to

$$R = 2B\log_2(n) \tag{9.1}$$

This formula states that as n tends to infinity, so does the information rate.

Is there a limit on the number of levels? The limit is set by the presence of noise. If we continue to subdivide the magnitude of the changes into ever decreasing intervals, we reach a point where we cannot distinguish the individual levels because of the presence of noise. Noise therefore places a limit on the maximum rate at which we can transfer information. Obviously, what really matters is the signal-to-noise ratio SNR.

There is a theoretical maximum to the rate at which information passes error free over the channel. This maximum is called the channel capacity C. The famous Hartley-Shannon Law states that the channel capacity C is given by

$$\boxed{C = B\log_2(1 + SNR) \tag{9.2}}$$

Note that SNR is linear in this expression!

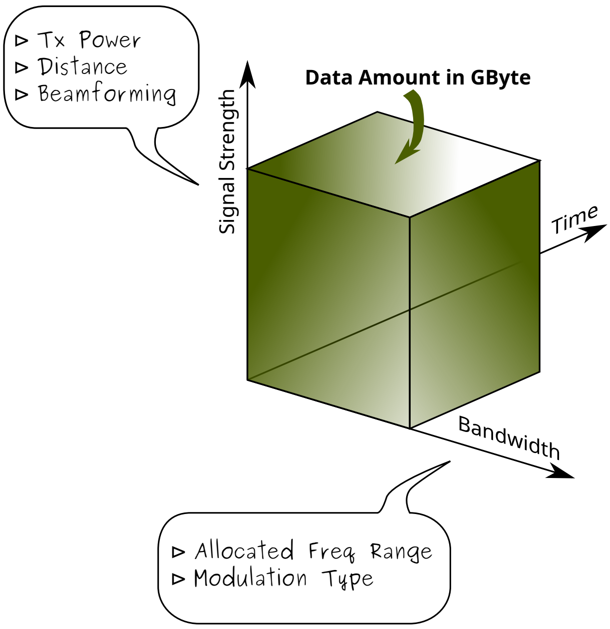


Figure 9.1: Shannon Box

Example 13:

A 10KHz channel operating in a SNR of 15dB has a theoretical maximum information rate of

$C = 10,000\frac{1}{\text{s}}\log_2(1+10^{\frac{15}{10}}) = 50278\frac{\text{Bits}}{\text{s}}$

For an SNR of 7 dB we need more Bandwidth in order to transmit the same amount of information per second, namely

$B = \frac{C}{\log_2(1+SNR)} = 19.4\text{kHz}$

Remember, that the noise in the Channel is given by the noise power spectral density n_o (the noise power per Hertz) times the Bandwidth $N = n_o B$. Hence with constant signal power S and in the presence of Gaussian white noise N, the channel capacity approaches the upper limit with increasing bandwidth.

$$\begin{aligned} C &= B\log_2(1+\frac{S}{n_o B}) \\ &= \frac{S}{n_o}\log_2\left[1+\frac{S}{n_o B}\right]^{\frac{n_o B}{S}} \\ \lim_{B\to\infty} C &= \frac{S}{n_o}\log_2 e \approx 1.44\frac{S}{n_o} \end{aligned} \tag{9.3}$$

There is a lower bound on the received signal power in a system for a given capacity, irrespective of the bandwidth utilized. For a given capacity

$$S = \frac{Cn_o}{1.44} \tag{9.4}$$

Once the minimum received signal power is exceeded, the bandwidth is traded of with SNR and vice verse.

Example 14:

For a received noise from an antenna at $T_A = 290\text{K}$ we have $n_o = kT_A = 4.0 \cdot 10^{-21}\,\frac{W}{\text{Hz}}$

We need for a channel capacity of 50278 bits/s (see last example) $S = \frac{C\eta}{1.44} = 1.3973e - 16\text{W}$. This is the power needed for an infinite bandwidth available.

For smaller Bandwidth we can choose for example:

SNR=7dB (=5) needs 19.5 kHz bandwidth

SNR=15dB (=31.6) needs 10 kHz bandwidth (as shown in the example above)

In order to cut the bandwidth by roughly half (19.5/10) we have to increase the signal power by the factor 6.3 (31.6/5)

The Shannon-Hartley-Theorem makes no statement as to how the channel capacity is achieved. In fact, channels only approach this limit. The task of providing high channel efficiency is the goal of coding techniques. The failure to meet perfect performance is measured by the bit-error-rate (BER).

9.2 Symbol Rate

In digital modulation, an analog carrier signal is modulated by a discrete signal. Digital modulation methods can be considered as digital-to-analog conversion, and the corresponding demodulation or detection as analog-to-digital conversion. The changes in the carrier signal are chosen from a finite number of M alternative symbols (the modulation alphabet).

In the transmitter a symbol generator generates symbols with M states, where $M = 2^m$, from m consecutive bits of the input bit stream.

For a bit rate R_b (bit/s) at the modulator input, the symbol rate R_s at the modulator output (the number of changes of state of the carrier per second) is given by:

$$Rs = \frac{R_b}{m} = \frac{R_b}{\log_2 M} \tag{9.5}$$

9.3 Digital Modulation Schemes

Example 15:

An amplitude modulation uses four different amplitude levels to encode two bits at a time, e.g.

$$\begin{aligned} 00 &\rightarrow 0\,\text{V} \\ 01 &\rightarrow 0.5\text{V} \\ 10 &\rightarrow 1.0\text{V} \\ 11 &\rightarrow 1.5\text{V} \end{aligned}$$

Two bits form one symbol, i.e. $m = 2$ with $M = 2^m = 4$ states.

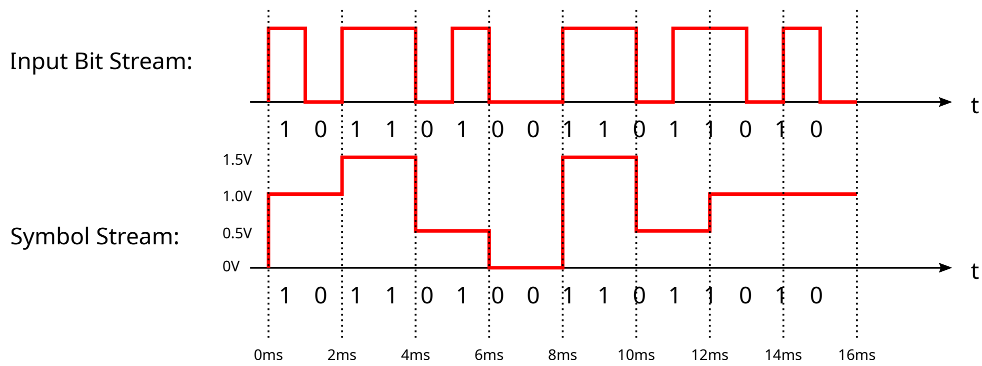


Figure 9.2: Bit Rate vs. Symbol Rate from Example above

The input stream of data has a bit rate of R_b =10kBits/s following a symbol rate of

$$R_s = \frac{10\text{kBits/s}}{2} = 5\text{kBits/s} \tag{9.6}$$

The symbol rate is half the bit rate as two bits are transferred simultaneously. See also Figure

Modulation Schemes

The main modulation schemes are phase modulation (phase shift keying PSK) and quadrature amplitude modulation (QAM).

Phase modulation,is particularly well suited to satellite links. In fact, it has the advantage of a constant envelope and, in comparison with frequency shift keying (FSK), it provides better spectral efficiency (number of bits/s transmitted per unit of radio-frequency bandwidth—see Section 9.5). Depending on the number m of bits per symbol, different M-ary phase shift keying modulations are considered:

- The simplest form is basic two-state modulation ($M = 2$), called 'binary phase shift keying' (BPSK) with standard direct mapping. When differential encoding is considered it is called 'deferentially encoded BPSK' (DE-BPSK). It is of interest because it enables a simplified demodulation process.
- If two consecutive bits are grouped to define the symbol, a four state modulation ($M = 4$) is defined, called 'quadrature phase shift keying' (QPSK) with direct mapping. Deferentially encoded QPSK (DE-QPSK) could be envisioned, but it is not used in practice (except for the specific case of \$\pi/4$-QPSK) as differential demodulation displays significant performance degradation compared to standard coherent demodulation when M is larger than 2.
- Higher-order modulations ($= 8, 16, 32, \dots$) are obtained with $m = 3, 4, \dots$ etc. bits per symbol. As the order of the modulation increases, the spectral efficiency increases as the number of bits per symbol. On the other hand, higher-order modulations require more energy per bit (Eb) to get the same bit error rate (BER) at the output of the demodulate.

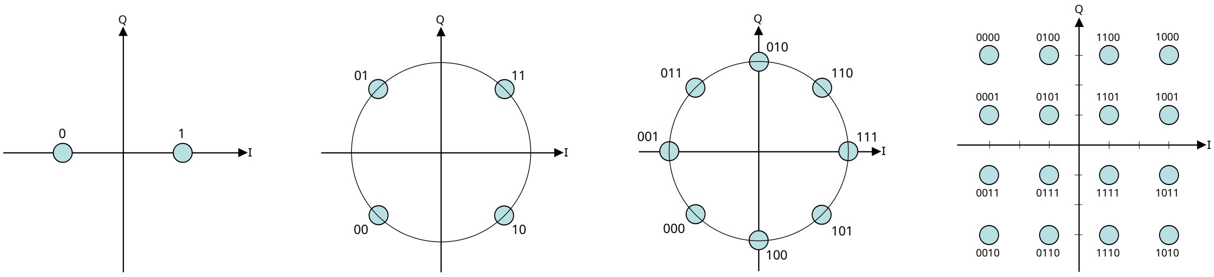


Figure 9.3: Some Constellations of Digital Vector Modulation Schemes: BPSK, QPSK 8PSK, 16QAM

With a modulation of high order (M equal to or larger than 16), better performance is achieved by considering hybrid amplitude and phase shift keying (APSK). States of the carrier correspond to given values of carrier phase and carrier amplitude (two values for 16APSK, three values for 32APSK).

9.4 Bit Error Rate BER

(see also [2, p.128])

The bit error rate or bit error ratio (BER) is the number of bit errors divided by the total number of transferred bits during a studied time interval. BER is a unitless performance measure, often expressed as a percentage.

When the noise superimposes the signal, there is a certain chance, that the signal is detected wrongly, i.e. a transmitted “1” is detected as “0” or vise verse.

In a noisy channel, the BER is often expressed as a function of the ratio E_b/N_o.

The ratio E_b/N_o arises in the expression for error probability, where E_b is the energy per channel bit. This is the product of the power of the received carrier for the duration of one bit, namely $E_b = P_b \cdot T_b = \frac{P_b}{R_b}$. The P_b is here the carrier power, which we referred to as the received signal power S in the last Section. n_o refers to the noise power density in W/Hz. The total received noise is hence $N = B \cdot n_o$

Hence:

$$E_b/n_o = \frac{S/R_b}{n_0} = \frac{S}{N}\frac{B}{R_b} = SNR\frac{B}{R_b} \tag{9.7}$$

For example, in the case of QPSK modulation and AWGN channel, the BER as function of the SNR is given by:

for BPSK, QPSK

$$BER = \frac{1}{2}\text{erfc}\sqrt{\frac{E_b}{n_0}} \tag{9.8}$$

for M-PSK with m bits per symbol

$$BER = \frac{1}{m}\text{erfc}\left(\sqrt{m\frac{E_b}{n_0}}\sin\frac{\pi}{M}\right) \tag{9.9}$$

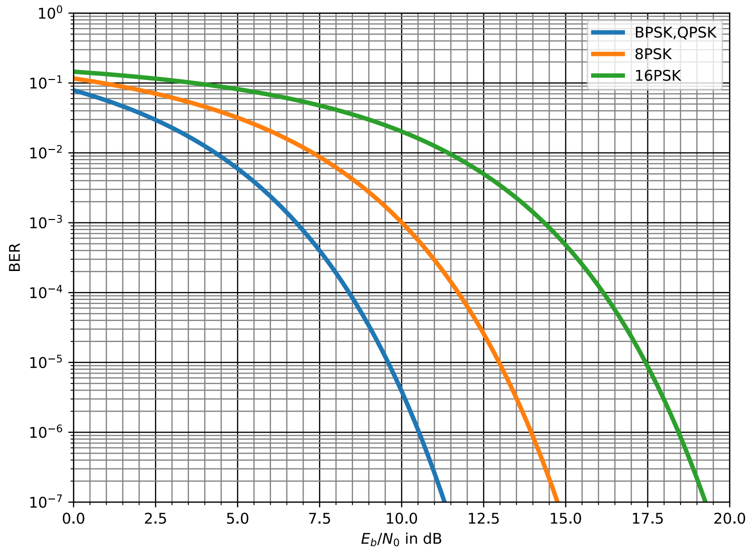


Figure 9.4: Bit Error Ratios for various Modulation Schemes

for D-QPSK

$$BER = \frac{1}{2}\exp(-E_b/n_o) \tag{9.10}$$

Where erfc is the complementary error function defines as

$$\text{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_0^x e^{-z^2} dz \tag{9.11}$$

Note, this function is often defined slightly different as Q-function (see [1]) as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{\frac{-z^2}{2}} dz = \frac{1}{2}\text{erfc}(\frac{x}{\sqrt{2}}) \tag{9.12}$$

Detailed information for all kinds of modulations can be find in e.g. [1]

9.5 Spectral Efficiency

Modulation spectral efficiency can be defined as the ratio of the transmitted bit rate R_b to the bandwidth B occupied by the carrier. The bandwidth occupied by the carrier depends on the spectrum of the modulated carrier and the filtering it undergoes. Hence, the spectral efficiency Γ describes how many bits per 1 Hz bandwidth can be transmitted.

$$\Gamma = \frac{R_b}{B} \tag{9.13}$$

Therefore the spectral efficiency Γ for an M-ary modulation scheme is following eqn. **??**:

Digital Modulation			
Modulation		QPSK	
Bitrate	Rb=	300	Mbit/s
Symbol Rate	Rs=	150	Msym/s
Symbol duration	Ts=	6.67E-009	S
Filterung		Raised-cos	
Roll-Off Faktor	Alpha=	0.35	
Bandbreitenbedarf	B=	202.5	Mhz
Spektraleffizienz	Gamma=	1.5	bit/Hz
E/No für QPSK	E/No=	9.78	dB
Bit Error Probability	BEP=	6.5E-006	
C/No	C/No=	94.6	dBHz

Figure 9.5: Link Budget (Digital Issues) of the Terra SAR X Satellite

Symbol	Quantity	Unit
R_b	Bit Rate in	bits/s
R_s	Symbol Rate	sym/s
T_b	Duration per Symbol	s
T_s	Duration per Bit	s
E_b	Energy per Bit	W/bit
n_0	Noise Power per 1 Hz Bandwidth	W/Hz
SNR	Signal to noise Ratio	1
B	Bandwidth of Channel	Hz
Γ	Spectral Efficiency	$\frac{\text{Bits}}{\text{s}\cdot\text{Hz}}$

Table 9.1: Summary of used Symbols and Quantities

$$\Gamma = \frac{R_b}{B} = \frac{R_b T_S}{1+\alpha} = \frac{\log_2(M)}{1+\alpha} \tag{9.14}$$

where $m = \log_2 M$ is the number of bits per symbol.

Example 16:

Roll-off Factor $\alpha = 0.35$ required bandwidth is $1.35/T_S$ and the spectral efficiency is $\Gamma = 0.7\frac{\text{bits/s}}{Hz}$ for BPSK and $\Gamma = 1.5\frac{\text{bits/s}}{Hz}$ for QPSK

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Satellite Systems 10

10.1 GPS

[from: `http://www.radio-electronics.com/info/satellite/gps/gps-technical-tutorial.php`]

Today the Global Positioning System or GPS is well established for military, commercial and private use, and although complicated, the GPS technical aspects are well understood and the levels of performance are high. Small GPS receivers are available at very reasonable prices. They are available at these prices because the Navstar satellite system that forms the basis of GPS is owned and run by the US Department of Defense. A further reason that GPS receivers are available at such low prices is because of the significant advances made in integrated circuit and digital signal processing techniques.

The fully operational GPS satellite system consists of a constellation of 24 operational satellites with a few more in orbit as spares in case of the failure of one. The GPS satellites are in one of six orbits. These are in planes that are inclined at approximately 55 degrees to the equatorial plane and there are four satellites in each orbit. This arrangement provides the earth user with a view of between five and eight satellites at any time from any point on the Earth. When four satellites are visible, sufficient information is available to be able to calculate the position on Earth.

10.1.1 GPS control network

The GPS satellites need to be monitored and controlled from the ground and it is necessary to be in contact with each satellite for most of the time to be able to maintain the level of performance required. To achieve this there is a master station located at Falcon Air Force Base, Colorado Springs, USA. However there are other remote stations are located on Hawaii, Ascension Island, Diego Garcia and at Kwajalein. Using all these stations the satellites can be tracked and monitored for 92% of the time. This results in two 1.5 hour periods each day when the satellite is out of contact with the ground stations.

Using the network of ground stations the performance of the GPS satellites is monitored very closely. The information that is received at the remote stations is passed to the main operational centre at Colorado Springs and the received information is assessed. Parameters such as the orbit and clock performance are monitored and actions taken to re-position the satellite if it is drifting even very slightly out of its orbit, or any adjust the clock if necessary or more usually provide data to it indicating its error. This information is passed to three uplink stations co-located with the downlink monitoring stations at Ascension Island, Diego Garcia and Kwejalein.

10.1.2 GPS operation

GPS operates by a process of triangulation. Each GPS satellite transmits information about the time, and its position. By comparing the signals received from four satellites the receiver is able to deduce how long it has taken for the signals to arrive and from knowledge of the position of the satellites it can calculate its own position.

The GPS satellites transmit two signals on different frequencies. One is at 1575.42 MHz and the other at 1227.6 MHz. These provide two services, one known as course acquisition (C/A) and the other is a precision (P) signal. The precision signal is only available for the military, but the C/A elements of GPS are open to commercial use, although initially a random "wobble" was put onto this to degrade its accuracy for civilian use. This facility known as Selective Availability (S/A) was discontinued in May 2000.

Both signals are transmitted using direct sequence spread spectrum (DSSS), and this enables all the satellites to use the same frequency. They can be separated in the GPS receiver by the fact that they use different orthogonal spreading codes, and this works in exactly the same way as the CDMA cell phone systems. The spreading codes are accurately aligned to GPS time to enable decoding of the signals to be facilitated.

The coarse acquisition signal at 1.5 GHz uses a 1.023 MHz spreading or chip code, while the precision signal is transmitted at 1.2 GHz using a 10.23 MHz code. This precision signal is encrypted and uses a higher power level. Not only does this assist in providing a higher level of accuracy, it also improves the reception in buildings.

All the GPS satellites continually transmit information. This includes what are termed ephemeris data, almanac data, satellite health information, and clock correction data. Correction parameters for the ionosphere and troposphere are also transmitted as these have a small but significant effect on signals even at these frequencies.

The ephemeris data is information that enables the precise orbit of the GPS satellite to be calculated. The almanac data gives the approximate position of all the satellites in the constellation and from this the GPS receiver is able to discover which satellites are in view. Although each satellite contains an atomic clock, they all drift to a small extent and as a result details of the clock offsets are transmitted. It is found that it is more effective to measure the error and transmit this data than maintain the clock exactly on time.

10.1.3 Transmitted data

The data transmitted by the GPS satellite is formatted into 25 frames, each 1500 bits in length. The frames are divided equally into five sub-frames. At a rate of 50 bits per second data transmission rate it takes six seconds to transmit a sub-frame, 30 seconds to transmit a frame and 12.5 minutes to transmit the complete set of 25 frames.

Sub-frames 1, 2, and 3 are the same for all data frames and contain critical satellite specific information. This allows the receiver to determine a single satellite clock correction and ephemeris within 30 seconds. Sub-frames 4 and 5 contain less critical data that applies to the complete satellite constellation and this data is distributed throughout the 25 frames.

10.1.4 GPS reception

The strength of the GPS signal that is received on the surface of the earth is very low. Typically it is around -127 dBm (for 0dB gain antennas) although there is a variation on this arising from the elevation of the GPS satellite. This may reduce the signal level by up to about 3 dB. To receive the signal a receiver bandwidth of around 2 MHz is often used even though a chip rate of 1.023 MHz is used. The reason for the use of the wide bandwidth is that it reduces differential group delay that would cause positional errors.

With the wide bandwidth and low signal levels this means that the actual received signal is below the thermal noise level. The only way to recover the signal is by correlation over a large number of chips. Commonly, the correlation is done over a complete code cycle of 1023 chips, giving a correlation time of 1ms. Using these techniques the best receivers may receive signals down to levels of around -142 dBm.

From a cold start, the GPS receiver chooses a satellite to look for and tries all possible code phases to see if correlation is achieved. The problem is compounded by the Doppler shift on the satellite which forces the receiver to look in a number of Doppler 'bins' for each code phase, thus increasing the search time. If no satellite is found then the search is repeated for the next satellite. Modern receivers speed up this search by using large numbers of correlators in parallel. Tens of thousands of correlators are typically used.

Once the first signal has been correlated, the GPS receiver can then demodulate the data the signal carries. With the almanac data available the GPS receiver is able to deduce which satellites are visible, and hence which ones to receive. In addition to this it enables the receiver to correlate the signals more quickly.

The receiver measures the relative phases of the signals from each of the satellites to provide what are termed "pseudoranges". Then the GPS receiver uses the emphemeris data and also compensates for elements such as the clock offsets, effects of the ionosphere and troposphere and even relativity. The receiver uses all of this information to calculate its own clock error and position. The overall calculation is somewhat involved and uses iterative processes to reach the final result.

In view of the time taken to correlate with the GPS satellites, as well as the time taken to transmit the data, the Time To First Fix (TTFF) is usually in excess of 12.5 minutes. Faster TTFF times from what is termed a cold start are often achieved by using a vast number of correlators, and by using this approach it is not always necessary to wait until all the data has been received before the first fixes can be made.

10.1.5 Summary

The Global Positioning System, GPS, has been in use for a number of years now and has proved to be very successful, with the GPS technical aspects being well understood by the various companies designing, manufacturing and selling GPS or satnav systems.

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