Digital Communication

October 22, 2023

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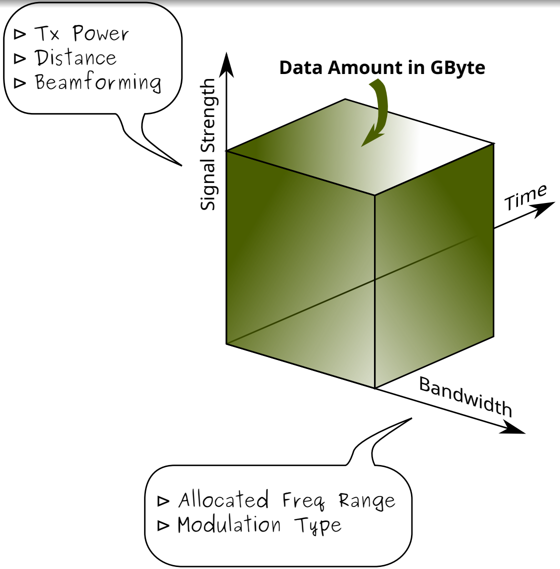
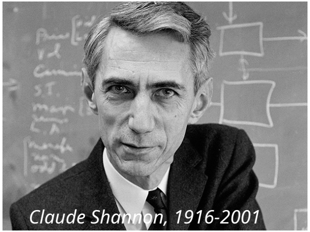
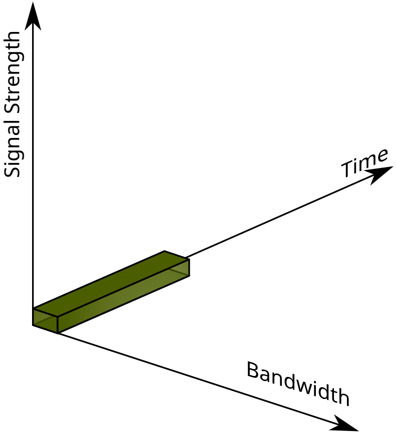
Outline





$$C = B\log(1+\frac{S}{N})$$


Signal Strength
Time
Bandwidth

Example

Optimal Capacity

A 10KHz channel operating in a SNR of 15dB

The Channel has a theoretical maximum information rate of

MICROWAVE LAB

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$$C = 10,000 \frac{1}{\mathrm{s}} \log_2(1 + 10^{\frac{15}{10}}) = 50278 \frac{\mathrm{Bits}}{\mathrm{s}}$$

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Optimal Capacity

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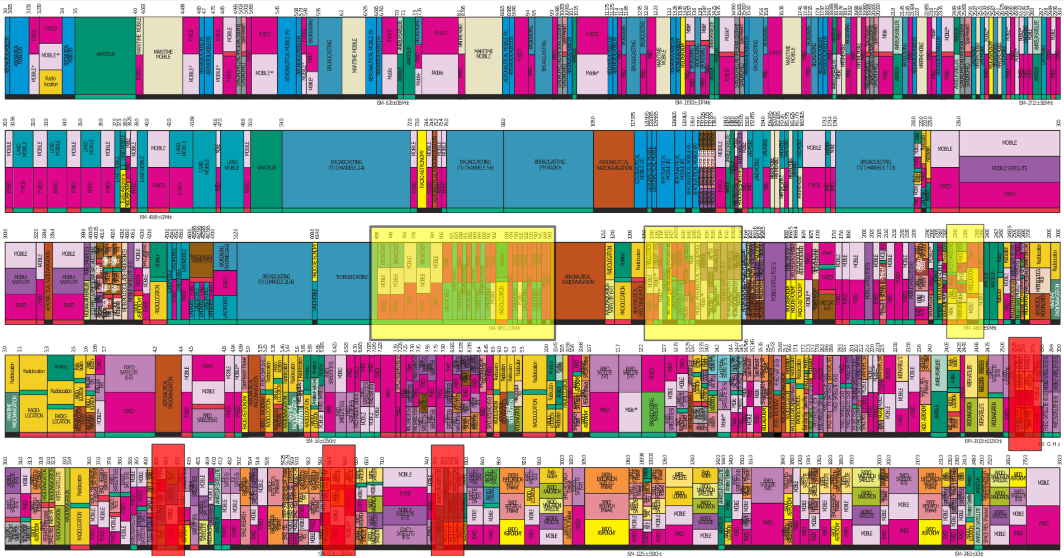
$$B = \frac{C}{\log_2(1+SNR)} = 19.4\text{kHz}$$

How to Transmit the Amount of X Bytes?

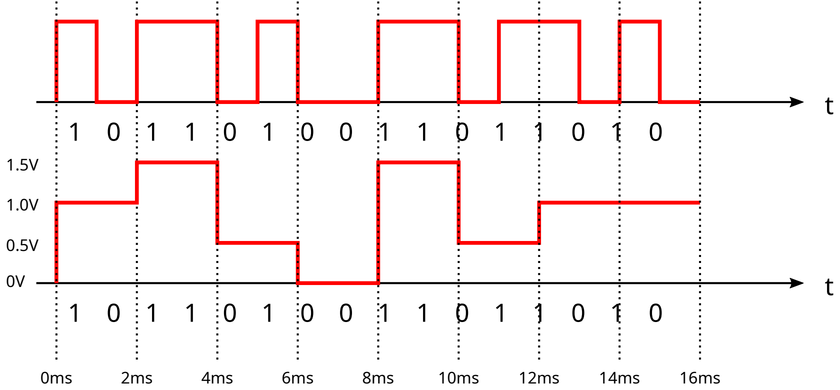
- Bandwidth?
- Signal Strength?
- Noise Considerations?



Bit Rate vs. Symbol Rate

Input Bit Stream:

Symbol Stream:



Symbol Rate, Bit Rate

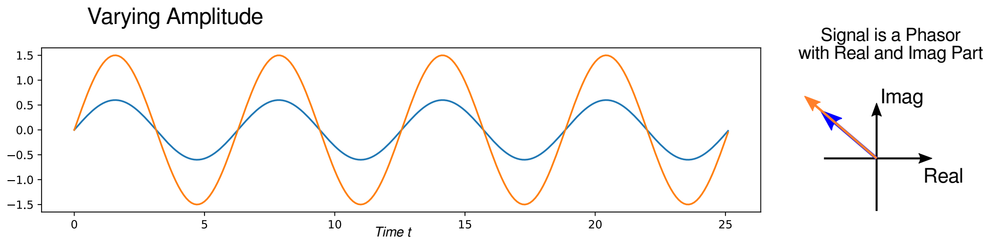
The bit Rate R_b describes the bits transmitted per second.

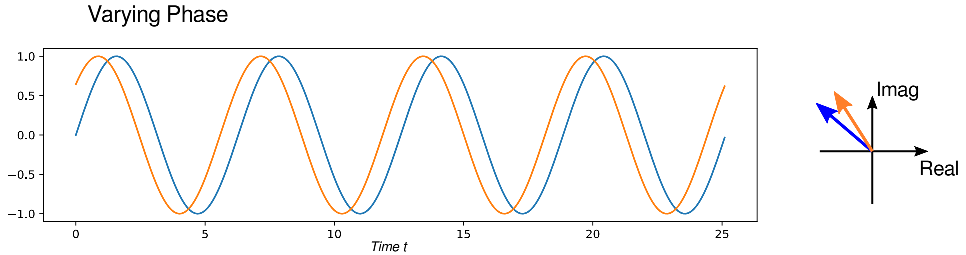
In the transmitter a symbol generator generates symbols with M states, where $M = 2^m$, from m consecutive bits of the input bit stream.

For a bit rate R_b (bit/s) at the modulator input, the symbol rate R_s at the modulator output (the number of changes of state of the carrier per second) is given by:

$$R_s = \frac{R_b}{m} = \frac{R_b}{\log_2 M}$$

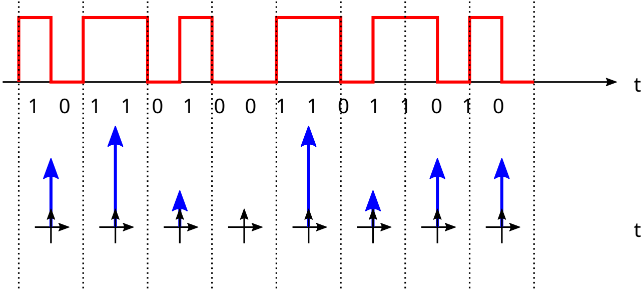
Signals at Frequency *f* have two “Variables”





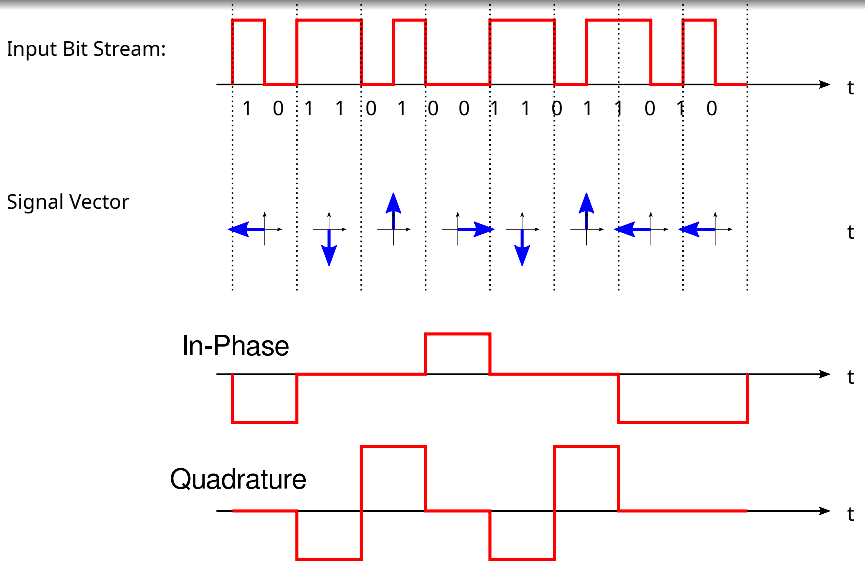
Input Bit Stream:

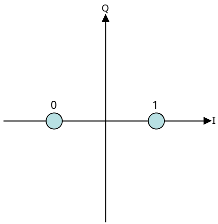
Signal Vector



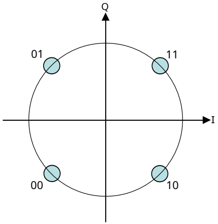
Code Mapping:

Phase Modulation

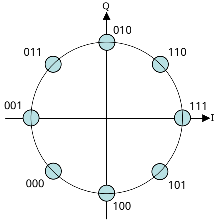




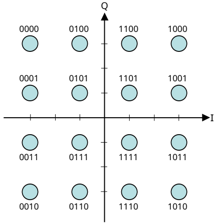
Q
I
0
1



Q
I
01
11
00
10



Q
I
010
011
110
001
111
000
101
100



Q
I
0000
0100
1100
1000
0001
0101
1101
1001
0011
0111
1111
1011
0010
0110
1110
1010

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  - P_b is here the carrier power, which we referred to as the received signal power S in the last Section.
  - n_o refers to the **noise power density in W/Hz**, where The total **received noise power** is hence $N = B \cdot n_o$

E_b/n_o

$$E_b/n_o = \frac{S/R_b}{n_0} = \frac{S}{N}\frac{B}{R_b} = SNR\frac{B}{R_b}$$

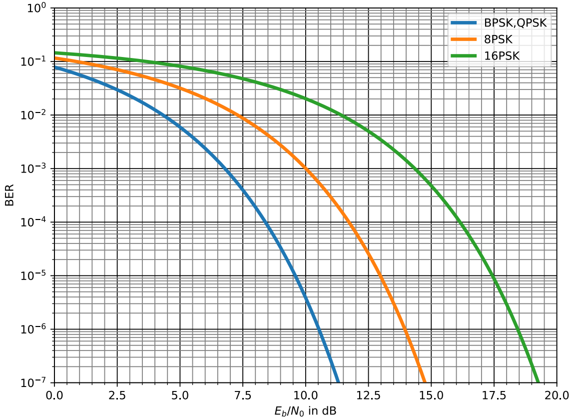
BPSK, QPSK

$$BER = \frac{1}{2}\operatorname{erfc}\sqrt{\frac{E_b}{n_0}}$$

for M-PSK with m bits per symbol

$$BER = \frac{1}{m}\operatorname{erfc}\left(\sqrt{m\frac{E_b}{n_0}}\sin\frac{\pi}{M}\right)$$

Bit Error Ratios for Various Modulation Schemes



Outline

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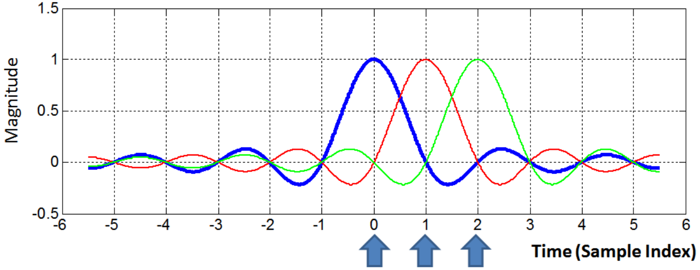
Pulse Shaping



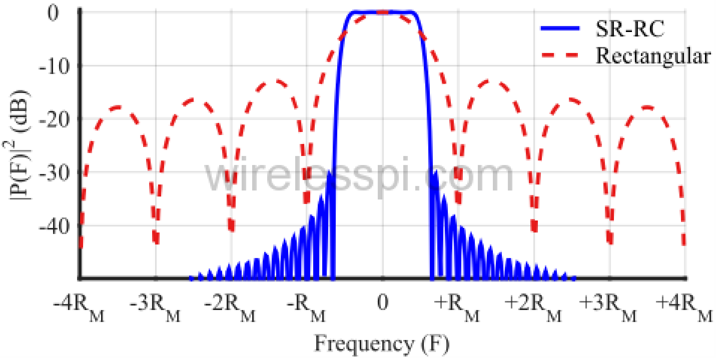
Zero ISI at the sampling instances for three consecutive symbols using a Nyquist Sinc pulse shape

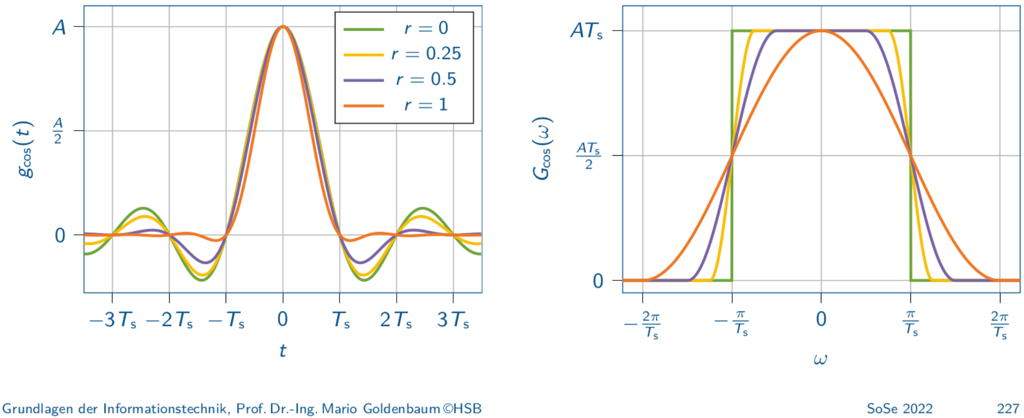
The Nyquist Sinc pulse, extending from negative infinity to infinity, results in a brickwall spectrum of 1/T wide. (Not realizable)

4/15/2012

75

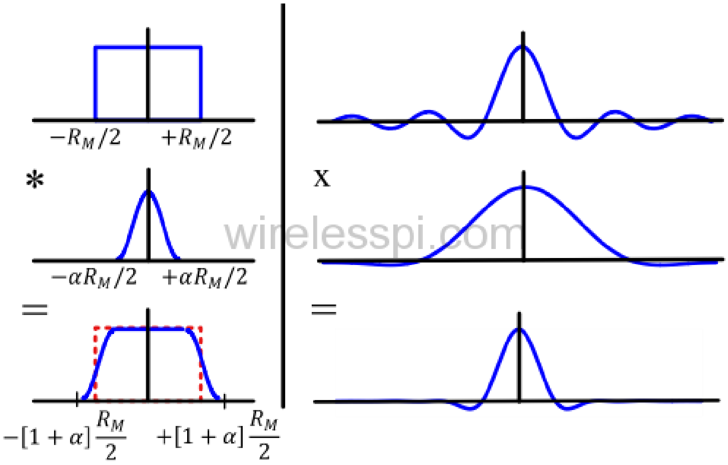
RRC Filtering





$$B = 2 \cdot (1+\alpha)\frac{R_s}{2} = \frac{(1+\alpha)}{T_s} \tag{1}$$

Impact of α



Spectral Efficiency

the spectral efficiency Γ describes how many bits per 1 Hz bandwidth can be transmitted.

$$\Gamma = \frac{R_b}{B}$$

Therefore the spectral efficiency Γ for an M-ary modulation scheme is following eqn. 1:

$$\Gamma = \frac{R_b}{B} = \frac{R_b T_S}{1+\alpha} = \frac{\log_2(M)}{1+\alpha}$$

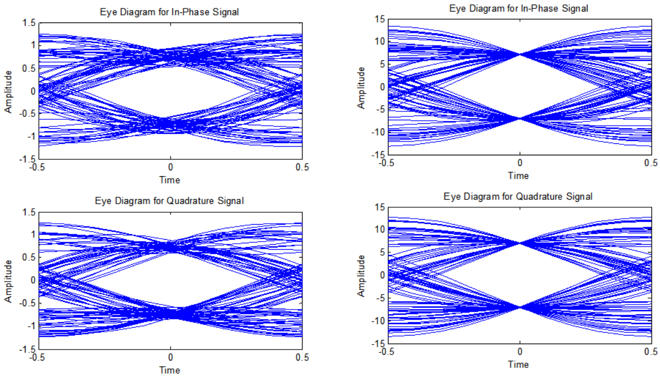
where $m = \log_2 M$ is the number of bits per symbol.

Example

Roll-off Factor $\alpha = 0.35$

- required bandwidth is $1.35/T_S$ and
- the spectral efficiency is $\Gamma = 0.7\,\frac{\text{bits/s}}{Hz}$ for BPSK and
- $\Gamma = 1.5\,\frac{\text{bits/s}}{Hz}$ for QPSK

Eye Diagram - QPSK



Alpha = 0.25 RRC Filter

After 2nd RRC (in receiver): RC response

Example

Digital Modulation			
Modulation		QPSK	
Bitrate	Rb=	300	Mbit/s
Symbol Rate	Rs=	150	Msym/s
Symbol duration	Ts=	6.67E-009	S
Filterung		Raised-cos	
Roll-Off Faktor	Alpha=	0.35	
Bandbreitenbedarf	B=	202.5	Mhz
Spektraleffizienz	Gamma=	1.5	bit/Hz
E/No für QPSK	E/No=	9.78	dB
Bit Error Probability	BEP=	6.5E-006	
C/No	C/No=	94.6	dBHz