Constants

Permittivity of Free Space: $\varepsilon_0 = 8.854 \cdot 10^{-12} \frac{\text{As}}{\text{Vm}}$

Permeability of Free Space: $\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$

Velocity of Light: $c = 2.998 \cdot 10^8 \frac{\text{m}}{\text{s}}$

Boltzman Constant: $k = 1.381 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$

Stefan-Boltzman-Constant of heat $\sigma = 56,7E - 9\text{W/K}^4\text{m}^2$

Universal gravitational constant $G = 6.6730 \cdot 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$

Mass of Earth $M_E = 5.9742 \cdot 10^{24}\text{kg}$

Standard gravitational parameter $\mu = GM_E = 3.986 \cdot 10^{14} \frac{\text{m}^3}{\text{s}^2}$

Radius Earth $R_E = 6\,378\,135\text{m}$

Escape Velocity $v_{esc} = 11.2\text{km/s}$

Orbital Mechanics

Orbital period $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

semi-major-axis $a = \frac{r_a + r_p}{2}$

Perigee Distance $r_p = a(1-e)$

Apogee Distance $r_a = a(1+e)$

Instantaneous velocity at distance r $v = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$

Instantaneous distance at velocity v $r = \sqrt{\frac{2a\mu}{(a\,v^2 + \mu)}}$

Escape velocity $v = \sqrt{\frac{2\mu}{r}}$

Apogee velocity $v_{apo} = \sqrt{\frac{2\mu}{r_a} - \frac{2\mu}{r_p + r_a}}$

Perigee velocity $v_{per} = \sqrt{\frac{2\mu}{r_p} - \frac{2\mu}{r_p + r_a}}$

Apogee distance from known velocities $r_a = \frac{r_p^2 v_{per}^2}{2\mu - r_p v_{per}^2}$

Perigee distance from known velocities $r_p = \frac{r_a^2 v_{per}^2}{2\mu - r_a v_{per}^2}$

Circular orbit velocity $v_{circ} = \sqrt{\frac{\mu}{r}}$

Mean motion $n = \frac{2\pi}{T}$

Mean anomaly $M = \frac{2\pi}{T}(t - t_p) = n(t - t_p)$

where t_p is the instant of passing through the perigee.

Eccentric anomaly $M = E - e\sin E$

Distance from anomalies $r = a(1 - e\cos E)$

Sideral day $T_{sid} = 23\text{h}\,56\,\text{min}\,4.1\,\text{s} = 86164.1\text{s}$

Solar day $T_{solar} = 24\text{h} = 86400\text{s}$

Latitude of sub-sat point $\varphi = \arcsin\left[\sin(i) \cdot \sin(\omega + \nu)\right]$

Longitude of sub-sat point

$$\lambda = \arctan\left[\tan(\omega + v) \cdot \cos(i)\right] - \left[\frac{\Omega_E}{n}(E - e \cdot \sin E) \quad \underbrace{-\frac{\Omega_E}{n} \cdot (E_N - e \cdot sinE_N)}_{\text{Offset due to launch time}}\right]$$

distance observer-satellite $R = \sqrt{R_E^2 + r^2 - 2R_E\,r\,\cos\phi}$

with R_E being the radius of the Earth and

$\cos\phi = \cos L \cdot \cos\varphi\cos l + \sin\varphi \cdot \sin l$

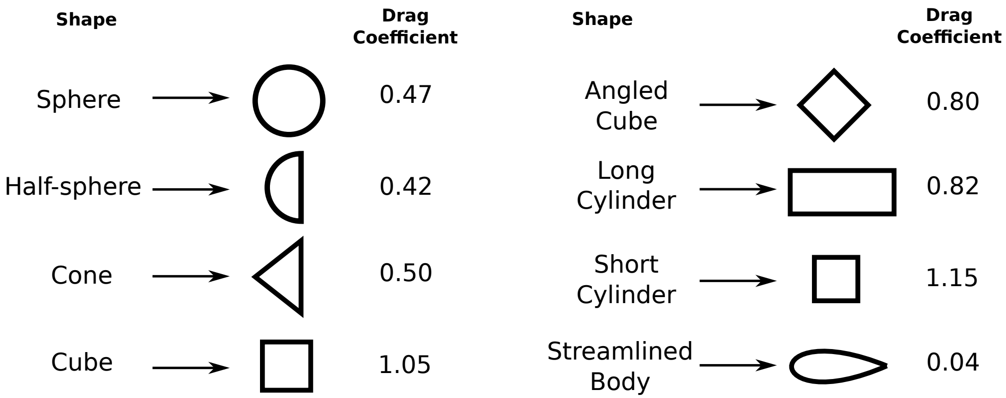
Elevation $E = \arccos(\frac{r}{R}\sin\phi)$

where $\sin\phi = \sqrt{1 - \cos^2\phi}$.

Azimuth direction $\sin a = \frac{\sin L \cos\varphi}{\sin\phi}$

Pertubations

The drag force $F_D = \frac{1}{2} C_D \rho v^2 A$



Propulsion

Specific impulse is the ratio of Thrust T to rate of fuel ejection

$$I_{sp} = \frac{T}{\frac{dm}{dt} g_0} = \frac{v_{ex}}{g_0}$$

Thrust $T = v_{ex} \cdot \frac{dm}{dt}$

Tsiolkovsky's rocket equation, M_0: Start Mass, M_1 :End Mass $\Delta v = v_{\text{ex}} \ln \frac{M_0}{M_1}$

Thermal Management

Heat absorption $Q_{in} = A_{ill} \cdot G \cdot \alpha$

Radiated power $Q_{out} = Q_{rad} = \epsilon \sigma A_{sur} T^4$

sun incident power density $G_s = 1367 \frac{\text{W}}{\text{m}^2}$

Albedo incident power density $G_a = G_s \cdot 0,34$

earth IR incident power density $G_{earth} = 237 \frac{\text{W}}{\text{m}^2}$

	Measurement Temperature	Surface Condition	Solar Absorptivity (VIS/NIR)	Infrared Emissivity (TIR)	Absorptivity/ Emissivity Ratio	Eqilibrium Temp.
Material	**T (K)**		**alpha**	**epsilon**	**a/e**	**T (K)**
Gold	294	As Received	0,299	0,023	**13,00**	**749**
Alu (6061-T6 @22°C)	294	As Received	0,379	0,0346	**10,95**	**717**
Alu (6061-T6 @150°C)	422	As Received	0,379	0,0393	**9,64**	**695**
Alu polisched (6061-T6 @22°C)	294	Polished	0,2	0,031	**6,45**	**628**
Alu polisched (6061-T6 @150°C)	422	Polished	0,2	0,034	**5,88**	**614**
Black Paint	295	Al. Substrate	0,975	0,874	**1,12**	**405**
Solar Cell-Fused Silica Cover	295		0,805	0,825	**0,98**	**392**
White Epoxy (@150°C)	422	Al. Substrate	0,248	0,888	**0,28**	**287**
White Epoxy (@22°C)	294	Al. Substrate	0,248	0,924	**0,27**	**284**
Aluminized Teflon	295		0,163	0,8	**0,20**	**265**
Silvered Teflon	295		0,08	0,66	**0,12**	**233**
OSR	295		0,077	0,79	**0,10**	**220**

Thermal Noise

Noise Power $P_n = \frac{V_n^2}{4R} = kTB$

with $k = 1.38 \cdot 10^{-23} \frac{\text{J}}{\text{K}}$ (Boltzman's constant) and B bandwidth of System

Spectral noise power density $S_n(\omega) = \frac{P_n}{2B} = \frac{kT}{2} = \frac{n_0}{2}$

with $n_o = kT$

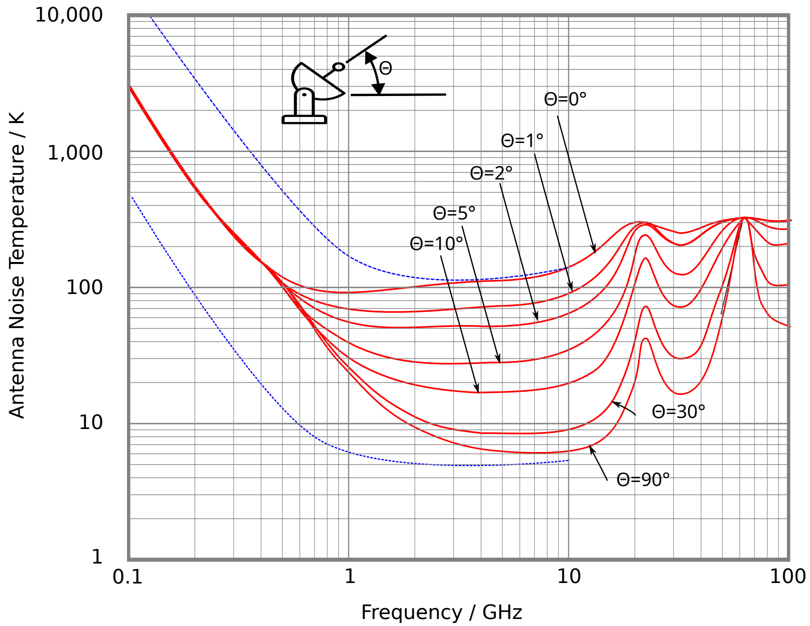
Gaussian Distribution $f_n(n) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-n^2/2\sigma^2}$

where σ is the effektive noise voltage

Bandlimited Noise $P_n = \Delta f \, n_o$

T_a is the Temp. T of a black body

Antenna Noise Temperature



Noise temperature at ground $T_A = 290\text{K}$

Noise Figure $F = \frac{S_i/N_i}{S_o/N_o} \geq 1$

with the condition: antenna noise temperature $T_a = T_0 = 290\text{K}$

Noise Temperature $T_e = (F-1)T_0$

Noise Temperature of passive components with loss $LF = 1+\frac{T_e}{T_0} = 1+(L-1)\frac{T}{T_0}$

at room temperature: $F = L$

Cascaded Two-Ports $F_{tot} = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1 G_2} + \cdots$

using noise temperature $T_{e,tot} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots$

Gain $G_{tot} = G_1 \cdot G_2 \cdot G_3$

G over T $G/T = \frac{\text{Antenna Gain}}{\text{System Noise}} = \frac{G_{ant}}{T_a + T_e}$

$$SNR = \frac{S_0}{N_0} = \frac{EIRP \cdot \frac{1}{L_p} \cdot G_{ant}}{k\,(T_a+T_e)\,B} = EIRP\,\frac{1}{L_p}\,\frac{1}{k}\,\frac{1}{B}\,\frac{G}{T}$$

$$SNR_{dB} = EIRP_{dBW} - L_{p,dB}\,\underbrace{-10\log(k)}_{+228.6\text{dB}}\,-10\log(B) + \left.\frac{G}{T}\right|_{dB}$$

Antennas

Power flux density $S_0(r) = \frac{G_t P_t}{4\pi\, r^2}$ or $\vec{S}(r) = \vec{E} \times \vec{H} = \frac{|E|^2}{\eta_0}\frac{\vec{r}}{|r|}$

EIRP $EIRP = G_t P_t$

Antenna gain $G(\phi,\theta) = \frac{S(r,\phi,\theta)}{S_0(r)}$

in Decibel $G_{dB} = 10\log G$

Directivity $D = G/\eta$

$D = G$ for loss-less antennas

Krauss' Equation $D = \frac{4\pi}{\Theta_{BW}\Phi_{BW}}$

Effective Antenna area $A_e = \frac{P_e}{S_e}$

Effective Antennenarea using Gain $A_e = \frac{G_{max}\cdot\lambda^2}{4\pi}$

Antenna Efficiency $\eta_{Ant} = \frac{A_e}{A_{phy}}$

Available Power $P_r = S_r\,A_r = S_r G \frac{\lambda^2}{4\pi}$

Wireless Link

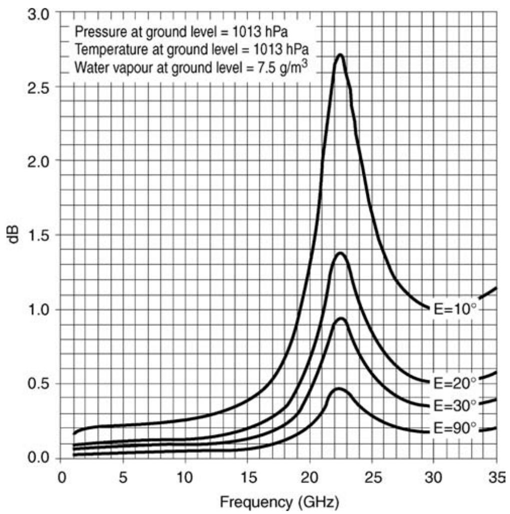
Transmit Power P_t generates power flux density $S_e = \frac{P_t G_t}{4\pi r^2}$

Friis' Formula: $P_r = P_t\,G_t\,G_r\,\frac{\lambda^2}{(4\pi r)^2}$

$$P_{r,dBm} = P_{t,dBm} + G_{t,dBi} + G_{r,dBi} - L_p$$

with Path Loss $L_p = 20\log\frac{4\pi r}{\lambda}$

Attenuation from Atmosphere



Rain Attenuation

Digital Transmission

Symbol	Quantity	Unit
m	Bits per Symbol	
M	States per Symbol $M = 2^m$	
R_b	Bit Rate in	bits/s
R_s	Symbol Rate	sym/s
T_b	Duration per Symbol	s
T_s	Duration per Bit	s
E_b	Energy per Bit	W/bit
n_0	Noise Power per 1 Hz Bandwidth	W/Hz
SNR	Signal to noise Ratio	1
B	Bandwidth of Channel	Hz
Γ	Spectral Efficiency	$\frac{\text{Bits}}{\text{s}\cdot\text{Hz}}$

Channel Capacity $$C = B\,\log_2(1+SNR)$$

Symbol Rate $$R_s = \frac{R_b}{m} = \frac{R_b}{\log_2 M}$$

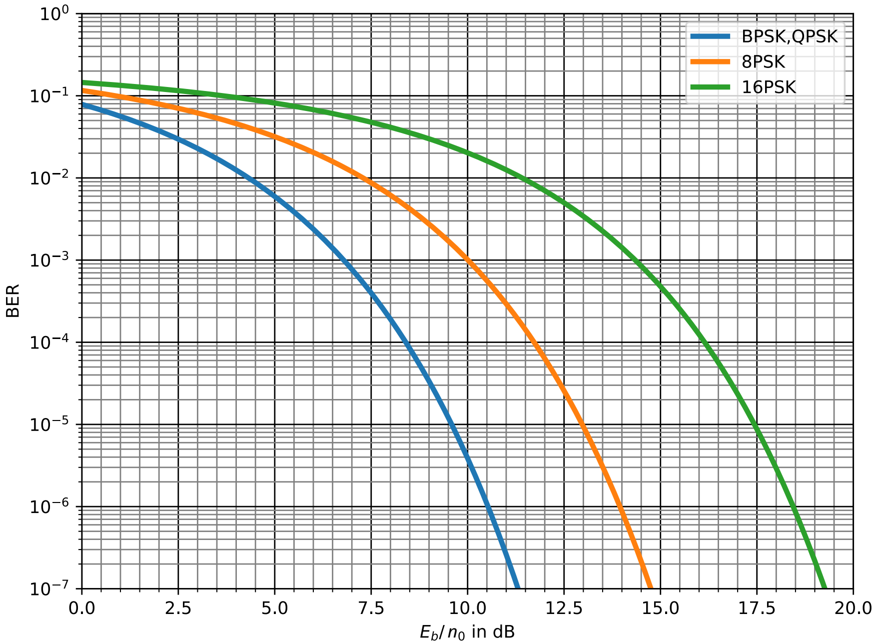
Bit Energy per Noise $$E_c/n_o = SNR\frac{B}{R_b}$$

Spectral Efficiency $$\Gamma = \frac{R_b}{B} = \frac{R_b T_S}{1+\alpha} = \frac{\log_2(M)}{1+\alpha}$$

BER for BPSK, QPSK $$BER = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{n_0}}\right)$$

BER for 8PSK $$BER = \frac{1}{3}\text{erfc}\left(\sqrt{3\frac{E_b}{n_0}}\sin\frac{\pi}{8}\right)$$

BER for D-QPSK $$BER = \frac{1}{2}\exp(-E_b/N_o)$$



Power Management

Solar Cell Types

Cell type	Efficiency, BOL 28 °C %	Efficiency, BOL 28 °C KW/m^2	Efficiency, EOL 1E15, 60 °C %	Efficiency, EOL 1E15, 60 °C KW/m^2	Cell weight (kg/m^2)
Si (200 µm)	12.6	0.170	8.7	0.118	0.464
Si (67 µm)	15.0	0.203	9.2	0.124	0.156
Si (100 µm) with diode	17.3	0.234	12.5	0.169	0.230
GaAs/Ge (137 µm)	19.6	0.265	14.7	0.199	0.720
DJ cascade (137 µm)	21.8	0.295	18.1	0.245	0.720
TJ standard (140 µm)	26.0	0.352	21.0	0.284	0.840
TJ improved (140 µm)	29.9	0.393	25.1	0.340	0.840
Thin film	12.6	0.170	9.5	0.128	0.100

BOL = beginning of life, EOL = end of life (for 1E15 1 MeV equivalent electron fluence), DJ = double junctions, TJ = triple junctions, solar flux = 135.3 mW/cm^2.

Batteries

Secondary Cell Types

	NiCd	NiH$_2$	Li-ion	System impact of Li-ion
Energy density (Wh/kg)	30	70	165	Weight saving
Energy efficiency (%)	72	70	96	Reduction of charge power
Thermal power (on a scale of 1–10)	8	10	3	Reduction of radiator, heat pipe sizes
Self discharge (%/day)	1	10	0.3	No trickle and simple management at launch pad
Temperature range (°C)	0 to 40	−20 to 30	0 to 40	Easy management
Memory effect	Yes	Yes	No	No reconditioning
Energy gauge/monitor	No	Pressure	Voltage	Better observation for states of charge
Charge management	Constant current	Constant current	Constant current then constant voltage	Weight saving
Modularity	No	No	Yes	Ability to put cells in parallel

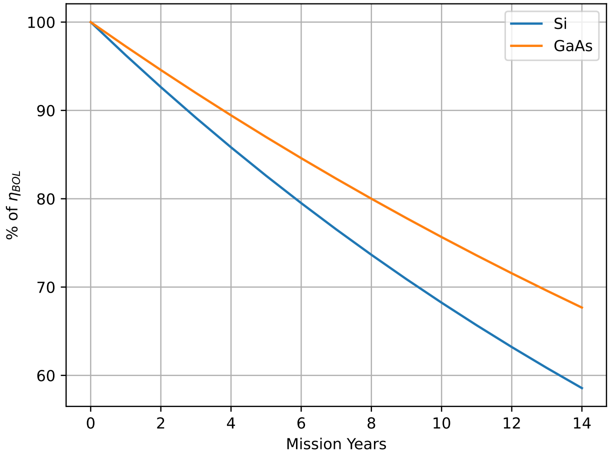
V_d	discharge voltage	normal Voltage while discharging
V_{low}	low end voltage	minimum allowed voltage while discharging
V_{high}	high end voltage	maximum allowed voltage while charging
I_d	discharge current	discharge current
E_d	discharge energy	total energy that can be recovered
C	Capacity	Capacity of cell, $E_d = C \cdot V_d$
E_{st}	stored energy	stored energy in cell
η_d	discharge efficiency	ratio of recovered energy to stored energy $= \frac{E_{st}}{E_d}$
DOD	depth of discharge	percentage of used energy in cell
SOC	state of charge	“fill level” of cell often in percent
I_c	charging current	charging current
V_c	charging voltage	charging voltage
E_c	charging energy	energy required for charging
η_d	charging efficiency	ratio of stored energy to charging energy $= \frac{E_c}{E_{st}}$

Stored energy $E_{st} = C\,V_d$

Discharge energy $E_d = C\,V_d\,DOD \cdot \eta_d$

Energy required for charging $E_c = E_{st}/\eta_c$

Cell Degradation $L_d = (1 - degradation/yr)^{T_d} \quad \eta_{EOL} = \eta_{BOL} L_d$



Solar power Flux Constant in LEO $S_{sol} = 1370\text{Wm}^{-2}$

Eclipse Times for Circular orbits $T_{ecl} = T_{orb}\frac{2\lambda}{2\pi} = T_{orb}\frac{1}{\pi}\arcsin\frac{R_e}{R_{orb}}$