# Chapter 3 Image Enhancement in the Spatial Domain

- ***The principal objective of enhancement***
  - to process an image so that the result is more suitable than the original image for a specific application.
- ***Enhancement methods***
  - Spatial Domain (in this chapter)
    - based on direct manipulation of pixels in an image
  - Frequency Domain (in chapter 4)
    - based on modifying the Fourier transform of an image
- ***The viewer is the ultimate judge of how well of a particular method works.***





# Chapter 3 Image Enhancement in the Spatial Domain

# 3.1 Background

- ***Spatial domain methods operate directly on image pixels***

$$g(x,y)=T\left[f(x,y)\right] \qquad (3.1\text{-}1)$$

- ***T* operates only for one pixel**

$$s=T(r) \qquad (3.1\text{-}2)$$

  - **Mapping**
  - **Contrast Stretching**
  - **Thresholding**
  - **Binary Image**



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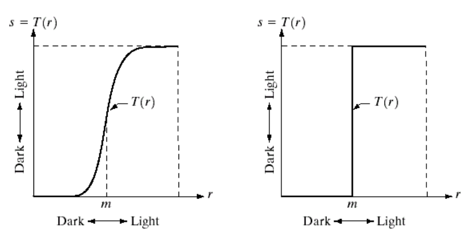
# 3.1 Background



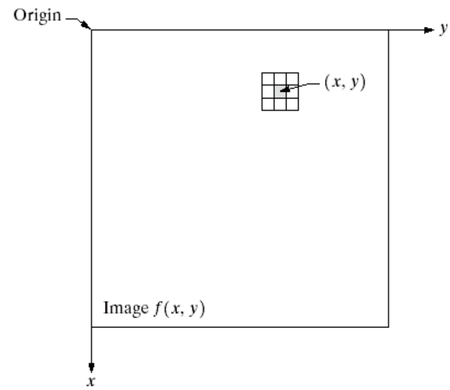

a b

**FIGURE 3.2** Gray-level transformation functions for contrast enhancement.

# 3.1 Background

- *T* operates with neighboring pixels
  - Mask (filter, kernel, template, window)

**FIGURE 3.1** A $3 \times 3$ neighborhood about a point $(x, y)$ in an image.








# 3.1 Background

$$Mask = \begin{bmatrix} m_{00} & m_{01} & m_{02} \\ m_{10} & m_{11} & m_{12} \\ m_{20} & m_{21} & m_{22} \end{bmatrix}$$

$$\begin{aligned} T[f(x,y)] = {} & f(x-1,y-1)\times m_{00} + f(x-1,y)\times m_{01} + f(x-1,y+1)\times m_{02} + \\ & f(x,y-1)\times m_{10} + f(x,y)\times m_{11} + f(x,y+1)\times m_{12} + \\ & f(x+1,y-1)\times m_{20} + f(x+1,y)\times m_{21} + f(x+1,y+1)\times m_{22} \end{aligned}$$

# 3.1 Background

$$Mask = \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

**Original Image**

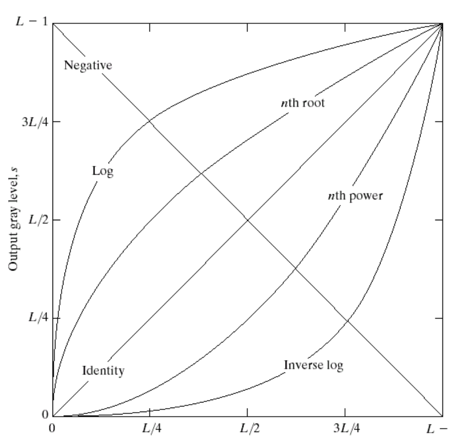
**Processed Image**





# 3.2 Some basic gray level transformations

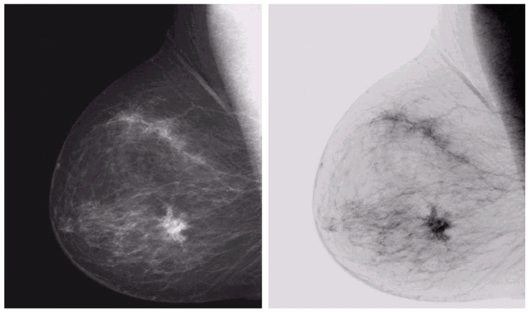
**FIGURE 3.3** Some basic gray-level transformation functions used for image enhancement.



# 3.2 Some basic gray level transformations

## Image Negative

$$s = (L-1) - r \qquad (3.2\text{-}1)$$



a b

**FIGURE 3.4** (a) Original digital mammogram. (b) Negative image obtained using the negative transformation in Eq. (3.2-1). (Courtesy of G.E. Medical Systems.)



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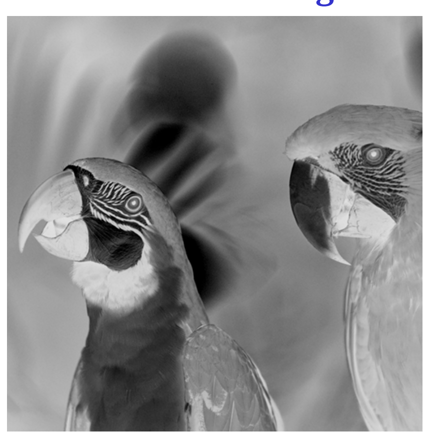
# 3.2 Some basic gray level transformations

**Original Image**

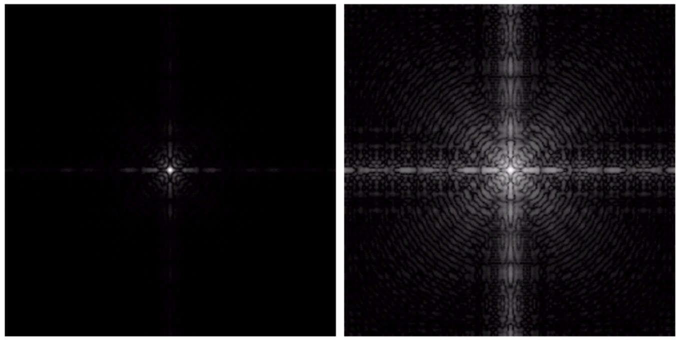
**Processed Image**



# 3.2 Some basic gray level transformations

## Log Transformation

$$s = c\log(1+r) \qquad (3.2\text{-}2)$$



a b

**FIGURE 3.5** (a) Fourier spectrum. (b) Result of applying the log transformation given in Eq. (3.2-2) with $c = 1$.



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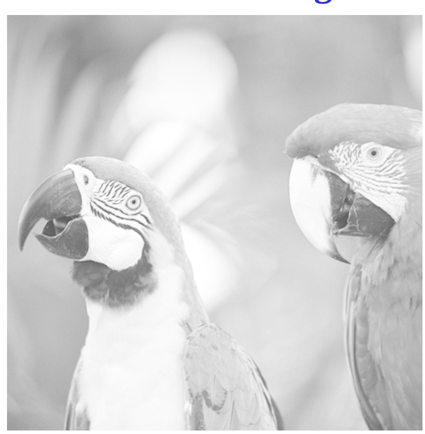
# 3.2 Some basic gray level transformations

## Log Transformation

**Original Image**

**Processed Image**



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# 3.2 Some basic gray level transformations

## Power-Law Transformation

$$s = cr^{\gamma} \qquad (3.2\text{-}3)$$

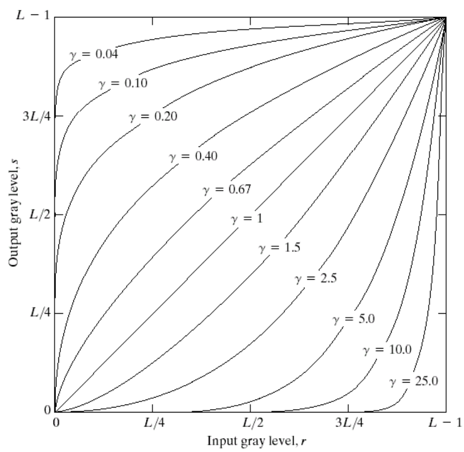


**FIGURE 3.6** Plots of the equation $s = cr^{\gamma}$ for various values of $\gamma$ ($c = 1$ in all cases).



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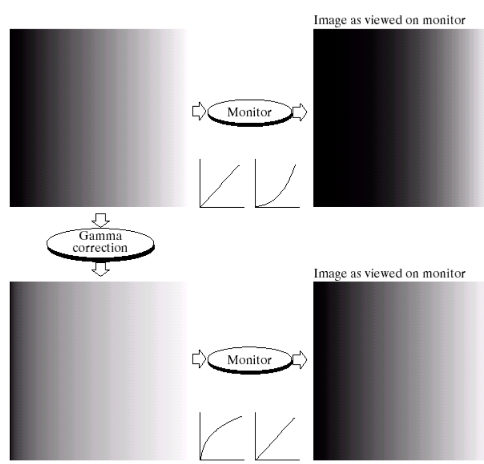
# 3.2 Some basic gray level transformations

## Power-Law Transformation

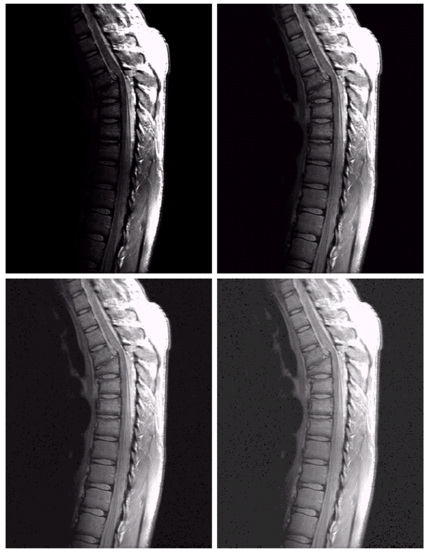
a b
c d

**FIGURE 3.7** (a) Linear-wedge gray-scale image. (b) Response of monitor to linear wedge. (c) Gamma-corrected wedge. (d) Output of monitor.



# 3.2 Some basic gray level transformations

## Power-Law Transformation



a b
c d

**FIGURE 3.8** (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and $0.3$, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)



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# 3.2 Some basic gray level transformations

## Power-Law Transformation

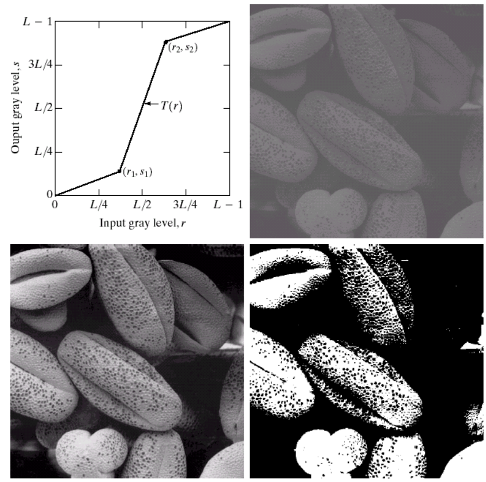
a b
c d

**FIGURE 3.9** (a) Aerial image. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 3.0, 4.0,$ and $5.0$, respectively. (Original image for this example courtesy of NASA.)

# 3.2 Some basic gray level transformations

## Piecewise-Linear Transformation Functions

### Contrast Stretching




a b
c d

**FIGURE 3.10** Contrast stretching. (a) Form of transformation function. (b) A low-contrast image. (c) Result of contrast stretching. (d) Result of thresholding. (Original image courtesy of Dr. Roger Heady, Research School of Biological Sciences, Australian National University, Canberra, Australia.)



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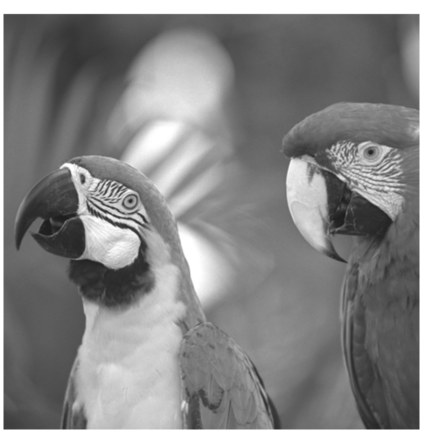
# 3.2 Some basic gray level transformations

## Piecewise-Linear Transformation Functions

### Original Image



### Processed Image

## 3.2 Some basic gray level transformations
## Piecewise-Linear Transformation Functions

### Gray-level slicing

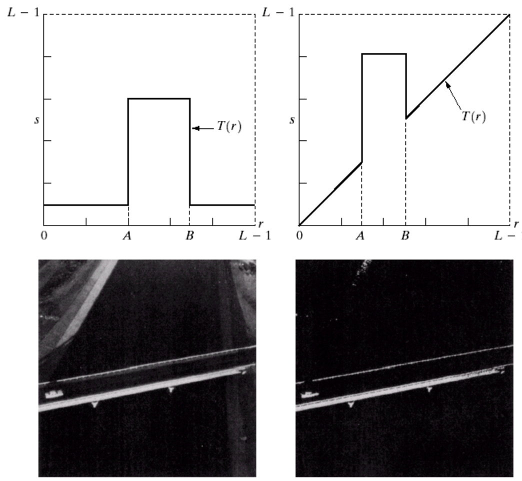



**FIGURE 3.11** (a) This transformation highlights range [$A$, $B$] of gray levels and reduces all others to a constant level. (b) This transformation highlights range [$A$, $B$] but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).





## 3.2 Some basic gray level transformations
## Piecewise-Linear Transformation Functions

### Original Image

### Processed Image

# 3.2 Some basic gray level transformations
## Piecewise-Linear Transformation Functions

### Bit-plane slicing

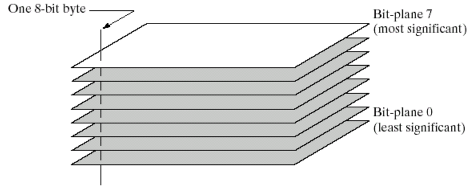



**FIGURE 3.12** Bit-plane representation of an 8-bit image.





# 3.2 Some basic gray level transformations
## Piecewise-Linear Transformation Functions

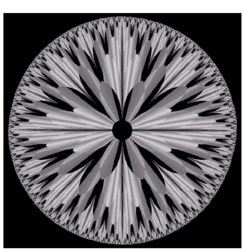


**FIGURE 3.13** An 8-bit fractal image. (A fractal is an image generated from mathematical expressions). (Courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA.)

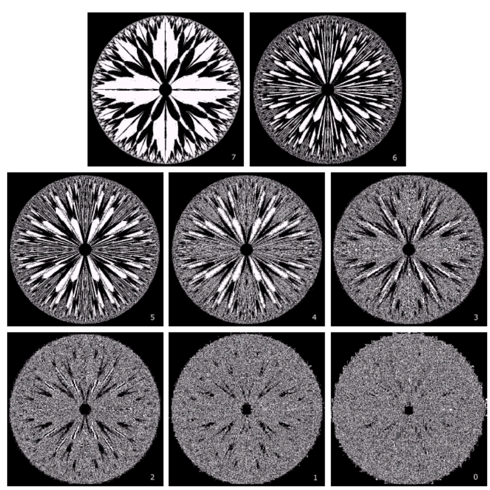
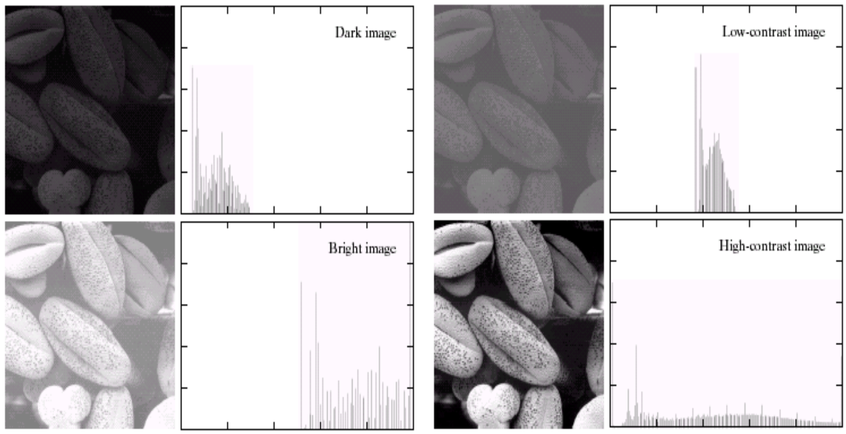



**FIGURE 3.14** The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

# 3.3 Histogram Processing

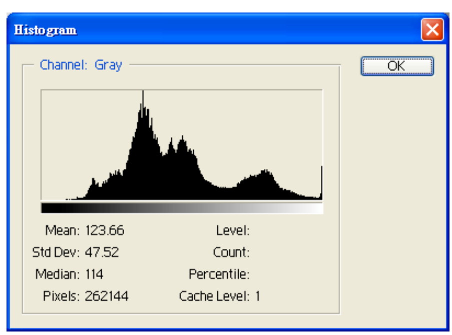
$$h(r_k) = n_k$$








# 3.3 Histogram Processing



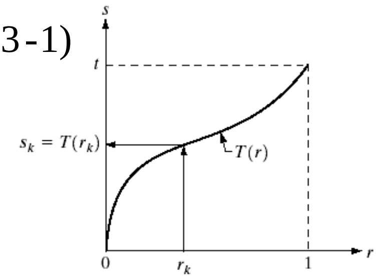
# 3.3 Histogram Processing

## Histogram Equalization

- **Transformation Function**

$$s = T(r) \qquad 0 \le r \le 1 \qquad (3.3\text{-}1)$$

Conditions:
(a) T(r) is single valued and monotonically increasing
(b) $0 \le T(r) \le 1$ for $0 \le r \le 1$




- **Inverse function**

$$r = T^{-1}(s) \qquad 0 \le s \le 1 \qquad (3.3\text{-}2)$$

*不一定符合條件(a)或(b)





# 3.3 Histogram Processing

## Histogram Equalization

- **PDFs(機率密度函式) of r and s**

$r$的機率密度函式：$p_r(r)$

$s$的機率密度函式：$p_s(s) = p_r(r)\left|\dfrac{dr}{ds}\right| \qquad (3.3\text{-}3)$

$s$的機率密度函式為所有經3.3-1公式轉換後會對映到$s$值之所有$r$值之機率密度函式的總和

# 3.3 Histogram Processing

## Histogram Equalization

If $s$ is the Cumulative Density Function (CDF) of $p_r$

$$s = T(r) = \boxed{\int_0^r p_r(w)\,dw} \qquad (3.3\text{-}4)$$

*符合條件(a)與(b)

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = \frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right] = p_r(r). \qquad (3.3\text{-}5)$$





# 3.3 Histogram Processing

## Histogram Equalization

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = \frac{d}{dr}\left[\int_0^r p_r(w)\,dw\right] = p_r(r).$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| = p_r(r)\left|\frac{1}{p_r(r)}\right| = 1 \qquad 0 \le s \le 1. \qquad (3.3\text{-}6)$$

$p_s(s)$ is a uniform PDF

Because $p_s(s)$ is 1 for any $p_r(r)$

# 3.3 Histogram Processing

## Histogram Equalization

- ***Discrete Version***

$$p_r(r_k) = \frac{n_k}{n} \qquad k = 0, 1, 2, \ldots, L-1 \tag{3.3-7}$$

$r_k$ = 第k階的灰階值, $n_k$ 是灰階值為 $r_k$ 之像素個數

$$\begin{aligned} s_k = T(r_k) &= \sum_{j=0}^{k} p_r(r_j) \\ &= \sum_{j=0}^{k} \frac{n_j}{n} \qquad k = 0, 1, 2, \ldots, L-1. \qquad 0 \le s_k \le 1 \end{aligned} \tag{3.3-8}$$

$$r_k = T^{-1}(s_k) \qquad k = 0, 1, 2, \ldots, L-1 \tag{3.3-9}$$





# 3.3 Histogram Processing

## Histogram Equalization

- ***Histogram Equalization***
  - *to produce an output image that has a uniform histogram*
  - *像素值均勻分佈可拉大影像明暗對比*
  - *procedures*
    - *求影像的Histogram*
    - *求影像的機率密度函數*
    - *產生CDF(累積密度函數)T()*
    - *使用T()處理每個像素*

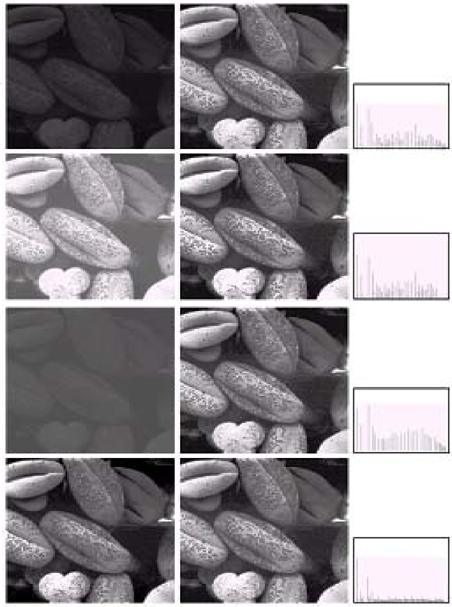
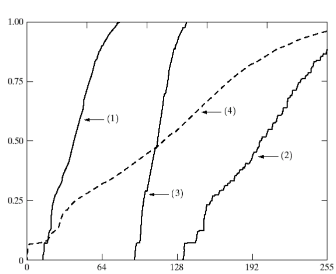
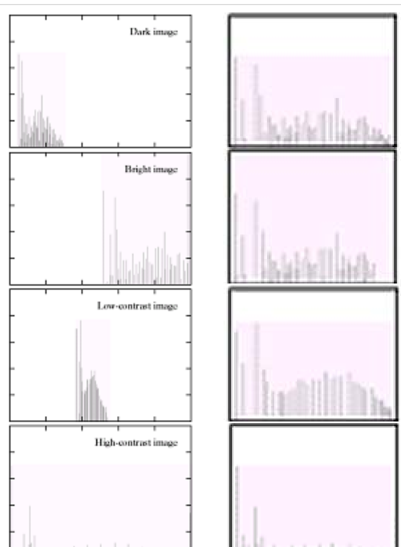


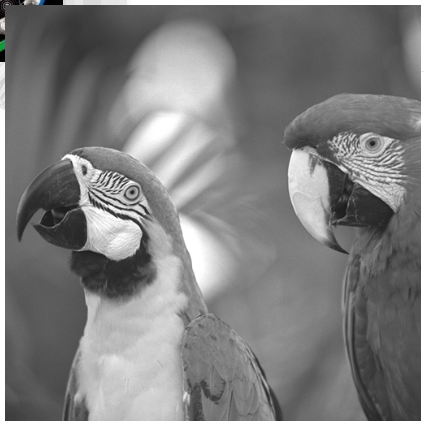


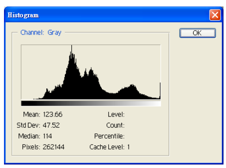


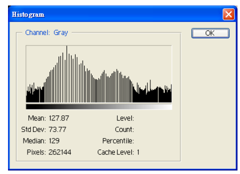
**FIGURE 3.18** Transformation functions (1) through (4) were obtained from the histograms of the images in Fig.3.17(a), using Eq. (3.3-8).












# 3.3 Histogram Processing

## Histogram Matching (Specification)

- **Development of the method**
  - r是來源影像灰階值
  - z是目的影像灰階值

**T( ): r -> s**
**G( ): z->s**

$$s = T(r) = \int_0^r p_r(w)\,dw \qquad (3.3\text{-}10)$$

$$G(z) = \int_0^z p_z(t)\,dt = s \qquad (3.3\text{-}11)$$

$$z = G^{-1}(s) = G^{-1}[T(r)]. \qquad (3.3\text{-}12)$$

# 3.3 Histogram Processing

## Histogram Matching (Specification)

- **Discrete Version**

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \qquad k = 0, 1, 2, \dots, L-1 \tag{3.3-13}$$

$$v_k = G(z_k) = \sum_{i=0}^{k} p_z(z_i) = s_k \qquad k = 0, 1, 2, \dots, L-1. \tag{3.3-14}$$

$$z_k = G^{-1}[T(r_k)] \qquad k = 0, 1, 2, \dots, L-1 \tag{3.3-15}$$

$$z_k = G^{-1}(s_k) \qquad k = 0, 1, 2, \dots, L-1. \tag{3.3-16}$$





# 3.3 Histogram Processing

## Histogram Matching (Specification)

a b
c

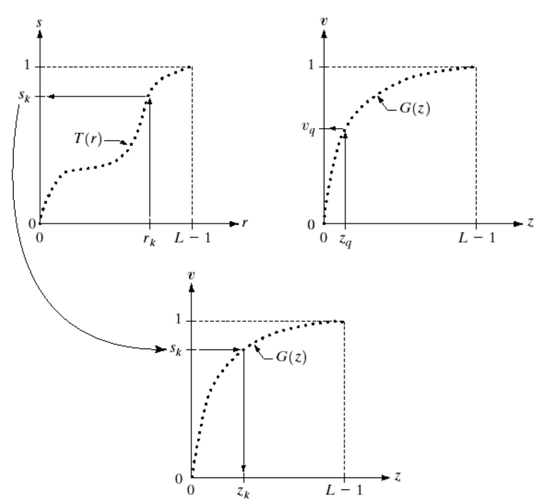



**FIGURE 3.19** (a) Graphical interpretation of mapping from $r_k$ to $s_k$ via $T(r)$. (b) Mapping of $z_q$ to its corresponding value $v_q$ via $G(z)$. (c) Inverse mapping from $s_k$ to its corresponding value of $z_k$.

# 3.3 Histogram Processing

## Histogram Matching (Specification)

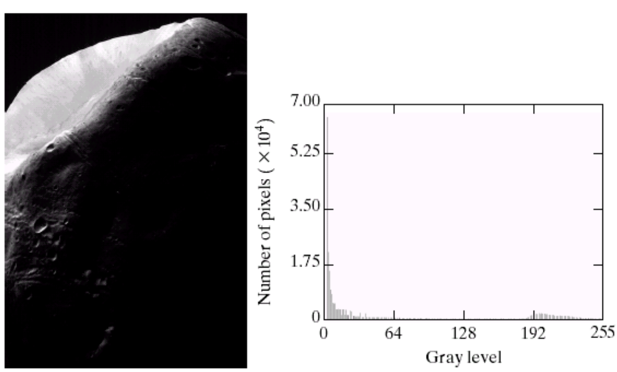
- **Implementation**

  1. 取得影像的Histogram
  2. 使用公式(3.3-13)計算每個 $s_k$
  3. 使用公式(3.3-14)取得pz(z)的轉換函式G
  4. 使用公式(3.3-17)計算每個 $z_k$
  5. 使用步驟2-4的結果處理每個像素
     - 取得 $r_k$ 的對映像素 $s_k$
     - 取得 $s_k$ 的對映像素 $z_k$



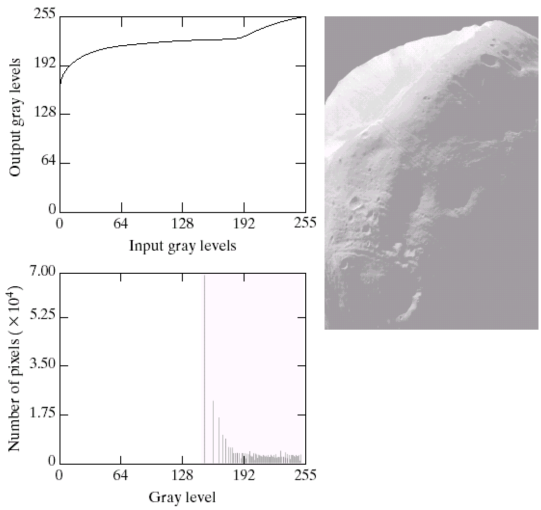


# Comparison between histogram equalization and histogram matching




a b

**FIGURE 3.20** (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)




a b
c

**FIGURE 3.21**
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).



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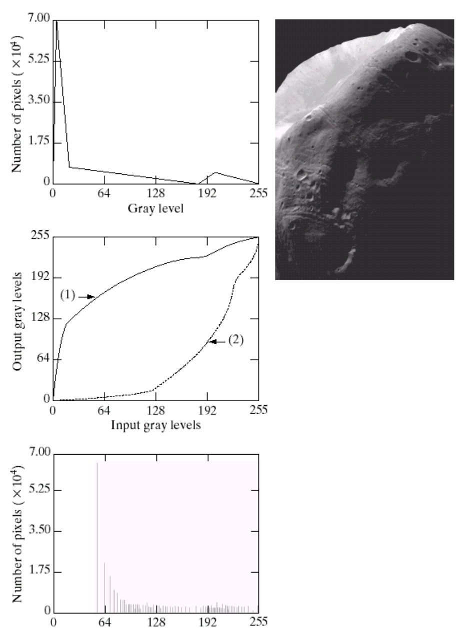
a c
b
d

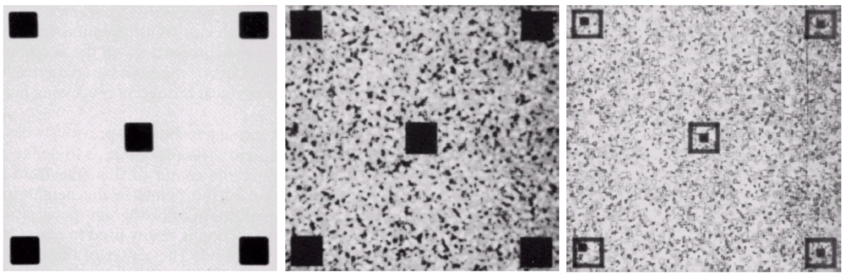
**FIGURE 3.22**
(a) Specified histogram.
(b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17).
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).



# 3.3 Histogram Processing

## Local Enhancement

步驟:1.定義方型或矩型的大小
2.針對每個像素
2.1.找出以目前像素為中心點之區塊內像素的histogram
2.2.使用histogram equalization/matching 方法取得像素對應值



a b c

**FIGURE 3.23** (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7 × 7 neighborhood about each pixel.



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# 3.3 Histogram Processing

## Use of histogram statistice for image enhancement

- 使用像素統計資訊

N-Norm

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i) \quad (3.3\text{-}18)$$

mean value

$$m = \sum_{i=0}^{L-1} r_i p(r_i). \quad (3.3\text{-}19)$$

2-Norm
Standard Deviation

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i). \quad (3.3\text{-}20)$$

子影像 $S_{x,y}$ 的區域統資訊

$$m_{S_{xy}} = \sum_{(s,t)\in S_{xy}} r_{s,t} p(r_{s,t}) \quad (3.3\text{-}21)$$

$$\sigma^2_{S_{xy}} = \sum_{(s,t)\in S_{xy}} \left[r_{s,t} - m_{S_{xy}}\right]^2 p(r_{s,t}). \quad (3.3\text{-}22)$$

# 3.3 Histogram Processing

## Use of histogram statistice for image enhancement

$$g(x,y) = \begin{cases} E \cdot f(x,y) & \text{if } m_{S_{xy}} \le k_0 M_G \text{ AND } k_1 D_G \le \sigma_{S_{xy}} \le k_2 D_G \\ f(x,y) & \text{otherwise} \end{cases}$$

**FIGURE 3.24** SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

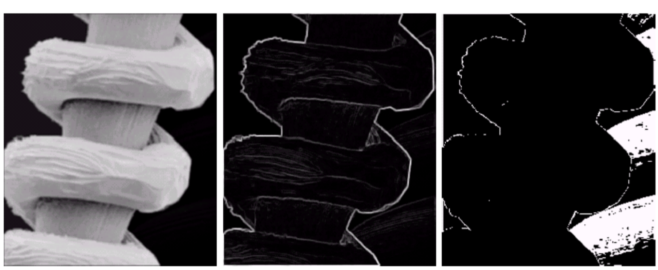




# 3.3 Histogram Processing

## Use of histogram statistice for image enhancement

*Block Size = 3x3*



a b c

**FIGURE 3.25** (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

# 3.3 Histogram Processing

## Use of histogram statistice for image enhancement

$$g(x,y) = \begin{cases} E \cdot f(x,y) & \text{if } m_{S_{xy}} \le k_0 M_G \text{ AND } k_1 D_G \le \sigma_{S_{xy}} \le k_2 D_G \\ f(x,y) & \text{otherwise} \end{cases}$$

**FIGURE 3.24** SEM image of a tungsten filament and support, magnified approximately 130×. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).

**FIGURE 3.26** Enhanced SEM image. Compare with Fig. 3.24. Note in particular the enhanced area on the right side of the image.

$E=4.0$, $k_0=0.4$, $k_1=0.02$, $k_2=0.4$

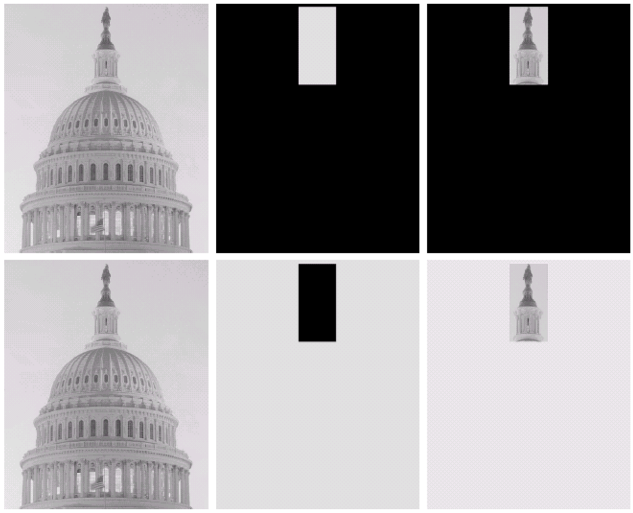




# 3.4 Enhancement using Arithmetic/Logic Operations

- *performed on a pixel by pixel basis between two or more images*
- *Logic Operations*
  - *AND*
  - *OR*
  - *NOT*
- *Arithmetic Operations*
  - *Substraction*
  - *Averaging*

# 3.4 Enhancement using Arithmetic/Logic Operations



a b c
d e f

**FIGURE 3.27** (a) Original image. (b) AND image mask. (c) Result of the AND operation on images (a) and (b). (d) Original image. (e) OR image mask. (f) Result of operation OR on images (d) and (e).



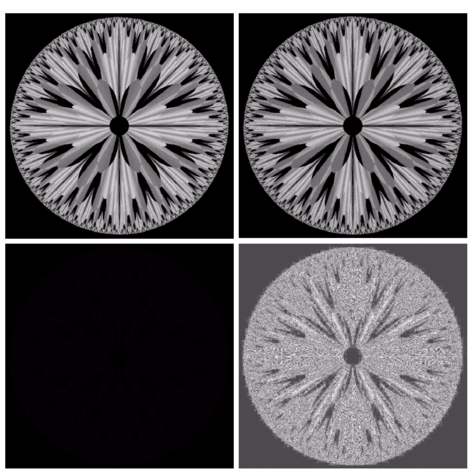


# 3.4 Enhancement using Arithmetic/Logic Operations

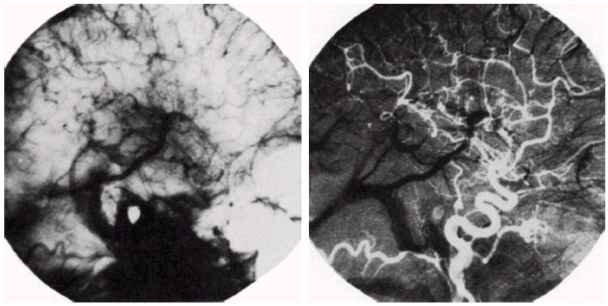
- *Image Substraction*

a b
c d

**FIGURE 3.28** (a) Original fractal image. (b) Result of setting the four lower-order bit planes to zero. (c) Difference between (a) and (b). (d) Histogram-equalized difference image. (Original image courtesy of Ms. Melissa D. Binde, Swarthmore College, Swarthmore, PA).



# 3.4 Enhancement using Arithmetic/Logic Operations



a b

**FIGURE 3.29** Enhancement by image subtraction. (a) Mask image. (b) An image (taken after injection of a contrast medium into the bloodstream) with mask subtracted out.



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# 3.4 Enhancement using Arithmetic/Logic Operations

取得影像 = 原始影像 + 雜訊

$$g(x, y) = f(x, y) + \eta(x, y) \tag{3.4-2}$$

平均結果

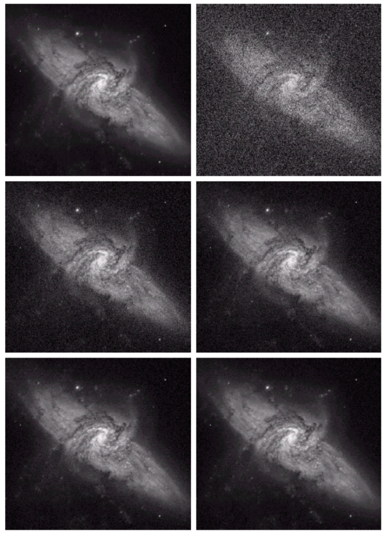
$$\bar{g}(x, y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x, y) \tag{3.4-3}$$

$$E\{\bar{g}(x, y)\} = f(x, y) \tag{3.4-4}$$

$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)} \tag{3.4-5}$$

$$\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}. \tag{3.4-6}$$

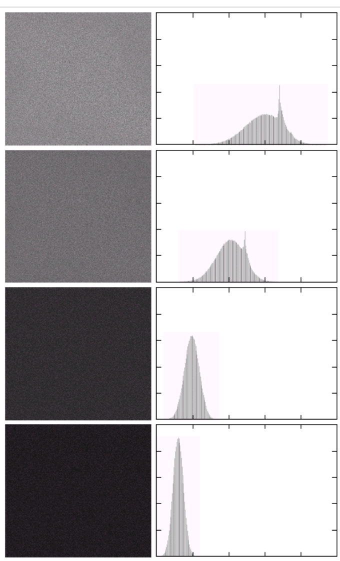
# 3.4 Enhancement using Arithmetic/Logic Operations - *Image averaging*







# 3.4 Enhancement using Arithmetic/Logic Operations - *Image averaging*

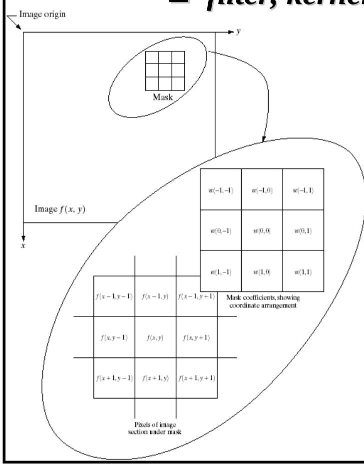


a b

**FIGURE 3.31** (a) From top to bottom: Difference images between Fig. 3.30(a) and the four images in Figs. 3.30(c) through (f), respectively. (b) Corresponding histograms.

# 3.5 Basics of spatial filtering

- **Mask**
  - *filter, kernel, templete, window*




$$R = w(-1,-1)f(x-1, y-1) + w(-1,0)f(x-1, y) + \cdots + w(0,0)f(x, y) + \cdots + w(1,0)f(x+1, y) + w(1,1)f(x+1, y+1),$$

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x+s, y+t) \qquad (3.5\text{-}1)$$



# 3.5 Basics of spatial filtering

- **Mask**
  - *filter, kernel, templete, window*
  - *converlution mask/kernel*

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn} = \sum_{i=1}^{mn} w_i z_i \qquad (3.5\text{-}2)$$

$$R = w_1 z_1 + w_2 z_2 + \ldots w_9 z_9 = \sum_{i=1}^{9} w_i z_i. \qquad (3.5\text{-}3)$$

**FIGURE 3.33** Another representation of a general 3 × 3 spatial filter mask.

| | | |
|---|---|---|
| $w_1$ | $w_2$ | $w_3$ |
| $w_4$ | $w_5$ | $w_6$ |
| $w_7$ | $w_8$ | $w_9$ |

# 3.6 Smoothing Spatial Filter

- ***Smoothing filters are used for***
  - ***blurring***
  - ***noise reduction***



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# 3.6 Smoothing Spatial Filter
## Smoothing Linear Filters

- ***Smoothing linear filters***
  - also called
    - averaging filter
    - lowpass filter

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)} \qquad (3.6\text{-}1)$$

$\frac{1}{9} \times$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$\frac{1}{16} \times$

| | | |
|---|---|---|
| 1 | 2 | 1 |
| 2 | 4 | 2 |
| 1 | 2 | 1 |

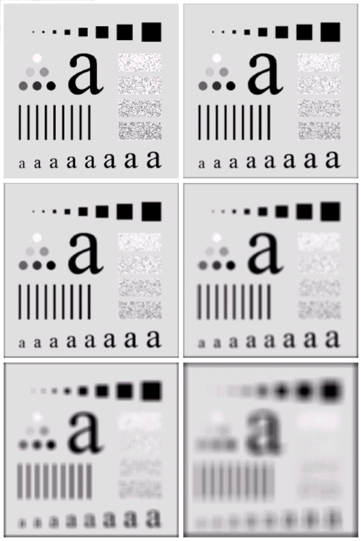
a b

**FIGURE 3.34** Two 3 × 3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

$$R = \frac{1}{9}\sum_{i=1}^{9} z_i,$$

# 3.6 Smoothing Spatial Filter

## Smoothing Linear Filters



Mask大小與影像平滑效果的關係

a b
c d
e f

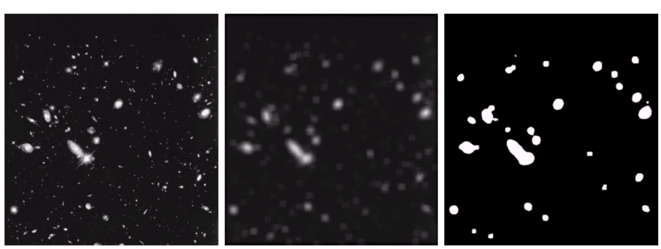
**FIGURE 3.35** (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.





# 3.6 Smoothing Spatial Filter

## Smoothing Linear Filters



a b c

**FIGURE 3.36** (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

# 3.6 Smoothing Spatial Filter
## Order-Statistics Filters

- *Non-linear filters*
  - *Median filter*
    - *example*
    - *unsorted (60,20,15,23,35,47,78,53,65)*
    - *sorted (15,20,23,35,47,53,60,65,78)*
    - *the median is 47*
  - *Max filter*
  - *Min filter*



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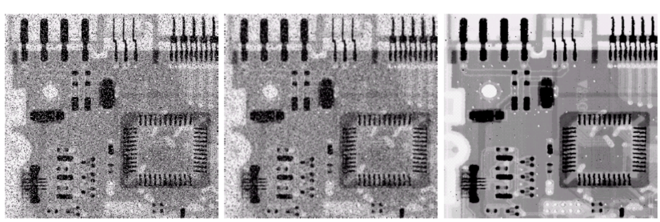
3.6 Smoothing Spatial Filter
Order-Statistics Filters



a b c

**FIGURE 3.37** (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

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# 3.7 Sharpening Spatial Filters

- *Sharpening Spatial filters are used to*
  - *highlight fine detail*
  - *enhance detail that has been blurred*



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# 3.7 Sharpening Spatial Filters
## Foundation

- *First-order derivative*

$$\frac{\partial f}{\partial x} = f(x+1) - f(x).$$

- *Second-order derivative*

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x).$$

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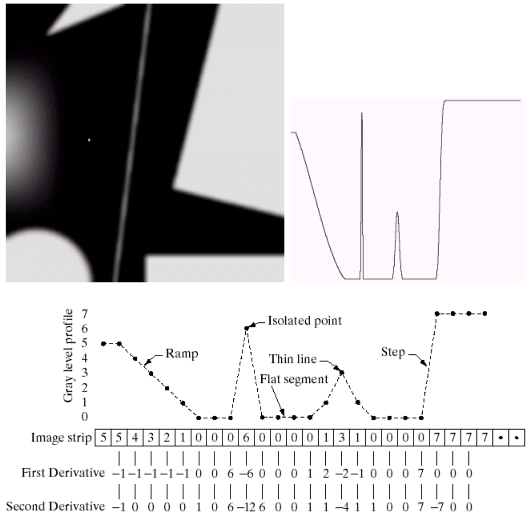
# 3.7 Sharpening Spatial Filters

## Foundation

a b
c

**FIGURE 3.38** (a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).








# 3.7 Sharpening Spatial Filters

## Use of second Derivatives for enhancement

- **The Laplacian**
  - ***Development of the method***

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}. \qquad (3.7\text{-}1)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y) \qquad (3.7\text{-}2)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y) \qquad (3.7\text{-}3)$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y). \qquad (3.7\text{-}4)$$

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases} \qquad (3.7\text{-}5)$$

# 3.7 Sharpening Spatial Filters

## Use of second Derivatives for enhancement

- **Filters**

(a)

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | −4 | 1 |
| 0 | 1 | 0 |

(b)

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | −8 | 1 |
| 1 | 1 | 1 |

(c)

| | | |
|---|---|---|
| 0 | −1 | 0 |
| −1 | 4 | −1 |
| 0 | −1 | 0 |

(d)

| | | |
|---|---|---|
| −1 | −1 | −1 |
| −1 | 8 | −1 |
| −1 | −1 | −1 |

a b
c d

**FIGURE 3.39** (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.



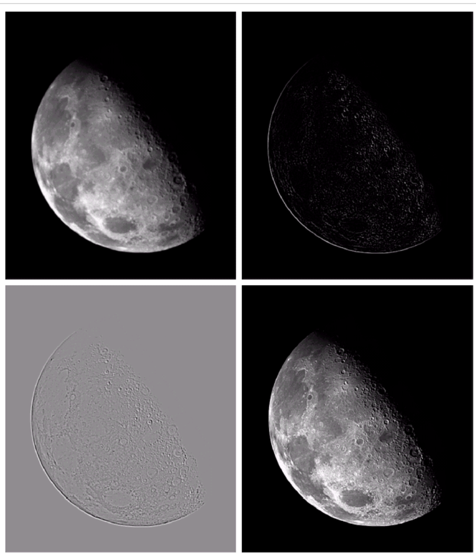


# 3.7 Sharpening Spatial Filters

## Use of second Derivatives for enhancement

a b
c d

**FIGURE 3.40** (a) Image of the North Pole of the moon. (b) Laplacian-filtered image. (c) Laplacian image scaled for display purposes. (d) Image enhanced by using Eq. (3.7-5). (Original image courtesy of NASA.)



# 3.7 Sharpening Spatial Filters
## Use of second Derivatives for enhancement

- ***Simplification***

$$
\begin{aligned}
g(x,y) &= f(x,y) - [f(x+1,y) + f(x-1,y) \\
&\quad + f(x,y+1) + f(x,y-1)] + 4f(x,y) \\
&= 5f(x,y) - [f(x+1,y) + f(x-1,y) \\
&\quad + f(x,y+1) + f(x,y-1)].
\end{aligned} \tag{3.7-6}
$$

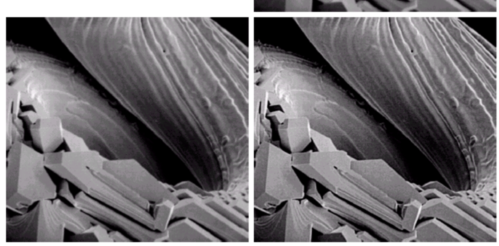




# 3.7 Sharpening Spatial Filters
## Use of second Derivatives for enhancement

| | | |
|---|---|---|
| 0 | −1 | 0 |
| −1 | 5 | −1 |
| 0 | −1 | 0 |

| | | |
|---|---|---|
| −1 | −1 | −1 |
| −1 | 9 | −1 |
| −1 | −1 | −1 |



a b c
d e

**FIGURE 3.41** (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

# 3.7 Sharpening Spatial Filters

## Use of second Derivatives for enhancement

- *Unsharp masking and high-boost filtering*

$$f_s(x, y) = f(x, y) - \bar{f}(x, y) \tag{3.7-7}$$

$$f_{hb}(x, y) = Af(x, y) - \bar{f}(x, y) \tag{3.7-8}$$

where $A \geq 1$ and, as before, $\bar{f}$ is a blurred version of $f$. This equation may be written as

$$f_{hb}(x, y) = (A - 1)f(x, y) + f(x, y) - \bar{f}(x, y). \tag{3.7-9}$$

By using Eq. (3.7-7), we obtain

$$f_{hb}(x, y) = (A - 1)f(x, y) + f_s(x, y) \tag{3.7-10}$$

as the expression for computing a high-boost-filtered image.



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# 3.7 Sharpening Spatial Filters

## Use of second Derivatives for enhancement

$$f_{hb} = \begin{cases} Af(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ Af(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases} \tag{3.7-11}$$

a

| | | |
|---|---|---|
| 0 | −1 | 0 |
| −1 | $A + 4$ | −1 |
| 0 | −1 | 0 |

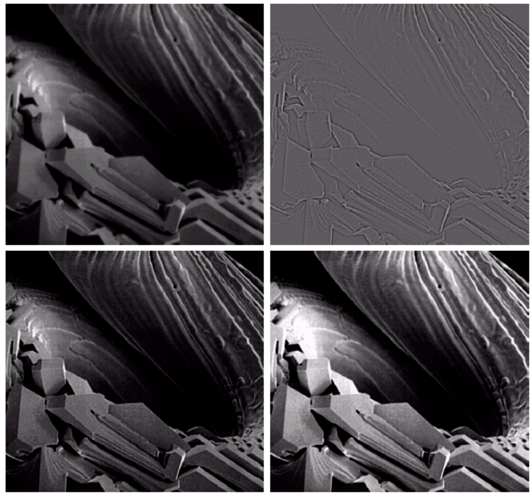
b

| | | |
|---|---|---|
| −1 | −1 | −1 |
| −1 | $A + 8$ | −1 |
| −1 | −1 | −1 |

**FIGURE 3.42** The high-boost filtering technique can be implemented with either one of these masks, with $A \geq 1$.

# 3.7 Sharpening Spatial Filters

## Use of second Derivatives for enhancement



a b
c d

**FIGURE 3.43** (a) Same as Fig. 3.41(c), but darker. (a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$. (c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.



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# 3.7 Sharpening Spatial Filters

## Use of First Derivatives for enhancement

- *The Gradient (坡度)*

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}. \quad (3.7\text{-}12)$$

$$\begin{aligned} \nabla f &= \operatorname{mag}(\nabla \mathbf{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}. \end{aligned} \quad (3.7\text{-}13)$$

$$\nabla f \approx |G_x| + |G_y|. \quad (3.7\text{-}14)$$

$$G_x = (z_9 - z_5) \quad \text{and} \quad G_y = (z_8 - z_6). \quad (3.7\text{-}15)$$

$$\nabla f = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2\right]^{1/2} \quad (3.7\text{-}16)$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|. \quad (3.7\text{-}17)$$

| | | |
|---|---|---|
| $z_1$ | $z_2$ | $z_3$ |
| $z_4$ | $z_5$ | $z_6$ |
| $z_7$ | $z_8$ | $z_9$ |

# 3.7 Sharpening Spatial Filters

## Use of First Derivatives for enhancement

$$\nabla f \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|. \quad (3.7\text{-}18)$$

for 3 x 3 masks

a

| | | |
|---|---|---|
| $z_1$ | $z_2$ | $z_3$ |
| $z_4$ | $z_5$ | $z_6$ |
| $z_7$ | $z_8$ | $z_9$ |

b

| | |
|---|---|
| −1 | 0 |
| 0 | 1 |

c

| | |
|---|---|
| 0 | −1 |
| 1 | 0 |

(b) and (c) are refered as **Roberts cross-gradient Operators**

d

| | | |
|---|---|---|
| −1 | −2 | −1 |
| 0 | 0 | 0 |
| 1 | 2 | 1 |

e

| | | |
|---|---|---|
| −1 | 0 | 1 |
| −2 | 0 | 2 |
| −1 | 0 | 1 |

(d) and (e) are called **Sobel Operators**

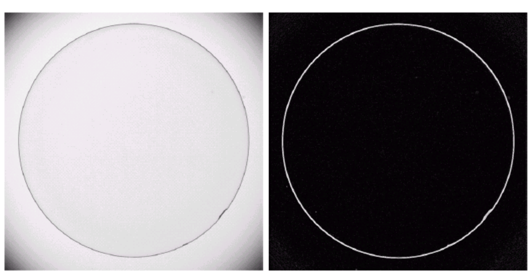
**FIGURE 3.44** A 3 × 3 region of an image (the $z$'s are gray-level values) and masks used to compute the gradient at point labeled $z_5$. All masks coefficients sum to zero, as expected of a derivative operator.





# 3.7 Sharpening Spatial Filters

## Use of First Derivatives for enhancement



a b

**FIGURE 3.45** Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

# 3.7 Sharpening Spatial Filters
## Use of First Derivatives for enhancement

水平強度

| | | |
|---|---|---|
| −1 | −2 | −1 |
| 0 | 0 | 0 |
| 1 | 2 | 1 |



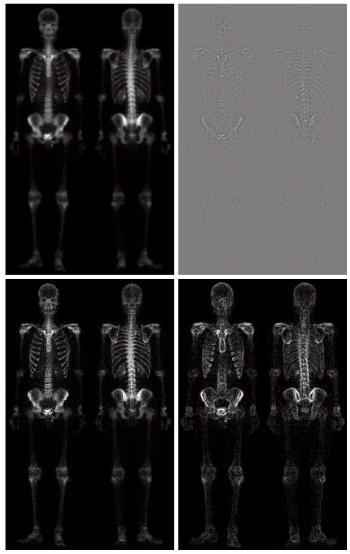


# 3.7 Sharpening Spatial Filters
## Use of First Derivatives for enhancement

垂直強度

| | | |
|---|---|---|
| −1 | 0 | 1 |
| −2 | 0 | 2 |
| −1 | 0 | 1 |

# 3.8 Combining Spatial Enhancement Methods



a b
c d

**FIGURE 3.46**
(a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).



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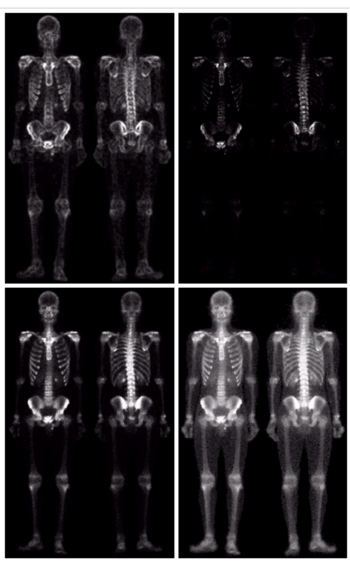
# 3.8 Combining Spatial Enhancement Methods



e f
g h

**FIGURE 3.46**
*(Continued)*
(e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

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