VATSAL SHREEKANT ; 500771363

February 1, 2018 8:54 AM

Q. $T(n) = T(n/2) + T(n/4) + T(n/8) + n$

$O(n) = ?$ $(d \geq n)$

$\sum_{i=1}^{n} x^k = \sum_{i=1}^{n} (7/8)^i n$

guess = n

$T(n) = d\left[\frac{n}{2} + \frac{n}{4} + \frac{n}{8}\right] + cn$

$T(n) \leq dn\left[\frac{7}{8}\right] + cn$

$dn\left[\frac{7}{8}\right] + \frac{dn}{8} - \frac{dn}{8} + cn$

$\boxed{dn} - \frac{dn}{8} + cn$

$-\frac{dn}{8} + cn \leq 0$

$d \geq 8c$

Q. $T(n) = T(n-1) + O(n)$

n

$n-1$

$n-2$

$\vdots$

arithmetic series :-

$\sum_{i=1}^{n} \cancel{\text{guess}} = n^2 \frac{n(n-1)}{2}$

$T(n) = (n-1)^2 + cn$

$\leq d(n^2 - 2n + 1) + cn$

$n \to \infty$

$\boxed{dn^2} - 2dn + d + cn$

$-2dn + cn \leq 0$

$d \geq c/2$

Q. $T(n) = 2T(n/2) + n^3$

n^3

$(n/2)^3 \quad (n/2)^3 \quad = n^3/4$

$(n/4)^3 \quad (n/4)^3 \quad (n/4)^3 \quad (n/4)^3 \quad = n^3/16$

geometric series $= \sum_{i=1}^{\log n} n^3\left[\frac{1}{4}\right]^i$

$\therefore$ guess $= n^3$

$T(n) = 2\left[\frac{dn^3}{8}\right] + cn^3$

$\leq \frac{dn^3}{4} + cn^3$

$\frac{dn^3}{4} + \frac{3}{4}dn^3 - \frac{3}{4}dn^3 + cn^3$

$\boxed{dn^3} - \left(\frac{3}{4}\right)dn^3 + cn^3$

$-\frac{3}{4}dn^3 + cn^3 \leq 0$

$d \geq \frac{4}{3}c$

Q. $T(n) = T(n/2) + T(n/4) + n^2$

n^2

$(n/2)^2 \quad (n/4)^2 \quad 5n^2/16$

guess $= n^2$

$(n/4)^2 \; (n) \; (n/8)^2 \quad \left(\frac{n}{2}\right)^2 + \left(\frac{n}{4}\right)^2 \quad (n/8)^2 + n^2 \quad (n/16)^2 \quad 25n^2/256$

geometric series $= n^2 \sum_{i=1}^{\log n} \left(\frac{5}{16}\right)^i$

$T(n) = \frac{dn^2}{4} + \frac{dn^2}{16} + cn^2$

$= \frac{5dn^2}{16} + \frac{11dn^2}{16} - \frac{11dn^2}{16} + cn^2$

$\boxed{dn^2} - \frac{11dn^2}{16} + cn^2 \leq 0$

$d \geq \frac{16}{11}c$

Q. $T(n) = T\left(\frac{9n}{10}\right) + n$

$n \to n(9/10)^0$

$9n/10 \to n(9/10)^1$

$81n/100 \to n(9/10)^2$

$\vdots$

geometric series $= n \sum_{i=1}^{\log n} (9/10)^i$

Q. $T(n) = 7T(n/3) + n^2$

$n^2 \qquad n^2$

$(n/3)^2 \quad (n/3)^2 \quad (n/3)^2 \quad (n/3)^2 \quad (n/3)^2 \quad (n/3)^2 \quad (n/3)^2 \qquad 7n^2/9$

$49n^2/81$

geometric series $= n^2 \sum_{i=1}^{\log n} (7/9)^i$

guess $= n^2$

$81n/100 \rightarrow n(9/10)^{\dots}$

geometric series $= n \sum_{i=1}^{\log n} (9/10)^i$

guess $= n$

$$T(n) = \frac{9dn}{10} + cn$$

$$\le \frac{9dn}{10} + \frac{dn}{10} - \frac{dn}{10} + cn$$

$$\le \boxed{dn} - \frac{dn}{10} + cn$$

$$-\frac{dn}{10} + cn \le 0$$

$$d \ge 10c$$

$49n/81$

geometric series $= n^2 \sum_{i=1}^{\log n} (7/9)^i$

guess $= n^2$

$$T(n) = 7\left((n/3)^2\right) + cn^2$$

$$= 7(n^2/9) + cn^2$$

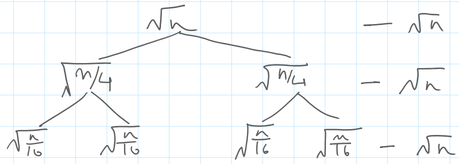
$$7dn^2/9 + cn^2$$

$$7dn^2/9 + 2dn^2/9 - 2dn^2/9 + cn$$

$$\le \boxed{dn^2} - 2dn^2/9 + cn$$

$$d \ge \frac{9}{2} c$$

Q) $T(n) = 2T(n/4) + \sqrt{n}$



guess $= \sqrt{n} \log n$

$$T(n) = 2\left[\sqrt{\frac{n}{4}} \log \frac{n}{4}\right] + c\sqrt{n}$$

$$\le d\sqrt{n} \log n/4 + c\sqrt{n}$$

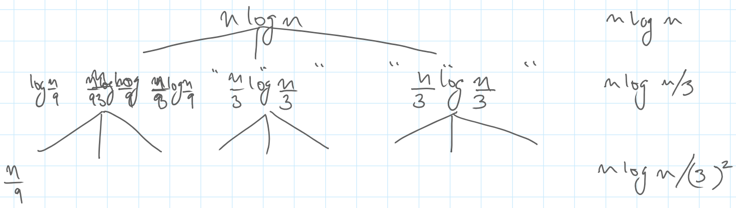
$$\le \boxed{d\sqrt{n} \log n} - d\sqrt{n} \log 4 + c\sqrt{n}$$

$$-d\sqrt{n} \log 4 + c\sqrt{n} \le 0$$

$$-d\sqrt{n} \log_4 4 + c\sqrt{n} \le 0$$

$$d \ge c$$

Q. $T(n) = 3T(n/3) + n \log n$



$$\sum_{i=1}^{\log n} n \log \frac{n}{(3)^i}$$

$$n \sum_{i=1}^{\log n} \log \frac{n}{(3)^i} = n \sum_{i=1}^{\log} \log n - \log(3)^i$$

$$n \sum_{i=1}^{\log} \log n - i \log 3 = n \sum_{i=1}^{\log} \log n - n \log 3 \sum_{i=1}^{\log} i$$

$$(n \log n)(\log n) - (n \log 3)(\log n) = (n \log n)[\log n - \log 3]$$

$$(n \log(n))(\log(n)) = n(\log n)^2 \longleftarrow \text{Guess}$$

$$T(n) = 3\left[d\left(\frac{n}{3}\right)\left(\log \frac{n}{3}\right)^2\right] + cn \log n$$

$$= (dn)(\log(n/3))(\log(n/3)) + cn \log n$$

$$(dn)(\log n - \log 3)(\log n - \log 3) + cn \log n$$

Assume log with base = 3

$$(dn)(\log n - 1)(\log n - 1) + cn \log n$$

$$dn(\log n/1)^2 + cn \log n$$

$$\boxed{dn(\log n)^2} + dn - 2dn \log n + cn \log n$$

$$dn - 2dn \log n + cn \log n$$

$$dn - n \log n(-2d + c)$$