

Q1:

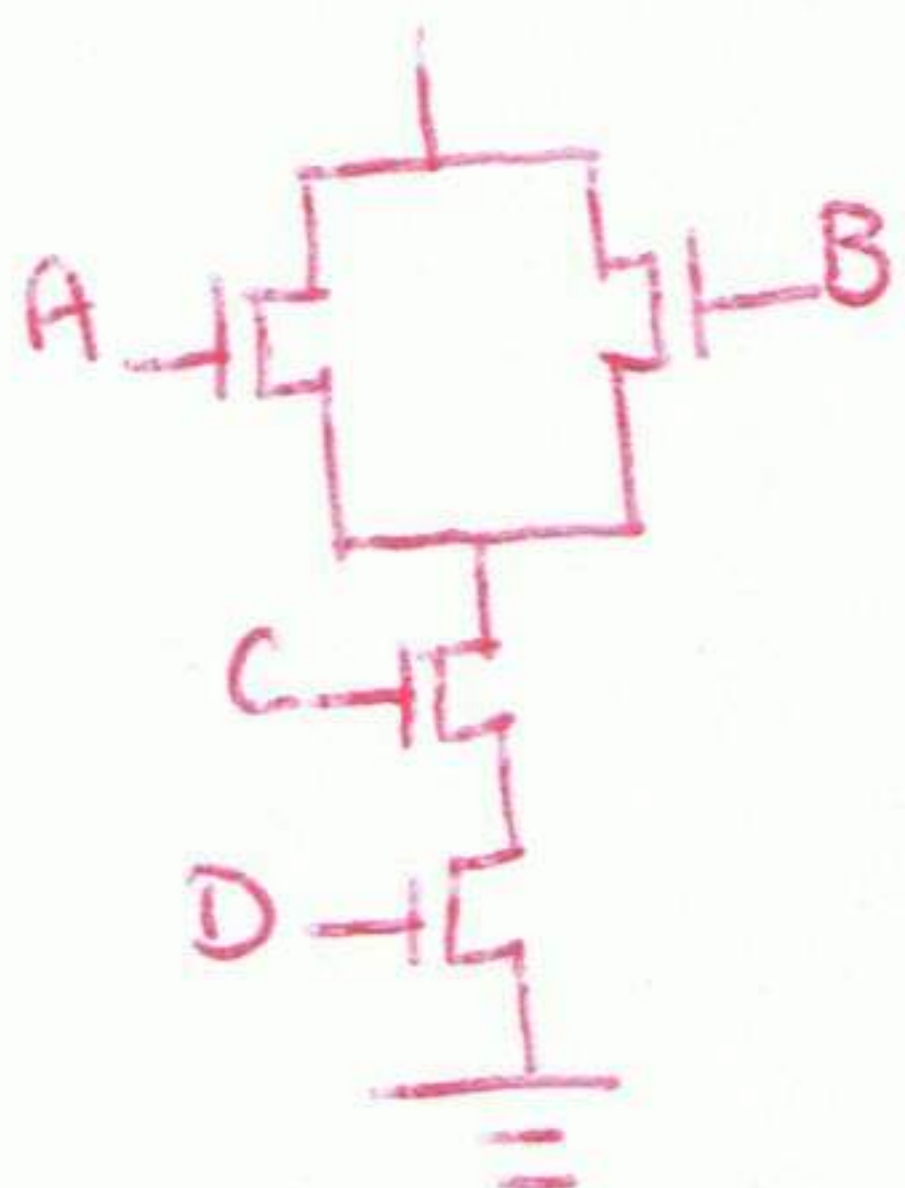
Design a static CMOS gate that has the following output.

$$Out = \overline{((A+B) * C * D)}$$

- Draw the transistor-level schematic of your design. Sizing is not required. (8 marks)
- Draw stick diagram of the gate that you have designed (*please minimize layout area by maximizing diffusion sharing* and clearly label metal, poly, n-diffusion and p-diffusion regions on your stick diagram). (8 marks)
- Estimate the area of the gate based on your stick diagram. (4 marks)

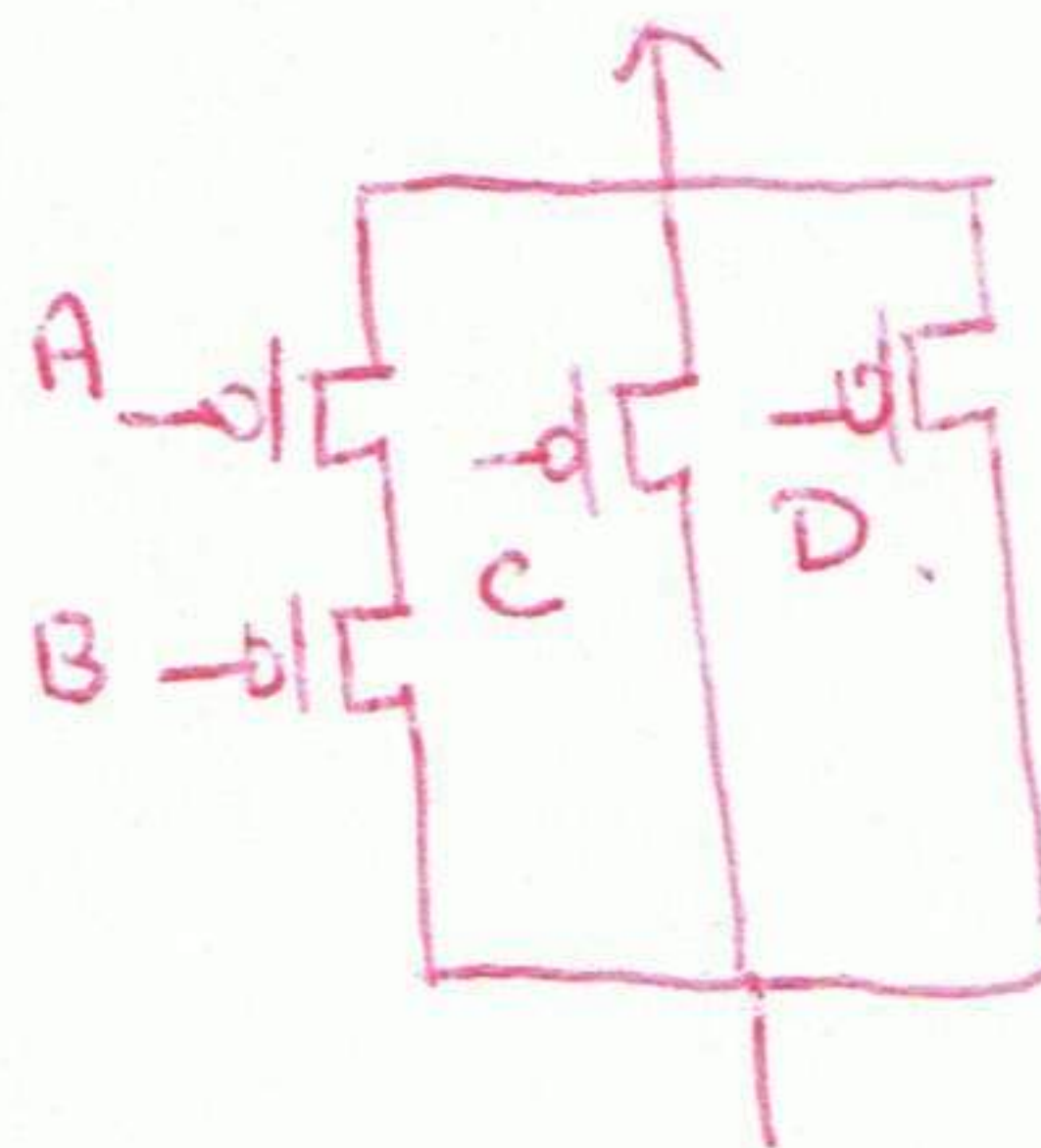
$$\overline{(A+B) * C * D} = \overline{(A+B)} + \overline{C} + \overline{D} = (\overline{A} * \overline{B}) + \overline{C} + \overline{D}$$

Pull down



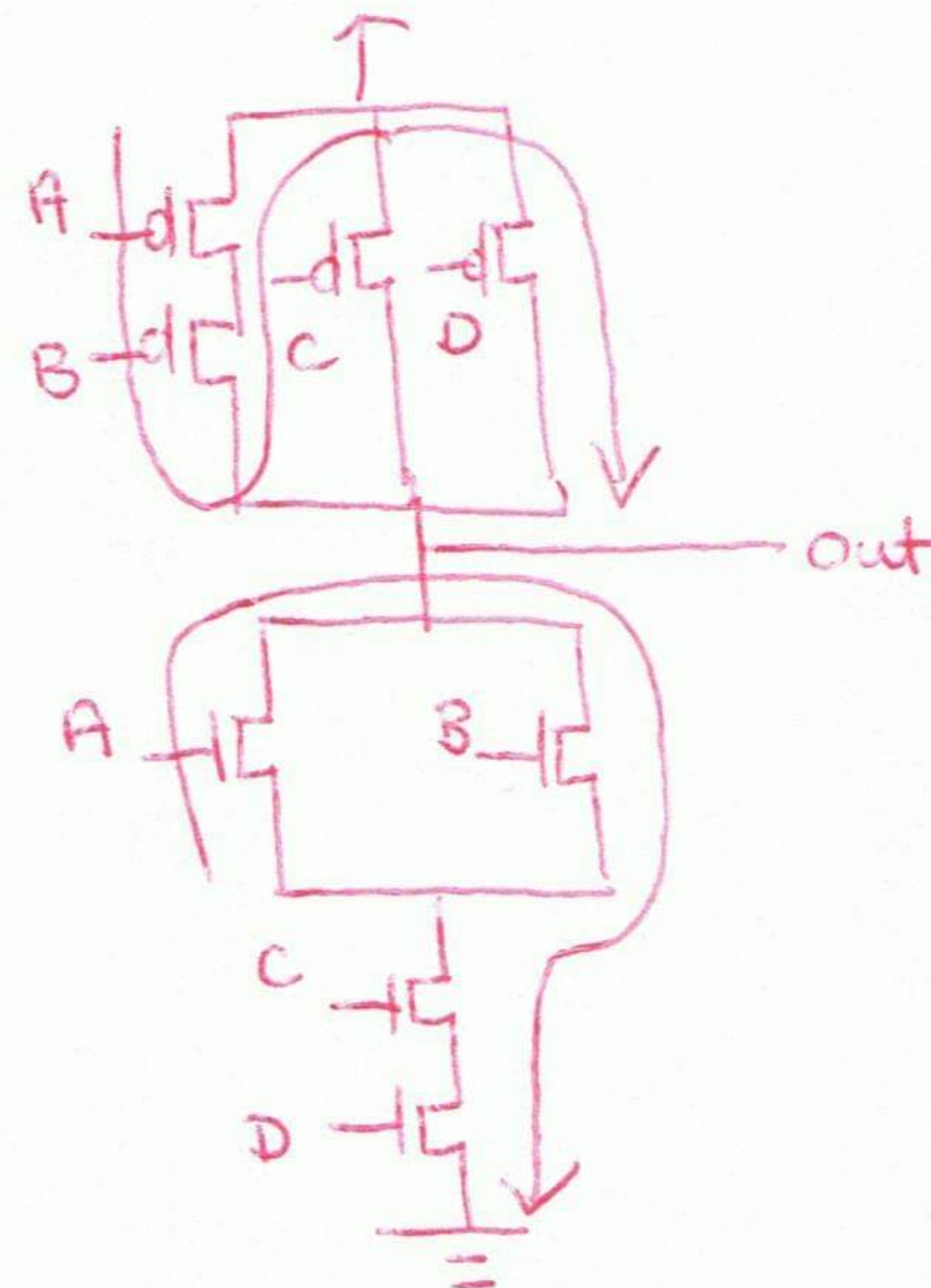
4 marks

Pull up



4 marks

Total Circuit

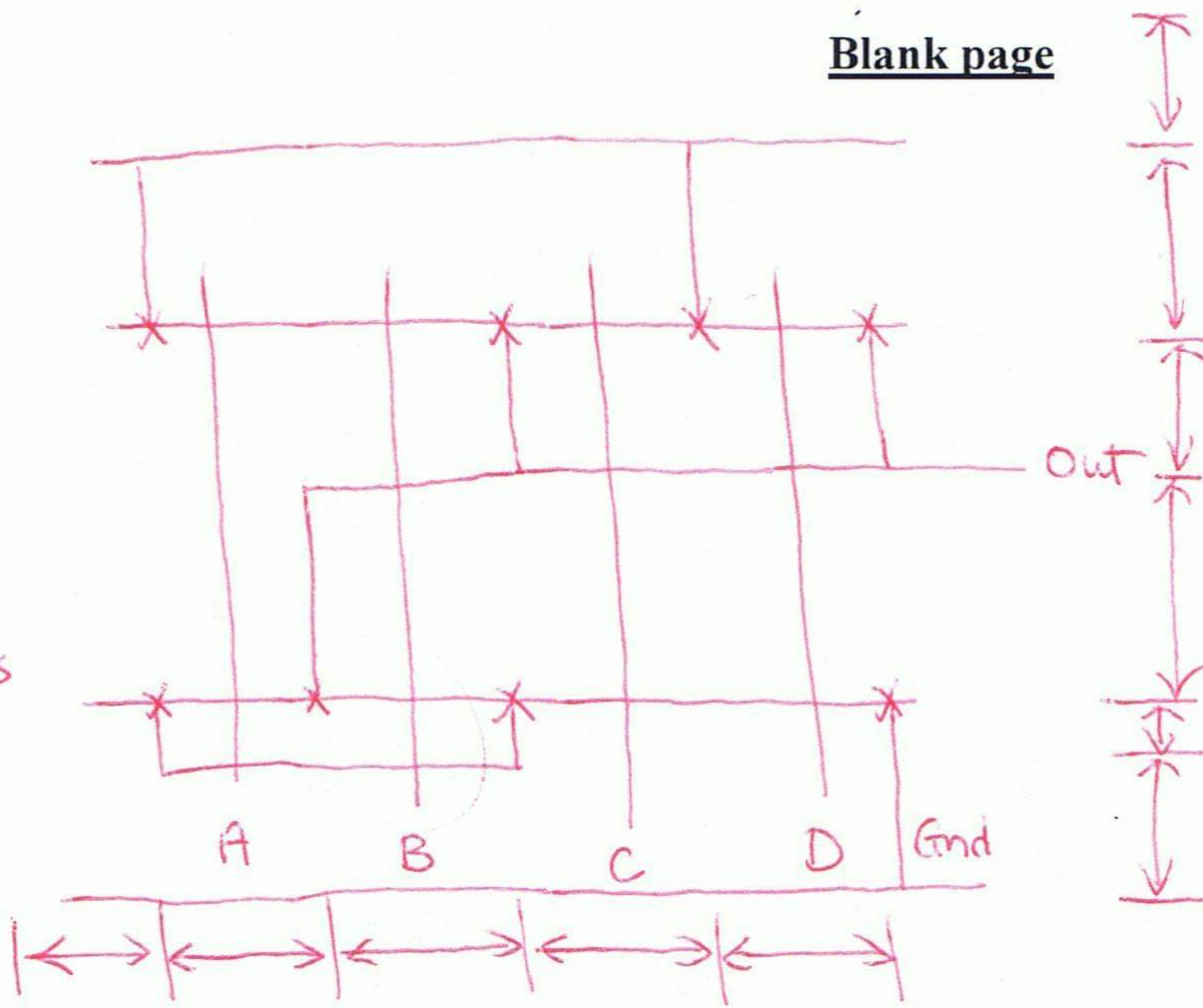




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4 marks  
pmos

4 marks  
nmos



6 tracks x 8λ each  
= 48λ

2 marks

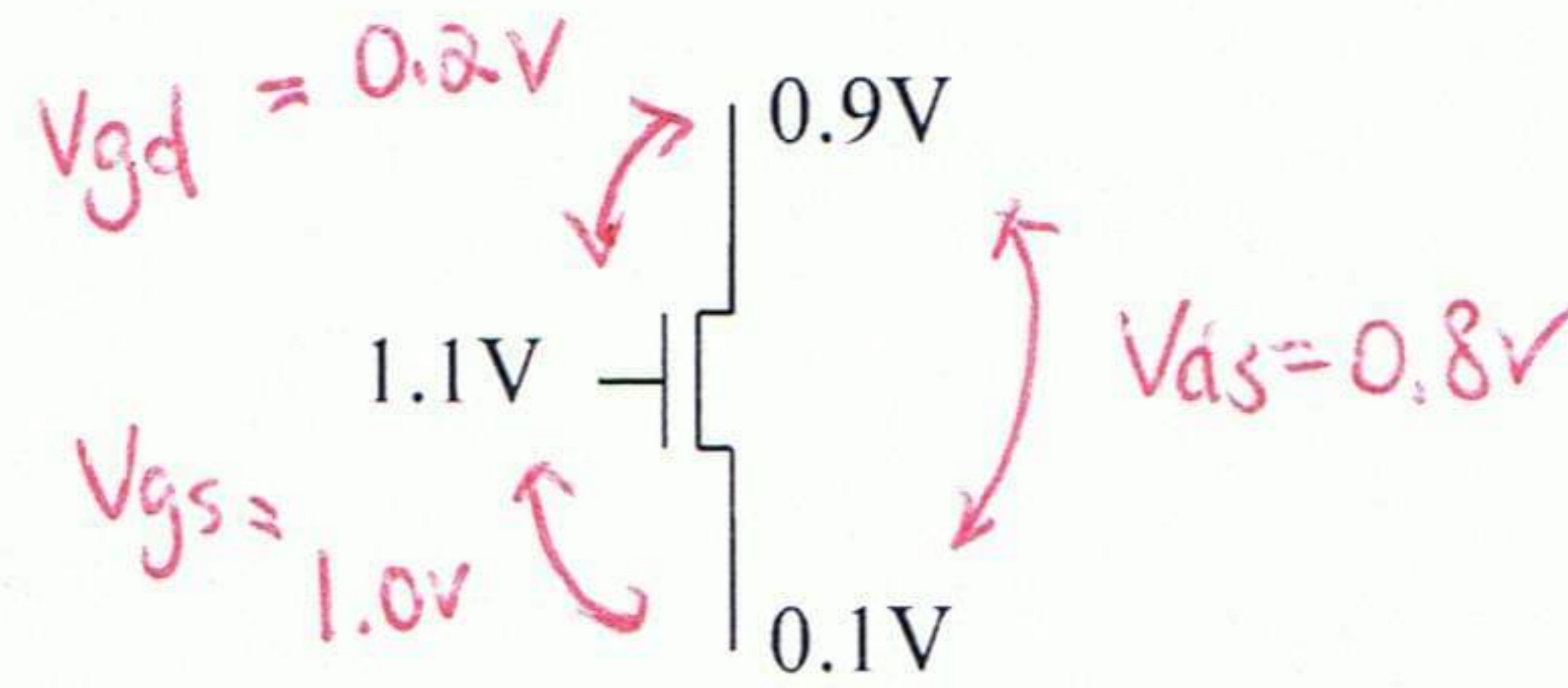
5 tracks x 8λ each  
= 40λ

2 marks

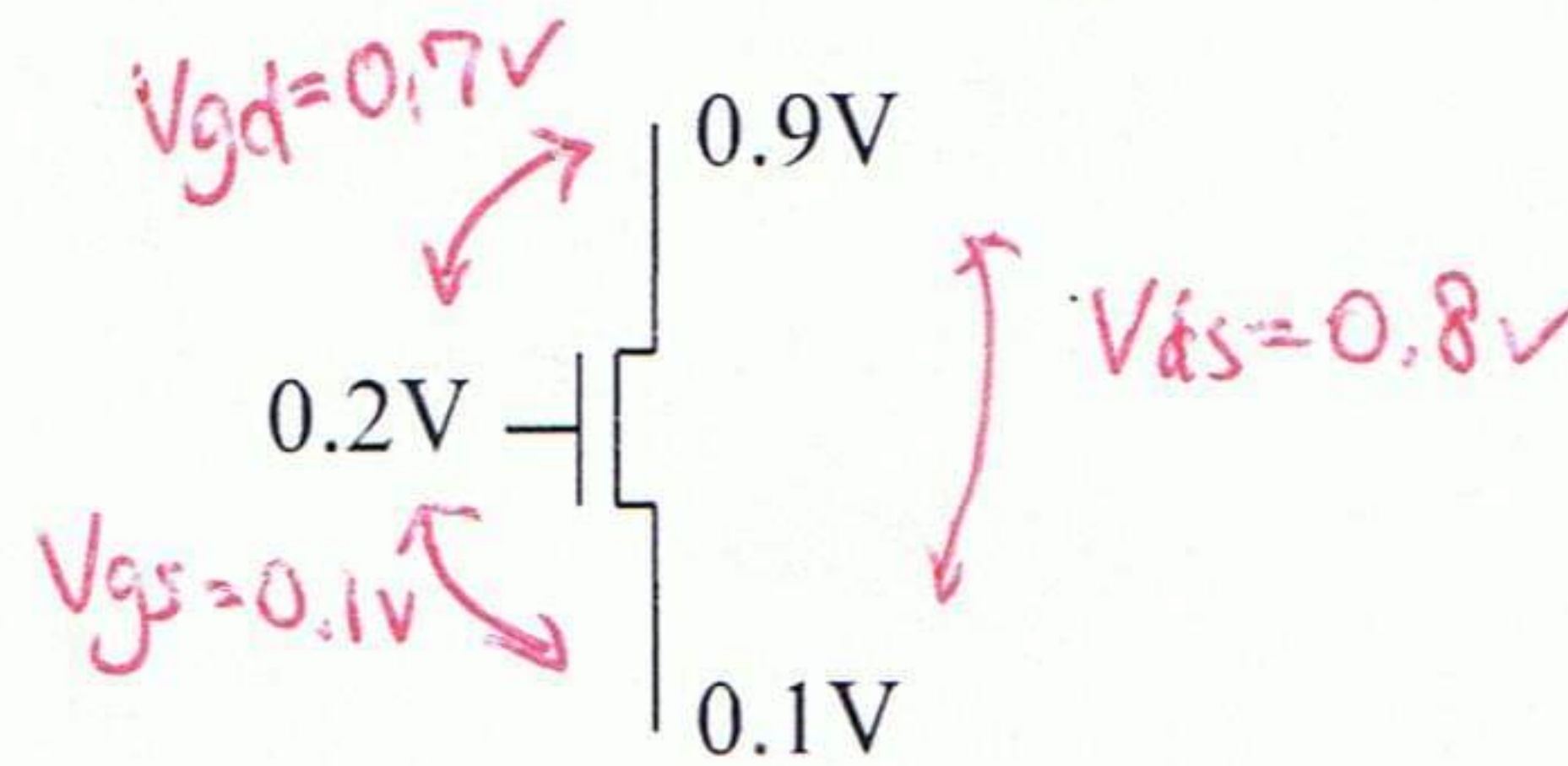


Q2: Assume  $V_t = 0.3V$ ,  $V_{sb} = -0.1V$ ,  $V_{dd} = 1.2V$ .

- a) Calculate  $I_{ds}$  for the following transistor using the Long-Channel model. (5 marks)



- b) Calculate subthreshold current for the following transistor. (5 marks)



- c) What should be the value of  $V_{sb}$  if one needs to reduce the subthreshold current to 20% of the value calculated in part (b) by changing the body voltage? (5 marks)  
 d) What will be the impact on the performance of the transistor shown in part (a) if  $V_{sb}$  is changed to the value calculated from part (c)? (5 marks)

$$a) I_{ds} = 1048 \mu A / V^2 \cdot (1.0 - 0.3)^2 V^2 \cdot \frac{1}{2} = \frac{513.53 \mu A}{2} = 256.76 \mu A.$$

5 marks

$$b) I_{ds} = 0.1 \mu A \cdot 10^{\frac{0.1 + 100 \text{ mV/V} (0.8 - 1.2) - 0.083(-0.1)}{100 \text{ mV/V}}} \left(1 - e^{-\frac{0.8}{26 \text{ mV}}}\right) \\ = 0.1 \mu A \cdot 10^{\frac{0.1 - 0.04 + 0.0083}{0.1}} \left(1 - e^{-\frac{0.8}{0.026}}\right) \\ = 0.1 \mu A \cdot 10^{0.683} (1 - e^{-30.77}) \\ = 0.1 \mu A \cdot 4.819 (1 - 4.33 \cdot 10^{-14}) \\ = 0.482 \mu A$$

5 marks

$$c) I_{ds} = I_{off} \cdot 10^{\frac{V_{gs} + \eta(V_{ds} - V_{dd})}{S}} \cdot 10^{\frac{-K_A V_{sb}}{S}} \left(1 - e^{-\frac{V_{ds}}{V_T}}\right)$$

$$10^{\frac{-K_A(-0.1)}{S}} \cdot 0.2 = 10^{\frac{-K_A(V_{sb})}{S}} \\ 10^{\frac{0.083 \cdot 0.1}{0.1}} \cdot 0.2 = 10^{\frac{-0.083(V_{sb})}{0.1}}$$

5 marks

$$0.242 = 10^{\frac{-0.083(V_{sb})}{0.1}}$$

$$-0.616 = \frac{-0.083(V_{sb})}{0.1}$$

$$V_{sb} = 0.74 \text{ volts}$$



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~~0.3 = V\_{t0} + 0.16(\sqrt{0.93} - \sqrt{0.1})~~

$$d) \quad 0.3 = V_{t0} + 0.16(\sqrt{0.93} - 0.1 - \sqrt{0.93})$$

$$0.3 = V_{t0} + 0.16(0.911 - 0.964)$$

$$0.3 = V_{t0} + (-0.00848)$$

$$V_{t0} = 0.30848$$

$$V_t = 0.30848 + 0.16(\sqrt{0.93 + 0.74} - \sqrt{0.93})$$

$$= 0.30848 + 0.16(\sqrt{1.67} - \sqrt{0.93})$$

$$= 0.30848 + 0.16 \cdot 0.328$$

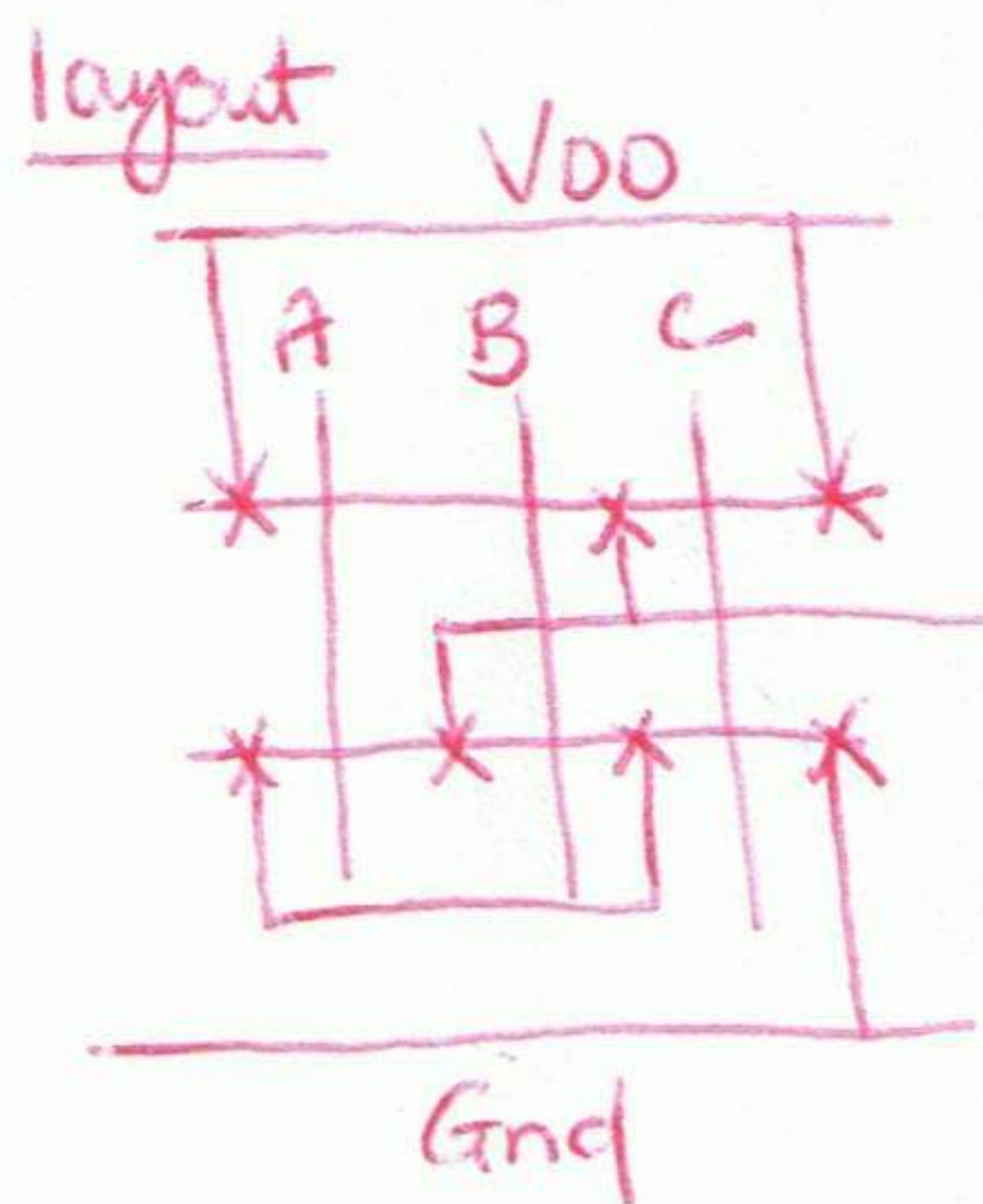
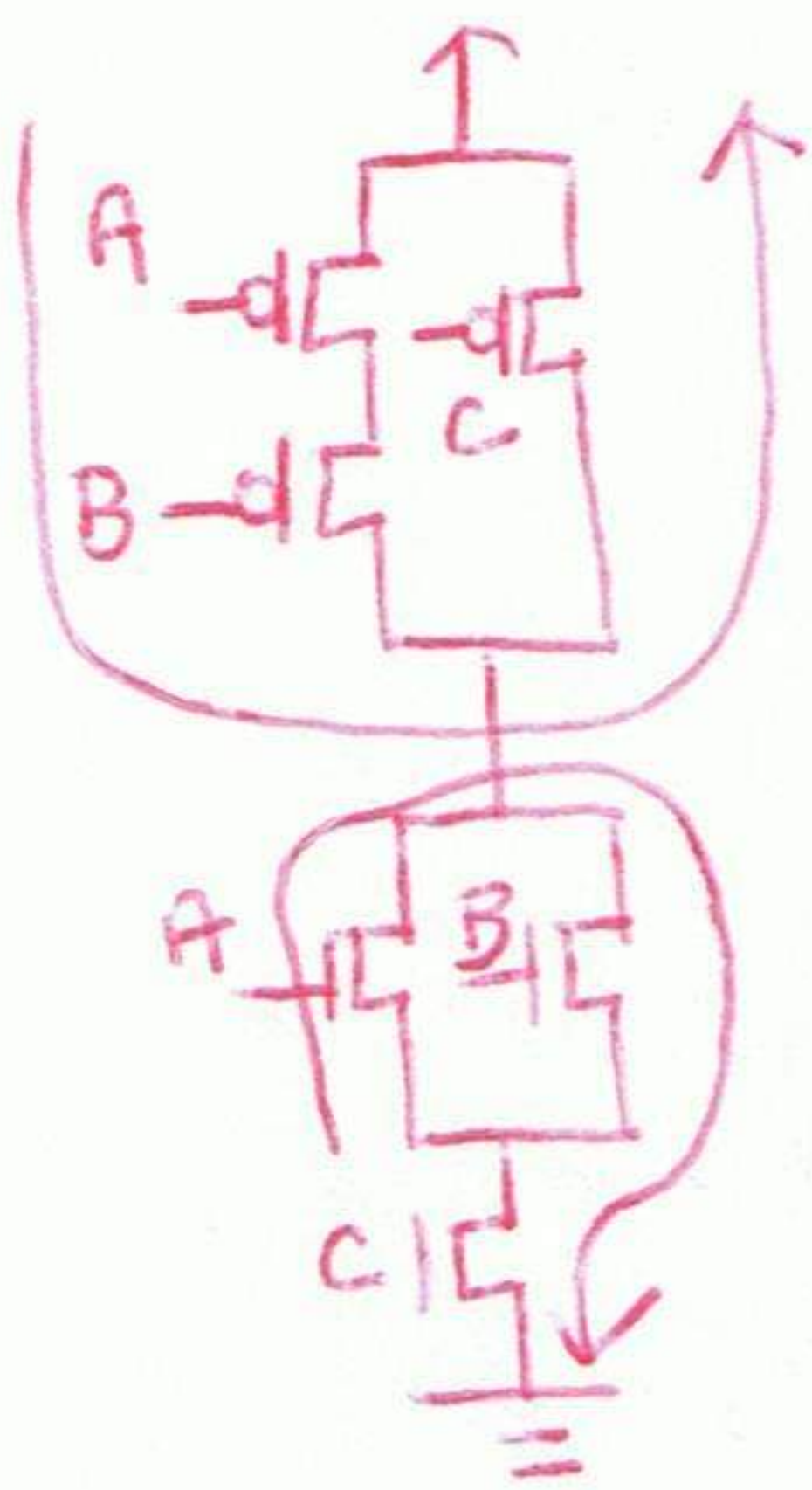
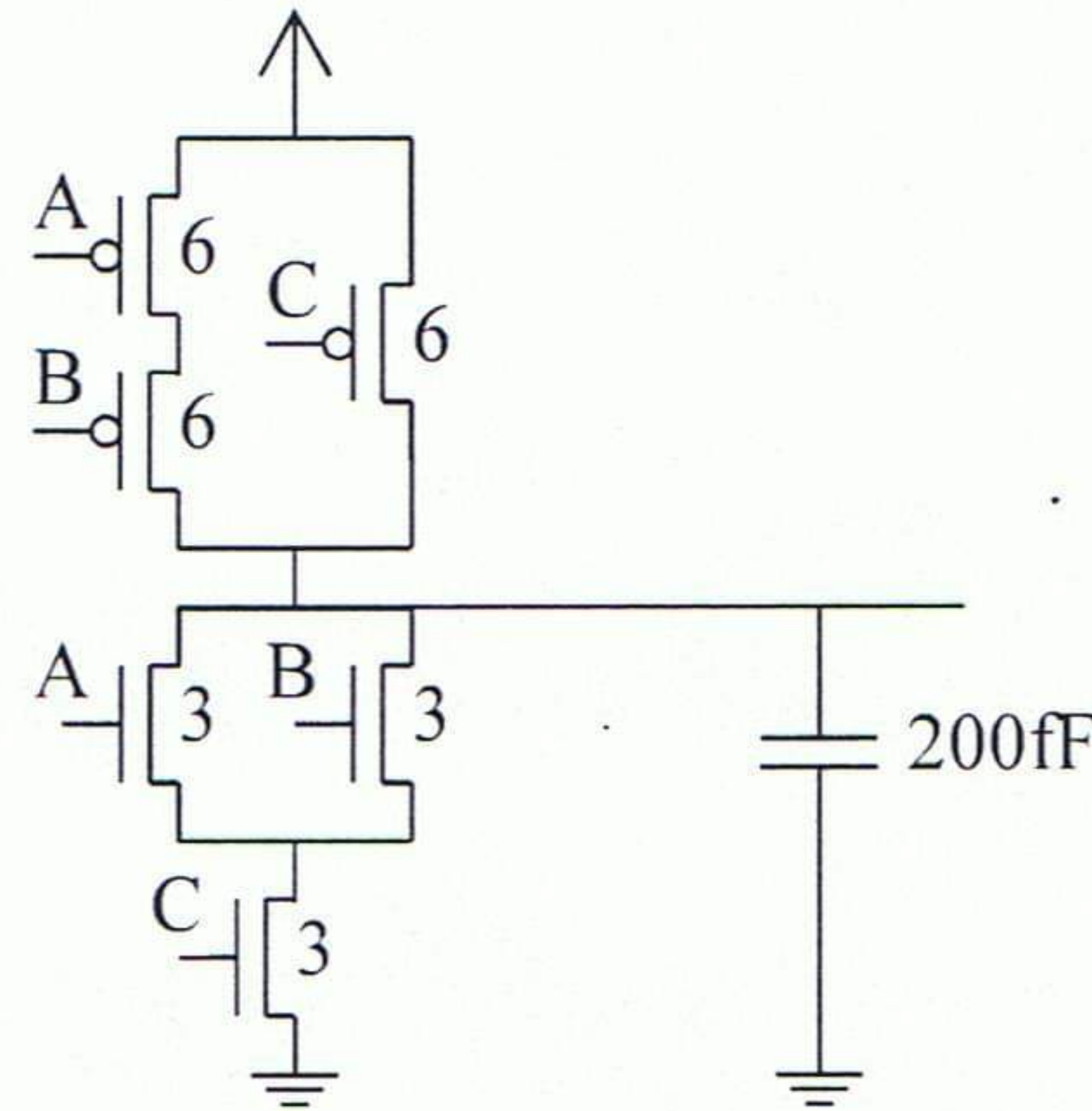
$$= 0.361$$

$$\text{Performance penalty} \Rightarrow \frac{\frac{\beta}{2}(1.0 - 0.361)^2}{\frac{\beta}{2}(1.0 - 0.3)^2} = 0.833 = 83\% \text{ performance.}$$

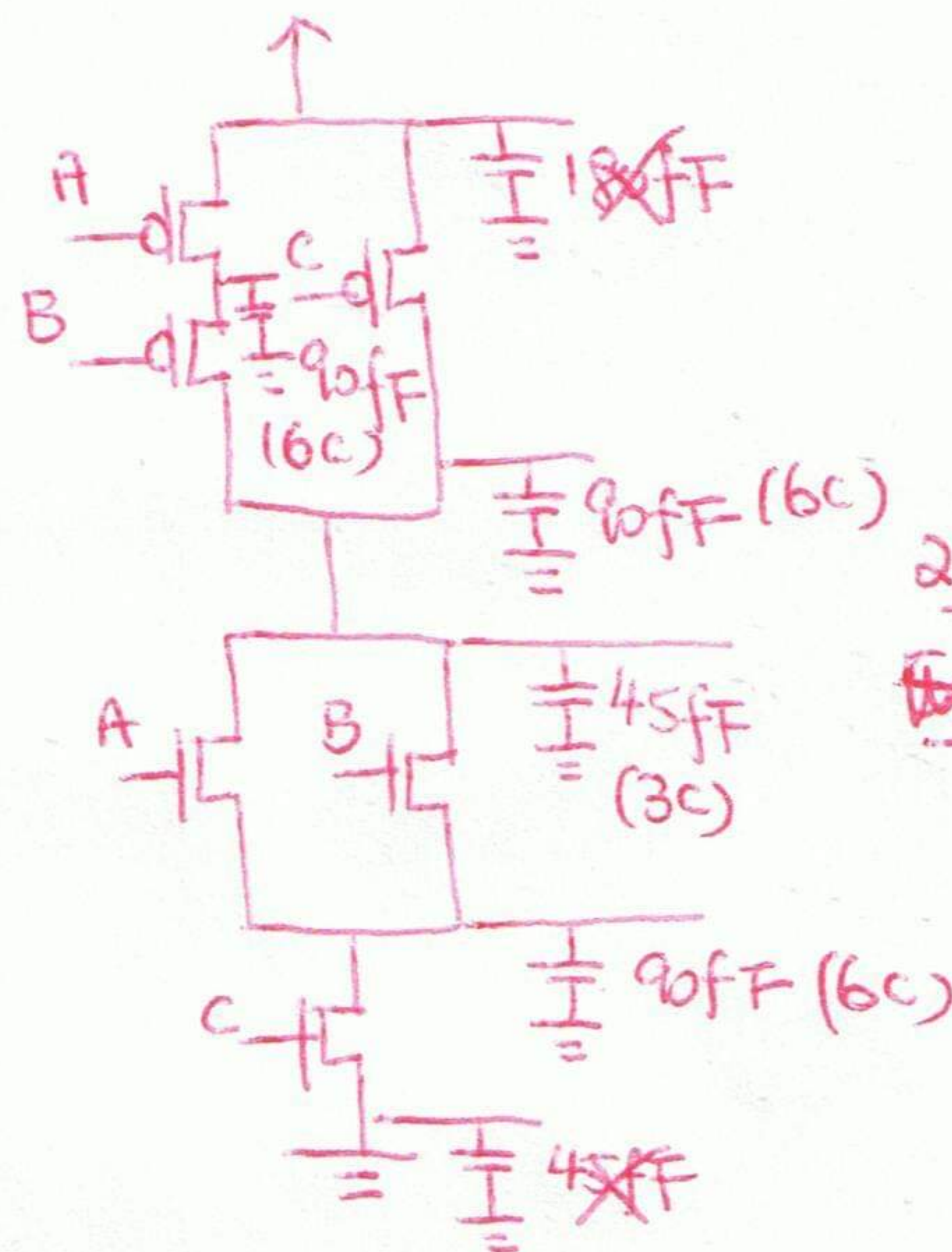


**Q3:**

Assume the on-resistance of a unit nMOS is 15Kohms and the on-resistance of a unit pMOS is 30Kohms. Also assume the drain, source, and gate capacitances of a unit transistor are all equal to 15fF. Using Elmore delay to calculate the delay of the gate when the input of the gate is transitioned from A=1, B=0, C=1 to A=1, B=0, C=0. **Please assume diffusion sharing is always used to minimize layout area of the gate.** (20 marks)



5 marks



2.5 marks

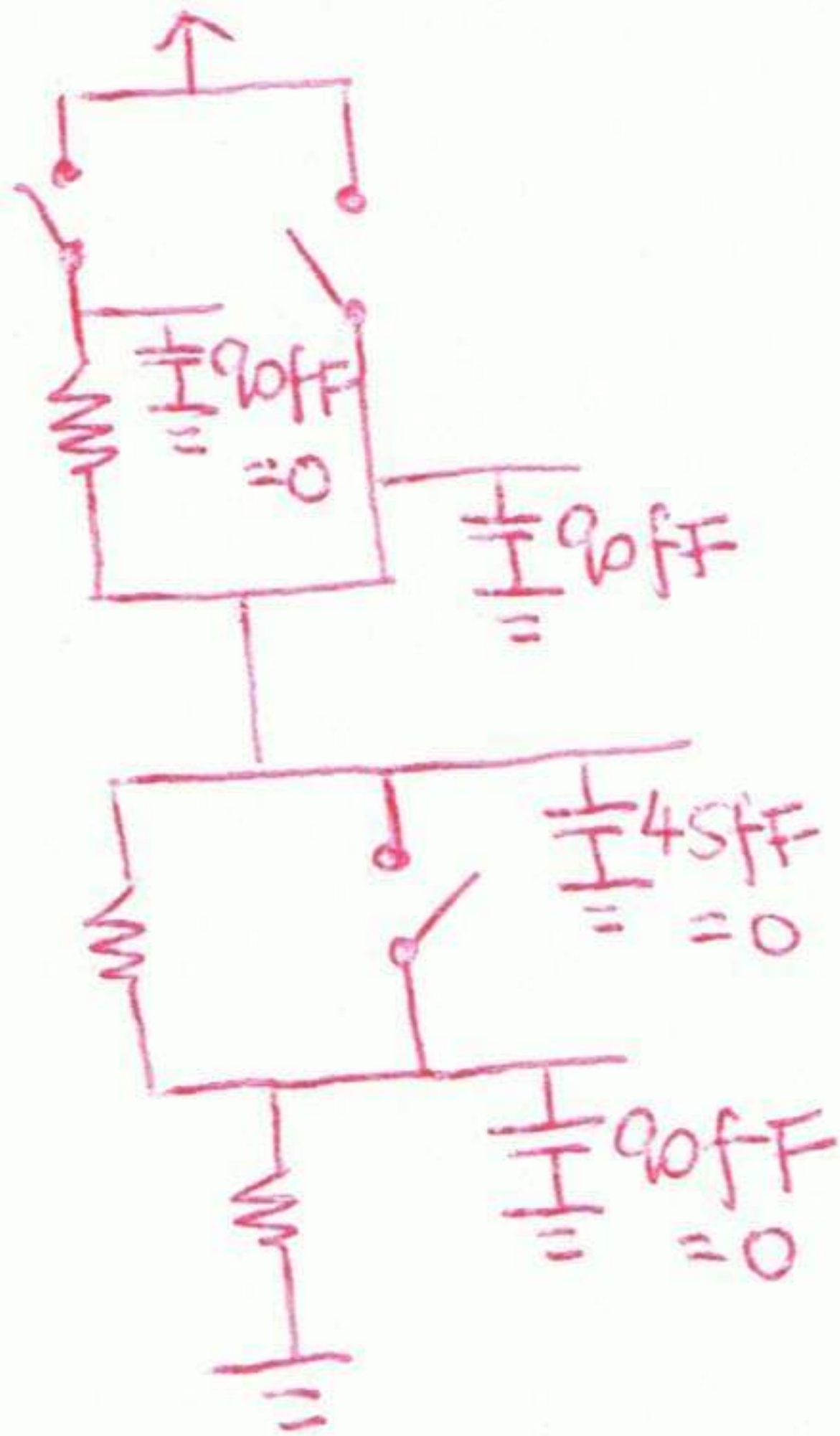
2.5 marks

out of 7.5 marks  
 2.5 some sort of RC tree  
 5 wrong diffusion sharing  
 7.5 everything right

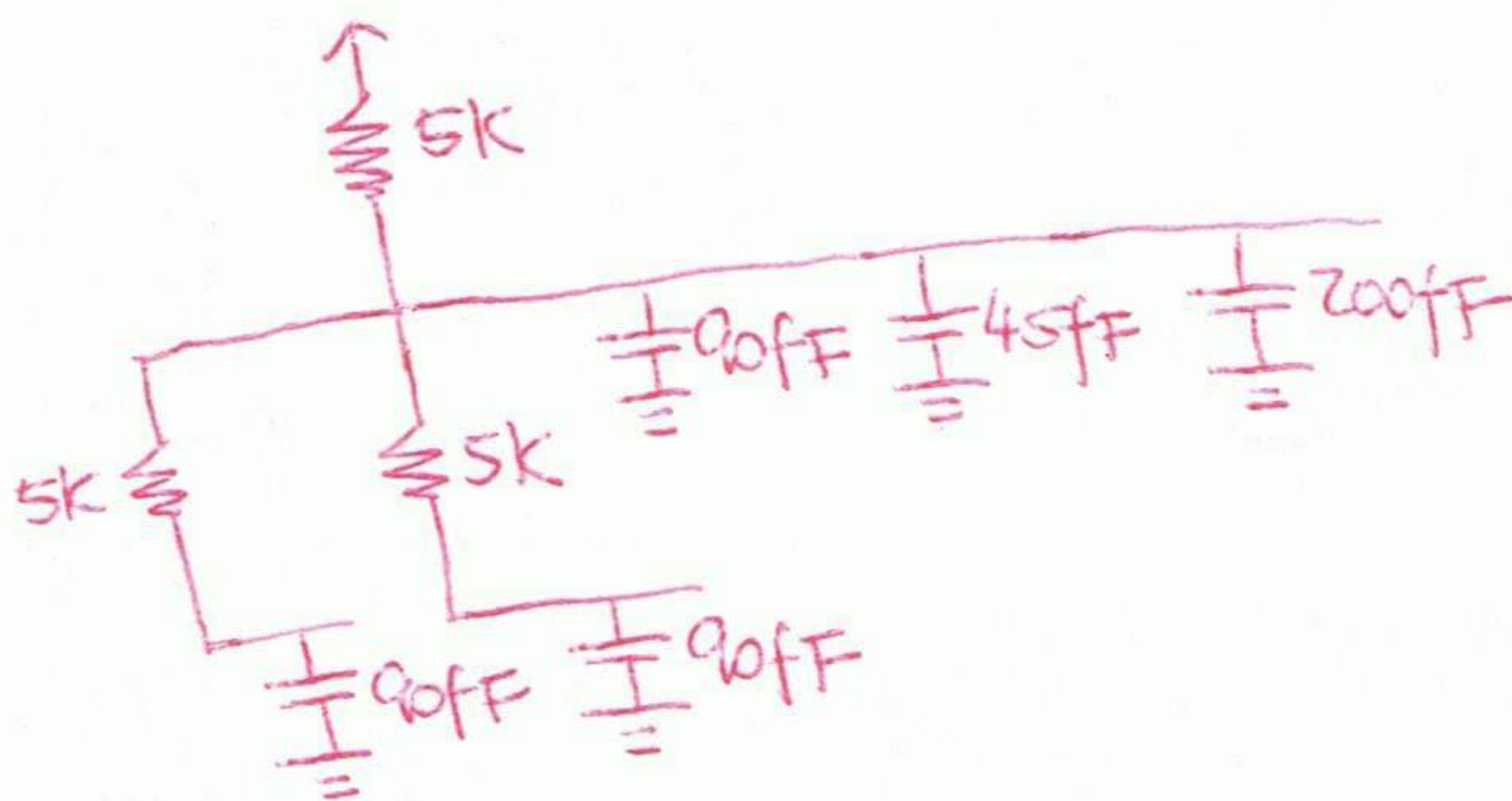
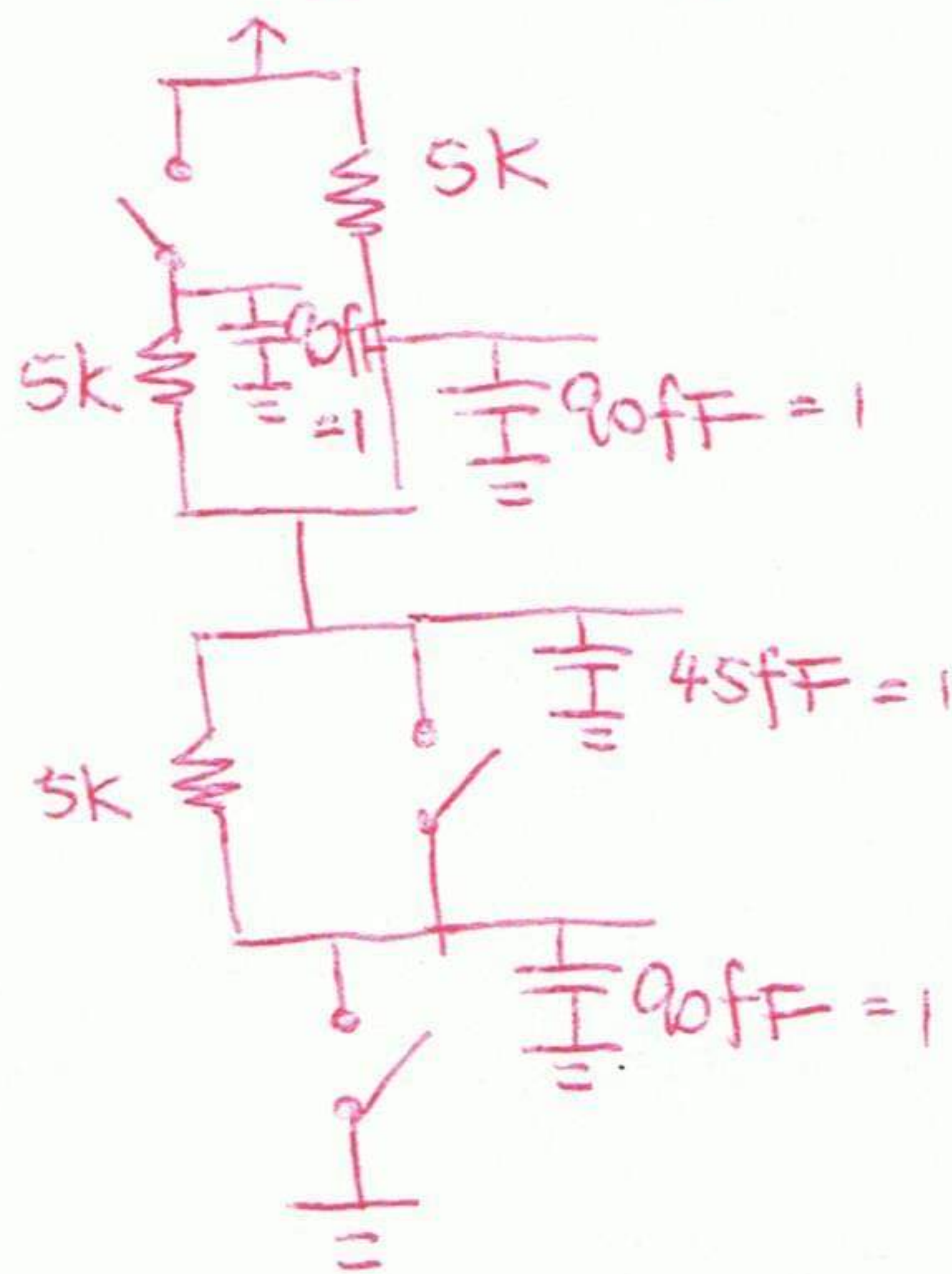


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A=1 B=0 C=1



A=1 B=0 C=0



5 marks

$$\begin{aligned} \text{delay} &= (90 + 45 + 200) \text{fF} \cdot 5\text{k} + 90 \text{fF} \cdot 5\text{k} + 90 \text{fF} \cdot 5\text{k} \quad \underline{5 \text{ marks}} \\ &= (90 + 45 + 200 + 90 + 90) \text{fF} \cdot 5\text{k} \\ &= 515 \text{fF} \cdot 5\text{k} \\ &= 2575 \text{ps} \\ &= 2.575 \text{ns} \end{aligned}$$

Extra marks  
2.5 marks

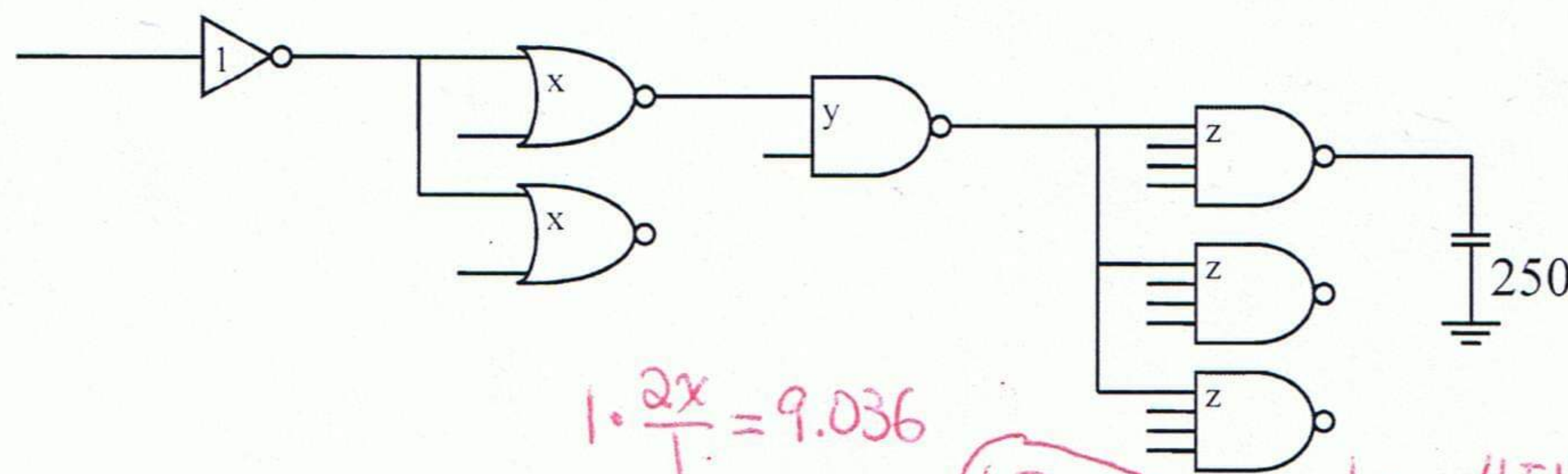
5 marks RC tree  
10 marks equation + RC tree  
12.5 marks everything correct

8 marks equation + RC tree  
with mistakes



**Q4:**

- Calculate the optimum path delay for the following circuit and the value of x, y, and z in order to achieve the optimum delay. (inverter:  $g=1, p=1$ ; 2-input nor:  $g=5/3, p=2$ ; 2-input nand:  $g=4/3, p=2$ ; 4-input nand:  $g=6/3, p=4$ ). (8 marks)
- Draw the transistor level diagram for the 2-input nor gate and clearly label the size of each transistor in the diagram. (3 marks)
- Draw the transistor level diagram for the 4-input nand gate and clearly label the size of each transistor in the diagram. (3 marks)
- Calculate the optimum path delay if additional inverter stages can be added between the 4-input nand gate and its load capacitance (2 marks).
- How many additional stages should be added in part d) in order to achieve the optimum delay and what should be the size of each stage (4 marks).



a)

$$F = GBH$$

$G$  = logic effort

$B$  = branch effort

$H$  = electrical effort

$$G = 1 \cdot \frac{5}{3} \cdot \frac{4}{3} \cdot \frac{6}{3}$$

$$B = 2 \cdot 1 \cdot 3 \cdot 1$$

$$H = \frac{250}{1}$$

$$F = GBH$$

$$= 1 \cdot \frac{5}{3} \cdot \frac{4}{3} \cdot \frac{6}{3} \cdot 2 \cdot 1 \cdot 3 \cdot 1 \cdot \frac{250}{1}$$

$$= \frac{20000}{3}$$

$$= 6666.67$$

$$(F)^{1/4} = 9.036 \quad (1.5 \text{ marks})$$

$$4 \cdot 9.036 + 1 + 2 + 2 + 4 = 45.144 \quad (1.5 \text{ marks})$$

$$1 \cdot \frac{2x}{1} = 9.036$$

$$x = 4.518 \quad (1.5 \text{ marks})$$

$$\frac{5}{3} \cdot \frac{y}{4.518} = 9.036$$

$$y = 24.49 \quad (1.5 \text{ marks})$$

$$\frac{4}{3} \cdot \frac{3z}{24.49} = 9.036$$

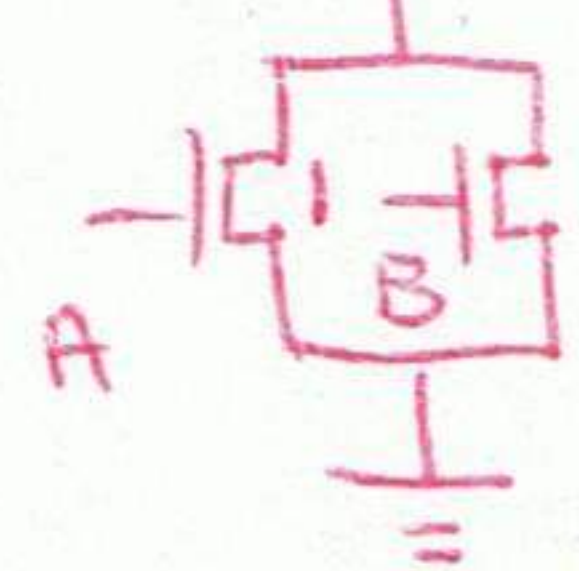
$$z = 55.32 \quad (1.5 \text{ marks})$$

$$\frac{6}{3} \cdot \frac{250}{55.32} = 9.038 \quad \checkmark$$

0.5 for effort

b) 4.518 units of unit inverter gate capacitance.

$$A \rightarrow 4 \quad 3C \Rightarrow 1$$



$$\frac{4.518 \cdot 3C}{5} = 2.7108C$$

$$4 \times 2.7108 = 10.84C$$

$$\frac{2.7108C}{3C} = 0.904 \quad \text{check} \quad \text{ok. nmos}$$

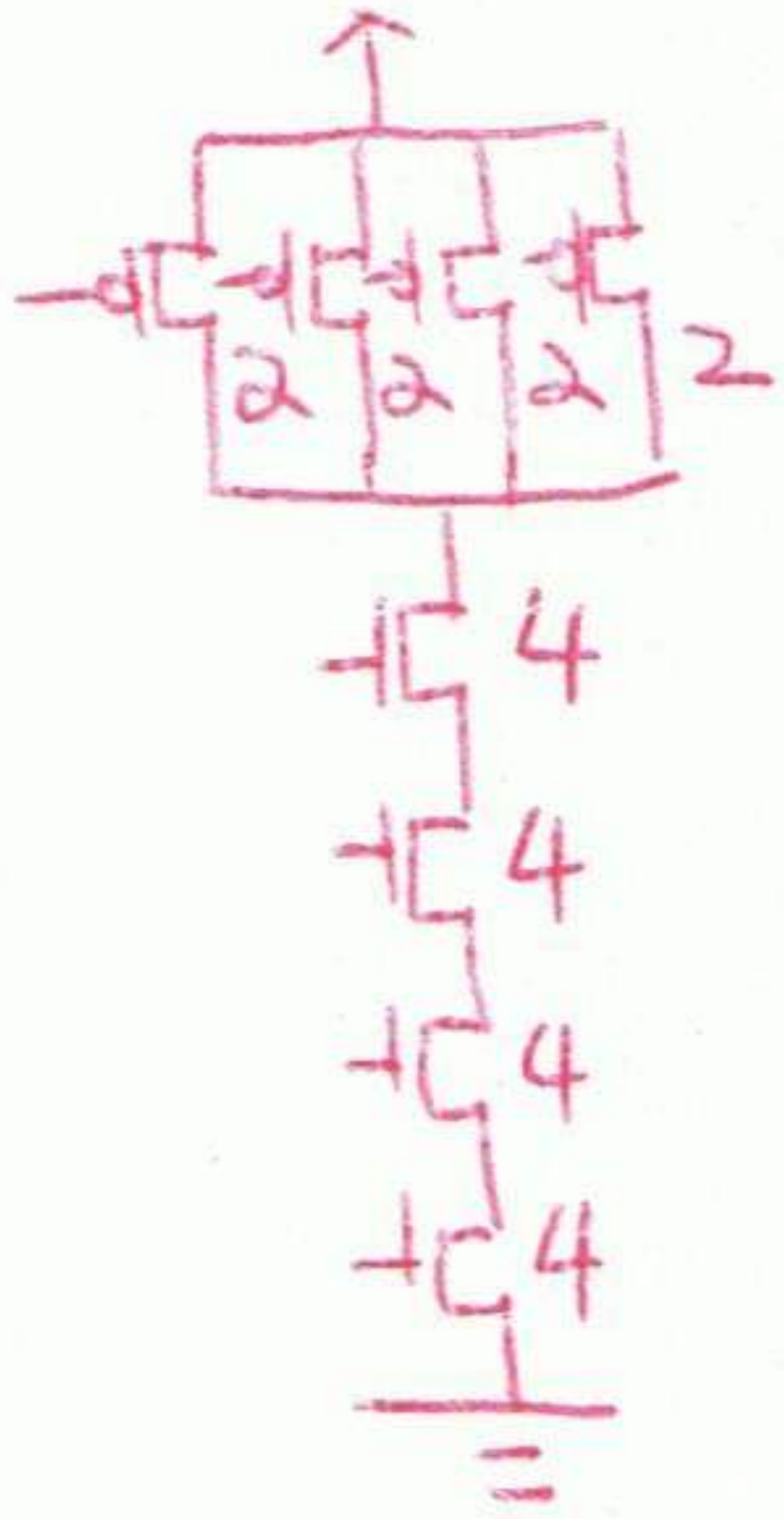
$$\frac{10.84C}{3C} = 3.613 \quad \text{check.} \quad \text{ok. pmos}$$

1.5 for pmos network

1.5 for nmos network.



c)



$Z = 55.32$  units of unit inverter gate capacitance

$$\frac{55.32 \cdot 3C}{6} \cdot 2 = 55.32C$$

$$\frac{55.32 \cdot 3C}{6} \cdot 4 = 110.64C$$

$$\frac{55.32C}{3C} = 18.44 \text{ check}$$

$$\frac{110.64C}{3C} = 36.88 \text{ check}$$

1.5 for pmos network

1.5 for nmos network

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d) ~~1.5~~

$$F = GBH$$

$$= 6666.67$$

$$(F)^{\frac{1}{N}} = 3.59$$

$$(6666.67)^{\frac{1}{N}} = 3.59$$

$$N = \frac{\log(6666.67)}{\log(3.59)}$$

$$= 6.89$$

$$N = 7$$

$$(6666.67)^{\frac{1}{6}} \cdot 6 \neq 1+2+2+4+2 = 37.03$$

$$(6666.67)^{\frac{1}{7}} \cdot 7 \neq 1+2+2+4+3 = 33.11$$

Optimal delay = ~~36.63~~

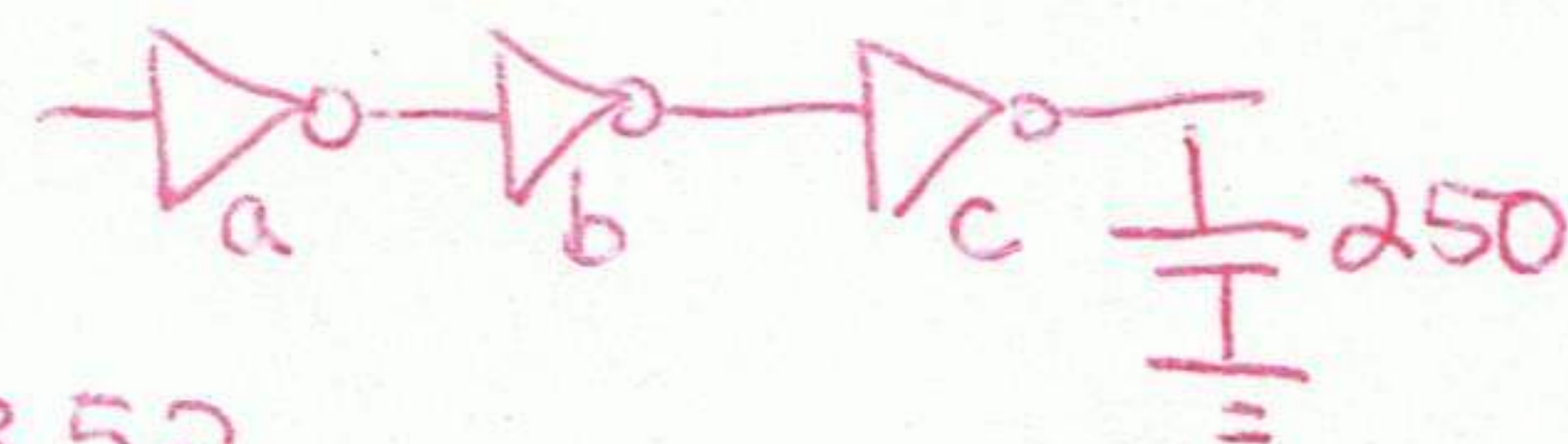
2 marks

$$(6666.67)^{\frac{1}{7}} = 3.52$$

$$3.5178$$

3 stages

1 mark



$$1 \cdot \frac{250}{C} = 3.52$$

$$C = \frac{250}{3.52} = 71.02$$

71.071

1 mark

$$\frac{6}{3} \frac{5.73}{Z} = 3.53$$

$$1 \cdot \frac{71.02}{b} = 3.52$$

1 mark

$$Z = 3.248$$

3.26298

$$b = 20.18$$

20.1895

$$\frac{4}{3} \frac{3 \cdot 3.248}{y} = 3.53$$

$$1 \cdot \frac{20.18}{a} = 3.52$$

1 mark

$$y = 3.68$$

3.71

$$a = 5.73$$

5.7392

$$\frac{5}{3} \frac{3.68}{x} = 3.53$$

$$x = 1.737$$

1.7578

$$1 \cdot \frac{2 \cdot 1.737}{1} = 3.47$$

3.5157

0.15



### Formulae and Constants

$$\beta = 1048 \mu A / V^2$$

$$I_{ds} = \begin{cases} 0 & V_{gs} < V_t & \text{Cutoff} \\ \beta(V_{GT} - V_{ds}/2)V_{ds} & V_{ds} < V_{dsat} & \text{Linear} \\ \frac{\beta}{2}V_{GT}^2 & V_{ds} > V_{dsat} & \text{Saturation} \end{cases}$$

$$\eta = 100 mV / V$$

$$k_\lambda = 0.083$$

$$S = 100 mV / V$$

$$v_T = 26 mV$$

$$I_{off} = 0.1 \mu A$$

$$I_{ds} = I_{off} 10^{\frac{V_{gs} + \eta(V_{ds} - V_{dd}) - k_\lambda V_{sb}}{S}} (1 - e^{\frac{-V_{ds}}{v_T}})$$

$$\gamma = 0.16$$

$$\phi_s = 0.93 V$$

$$V_t = V_{t0} + \gamma(\sqrt{\phi_s + V_{sb}} - \sqrt{\phi_s})$$