Lab # 2

Performance of Control Systems under Proportional, PI, PD and PID Control

Introduction

Project Objectives

This Lab project is a Simulink simulation to analyze performance of systems under various controllers. In the first part of the Project, you will investigate three basic modes of Controller operation: Proportional (P) Control, Proportional + Integral (PI) Control, and Proportional + Derivative (PD) Control. In the second part of the Project you will design/tune a full three-mode PID Controller. Your objectives are therefore to:

- Increase your understanding of basic operations of a control system under the three modes of control (P, PI and PD), including tracking of a reference signal both in the transient state and in the steady state, and of implications of the different modes of control on the performance of closed loop systems.

- Achieve an improvement of the closed loop system response by finding the "best" settings of the PID Controller so that the defined response specifications are met.

Logistics

During the regular semester this Lab is completed over a period of four weeks, in 1.5-hour per week sessions. Should you require additional time, you are expected to complete all simulations and calculations outside the lab time (in lieu of a Computer Assignment that used to be one of the course deliverables). You will not have access to the Controls Lab (ENG413) outside your scheduled hours.

You are allowed to work with a partner of your choice, as long as he/she is registered in the same lab section as you are. **If you continue working with your partner from Project # 1, use the previously assigned data set.** If you changed partners, your new partner has worked with a different data set - consult with the Instructor as to which data set you should continue with. All partnerships will be finalized and approved by your Lab Instructor during the in-lab session.

Lab # 2 reports are due at the beginning of the first Lab # 3 session – check the Course Schedule posted on D2L website for the exact due date for your lab section. The written report is a collaborative part and you will receive 80 points (out of 100) for it. The remaining 20 points is an individual component and will depend on how you answer questions in a short interview with the Lab Instructor. You have to be in the lab in person for the interview, or else you will receive a zero for it. The interview will take place during the session when the reports are due. Should you miss the interview with a valid reason (i.e. a medical note handed to the Main Office in the Department), please make sure that your partner hands in the report and then you can make arrangements with the Lead TA for a make-up interview.

Pre-Lab: Creating SIMULINK Simulation Diagram

This part of the project is to be completed individually before coming to the Controls Lab.

The Pre-Lab consists of creating a functional SIMULINK simulation diagram for comparisons between different modes of Control (P, PI, and PD) - use the simulations you created for Project # 1 as a starting point. Figure 1 shows an example of such comparison.

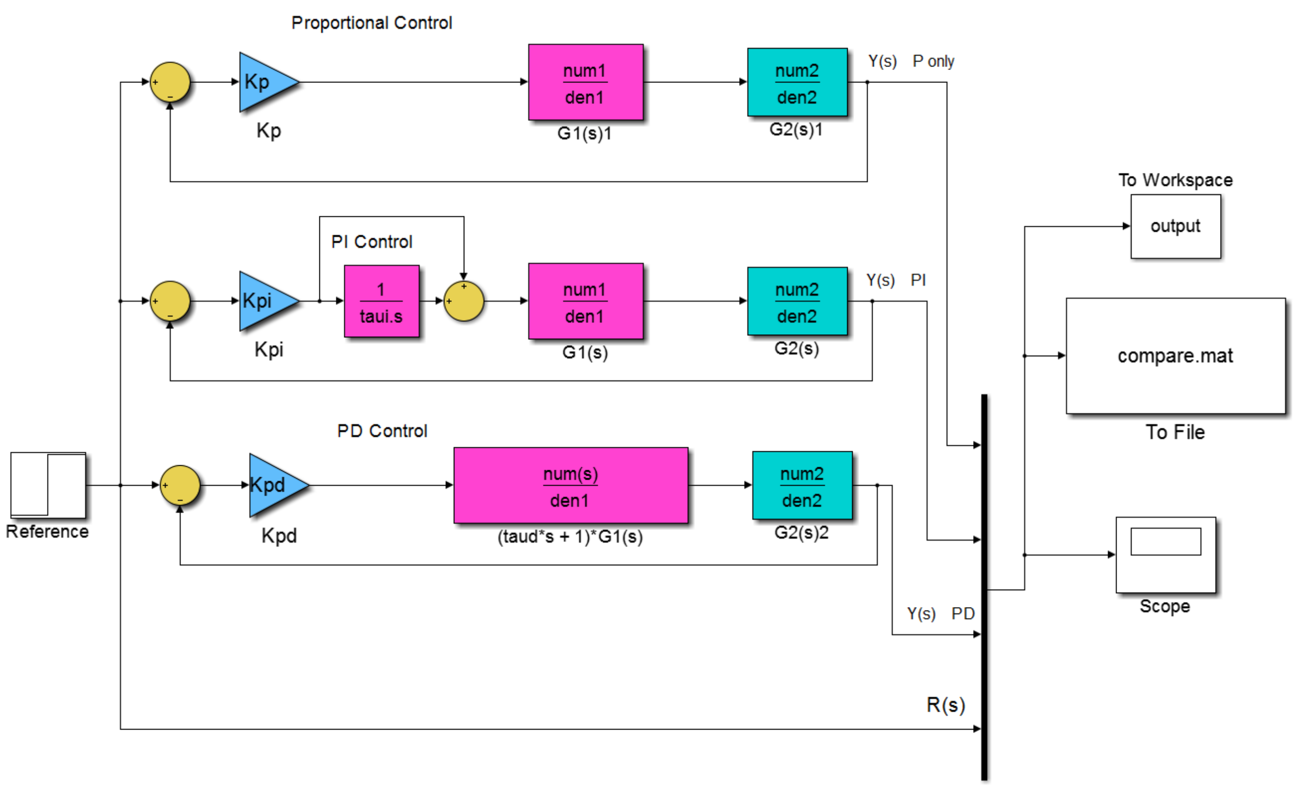

Figure 1: An Example of a Simulink Diagram for Comparisons between Various Modes of Control

To demonstrate the functionality of your simulation, set the Integral Time Constant τ_i to 5 seconds and the Derivative Time Constant τ_d to 2 seconds, and use the Proportional Gain setting such that you operate safely within the stable range of all three controller configurations, as calculated in Lab # 1.

The Pre-Lab is individual, and both you and your partner will need to demonstrate your simulation diagrams to the Lab Instructor, who will verify that simulations are working by signing in appropriate boxes on your Lab # 2 Grading Sheet - the signature should be in the box under your name.

Each one of the partners needs to get **an individual sign-off** on the Grading Sheet to verify that he/she completed the Pre-Lab. If the Instructor's signature is missing, 20 points for the missing Pre-Lab will be deducted from your individual lab report total.

The Lab Instructor will not check pre-lab simulations outside of the first lab session scheduled for this Project. Should you miss that lab session with a valid reason (i.e. a medical note handed to the Main Office in the Department), please contact your TA to make arrangements for the Pre-Lab demonstration.

Once you obtained the Instructor sign-off on the pre-Lab, you can proceed with the lab.

Part 1: Exploring Control Modes (P, PI and PD)

Proportional Control

Consider the system under Proportional Control, as shown in Figure 2, that you are familiar with from Project # 1.

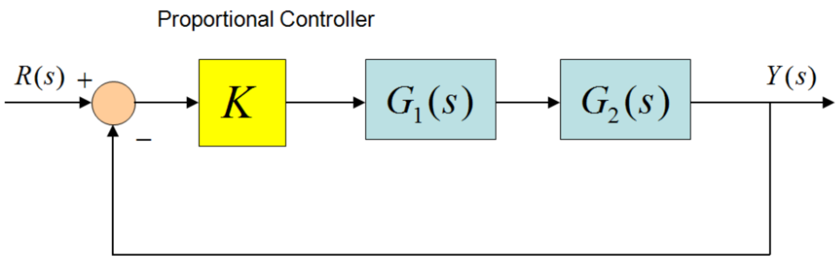

Figure 2: Unit Feedback System under Proportional Control

You already have a working SIMULINK simulation diagram for this system, which you can continue using for now. Alternatively, you can start working with the simulation diagram that you created in this Pre-Lab – if you do so, please remember to disconnect the PI and PD parts of your simulations.

You will be experimenting with the Proportional Controller Operating Gains, K_{op} within the stable range, i.e. $0 < K_{op} < K_{crit}$. Your objectives are to determine the effect of gain changes on the *steady state errors* as well as on the *transient response*. You will use two standard normalized unit inputs for the steady state error analysis: a unit step and a unit ramp. You will only use the step input to explore the transient response.

1. Determine how the Proportional Controller Gain changes affect the Steady State Error (in %), $e_{ss(step)\%}$ when a unit step input is used. Make sure that you can see the system response setting into a *steady state*.
2. Determine how the Proportional Controller Gain changes affect the Steady State Error, $e_{ss(ramp)}$, when a unit ramp input is used. Make sure that the observation (simulation) time is long enough to ascertain what happens to the ramp error. Check if that is consistent with what you know about the Type 0 system response. See Chapter 5 of the Course Notes for definitions and steady state error analysis.
3. Determine how the Proportional Controller Gain changes affect the following Transient Specifications: Percent Overshoot, PO, Settling Time, $T_{settle(\pm 2\%)}$, and Rise time, $T_{rise(0-100\%)}$. See Chapter 4 of the Course Notes for definitions of transient specifications.
4. Collect and save data for representative values of the Controller gain (for example, low-medium-high), to illustrate your conclusions for the report.

"Benchmarking" the System

You will now establish a "benchmark" for your system performance, which you will later attempt to improve by applying different modes of Control. To obtain the benchmark, set the Operating Proportional Controller Gain, K_{op} such that the so-called "Quarter Decay" response is obtained. Consult the Appendix on what the "Quarter Decay" response is.

Save the step response data for that controller setting, plot it in MATLAB and take the record of: Percent Overshoot, PO, Settling Time, $T_{settle(\pm 2\%)}$, and Rise time, $T_{rise(0-100\%)}$. Obtain the unit ramp response for that controller gain setting to illustrate the Steady State Error, $e_{ss(ramp)}$.

NOTE: You may find it useful to use the MATLAB files posted on the course website ("stepeval" and "rampeval") which will allow you to easily obtain the numerical values of the specs. Please use them in a mindful way, to verify your results – you still need to demonstrate in your lab report how you found, say the Percent Overshoot, by showing it on a plot. As well, please do not use screen captures from MATLAB, showing numerical values of the specs as computed by MATLAB – instead, word-process and tabulate the specs.

Establishing System Performance Specifications

When an improvement in the system performance is required, we work to meet a set of “desired” specifications for the system response, defined based on the user requirements and on your knowledge, as a designer, of what constitutes an “acceptable” response of a control system in question. For this Project, the performance specifications are defined as follows:

- Steady State Error (in %) of the response to a unit step input is to be equal to zero: $e_{ss(step)\%} = 0\%$;
- Steady State Error of the response to a unit ramp input is to be as small as possible: $e_{ss(ramp)} \rightarrow 0$;
- Percent Overshoot of the response to a unit step input is to be less than 15%;
- Settling Time, $T_{settle(\pm 2\%)}$ of the response to a unit step input is to be one-half, or less, of the Settling Time as measured for the “benchmarked” response.

NOTE 1: The Percent Overshoot requirement is the same for all data sets and will result in the same value of the equivalent closed loop damping ratio ς – see Chapter 7 of the Course Notes for a full explanation. However, the Settling Time depends both on the equivalent closed loop damping ratio, ς as well as on the equivalent closed loop frequency of natural oscillations, ω_n which will be different for each different process. As a result some of the data sets describe uncompensated systems that are much slower than others. Setting a single value of the Settling Time specification for all groups to meet may thus prove challenging for those working with data sets representing slower systems. To make the “Improvement Challenge” more equitable across all the different data sets, we only require that the Settling Time improvement is against your own “benchmarked” response, hence the “relative” definition of the requirement as one-half of the “benchmarked” Settling Time value.

NOTE 2: It should be already clear to you that Proportional Control will not allow you to meet all of these requirements at the same time. However, exploring the properties of the PI and PD modes of Control will show you that these seemingly conflicting requirements can indeed all be met with the “right” combination of PID Controller parameters. This will become your task for Part 2 (PID Design) of your Project.

Proportional + Integral (PI) Control

Next, consider the system under Proportional + Integral Control (where τ_i is the Integral Time Constant), shown in Figure 3, that you are already familiar with from Project # 1.

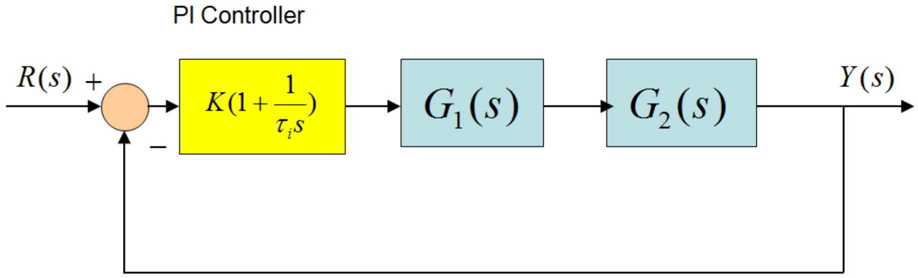

Figure 3: Unit Feedback System under PI Control

In this part of the project, you will be experimenting with both the Proportional Gain, K_P and with the Integral Time Constant, τ_i of the PI Controller, to find out what their effect is on the system performance.

Your objectives are to:

1. Determine the effect of the Controller parameters on the *steady state errors* as well as on the *transient response*. You will use two standard normalized unit inputs for the steady state error analysis: a unit step and a unit ramp. You will only use the step input to explore the transient response.
2. Compare the performance of the system under PI Control with the system under Proportional only Control. To do so, select and save some representative samples of PI system responses. Make comparisons with reference to the "benchmarked" response obtained under Proportional only Control.
3. Attempt to improve the "benchmarked" performance with respect to the following specifications: Step Steady State Error, $e_{ss(step)\%}$, Ramp Steady State Error, $e_{ss(ramp)}$, Percent Overshoot, PO, Settling Time, $T_{settle(\pm 2\%)}$, and Rise time, $T_{rise(0-100\%)}$. Is it possible to meet the required performance specifications? What are the issues here?

In order to facilitate easy comparisons between PI Control and Proportional Control, use the simulation diagram you created in the Pre-Lab. Make sure to "disable" the PD part of the simulation. Set the Proportional Gain in Proportional only Control part of the simulation to the value found in the "benchmarking" exercise, i.e.K_{op}.

When experimenting with changing the value of the Integral Time Constant, be aware that your relative stability range will also change, since your Project # 1 calculations were done for $\tau_i =$ 5 seconds. In your simulations, when using a different value of the Integral Time Constant, adjust the Proportional Gain accordingly so that you always operate within the stable range of system responses.

Recall from Chapter 5 that most of the "heavy-lifting" of your Controller work should be done by the Proportional part of the Controller, as it is the Proportional Gain value that determines the accuracy of tracking in the steady state. Integral and Derivative modes should be used sparingly. Think of Proportional control as a "main meal" and of Integral and Derivative as condiments to improve its taste ☺.

Proportional + Derivative (PD) Control

Next, consider the system under Proportional + Derivative Control (where τ_d is the Derivative Time Constant), shown in Figure 4, that you are already familiar with from Project # 1.

In this part of the project, you will be experimenting with both the Proportional Controller Operating Gains, K_{op}, and with the Derivative Time Constant, τ_d, to find out what their effect is on the system performance.

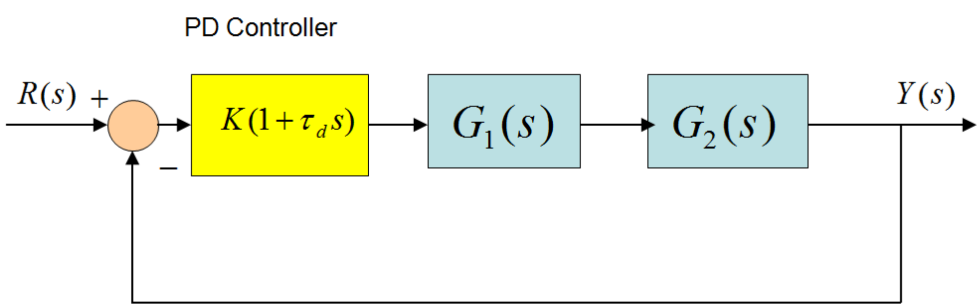

Figure 4: Unit Feedback System under PD Control

Your objectives are to:

1. Determine the effect of the Controller parameters on the *steady state errors* as well as on the *transient response*. You will use two standard normalized unit inputs for the steady state error analysis: a unit step and a unit ramp. You will only use the step input to explore the transient response.
2. Compare the performance of the system under PD Control with the system under Proportional only Control. To do so, select and save some representative samples of PD system responses. Make comparisons with reference to the "benchmarked" response obtained under Proportional only Control.
3. Attempt to improve the "benchmarked" performance with respect to the following specifications: Step Steady State Error, $e_{ss(step)\%}$, Ramp Steady State Error, $e_{ss(ramp)}$, Percent Overshoot, PO, Settling Time, $T_{settle(\pm 2\%)}$, and Rise time, $T_{rise(0-100\%)}$. Is it possible to meet the required performance specifications? What are the issues here?

In order to facilitate easy comparisons between PD Control and Proportional Control, use the simulation diagram you created in the Pre-Lab. Make sure to “disable” the PI part of the simulation. Set the Proportional Gain in Proportional only Control part of the simulation to the value found in the “benchmarking” exercise, i.e.K_{op}.

When experimenting with changing the value of the Derivative Time Constant, be aware that your relative stability range will also change, since your Project # 1 calculations were done for $\tau_d = 2$ seconds. In your simulations, when using a different value of the Derivative Time Constant, adjust your Proportional Gain accordingly so that you always operate within the stable range of system responses.

Part 2: PID Control

In the second part of this Project, your task is to come up with a significant improvement of your system closed loop response by implementing the PID Controller, combining the observed benefits of both previously explored controller modes: PI and PD. Your objective is to meet (or exceed) the Performance Specifications established in Part 1. You will demonstrate the quality of your design by showing your system compensated response vs. the “benchmarked” response from Part 1.

You will have a large degree of latitude in deciding how best to accomplish that, and in the process of doing it, to demonstrate the mastery of the learned course material. The quality of your design will be judged by two factors:

- How dramatic the improvement is, compared with the Proportional only Control for your “benchmark” Operating Proportional Gain setting;
- How in-depth your approach is, i.e. how are you able to connect it with the theory learned in the course.

The simplest, and least-insightful, approach to finding the “best” PID Controller setting that improve the system response is to follow a “Trial-and-Error” approach.

A more organized empirical approach to finding the “best” PID Controller setting that improve the system response is to follow a “tuning” method (either Ziegler-Nichols “Ultimate Gain” method or Ziegler-Nichols “Process reaction” method – refer to the Appendix for their detailed description.

Finally, your design could be informed by making references to theory, for example using the concept of a “dominant poles” closed loop model, perhaps showing the Root Locus analysis of your compensated system, and performing calculations of the Proportional Gain, based on the Root Locus theory. Since at this point of the course the Root Locus theory has yet to be introduced, this last approach is only recommended to students who feel comfortable studying this material on their own. **Should this approach be used, it will be considered as a bonus, with the expected approaches being either the “trial & error” approach or one of the Ziegler-Nichols methods.**

Depending on what strategy you decide to adopt for this "Performance Improvement Challenge", you can implement either a parallel or a series structure of the PID Controller – both are acceptable. Please refer to the Appendix for an explanation of the differences between the two.

It is however suggested that, if you decide to simply "tune" your PID Controller, you should choose the parallel structure, which is the most intuitive to use for direct time domain experimentation. The "Trial-and-Error" approach as well as both Ziegler-Nichols methods described in the Appendix define parameter values for the parallel structure, so you should work with the parallel structure of the PID if you choose one of the empirical tuning methods.

Should you decide to use a more informed, analytical approach making references to the pole locations in the s-plane, and performing the Root Locus analysis, you are free to use either the parallel or the series structure. Each has its own benefits so the choice is left to you – please see the Appendix for more explanations.

PID Controller Simulation Diagram

In order to facilitate easy comparisons between responses of your system under PID Control and under "benchmarked" Proportional Control, you will have to modify the simulation diagram you created in the Pre-Lab. You can either remove the PI and PD parts of your simulation, replacing them with a full PID Controller, or simply add the fourth component to the diagram, representing the PID Controller.

Make sure to set the Proportional Gain in Proportional only Control part of the simulation to the value found in the "benchmarking" exercise, i.e.K_{op}.

Note that the PID Controller transfer function, whether in a parallel or in a series configuration, will present the same accuracy challenges as the PD Controller, because of the way SIMULINK Derivative Block is implemented. The best way to implement the PID transfer function is the same work-around solution described in Project 1 Appendix w.r.t. PD transfer function, where the PID Controller transfer function will be combined with the transfer function $G_1(s)$.

Let us consider the parallel structure of the PID Controller:

$$G_{PID}(s) = K_p\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) = K_p\left(\frac{\tau_d \tau_i s^2 + \tau_i s + 1}{\tau_i s}\right)$$

The transfer function $G_{PID}(s)$, followed by the transfer function $G_1(s)$, is equivalent to the gain block K_P followed by a transfer function block $\left(\frac{\tau_d \tau_i s^2 + \tau_i s + 1}{\tau_i s}\right) \cdot G_1(s)$.

The coefficients of the PID Controller are now combined with the coefficients of the first process transfer function block $G_1(s)$ to create a proper transfer function with two zeros, belonging to the PID Controller, and three poles, one (the integrator) belonging to the PID Controller, and two belonging to the process $G_1(s)$. Note that we can use the convolution (MATLAB function *conv*) to multiply the two s-polynomials in the denominator of the new block.

We can deal in the same way with the series structure of the PID Controller:

$$G_{PID(series)}(s) = K_p\left(1 + \frac{1}{\tau_i s}\right)(1 + \tau_d s) = K_p\,\frac{(\tau_i s + 1)(\tau_d s + 1)}{\tau_i s}$$

$$G_{PID(series)}(s) = K_p\left(\frac{\tau_d \tau_i s^2 + (\tau_d + \tau_i)s + 1}{\tau_i s}\right)$$

The transfer function $G_{PIDseries}(s)$, followed by the transfer function $G_1(s)$, is equivalent to the gain block K_P followed by a transfer function block $\left(\frac{\tau_d \tau_i s^2 + (\tau_d + \tau_i)s + 1}{\tau_i s}\right) \cdot G_1(s)$.

As with the parallel structure, the coefficients of the series PID Controller are now combined with the coefficients of the first process transfer function block $G_1(s)$ to create a proper transfer function with two zeros, belonging to the PID Controller, and three poles, one (the integrator) belonging to the PID Controller, and two belonging to the process $G_1(s)$.

Discussion

In this section you should analyze the steady state as well as the transient response of your control system under the three different configurations of the controller (P, PI and PD), and describe the combined benefits of these as illustrated by your PID Controller design. The issues to discuss include:

1. How does each of the Control modes affect the steady state error in the step response? In the ramp response? Which of the Controller parameters need to be adjusted to control the errors, and in what way? Connect your observations to theory. What are the implications for choosing the “best” settings of parameters?
2. How does each of the Control modes affect the percent overshoot in the step response? Which of the Controller parameters need to be adjusted to control the overshoot, and in what way? Connect your observations to theory. What are the implications for choosing the “best” settings of parameters?
3. How does each of the Control modes affect the settling time in the step response? Which of the Controller parameters need to be adjusted to control the settling time, and in what way? Connect your observations to theory. What are the implications for choosing the “best” settings of parameters?
4. In your PID design, analyze your compensated system behaviour based on theory learned in the course. Did you meet the required performance specifications? If yes, explain how you did it. If not, what do you think that is?

Instructions for the Write-up

Following are some general guidelines on the write-up for this Project.

1. The written report is to be word-processed, including any formulae that you may need. The first page is a standard cover page (posted on the website) which has to be signed by both partners. The report will not be accepted without both original, hand-written signatures. Please note that electronic signatures are not acceptable.
2. The second page is a grading sheet - please write the names of both partners in appropriate boxes, and the assigned data set number. Each of the partners has to complete the Pre-Lab and to obtain a signature from the Lab Instructor verifying that he/she demonstrated a working simulation diagram for comparisons between Proportional, PI and PD Control. If the Instructor's sign-off on the Pre-Lab is missing, 20 points will be deducted from your individual lab report score.
3. The third page is a one-page Executive Summary - your "bottom line" results and observations have to be stated here, and then expanded on in the discussion. It is best to write the Executive Summary last, when you have a clear overview of the entire report.
4. A maximum of sixteen (16) single-sided pages of content (i.e. not counting the cover, grading sheet and Executive Summary pages) are allowed. To make your report easy to follow, use separate headers for each part of the project. Use page numbering! Use at least point 11 font size throughout the report.
5. Figures have to be properly labeled and referred to in the body of the report. Plots have to be created in MATLAB, with all traces properly labeled, large enough to be easily legible, and included in the main body of the report.
6. Quality of writing, spelling, clarity and overall presentation is very important in an engineering report and the Canadian Engineering Accreditation Board (CEAB) requires that we pay attention to developing good writing skills. Accordingly, a significant part of your mark will depend on how you write your observations and conclusions, not just on whether your numerical results are correct.

CHECK LIST

Before you hand in the report, please go through this check list to make sure you have not missed something important. Note that this check list is just for you and your partner and you do not submit it in your report.

Form: layout, figures, etc.

- [ ] Did you use the correct parameter set? The TA will check if you used the set assigned to you, and if you used the wrong set, you will receive a zero mark for the report.
- [ ] On your Grading Sheet, do you have the signatures of your Lab Instructor confirming that both of you completed the Pre-Lab? If you do not have that signature, your individual lab report mark will be discounted by 20 points.
- [ ] Did both partners sign the Cover page? The report **will not be marked** without these signatures.
- [ ] Are the content pages of your lab report numbered? If not, you will lose points.
- [ ] Did you count content pages? If you have more than 16 content pages you will lose points.
- [ ] Did you write your data set number in the appropriate space on the Grading Sheet? If it is missing, you will lose points.
- [ ] Did you write and include a one-page Executive Summary?
- [ ] General impression: is your report easy to read and visually pleasing? Are the results easily located? Do you have separate headings for each of the parts? Consider using Tables to present your results. Break up large blocks of text into separate paragraphs, make sure that there is enough "white space" in your report to make it easy to read. If you are not sure, ask a friend to have a look at your report and get his/her feedback.
- [ ] Are all your numerical results and equations properly type-set? Any hand-written equations, as well as for equations copied from MATLAB will cause deductions.
- [ ] Are all the Figures you included in the report essential and contributing to your discussions? You will lose points for any Figures that are not discussed in the report.
- [ ] Do your Figures have proper time scales? Are the traces smooth? If not, you will lose points.
- [ ] Are all traces in your Figures easily identifiable? Do all your Figures have legends to identify each trace? If not, you will lose points.

- [ ] Are all your Figures properly labeled? Are they included in the body of your report, not at the end? Also remember that any reference to a Figure must be made before the Figure appears in the text, not after.

- [ ] Did you check your report for spelling, punctuation, grammar errors and logical flow of your narrative? There will be deductions for poor basic writing skills.

Content: calculations, discussions, observations, etc.

- [ ] Did you include the value of the Proportional Gain that resulted in the "Quarter-Decay" response, and the steady state and transient specifications for your "benchmarked" system response?

- [ ] Did you discuss P, PI and PD Control in terms of how they affect the steady state and the transient responses?

- [ ] For the steady state analysis, did you calculate, and show, the step as well as the ramp errors?

- [ ] Did you present the PID compensated system response specifications?

- [ ] Did you present the details of your PID design?

Academic Integrity

- [ ] Are all figures, text and equations included in this report created by you and your partner and 100% original? Remember that your signature on the Cover Page signifies that it is, and that you are fully aware of the consequences should it be otherwise.

- [ ] Are any of the components of your report copied from the Course Notes, Lecture Slides or Lab Instructions? If so, this is your last chance to remove them, because you do not have permission to copy any of the course materials, whether or not you make a reference to the source.