Appendix:

PID Controller, its Tuning and Design

The Classical Three-Mode Controller: Proportional + Integral + Derivative (PID)

Consider the following single feedback loop system, shown in Figure 1.

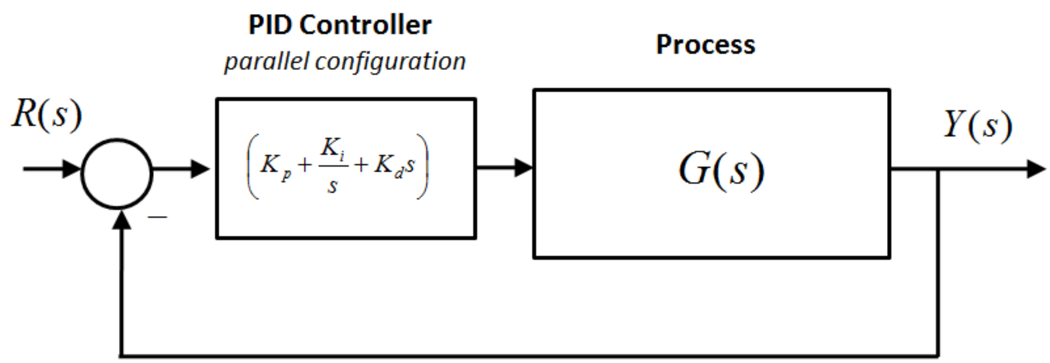

Figure 1: Basic Parallel Structure of the PID Controller

The control scheme in this system utilizes one of the most versatile controller configurations, a PID Controller. Its name stands for Proportional + Integral + Derivative. In its most basic form, the parallel configuration is realized as follows:

$$G_{PID}(s) = K_p + \frac{K_i}{s} + K_d s$$
$$u(t) = K_p \cdot e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt}$$

In the equation above, $e(t)$ is error, and $u(t)$ is the Controller output, actuating the Process. In the parallel PID structure, the three modes of controller operation are simply added - the controller output is a sum of the three control channels: P, I and D. The three parallel channels, Integral, Derivative and Proportional, can be implemented with independent gains K_p, K_i and K_d, as shown in the equation above.

A slightly different and more practical form of the parallel structure is widely used, with the Proportional Controller Gain K_p, implemented as a single multiplier, and with the Integral and Derivative channel parameters using the so-called time constants, rather than separate gains. They are called the Integral Time Constant τ_i and a Derivative Time Constant τ_d, respectively:

$$G_{PID}(s) = K_p\left(1 + \frac{1}{\tau_i s} + \tau_d s\right)$$

The parallel structure is very easily implemented digitally as a sum of three separate signal channels. It is also easy to visualize the Proportional, the PI Control and the PD Control, which you explored earlier, as a "subset" of this structure:

- Proportional Control: set the Integral Time Constant to infinity: $\tau_i = \infty$, and set the Derivative Time Constant to zero: $\tau_d = 0$.
- PI Control: set the Derivative Time Constant to zero: $\tau_d = 0$.
- PD Control: set the Integral Time Constant to infinity: $\tau_i = \infty$.

The parallel PID Controller is very intuitive to adjust empirically (i.e. by experimentation), where the direct impact of each "mode" of control (i.e. Proportional, Integral and Derivative) on the closed loop system time response is observed, and adjustments are made accordingly. It can be accomplished by a simple "trial-and-error" adjustments, as well as by a more structured, step-by-step adjustments referred to as Ziegler-Nichols Tuning.

NOTE: It is important to recognize the limitations of the empirical approach – it may not always allow you to meet specific performance requirements. However, it will result in a closed loop system response that is stable, and will allow you to exert a reasonable level of control over, and an overall improvement of, the shape of the response.

An alternative configuration of the PID Controller can be implemented, called the series PID Controller structure and shown in Figure 2.

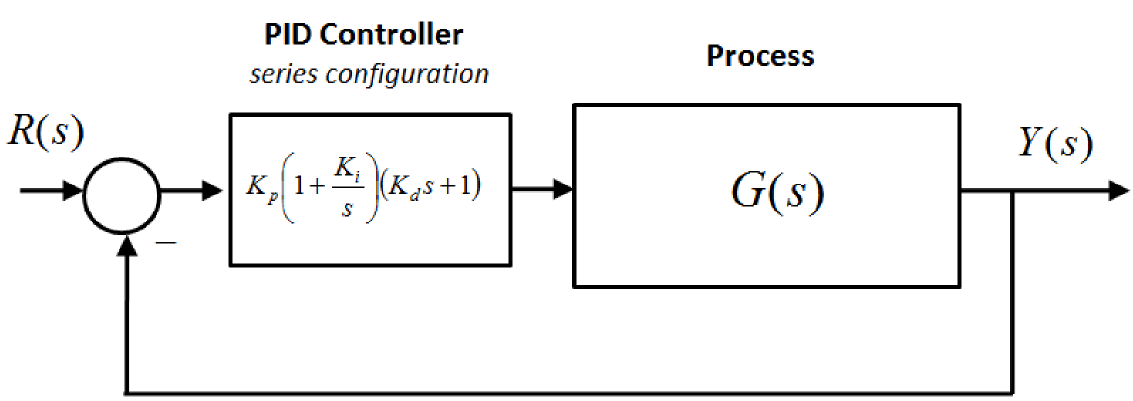

Figure 2: Series Structure of the PID Controller

The zeros of the controller transfer function are now independent and can be moved on the Root Locus one at a time. In the series PID structure, the PI and PD structures are cascaded rather than put in parallel:

$$G_{PID}(s) = K_p\left(1 + \frac{K_i}{s}\right)(1 + K_d s)$$

Again, it is traditional to use the Proportional Controller Gain K_p as a single multiplier, with the Integral Time Constant τ_i and a Derivative Time Constant τ_d used:

$$G_{PID(series)}(s) = K_p\left(1 + \frac{1}{\tau_i s}\right)(1 + \tau_d s)$$

You can check that the Proportional, the PI Control and the PD Control are “subsets” of the series structure as well:

- Proportional Control: set the Integral Time Constant to infinity: $\tau_i = \infty$, and set the Derivative Time Constant to zero: $\tau_d = 0$.
- PI Control: set the Derivative Time Constant to zero: $\tau_d = 0$.
- PD Control: set the Integral Time Constant to infinity: $\tau_i = \infty$.

PID Controller Tuning

The PID Controller can be tuned directly, based on its responses in time domain, and following certain sets of tuning rules, formulated from empirical (i.e. experimental) observations of the different controller modes and their impact on the system stability and transient response. The tuning rules can be listed as:

- Intuitive Tuning (also known as "The Seat of the Pants" tuning approach)
- Ziegler-Nichols Ultimate Gain Method
- Ziegler-Nichols Process Reaction Method

Both PID Controller structures (i.e. parallel and series) can be tuned empirically. However, the parallel PID Controller is more intuitive to tune this way, because the direct impact of each “mode” of control (i.e. Proportional, Integral and Derivative) can be observed directly on the closed loop system time response, and adjustments are made accordingly. Why is that? Because the three parallel channels for the control signal are added in s-domain as well as in time domain. Thus, they do not interfere with one another in time domain, and are simply superimposed, i.e. added.

Tuning of the parallel PID structure can be accomplished by a simple "trial-and-error" approach as well as by more structured, step-by-step adjustments referred to as the so-called Ziegler-Nichols tuning methods.

In the series PID structure, because the PI and PID modes of control are multiplied in s-domain, instead of added, the equivalent effect in time domain is the signal **convolution**, instead of a simple addition, as was the case with the parallel structure. What this means is the effect of parameter changes on the time domain response is more, shall we say, convoluted, pun definitely intended ☺ It is still there, but not in such direct way as for the parallel structure.

As a result, it is suggested that, if you decide to simply "tune" your PID Controller, you should use the parallel structure, which is the most intuitive to use for trial and error experimentation. As well, both Ziegler-Nichols methods define parameter values for the parallel structure, so you should work with the parallel structure of the PID if you choose either one of the Z-N methods.

NOTE: It is important to recognize the limitations of the empirical approach – it may not always allow you to meet specific performance requirements. However, it will result in a closed loop system response that is stable, and will allow you to exert a reasonable level of control over, and an overall improvement of, the shape of the response.

Method 1: Intuitive Tuning, a.k.a. "Seat of the Pants", or "Trial-and-Error", Approach

From your experimentation with the P, PI and PD Controllers in the first part of this Project, you should recall that a strong PI action reduces steady state errors, but that oscillations are the downside of using PI. On the other hand, a PD action increases the system damping, and can therefore be used to reduce the oscillatory transients.

1. Start the tuning process with both the Integral and Derivative modes disabled, (Proportional Control Mode only) and the Proportional Gain K_p at a low level so as not to destabilize the system. This is because typical industrial processes are stable in the open loop configuration and therefore also closed loop stable for low values of the controller gain.
2. Increase the Derivative Time Constant τ_d until oscillations are observed, then back off slightly.
3. Increase the proportional gain K_p until oscillations are observed, then back off slightly.
4. Set the Integral Time Constant τ_i to a large value (say, 100), then start reducing it, until oscillations are observed, then back off slightly.
5. Fine-tune the response by adjusting slightly K_p and τ_d.

The general rule is to use Proportional Control as a “muscle” doing work, use Integral Control to improve tracking, and use Derivative Control to damp out oscillations.

HINT: Think of the Proportional Mode as the "main meal", and of the Integrator Mode and the Derivative Mode as "salt" and "pepper".

The “Seat of the Pants” tuning method doesn't use any clues to make it easier to discover at what range of parameters the PID action is most effective.

The following two tuning methods, named after their inventors John G. Ziegler and Nathaniel B. Nichols, were formulated in 1940’s based on typical behaviour of industrial systems, and are still widely used in the industry to accomplish quick improvements in the quality of the closed loop system response. Both methods aim at finding the approximate range of the controller parameters by getting a sense of the open loop system dynamics. The controller will be most effective if its time constants are comparable with the time constants of the open loop process. This is apparent once you consider the concepts of Gain Margin and Phase Margin, to be studied in class.

Method 2: Ziegler-Nichols "Ultimate Gain" Method

In this method, the open loop system dynamics are estimated by establishing the system bandwidth. When the closed loop system oscillates at the critical gain, the frequency of its oscillations is a measure of the system bandwidth. Open loop system time constants are inversely proportional to the bandwidth. Again, this will become more apparent once the concepts of Gain Margin and Phase Margin are introduced.

Procedure:

1. Set up the closed loop system with a Proportional-only controller, with both the Integral and Derivative modes disabled.
2. Gradually increase the controller gain K_p until the step response exhibits persistent oscillations. If the actual process is used, care must be taken, as this value of gain K_p is the critical gain; and instability occurs when the gain is increased further.
3. Denote this value of the gain as K_u (where “u” stands for “ultimate”), and measure the period P_u of the oscillations, as shown in Figure 2.
4. Use the recommended Ziegler-Nichols settings, as shown in Table 1.

Since in practice bringing the process to the brink of instability is risky, a **Modified Ziegler-Nichols Method** is suggested, where the proportional gain is only increased until the so-called **“Quarter-Decay”** response is obtained. The quarter-decay type of response is characterized by the fact that the amplitude of each subsequent oscillation is approximately equal to 1/4 of the previous one. The system response shown in Figure 3 exhibits the quarter-decay response.

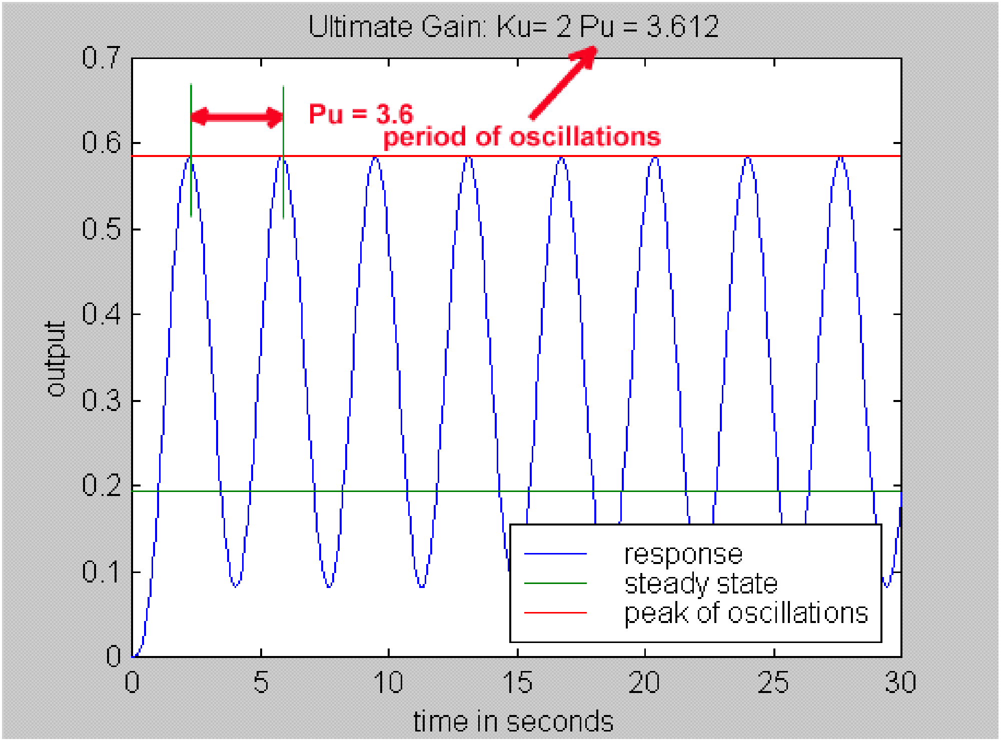

Figure 2: Z-N Ultimate Gain Method

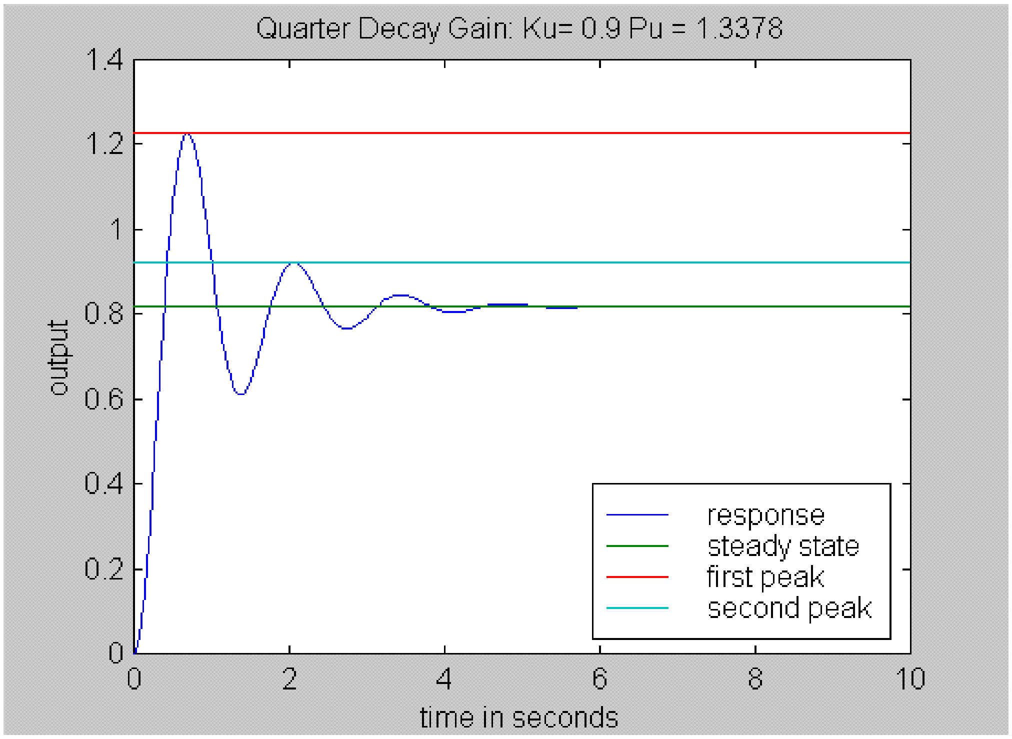

Figure 3: Z-N Modified 1/4 Decay Method

Table 1: Settings for Ultimate Gain Ziegler-Nichols PID Tuning Method

Type of Controller	Optimum Parameter Settings	
Proportional	$K_p = 0.5K_u$	
PI	$K_p = 0.45K_u$ $T_i = P_u / 1.2$	
PID	Original Method $K_p = 0.6K_u$ $T_i = P_u / 2$ $T_d = P_u / 8$	Modified method: $K_p = K_u$ $T_i = P_u / 1.5$ $T_d = P_u / 6$

Method 3: Ziegler-Nichols "Process Reaction" Method

In this method, the open-loop system dynamics are estimated to determine the time constants in the case of a second-order over-damped system using the slope of the open-loop system response. This is a good approach for a process that exhibits a significant time lag - transportation delay. However, it can also be used for the system with no discernible time delay. Note that in such case the line with the slope M does not have to start at t = 0 – we can create an artificial delay, based on how the slope line is drawn.

Procedure:

1. Start the tuning process with both the Integral and Derivative modes disabled, the Proportional Gain K_p set to 1 – this is equivalent to having no Controller in the open loop. Generate a step response from the open loop process to obtain its over-damped response.
2. Measure the slope M of the step response and the equivalent time lag L, as shown in Figure 4.
3. Use the recommended Zeigler-Nichols settings from Table 2.

Table 2: Settings for Process Reaction Ziegler-Nichols PID Tuning Method

Type of Controller	Optimum Parameter Settings
Proportional	$K_p = \frac{1}{ML}$
PI	$K_p = \frac{0.9}{ML}$ $T_i = \frac{L}{0.3}$
PID	$K_p = \frac{1.2}{ML}$ $T_i = 2L$ $T_d = 0.5L$

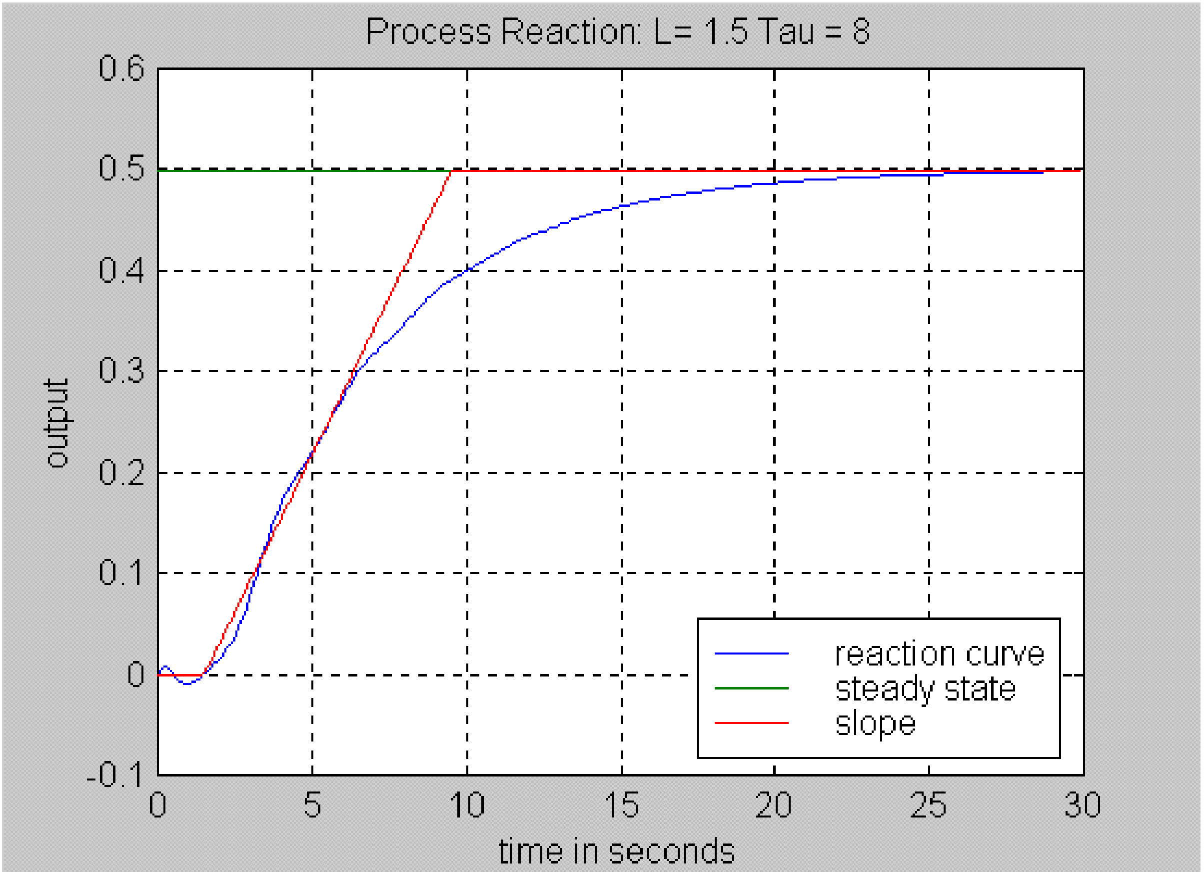

Figure 4: Z-N Process Reaction Method

PID Controller – Analytical Design Approach

There are several ways to approach the PID Controller in a more analytical way. The analytical approach is informed by assuming that the closed loop system can be modeled by a pair of the dominant closed loop poles (see Chapters 7 and 8 of the Course Notes).

In some cases of simple systems, the so-called "top-down", or "pole-placement", approach can be used to place the closed loop poles exactly at the desired locations, and to calculate values of the PID Controller parameters – see examples of such design in class and in course notes. In most cases however, the exact pole placement, as applied to systems of orders higher than two, requires multiple feedback loops and can only be accomplished by the so-called State Variable Feedback, which is beyond the scope of this course.

However, an effective improvement of the system closed loop response, often much more dramatic than the tuning approach, can be achieved by choosing the PID Controller parameters aided by one of the two approaches you will study in this course: the Root Locus method for analysis and design in the s-domain (see Chapter 10 of the Course Notes) and the Open Loop Bode Plots method for analysis and design in the frequency domain (see Chapters on Frequency

Domain Design in the Course Notes). For practical reasons of timing of this project, only the Root Locus-aided analytical approach is feasible for this Project. Please refer to the Course Notes and examples in class for more information on the Root Locus method.

The Root Locus method requires the presence of a single Proportional Gain of the whole open loop transfer function that can be varied, as required. Both the modified parallel structure meets this criterion.

However, the problem with the parallel PID structure is that the zeros of the controller transfer function interact, since they are solutions of a quadratic term - when one zero location in the s-plane is changed as a results of one of the time constants changed, the other zero moves as well:

$$G_{PID}(s) = K_p\left(1 + \frac{1}{\tau_i s} + \tau_d s\right) = K_p\left(\frac{\tau_d \tau_i s^2 + \tau_i s + 1}{\tau_i s}\right)$$

This interaction in s-domain is not convenient as it does not give us the ability to only make one adjustment to position controller zeros in the s-plane one at a time. It can be avoided if the series controller structure is used for the s-domain analysis:

$$G_{PID(series)}(s) = K_p\left(1 + \frac{1}{\tau_i s}\right)(1 + \tau_d s) = K_p\,\frac{(\tau_i s + 1)(\tau_d s + 1)}{\tau_i s}$$

Because the PD and PI terms are now multiplied, when an adjustment to the positioning of the PI zero is made, it doesn’t affect the positioning of the PD zero and vice versa. As a result, you may find it easier to perform s-plane analysis (Root Locus) and adjustments with the series PID structure than with the parallel PID structure.