Reply to Referee 2

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The authors have made some improvements to the manuscript based on the feedback provided, however some concerns were not fully addressed, as described below. I will use the same number scheme as in the prior report for consistency. I am content with the response for points that are not re-addressed below.

We thank the referee for carefully reading our manuscript and providing valuable feedback and suggestions. We have reviewed the referee's comments and made changes based on their suggestions. Below, we offer a point-to-point reply to each of the referee's comments and include the changes made in the manuscript. The referee's comments are in italics, and our replies are in plain black. Changes in the manuscript are highlighted in magenta.

• While I appreciate the inclusion of two additional force datasets, no new droplet shape profiles were included. For a paper that has a significant experimental component, I am surprised with the lack of direct visualization of the droplet dynamics. At minimum, I would recommend adding droplet shape profile comparisons for the two new data sets. Furthermore, these visualizations (including the one already included) should include a scale bar. Additionally, I would suggest including some experimental videos as supplementary materials.

We have modified figure 2 of the main text (see figure R1) to include direct visualization of the droplet dynamics. We have also added three supplementary videos: SM1-SM3.

• 2. I appreciate the clarification regarding the error bars. However, a proper characterization (and description of the procedure by which they are determined) of the estimated parametric errors is still incomplete. Rather than including additional discussion, the authors reference the supplementary materials of a prior work (Zhang et al. (2022)). In reviewing this work, I cannot identify error assessments on key parameters (such as impact velocity). An error is now included on the droplet size, but there is no discussion of how drop size is determined nor how this error determined. Additionally, the related point regarding horizontal error bars was not addressed. Given that there is no length limit, I would suggest the authors include all details in the present manuscript rather than referring to incomplete discussions in prior work. (Also a minor point: the new horizontal line associated with the force sensor resolution should be described in the caption.)

We appreciate the reviewer's insistence on a comprehensive error characterization. We have addressed this concern by adding a new appendix to the manuscript, titled "Note on the experimental error characterization". We have also modified the caption of figure 2 of the main text (see figure R1).

Throughout the manuscript, the error bars account for repeated trials and are visible if they are larger than the marker size. We refer the readers to the supplementary material of Zhang et al. (2022) and appendix A for further details of the experimental setup and error characterization of the dimensionless control parameters, respectively.

Appendix A. Note on the experimental error characterization

This appendix outlines the methodology for characterizing experimental errors in quantification of the drop's size and impact velocities which is crucial for accurate calculation of dimensionless control parameters, We and Oh. The drop diameter determination involved multiple steps. First, we measured the total mass (M_{100}) of 100 drops using an electric balance. From this mass, the drop density, and assuming spherical shape, we calculated the drop diameter (D_0) . We repeated this process five times, yielding $D_{0,1}$ through $D_{0,5}$. The average of these measurements provided the final drop diameter (D_0) and its standard error. For impact velocity determination, we extracted data from experimental high-speed imagery. By tracking the drop center's position in successive frames prior to substrate contact, and knowing the frame rate, we calculated the impact velocity. We repeated this process for five trials, obtaining $V_{0,1}$ through $V_{0,5}$. The average of these values gave the final impact velocity (V_0) and its standard error.

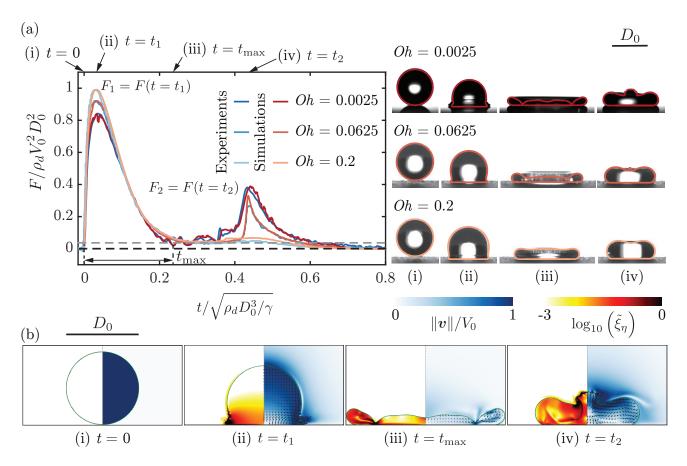


Figure R1: Comparison of the drop impact force F(t) obtained from experiments and simulations for the three typical cases with impact velocity $V_0 = 1.2 \,\mathrm{m/s}, 0.97 \,\mathrm{m/s}, 0.96 \,\mathrm{m/s}, \mathrm{diameter} \ D_0 = 2.05 \,\mathrm{mm}, 2.52 \,\mathrm{mm}, 2.54 \,\mathrm{mm},$ surface tension $\gamma = 72 \,\mathrm{mN/m}, 61 \,\mathrm{mN/m}, 61 \,\mathrm{mN/m}$ and viscosity $\eta_d = 1 \,\mathrm{mPa}\,\mathrm{s}, 25.3 \,\mathrm{mPa}\,\mathrm{s}, 80.2 \,\mathrm{mPa}\,\mathrm{s}$. These parameters of the parameters of rameter give Oh = 0.0025, 0.0625, 0.2 and We = 40. For the three cases, the two peak amplitudes, $F_1/\rho_d V_0^2 D_0^2 \approx$ $0.82, 0.92, 0.99 \text{ at } t_1 \approx 0.03 \sqrt{\rho_d D_0^3/\gamma} \text{ and } F_2/\rho_d V_0^2 D_0^2 \approx 0.37, 0.337, 0.1 \text{ at } t_2 \approx 0.42 \sqrt{\rho_d D_0^3/\gamma}, \text{ characterize the}$ inertial shock from impact and the Worthington jet before takeoff, respectively. The drop reaches the maximum spreading at t_{max} when it momentarily stops and retracts until $0.8\sqrt{\rho_d D_0^3/\gamma}$ when the drop takes off (F=0). The black and gray dashed lines in panel (a) mark F=0 and the resolution $F=0.5\,\mathrm{mN}$ of our piezoelectric force transducer, respectively. (b) Four instances are further elaborated through numerical simulations for (We = 40, Oh = 0.0025), namely (i) t = 0 ms (touch-down), (ii) t = 0.37 ms (t_1), (iii) t = 2.5 ms (t_{max}), and (iv) $t = 4.63 \,\mathrm{ms}$ (t_2). The insets of panel (a) exemplify these four instances for the three representative cases illustrated here. The experimental snapshots are overlaid with the drop boundaries from simulations. We stress the excellent agreement between experiments and simulations without any free parameters. The left part of each numerical snapshot shows the dimensionless local viscous dissipation function $\tilde{\xi}_{\eta} \equiv \xi_{\eta} D_0 / (\rho_d V_0^3) = 2Oh(\tilde{\mathcal{D}} : \tilde{\mathcal{D}}),$ where, \mathcal{D} is the symmetric part of the velocity gradient tensor, on a \log_{10} scale and the right part the velocity field magnitude normalized with the impact velocity. The black velocity vectors are plotted in the center of mass reference frame of the drop to clearly elucidate the internal flow. Also see supplementary videos SM1-SM3.

The standard errors for drop diameters did not exceed 0.13 mm. For drops with Ohnesorge numbers of 0.0025, 0.0625, and 0.2, the diameters were 2.05 ± 0.13 mm, 2.52 ± 0.11 mm, and 2.54 ± 0.09 mm, respectively. The standard errors for impact velocities did not exceed $0.02\,\text{m/s}$. For the same Oh values, the impact velocities were $1.2\pm0.002\,\text{m/s}$, $0.97\pm0.01\,\text{m/s}$, and $0.96\pm0.01\,\text{m/s}$, respectively. The combined errors in D_0 and V_0 resulted in approximately $\pm7\%$ error in Weber number We and $\pm3\%$ error in Ohnesorge number Oh. Consequently, the horizontal error bars, which relate to errors in the control parameters, are smaller than the symbol sizes in our figures.

3.

- In the experimental methods, the authors now describe that the Bond number is fixed at 1. Given the change in fluid properties between different solutions (with fixed droplet size), this cannot be the case. This needs to be made more precise. Furthermore, in the authors' prior work (Zhang et al. (2022)), it is mentioned that the Bo is fixed at 0.5 in simulation for similar experimental parameters. Why the change?
 - To discuss: The results are Bond invariant in the leading order. Therefore, we chose a representative value of Bond = 1: this if for a diameter $0.00256 \,\mathrm{mm}$, density $1000 \,\mathrm{kg/m^3}$, acceleration due to gravity $10 \,\mathrm{m/s^2}$, surface tension $0.06 \,\mathrm{N/m}$. The dimensionless number that matter is the Froude number $Fr = V^2/gD_0 = We/Bo$ which is larger than 1 throughout the paper. The difference as compared to Zhang et al. (2022) is in the length scale (2 mm in PRL and 2.56 mm in this work).
- By now providing the drop size used, the Oh and Bo now can be used to define the parameters. However, it is not clear that the non-dimensional values are correct. For instance, Figures 2 and 3 both mention experimental conditions where Oh = 0.0025 whereas apparently different droplet sizes were used (2.05 mm in Figure 2, and 2.54 mm in Figure 3). Was a different fluid used between these figures such that the Oh could be held fixed? To clarify all of these issues, and for the ease of the readers, I strongly suggest the authors add the dimensional parameters for all experiments to the captions (as they have now done for Figure 2), an appendix, or make experimental data available in a supplementary data set.
- Given the significant amount of data overlap in the various figures (e.g. Figure 3(b)), I am more convinced now that the parameters and raw data need to be provided in a supplementary dataset for reproducibility and to facilitate further comparisons.
- The authors mention that their mixtures maintain "a fairly constant surface tension and density, around 61 mN/m and 1000 kg/m³, respectively." Since the values are now clearly specified in the table now, what is the meaning of these particular characteristic values which do in fact vary by around 20%?

We can rephrase to address this. I will take care.

- 5. The new title is more appropriate, but it should be mentioned in the title that there are specific restrictions/assumptions on the substrate (i.e. non-wetting), as the results are likely to depend on the surface wettability.
 - The first impact peak is independent of wetability. The second peak, of course, depends on wetability as it only shows up on non-wetting surfaces. Any suggestions?
- 7. If the experiment is repeatable, for a given drop height, the oscillation phase at which the droplet arrives at the surface should not vary, and thus is unlikely to be captured by the error bars as claimed. Some additional quantification of the non-sphericity of the droplets should be included.
 - What to do? There are other more detailed works on impact of non-sphericity. Mostly, they show that the things do not depend on whether the drop is prolate or oblate... At least not in the leading order. Also see: Sanjay et al. (2023).
- 11. While moving to a different fluid is one viable option, using smaller radii could also allow for smaller Oh

Yes, using smaller radii could also allow for larger Oh. We have added this in the revised manuscript. $\S 2.1$:

We note that using liquids such as silicone oil can provide a broader range of viscosity variation when paired with a superamphiphobic substrate (Deng et al., 2012). Additionally, employing drops of smaller radii facilitates the exploration of higher Ohnesorge numbers (Oh).

• 13. While the authors have done a good job clarifying their theoretical arguments, I am not sure that citation to unpublished (and currently inaccessible) work by the same group is appropriate or really necessary.

Easily fixed: we can submit the other work to arxiv before we resubmit this work. :) Or, remove the citation.

Regardless of the remaining critical feedback, I still believe this work is valuable and will be of interest to the community working on impacting droplets. However, I still have reservations on the current version given the persistent lack of details on the experiments.

We appreciate the reviewer's belief in the value and interest of our work to the impacting droplets community. We acknowledge the reservations regarding the level of experimental detail. We have that the details added in the revised manuscript will address the reservation of the reviewer.

References

- Deng X., Mammen L., Butt H.-J., and Vollmer D. 2012. Candle soot as a template for a transparent robust superamphiphobic coating. *Science* 335.6064, pp. 67–70.
- Sanjay V., Lakshman S., Chantelot P., Snoeijer J. H., and Lohse D. 2023. Drop impact on viscous liquid films. J. Fluid Mech. 958, A25.
- Zhang B., Sanjay V., Shi S., Zhao Y., Lv C., and Lohse D. 2022. Impact forces of water drops falling on superhydrophobic surfaces. *Phys. Rev. Lett.* 129, p. 104501.