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Artificial Intelligence

BTech ICT/CSE Sem VI, Winter Semester 2020

Lab2: Implementing basic Logic functions using Single Neuron Model

Preamble

An Artificial Neural Network (ANN) is a computational paradigm that is inspired by the way biological nervous systems, such as the brain, process information. ANN is composed of a large number of highly interconnected simple processing elements (neurons) working together to solve specific problem. An ANN is configured for a specific application, such as pattern recognition or data classification or regression, through a learning process. Learning in biological systems involves adjustments to the weighted connections that exist between the neurons.

Biological Neuron

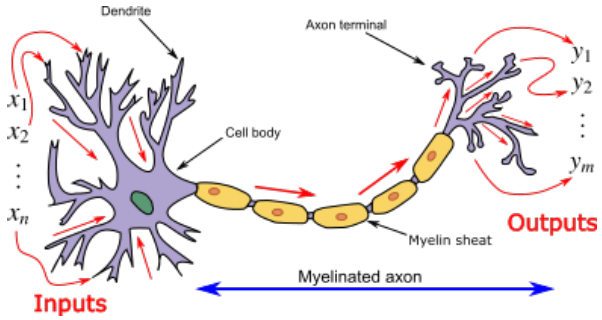
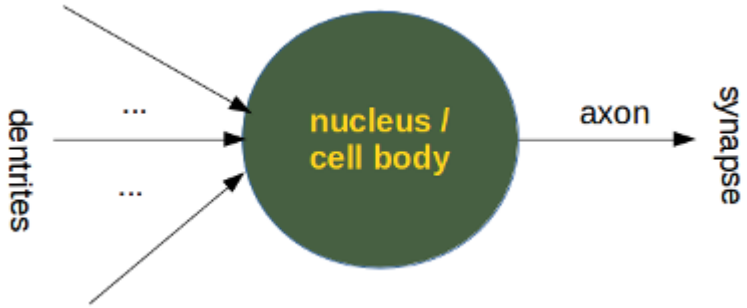
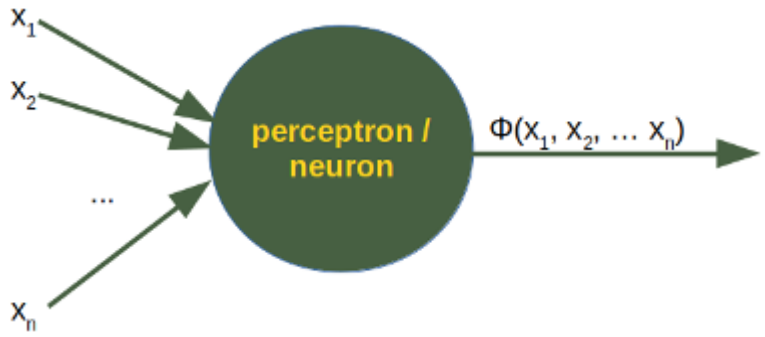


Figure 1: Biological Neuron of Human Brain (Image Source: Biological Neuron Wikipedia)

Even though the above image is already an abstraction for a biologist, we can further abstract it:



A perceptron of artificial neural networks is simulating a biological neuron.



A Simple Artificial Neuron Model

An artificial neuron is a mathematical model with a set of inputs $X = \{x_1, x_2, \ldots x_n\}$, a set of weights and an activation function. The neuron translates these inputs into a single output as shown in the following figure.

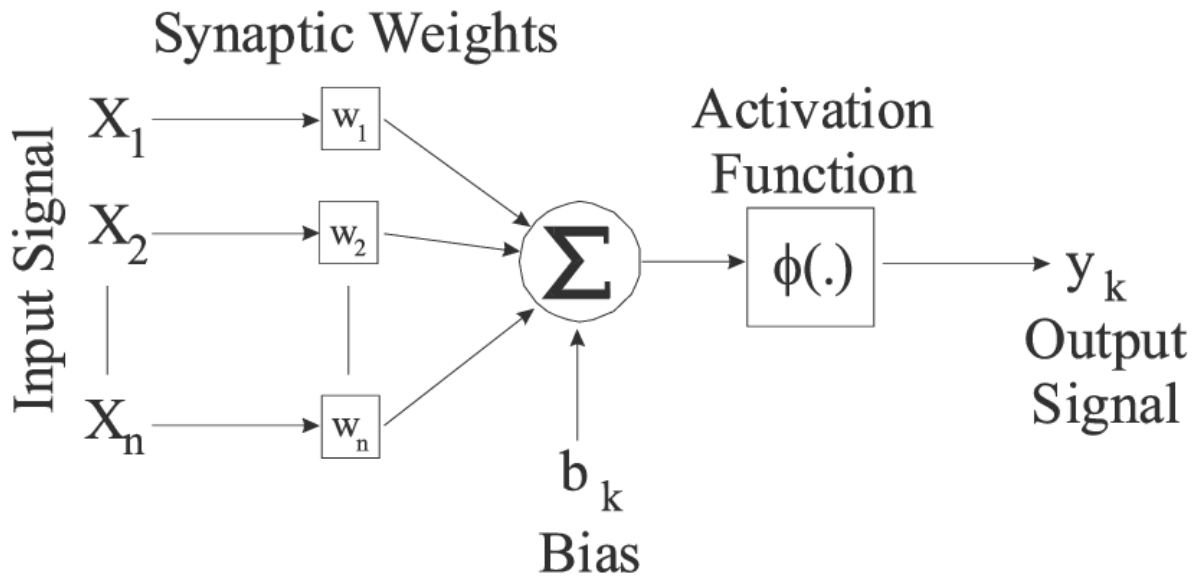


Figure 2: Simple Neuron Model (Image Source: Simon Haykin)

The input and weight forms linear term f as follows.

$$f = \sum_{i=1}^{n} w_i x_i$$

Here, $w_1, w_2, \ldots w_n$ represent the strength of inputs which may amplify or de-amplify the input connection. The neuron can also have a bias term making input to activation function as $f+b$.

Considering a thresholding activation function;

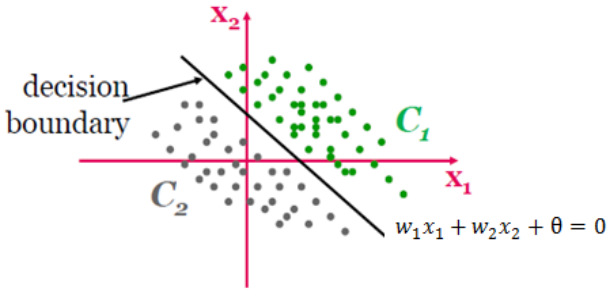
The output =1, if $f+b >= \theta$ (threshold), else it is 0, where θ is the threshold value.

For simplicity, let us assume there are two inputs x_1 and x_2. Let the weights for x_1 and x_2 be w_1 and w_2 respectively. Then, the output of the neuron can be written as follows.

$$Output = \{w_1 x_1 + w_2 x_2 > \theta\}$$

$$\{w_1x_1 + w_2x_2 \leq \theta\}$$

$w_1x_1 + w_2x_2 > \theta$ and $w_1x_1 + w_2x_2 \leq \theta$ represents two regions on the x_1x_2 plane separated by the line $w_1x_1 + w_2x_2 + \theta = 0$. If we consider the input (x_1, x_2) as a point on a plane, then the single neuron model actually tells us which region on the plane to which this point belongs. Such regions, since they are separated by a single line, are called linearly separable regions which are then used for binary classification problems. The line which separates these regions is called as decision boundary as shown in the following figure.



The neuron model has to be trained to learn appropriate weight vector in order to separate the given input instances. Once the model is trained, it can then be used for specific classification problem.

Exercise I

Let w_1 and w_2 be the weights of a perceptron (i.e. simple neural network) with two inputs x_1 and x_2. Let AND(x_1, x_2) be the "logical and" function, let OR(x_1, x_2) be the "logical or" function and let NOT(x) be the logical NOT function.

x_1	x_2	Y
0	0	0
0	1	0
1	0	0
1	1	1

AND

x_1	x_2	Y
0	0	0
0	1	1
1	0	1
1	1	1

OR

X	Y
0	1
1	0

NOT

1. Plot constellation diagram for these logic gates. Are the data points linearly separable? If yes, what is its significance with respect to perceptron?

Exercise II

1. Design three single-neuron models which correspond to the logical AND, OR and NOT functions respectively. Assume threshold activation function at the output of the Neuron.

Write the Scilab script to compute the weights automatically. You may assume suitable value of bias and threshold.

Exercise III

1. Can we design a model that implements XOR(x_1, x_2) logic function with single neuron? Justify your answer with necessary decision graph.

Exercise IV

Visit https://ml-playground.com to visualize binary classification using different models (Perceptron, K nearest neighbours, Support vector machine(SVM), Artificial neuron network(ANN), and Decision Tree.