

# Oligarchic Growth of Protoplanets

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We investigate the growth and the orbital evolution of protoplanets embedded in a swarm of planetesimals using three-dimensional  $N$ -body simulations. We find that among protoplanets, larger ones grow more slowly than smaller ones, while the growth of protoplanets is still faster than that of planetesimals. As a result, in the stage after rapid runaway growth, protoplanets with the same order mass grow oligarchically, while most planetesimals remain small. While the protoplanets grow, orbital repulsion keeps their orbital separations wider than about 5 Hill radius of the protoplanets. The typical orbital separation is about 10 Hill radius, which only weakly depends on the mass and the semimajor axis of protoplanets. We explain how this self-organized protoplanet–planetesimal system forms. © 1998 Academic Press

**Key Words:** planetary formation; planetary accretion; planetesimals;  $N$ -body simulation.

## 1. INTRODUCTION

Terrestrial planets and solid cores of Jovian planets are formed through the accretion of planetesimals whose initial masses are  $10^{18-21}$  g (10–100 km) in the standard scenario of planetary formation. The mode of planetary accretion is an important problem since the formation time and the structure of a planetary system may depend on it. A pioneering work by Greenberg *et al.* (1978) found that in the early stage of planetary accretion, larger planetesimals grow more rapidly than smaller ones, resulting in runaway growth of the largest planetesimal. Wetherill and Stewart (1989) also showed runaway growth of planetesimals, explicitly noting the role of the dynamical friction term in determining the velocity evolution of planetesimals. Their models were the gas model where planetesimals are treated as gas molecules in a free space. Dynamical friction in a planetesimal system is also studied through three-body

and  $N$ -body calculations (e.g., Ida 1990, Ida and Makino 1992b). It is shown that dynamical friction is effective in a Keplerian potential as well as in a free space. Kokubo and Ida (1996), hereafter referred to as Paper I, performed 3-D  $N$ -body simulation of planetary accretion, including both gravitational interaction and accretion between planetesimals. They directly showed that runaway growth is an inevitable outcome of a 3-D planetesimal system where gravitational focusing and dynamical friction are effective.

Runaway growth is a kind of instability phenomenon. It is not likely that runaway growth continues till the end of planetary accretion, since runaway growth itself breaks the uniformity of the velocity and the spatial distributions of planetesimals on which runaway growth is based. It is necessary to consider the back reaction of a planetesimal system against runaway growth to follow realistic planetary accretion. Ida and Makino (1993) pointed out that the growth of protoplanets (runaway planetesimals) slows down when protoplanets grow large enough to increase the velocity dispersion of neighbor planetesimals. Lissauer (1987) also suggested that the depletion of planetesimals through accretion by protoplanets would limit the growth of protoplanets. We call this stage after the initial rapid runaway growth as post-runaway stage. It is an open question how protoplanets grow in the post-runaway stage and, as a result, how the surrounding planetesimal system evolves.

Some pioneering works dealt with the late stage of planetary accretion. Lecar and Aarseth (1986) performed 2-D  $N$ -body simulations in which protoplanets started with equal masses and relatively small orbital separations, and there were no smaller bodies present. Greenberg (1989) presented two hypotheses: the isolated embryos (protoplanets) hypothesis in which the embryos are isolated in the depleted feeding zones and the runaway growth hypothesis in which protoplanets continue to feed on small planetesi-

imals till they become full-size planets. These hypotheses remain open.

Kokubo and Ida (1995) investigated the orbital evolution of protoplanets embedded in a swarm of planetesimals by  $N$ -body calculation. They focused on the dynamical evolution of the system by gravitational scattering; in other words, they did not include the accretion of planetesimals. They found the orbital repulsion of protoplanets that is a coupling effect of scattering between protoplanets and dynamical friction by planetesimals. Protoplanets repel each other and expand their orbital separation if the orbital separation is smaller than about  $5r_H$ , where  $r_H$  is the Hill radius of the two protoplanets given by

$$r_H = \left( \frac{M_1 + M_2}{3M_\odot} \right)^{1/3} a, \quad (1)$$

where  $M_1$  and  $M_2$  are the masses of two protoplanets,  $M_\odot$  is the solar mass, and  $a$  is the semimajor axis of the system. As the next step, we should consider the accretion of planetesimals simultaneously with the orbital evolution.

In the present paper, we explore the post-runaway stage of planetary accretion by 3-D  $N$ -body simulation.  $N$ -body simulations offer a powerful and reliable method for exploring the post-runaway stage where it is necessary to calculate the evolution of spatial and velocity distributions simultaneously to follow the realistic evolution of mass distribution. We focus on the accretion under the velocity dispersion of planetesimals smaller than their surface escape velocity, which is realized by gas drag in the solar nebula.

We summarize the method of calculation in Section 2. In Section 3, we present the results. We first investigate the planetesimal system with two seed protoplanets. We focus on the orbital evolution of the two growing protoplanets interacting with each other. We find that the protoplanets grow keeping their orbital separations wider than the critical value of about  $5r_H$  as suggested by Kokubo and Ida (1995). The typical value of orbital separation is about  $10r_H$ . The orbital separation is scaled by their Hill radius, which means that the orbital separation increases with the mass and the semimajor axis of protoplanets. Among the two protoplanets, the larger one grows more slowly than the smaller one, while the growth of the protoplanets is still faster than that of the planetesimals. As a result, the two protoplanets grow oligarchically. Their mass ratio approaches unity; in other words, they grow to the same size. We explain how this self-organized protoplanet-planetesimal system forms. In order to support the oligarchic growth theory, we show the result of a simulation that is larger in radial extent than the two-protoplanet case and that starts with equal-mass planetesimals without seed protoplanets. We show that several protoplanets with the

same order masses are formed with orbital separations of  $5\text{--}10r_H$ . Section 4 is devoted to conclusion and discussion. We consider the application of the results to the formation of jovian planets.

## 2. METHOD OF CALCULATION

### 2.1. Force Calculation

The orbits of planetesimals are numerically integrated by using the fourth-order Hermite scheme (Makino and Aarseth 1992) with the individual and hierarchical timestep (Makino 1991). The most expensive parts of the Hermite scheme is the calculation of the force and its time-derivative whose cost increases in proportion to the square of the number of bodies. We calculate the force and its first time-derivative directly by summing up interactions of all pairs on the special-purpose computer for  $N$ -body simulation, HARP-2 (Makino *et al.* 1993), whose peak speed is about 1 Tflops.

We do not take account of gas drag due to the gas component of the solar nebula in order to see the basic dynamical process clearly. The neglect of gas drag would not affect results since gas drag is very weak and calculation time is relatively short.

### 2.2. Accretion

We assume that two planetesimals always accrete when they contact. The lack of collisional fragmentation or rebound seems to make no significant change in the growth mode of protoplanets (Wetherill and Stewart 1993). This would be because the relative velocity between protoplanets and planetesimals is usually smaller than the surface escape velocity of the protoplanets, so that most collisions with the protoplanets lead to accretion (Ohtsuki 1993). Collisions between small bodies may result in disruptive fragmentation and produce a large amount of fragments in the late runaway stage. The fragments suffering strong gas drag may play an important role in growth of the largest bodies (Wetherill and Stewart 1993, Tanaka and Ida 1997). The inclusion of this effect is a future subject.

We save computational time by accelerating the accretion process by using larger physical radii of bodies, which changes only the time scale but not the qualitative feature of accretion (for details, see Paper I). The reduction of gravitational scattering cross-section by cutting of close encounters due to  $f$ -fold radii is estimated by

$$\frac{S_{\text{collision}}}{S_{\text{scattering}}} \simeq 9 \times 10^{-2} \left( \frac{f}{4} \right) \left( \frac{\tilde{v}_{\text{rel}}}{4} \right)^2 \left( \frac{\ln \Lambda}{3} \right)^{-1}, \quad (2)$$

where  $\tilde{v}_{\text{rel}}$  is the relative velocity of bodies measured by the Hill velocity  $r_H\Omega$ , where  $\Omega$  is the local Kepler angular

velocity, and  $\Lambda = p_{\max}/p_{\min}$ , where  $p_{\max}$  and  $p_{\min}$  are the maximum and the minimum impact parameters, respectively, and  $\ln \Lambda$  has the typical value of 3–5 in planetesimal systems (Ida 1990).

In most cases, we adopt four-fold enlargement ( $f = 4$ ). In our calculation,  $\tilde{v}_{\text{rel}} \approx 4$  for protoplanets. With these parameters the reduction of gravitational scattering is less than 10%. The ratio of collisional damping due to increased  $f$  to that due to dynamical friction is also estimated by Eq. (2), which shows that energy damping is dominated by not collisional damping but dynamical friction in the same way as the realistic system with  $f = 1$ . Thus, the increased collisional damping with  $f = 4$  hardly changes the dynamics of largest bodies. The random velocity of planetesimals depends on the strength of gravitational stirring and collisional damping (Safronov 1969). The reduction of gravitational scattering and the increased collisional damping may make random velocities of planetesimals smaller than the realistic value. However, we are interested in a relatively small random velocity case in the solar nebula where gas drag reduces the random velocities. The damping due to increased  $f$  would be comparable to damping by gas drag. It would not significantly affect the spatial distribution and dynamical properties of growing protoplanets that we are interested in. Note that the growth time scale presented here is at most  $f^{-2}$  times shorter than the realistic time scale.

### 2.3. Initial Conditions

We perform two sets of calculations:

(i) 4000 equal-mass bodies ( $m = 1.5 \times 10^{23}$  g) + 2 seed protoplanets ( $M_1 = M_2 = 40m$ ),

$$\Delta a \approx 0.042 \text{ AU}, f = 4,$$

(ii) 4000 equal-mass bodies ( $m = 3.0 \times 10^{23}$  g),

$$\Delta a \approx 0.085 \text{ AU}, f = 6,$$

where  $\Delta a$  is the radial extent of the simulation region. In set (i), the two protoplanets are introduced to see the dynamical evolution of growing protoplanets interacting with each other embedded in a swarm of planetesimals. Initially, the two protoplanets are on circular orbits and their orbital separation is  $b = a_2 - a_1 = 8r_{\text{H}} = 0.01 \text{ AU}$ , where  $a_1$  and  $a_2$  are the semimajor axes of the two seed protoplanets.

In both cases, planetesimals are initially distributed in a ring-like region around  $a = 1 \text{ AU}$  with the width of  $\Delta a$ . The boundaries of the ring is open. This boundary condition allows bodies to diffuse. However, the diffusion proceeds very slowly compared with accretion and hardly changes the growth mode of bodies. The width of the ring expands by about 50% in set (i) in 10,000 years. We set

the surface mass density of the planetesimal ring as  $\Sigma = 10 \text{ g cm}^{-2}$ , which is similar to the minimum-mass solar nebula model (Hayashi 1981). The initial eccentricities  $e_m$  and inclinations  $i_m$  of planetesimals are given by Rayleigh distribution (Ida and Makino 1992a)

$$f(e_m, i_m) de_m di_m \propto e_m \exp\left(-\frac{e_m^2}{\langle e_m^2 \rangle}\right) i_m \exp\left(-\frac{i_m^2}{\langle i_m^2 \rangle}\right) de_m di_m, \quad (3)$$

with dispersions  $\langle e_m^2 \rangle^{1/2} = 2\langle i_m^2 \rangle^{1/2} = 4h_m$ , where  $h_m$  is the reduced Hill radius of the planetesimals defined by

$$h_m = \left(\frac{2m}{3M_{\odot}}\right)^{1/3}. \quad (4)$$

The initial dispersions of  $e_m$  and  $i_m$  are chosen so that  $e_m$  and  $i_m$  are in a dispersion-dominated region (Ida 1990). Note that eccentricity and inclination are related to the velocity dispersion  $v_m$  as

$$v_m \approx (e_m^2 + i_m^2)^{1/2} v_c, \quad (5)$$

where  $v_c$  is Keplerian circular velocity.

From the results of set (i) calculations, it is expected that a group of protoplanets with orbital separation  $\sim 10r_{\text{H}}$  would be formed if the calculation range is wider. In set (ii), we confirm it through more general calculation starting from equal-mass bodies without seed protoplanets in the wider region. We will show how protoplanets are formed from the equal-mass bodies and how they interact with one another to keep their orbital separation  $\sim 10 r_{\text{H}}$ . Since this is relatively long term calculation, we adopt  $f = 6$ . The results obtained here can be applied to other  $a$  and  $\Sigma$  with adequate scaling of time (Ida and Makino 1992a).

## 3. RESULTS

We show the growth and the orbital evolution of protoplanets that are gravitationally interacting with each other in a swarm of planetesimals. First, we perform the calculation of two-seed protoplanets embedded in a swarm of planetesimals. The basic process of the evolution of two interacting large bodies is investigated. Second, we perform the calculation with the wider region without seed protoplanets, where several protoplanets are formed and interact with one another.

In Fig. 4 of Paper I, we showed that two protoplanets (runaway bodies) always appear in the region with  $\Delta a = 0.04 \text{ AU}$  centered about  $a = 1 \text{ AU}$  at 20,000 years from the initial 3000 equal-mass ( $10^{23}$  g) planetesimals in spite of the difference in the initial distributions of the

semimajor axis, the eccentricity, and the inclination of orbits. The detailed way in which two protoplanets appear differs in each simulation, since it is a stochastic process. However, the results are qualitatively the same in all simulations: the masses of two protoplanets are of the same order of  $10^{25}$  g and their orbital separation is about  $10r_H$ . We investigate in detail how this property of two protoplanets is realized.

In Figs. 1–3, we present the results of set (i) simulation. We performed 10 simulations of set (i) with different random numbers for the initial distributions of bodies. The results are qualitatively the same in all the simulations. We also performed the simulations with  $f = 3$  and 5 and found that the results are qualitatively the same as that of  $f = 4$ . The relative energy error taking into account the dissipation due to accretion is  $10^{-4}$  at  $t = 10,000$  years. Figure 1 shows an example of the snapshots of the system on the  $a - e$  plane. The two-seed protoplanets grow rapidly, compared with planetesimals. Due to dynamical friction from small bodies, the eccentricities of the two seed protoplanets are kept small. The RMS eccentricity and inclination of planetesimals at  $t = 10,000$  years are  $\langle e_m^2 \rangle^{1/2} = 0.01$  and  $\langle i_m^2 \rangle^{1/2} = 0.0045$ , respectively. It is shown in Fig. 2 that the mass ratios of the two-seed protoplanets to the mean mass of the system increases with time. On the other hand, there is no secular increase in the mass ratio between the two-seed protoplanets. They grow keeping the same order masses. The masses of the protoplanets reached about eight times of their initial masses while the mean mass of the system increase by a factor of 1.6. This property of growth of protoplanets is explained by the self-limiting nature of runaway growth (Lissauer 1987, Ida and Makino 1993). It is the relative growth rate  $(1/M) dM/dt$ , where  $M$  is the mass of a protoplanet, that determines the growth mode (Paper I). When the relative growth rate increases with mass, runaway growth occurs, and when it decreases with mass, orderly growth occurs where all the bodies tend to grow equally. In a planetesimal system where gravitational focusing and dynamical friction are effective, the relative growth rate is given

$$\frac{1}{M} \frac{dM}{dt} \propto \Sigma M^{1/3} e_m^{-2}, \quad (6)$$

where  $\Sigma$  is the surface density of the solid material of the solar nebula and  $e_m$  is the RMS eccentricity of the planetesimals (for details, see Paper I). In the early stage of planetary accretion,  $\Sigma$  and  $e_m$  are independent of  $M$  (Ida and Makino 1993), so that  $(1/M) dM/dt \propto M^{1/3}$ , which leads to runaway growth. When the mass of a protoplanet  $M$  exceeds the critical value  $\sim 50m$  in our simulation, the protoplanet heats up the velocity dispersion of neighbor planetesimals and the velocity dispersion of its neighborhood comes to depend on its mass as  $v_m \propto M^{1/3}$  (Ida and

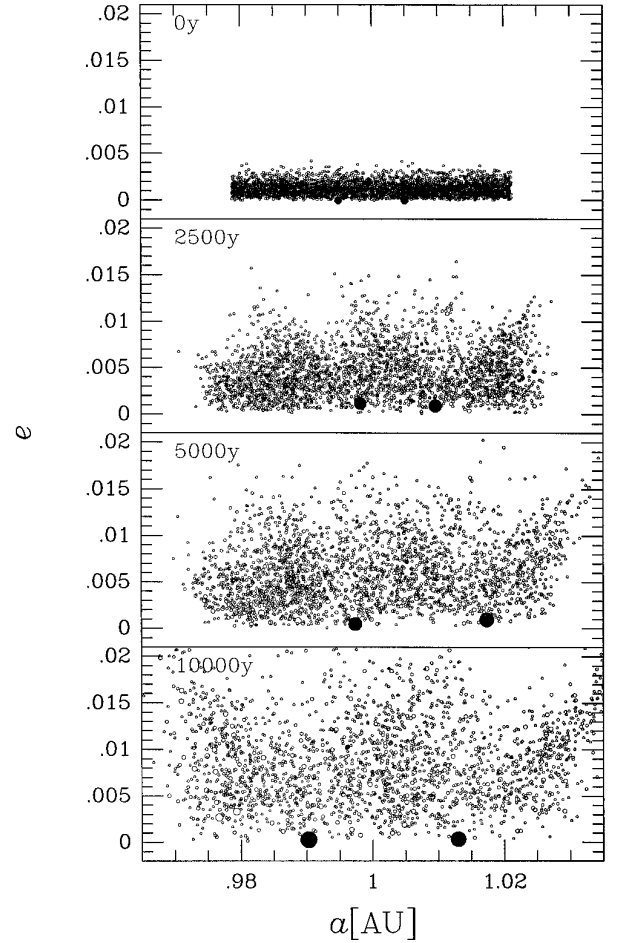


FIG. 1. Time evolution of a planetesimal system on the  $a - e$  plane. The circles represent planetesimals and their radii are proportional to the radii of planetesimals. The filled circles represent protoplanets. The system initially consists of 4000 planetesimals ( $m = 1.5 \times 10^{23}$  g) and two-seed protoplanets ( $M_1 = M_2 = 6 \times 10^{24}$  g). We used the four times larger radii of planetesimals than the realistic ones. The numbers of planetesimals are 4000 ( $t = 2500$  years), 2880 ( $t = 5000$  years), 2537 ( $t = 7500$  years), and 2176 ( $t = 10,000$  years).

Makino 1993). In this case, the relative growth rate becomes

$$\frac{1}{M} \frac{dM}{dt} \propto \Sigma M^{-1/3} \propto M^{-\alpha}, \quad (7)$$

where we used  $e_m \propto v_m$ . The power index  $\alpha$  could be larger than  $1/3$ , since the local  $\Sigma$  decreases through accretion of planetesimals by the protoplanet as  $M$  increases (Lissauer 1987). This means that in the post-runaway stage, the growth mode among protoplanets is orderly; in other words, the mass ratios between protoplanets tend toward unity rather than increase. Thus, the two protoplanets grow keeping similar masses. Note that still in the post-runaway

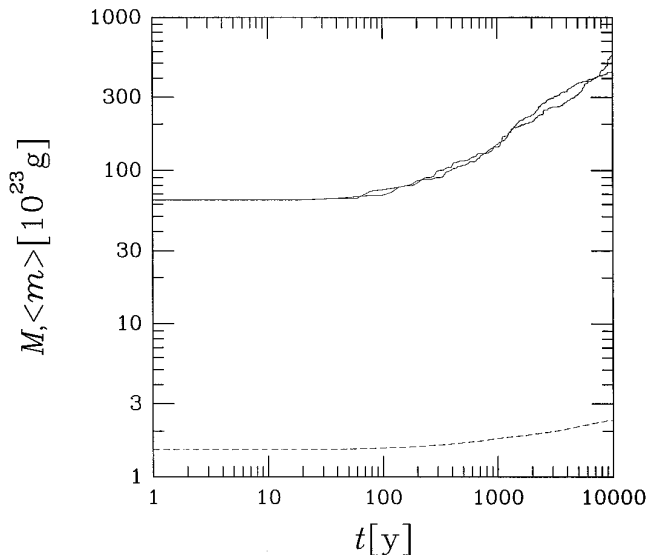


FIG. 2. Time evolution of the masses of the two protoplanets (solid curves) and the mean mass of the planetesimals (dashed curve).

stage, the mass ratio of a protoplanet to its neighbor planetesimals increases since for its neighbor planetesimals with mass  $m$ ,  $(1/m) dm/dt \propto \Sigma m^{1/3} e_m^{-2}$ , which shows that the relative growth rate of the protoplanet is  $(M/m)^{1/3}$  larger than that of the planetesimals even when  $e_m \propto M^{1/3}$ . The mass distribution of planetesimals relaxes to the power-law distribution  $n \propto m^\beta$ , where  $n$  is the number of planetesimals in a linear mass bin and  $\beta \simeq -2.5$  (Paper I).

Figure 3 shows the time evolution of the Hill radius and the orbital separation of the protoplanets averaged over 10 set (i) simulations. The solid curve indicates the mean value and the two dashed curves surrounding the mean the  $1\sigma$ -variation. The orbital separation  $b$  (middle panel) expands as the Hill radius  $r_H$  (upper panel) increases, in other words, as the protoplanets grow, though there are fluctuations due to protoplanet–planetesimal scattering. The orbital separation almost doubles in 10,000 years. On the other hand, there is no appreciable secular change in the orbital separation normalized by the Hill radius,  $\tilde{b} = b/r_H$  (lower panel). It is always larger than about 5 and has the typical value of 8–10. There seems a potential barrier at  $\tilde{b} = 5$ . This expansion of the orbital separation is explained by the orbital repulsion (Kokubo and Ida 1995). The orbital repulsion is a coupling effect of scattering between large bodies and dynamical friction from small bodies. Scattering between two protoplanets on nearly circular orbits increases their eccentricities and orbital separation. After scattering, dynamical friction reduces the eccentricities while the orbital separation remains almost the same. Thus, the orbital separation of two protoplanets increases keeping nearly circular orbits. If the separation is less than  $5r_H$ , relatively strong scattering often occurs

and the separation expands rapidly. As protoplanets grow, their orbital separation normalized by the Hill radius becomes small, since  $r_H \propto M^{1/3}$ . This implies that they repeat the orbital repulsion while growing. Consequently, the orbital separation of protoplanets keeps always larger than about  $5r_H$ .

We can make rough estimation of the typical orbital separation of protoplanets by semianalytical argument. The scaled orbital separation,  $\tilde{b}$ , is increased by orbital repulsion, while it is decreased by growth of the protoplanets since  $r_H \propto M^{1/3}$ . The increase rate is small for large  $\tilde{b}$ , which means that there is an equilibrium value of  $\tilde{b}$ . We regard this value as the typical separation distance.

For simplicity, we assume that the eccentricities of protoplanets are reduced to zero in synodic time; in other words, two protoplanets are always on circular orbits when they encounter. In this case, scattering between protoplanets always expands the orbital separation.

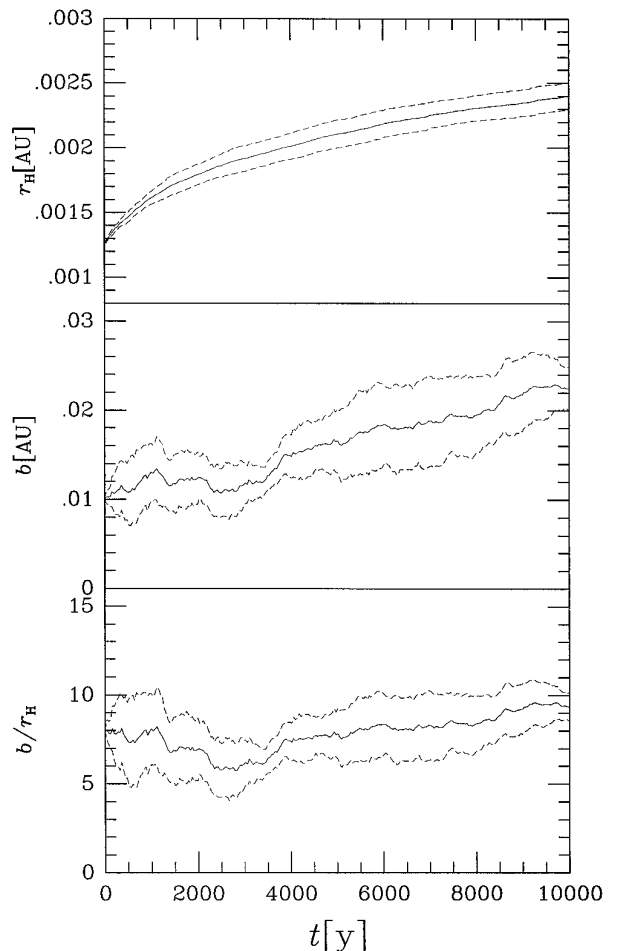


FIG. 3. The Hill radius of the two protoplanets  $r_H$ , the orbital separation  $b$ , and the orbital separation normalized by  $r_H$  averaged over 10 simulations are plotted against time. The solid curve shows the mean value and the two dashed curves surrounding the mean show the  $1\sigma$ -variation.

The expanding rate is given by

$$\frac{d\tilde{b}}{dt} \simeq \frac{\Delta\tilde{b}}{\Delta t_s} \simeq \frac{7h\Omega}{\tilde{b}^4}, \quad (8)$$

where  $\Delta\tilde{b} \simeq 30/\tilde{b}^5$  is the change of  $\tilde{b}$  by a scattering (Petit and Hénon 1986, Hasegawa and Nakazawa 1990),  $\Delta t_s = 4\pi/(3\tilde{b}h\Omega)$  is the synodic time, and  $h = r_H/a$  is the reduced Hill radius of the protoplanets. The time scale of orbital repulsion is estimated by

$$\begin{aligned} T_{\text{repe}} &\equiv \frac{\tilde{b}}{d\tilde{b}/dt} = \frac{\tilde{b}^5}{7h\Omega} \\ &= 2 \times 10^4 \left( \frac{M}{10^{26} \text{ g}} \right)^{-1/3} \left( \frac{\tilde{b}}{5} \right)^5 \left( \frac{a}{1 \text{ AU}} \right)^{3/2} [\text{years}]. \end{aligned} \quad (9)$$

On the other hand, the time scale for the Hill radius to increase due to growth of protoplanets is given by (see e.g., Kokubo and Ida 1995)

$$T_{r_H} \equiv \frac{r_H}{dr_H/dt} = 3T_{\text{grow}} \simeq \begin{cases} 5 \times 10^4 \tilde{e}_m^2 f^{-1} \left( \frac{\Sigma}{10 \text{ g cm}^{-2}} \right)^{-1} \left( \frac{M}{10^{26} \text{ g}} \right)^{1/3} \left( \frac{a}{1 \text{ AU}} \right)^{1/2} [\text{years}] & a < 2.7 \text{ AU} \\ 3 \times 10^5 \tilde{e}_m^2 f^{-1} \left( \frac{\Sigma}{4 \text{ g cm}^{-2}} \right)^{-1} \left( \frac{M}{10^{26} \text{ g}} \right)^{1/3} \left( \frac{a}{5 \text{ AU}} \right)^{1/2} [\text{years}] & a > 2.7 \text{ AU}, \end{cases} \quad (10)$$

where  $\tilde{e}_m$  is the normalized eccentricity given by  $\tilde{e}_m = e_m/h$ . Note that in Eq. (10), the  $a$ -dependence of the surface mass density is not included.

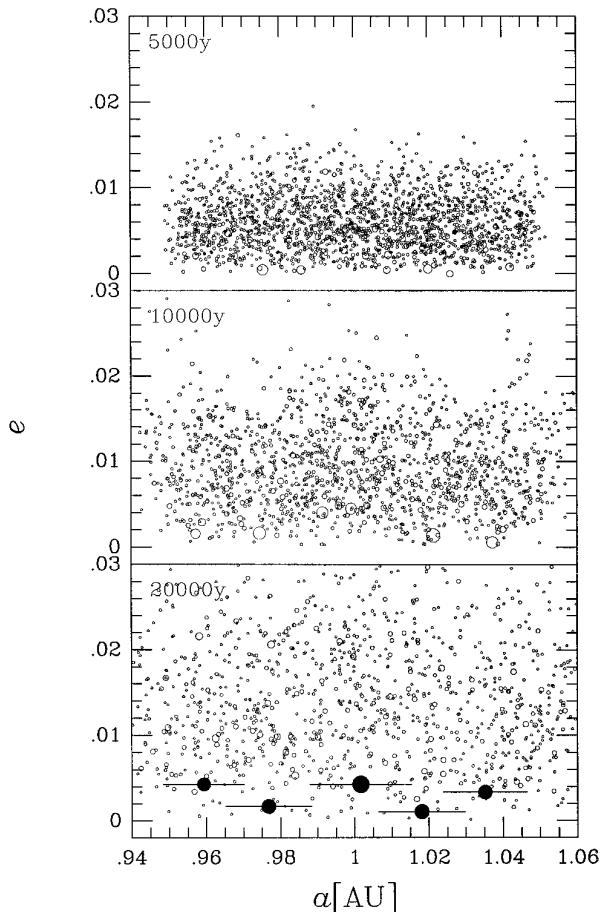
Equating  $T_{\text{repe}}$  with  $T_{r_H}$ , we obtain the typical orbital separation kept while growing as

$$\tilde{b}_{\text{typical}} \simeq \begin{cases} 11f^{-1/5} \left( \frac{\tilde{e}_m}{5} \right)^{2/5} \left( \frac{M}{10^{26} \text{ g}} \right)^{2/15} \left( \frac{\Sigma}{10 \text{ g cm}^{-2}} \right)^{-1/5} \left( \frac{a}{1 \text{ AU}} \right)^{-1/5} & a < 2.7 \text{ AU} \\ 10f^{-1/5} \left( \frac{\tilde{e}_m}{5} \right)^{2/5} \left( \frac{M}{10^{26} \text{ g}} \right)^{2/15} \left( \frac{\Sigma}{4 \text{ g cm}^{-2}} \right)^{-1/5} \left( \frac{a}{5 \text{ AU}} \right)^{-1/5} & a > 2.7 \text{ AU}. \end{cases} \quad (11)$$

It should be noted that the orbital separation measured by  $r_H$  only weakly depends on all its component parameters. At  $t = 10,000$  years of the above  $N$ -body simulation, the masses of the two protoplanets are about  $5 \times 10^{25} \text{ g}$ , the RMS eccentricity of planetesimals measured by the reduced Hill radius of the protoplanets is  $\tilde{e}_m \simeq 4$ , and  $f = 4$ . Substituting these values into Eq. (11), we obtain  $b_{\text{typical}} \simeq 7$ , which almost agrees with the result of the  $N$ -body simulation. Note that the orbital separation is scaled by  $r_H$ , which means the orbital separation increases with  $a$  and  $M$ .

The results of the two-protoplanet system show that in the post-runaway stage adjacent protoplanets grow keeping the same order masses and the orbital separations larger than about  $5r_H$ . The typical orbital separation is about  $10r_H$  that is almost independent of the mass and the semimajor axis of the protoplanet. The mass distribution becomes bimodal, namely, small number of protoplanets and large number of planetesimals. We call this growth of protoplanets as “oligarchic growth” in the sense that not only one but several protoplanets dominates the planetesimal system.

In order to reinforce the oligarchic growth theory, we perform simulations with wider  $\Delta a$  in which several protoplanets are expected to be formed with the orbital separations of about  $10r_H$ . The simulations start from equal-mass planetesimal system without seed protoplanets. We performed four simulations of set (ii) with different random numbers for the initial distribution of bodies. All the results are qualitatively the same. Figure 4 shows an example of the time evolution of the system of set (ii). First, runaway bodies appear randomly in the semimajor axis (the panel of  $t = 5000$  years of Fig. 4). As they grow, they start to interact with one another. Protoplanets expand their orbital separations to  $10r_H$  if there is a room for orbital repulsion (the panel of  $t = 10,000$  years of Fig. 4). Sometimes, relatively small protoplanets in between two large protoplanets and with no room for orbital repulsion are thinned out by collision with one of the adjacent large protoplanets. Through the above two processes, protoplanets grow keeping their orbital separations wider than about  $5r_H$ . In the panel of  $t = 20,000$  years of Fig. 4, oligarchic growth is clearly shown. Protoplanets with the same order mass ( $M \sim 10^{26} \text{ g}$ ) on nearly circular orbits



**FIG. 4.** The same as Fig. 1 but for the system initially consists of 4000 equal-mass planetesimals ( $m = 3 \times 10^{23}$  g). The radius increase factor is 6. In the final frame, the filled circles represent protoplanets and lines from the center of the protoplanets to both sides have the length of  $5r_H$ . The protoplanets are selected if their masses are larger than  $1/5$  of the maximum mass of the system. The numbers of planetesimals are 1977 ( $t = 5000$  years), 1514 ( $t = 10,000$  years), and 1116 ( $t = 20,000$  years).

are formed, while most planetesimals remain small. The five protoplanets have the 34% of the total mass of the system. The lines with the length of  $5r_H$  are drawn from the center of the protoplanets to both sides in the final frame. This Hill radius is slightly modified to include only the mass of a protoplanet. The separations are roughly constant with the typical value of  $5\text{--}10r_H$ , which agrees well with the result of the two-protoplanet system and the analytical estimation.

#### 4. CONCLUSION AND DISCUSSION

We have shown the oligarchic growth of protoplanets in the post-runaway stage. Protoplanets with the same order masses with the orbital separation larger than about  $5r_H$  is the inevitable outcome of planetary accretion in the post-

runaway stage, while most planetesimals remain small. The typical orbital separation of protoplanets kept while growing is about  $10r_H$ . This value depends only weakly on the mass of protoplanets, the surface density of the solid material, and the semimajor axis. This self-organized structure is a general property of self-gravitating accreting bodies in a disk when gravitational focusing and dynamical friction are effective.

If we assume that the oligarchic growth continues till the final stage of planetary accretion, the mass of protoplanets is estimated by  $M = 2\pi ab\Sigma$ . In the solar nebula model that is 50% more massive than the minimum mass model, the surface mass density of the solar nebula is given by

$$\Sigma = \begin{cases} 10 \left( \frac{a}{1 \text{ AU}} \right)^{-3/2} [\text{g cm}^{-2}] & a < 2.7 \text{ AU} \\ 4 \left( \frac{a}{5 \text{ AU}} \right)^{-3/2} [\text{g cm}^{-2}] & a > 2.7 \text{ AU}. \end{cases} \quad (12)$$

Adopting this  $\Sigma$  and  $b = 10r_H$ , we have  $M \simeq 0.2M_\oplus$  and  $b \simeq 0.07 \text{ AU}$  at  $1 \text{ AU}$  ( $\Sigma = 10 \text{ g cm}^{-2}$ ),  $M \simeq 7M_\oplus$  and  $b \simeq 2 \text{ AU}$  at  $7 \text{ AU}$  ( $\Sigma = 2.4 \text{ g cm}^{-2}$ ), and  $M \simeq 17M_\oplus$  and  $b \simeq 8 \text{ AU}$  at  $25 \text{ AU}$  ( $\Sigma = 0.36 \text{ g cm}^{-2}$ ), where  $M_\oplus$  is the Earth mass. In the terrestrial planet region, the estimated mass and the orbital separation of protoplanets are still smaller than the present planets. This may suggest that oligarchic growth does not continue till the final stage of planetary accretion in the terrestrial planet region. The orbital separation may get larger in the terrestrial planet region, if the radial excursion of planetesimals  $ea$  that is proportional to the random velocity gets larger than  $10r_H$  due to, for example, the clearance of solar nebula gas in the late stage of planetary accretion. The absence of gas drag leads to the higher velocity dispersion and thus wider radial excursion.

In the jovian planet region, however, the oligarchic growth may be consistent with the formation of the present planets. As for Jupiter and Saturn, which have massive gas envelopes, the estimated mass of protoplanets is as large as the critical mass to onset the gas accretion onto the protoplanets. As for Uranus and Neptune, which consist mainly of solid material, the estimated mass of protoplanets and the orbital separation are consistent with their present values. These results suggest that jovian planets may have been formed along the line of oligarchic growth. However, we should be careful when we apply oligarchic growth to the jovian planet region. Oligarchic growth is obtained from the local area simulation where the semimajor axis is much larger than the width of the simulation region. It is uncertain that oligarchic growth takes place in the wide jovian planet region in the same way as the local area simulation. Further work on this issue is required.

Our model does not take into account fragmentation of bodies. In the final stage, however, it must play an important role in the evolution of mass distribution.

We find that a bimodal protoplanet–planetesimal system is formed in the post-runaway stage. For planetary formation, we should consider the final stage starting from this protoplanet–planetesimal system. The orbital separation between protoplanets is about  $10r_H$ . This is three times as large as  $2\sqrt{3}r_H$ , which is sometimes referred to (Lissauer 1987, Lissauer and Stewart 1993). Large amount of small planetesimals may fill the region between the protoplanets. Hence, small bodies may play important roles in the growth of protoplanets even in the final accretion stage: protoplanets may grow through sweeping small bodies as well as collisions between the protoplanets until the end of accretion, and relatively strong gas drag on small bodies may cause rapid orbital migration of protoplanets (Tanaka and Ida 1997). To explore the final accretion stage, we need careful  $N$ -body simulations with large number of small bodies in nebula gas, rather than simulations which start from the system of only similar sized protoplanets with small orbital separations.

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