N-body Simulations of Star-Star Encounters

150193993

(DATE)

Abstract

The basic N-body problem and the applications of it to problems in astrophysics are explained in the literature review. The progress section gives a brief description of the second-order code written for the system along with a mention of the values selected for the time-step and the run-time. Results produced from the energy conservation and stability checks also included. A project plan is added at the end with all the tasks for semester two in a gannt chart. Along with it, a short explanation of the all the tasks is included. A conclusion summarises all the sections mentioned in the report.

1. Introduction

The basic N-body problem is described for a system where only gravitational interactions take place. These interactions cause the positions and velocities of the particles to change with time. For a stellar cluster where Newtonian gravity dominates, these changes can be simulated. The aim of this project lies in developing a working N-body code that tracks the evolution of a star formation region.

The motivation of writing this N-body code lies in the problems of astrophysics. Star cluster evolution present as the perfect area for applying the N-body problem. An important phase of this evolution comes during mass segregation. Consider a star cluster with two different stellar mass groups. Gravitational interactions between the stars from the heavy and light groups cause a net movement of the heavy component to the centre. This causes a thermal energy outflow as energy is transferred from one component to the other. This leads to mass segregation in the star cluster with the high mass stars located at the centre.

Another application comes from the planet migration theory explained in the Nice model. The current shape and structure of the Kuiper belt, existence of Kuiper belt objects, the highly eccentric orbit of pluto and the location of the orbits of the four giant planets is explained by this theory. After the dissipation of the initial planetesimal disk, gravitational interactions occur between the giant planets and the remnant planetesimals. Exchange of angular momentum caused the orbits to drift outwards causing the giant planets to migrate. This situation of orbital migration can be recreated using N-body simulations. Similarly, other problems like the interactions between binary systems and individual stars, merger between two galaxies causing a stream of gas and other particles to appear etc. can be solved using a similar N-body code.

For the initial part of the project, a simple second order code is created for the solar system. This allows to look at the orbital evolution of the planets due to the planet-planet interactions. The blocks of the code are explained later. Energy conservation and stability checks are done on the system. Results from the tests give an idea on how good is the chosen time-step.

2. Literature review

2.1 N-body problem

Newton's law of gravitation describes how a group of stars interact in a star cluster. These gravitational interactions cause the dynamical properties (velocity, position, acceleration) of the stars to change. Due to this, the dynamical properties of the region inhabiting the stars is affected. Therefore, N-body simulations are useful to observe the evolution of region. The basic N-body problem is ingrained in the prediction of the future values of the positions and velocities of the particles in the system. The initial properties of the particles in the system are known. In a system of N particles, the acceleration of a particle can be defined as,

$$\boldsymbol{a_i} = \sum_{i=1.i\neq j}^{N} \frac{Gm_j}{|r_{ij}|^2}\widehat{\boldsymbol{r_{ij}}} \qquad -1$$

Where, m_j − mass of the test particle

$\widehat{\boldsymbol{r_{ij}}}$ −unit vector giving a direction of the force exerted by body j on i

$|r_{ij}|^2$ − modulus square of the distance between the bodies

Integrating equation (1) provides the position and velocity of a particle at any time *t*. For *N*=2 the above equation is analytically solvable. Since we are considering stellar clusters as our system, where N≥2, numerical methods become reliable.

Numerical integration of equation 1 provides the solutions for the position and velocity.

$$\boldsymbol{r_1} = \boldsymbol{r_0} + \boldsymbol{v_0}dt + \frac{\boldsymbol{a_0}}{2}(dt)^2 + \frac{\dot{\boldsymbol{a_0}}}{3!}(dt)^3 + \frac{\ddot{\boldsymbol{a_0}}}{4!}(dt)^4 + \cdots \qquad -2$$

$$\boldsymbol{v_1} = \boldsymbol{v_0} + \boldsymbol{a_0}dt + \frac{\dot{\boldsymbol{a_0}}}{2}(dt)^2 + \frac{\ddot{\boldsymbol{a_0}}}{3!}(dt)^3 + \frac{\dddot{\boldsymbol{a_0}}}{4!}(dt)^4 + \cdots \qquad -3$$

Where, $\boldsymbol{r_1}, \boldsymbol{v_1}$ − new position and velocity of the particles.

$\boldsymbol{r_0}, \boldsymbol{v_0}$ − initial position and velocity of the particles

$\boldsymbol{a_0}$, $\dot{\boldsymbol{a_0}}$, $\ddot{\boldsymbol{a_0}}$, $\dddot{\boldsymbol{a_0}}$ − initial acceleration of the particles with the latter three being the 1st, 2nd, 3rd time derivatives respectively

dt − Time-step for the simulation

The time-step *dt* determines the accuracy of the values of the future position and velocity of the particles. It goes inversely with the computational time. As *dt* drops there is a rise in the number of calculations done in a single simulation.

Accuracy in predicting the motion of stars in the cluster is the main task. The error in the solution is proportional to the n^{th} power of the time-step (where n- order number). As we go to higher orders, a small drop in *dt* will imply a large reduction in the error. The timescales for evolution of a star cluster lies are million years. The second order code would take a huge amount of computational time to observe these evolutionary features. Hence, the fourth-order method is the preferred option when simulating a star cluster evolution as it offers higher accuracy than other ones. The fourth order predictor-corrector method will be implemented in the second semester.

For our project, we consider the second order method, with the equations used given below.

$$\boldsymbol{r_1} = \boldsymbol{r_0} + \boldsymbol{v_0}dt + \frac{\boldsymbol{a_0}}{2}(dt)^2 \qquad -4$$

$$\boldsymbol{v_1} = \boldsymbol{v_0} + \boldsymbol{a_0}dt + \frac{\dot{\boldsymbol{a_0}}}{2}(dt)^2 \qquad -5$$

2.2 Astrophysics applications of N-body method

The motivation of writing this N-body code come from its application to certain astrophysics problems. The problems like mass segregation in star clusters, orbital migration of the giant planets in the planetesimal disk (Nice model), presence of boxy and peanut-shaped bars in the bar evolution phase etcetera. In these problems, the system considered (stellar cluster or an ensemble of planets and planetesimals) is self-gravitating.

Star cluster evolution is a difficult scenario to build in a numerical simulation. The interaction between the stars determine the evolution of the cluster. Khalisi, E., Amaro-Seoane, P. and Spurzem, R. (2006, p. 703-720) observed dynamical evolution of a star cluster consisting of two different stellar mass groups (heavy and light components). In dynamical equilibrium, the cluster achieving a thermal velocity distribution determines its evolution. Reaching a maxwellian velocity distribution through small changes in velocity of the stars is termed as relaxation. Relaxation forms the major part in shaping the structure of the cluster. The important phase in the evolution of the cluster comes during the central core collapse. Three processes are able to reach this stage: equipartition, evaporation and gravothermal instability. Star-star encounters allow mass segregation in the central regions. These encounters cause a transfer of kinetic energy from the heavy to light component. This leads to movement in the heavy and low mass components to and away from the centre of the star cluster. There is a thermal energy outflow from the center to the outer regions of the cluster. Significant evidence for mass segregation in star clusters was found in Infra-red observations of the trapezium cluster in Orion (Khalisi, E., Amaro-Seoane, P. and Spurzem, R., 2006, p.703-720)

A simplest approximation of two-mass component simulations is considered. To carry out the simulations, they considered a plummer sphere model. This model describes the density distribution in the cluster. On using this model, the particles can be considered point-like, and large scattering events between the particles at small distances are avoided. In addition, other stellar evolution scenarios, like primordial binaries and tidal fields are neglected. Khalisi, E., Amaro-Seoane, P. and Spurzem, R. (2006, p. 703-720) used two parameters namely, fraction of heavy mass component, $q = M_h/M_{clu}$ and the mass fraction of each particle, $\mu = m_h/m_l$ to describe a set of models. A number of runs were assigned to each model, with each of them having a different setup of initial positions and velocities for all the particles. A strong approach was done to produce accurate N-body simulations with a large number of particles, keeping in mind about a low computing power. Similar to mass segregation in star clusters, simulations are possible for the orbital migration of the giant planets in the remnant planetesimal disk.

The Nice model explains the orbital migration of the giant planets and the current structure of the outer Solar System. Specifically the orbital migration of Neptune caused the majority of Kuiper Belt Objects (KBOs) to be at Neptune's mean motion resonance of 5:2. The large eccentricity and inclination in Pluto's orbit, along with the occurrence of the Oort cloud at distances>2000 AU are result from this migration. The Kuiper belt inhabits a vast number of residual planetesimals, which did not coalesce to in the giant planet formation stage. These Kuiper Belt Objects can give the history of the outer solar system. The interaction between the planetesimal disk and the planets led to exchange of angular momentum thereby affecting their orbits.

N-body simulations of the interactions between giant planets and the planetesimal disk was carried out by Hahn, J.M. and Malhotra, R. (2005, p. 2392). The simulations included 10^4 massless particles surrounding the migrating giant planets. With a timescale of the age of the solar system, the orbital evolution was tracked. Two scenarios were considered for the initial disk: when it was dynamically cold, and when it was destabilized before migration. They used a MERCURY6 N-body integrator for the particular evolution process. The migration was demonstrated by applying an external torque to the semi-major axis of the orbit. Below is the time-dependent form of the semi-major axis,

$$a_j(t) = a_{f,j} - \Delta_j e^{-\frac{t}{\tau}}$$

Where, $a_{f,j}$ −the final semi-major axis of the planet
Δ_j −planet's net radial displacement
τ −e-fold timescale for planet migration

For the initial conditions, the current masses and orbits of the planets was considered. In addition to that, the initial semi-major axis was modified by $-\Delta_j$. Due to this, the applied torque allowed the orbits to return to the present configurations. Various other constraints were used for Δ_j for each planet's orbit depending on the different resonances. A time-step of 0.5 years was used, with the massless particles randomly distributed over a range of 20-80 AU. A more extensive approach for the N-body simulations was done by Hahn, J.M. and Malhotra, R. (1999, p. 3041) to look at the orbital migration of the giant planets in the remnant planetesimal disk. Moreover, different cases for migration were considered based on the mass of the planetesimal disk.

The advent of oligarchic growth in planet formation is another scenario for an N-body problem. The core accretion theory for planet formation gives a good description of the runaway and oligarchic growth stages. Kokubo, E. and Ida, S. (1998, p.171) explored the orbital evolution of the proto-planets embedded in the planetesimal disk using three-dimensional N-body simulations. They considered was a planet-planetesimal system. They observed the growth of the protoplanets to be slower among themselves. However, it was faster in comparison to that of the planetesimals. From orbital migration, to the formation of magellanic stream, they represent good scenarios for N-body simulations.

Galactic cannibalism of the Magellanic Clouds by the tidal field created from the merger with the Milky Way led to stream of gas called the magellanic stream. This scenario was reproduced in numerical simulations done by Maddison et. al (2002, pp.421-422). They compared two types of numerical simulations: one, which looked at N-body only merger and the other type, hydrodynamic. For the latter type, the factors of star formation, supernova feedback and metal enrichment were included into the simulations along with the gravitational interactions. The results from the N-body only simulations showed the Small Magellanic Cloud (SMC) being tidally stripped, thereby showing the presence of the magellanic stream. However, from the second simulation the SMC was strongly disrupted, with the stream devoid of stars. Figure 1 shows these results. The astronomical observations of the magellanic stream show the stream consisting of only gas.

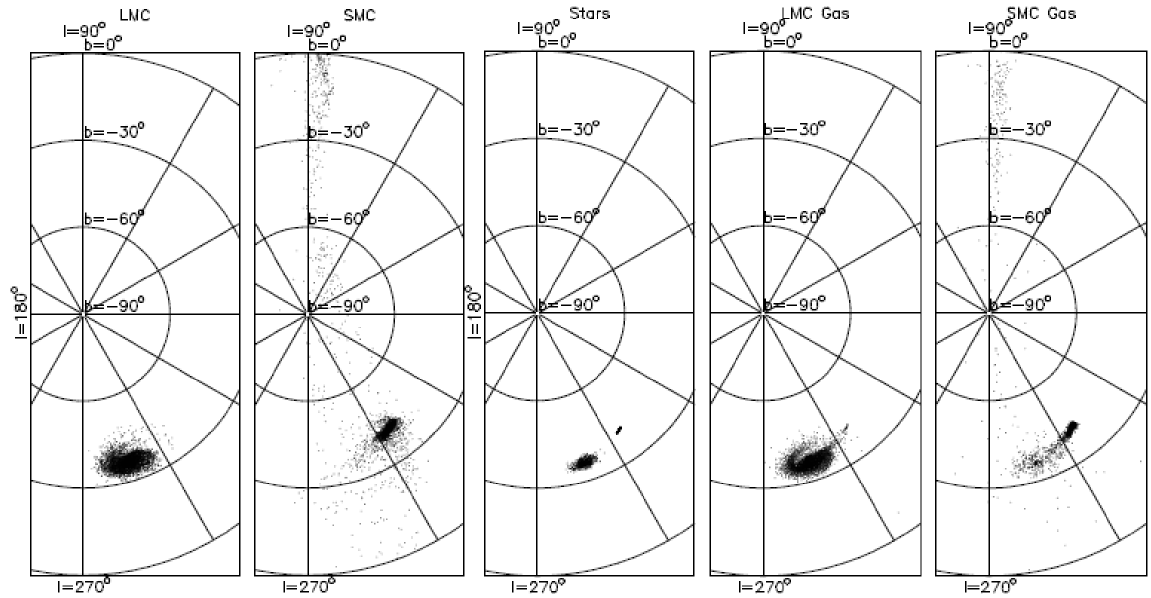


Figure 1: Different panels show the results from the N-body only and hydrodynamical simulations of the three-galaxy merger. Left two panels- N-body-only results showing the magellanic stream consisting of stars. Three right panels- hydrodynamical simulation results show the presence of mostly gas in the magellanic stream.

3. Progress on project

The initial work on the project was done by constructing a simple second order code. Equation (5) can be corrected by considering an assumption given below.

$$\dot{\boldsymbol{a}}_{\boldsymbol{0}} \sim \frac{\boldsymbol{a}_{\boldsymbol{1}} - \boldsymbol{a}_{\boldsymbol{0}}}{dt}$$

$$\boldsymbol{v}_{\boldsymbol{1}} = \boldsymbol{v}_{\boldsymbol{0}} + \frac{(\boldsymbol{a}_{\boldsymbol{0}} + \boldsymbol{a}_{\boldsymbol{1}})}{2} dt \qquad -6$$

Initially a three-body system of Sun-Earth-Jupiter was considered for the problem, with the rest of the planets added later.

For the first block of the code, the declaration of all the variables was done. Known variables like the gravitational constant *G*, the mass, initial velocity and initial position of the planets were initialised with the values taken from the planetary fact sheet on the NASA website. Certain initial conditions for the position and velocity are mentioned below. These conditions forced the planets to orbit in the XY plane. This reduced the complexity in the simulations.

$$r_y, r_z = 0$$

$$v_x, v_z = 0$$

Initially for the three body system of the Sun-Jupiter-Earth, their orbits were produced. At this point the centre-of-mass of the system was not considered. The time-step was chosen to be 100 seconds and the run-time for the simulations was 30 years. Without including the centre-of-mass, the system would seem to drift away from the origin. Figure 2 shows the minor drift of the system.

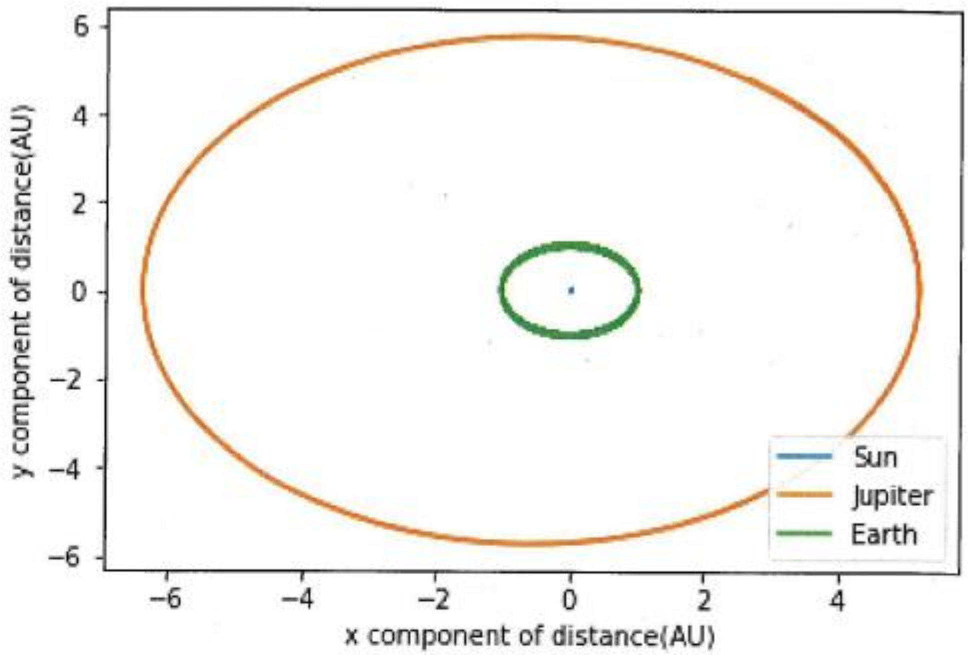


Figure 2: Orbits of the Sun-Jupiter-Earth with the simulations running for 30 yrs. The orbits are not corrected for the centre-of-mass and velocity.

To correct this, the objects' positions and velocities must be corrected for the centre-of-mass. Therefore, at this point the code which doing the corrections is added. The centre-of-mass and centre-of-velocity in the x direction is calculated as below,

$$COM_x = \frac{\sum_{i=1}^{N} m_i x_i}{m_{tot}} \qquad -7$$

$$COV_x = \frac{\sum_{i=1}^{N} m_i v_i}{m_{tot}} \qquad -8$$

Where, $m_{tot} = \sum_{i=1}^{N} m_i$, is the total mass of the system

An infinite loop was created which would run the simulations to a time t_{out}, and with a time-step *dt.* Future positions and velocities determined using equation (4) and (6). Modified form of equation (1) was used to calculate the future accelerations.

$$\boldsymbol{a_i} = \sum_{i=1.i\neq j}^{N} \frac{Gm_j.\boldsymbol{dr_{ij}}}{|r_{ij}|^3} \qquad -9$$

Where, $\boldsymbol{dr_{ij}} = \boldsymbol{r_i} - \boldsymbol{r_j}$

At certain points, small tests are given to the code to check for any logical errors. With all the positions, velocities and accelerations determined, the orbits of the planets were produced. The run-time for the simulations was upgraded to 1000 years with the time-step of 1000 sec. Due to the large runtime of 1000 years, the compilation took around 4-5 minutes. Figure 3 shows the orbits of all the planets in the Solar System.

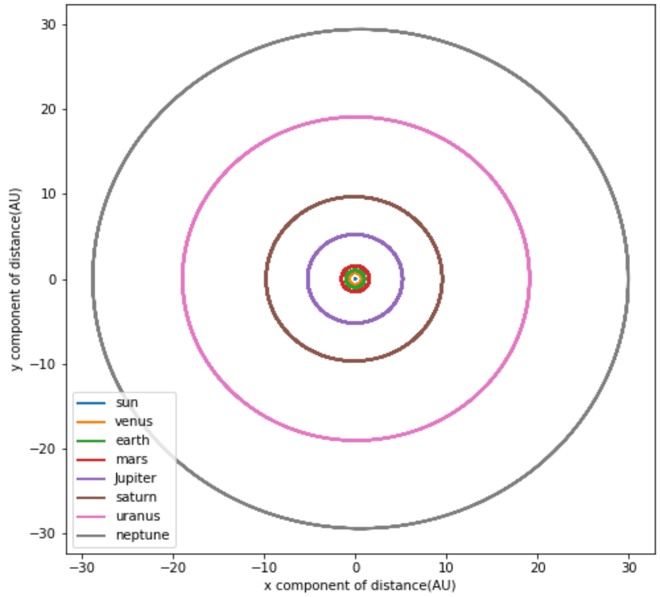


Figure 3: Orbits of the planets in the solar system. The time-step was chosen to be 1000 seconds with a run-time of 1000 years.

To check the stability of this system, an energy conservation check was done. This required the fractional energies to be determined using the equations given below. A counter, t_{count} was set so that after every 6 months this calculation was possible. This helped in speeding up the compilation.

$$PE = -\sum_{i=1, i \neq j}^{N} \frac{Gm_i m_j}{r} \qquad -10$$

$$KE = \sum_{i=1}^{N} \frac{1}{2} m_i \, v^2 \qquad -11$$

$$\Delta E = \frac{E_t - E_0}{E_0} \qquad -12$$

Where, $r = (x^2 + y^2 + z^2)^{1/2}$

$v^2 = (v_x^2 + v_y^2 + v_z^2)$

$E_t -$ total energy of the system at time t

ΔE and E_0 −the fractional energy and the initial energy of the system respectively

Figure 4 shows the fractional energy of the system varying with time was produced for a time step, $dt =$ 1000 seconds. The time-step was then reduced to 500 seconds to look at the effects on the fractional energy. This caused a lot more computational time due to the large number of data points produced.

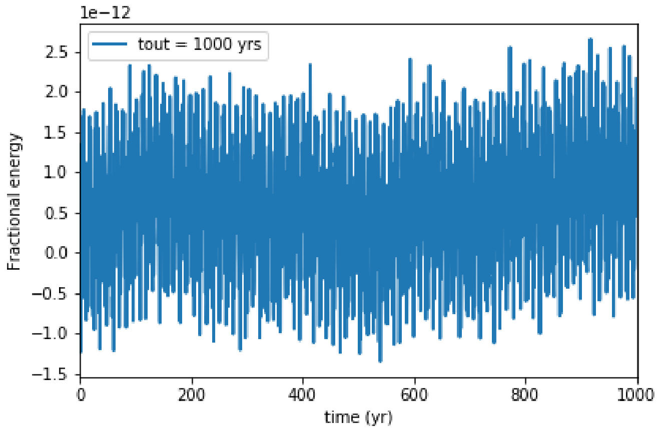


Figure 4: Fractional energy of the system against time (in years). The simulation was run for 1000 yrs with a time-step of 1000 seconds.

The fractional energy of the system depends on the time-step, *dt*. For good accuracy, the value of the time-step should be very low. This significantly increases the computational time, making the second order code slow. For a million year timescale the second order method is ineffective.

Finally, another test on the code was done by looking at how the distances from the sun varied with time for the planets. This gave an idea of the stability of the system. The run-time and the time-step were left unchanged. But to avoid a huge amount of data points, the distance from the Sun was calculated for every thousandth iteration (i.e. after every million seconds). Figure 5 shows the variations in the separation from the Sun for the planets.

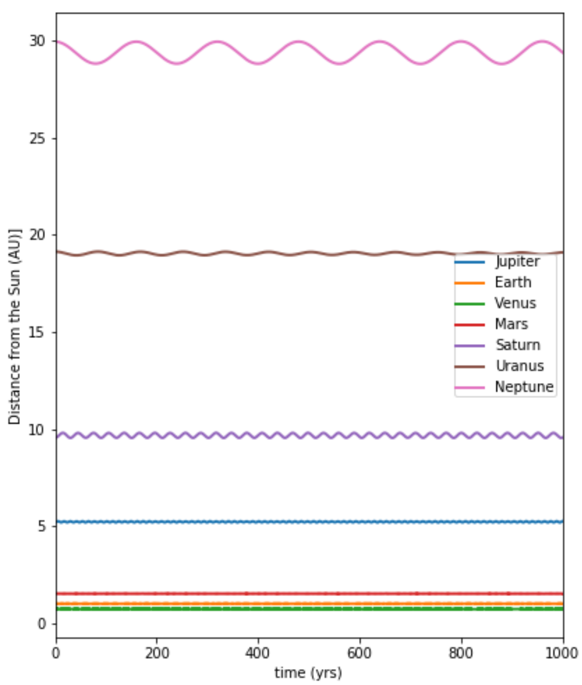
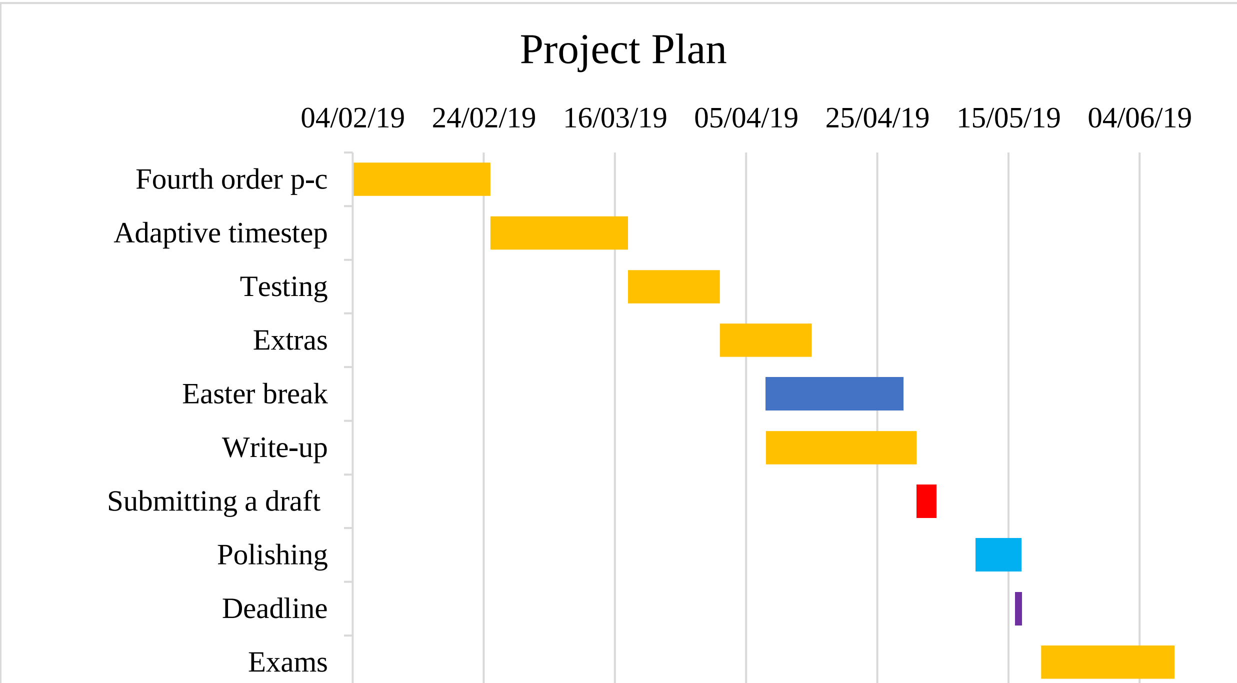


Figure 5: The separation from the sun (in AU) against time (in years). The periodic variations present in the curves is due to the interactions between the planets.

Therefore, the self-gravitating system in the code is the current the Solar System. The gravitational interactions between the bodies caused their dynamical properties to vary with time. N-body simulations showed how the dynamical properties of the objects in the Solar System evolve with time.

4. Project plan



At the start of week 1, the core part of the work in semester two begins with task-1, i.e., building a basic fourth order predictor-corrector code. A time of two-three weeks is given for this task based on its difficulty. After forming the base, we require an adaptive time-step for the code (task-2). Depending on the errors obtained from the energy checks, the code either doubles or halves the time-step, *dt*. A similar amount of time of three weeks is assigned to this task. Now, the two main components of the code are ready. This leads to task-3 of testing the code. The results obtained from these tests determine whether the code is working correctly. For example, milankovic cycles should be present in the time-varying plot of eccentricity of the orbits of the planets in our Solar System. A time of two weeks is set for this work.

Task-4 provides the motivation of writing this piece of code. This code is applied to any astrophysics problem mentioned in the previous sections. In this task, certain extra bits are added to the code. Moreover, certain tweaks are applied at this point to increase the speed of the code and to make it more efficient. With a time of two weeks, it overlaps with the Easter break. This leads to the most important task-5, the write-up of the report. All the figures and results produced in tasks 3 and 4 are included in the report. Tasks 4 and 5 overlap at the beginning of Easter break. A time of three-four weeks is assigned so that a draft of the final report can be submitted to the supervisor approximately two weeks before the deadline (17/05/2019). The final task involves in refining the report. Within a week the report is checked for any mistakes before the final submission.

5. Conclusion

A literature review was done on the basic N-body problem and the applications of the code to certain astrophysics problems. A simple second order code for the N-body simulations was constructed to apply on the current Solar System. With the necessary initial conditions given to the system and applying the centre-of-mass corrections, the orbits of the planets were produced. A check on the energy conservation and stability were applied on the system. Finally, a project plan for semester two was added which gave an overview of all the tasks to be done for the project.

References

Khalisi, E., Amaro-Seoane, P. and Spurzem, R., 2006. A comprehensive N-BODY study of mass segregation in star clusters: energy equipartition and escape. *Monthly Notices of the Royal Astronomical Society*, *374*(2), pp.703-720.

Hahn, J.M. and Malhotra, R., 2005. Neptune’s migration into a stirred-up Kuiper belt: A detailed comparison of simulations to observations. *The Astronomical Journal*, *130*(5), p.2392.

Hahn, J.M. and Malhotra, R., 1999. Orbital evolution of planets embedded in a planetesimal disk. *The Astronomical Journal*, *117*(6), p.3041.

Kokubo, E. and Ida, S., 1998. Oligarchic growth of protoplanets. *Icarus*, *131*(1), pp.171-178.

Maddison, S.T., Kawata, D. and Gibson, B.K., 2002. Galactic cannibalism: The origin of the Magellanic Stream. In *The Evolution of Galaxies* (pp. 421-422). Springer, Dordrecht.

https://nssdc.gsfc.nasa.gov/planetary/factsheet/