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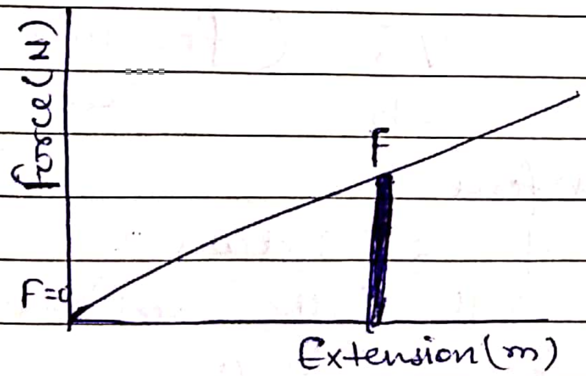
MEC31 (Solid mechanics)

Term paper topic → Elastic Strain ~~energy for~~ shearing stresses.

Elastic Strain ~~energy~~ energy

The external work done on an elastic member in causing it to distort from its unstressed state is transformed into strain energy which is a form of potential energy. The strain energy in form of elastic deformation is mostly recoverable in the form of mechanical work.

Elastic strain energy is the area under a force extension graph.

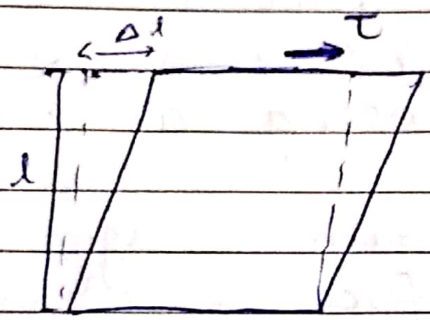


work done = force × displacement

Average force = $\frac{1}{2}F$

■ Shearing stresses →

Shear stresses, force tending to Cause deformation of a material by Slip page along a plane or planes parallel to the impose Stress. The result of Shear is of greater imposed in nature, ~~beig~~ being intimately related to the downslope movements of earth materials and to earthquake.



$$q = \frac{VQ}{Ib}$$ (formula of Shear stress)

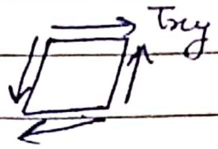
$V \rightarrow$ Shear force

I = I_{NA} of the section

b = The width of the sec at a depth y for N.A

Q = Frist moment of Area befor or above the level where Stress is required.

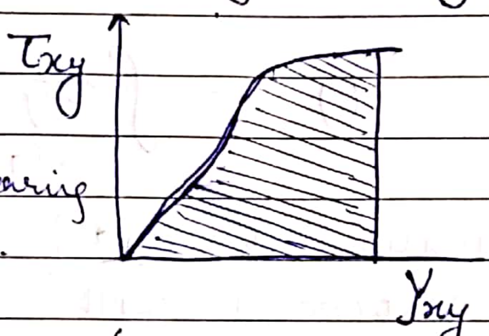
When a material is subjected to a plane shearing stress τ_{xy} the strain-energy density at a given point can be expressed as

(a) 

$$U = \int_0^{\gamma_{xy}} \tau_{xy}\, d\gamma_{xy} \quad \text{—①}$$

γ_{xy} - shearing strain corresponding to τ_{xy}

U = Area under the shearing stress-strain diagram.



(strain energy due to due to shear).

For value of τ_{xy} within the proportional limit,
we have $\tau_{xy} = G\gamma_{xy}$ into equation ①

Now doing integration

$$U = \frac{1}{2} G \gamma_{xy}^2 = \frac{1}{2}\tau_{xy}\gamma_{xy} = \frac{\tau_{xy}^2}{2G} \quad \text{—②}$$

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Strain energy U of a body subjected to plane shearing stresses can be obtained by ~~recalling~~

$$u = \frac{dU}{dV} \quad \text{--- (3)}$$

Substituting u in equation (2) and the integration Both ~~equals~~ member.

$$U = \int \frac{\tau_{xy}^2}{2G} \, dV \quad \text{--- (4)}$$

equation (4) defines the elastic strain associated with the shear deformation of the body.

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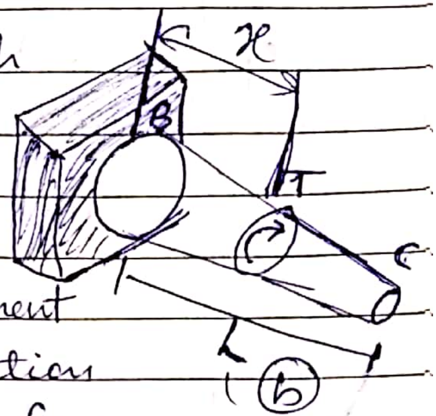
Strain Energy in Torsion ⇒



Consider a shaft BC of length L subjected to one or several twisting couples.

Denotings by J the polar moment of inertia of the cross section location at a distance x from B (Fig. b) and by T the ~~a.~~ internal torque in that section.

Shearing stresses is $\tau_{xy} = T\rho / J$

Substitute for τ_{xy} into (ω) eqn.

we have
$$U = \int \frac{\tau_{xy}^2}{2G}\, dV = \int \frac{T^2 \rho^2}{2GJ^2}\, dV$$

Setting $dV = dA\, dx$, where dA represent an element of the cross section area and obtaining that $\frac{T^2}{2GJ^2}$ is a function of x alone

We write
$$U = \int_0^L \frac{T^2}{2GJ^2}\left(\int \rho^2 dA\right) dx$$

The integration within the parentheses represent the polar moment of inertia J of the cross-section, we have

$$U = \int_0^L \frac{T^2}{2GJ} \, dx \quad — (5)$$

In the case of a shaft of uniform cross section subjected at its ends to equal and opposite couples of magnitue T

$$U = \frac{T^2 L}{2GJ} \quad — (6)$$