Ex- 2.4

For a uniform distribution:

$$p(x) = \begin{cases} 1/b-a & ; \quad a \leq x \leq b \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$\mathbb{E}[x] = \int_a^b x \cdot \frac{1}{b-a} \, dx = \frac{a+b}{2}$$

$$\mathbb{E}[x^2] = \frac{(b-a)^2}{12}$$

$\Rightarrow$ These values depend only on the interval, not the actual value of x.

Guassian Distribution

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}$$

$$\mathbb{E}[x] = \int_{-\infty}^{\infty} x \, p(x) \, dx$$

$$\mathbb{E}[x] = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

substitute $x - \mu = y \Rightarrow x = y + \mu$ & $dy = dx$

$$= \int_{-\infty}^{\infty} (y+\mu) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} dy$$

$$= \underbrace{\int_{-\infty}^{\infty} y \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} dy}_{\rightarrow 0} + \mu \underbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} dy}_{\rightarrow 1}$$

(total probablity)

$$= 0 + \mu \times 1 = \mu$$

Ex - 2.8

$$\mathrm{var}[f] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2] \qquad (2.44)$$

$$\mathbb{E}[(f(x)-\mathbb{E}[f(x)])^2] = \mathbb{E}[f(x)^2 - 2f(x)\mathbb{E}[f(x)] + \mathbb{E}[f(x)]^2]$$

$$= \mathbb{E}[f(x)^2] - 2\mathbb{E}[f(x)]\mathbb{E}[f(x)] + \mathbb{E}[f(x)]^2$$

$$= \mathbb{E}[f(x)^2] - \mathbb{E}[f(x)]^2 \qquad (2.45)$$

as required.

Ex - 2.9

$$\mathrm{cov}[x,y] = \mathbb{E}[xy] - \mathbb{E}[x].\mathbb{E}[y] \qquad (2.47)$$

using (2.38)

$$\mathbb{E}[xy] = \sum_x \sum_y p(x,y)\, xy$$

since x & y are independent $p(x,y) = p(x)p(y)$

$$\Rightarrow \qquad = \sum_x p(x)x \sum_y p(y)y$$

$$= \mathbb{E}[x].\mathbb{E}[y]$$

$$\Rightarrow \quad \mathrm{cov}(x,y) = 0$$

Created with Notewise