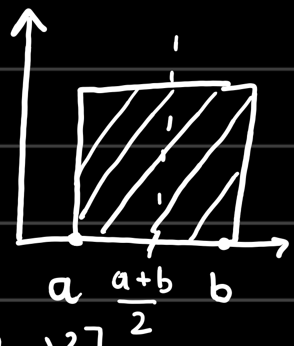
For a uniform distribution:

$$p(x) = \begin{cases} 1/b-a & ; \quad a \leq x \leq b \\ 0 & ; \quad \text{otherwise} \end{cases}$$



$$\mathbb{E}[x] = \int_a^b x \cdot \frac{1}{b-a}\, dx = \frac{a+b}{2}$$

$$\mathbb{E}[x^2] = \frac{b^3 - a^3}{3(b-a)}$$

(second moment)

$$\text{var}(X) = \mathbb{E}[(x-\mu)^2]$$

$$\text{var}(X) = \mathbb{E}[(x-\mu)^2]$$

$$= \int_a^b \left(x - \left(\frac{a+b}{2}\right)\right)^2 \cdot \frac{1}{b-a}\, dx$$

$$= \frac{1}{b-a}\left[\int_a^b \left(x^2 - (a+b)x + \frac{(a+b)^2}{4}\right) dx\right.$$

1) $\int_a^b x^2 \, dx = \frac{b^3 - a^3}{3}$

2) $\int_a^b x \, dx = \frac{b^2 - a^2}{2}$

3) $\int_a^b 1 \, dx = b - a$

$$\Rightarrow \; = \frac{1}{b-a}\left[\frac{b^3 - a^3}{3} - (a+b)\left(\frac{b^2 - a^2}{2}\right) + \frac{(a+b)^2}{4}(b-a)\right]$$

$$= \frac{1}{b-a}\left[\frac{(b-a)(b^2 + ab + a^2)}{3} - \frac{(a+b)(b+a)(b-a)}{2}\right.$$

$$= \frac{b^2 + ab + a^2}{3} - \frac{(a+b)^2}{2} + \frac{(a+b)^2}{4}$$

$$= \frac{b^2+ab+a^2}{3} - \frac{(a+b)^2}{4}$$

$$= \frac{4b^2+4ab+4a^2-3a^2-6ab-3b^2}{12}$$

$$= \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12}$$

For Guassian (Normal) Distribution

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$E[x] = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \cdot dx$$

(Expected value)

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}} \cdot dx$$

replace $\frac{x-\mu}{\sqrt{2}\sigma} = y \Rightarrow x-\mu = y\cdot\sqrt{2}\sigma$

$$dx = \sqrt{2}\sigma\, dy$$

$$= \frac{1}{\cancel{\sigma\sqrt{2}}\sqrt{\pi}} \int_{-\infty}^{\infty} (\sqrt{2}\sigma y + \mu) \cdot e^{-y^2} \cdot dy \cdot \cancel{\sqrt{2}\sigma}$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \sqrt{2}\sigma y \cdot e^{-y^2} \cdot dy + \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \mu \cdot e^{-y^2} dy$$

$$= \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \underbrace{\overset{0}{\cancel{\int_{-\infty}^{\infty} y \cdot e^{-y^2} \cdot dy}}}_{\text{odd}} + \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2} dy$$

$$= \frac{2\mu}{\sqrt{\pi}} \int_{0}^{\infty} e^{-y^2} dy = \frac{\cancel{2}\mu}{\cancel{\sqrt{\pi}}} \cdot \frac{\cancel{\sqrt{\pi}}}{\cancel{2}} = \mu$$

$$\text{Var}(x) = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \cdot dx$$

(variance)

$$= \frac{1}{\cancel{\sigma\sqrt{2}}\sqrt{\pi}} \int_{-\infty}^{\infty} 2\sigma^2 y^2 \cdot e^{-y^2} \cdot \cancel{\sqrt{2}\sigma}\, dy$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} y^2 \cdot e^{-y^2} \cdot dy$$

let $y^2 = t \Rightarrow 2y\,dy = dt$

$$= \frac{\cancel{2}\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t \cdot e^{-t} \cdot \frac{dt}{\cancel{2}\sqrt{t}} = \frac{\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t \cdot \frac{e^{-t}}{t^{1/2}}\, dt$$

$$= \frac{\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t^{1/2} e^{-t}\, dt$$

$$= \frac{\sigma^2}{\sqrt{\pi}} \cdot \sqrt{\pi} = \sigma^2$$