Session 17 Group Quiz

(a) For the likelihood function, we need to multiply all the values of Beta density for each of the observations

$$L(\theta, x) = f_x(\theta, x_1) \cdot f_x(\theta, x_2) \cdots f_x(\theta, x_N)$$
$$= (\theta x_1^{\theta-1}) \cdot (\theta x_2^{\theta-1}) \cdots (\theta x_3^{\theta-1})$$
$$= \prod_{i=1}^{n} \theta x_i^{\theta-1} \qquad 0 < x \leq 1$$

For any other values of x, $L(\theta, x) = 0$

(b) $\ln L(\theta, x) = \sum_{i=1}^{n} \ln \theta + \sum_{i=1}^{n} x_i^{\theta-1} \ln$

$$= n \ln \theta + (\theta - 1) \sum_{i=1}^{n} \ln(x_i)$$

$$\frac{\partial \ln L}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \ln(x_i)$$

Now, we know $\frac{\partial \ln L}{\partial \theta} = 0$

$$= \quad 0 = \frac{n}{\theta} + \sum_{i=1}^{n} \ln(x_i) \qquad \Bigg\| \qquad \frac{n}{\theta} = -\sum_{i=1}^{n} \ln(x_i)$$

$$\Rightarrow \quad \theta = -\frac{n}{\sum_{i=1}^{n} \ln(x_i)}$$

$\theta < 0$

because summation of logs is always > 0 an $n > 0$

$\therefore$ Since $\theta < 0$, this is a maximum.