

f-worksheet - Conditional Probability

1. With the monsoon season, we can have more cases of dengue fever, a mosquito-borne tropical disease caused by the dengue virus. Antibody tests are recommended during a dengue outbreak. However, the presence of other viruses in the human body can have cross-reactive results yielding a high false positive rate. Assume a false positive rate of 10% and a false negative rate of 1%.

- (a) Given that a person has dengue, what is the probability of a positive test. Demonstrate correct use of conditional probability notation.

$$P\{\text{test+} \mid \text{has d}\} = 1 - P\{\text{test-} \mid \text{has d}\} = 1 - 0.01 = 0.99$$



Figure 1: A koala bear

- (b) If two percent of a population has dengue, what fraction of the population will test positive. Hint: The probability of having a positive test is the same as summing the probabilities of having a positive test with dengue and a positive test without dengue.

$$\begin{aligned} P\{\text{test+}\} &= P\{\text{test+ AND has d}\} + P\{\text{test+ AND no d}\} = P\{\text{test+} \mid \text{has d}\} * P\{\text{has d}\} + \{\text{test+} \mid \text{no d}\} \\ &* P\{\text{no d}\} = 0.99 * 0.02 + 0.1 * (1-0.02) = 0.1178 \end{aligned}$$

- (c) If the individual tests positive, what is the probability that this individual has dengue? Assume the same population dengue rate as in (b).

$$\begin{aligned} P\{\text{has d} \mid \text{test+}\} &= P\{\text{has d AND test+}\} / P\{\text{test+}\} \\ &= P\{\text{test+} \mid \text{has d}\} * P\{\text{has d}\} / P\{\text{test+}\} \\ &= 0.99 * 0.02 / 0.1178 \\ &= 0.1681 \end{aligned}$$

- (d) Produce a two-way table showing the numbers of people with and without dengue and with and without a positive test out of a population of 10,000 (don't have to show calculations, just fill in the table). Assume the same population dengue rate as in (b). How can your answer in part (c) be seen in the two-way table?

Population	no dengue	has dengue	total
test +	980	198	1178
test -	8820	2	8822
total	9800	200	10000

so $P\{\text{has dengue} \mid \text{test}+\} = P\{\text{has dengue AND test}+\}/P\{\text{test}+\} = 198/1178 = 0.1681$ or 16.81%.

- (e) In a nearby country a third of those tested have dengue. For that country, what is the probability that an individual testing positive actually has dengue?

$$\begin{aligned}
 P\{\text{has D} \mid \text{test}+\} &= P\{\text{has D AND test}+\}/P\{\text{test}+\} \\
 &= P\{\text{test}+ \mid \text{has d}\} * P\{\text{has d}\} / P\{\text{test}+\} \\
 &= \frac{P\{\text{test}+ \mid \text{has d}\} * P\{\text{has d}\}}{P\{\text{test}+ \mid \text{has d}\}P\{\text{has d}\} + P\{\text{test}+ \mid \text{no d}\}P\{\text{no d}\}} \\
 &= \frac{0.99 * (1/3)}{0.99*(1/3) + 0.1*(2/3)} \\
 &= 0.8319 = 83.19\%
 \end{aligned}$$