DATA 363: Checkpoint H2 H3

H2-6 $EX^2 = ?\quad EX = ?$

We know $Eg(X) = \int_{-\infty}^{\infty} g(x) f_X(x)\,dx$

We are told limits $\Rightarrow [0,1]$ $\quad g(X) = X^2\ \&\ X$

$$EX = \int_0^1 x \cdot 2x\,dx = \int_0^1 2x^2\,dx = \left.\frac{2x^3}{3}\right|_0^1 = \boxed{\frac{2}{3}}$$

$$EX^2 = \int_0^1 x^2 \cdot 2x\,dx = \int_0^1 2x^3\,dx = \left.\frac{2x^4}{4}\right|_0^1 = \left.\frac{1}{2}\cdot x\right|_0^1 = \boxed{\frac{1}{2}}$$

H2-8 $F_X(x) = x^2 \quad$ So, $f_X(x) = 2x$

$$EX = \int_0^1 x \cdot 2x\,dx = \int_0^1 2x^2\,dx = \left.\frac{2x^3}{3}\right|_0^1 = \boxed{\frac{2}{3}}$$

H3-5 (a) If Bernoulli trial has prob of success $p \Rightarrow$

$EX = (1-p)\cdot 0 + p = p$

$EX^2 = 0^2(1-p) + 1^2 \cdot p = p$

$Var(X) = EX^2 - (EX)^2 = p - p^2$

$SD(X) = \sqrt{p-p^2}$

(b) $EX = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6}$

$EX^2 = \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6} = \frac{91}{6}$

$Var(X) = EX^2 - (EX)^2 = \left(\frac{91}{6}\right) - \left(\frac{21}{6}\right)^2 = \frac{105}{36} = \frac{35}{12}$

$SD(X) = \sqrt{\frac{35}{12}}$

(c) Using our results from H2-6, we know

$EX = \frac{2}{3}$ $\quad EX^2 = \frac{1}{2}$

$Var(X) = \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$

$SD(X) = \sqrt{\frac{1}{18}}$

(d) $EY = E(aX+b) \Rightarrow aEX$

$EY^2 = E(aX+b)^2 \Rightarrow E(a^2X^2 + 2abX + b^2)$

$\Rightarrow a^2 EX^2$

$Var(Y) = EY^2 - (EY)^2 = a^2 EX^2 - a^2 (EX)^2$

$= a^2 (EX^2 - (EX)^2)$

$= a^2 (Var X)$

$SD(Y) = \sqrt{Var(Y)} = a\sqrt{Var(X)} = aSD(X)$