

## g1-worksheet - Random Variables and Distribution Functions

1. A Gumbel random variable  $X$  has distribution function  $F_X(x) = \exp(-e^{-x})$ .
  - (a) Give a graph of  $F_X(x)$  and explain using this plot why  $F_X$  is a valid cumulative probability distribution function.

This is a valid cumulative probability function because the y values range from 0 to 1 and the graph doesn't decrease for increasing values of x. Finally, we can also see that it is right continuous

- (b) Find the values of the first and third quartiles and median of  $X$  and show their values on the graph (you can show only one graph for (a) and (b) if you prefer).

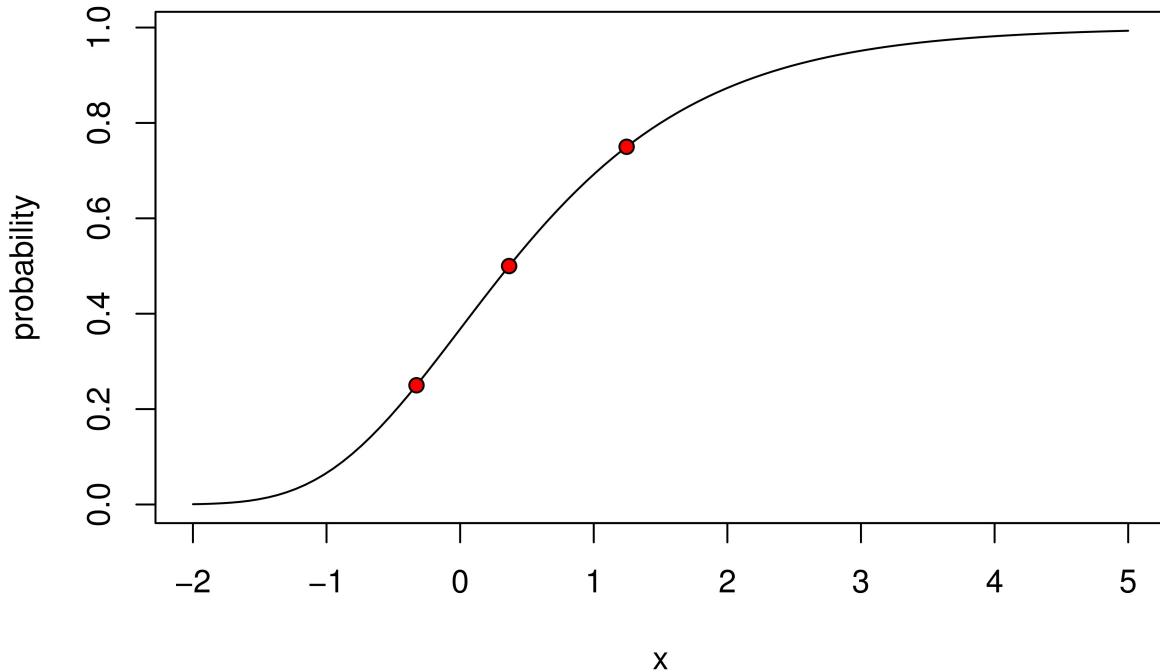
```
# (a) Plot should show enough of the function for the reader to see the
# reasons why Fx is a valid distribution function

# Two options for the plot:
# 1. Use plot(x,y) where you manually specify the x-values to plot at
#x <- seq()
#y <- # code up distribution function
#plot(x,y,xlab="",ylab "") # and set xlab, ylab appropriately

# 2. Use curve() and let R figure out the values of x for you
# remember to set xlab and ylab appropriately
curve(exp(-exp(-x)),from = -2, to = 5,xlab="x",ylab="probability")

# (b) Use the distribution function and solve for the quartiles
# (this is similar to one of the checkpoints), you may need a
# logarithm or two.

points_yvals <- c(0.25,0.5,0.75)
points_xvals <- -log(-log(points_yvals))
points(points_xvals,points_yvals,pch=21,bg="red")
```



(c) Make a table of  $x$  and  $F_X(x)$  for  $x$  equal to integers from -2 to 5.

```
# Here's how to make a function in R
pgumbel <- function(x)
{
  gx <- exp(-exp(-x))
  return(gx)
}

# set xs and call pgumbel - the p is the R shorthand for a distribution function
xs <- c(-2, -1, 0, 1, 2, 3, 4, 5)
F_x <- pgumbel(xs)

data.frame(xs, F_x)

##   xs      F_x
## 1 -2 0.000617979
## 2 -1 0.065988036
## 3  0 0.367879441
## 4  1 0.692200628
## 5  2 0.873423018
## 6  3 0.951431993
## 7  4 0.981851073
## 8  5 0.993284702
```

- (d) Find the probabilities  $P\{-1 < X \leq 4\}$  and  $P\{X > 4\}$  using the table from part (c). You can type this out or insert a scan.

$$P\{-1 < x \leq 4\} = F_x(4) - F_x(-1) = 0.981851073 - 0.065988036 = 0.915863$$

$$P\{X > 4\} = 1 - F_x(4) = 1 - 0.981851073 = 0.018149$$

- (e) Find the probability density for this distribution function.

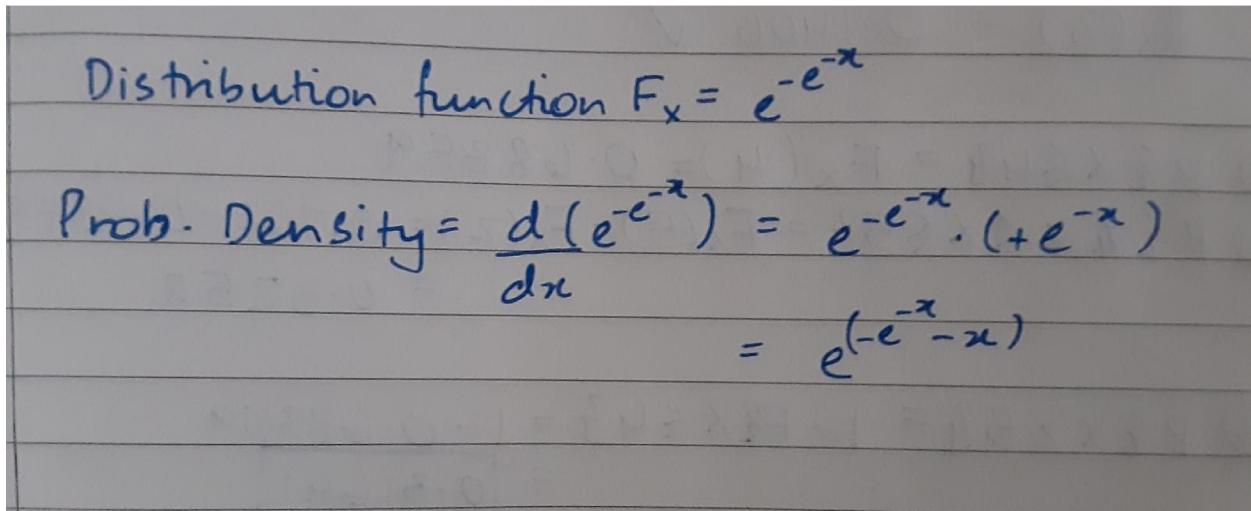


Figure 1: Part e

- (f) Sketch the distribution function along with a sketch of the density function indicating  $P\{-1 < X \leq 4\}$  on both plots.

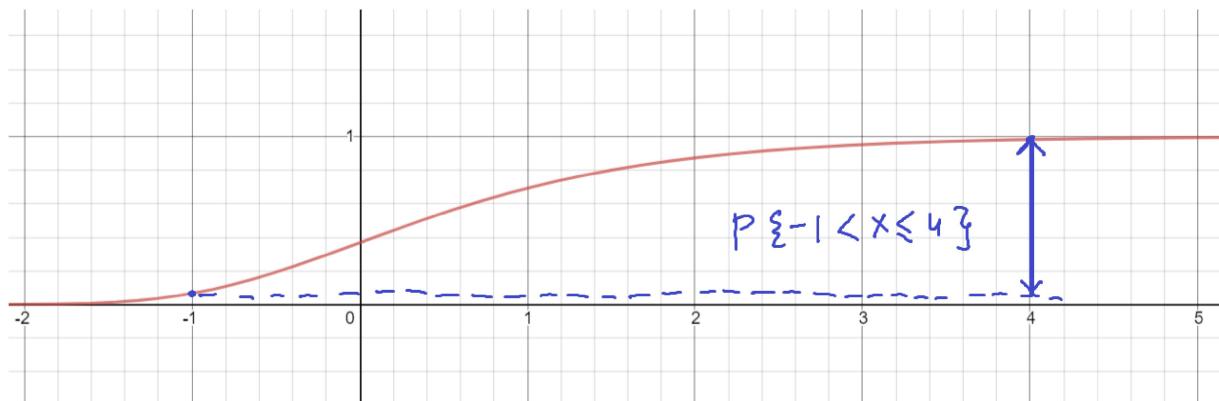


Figure 2: Distribution Function

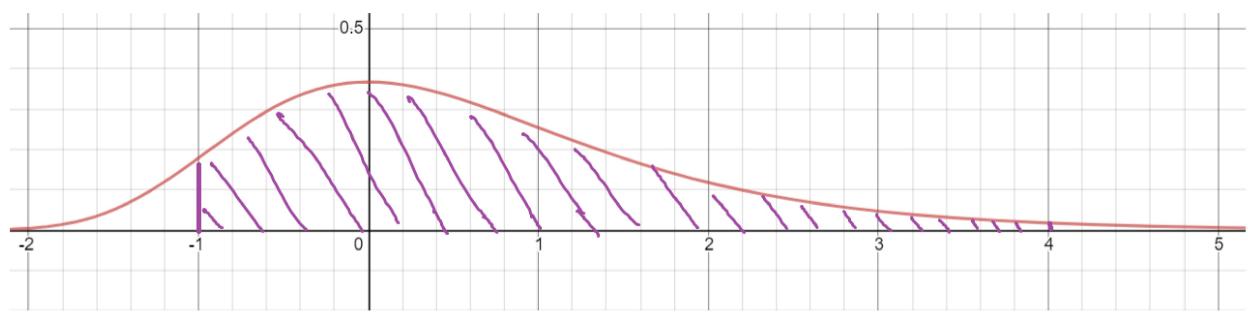


Figure 3: Density Function