VATSAV SETHUPATHI

Date....../....../......

Page......

MATH-DATA 363 Exam 2

① (a) We are told that $S_{100} = \bar{X} \cdot 100$

$\therefore$ Mean of $S_{100} = 2740$ pounds

$SD = \frac{12.5}{\sqrt{100}} \times 100 = \cancel{1.25}$ pounds
125

(b) $z = \cancel{} \frac{2800-2740}{1225} = 0.48$

normcdf (~~0.231~~ 0.48, ∞, 0, 1) = 0.3156

(c) $z_{up} = \frac{28-27.4}{1.25} = 0.48$

$z_{low} = \frac{26-27.4}{1.25} = -1.12$

normcdf(-1.12, 0.48, 0, 1) = 0.553

(d) Our answer in (c) increases because by increasing the sample size, our standard deviation decreases, which increases the size of the interval, thereby increasing the area under the distribution that we consider.

Date / /

Page

(2) (a) Mean = $\frac{S}{f}$ Hz

SD = $\frac{10}{\sqrt{16}} = \frac{10}{4} = 2.5$ Hz [From Law of large Numbers]

(b) $\hat{l} = \frac{S}{f}$

(c) Mean of $\hat{l} = \frac{340}{400} = 0.85$ meters

Var of $\hat{l} = (\hat{l}')^2 \cdot$ Var of S

$= \left(\frac{-S}{(f^2)}\right)^2 \cdot 2.5 \cdot 2.5$

$= \left(\frac{-340}{(400)^2}\right)^2 (2.5)(2.5)$

$= 0.000028$

SD of $\hat{l} = \sqrt{\text{Var of } \hat{l}} = 0.0053$ m

3 (a) $L(\lambda | t_1, \dots, t_n) = \lambda e^{-\lambda t_1} \cdot \lambda e^{-\lambda t_2} \cdots \lambda e^{-\lambda t_n}$

$$= \lambda^n e^{-\lambda(t_1+t_2+\dots+t_n)}$$

(b) $\ln L(\lambda | t_1 \dots t_n) = n \ln \lambda - \lambda(t_1 + t_2 + \dots + t_n)$

$$= n \ln \lambda - \lambda \sum_{i=1}^{n} t_i$$

$$\frac{\partial \ln L(\lambda | t_1 \dots t_n)}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} t_i$$

But we will set the derivative $= 0$

$$\frac{n}{\lambda} - \sum_{i=1}^{n} t_i = 0 \quad \Bigg\| \quad \sum_{i=1}^{n} t_i = \frac{n}{\lambda} \quad \Bigg\| \quad \hat{\lambda} = \frac{n}{\sum_{i=1}^{n} t_i}$$

$$\Rightarrow \boxed{\hat{\lambda} = \frac{1}{\bar{t}}}$$

[$\frac{\partial^2 \ln L}{\partial \lambda^2} = -\frac{n}{\lambda^2}$ which is less than 0, meaning this is a maximum]

(c) $\bar{t} = \frac{3.8 + \cancel{4}1.6 + 4.3 + 6.8 + 1.0 + 4.1 + 0.3 + 6.5 + 5.0 + 1.7}{10}$

$= 3.51$

$\hat{\lambda} = \frac{1}{3.51} = 0.285$

(d) $F_t(3 | \hat{\lambda}) = 1 - e^{-0.285 \cdot 3}$

$= 0.5747$

$\therefore$ The probability that the lifetime of the Samsung phone is 3yrs or less is 0.5747

(4) (a) $H_0: p <= p_0$
$H_1: p > p_0$

Where p_0 is the given probability 0.4
p is the probability of an individual in Tucson having Otve Hood.

(b) $\hat{p} = 200/450 \Rightarrow 0.44$ $\quad p_0 = 0.4$

test statistic $\rightarrow$ $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$ [From cheat sheet]

$$= \frac{0.44 - 0.4}{\sqrt{\dfrac{(0.4)(0.6)}{450}}} = 1.732$$

Func^n I used on my calcu

(c) p-value $\Rightarrow$ normalcdf$(1.732, \infty, 0, 1)$
$\Rightarrow 0.0416$

This means that we can reject the null hypothesis at a 5% significance level since our p-value is less than 0.05, but we fail to reject H_0 at a 1% significance level since $0.0416 > 0.01$

(d) $C.I = \hat{p} \pm z^* \sqrt{\dfrac{\hat{p}_0(1-\hat{p}_0)}{n}}$

[$z^* \Rightarrow$ inv norm$(0.975, 0, 1)$] $\leftarrow$ from calculator

$$\Rightarrow 0.44 \pm 1.96 \sqrt{\frac{0.44(0.56)}{450}}$$

$\Rightarrow$ ~~0.44~~ ± 0.0459 $\Rightarrow (0.3985, 0.4903)$
0.4444

Date ..../..../....
Page............

(5) (a) $H_0 : \mu = 50$ || where μ is the depth of the groove in nanometers
$H_1 : \mu \neq 50$

(b) Since we are given s, we will use 't^*' and not 'z^*'

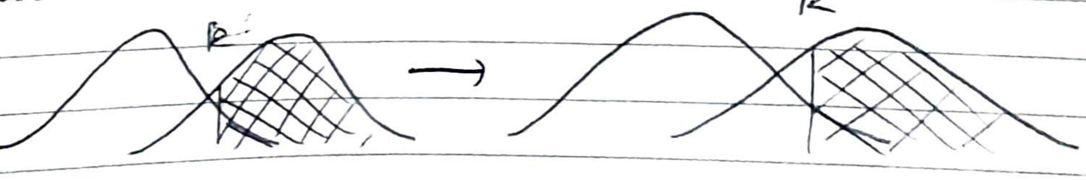
$CI = 53.5 \pm t^* \left(\frac{s}{\sqrt{n}}\right)$ [From cheat sheet]

$= 53.5 \pm 2.201 \left(\frac{10.6}{\sqrt{12}}\right)$ [t^* from table, $df = 11$ & $C = 95\%$]

$= 53.5 \pm 6.73$
$= (46.77, 60.23)$

(c) Since 50 is in the confidence interval, we fail to reject the null hypothesis at the 5% ~~sig~~ significance level.

(d) (i) In this case, the power will decrease since the data will be more spread out, therefore, the critical region becomes much smaller

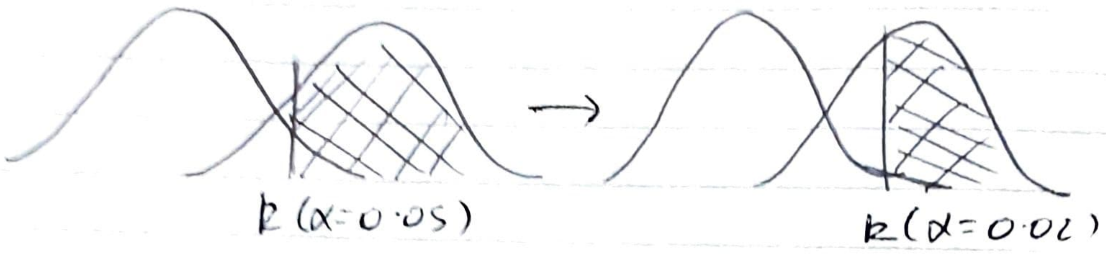


(ii) In this case, the power decreases since the distributions are ~~farther away from each of~~ closer to each other, meaning the area to the right of the critical point ~~become~~ decreases, meaning the power decreases.

Date......../......../......

Page............

iii When we decrease α, the value of the critical point increases, meaning the area under the critical region decreases, thereby decreasing the power.



Date......./........./.......

Page...................

Mean with σ known

1 sample ⇒ $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ $\bar{x} + z^* \dfrac{\sigma}{\sqrt{n}}$

2 sample $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$\bar{x}_1 - \bar{x}_2 \pm z^* \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Mean with σ unknown

1 sam ⇒ $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ $\bar{x} + t^* \dfrac{s}{\sqrt{n}}$

2 sam ⇒ $t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$

Matched pairs:

$d = x_1 - x_2$ $\bar{d} = \bar{x}_1 - \bar{x}_2$ $sd = sd(x_1 - x_2)$

Proportions (Only z)

1 sample $z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$ CI $\hat{p} \pm z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

2 sample $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n} - \frac{1}{n_2}\right)}}$ [Here, $\hat{p} = \dfrac{x_1 + x_2}{n + n_2}$]

Calc commands ⇒ normcdf, invcdf

CI $\hat{p}_1 - \hat{p}_2 \pm z^* \sqrt{\dfrac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \dfrac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

$\boxed{* - (1-\alpha)/2}$