$E[X] = \sum_{i=1}^{n} x_i \cdot f_X(x_i)$; $E[X^2] = \sum_{i=1}^{n} x_i^2 \cdot f_X(x_i)$; $\mathrm{var}(X) = E[X^2] - (E[X])^2$

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Session 15 Group Quiz

Assume that X, the value an unfair die, has the following probability mass function.

x	1	2	3	4	5	6
$f_X(x\|\alpha)$	$\frac{1}{6}(1-3\alpha)$	$\frac{1}{6}(1-2\alpha)$	$\frac{1}{6}(1-\alpha)$	$\frac{1}{6}(1+\alpha)$	$\frac{1}{6}(1-2\alpha)$	$\frac{1}{6}(1+3\alpha)$

a. What values for α are possible?

b. Find the mean $\mu = E_\alpha X$ and the variance $\sigma^2 = \mathrm{Var}_\alpha(X)$.

a. alpha could be positive or negative
positive: 0<=1/6(1-3a) and 1/6(1+3a)<=1
then 0<=a<=1/3
negative:0<=1/6(1+3a) and 1/6(1-3a)<=1
then -1/3<=a<=0
in sum -1/3<=a<=1/3

mean=1*(1/6-1/2a)+2*(1/6-1/3a)+3*(1/6-1/6a)+4*(1/6+1/6a)+5*(1/6+1/3a)+6*(1/6+1/2a)=3.5+(11/3)a

E(x^2)=1/6(1-3a) + 4/6(1-2a)+ 9/6(1-a)+ 16/6(1+a)+ 25/6(1+2a)+ 36/6(1+3a)=91/6 +a(-3/6-8/6-9/6+16/6+50/6+108/6 = 91/6 +(77/3)a
var=91/6 +(77/3)a -(3.5+11/3a)^2= (-121/9)a^2 + 35/12