S2S3Chpt

S2-10

1. The 95% confidence interval for the difference in proportions is [0.006942351, 0.163081746]. This doesn't contain the value 0.

```r
prop.test(c(250, 167), c(332, 250), conf.level = 0.98)
```

```
##
##  2-sample test for equality of proportions with continuity correction
##
## data:  c(250, 167) out of c(332, 250)
## X-squared = 4.664, df = 1, p-value = 0.0308
## alternative hypothesis: two.sided
## 98 percent confidence interval:
##  -0.006996134  0.177020231
## sample estimates:
##   prop 1   prop 2
## 0.753012 0.668000
```

2. The 98% confidence interval for the difference in proportions is [-0.006996134, 0.177020231]. This DOES contain the value 0.

3. The p-value in both cases is the same, that is, 0.0308. This is because the data that we provide in both cases was the same. The 98% interval contains 0 because the size of the interval is larger, and since the 95% interval was already very close to zero, the 98% interval covers just a little larger radius, thereby incorporating the value 0 into the interval.

4. A more severe winter must have less than 75.3012% hives surivive it. $H0 : p2 >= 0.753012$, that is, the survival rate of the hives in the second winter is greater than or equal to the rate in the first winter, which was 250/332. $H1 : p2 < 0.753012$, that is, the second winter was more severe.

```r
prop.test(c(250, 167), c(332, 250), alternative = c("less"))
```

```
##
##  2-sample test for equality of proportions with continuity correction
##
## data:  c(250, 167) out of c(332, 250)
## X-squared = 4.664, df = 1, p-value = 0.9846
## alternative hypothesis: less
## 95 percent confidence interval:
##  -1.0000000  0.1510939
## sample estimates:
##   prop 1   prop 2
## 0.753012 0.668000
```

Here, we can see that the p-value is 0.9846 which is quite high. This means that it is highly likely for the second winter to have a survival rate of 66.8% (as indicated by p2 = 66.8%).

S2-12

```
# The original test with alpha = 0.05
power.prop.test(n=c(250, 350, 450, 550),
p1=0.7, p2=0.6, sig.level = 0.05,
alternative = c("one.sided"))
```

```
##
##      Two-sample comparison of proportions power calculation
##
##               n = 250, 350, 450, 550
##              p1 = 0.7
##              p2 = 0.6
##       sig.level = 0.05
##           power = 0.7589896, 0.8717915, 0.9342626, 0.9672670
##     alternative = one.sided
##
## NOTE: n is number in *each* group
```

```
# Now the modified test with the value of alpha = 0.02
power.prop.test(n=c(250, 350, 450, 550),
p1=0.7, p2=0.6, sig.level = 0.02,
alternative = c("one.sided"))
```

```
##
##      Two-sample comparison of proportions power calculation
##
##               n = 250, 350, 450, 550
##              p1 = 0.7
##              p2 = 0.6
##       sig.level = 0.02
##           power = 0.6148167, 0.7653872, 0.8637122, 0.9237709
##     alternative = one.sided
##
## NOTE: n is number in *each* group
```

```
# To calculate the difference in powers
c(0.7589896, 0.8717915, 0.9342626, 0.9672670) - c(0.6148167, 0.7653872, 0.8637122, 0.9237709)
```

```
## [1] 0.1441729 0.1064043 0.0705504 0.0434961
```

We see that there is quite a significant decrease in power when we go from alpha = 0.05 to 0.02. We can also notice that as the sample size increases, the difference between the 2 reduces. The power decreases since when we decrease alpha, the critical value increases. meaning the probability to get a Type II error increases. Therefore, since it is more likely to get a Type II error, the power decreases.

$$L(\lambda_0 | n_m, n_f) = \frac{\lambda_0^{n_m+n_f}}{n_m!\, n_f!} e^{-2\lambda_0}$$

$$\ln L = (n_m + n_f) \ln(\lambda_0) - \ln(n_m!\, n_f!) - 2\lambda_0$$

$$\frac{\partial \ln L}{\partial \lambda_0} = \frac{n_m + n_f}{\lambda_0} - 0 - 2 = \frac{n_m + n_f}{\lambda_0} - 2$$

$$\frac{\partial \ln L}{\partial \lambda_0} = 0 = \frac{n_m + n_f}{\lambda_0} - 2$$

$$\boxed{\hat{\lambda}_0 = \frac{\cancel{2}\, n_m + n_f}{2}}$$

$$\frac{\partial^2}{\partial \lambda_0^2} \ln L = -\frac{(n_m + n_f)}{\lambda_0^2}$$

Since this double derivative is -ve, it is a maximum

Figure 1: S3 Chpt