s-worksheet - Extensions on the Likelihood Ratio Test

1. Snell's law tell us how light bends at an interface - the angle of incidence versus the angle of refraction - based on the ratio of the velocities of light in the two isotropic media. If the angle of incidence of a laser beam in a vacuum is θ_1 radians and the angle of refraction in an unknown medium is θ_2, then

$$n = \frac{\sin\theta_1}{\sin\theta_2}$$

is called the **index of refraction** ($\beta = 1/n$ is the velocity of light in the medium as a fraction of the speed of light in a vacuum.) Make repeated independent measurements in radians, $\theta_{1,1}, \theta_{1,2}, \cdots, \theta_{1,16}$ of the angle of incidence in vacuum and $\theta_{2,1}, \theta_{2,2}, \cdots, \theta_{2,16}$, of the angle of refraction in the second medium. If these measurements have

- sample mean $\overline{\theta}_1 = 0.786$ radians and standard deviation 0.03 radians in vacuum and
- sample mean $\overline{\theta}_2 = 0.326$ radians and standard deviation 0.06 radians in the unknown medium.

a. Snell's law gives an estimate $\hat{n}$ based on the values of $\overline{\theta}_1$ and $\overline{\theta}_2$. Use the delta method to estimate the mean and standard deviation of $\hat{n}$.

b. You suspect that the substance is cubic zirconia ($n_z = 2.165$) and not the claimed material, diamond ($n_d = 2.418$). For the hypothesis

$$H_0 : n = n_d \quad \text{versus} \quad H_1 : n = n_z,$$

use the information above to devise a z-test for the hypothesis and report a p-value for the test.

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# First we need to find the z-value (Which is wrt H0)
# using n = 2.209 and sd = 0.0994 which we found in Part (a)
# z = (n-n_d)/sd
z<-(2.209 - 2.418)/0.0994

# Now find the p-value using the pnorm function on z
(p<-pnorm(z))
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## [1] 0.01774969
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c. Describe what the p-value communicates in this case.

In this case, the p-value tells us that we can reject the hypothesis at alpha = 0.02, or the 2% level. This means that we can be 98% certain that the sample we received is not actually diamond.