(b) $P\{X > 2\alpha\}$

$$= \int_{2\alpha}^{\infty} f_X(x)\,dx = \lim_{t\to\infty} \int_{2\alpha}^{t} f_X(x)\,dx$$

$$\Rightarrow \underline{\beta \alpha^{\beta}} \lim_{t\to\infty} \int_{2\alpha}^{t} 1/x^{+\beta+1}\,dx$$

$$\Rightarrow \beta\alpha^{\beta} \lim_{t\to\infty} \int_{2\alpha}^{t} x^{-\beta-1}\,dx$$

$$\Rightarrow \beta\alpha^{\beta} \lim_{t\to\infty} \left[\frac{x^{-\beta}}{-\beta}\right]_{2\alpha}^{t}$$

$$\Rightarrow -\alpha^{\beta} \lim_{t\to\infty} \left[x^{-\beta}\right]_{2\alpha}^{t}$$

Goes to 0 because $\beta > 0$

$$\cancel{\Rightarrow -\alpha^{\beta} \lim_{t\to\infty} [t^{-\beta} - x^{-2\alpha}]}$$

$$\Rightarrow -\alpha^{\beta} \left[\boxed{t^{-\beta}} - 2\alpha^{-\beta}\right]$$

$$\Rightarrow (-\alpha^{\beta})(-(2\alpha)^{-\beta}) \Rightarrow \frac{\alpha^{\beta}}{(2\alpha)^{\beta}} = \boxed{\frac{1}{2^{\beta}}}$$

(c) Mean $= EX = \int_{\alpha}^{\infty} x\, f_X(x)\,dx$

$$= \lim_{t\to\infty} \int_{\alpha}^{t} \frac{\beta\alpha^{\beta}}{x^{\beta}}\,dx$$

$$= \beta\alpha^{\beta} \lim_{t\to\infty} \int_{\alpha}^{t} x^{-\beta}\,dx = \beta\alpha^{\beta} \lim_{t\to\infty}\left[\frac{x^{-\beta+1}}{-\beta+1}\right]$$

$$= \frac{\beta\alpha^{\beta}}{-\beta+1}\left[\lim_{t\to\infty} t^{-\beta+1} - \alpha^{-\beta+1}\right]$$

$$= \frac{\beta\alpha^{\beta}}{-\beta+1}\left[\lim_{t\to\infty} \frac{1}{t^{+\beta-1}} - \frac{1}{\alpha^{\beta-1}}\right]$$

= Note that $\beta > 1$ for it to converge

$$= \frac{\beta\alpha^{\beta}}{-\beta+1}\left(\frac{-1}{\alpha^{\beta-1}}\right) = \frac{\beta\alpha^{\beta}}{(\beta-1)(\alpha^{\beta-1})}$$

$$\cancel{= \frac{\beta}{(\beta-1)\alpha^{-1}}} = \boxed{\frac{\alpha\beta}{(\beta-1)}}$$

$$EX^2 = \int_{\alpha}^{\infty} x^2 \frac{\beta \alpha^\beta}{x^{\beta+1}}\, dx$$

$$= \int_{\alpha}^{\infty} \frac{\beta \alpha^\beta}{x^{\beta-1}}\, dx = \lim_{t\to\infty} \int_{\alpha}^{t} \frac{\beta\alpha^\beta}{x^{\beta-1}}\, dx$$

$$= (\beta\alpha^\beta) \lim_{t\to\infty} \int_{\alpha}^{t} x^{-\beta+1}\, dx$$

$$= (\beta\alpha^\beta) \lim_{t\to\infty} \left[\frac{x^{-\beta+2}}{-\beta+2}\right]_{\alpha}^{t}$$

$$= \frac{(\beta\alpha^\beta)}{(-\beta+2)} \lim_{t\to\infty} \left[t^{-\beta+2} - \alpha^{-\beta+2}\right]$$

$$= \left(\frac{\beta\alpha^\beta}{-\beta+2}\right) \lim_{t\to\infty} \left[\frac{1}{t^{\beta-2}} - \frac{1}{\alpha^{\beta-2}}\right]$$

$$= \frac{\beta\alpha^\beta}{-\beta+2}\left[\frac{-1}{\alpha^{\beta-2}}\right] \quad (\text{If } \beta > 2)$$

$$= \frac{\beta\alpha^\beta}{(\beta-2)(\alpha^{\beta-2})} = \boxed{\frac{\beta\alpha^2}{\beta-2}}$$

Variance $\Rightarrow EX^2 - (EX)^2$

$$\Rightarrow \frac{\beta\alpha^2}{\beta-2} - \left(\frac{\alpha\beta}{\beta-1}\right)^2$$

(d) $F_X(x) = \int_1^x f_X(t)\,dt$

We know $\alpha = 1$, $\beta = 4$

$$= \int_1^x \frac{4}{t^5}\,dt = \left[-\frac{1}{t^4}\right]_1^x$$

$$= -\frac{1}{x^4} + 1 \Rightarrow \boxed{1 - \frac{1}{x^4}}$$

$F_X^{-1}(x) \Rightarrow x = 1 - \frac{1}{u^4}$

$$\frac{1}{u^4} = 1 - x$$

$$u^4 = \frac{1}{1-x} = \boxed{u = \frac{1}{\sqrt[4]{1-x}}}$$

(e) Mean $\Rightarrow EX = \dfrac{\alpha\beta}{(\beta-1)} = \dfrac{(1)(4)}{(4-1)} = \boxed{\dfrac{4}{3}}$

$$SD(x) = \sqrt{Var(X)} = \sqrt{\frac{\beta\alpha^2}{\beta-2} - \left(\frac{\alpha\beta}{\beta-1}\right)^2}$$

$$= \sqrt{\frac{(4)(1)}{2} - \left(\frac{1 \cdot 4}{3}\right)^2}$$

$$= \sqrt{2 - 4/3} \Rightarrow \boxed{\sqrt{2/3}}$$