

s-worksheet - Extensions on the Likelihood Ratio Test

1. Snell's law tell us how light bends at an interface - the angle of incidence versus the angle of refraction - based on the ratio of the velocities of light in the two isotropic media. If the angle of incidence of a laser beam in a vacuum is θ_1 radians and the angle of refraction in an unknown medium is θ_2 , then

$$n = \frac{\sin \theta_1}{\sin \theta_2}$$

is called the **index of refraction** ($\beta = 1/n$ is the velocity of light in the medium as a fraction of the speed of light in a vacuum.) Make repeated independent measurements in radians, $\theta_{1,1}, \theta_{1,2}, \dots, \theta_{1,16}$ of the angle of incidence in vacuum and $\theta_{2,1}, \theta_{2,2}, \dots, \theta_{2,16}$, of the angle of refraction in the second medium. If these measurements have

- sample mean $\bar{\theta}_1 = 0.786$ radians and standard deviation 0.03 radians in vacuum and
 - sample mean $\bar{\theta}_2 = 0.326$ radians and standard deviation 0.06 radians in the unknown medium.
- a. Snell's law gives an estimate \hat{n} based on the values of $\bar{\theta}_1$ and $\bar{\theta}_2$. Use the delta method to estimate the mean and standard deviation of \hat{n} .
 - b. You suspect that the substance is cubic zirconia ($n_z = 2.165$) and not the claimed material, diamond ($n_d = 2.418$). For the hypothesis

$$H_0 : n = n_d \quad \text{versus} \quad H_1 : n = n_z,$$

use the information above to devise a z -test for the hypothesis and report a p -value for the test.

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# First we need to find the z-value (Which is wrt H0)
# using n = 2.209 and sd = 0.0994 which we found in Part (a)
# z = (n-n_d)/sd
z<-(2.209 - 2.418)/0.0994

# Now find the p-value using the pnorm function on z
(p<-pnorm(z))
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## [1] 0.01774969
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- c. Describe what the p -value communicates in this case.

In this case, the p -value tells us that we can reject the hypothesis at $\alpha = 0.02$, or the 2% level. This means that we can be 98% certain that the sample we received is not actually diamond.