① $\ln(f(x|\mu,\sigma)) = \ln\left(\frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}\right)$

$= \ln\left(\frac{1}{\sigma\sqrt{2\pi}}\right) - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$

$\frac{\partial}{\partial\mu} \ln f(x|\mu,\sigma) = 0 - \frac{2\cdot(-1)}{2}\left(\frac{x-\mu}{\sigma}\right)\left(\frac{1}{\sigma}\right)$

$= \frac{x-\mu}{\sigma^2}$

$\frac{\partial^2}{\partial^2\mu^2} \ln f(x|\mu,\sigma) = \frac{-1}{\sigma^2}$ | $I(\mu) = -\left(-\frac{1}{\sigma^2}\right) = \frac{1}{\sigma^2}$

② $var(\bar{X}) = var\left(\frac{1}{n}(X_1 + \cdots + X_n)\right) = \frac{1}{n^2}(var X_1 + \cdots + var X_n)$

$= \frac{1}{n^2}(n \, var\, X) = \frac{\sigma^2}{n} = \frac{1}{n^{-1}/\sigma^2} = \frac{1}{n I(\mu)}$

$= \frac{1}{I_n(\mu)}$

$$\frac{\partial^2 \ln L(p|\bar{x})}{\partial p^2} = \frac{\partial}{\partial p}\left[\frac{n\bar{x} - p}{p(1-p)}\right] = \frac{n(2p\bar{x} - p^2 - \bar{x})}{p^2(1-p)^2}$$

When $p = \bar{x}$ we get $= \dfrac{n(2\bar{x}^2 - \bar{x}^2 - \bar{x})}{\bar{x}^2(1-\bar{x})^2}$

$$= \frac{n(\bar{x}^2 - \bar{x})}{\bar{x}^2(1-\bar{x})^2}$$

Now since $\bar{x}^2 - \bar{x} < 0$

This term is < 0 because $\bar{x}^2 - \bar{x} < 0$ & $\bar{x}^2(1-\bar{x})^2 > 0$

Therefore, this is a maximum