i-worksheet: Examples of Mass Functions and Densities

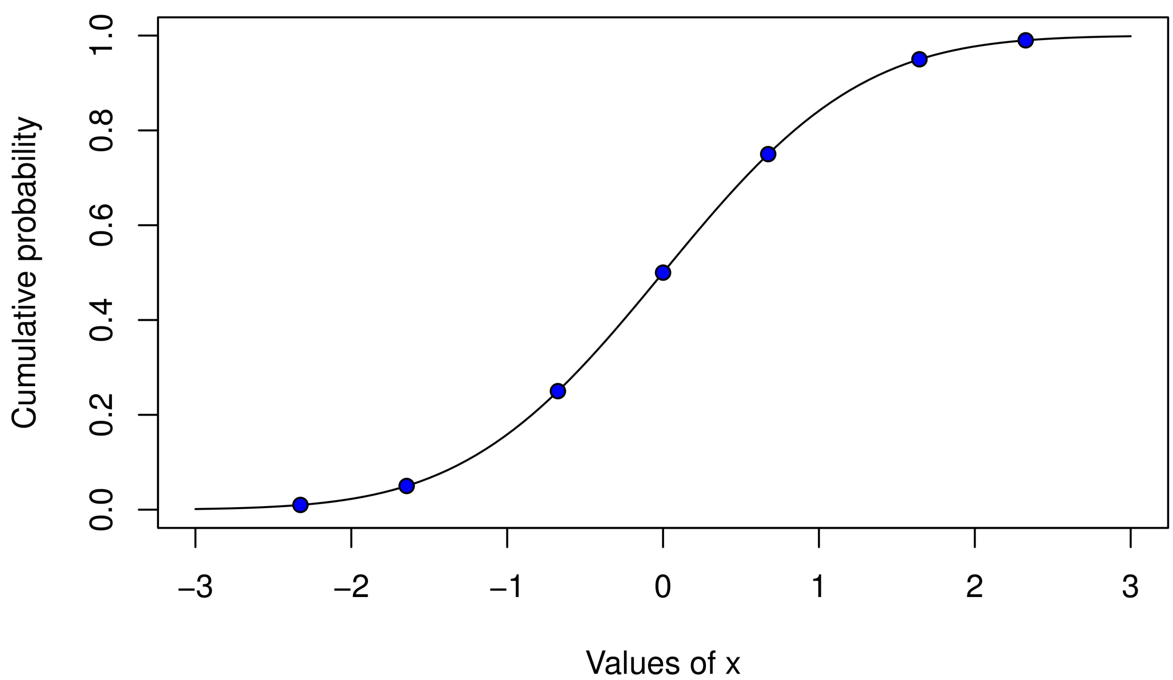
1. In this problem, we shall use **R** to calculate probabilities and quantiles for random variables.

(a) For Z a standard normal, find values for z so that $P\{Z \leq z\} = 0.01, 0.05, 0.25, 0.50, 0.75, 0.95, 0.99.$. Indicate these values on a plot of the distribution function for Z.

```
quarts<-c(0.01, 0.05, 0.25, 0.50, 0.75, 0.95, 0.99)
z<-qnorm(quarts)

curve(pnorm(x),
      from = -3,
      to = 3,
      xlab = "Values of x",
      ylab = "Cumulative probability")

points(z, quarts, pch = 21, bg = "blue")
```



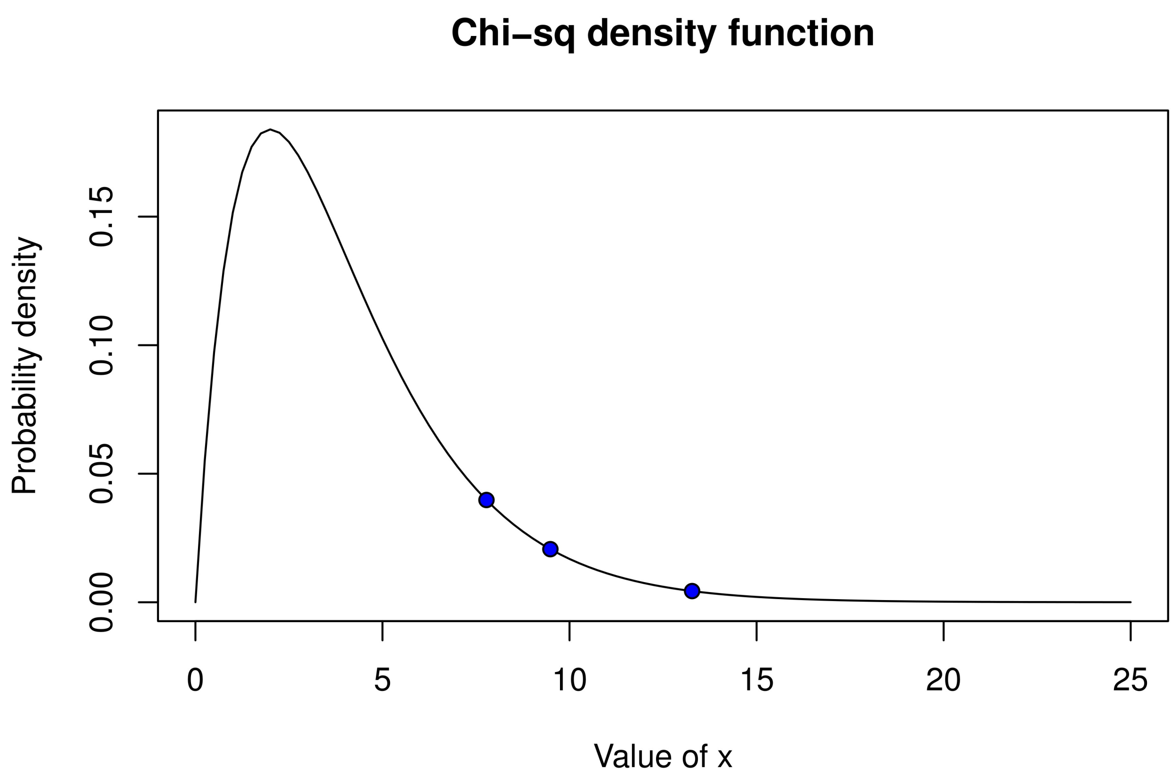
(b) For X a χ^2_4 random variable find values for x so that $P\{X > x\} = 0.10, 0.05, 0.01$. Indicate these values on a plot of the density function for X.

```
prob<-c(0.10, 0.05, 0.01) # Storing the values of x that we need

qvals<-qchisq(1-prob, df=4) # We are doing 1 - the values because we need X > x but we have X <= x
```

```r
curve(dchisq(x, df=4), 0, 25,
      main = "Chi-sq density function",
      xlab = "Value of x",
      ylab = "Probability density")

points(qvals, dchisq(qvals, df=4), pch=21, bg="blue")
```



(c) Simulate 1000 independent beta random variables with $\alpha = 2$ and $\beta = 4$. Find the mean and variance of this sample and compare it to the distributional mean and variance:

$$E[X] = \frac{\alpha}{\alpha+\beta}, var(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

```r
sim<-rbeta(1000, 2, 4)
mean(sim)
```

```
## [1] 0.339675
```

```r
var(sim)
```

```
## [1] 0.03246171
```

Now when we substitute the value of alpha and beta into the given equations for the mean and variance, we get:

Mean = 2/(2+4) = 1/3 = 0.33333 which we see is roughly equal to the value we got from our simulation

Variance = (2x4)/(2+4)^2 x (2 + 4 + 1) = 8/252 = 0.031746 which we see is roughly the same as the value we got from our simulation