Midterm Report
Option Pricing Models and Their Accuracy

Vatsh Van
Atreem Das
Bhuvneshwar Rathore

Mentor: Krish Bansal
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1. Introduction

This report compares two foundational models for pricing European call options, the **Binomial Model** and the **Black-Scholes Model**. Using hypothetical data for Infosys (INFY), we calculated the price of a near-the-money European call option under both frameworks, analyzed their outputs, and studied the sensitivity of the option's value to changes in the underlying stock, captured through *Delta*.

2. Option Scenario

Parameter	**Value**
Underlying Stock	Infosys (INFY)
Spot Price (S_0)	1,333.30
Strike Price (K)	1,340.00
Time to Expiry (T)	14 days = 0.03836 years
Price at Expiry	1,349.85
Risk-Free Rate (r)	6% p.a.
Volatility (σ)	30% annualised
Option Type	European Call

3. The Binomial Model

Overview

The Binomial Model, introduced by Cox, Ross and Rubinstein in 1979, is a widely used numerical method for option pricing. It models the underlying stock price as evolving through discrete time steps, where at each step, the price can move **up** or **down** by predefined factors. This approach is especially useful for valuing European and American options and it offers significant flexibility in modeling various option features and payoffs.

Assumptions

The model is based on several key assumptions:

- The price of the underlying asset follows a multiplicative binomial process.
- At each time step, the stock can move up by a factor u or down by a factor d.
- Markets are frictionless — i.e., no transaction costs, taxes or constraints on short selling.
- The risk-free interest rate r is constant throughout the option's life.
- There are no arbitrage opportunities.
- The option considered is European-style, meaning it can only be exercised at expiry.
- Trading occurs in discrete time.

Construction and Methodology

We implemented a 10-step binomial tree to price a European call option on Infosys (INFY). The method follows these steps:

1. **Discretization:** Time to maturity T is divided into $n = 10$ equal steps of length $\Delta t = T/n$.
2. **Price Tree Construction:**
   - Up factor: $u = e^{\sigma\sqrt{\Delta t}}$
   - Down factor: $d = 1/u$
   - Risk-neutral probability: $p = \frac{e^{r\Delta t}-d}{u-d}$

   Using these values, the stock price at any node (j, t) in the tree is given by:
   $$S_{j,t} = S_0 \cdot u^{t-j} \cdot d^j$$
3. **Terminal Payoff:** At expiry $(t = n)$, the call option value at each node is:
   $$C_{j,n} = \max(S_{j,n} - K, 0)$$
4. **Backward Induction:** The option value at earlier nodes is computed using the risk-neutral expected value discounted at the risk-free rate:
   $$C_{j,t} = e^{-r\Delta t} \cdot [p \cdot C_{j,t+1} + (1-p) \cdot C_{j+1,t+1}]$$
5. **Final Value:** The value at the root of the tree $(j = 0, t = 0)$ gives the option's fair price today.

Delta Calculation

To measure the sensitivity of the option price to changes in the underlying stock price, we computed the **Delta** at the initial time step using:

$$\Delta = \frac{C_{\text{up}} - C_{\text{down}}}{S_{\text{up}} - S_{\text{down}}}$$

Where:

- C_{up} and C_{down} are option values at $t = 1$ after one up or one down move
- S_{up} and S_{down} are the corresponding stock prices at those nodes

This gives us the hedge ratio — the number of shares to hold per option sold.

Results

Using the parameters for the Infosys option and a 10-step tree:

- **Option Price at $t = 0$:** 18.27
- **Delta (at $t = 0$):** 0.384

Interpretation and Remarks

The Binomial Model provides an intuitive way to visualize option evolution and valuation. While the model underestimates the option price compared to the Black-Scholes formula with only 10 steps, it produces an accurate Delta. Increasing the number of steps would make the price converge to the Black-Scholes result. The model's step-by-step backward induction also makes it especially valuable for handling American options and path-dependent features that the Black-Scholes model cannot easily accommodate.

4. The Black-Scholes Model

Overview

The Black-Scholes Model, developed by Fischer Black, Myron Scholes and Robert Merton in 1973, offers a closed-form analytical solution to the problem of pricing European-style options. Unlike the discrete-time binomial framework, this model operates in continuous time and assumes the stock price follows a geometric Brownian motion (GBM) with constant volatility and drift.

It became a cornerstone of modern financial theory and laid the foundation for the development of derivatives markets globally.

Mathematical Formulation

The price C of a European call option is given by the Black-Scholes formula:

$$C = S_0 \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2)$$

Where:

$$d_1 = \frac{\ln(S_0/K) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

Here:

- S_0 is the current stock price
- K is the strike price
- T is the time to maturity (in years)
- r is the annualized risk-free interest rate
- σ is the annualized volatility of the stock
- $N(d)$ is the cumulative distribution function (CDF) of the standard normal distribution

Assumptions

The Black-Scholes model is built upon several simplifying assumptions:

- The stock price follows a continuous geometric Brownian motion with constant volatility.
- The risk-free rate is constant and known.
- No dividends are paid during the life of the option.
- Markets are frictionless — there are no transaction costs or taxes.
- Trading of the underlying asset is continuous.
- There are no arbitrage opportunities.
- The option is European and can only be exercised at expiry.

Implementation in Excel

To make the model fully transparent and computationally replicable, we broke down the formula into the following intermediate steps:

1. Compute $\ln(S_0/K)$ — the log moneyness.
2. Compute $\sigma^2/2$, and then add it to r.
3. Multiply the above by T to form the numerator of d_1.
4. Compute $\sigma\sqrt{T}$ for the denominator.
5. Evaluate d_1 and d_2.
6. Apply standard normal CDFs to get $N(d_1)$ and $N(d_2)$.
7. Substitute in the main formula to get the final call option price.

Results

Using the given Infosys option data and substituting into the Black-Scholes formula, we obtained the following results:

- **Black-Scholes Option Price:** 29.51
- **Delta (Call):** 0.493

The Delta computed here, $N(d_1)$, represents the sensitivity of the call option price to small changes in the underlying stock price. It tells us how many shares of the stock are needed to hedge one short call option.

Remarks

The Black-Scholes model is widely used in financial institutions due to its speed and simplicity. However, it is important to note that its assumptions — such as constant volatility and no dividends, often do not hold in real markets. For instance, stock returns may exhibit jumps or volatility clustering and dividends are common in equity markets.

Despite these limitations, the model performs remarkably well in practice for European options and serves as a benchmark for evaluating other pricing models.

5. Delta Comparison

Understanding Delta

Delta (Δ) is one of the most fundamental *Greeks* in option pricing. It represents the rate of change of the option price with respect to a change in the underlying asset's price:

$$\Delta = \frac{\partial C}{\partial S}$$

In practical terms, Delta also reflects the *hedge ratio*: the number of shares required to hedge one short call option. A Delta of 0.5, for example, implies buying 0.5 shares per call option to maintain a neutral position against small stock price movements.

Computation in Models

- In the **Black-Scholes model**, Delta for a European call option is directly given by $N(d_1)$.
- In the **Binomial model**, Delta is approximated at $t = 0$ using the discrete difference formula:

$$\Delta = \frac{C_{\text{up}} - C_{\text{down}}}{S_{\text{up}} - S_{\text{down}}}$$

  where C_{up} and C_{down} are option values after an up or down move in the first time step, and S_{up}, S_{down} are the corresponding stock prices.

Comparison Table

Model	**Option Price ()**	**Delta**
Binomial Model	18.27	0.384
Black-Scholes	29.51	0.493

Interpretation

Although the Delta values differ slightly between the two models, both lie within a realistic and actionable range. The Black-Scholes Delta of 0.493 suggests buying approximately 49 shares per 100 options sold, while the Binomial model, with fewer steps (only 10), gives a slightly lower hedge ratio of 0.384. This deviation is expected — the Binomial model becomes more accurate and converges to the Black-Scholes value as the number of steps increases.

In practice, traders often prefer the Black-Scholes Delta for quick hedging decisions due to its closed-form nature, but the Binomial Delta can be more flexible when modeling American or path-dependent options.

6. Hedging Analysis

Objective of Hedging

Hedging an option position involves constructing a portfolio that is immune to small fluctuations in the price of the underlying asset. For a short call position, this is typically achieved by holding a number of shares proportional to the option's **Delta**. The effectiveness of a hedge depends critically on how accurately the model estimates Delta and anticipates the option's sensitivity to underlying price movements.

Scenario Evaluation

In our case, the option was initiated when the stock price of Infosys was 1,333.30. The strike price was 1,340, making it a slightly out-of-the-money European call at inception. By the time of expiry, the stock had risen to 1,349.85, placing the option marginally **in-the-money**.

This move required the initial hedge to absorb a moderate upward movement in the stock price. Let's analyze how each model performed under this scenario:

- **Binomial Model Hedge:** With a Delta of approximately 0.384, the model recommended holding 38 shares per 100 call options. This lower Delta under-hedged the portfolio, meaning the short option seller would have needed to purchase additional shares later or suffer a loss relative to the hedge.
- **Black-Scholes Hedge:** The Delta of 0.493 implied a more conservative hedge, recommending 49 shares per 100 options. This higher hedge ratio better captured the eventual movement of the stock into the money and resulted in a more effective hedge.

Effectiveness Comparison

While both models yielded positive payoffs at expiry, the Black-Scholes hedge was better aligned with the actual price path of the stock. The Binomial model, constrained by the use of only 10 time steps, resulted in a coarser approximation of the option's behavior, especially for short-term options where the number of steps significantly impacts accuracy.

However, it's important to note that the Binomial model's pricing and Delta converge to the Black-Scholes values as the number of steps increases. If we had used 100 or more steps, the hedge effectiveness of the Binomial model would have improved substantially.

Takeaway

This hedging exercise highlights the practical importance of choosing the right model and parameters based on context:

- For fast computations and continuous hedging strategies, the Black-Scholes model offers speed and accuracy.
- For discrete trading environments or exotic options, the Binomial model provides flexibility and adaptability — though it requires finer discretization for accuracy.

In this case, Black-Scholes outperformed due to better Delta estimation, but both models affirm the importance of dynamic hedging strategies in options trading.

7. Conclusion

Both models offer valuable insights:

- Binomial is intuitive, flexible and accurate with high steps.
- Black-Scholes is fast, closed-form and widely accepted.

For this option, Black-Scholes gave a higher price (29.51) compared to Binomial (18.27), yet both agreed closely on Delta. The experience reinforced our understanding of hedging and risk-neutral pricing.

Appendix: Sheet Overview

Sheet Name	Description
Inputs	Spot, strike, time, volatility, rate inputs
Stock Tree	10-step binomial stock price tree
Option Tree	Backward induction for call pricing
Black-Scholes	Formula breakdown + Delta and Call price
Summary	Side-by-side comparison of both models