Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2021

Test 3 – (10%)

3.2.5, 3.3.1 to 3.3.16, 4.1.1 to 4.1.8

Time Allowed 25 minutes	**First Name**	**Surname**	**Marks** 26 marks

Circle your Teacher's Name:

Mrs Alvaro	Mrs Bestall	Ms Chua
Mr Gibbon	Mrs Greenaway	Mr Luzuk
Mrs Murray	Ms Robinson	Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Not Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART A – CALCULATOR FREE

Question 1

[2, 2 – 4 marks]

a) Simplify the expression $\log_2 12 + \log_2 10 - \log_2 15$.

$= \log_2(120) - \log_2(15)$

$= \log_2(120 \div 15)$

$= \log_2(8)$

$= 3$

b) If $\log_3 y = 3\log_3 x + 2$, express y in terms of x.

$\log_3 y = \log_3 x^3 + \log_3 9$

$\log_3 y = \log_3 x^3 \times 9$

$y = 9x^3$

Question 2 [2, 2 – 4 marks]

Solve for x.

a) $2\ln x - \ln 9 = 0$

$$2\ln x = \ln 9$$
$$\ln x = \frac{\ln 9}{2}$$
$$\ln x = \frac{1}{2}\ln 9$$
$$\ln x = \ln 3$$
$$x = 3$$

b) $5\log_2(4x - \frac{1}{2}) = -5$

$$\log_2\left(4x - \frac{1}{2}\right) = -1$$
$$4x - \frac{1}{2} = \frac{1}{2}$$
$$4x = 1$$
$$x = \frac{1}{4}$$

Question 3 [3 marks]

For each of the following scenarios, state whether the random variable has a binomial distribution:

a) The number of times a coin is tossed before a head is observed. No

b) The number of sixes observed when a die is rolled 10 times. Yes

c) The height in centimetres of a randomly chosen student. No

Question 4 [2 marks]

Let X be the number of times a shooter hits a target in a competition. If the distribution of X is binomial and the probability of success is 0.67, state an expression for the probability of the shooter hitting the target three times out of five attempts (do not evaluate the expression).

$X \sim Bin(5, 0.67)$

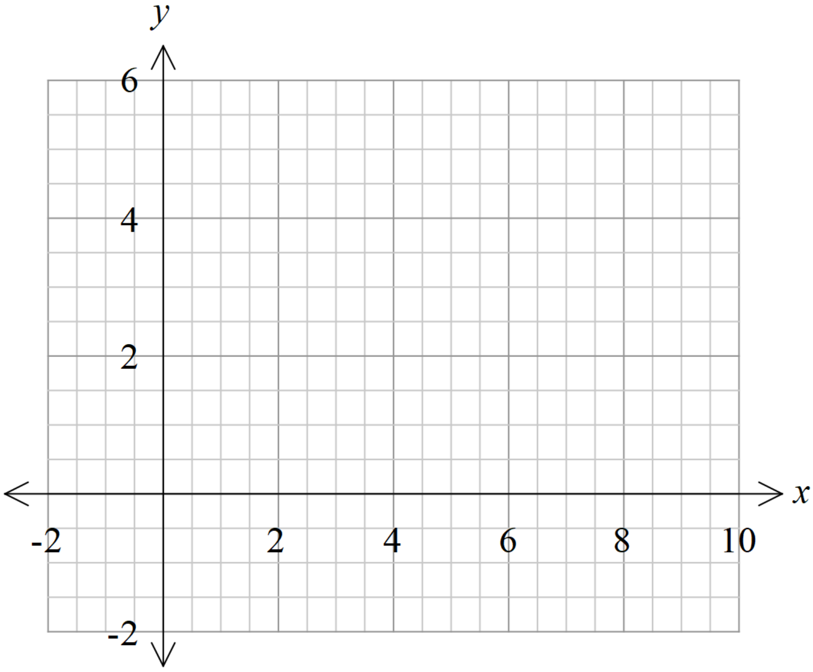
where x is number of times a shooter hits a target.

$$\begin{array}{r} 0\ \cancel{1}\cancel{0}0 \\ 67 \\ \hline 33 \end{array}$$

$$P(x=3) = {}^5C_3 \times 0.67^3 \times (1-0.67)^2$$
$$= {}^5C_3 \times 0.67^3 \times 0.33^2$$

Question 5 **[3 marks]**

Sketch the graph of $y = \ln(x-2)+3$ on the axes provided.



Question 6 **[2, 2 – 4 marks]**

A random variable X is such that its mean is 4 and its standard deviation is 3.

a) Find $E(X^2)$.

$SD(x) = \sqrt{E(x^2) - [E(x)]^2}$

$3 = \sqrt{E(x^2) - 4^2}$

$9 = E(x^2) - 16$

$E(x^2) = 25$

b) If $Y = 2X - 7$, find $E(Y)$ and $Var(Y)$.

$E(Y) = 2E(x) - 7$

$= 2 \times 4 - 7$

$= 1$

$Var(Y) = 2^2 \times Var(x)$

$= 2^2 \times 3^2$

$= 36$

Question 7 **[$\cancel{2}$, 4 – 6 marks]**

a) If $f'(x) = -8\sin 4x$ and $f\left(\frac{3\pi}{4}\right) = 1$, find $f(x)$.

(

$$f(x) = \int f'(x)$$

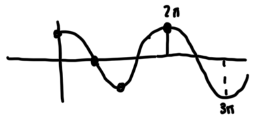
$$= \int -8\sin 4x$$

$$= 2\cos 4x + c \quad \Rightarrow \text{sub } \left(\tfrac{3\pi}{4}, 1\right)$$

$$1 = 2\cos(3\pi) + c$$

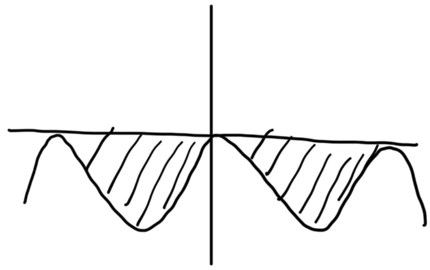
$$1 = -2 + c$$

$$c = 3$$



$\therefore \; f(x) = 2\cos 4x + 3$

b) Calculate the exact area enclosed between the curve $y = 3\cos(2x) - 3$ and the x-axis between the lines $x = -\pi$ and $x = \pi$.



$$A = \int_{-\pi}^{\pi} 3\cos(2x) - 3\, dx$$

$$= \left[\tfrac{3}{2}\sin(2x) - 3x\right]_{-\pi}^{\pi}$$

$$= -3\pi - (0 + 3\pi)$$

$$= -6\pi$$

$\therefore \; A = 6\pi \; u^2$

Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2021

Test 2 – (10%)

3.2.5, 3.3.1 to 3.3.16, 4.1.1 to 4.1.8

Time Allowed 25 minutes	**First Name**	**Surname**	**Marks** 26 marks

Circle your Teacher's Name:

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Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART B – CALCULATOR ALLOWED

Question 8 **[3, 1, 1 – 5 marks]**

In order to assess the quality of a handheld game console, the manufacturer selects eight consoles at random before they are packaged and inspects them. It is known that 5% of the consoles are defective. Let X be the number of defective consoles in the selection.

a) Find the probability that exactly one of the consoles in the sample is defective, given that at least one of the consoles is defective (round to 4dp).

$X \sim Bin(8, 0.05)$

where X is number of defective consoles

~~$P(X=1) = BinPDF(1, 8, 0.05)$~~
~~$= 0.2793$~~

$P(X=1 \mid X \geq 1) = \dfrac{P(X=1)}{P(X \geq 1)} = \dfrac{0.2793}{0.336579}$

$= \dfrac{BinPDF(1,8,0.05)}{BinCDF(1,8,8,0.05)}$

$= 0.82981 \approx 0.8298$

b) Find the expected number of defective consoles in the sample.

$E(X) = np$
$= 8 \times 0.05$
$= 0.4$

c) Find the standard deviation of the number of defective consoles in the sample.

$SD(X) = \sqrt{np(1-p)}$
$= \sqrt{0.4 \times 0.95}$
$= 0.6164$

Question 9 **[3, 5, 4 – 12 marks]**

Sasha has a biased die. When it is rolled, the probability of obtaining a particular outcome (X) is shown in the following table:

x	1	2	3	4	5	6
$P(X=x)$	0.1	0.2	0.3	0.2	0.1	p

a) Find:

i. The value of p

$p = 0.1$

ii. $E(X)$

$E(x) = 3.3$ C.A.S

iii. The standard deviation of X (to 4dp)

$SD(x) = 1.4177$

b) Suppose that Sasha has two fair dice, as well as the biased one described above. She puts all three in a bag then asks her friend to select a die and roll it.

i. Show that the probability that a six is rolled is $\frac{13}{90}$.

$P(x=6) = \left(\frac{1}{3} \times \frac{1}{6}\right) \times 2 + 0.1 \times \frac{1}{3}$

$= \frac{1}{9} + \frac{1}{30}$

$= \frac{13}{90}$

ii. What is the probability that, if the die did show a six, it came from the biased die?

$P(X = 6 \mid X = \text{Biased}) = \frac{0.1}{1/3}$

$= 0.3$

c) To make the game from part (b) more interesting, Sasha offers to pay her friend $10 if a six is rolled, otherwise she pays nothing. How much, to the nearest cent, should Sasha ask as a fee to play the game if the game is to be fair? If a six is rolled, the fee is refunded to the player, in addition to the $10 winnings.

x	x	-10
$P(X=x)$	$\frac{77}{90}$	$\frac{13}{90}$

where x is profit.

fair game $E(x)=0$

$E(x)=0$

$0 = x \times \frac{77}{90} + -10 \times \frac{13}{90}$

$x = \frac{130}{77}$

$= \$1.69$

Question 10 **[4, 2, 1, 2 – 9 marks]**

A particle, initially at the origin with a velocity of $18 m/s$, moves in a straight line such that its acceleration after t seconds is given by $a(t) = -27\sin\left(\frac{3t}{2}\right)$.

a) Find an expression for the displacement $x(t)$.

$v(t) = \int -27\sin\left(\frac{3t}{2}\right)dt$

$= 18\cos\left(\frac{3t}{2}\right) + c \Rightarrow \text{Sub } (0, 16)$

$18 = 18 + c$

$c = 0$

$x(t) = \int v(t)$

$= \int 18\cos\left(\frac{3t}{2}\right)dt$

$= 12\sin\left(\frac{3t}{2}\right) + c \Rightarrow \text{Sub } (0,0)$

$0 = 12\sin\left(\frac{3t}{2}\right) + c$

$c = 0$

$\therefore x(t) = 12\sin\left(\frac{3t}{2}\right)$

b) At what time does the particle first return to the origin?

$12\sin\left(\frac{3t}{2}\right) = 0$

$t = \frac{2\pi}{3}$

c) State the maximum distance of the particle from the origin during the motion.

Maximum displacement is 12 m

d) How far does the particle travel in the first 3 seconds?

distance $= \int_0^3 \left|18\cos\left(\frac{3t}{2}\right)\right| dt$

$= 35.73$ m