

FACULTY
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AND PHYSICS
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BACHELOR THESIS

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Martin Vavřík

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**Simulation and Reconstruction
of Charged Particle Trajectories
in an Atypic Time Projection Chamber**

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Institute of Particle and Nuclear Physics

6

Supervisor of the bachelor thesis: Mgr. Tomáš Sýkora, Ph.D.

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Study programme: Physics

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Prague 2025

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Title: Simulation and Reconstruction of Charged Particle Trajectories in an Atypical Time Projection Chamber **Added hyphen to avoid overfull hbox**

Author: Martin Vavřík

Institute: Institute of Particle and Nuclear Physics

Supervisor: Mgr. Tomáš Sýkora, Ph.D., Institute of Particle and Nuclear Physics

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56 Motivation

57 A Time Projection Chamber (TPC) [refs] is a type of gaseous detector that detects
58 charged particle trajectories by measuring the positions and drift time of ions cre-
59 ated in the gas. The energies of these particles can be inferred from the curvatures
60 of their trajectories in the magnetic field (specific field inside the TPC).

61 The goal of this thesis is to develop an algorithm for the reconstruction of
62 charged particle trajectories and energy in an *atypic* TPC with orthogonal elec-
63 tric and magnetic fields, hereafter referred to as the Orthogonal Fields TPC
64 (OFTPC), used in the X17 project at the Institute of Experimental and Applied
65 Physics, Czech Technical University in Prague (IEAP CTU). Furthermore, we
66 present the results of testing of several (gradually improving) developed algo-
67 rithms with different samples of simulated data. Put this somewhere, (maybe
68 just the abstract?). We use the Garfield++ toolkit [1] for simulations in combina-
69 tion with the ROOT framework [2] for data analysis and visualization. Some of
70 our more demanding simulations are run on the MetaCentrum grid [3].

71 The X17 project in IEAP CTU aims to reproduce measurements of anomalous
72 behavior in the angular correlation distribution of pairs produced by the Internal
73 Pair Creation (IPC) mechanism [4] during the decay of certain excited nuclei
74 (${}^8\text{Be}$, ${}^{12}\text{C}$, and ${}^4\text{He}$) observed by a team at ATOMKI in Hungary. I would leave
75 this here as a short summary before I explain it in more detail in the sections
76 below.

77 Add citations: X17 project, VdG. Maybe also TPC, etc.

78 0.1 ATOMKI Anomaly

79 Many different theories propose the existence of *new light bosons* that are weakly
80 coupled to ordinary matter [5]. These particles are potential dark matter candi-
81 dates and could solve other issues with the Standard Model, such as the strong
82 CP problem and the anomalous muon magnetic moment.

83 A possible way of detecting such bosons with a short lifetime is to observe
84 nuclear transitions of excited nuclei. If a boson was emitted during the transition
85 and subsequently decayed into an electron-positron pair, we could observe this
86 as a peak on top of the standard e^+e^- angular correlation from the Internal Pair
87 Creation (IPC) and the External Pair Creation (EPC).

88 0.1.1 ATOMKI Measurements

89 Historically, there were several measurements of the IPC in nuclear transitions
90 in ${}^8\text{Be}$ at Institute für Kernphysik (Frankfurt) [6, 7, 8] and at ATOMKI [9, 10]
91 resulting in different anomalies with invariant mass in the range 5–15 MeV. This
92 prompted a development of a better spectrometer at ATOMKI.

93 In 2015, a group at ATOMKI led by Attila Krasznahorkay observed an anom-
94 arious IPC in ${}^8\text{Be}$ [11]. They used the ${}^7\text{Li}(p, \gamma){}^8\text{Be}$ reaction at the $E_p = 1030$ keV
95 proton capture resonance to prepare the 18.15 MeV excited state ($J^\pi = 1^+, T =$
96 = 0). This state decays predominantly through M1 transitions to the ground
97 state ($J^\pi = 0^+, T = 0$) and to the 3.03 MeV state ($J^\pi = 2^+, T = 0$) [12].

98 The angular correlation of the e^+e^- pairs created internally in these transitions
99 were measured and compared to the simulation; results from a narrow $E_{\text{sum}} =$
100 $= 18$ MeV region are shown in Figure 0.1a. The simulation includes boson decay
101 pairs for different boson masses. The disparity parameter y is defined as

$$y = \frac{E_{e^-} - E_{e^+}}{E_{e^-} + E_{e^+}}, \quad (0.1)$$

102 where E_{e^-} and E_{e^+} are the kinetic energies of the electron and positron.

103 Their experimental setup was later upgraded ([details?](#)) and used for new mea-
104 surements. In 2022 the ${}^8\text{Be}$ anomaly was also measured using the $E_p = 441$ keV
105 resonance to produce the 17.64 MeV excited state ($J^\pi = 1^+$, $T = 1$) which again
106 decays primarily to the ground state and the 3.03 MeV state [12]. The anomaly
107 was also measured for $E_p = 650$ and 800 keV where E1 transitions from the direct
108 proton capture dominate [13]. The results for e^+e^- with $E_{\text{sum}} \in [13.5, 20]$ MeV
109 are shown in Figure 0.1b.

110 The newer setup was also used in 2021 to study the ${}^3\text{H}(p, e^+e^-){}^4\text{He}$ reaction at
111 $E_p = 510, 610$ and 900 keV [14], inducing direct and resonant capture populating
112 the overlapping first 20.21 MeV ($J^\pi = 0^+$) and second 21.01 MeV ($J^\pi = 0^-$)
113 excited states [15]. The comparison of simulated and measured e^+e^- pair angular
114 correlations in the $E_{\text{sum}} \in [18, 22]$ MeV region is shown in Figure 0.1c.

115 In 2022, another anomaly was measured in the ${}^{11}\text{B}(p, e^+e^-){}^{12}\text{C}$ process [16].
116 The $E_p = 1388$ keV resonance was used to populate the 17.23 MeV excited state
117 ($J^\pi = 1^-, T = 1$) with a large width $\Gamma = 1.15$ MeV [17]. This state decays
118 mainly through E1 transitions to the ground state $J^\pi = 0^+$ and to the 4.44 MeV
119 state $J^\pi = 2^+$. To compensate for energy losses in the target, five energies in
120 the range $E_p = 1.5\text{--}2.5$ MeV were used. The experimental angular correlation for
121 the 17.23 MeV transition to the ground state is shown in Figure 0.1d.

122 Possible explanations of the anomaly include experimental effects, higher or-
123 der processes in the Standard Model [18, 19] or even a protophobic fifth force
124 mediated by a new 17 MeV boson X17 [20]. [Zhang and Miller: <https://www.sciencedirect.com/science/article/pii/S0370269321000010>](https://www.sciencedirect.com/science/article/pii/S0370269321000010)

126 0.1.2 Other Experiments

127 Since the ATOMKI measurements, several experiments have been initiated to
128 attempt to replicate the results and search for the hypothetical X17 particle.
129 [Here are a few with results. Could cite the ATOMKI review paper here.](#)

130 Two-arm e^+e^- spectrometer in Hanoi

131 The anomaly in ${}^8\text{Be}$ has been observed with $> 4\sigma$ confidence by a team at the
132 Hanoi University of Sciences for $E_p = 1225$ keV [21]. They built a two-arm
133 spectrometer in collaboration with ATOMKI and calibrated it using the 17.6 MeV
134 M1 transition. The results are shown in Figure 0.2.

135 Collisions at Nuclotron in Dubna

136 At the Joint Institute for Nuclear Research in Dubna, signal in the form of en-
137 hanced structures in the $\gamma\gamma$ spectra at ~ 17 and ~ 38 MeV invariant masses

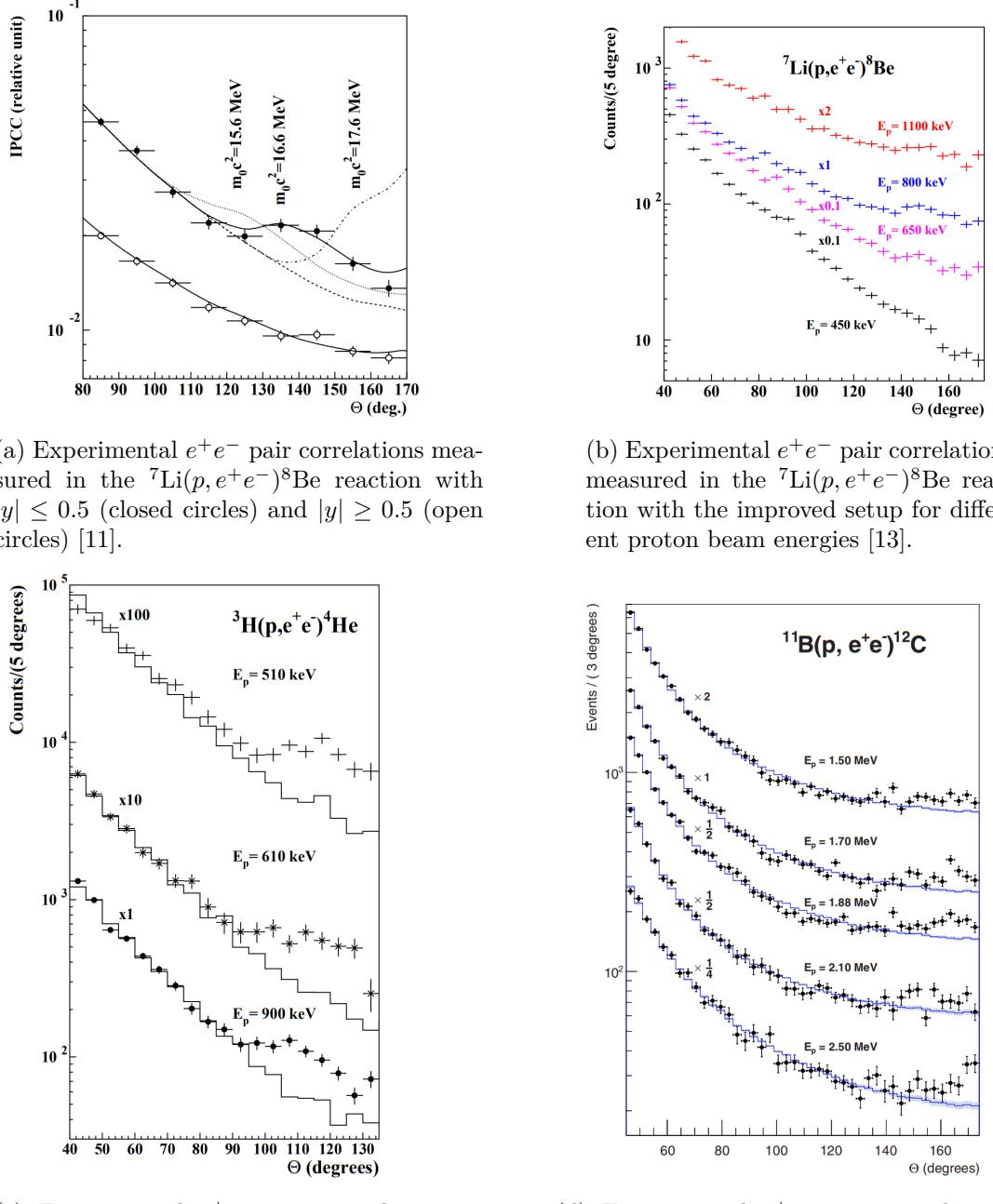


Figure 0.1: The ATOMKI anomalous IPC measured for different nuclei.



Figure 0.2: Results from the Hanoi spectrometer – angular e^+e^- pair correlations measured in the ${}^7\text{Li}(p, e^+e^-){}^8\text{Be}$ reaction at $E_p = 1225$ keV [21].

for $p + \text{C}$, $d + \text{C}$ and $d + \text{Cu}$ reactions at momenta 5.5, 2.75, and 3.83 GeV per nucleon [22]. Monte Carlo simulations support the conclusion that the signals are a consequence of a decay of unknown particles X17 and E38.

141 The MEG II (Muon Electron Gamma) experiment

142 Experiments using the ${}^7\text{Li}(p, e^+e^-){}^8\text{Be}$ reaction were carried out at the Paul Scherrer Institute with the MEG II superconducting solenoid spectrometer [23].
 143 Analysis of the data with $E_p = 1080$ keV exciting both of the resonances (beam
 144 fully stopping in the target) found no significant evidence supporting the X17
 145 hypothesis, results are shown in Figure 0.3. An upper bound (at 90% confidence)
 146 on the X17-to- γ branching ratio was set at $1.2 \cdot 10^{-5}$ for the 18.15 MeV state
 147 (larger than the ratio $5.8 \cdot 10^{-6}$ obtained by ATOMKI in 2016). Could add their
 148 90% C.L bounds figure also.
 149

150 0.2 X17 Project at IEAP CTU

151 The aim of the X17 project at the Van der Graaff facility of the Institute of
 152 Experimental and Applied Physics, Czech Technical University in Prague is to
 153 repeat the original ATOMKI experiments with ${}^7\text{Li}$ and ${}^3\text{H}$ targets using an inde-
 154 pendent e^+e^- spectrometer. In order to effectively measure the anomaly, we need
 155 to reconstruct both the energy and the angular correlation of the e^+e^- pairs. The
 156 spectrometer will use three layers of detectors to achieve this – Timepix 3 (Tpx3)
 157 silicon pixel detector and Multi-Wire Proportional Chamber (MWPC) layers for
 158 the angle reconstruction and a Time Projection Chamber (TPC) layer for the en-
 159 ergy reconstruction. The schematics of the prepared detector is in Figure 0.4
 160 Spectrometer CAD drawing (coordinates here or next chapter?). Cite some VdG
 161 paper, mention grant? Using https://cernbox.cern.ch/pdf-viewer/public/rf0oU1nqVLN3acZ/LuzH_submitted.pdf.
 162

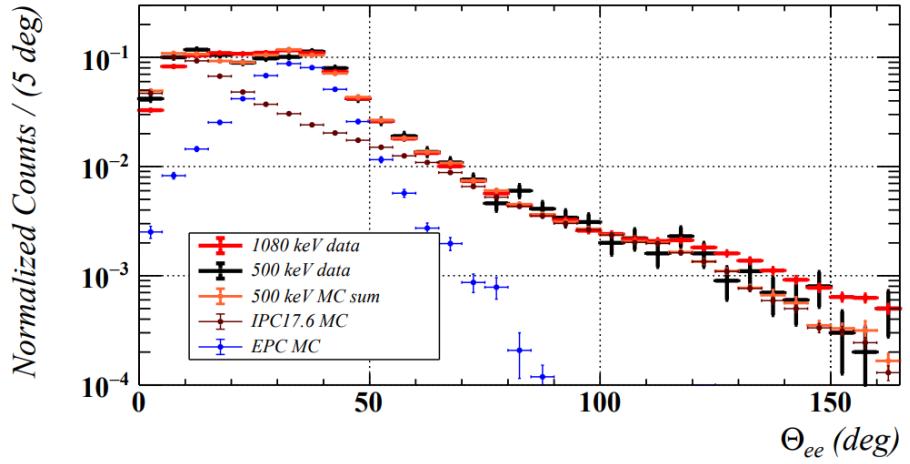


Figure 0.3: Results from the MEG II experiments – angular correlation of e^+e^- pairs with $E_{\text{sum}} \in [16, 20]$ MeV measured in the ${}^7\text{Li}(p, e^+e^-){}^8\text{Be}$ reaction with proton beam energies 500 and 1080 keV. The 500 keV dataset is fitted with Monte Carlo of both the IPC deexcitation and the EPC produced by gammas [23].

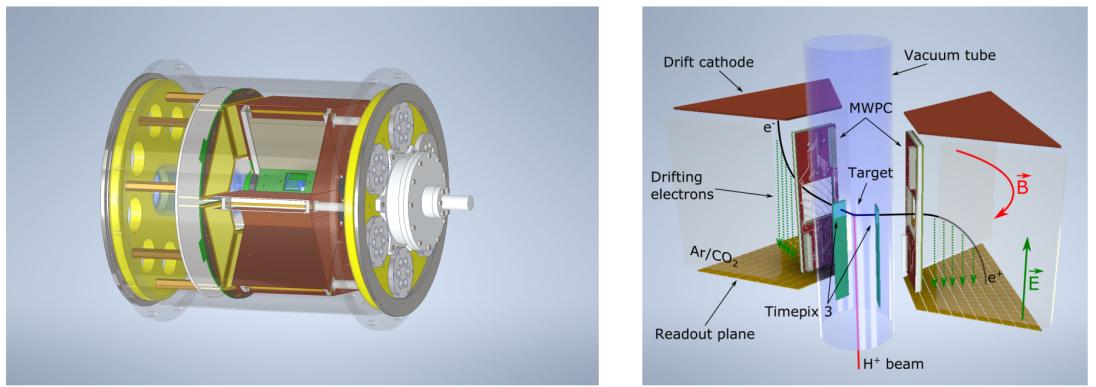


Figure 0.4: Schematics of the detector at the Van der Graaff facility at IEAP CTU.

163 The energy of e^+e^- pair produced in the reaction is given by the energy
164 available E_r in the reaction and can be distributed between them arbitrarily.
165 Nonetheless in the decay of the hypothetical X17 particle, electron and positron
166 should have similar energy and we can therefore use a disparity cut $|y| \leq 0.5$
167 for the disparity parameter (defined in Equation 0.1). Interesting events should
168 rarely have a particle with an energy below $E_r/4$ (roughly 4 MeV). Electrons with
169 such low energies are scattered significantly by even a thin layer of relatively light
170 material, for this reason the Tpx3 layer will be inside of the vacuum tube and the
171 tube will have a thinned aluminum segment or KaptonTM windows.

172 Tpx3 can measure (in each $55 \times 55 \mu\text{m}$ pixel of its 256×256 grid) time-of-arrival
173 (ToA) with 1.6 ns precision and time-over-threshold (ToT) which reflects the de-
174 posited energy. This potentially allows 3D tracking if we increase the chip thick-
175 ness at the cost of increased scattering. The layer can reconstruct the reaction
176 vertex and the angular correlation with high precision.

177 The layer of MWPCs with sensitive area $40 \times 38 \text{ mm}^2$ will be outside of
178 the beam pipe. It will provide an extra point on the particle trajectory which can
179 help with the estimation of the reaction vertex and improve the TPC performance
180 by providing its entry point.

181 The TPCs, which are a subject of this theses, are in a magnetic field of per-
182 manent magnets positioned between them and provide 3D track reconstruction
183 and subsequent momentum and particle identification (its charge, or even type
184 based on its stopping power). They avoid radiative losses thanks to the small
185 interaction with the incident particle. For the readout, triple Gas Electron Mul-
186 tiplier (GEM) will be used. The magnetic field layout in our TPCs is atypical –
187 orthogonal to the electric field inside the chamber, this is why we call them Or-
188 thogonal Fields TPC (OFTPC). Further details about our OFTPCs are provided
189 in section 1.3.

190 1. Time Projection Chamber

191 Using (2010 – a little old) <https://cds.cern.ch/record/1302071/files/CERN-PH-EP-2010-047.pdf>

193 A Time Projection Chamber (TPC) is a type of gaseous detector that uses
194 the drift in an electric field of free charges (electrons and cations, **also anions**
195 **if attachment is considered**) produced by an ionizing particle to reconstruct its
196 3D trajectory. When placed inside a magnetic field, the momentum of the incident
197 particle can be inferred from the curvature of its trajectory. Particle identification
198 is also possible using the ionization energy loss inside the TPC.

199 The original TPC used in the PEP-4 experiment at SLAC (Figure 1.1) was
200 a 2×2 m cylinder with a central cathode that produced a strong electric field,
201 making the ionization electrons drift towards one of the bases. The readout
202 consisted of MWPCs, where electrons are accelerated towards the anode wires
203 enough to further ionize the gas and cause an avalanche.

204 When a charged particle crosses the volume of a TPC, it loses energy by ex-
205 citation and ionization of the detector gas (**how much – from dE/dx + density**
206 **→ footnote?**). Most ionizing collision produce a single ionization electron, some-
207 times a few secondary electrons are produced close to the collision vertex. In
208 rare cases, the ionization electron has energy large enough to create a measurable
209 track, such an electron is called a δ -electron (**terminology, just like bellow – tech-**
210 **nically it's a (primary) ionization electron causing other (secondary) ionization**).
211 Penning transfer (collisions, light – factor 10 for gas gain in Ar/CO₂ viz PDG
212 CERN)?

213 CERES/NA45 – very inhomogeneous magnetic field

214 1.1 Charge transport in gases

215 1.1.1 Drift

216 Produced ionization electrons (**terminology – called ionization electrons in the**
217 **rest of the thesis**) are accelerated towards the readout by the electric field in-
218 side the chamber. At the same time, they lose speed by colliding with the gas
219 particles, quickly reaching a constant (for a given field \mathbf{E}, \mathbf{B}) mean drift velocity.
220 The electrons might be absorbed by electronegative impurities, such as halides
221 and oxygen.

222 In many gases (called "hot", e.g., Ar or CH₄), the drift velocity is much greater
223 than that of their thermal motion thanks to a high proportion of elastic collisions.
224 On the other hand, "cold" gases like CO₂ have a higher proportion of inelastic
225 collisions (e.g., thanks to the excitation of rotational and vibrational states) and
226 therefore much lower (**value? magnitude (implied)?**) drift velocity. (**def?**)

227 The ions produced by the ionization lose a significant portion of their energy
228 during each collision since their mass is close to the mass of the gas particles (**see**
229 **the source material – average energy loss during collision** $\Delta E = \frac{2m_i M}{(m_i + M)^2}$, **this way**
230 **it's more accurate**). This, together with their large collision cross section, makes
231 their drift velocity much smaller and their energy is close to thermal. Since their
232 momenta aren't randomized to such an extent during collisions, their diffusion

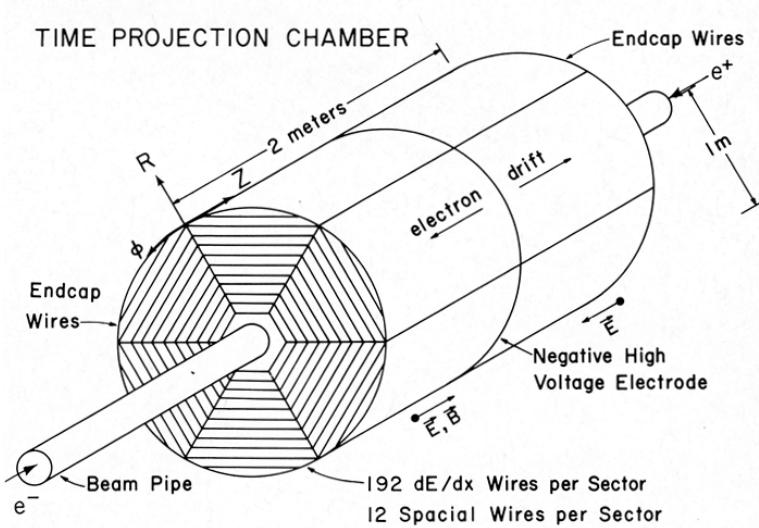


Figure 1.1: Schematic view of the PEP-4 TPC [24].

233 is smaller (more in the sense of distribution of positions, could move this to the
234 diffusion subsection).

235 The drift is also influenced by the magnetic field. Langevin derived a good
236 approximation for the drift velocity vector:

$$\mathbf{v}_d = \left(\frac{\mathbf{E}}{\|\mathbf{E}\|} + \omega\tau \frac{\mathbf{E} \times \mathbf{B}}{\|\mathbf{E}\| \|\mathbf{B}\|} + \omega^2\tau^2 \frac{\mathbf{E} \cdot \mathbf{B}}{\|\mathbf{E}\| \|\mathbf{B}\|} \cdot \frac{\mathbf{B}}{\|\mathbf{B}\|} \right) \frac{q\tau}{m(1 + \omega^2\tau^2)} \|\mathbf{E}\|, \quad (1.1)$$

237 where q is the charge of the particle, m is its mass, τ is the mean time between
238 collisions and $\omega = \frac{q}{m} \|\mathbf{B}\|$ is the Larmor frequency. In a standard TPC, \mathbf{E} is
239 nearly parallel to \mathbf{B} and the influence of the magnetic field on the drift is minimal.
240 The drift of ions is only negligibly influenced by the magnetic field ($\omega\tau \sim 10^{-4}$
241 is small due to the low drift velocity – better because it takes τ into account
242 and differs only by E/B ratio). Lorentz angle for orthogonal fields $\tan \psi = -\omega\tau$
243 (deviation from electric field) – maybe mention in the OFTPC section. Without
244 magnetic field, we can write

$$\mathbf{v}_d = \frac{q\tau}{m} \mathbf{E} = \mu \mathbf{E}, \quad (1.2)$$

245 where μ is called charge mobility.

246 1.1.2 Diffusion

247 Due to collisions a cloud of electrons or ions originating from the same point will
248 show a Gaussian density distribution at time t while drifting in the electric field
249 $\mathbf{E} = (0, 0, E_z)$:

$$\rho(x, y, z, t) = (4\pi Dt)^{-\frac{3}{2}} \exp \left(-\frac{x^2 + y^2 + (z - v_d t)^2}{4Dt} \right), \quad (1.3)$$

250 where the diffusion coefficient D can be expressed as

$$D = \frac{\lambda^2}{3\tau} = \frac{\lambda v_d}{3} = \frac{v_d^2 \tau}{3} = \frac{2\varepsilon\tau}{3m}, \quad (1.4)$$

251 where λ is the mean free path and ε the mean energy. The lateral diffusion width
252 σ_x after a drift distance L can be expressed as

$$\sigma_x^2 = 2Dt = \frac{4\varepsilon L}{3qE}. \quad (1.5)$$

253 The minimal diffusion width is given by the lowest possible energy of the particles
254 $\varepsilon_{\text{th}} = \frac{3}{2}kT$ (corresponding to thermal motion):

$$\sigma_{x,\min}^2 = \frac{2kTL}{qE}. \quad (1.6)$$

255 For electrons in "cold gases" (e.g., Ar/CO₂ mixture), the diffusion approaches
256 this limit up to a certain field intensity (~ 100 V/cm at 1 atm pressure)¹. In
257 reality, the transversal diffusion of electrons can differ significantly from their
258 longitudinal diffusion and simulations are necessary to get a precise result.

259 In most TPCs, the transversal (but not the longitudinal) diffusion is reduced
260 by the magnetic field, since it is parallel to the electric field and curves the dif-
261 fusing electrons around their mean trajectory:

$$\frac{D_T(B)}{D_T(0)} = \frac{1}{C + \omega^2 \tau_2^2}, \quad (1.7)$$

262 where C and τ_2 are parameters dependent on the gas used. At low intensity of
263 the magnetic field, we can use an approximation $C \approx 1$ and $\tau_2 \approx \tau$.

264 1.2 Readout

265 1.2.1 Multi-Wire Proportional Chamber

266 In most (2010 – almost all) TPCs operated in experiments Multi-Wire Proportion-
267 al Chamber (MWPC) was used for the readout. The electrons enter the cham-
268 ber through a cathode grid and get accelerated in the strong electric field towards
269 the thin anode wires and create a Townsend avalanche, multiplying the signal.
270 **Alternating with field wires?** The trajectory can be reconstructed using pulses
271 from each separate wire. Segmented cathode is also often used for the readout of
272 produced cations. **Gating grid (reduction of space charge effect, blocking backflow**
273 **of ions?, closed for electrons B=0, ΔV , static mode (loss of 25% el.) x opening on**
274 **trigger)? (gas amplification > 10000 required for good SNR, 100-200 ns shaping**
275 **time), figure?**

276 1.2.2 Gas Electron Multiplier

277 The Gas Electron Multiplier (GEM) is a thin metal-coated polymer sheet with
278 a high density of small holes. The amplification is achieved by applying voltage
279 on the metal layers, creating a strong electric field inside the holes and causing
280 avalanches. Double or triple stack of GEMs is usually used to create a sufficient
281 gain. From the last foil, the electrons drift to a segmented anode where the signal
282 is read. The backflow of cations is reduced compared to MWPC. An example
283 simulation of an avalanche inside GEM is shown in Figure 1.2. **Parameters?**

¹For us 0.45 mm, quite close to the actual diffusion 0.5-0.7 mm.

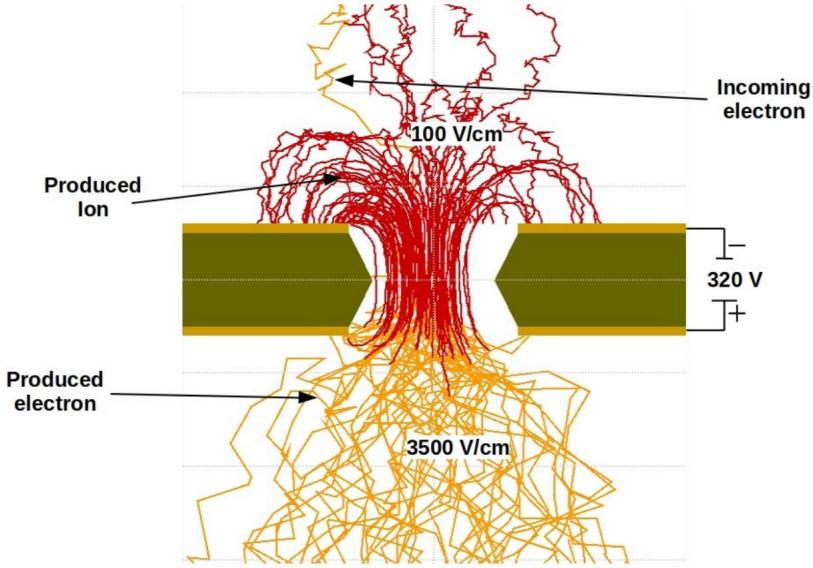


Figure 1.2: Garfield simulation of an avalanche in a GEM hole [25].

284 1.2.3 Micromegas

285 In a MICRO-MEsh GAseous Structure (Micromegas) electrons pass through a fine
 286 mesh (made out of very thin wires) into the narrow amplification gap where they
 287 are multiplied in the high field and read as signal on the segmented anode. Very
 288 high field ($30\text{-}80 \text{ kV/cm}^2$) is necessary to achieve sufficient gain. Cation backflow
 289 is heavily suppressed by the mesh.

290 1.2.4 Parallel Plate Chamber

291 ... micowell?

292 1.3 Orthogonal Fields TPC at IEAP CTU

293 At IEAP CTU, we are going to use six identical atypical TPCs with inhomogeneous
 294 toroidal magnetic field orthogonal to the electric field, hereafter referred to
 295 as Orthogonal Fields TPC (OFTPC). It has the shape of isosceles trapezoidal
 296 prism 16 centimeters high with triple-GEM readout on one of its bases. Dimen-
 297 sions of the OFTPC are discussed in detail in section 1.3.2 below. Throughout this
 298 thesis, we assume a uniform electric field along the z axis with $E_z = -400 \text{ V/cm}$.
 299 Gas mixture used in the detector (70/30) and its effect – some graph with the
 300 mixture.

301 1.3.1 Motivation and Associated Challenges

302 The reasons for the unusual field layout are mostly cost related:

- 303 • we use permanent magnets instead of a solenoid and parallel fields are
 304 difficult to accomplish this way,
- 305 • granularity of the TPC readout is limited in order to fit one SAMPA/SRS
 306 hybrid in each sector – parallel fields would bend the trajectories parallel

307 to the readout requiring more pads and different architecture.

308 In this thesis, we will show that such a setup can reach a similar energy resolution
309 as common cylindrical TPCs while reducing cost.

310 The layout introduces two complications to the track reconstruction – the
311 trajectory in inhomogeneous field is not circular and the drift is distorted by
312 the magnetic field (see Equation 1.1, in our case $\omega\tau \approx 0.08$ for 0.3 T assuming
313 $\mu \approx 0.25 \text{ T}^{-1}$, varies inside the detector). The diffusion in such setup is larger
314 since parallel orientation reduces diffusion by curling the electrons in the x - y
315 direction (see Equation 1.7) but for our relatively weak magnetic field and short
316 drift distance the difference is negligible.

317 1.3.2 Coordinate Systems and Dimensions

318 In order to describe events in our detector, we use three distinct spaces: the de-
319 tector space \mathcal{D} , the readout space \mathcal{R} and the pad space \mathcal{P} . Each space is later
320 used to represent ionization electrons at different stages of the detection process:
321 their creation in the gas, their final position when hitting the readout plane, and
322 finally their representation in the discrete pad space.

323 Detector Space

324 The detector space \mathcal{D} represents the physical space of our detector. We de-
325 scribe it using Cartesian coordinates (x, y, z) . The z -axis is the detector's axis of
326 symmetry, with its negative direction aligned with the proton beam. The origin
327 $(0, 0, 0)$ is located at the center of the irradiated target. The positive x -axis passes
328 through the center of one the OFTPCs along the intersection of its two planes
329 of symmetry. The y -axis is then chosen to maintain a right-handed coordinate
330 system.

331 Since the detector has a hexagonal symmetry, we use only one of its sectors
332 in this work – the first sector $\mathcal{D}_1 \subset \mathcal{D}$ which is defined by the condition:

$$(x, y, z) \in \mathcal{D}_1 \Leftrightarrow |y| \leq x \tan \frac{\pi}{6}. \quad (1.8)$$

333 Simulations in this sector can be applied to all sectors by rotating the coordinates
334 accordingly. The volume of the OFTPC in this sector, which has the shape of
335 a trapezoidal prism, has these boundaries:

$$x \in [x_{\min}, x_{\max}] = [6.51, 14.61] \text{ cm}, \quad (1.9)$$

$$z \in [z_{\min}, z_{\max}] = [-8, 8] \text{ cm}, \quad (1.10)$$

$$y_{\max}(x_{\min}) = -y_{\min}(x_{\min}) = 2.75 \text{ cm}, \quad (1.11)$$

$$y_{\max}(x_{\max}) = -y_{\min}(x_{\max}) = 7.45 \text{ cm}, \quad (1.12)$$

336 where $y_{\max}(x)$ is the maximal value of the y -coordinate for a given x . The read-
337 out is located at $z = 8$ cm; for some purposes, we also define the distance to
338 the readout $d_r = 8 \text{ cm} - z$ as an alternative to the z -coordinate. Keeping this
339 paragraph as it is because the OFTPC volume is distinct from the first sector
340 and some parts of this thesis use the space beyond this volume.

341 We also use spherical coordinates (r, θ, φ) with θ measured relative to the xy
342 plane.

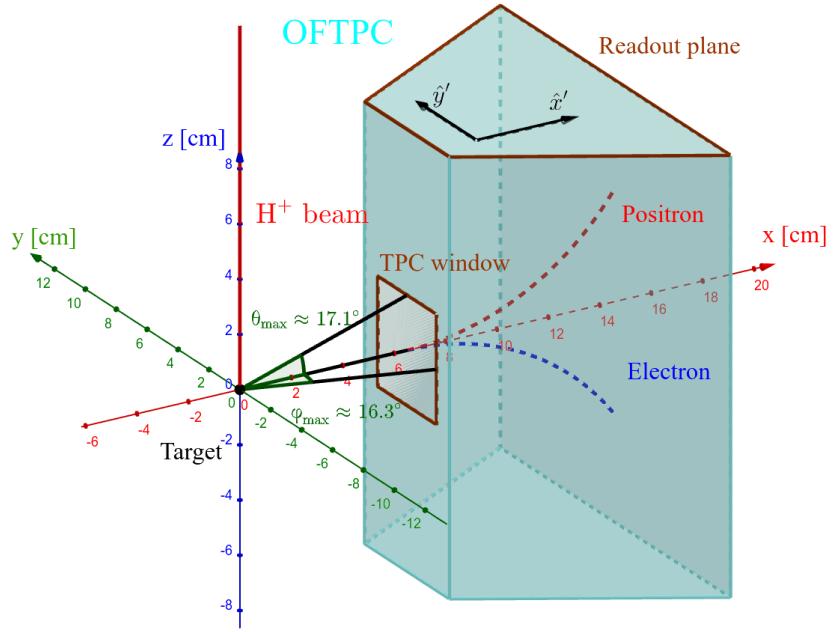


Figure 1.3: Schematics of the first sector OFTPC with detector space coordinates.

343 Readout Space

344 The readout space \mathcal{R} represents the drift time and final positions of ionization
 345 electrons as measured by an ideal continuous readout. We describe it using
 346 coordinates (x', y', t) , where x' and y' correspond to the detector coordinates at
 347 the readout plane ($z = 8 \text{ cm}$). Currently not entirely sure how to put this
 348 into a figure since only x' and y' correspond to the detector coordinates, it will
 349 make more sense when visualizing the map. The drift time t is approximately
 350 proportional to d_r .

351 Pad Space

352 The pad space \mathcal{P} represents the time bin and pad number of ionization electrons
 353 as measured by an ideal discrete readout:

$$\mathcal{P} = \{(n_{\text{pad}}, n_t) \in \mathbb{N}^2 \mid n_{\text{pad}} \leq 128\}. \quad (1.13)$$

354 Technically both values can be zero as defined in the code (max channel 127).
 355 It is not really a subspace of \mathcal{R} but there is a mapping from \mathcal{R} to \mathcal{P} . It is a
 356 discretization of a part of \mathcal{R} , the mapping can be adjusted depending on the sim-
 357 ulation. If we assume uniform electric field there will be gaps, we don't use gaps
 358 in the reconstruction since the electrons should be pulled towards the pads.

359 The readout of the OFTPC will consist (is the design final?) of 128 rectangular
 360 pads arranged in a staggered pattern. Parameters of the pad layout are shown
 361 in Figure 1.4. The bottom left corner of n -th pad has coordinates $(x_{1,n}, y_{1,n})$,
 362 the top right $(x_{2,n}, y_{2,n})$ and its center has coordinates $(x_{c,n}, y_{c,n})$. The gap
 363 between neighboring pads is $g = 0.08 \text{ cm}$. Time will be read out with 100 ns
 364 intervals (details?). Could also describe pad-related functions.

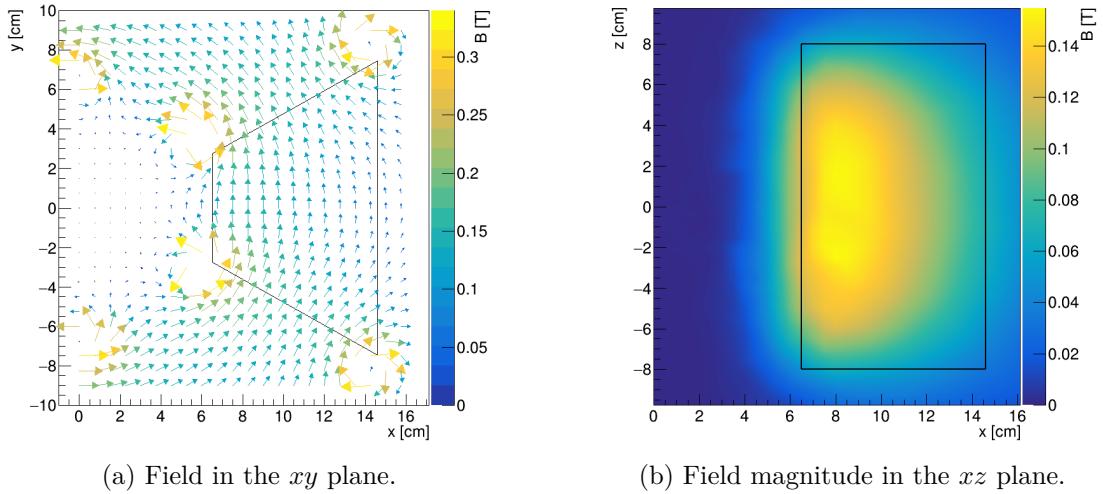


Figure 1.5: Magnetic field simulation results. OFTPC volume is marked with black lines.

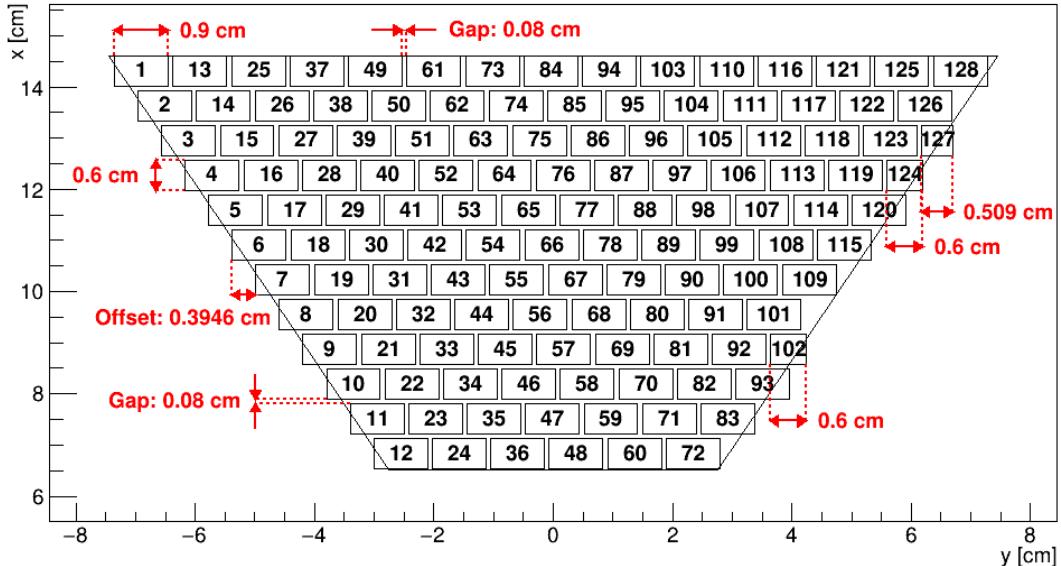


Figure 1.4: Pad layout of the OFTPC and its parameters. Pads 102, 124 and 127 are irregular, the rest has the same dimensions.

365 1.3.3 Magnetic Field Simulation

366 The magnetic field inside our detector is produced by six permanent magnets. It
 367 was simulated using Ansys Maxwell ([citation?](#)) which gives us values on a regular
 368 grid. Visualization of the magnetic field is shown in Figure 1.5. Whenever we
 369 need to work with values outside this grid, we use trilinear interpolation described
 370 below.

371 Trilinear Interpolation

372 Trilinear interpolation is a 3D generalization of linear interpolation. It can be
 373 used to interpolate a function whose values are known on a regular grid with

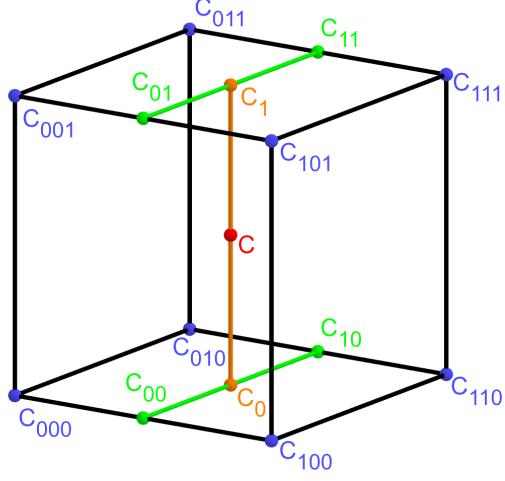


Figure 1.6: Visualization of trilinear interpolation as a composition of linear interpolations. **Image drawn in GeoGebra and inspired by a similar image on Wikipedia (which looks a bit worse) – is credit necessary?**

374 rectangular prism cells. We use this simple method for interpolating the magnetic
 375 field, and it is later used in Section 3.2.1 to interpolate the Ionization Electron
 376 Map, a key component of our track reconstruction algorithm. In both cases, we
 377 use a regular cubic grid (apparently it is also called a **Cartesian grid**).

378 Could put a paragraph about linear interpolation here if it is not clear from
 379 the equations below.

380 Let us consider a cell of our regular grid (a cube) with an edge of length a
 381 containing the point $\mathbf{C} = (x, y, z)$ where we want to interpolate a function
 382 $f: \mathbb{R}^3 \rightarrow \mathbb{R}$. We know the values of this function at the vertices of the cell
 383 $\mathbf{C}_{ijk} = (x_0 + ia, y_0 + ja, z_0 + ka)$, where $i, j, k \in \{0, 1\}$ are indices. We also define
 384 the points $\mathbf{C}_{ij} = (x, y_0 + ia, z_0 + ja)$ and $\mathbf{C}_i = (x, y, z_0 + ia)$. Then the interpolated
 385 value $\hat{f}(\mathbf{C})$ can be calculated as a composition of three linear interpolations (see
 386 Figure 1.6):

$$\hat{f}(\mathbf{C}_{ij}) = (1 - x_d) f(\mathbf{C}_{0ij}) + x_d f(\mathbf{C}_{1ij}), \quad (1.14)$$

$$\hat{f}(\mathbf{C}_i) = (1 - y_d) \hat{f}(\mathbf{C}_{0i}) + y_d \hat{f}(\mathbf{C}_{1i}), \quad (1.15)$$

$$\hat{f}(\mathbf{C}) = (1 - z_d) \hat{f}(\mathbf{C}_0) + z_d \hat{f}(\mathbf{C}_1), \quad (1.16)$$

387 where x_d , y_d , and z_d are given as follows:

$$x_d = \frac{x - x_0}{a}, \quad y_d = \frac{y - y_0}{a}, \quad z_d = \frac{z - z_0}{a}. \quad (1.17)$$

388 We can also write

$$\hat{f}(\mathbf{C}) = \sum_{i,j,k \in \{0,1\}} t_x^i t_y^j t_z^k f(\mathbf{C}_{ijk}), \quad (1.18)$$

$$t_\alpha \stackrel{\text{def}}{=} \begin{pmatrix} t_\alpha^0 \\ t_\alpha^1 \end{pmatrix} = \begin{pmatrix} 1 - \alpha_d \\ \alpha_d \end{pmatrix}, \quad (1.19)$$

389 where $\alpha \in \{x, y, z\}$ is an index. This gives a nice geometric interpretation to the
 390 trilinear interpolation as shown in Figure 1.7. Furthermore, we can write $\hat{f}(\mathbf{C})$

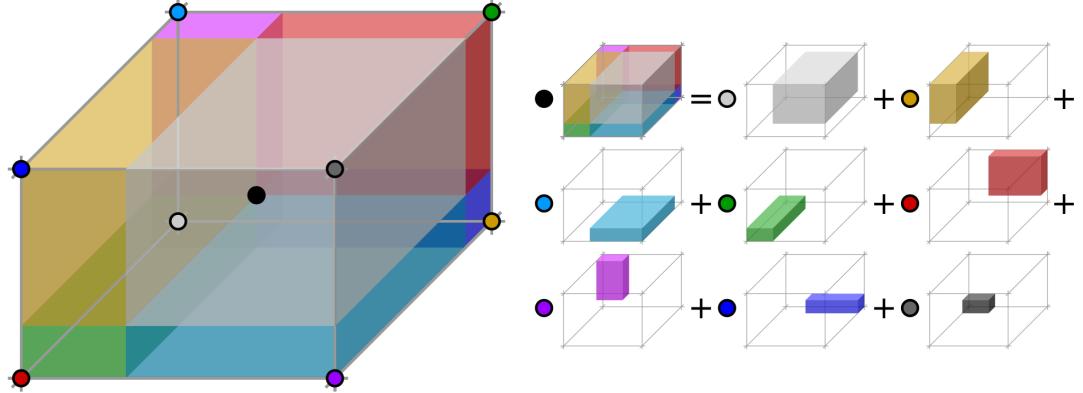


Figure 1.7: Geometric interpretation of trilinear interpolation. The colored dots represent the values in given points and the colored boxes represent the volume by which the corresponding values are multiplied. The black dot represents the interpolated value which is multiplied by the entire volume [26].

391 as a polynomial:

$$\hat{f}(\mathbf{C}) = \sum_{\alpha, \beta, \gamma \in \{0,1\}} \sum_{i=0}^{\alpha} \sum_{j=0}^{\beta} \sum_{k=0}^{\gamma} (-1)^{(\alpha-i) + (\beta-j) + (\gamma-k)} f(\mathbf{C}_{ijk}) x_d^{\alpha} y_d^{\beta} z_d^{\gamma}. \quad (1.20)$$

392 We take advantage of this form when generalizing trilinear interpolation to irregular grid in section 3.2.2.
393

394 Maybe a citation here, although I am not sure it is necessary since it could
395 be considered common knowledge. The last two equations are my own. Maybe
396 x_0 , etc. should be explicitly described.

2. Track Simulation

398 In order to develop and test the reconstruction algorithm, electron and positron
399 tracks are simulated inside the first sector \mathcal{D}_1 of our detector (see Section 1.3.2)
400 with different initial parameters. Two approaches are currently used to simulate
401 tracks, each of them for different purpose.

402 The **Microscopic Simulation** uses the Garfield++ toolkit [1]. Within this
403 toolkit, the High Energy Electro-Dynamics (HEED) program [27] is used to simu-
404 late the primary particle and the class *AvalancheMicroscopic* to simulate the drift
405 of secondary electrons created by ionization in the gas. This is the most precise
406 and time-consuming simulation used; our current goal is to be able to successfully
407 reconstruct its results and determine our best-case energy resolution.

408 The **Runge-Kutta Simulation** uses the 4th order Runge-Kutta numerical
409 integration ([add citation for Runge-Kutta](#)) to simulate the trajectory of the pri-
410 mary particle in the electromagnetic field inside the detector. It is relatively
411 fast since it does not simulate the secondary particles. It is used as part of our
412 reconstruction algorithm and for testing some parts of the reconstruction.

413 All of these simulations require the knowledge of the electromagnetic field
414 inside the detector. A uniform electric field of $400 \text{ V}\cdot\text{cm}^{-1}$ is assumed. The mag-
415 netic field was simulated in Maxwell (see Section 1.3.3). [add citation](#)

416 [Single track in positive x direction or initial parameter randomization. Im-](#)
417 [portance of gas composition, used gas compositions.](#)

418 2.1 Microscopic Simulation

419 The microscopic simulation, the most detailed simulation used in this work, is
420 performed using the Garfield++ toolkit [1].

421 The electron transport properties are simulated using the program Mag-
422 boltz ([Add citation](#)). Two different gas mixtures were used: 90% Ar + 10% CO₂
423 and 70% Ar + 30% CO₂. The second mixture will be used in our detector.
424 The temperature is set to 20 °C, the pressure is atmospheric.

425 The primary track is simulated using the program HEED [27], which is an
426 implementation of the photo-absorption ionization model. This program pro-
427 vides the parameters of ionizing collisions. HEED can also be used to simulate
428 the transport of delta electrons; we do not account for these in the current sim-
429 ulation but plan to include them in the future. The photons created in the atomic
430 relaxation cascade ([fluorescence reabsorption, ?](#)) are also not simulated.

431 Finally, we use the microscopic tracking provided by the class *AvalancheMicro-*
432 *scopic* to simulate the drift of the ionization electrons. Each electron is followed
433 from collision to collision using the equation of motion and the collision rates
434 calculated by Magboltz.

435 [First simulated track in the z direction should be described in detail here \(own](#)
436 [subsection?\). Figures.](#)

437 [Add more detailed and better description of HEED, and microscopic tracking](#)
438 [\(each their own subsection?\). Could also mention Monte Carlo \(requires gas file](#)
439 [generation - Magboltz\) and Runge-Kutta simulation implemented in Garfield,](#)

440 why we don't use them (another subsection? rename the section to Garfield++
441 simulation and mention all relevant parts?).

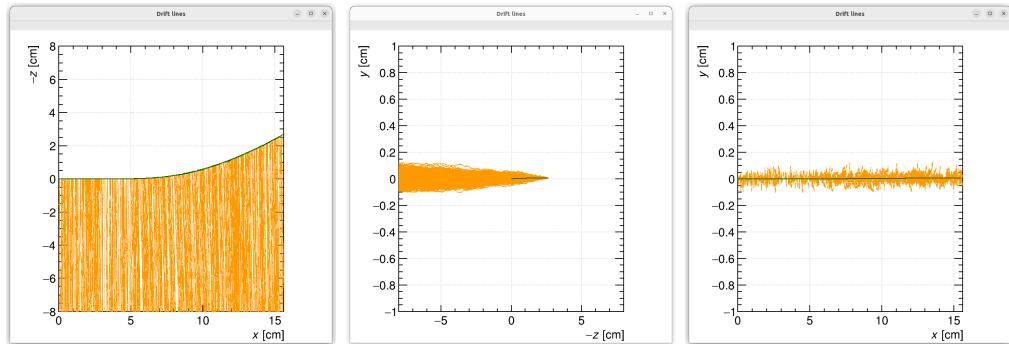


Figure 2.1: Example of a simulated electron track in 70 % argon and 30 % CO₂ atmosphere (on the left). Swap for better images, better zoom. Explain drift lines, primary particle.

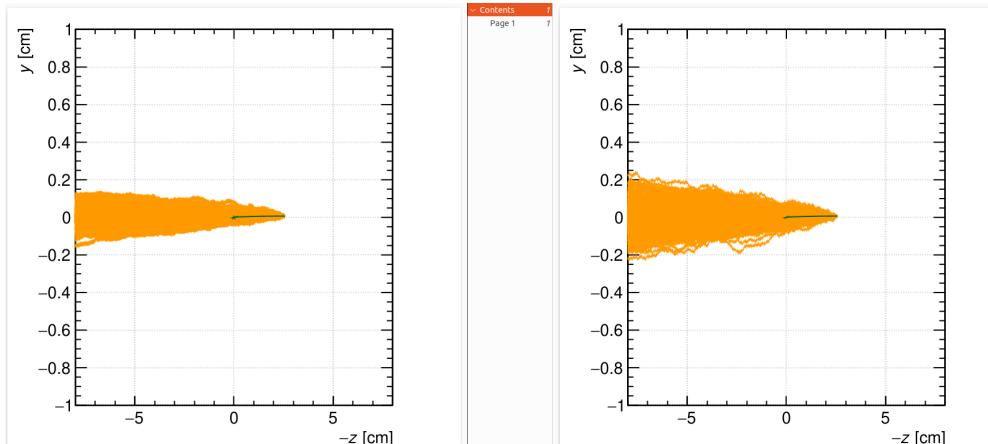


Figure 2.2: Comparison of diffusion in a simulated electron track in 70 % argon, 30 % CO₂ atmosphere and in 90 % argon, 10 % CO₂ atmosphere (on the right). Swap for better image, better zoom. Or put the same pictures for both comparisons in one subfigure, etc. Describe better.

442 2.2 Runge-Kutta Simulation

443 Trajectory simulation with 4th order Runge-Kutta. Relativistic equation that is
444 numerically integrated by the algorithm.

3. Track Reconstruction

In the first stage of the reconstruction algorithm, we reconstruct the track of a primary particle (either an electron or a positron). The result of this step is then used to determine the energy of the particle (Section 4).

The **Reconstruction Assuming Steady Drift** uses the standard TPC approach. With parallel fields, the drift inside a uniform electric field remains undistorted (reference to some future part of the TPC chapter). Therefore, we only need to reconstruct the z -coordinate from the drift time using the known drift velocity. We also assume that the readout coordinates (x', y', t) are known exactly, neglecting the pads and time bins.

Reconstruction using the **Ionization Electron Map** (from now on referred to as *the map*) uses a simulation of the drift of secondary (ionization) electrons within the detector volume. This simulation can then be used to interpolate the initial position of the secondary electrons. First attempts neglect the pads.

We use the map for reconstruction in two different ways. The first one uses gradient descent search along with trilinear interpolation (see Section 1.3.3) of the map. The second method uses interpolation on the irregular inverse grid with a linear polynomial.

The **Discrete Reconstruction** uses the map; instead of reconstructing the exact position of each electron, we reconstruct the center of each hit pad with the time corresponding to the midpoint of the time bin. The electron count in each TPC bin (consisting of the pad and the time bin) serves as the charge value, which is then used as a weight in the energy reconstruction fit.

3.1 Reconstruction Assuming Steady Drift

As the first step, we decided to try to reconstruct an electron track with a special set of initial parameters. The origin of the particle is given by the origin of our coordinate system. The initial direction is given by the positive x -axis. This means the magnetic field of our detector is perpendicular to the momentum of the particle at all times, and we can reduce the problem to two-dimensional space. As an example, we use a track simulated using the microscopic simulation (see Section 2.1) with a kinetic energy of 8 MeV. The gas composition used in this simulation is 90% Ar + 10% CO₂. Might be better to describe this track in Section 2.1.

In this approach to the reconstruction of the track, we decided to use the common method used in a standard TPC. This will allow us to explore the significance of the atypical behavior in our OFTPC. Additionally, we assume the readout is continuous to further simplify the problem. In this approximation, we reconstruct the initial position of each ionization electron.

The reconstruction is then defined by the following relations between the coordinates of the detector space and the readout space (see Section 1.3.2):

$$x = x', \quad (3.1)$$

$$y = y', \quad (3.2)$$

$$z = v_d t, \quad (3.3)$$

485 where v_d is the drift velocity of electrons in the given gas mixture. At a phe-
 486 nomenological level, this velocity can be considered as a function of the electric
 487 field \mathbf{E} and the magnetic field \mathbf{B} :

$$v_d = v_d(\mathbf{E}, \mathbf{B}). \quad (3.4)$$

488 Equation taken from Garfield user manual. The Garfield++ toolkit uses this
 489 fact to accelerate their drift simulation with non-microscopic approaches (could
 490 mention in the simulation chapter). Since we assume a uniform electric field in
 491 our detector and we want to neglect the effect of our unusual magnetic field, we
 492 consider the drift velocity to be constant in this scenario. We then approximate
 493 this velocity by fitting the dependence $z(t)$ taken from the simulated ionization
 494 electrons. This is in one of the provisional figures. Also, this description is
 495 not completely accurate; in reality, we fit $t1:8-y0$ with $a1*x+a0$ and then invert
 496 this and use $8-y0 = b1*t1+b0$ (old coordinates); $b1=1/a1$ functions as the drift
 497 velocity. Maybe also define this 8-z variable as an alternative to z in Section 1.3.2
 498 and then use it when correcting this.

499 Later, in a commit after this, I plotted some residues (provisional figure),
 500 which could be useful, but for some reason they are residuals from a spline fit of
 501 the track?! Probably redo this without the spline fit; just explore the difference
 502 in individual points.

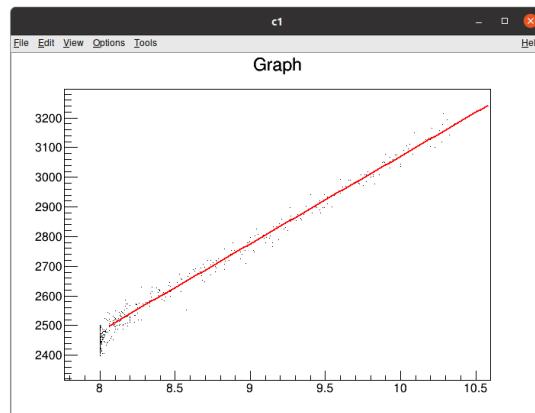


Figure 3.1: Dependence of the drift time on the z coordinate in 90 % argon and 10 % CO₂ atmosphere, fitted with a linear function. The fitted function gives us the average drift velocity in the gas and can be used for rough reconstruction in our TPC. Swap for better image with axis labels, etc. Maybe write the fitted equation.

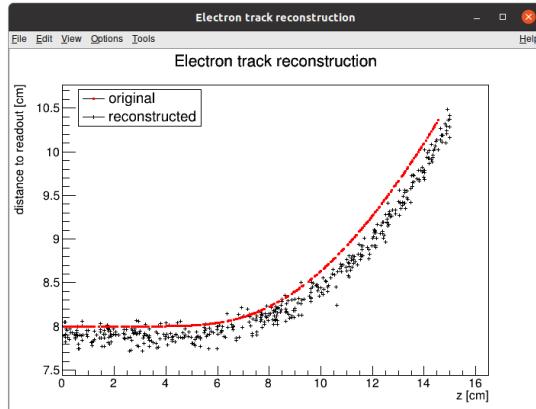


Figure 3.2: First attempt at a track reconstruction using only the drift velocity. This approach works well in a standard TPC (ideally cite some source?). 90 % argon and 10 % CO₂ atmosphere. Swap for better image, correct coordinates.

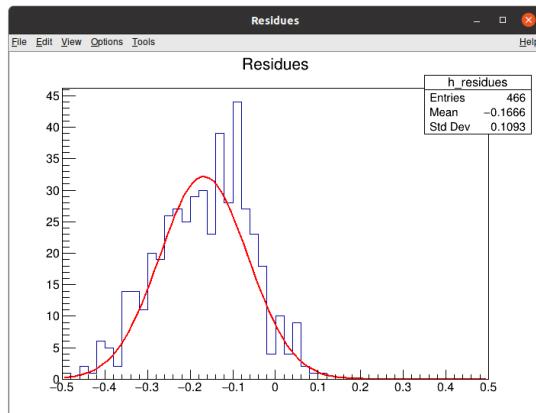


Figure 3.3: First attempt at a track reconstruction using only the drift velocity, residues. Swap for better image, correct coordinates. What's causing the shift? Explain details.

503 3.2 Ionization Electron Map

504 Inside an OFTPC, the drift of the secondary (ionization) electrons is significantly
 505 affected by its magnetic field (pictures of the distortion later, the effect is bigger
 506 for the 90/10 composition.). We need to take this into account for accurate
 507 reconstruction. In the first approximation, we assume a continuous readout (i.e.,
 508 we neglect pads). We can then reconstruct the original position of each ionization
 509 electron using its readout coordinates. For this purpose, we use the ionization
 510 electron map.

511 The ionization electron map represents a mapping from the detector space
 512 to the readout space (see Section 1.3.2). It tells us what readout coordinates
 513 (x', y', t) we can expect on average for an ionization electron created at the detec-
 514 tor coordinates (x, y, z). More precisely it is a mapping to the distributions on
 515 the readout space; we can simplify this as only the means $\bar{\mathcal{M}}$ and the covariance

516 matrices \mathcal{M}_{cov} , assuming Gaussian distribution.

$$\mathcal{M} : \mathcal{D} \longrightarrow \mathcal{R}, (x, y, z) \longmapsto (x', y', t). \quad (3.5)$$

517 To get an approximation of this mapping, we simulate the drift of ionization elec-
 518 trons generated on a regular grid inside the volume of our OFTPC¹. It is also
 519 useful to simulate multiple (100 in our case) electrons originating from the same
 520 position so we can get a better information about the average drift and its vari-
 521 ance. In order to get more accurate results, we use the microscopic simulation of
 522 these electrons described in Section 2.1. When evaluating the map inside the grid,
 523 we use trilinear interpolation (see Section 1.3.3). From now on, we will denote
 524 this interpolated simulation with the same symbol \mathcal{M} .

525 Finally, we need to invert the map to get the original detector coordinates
 526 (x, y, z) for the given readout coordinates (x', y', t) . In our case, we can reason-
 527 ably assume that the mapping $\overline{\mathcal{M}}$ is one-to-one (as seen in the simulations). We
 528 implemented two methods for this purpose: the gradient descent search (Sec-
 529 tion 3.2.1) and interpolation on the inverse grid (Section 3.2.2).

530 The simulation of the map is a computationally heavy task. For this reason,
 531 we use the MetaCentrum grid [3] to parallelize needed calculations. At first, this
 532 was done by evenly distributing the simulated electrons across the individual jobs
 533 in a simulation with only one electron per vertex in the regular grid with a spacing
 534 of one centimeter.

535 Later, a more efficient approach was implemented, accounting for the varying
 536 lengths of the drift of individual electrons. If we index the electrons in the order
 537 of increasing coordinates y, x, z ([picture?](#)), we can express the number n_l of full
 538 XY layers (i.e., electrons with the same z coordinate) of electrons with index less
 539 than or equal to i

$$n_l(i) = \left\lfloor \frac{i}{n_{xy}} \right\rfloor, \quad (3.6)$$

540 where n_{xy} is the number of electrons in each XY layer calculated simply by count-
 541 ing the electrons that satisfy boundary conditions for x and y . **These conditions**
 542 **should be mentioned above; sector condition + maximal x value.** The number of
 543 electrons remaining in the top layer is then

$$n_r(i) = i \bmod n_{xy}. \quad (3.7)$$

544 Finally, we can calculate the sum of the drift gaps of electrons up to index i

$$d_{\text{sum}} = (z_{\max} - z_{\min})n_{xy}n_l - \frac{n_l(n_l - 1)}{2}n_{xy}l + n_r(z_{\max} - z_{\min} - n_l l). \quad (3.8)$$

545 We then use a binary search algorithm to find the maximum index i such that
 546 the value of this sum is less than the fraction $\frac{\text{job id}}{\max \text{ job id}}$ of the total sum. This way
 547 we obtain the minimal and the maximal index of electrons simulated in the given
 548 job. **The spacing l should be probably defined above + picture of the simulating**
 549 **grid (1 layer). zmin zmax also**

550 After the simulation of the map, we calculate the mean readout coordinates
 551 assuming Gaussian distribution (i.e., we use averages). We also calculate standard

¹we do not take the detector walls into account and simulate even outside of the OFTPC which lets us interpolate even close to the walls

552 deviations in a later commit, should be upgraded to the covariance matrix. We
553 never actually plotted the distributions we get when simulating the same electron
554 multiple times, so we do not know if our assumptions are accurate (could also
555 run some statistical test to see how well the Gaussian distribution fits).

556 The obtained map is then stored in a custom class template *Field*, could
557 expand on that. Maybe earlier, since the same template is used for the magnetic
558 field.

559 Could insert a table here describing all 4 simulations of the map (gas composi-
560 tion, spacing, etc.). Simulation inside of one sector (at first double angle). Extra
561 space on the sensor. Edge cases not taken into account (TPC wall). Using qsub
562 (not sure if important). Add plots of distortion of the coordinates. Could also do
563 these plots in a different way (e.g., drawing all the endpoints of each ionization
564 electron or some error ellipse plot).

565

566 Images to add (comparison of both simulations):

- 567 • 3D visualization of the map, simulation example
- 568 • z vs. t plot
- 569 • XY plane distortion for different z values; with arrows and error bars, for
570 all z -layers with different colors
- 571 • XZ plane ($y = 0$) distortion in x (maybe not necessary?)
- 572 • XT plot ($y = 0$) showing (small) distortion in drift times

573

574 More images:

- 575 • Residuals of the continuous readout reconstruction.

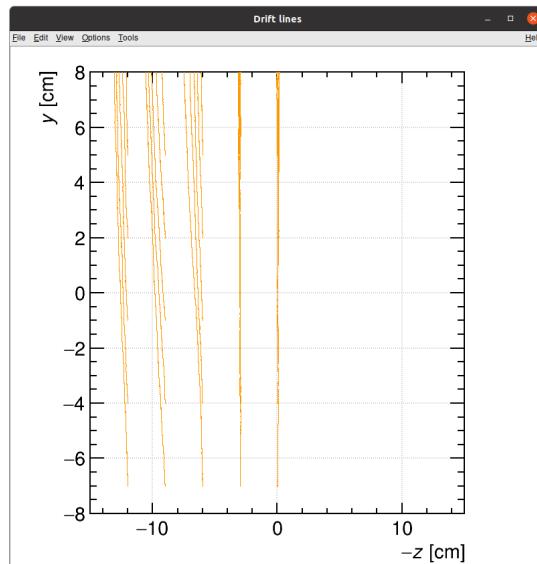


Figure 3.4: Example of map generation. Swap for better image, correct coordinates.

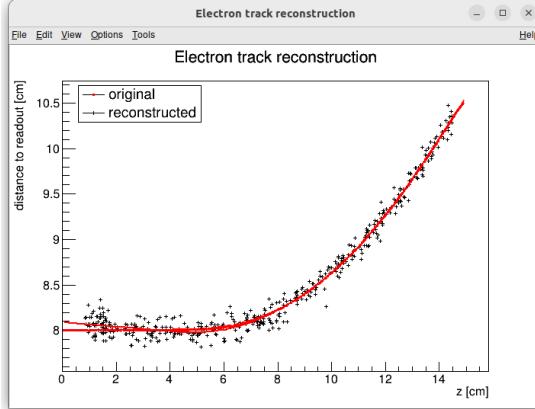


Figure 3.5: Example reconstruction with the map. Swap for better image, correct coordinates.

576 3.2.1 Gradient Descent Search

577 The first implemented method of reconstruction uses a gradient descent search
 578 to calculate an inversion of the map $\bar{\mathcal{M}}$ in a given point. Gradient descent is
 579 an iterative minimization algorithm for multivariate functions. Let $R \in \mathcal{R}$ be
 580 a point in the readout space; we want to find a point $D = (x, y, z) \in \mathcal{D}$ in
 581 the detector space such that

$$582 \quad \bar{\mathcal{M}}(D) = R = (x'_R, y'_R, t_R). \quad (3.9)$$

583 We define a function f_R in the readout space as a distance in this space:

$$584 \quad f_R(x', y', t) = \sqrt{(x' - x'_R)^2 + (y' - y'_R)^2 + v_d^2(t - t_R)^2}, \quad (3.10)$$

585 where v_d is an approximation of the drift velocity in the TPC, obtained from
 586 the reconstruction in Section 3.1 (there will be an image with the linear fit there).
 587 We make an initial guess (actually in the original code we just take $z = 0$):

$$588 \quad D_0 = (x'_R, y'_R, v_d t). \quad (3.11)$$

586 Assuming we have the n -th estimate D_n , we calculate the i -th component of
 587 the gradient of $f_R \circ \bar{\mathcal{M}}$ numerically using central differences:

$$588 \quad [\nabla(f_R \circ \bar{\mathcal{M}})]^i(D_n) \approx \frac{f_R(\bar{\mathcal{M}}(D_n + s \cdot e^i)) - f_R(\bar{\mathcal{M}}(D_n - s \cdot e^i))}{2s}, \quad (3.12)$$

588 where $e^i \in \mathcal{D}$ is the i -th coordinate vector and s is the step size. The step size
 589 should be sufficiently small; initially, we set it as a fraction of the map's grid
 590 spacing $s = \frac{l}{10}$. During the minimization, we check that $f_R(\bar{\mathcal{M}}(D_n)) < 10s$
 591 at all times. When using trilinear interpolation, it would be more efficient to
 592 calculate the gradient explicitly (\pm same result). This could be implemented
 593 inside the *Field* template class. The next iteration can be calculated as follows:

$$594 \quad D_{n+1} = D_n - \gamma \nabla(f_R \circ \bar{\mathcal{M}})(D_n), \quad (3.13)$$

594 where $\gamma \in \mathbb{R}^+$ is the damping coefficient. It should be set to a small enough
 595 value to ensure convergence, but large enough for sufficient converging speed.

596 The minimization stops either when the error $f_R(\bar{\mathcal{M}}(D_n))$ drops below a specified
 597 value or when the number of iterations exceeds a certain limit (in this case,
 598 a message is printed into the console). The parameters of this method can be
 599 further optimized (e.g., a better choice of γ , gradient computation); instead, we
 600 later decided to use the interpolation on the inverse grid described in the next
 601 section.

602 Measure reconstruction duration and compare it with the inverse grid inter-
 603 polation? Also compare the result? Not sure if this has to be cited.

604 3.2.2 Interpolation on the Inverse Grid

605 Interpolating between known points in the readout space. Gaussian elimina-
 606 tion, multivariate polynomial. Benefits compared to the gradient descent search
 607 method (one-time computation for the whole map is easy to achieve if needed).

608 The currently used baseline reconstruction method is the interpolation on
 609 the inverse grid. Rather than attempting to invert the trilinearly interpolated
 610 map as in the previous section, we take advantage of the fact that the map $\bar{\mathcal{M}}$
 611 is one-to-one (isomorphism is supposed to preserve structure, not sure how to
 612 interpret that here). Since we have simulated values of this map on a regular
 613 grid in the detector space \mathcal{D} , we also know the inverse map $\bar{\mathcal{M}}^{-1}$ on the irregular
 614 inverse grid in the readout space \mathcal{R} . To get an approximation of the inverse map
 615 in the entire readout space, we can use interpolation.

616 Since the inverse grid is irregular, trilinear interpolation cannot be applied.
 617 Given that the simulated map is dense enough to provide a good approxima-
 618 tion considering the size of our pads, we can adopt a similar approach (more
 619 complicated and computationally heavy alternative would be natural neighbor
 620 interpolation). As shown in Equation 1.20 in Section 1.3.3, trilinear interpolation
 621 can be expressed as a polynomial:

$$\hat{f}(x, y, z) = axyz + bxy + cxz + dyz + ex + fy + gz + h, \quad (3.14)$$

622 where a, b, c, d, e, f, g, h are coefficients uniquely determined by the values of
 623 the function at the vertices of the interpolation cell. We can generalize this
 624 for a function defined on an irregular grid. Given the function values at any eight
 625 points, we can write a system of eight linear equations

$$\begin{pmatrix} x_1y_1z_1 & x_1y_1 & x_1z_1 & y_1z_1 & x_1 & y_1 & z_1 & 1 \\ \vdots & \vdots \\ x_8y_8z_8 & x_8y_8 & x_8z_8 & y_8z_8 & x_8 & y_8 & z_8 & 1 \end{pmatrix} \begin{pmatrix} a \\ \vdots \\ h \end{pmatrix} = \begin{pmatrix} f(x_1, y_1, z_1) \\ \vdots \\ f(x_8, y_8, z_8) \end{pmatrix}, \quad (3.15)$$

626 which has a unique solution for the coefficients for most values of (x_n, y_n, z_n) and
 627 $f(x_n, y_n, z_n)$, where $n \in \{1, \dots, 8\}$.

628 This approach introduces a small complication: finding the correct pseudocell
 629 (i.e., the image of eight vertices forming a cubic cell in the regular grid) in
 630 the inverse grid. The eight irregularly spaced vertices of this pseudocell do not
 631 define a unique volume, so there are multiple possible ways to partition \mathcal{R} into
 632 pseudocells, with no obvious choice among them.

633 We are currently ignoring this problem and performing binary search along
 634 x, y, z (in this order). It shouldn't matter too much because the 70/30 map

635 doesn't cause such a big distortion and was even accidentally extrapolated for all
 636 z different from the central plane. Interpolation should be generally faster than
 637 the gradient descent since we don't need to iterate. We also don't need to optimize
 638 it to improve performance, if it's too slow we can even calculate the coefficients
 639 for the entire map before reconstruction.

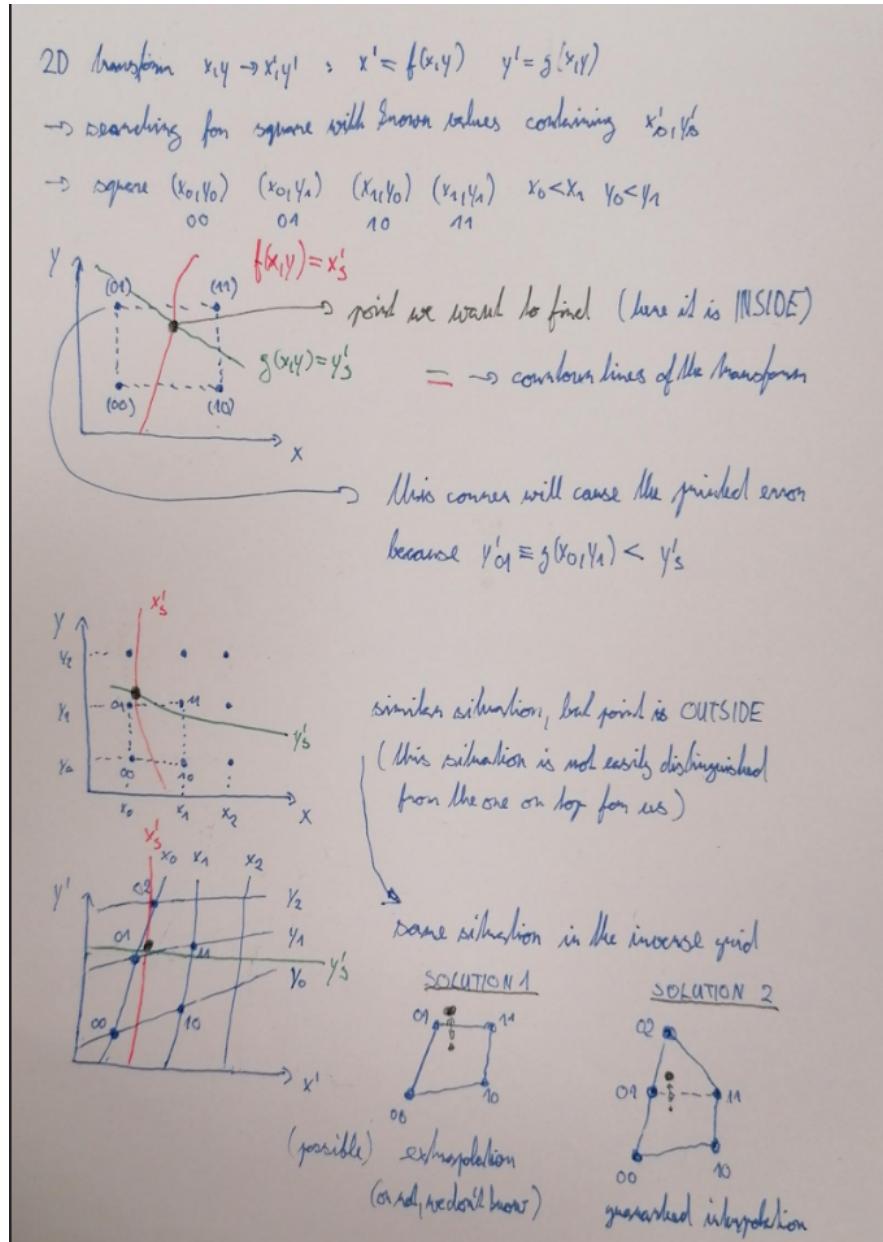


Figure 3.6: Selection of the points for interpolation. Create better images; use the explanation interpolation vs. extrapolation strange property. Solution 2 probably does not make much sense.

640 3.3 Discrete Reconstruction

641 Reconstruction with pads and time bins. Maybe testing different pads. Mapping
 642 the center of the pad (along with the midpoint of the time bin) isn't necessarily

643 the best approach since it might not correspond to the average parameters of
 644 an electron with these readout parameters (insignificant?).

645 It is also possible to make this a subsection of the map, making the previous
 646 subsections parts of a new subsection 'Map Inversion'

647 In order to get a more realistic representation of a track measured in the OFTPC,
 648 we need to take the discretization of the position and time data into account.
 649 The readout of the OFTPC will consist of 128 pads, their layout is shown in
 650 Figure 1.4. Time will be read out with $t_{\text{int}} = 100$ ns intervals.

651 As a first approximation, we can neglect the multiplication in the triple-GEM
 652 and assume an ideal charge readout. The time is started at the beginning of
 653 the electron/positron simulation (randomize this time a bit and see what it does
 654 to the reconstruction). The readout coordinates $(x', y', t) \in \mathcal{R}$ of each ionization
 655 electron can be mapped to the pad coordinates $(n_{\text{pad}}, n_t) \in \mathcal{P}$ (using the param-
 656 eters described in Section 1.3.2):

$$n_{\text{pad}} = n: (x', y') \in \left[x_{1,n} - \frac{g}{2}, x_{2,n} + \frac{g}{2} \right] \times \left[y_{1,n} - \frac{g}{2}, y_{2,n} + \frac{g}{2} \right], \quad (3.16)$$

$$n_t = \left\lceil \frac{t}{t_{\text{int}}} \right\rceil. \quad (3.17)$$

657 This way the closest pad is assigned to each readout position within the OFTPC
 658 volume². Makes sense since the pads attract the electrons, the inhomogeneity of
 659 electric field is neglected. The number of electrons in each pad (i.e., collected
 660 charge) is then counted and serves as a weight for the energy reconstruction.
 661 The reconstructed track consists of points for each $(n, n_t) \in \mathcal{P}$, we get these by
 662 reconstructing the position of a hypothetical electron with the readout coordi-
 663 nates of the pad/time bin center:

$$\mathcal{D} \ni (x, y, z) = \overline{\mathcal{M}} \left(x_{c,n}, y_{c,n}, \left(n_t - \frac{1}{2} \right) t_{\text{int}} \right). \quad (3.18)$$

²Some positions near the wall are not handled and some pads extend beyond the OFTPC volume.

664 4. Energy Reconstruction

665 The second stage is the reconstruction of the particle's energy using a fit of its
666 reconstructed track (see Section 3). We have tested three ways of reconstructing
667 the energy. Fitting is done using the MINUIT algorithm implemented in
668 ROOT [2]. **Cite some CERN article directly on MINUIT, can add a section.**

669 The **Cubic Spline Fit** is a tested and later rejected method of energy re-
670 construction. It uses smoothly connected piecewise cubic polynomials between
671 uniformly spaced nodes. Energy is calculated using the fit parameters by comput-
672 ing the radius of curvature in different points of the fitted curve using the known
673 magnitude of the magnetic field perpendicular to the trajectory. We rejected this
674 method because tuning of the fit to have a reasonably stable radius of curvature
675 turned out to be unpractical.

676 The **Circle and Lines Fit** was chosen as an alternative since this corre-
677 sponds to the shape of a trajectory of a charged particle crossing a finite volume
678 with a homogeneous magnetic field. The energy of the particle can be estimated
679 using the fitted radius and the magnitude of the perpendicular magnetic field in
680 the middle of the TPC.

681 The **Runge-Kutta Fit** uses the 4th order Runge-Kutta numerical integration
682 described in Section 2.2. Initial parameters of the track (including the particle's
683 energy) are optimized so that the integrated trajectory fits to the reconstructed
684 one. This fit can also be performed as a single parameter (i.e., energy) fit if we
685 get the initial position and orientation of the particle on the entrance to the TPC
686 from previous detectors (Tpx3 and MWPC, see Section 0.2).

687 4.1 Cubic Spline Fit

688 The first attempt to get an early estimate of the kinetic energy of the particle
689 uses a cubic spline fit. We use an electron track starting in the origin of our
690 coordinate system with an initial direction in the positive x axis. The example
691 track is simulated microscopically (see Section 2.1) with a kinetic energy of 8 MeV
692 in a gas mixture 90% Ar + 10% CO₂ (the same track was used in Section 3.1).
693 **This track should probably be described in the simulation chapter.**

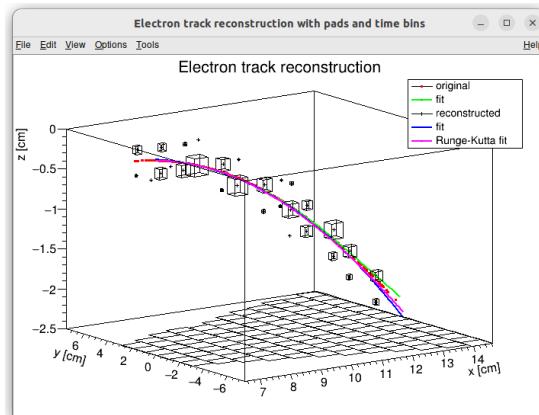


Figure 4.1: Example of a fitted reconstructed track. **Swap for better image.**

694 In order to calculate the spline, we use the class *TSpline3* from ROOT. This
 695 allows us to evaluate the spline using the coordinates (x_n, z_n) of each node and
 696 the derivatives d_1, d_2 in the first and the last node. We can fit these parameters
 697 of a fixed amount of nodes to the simulated trajectory. We use the IMPROVE
 698 algorithm provided by the *TMinuit* class in ROOT. This algorithm attempts to
 699 find a better local minimum after converging.

700 After the fit, we want to get an energy estimate. In order to calculate it, we
 701 need the radius of curvature, which we get from the fitted spline at every point
 702 of the trajectory. The part of the spline corresponding to a given node is defined
 703 as

$$z(x) = z_n + b\Delta x + c(\Delta x)^2 + d(\Delta x)^3, \quad (4.1)$$

704 where $\Delta x = x - x_n$ and b, c, d are coefficients. Using this equation, we derive
 705 the radius of curvature¹ as:

$$r(x) = \frac{(1 + z'^2(x))^{\frac{3}{2}}}{z''(x)} = \frac{(1 + (b + 2c\Delta x + 3d(\Delta x)^2)^2)^{\frac{3}{2}}}{2c + 6d\Delta x}. \quad (4.2)$$

706 Based on the geometry of the detector, we can assume the magnetic field
 707 $\mathbf{B}(x, 0, z) = (0, B(x, z), 0)$ for a track in the XZ plane. Since the electron is rela-
 708 tivistic, the effect of the electric field on its trajectory is negligible. The Lorentz
 709 force F_L is then always perpendicular to the momentum of the electron and acts
 710 as a centripetal force F_c :

$$\mathbf{F}_L = \mathbf{F}_c, \quad (4.3)$$

$$\|e\mathbf{v} \times \mathbf{B}\| = \frac{\gamma m_e v^2}{r}, \quad (4.4)$$

$$ec\beta B = \frac{E_{0e}\beta^2}{r\sqrt{1 - \beta^2}}, \quad (4.5)$$

$$\sqrt{1 - \beta^2} = \frac{E_{0e}\beta}{ecBr}, \quad (4.6)$$

711

$$\beta^2(x) = \left[1 + \left(\frac{E_{0e}}{ecB(x, z(x))r(x)} \right)^2 \right]^{-1}, \quad (4.7)$$

712 where e is the elementary charge, c is the speed of light in vacuum, m_e is the rest
 713 mass of electron, $E_{0e} = m_e c^2$ is the corresponding energy, γ is the Lorentz factor,
 714 \mathbf{v} is the velocity of the electron, and $\beta = \frac{v}{c}$. We can then finally get our estimate
 715 of the kinetic energy for a given point on the trajectory as follows:

$$E_{\text{kin}}(x) = \left(\frac{1}{\sqrt{1 - \beta^2(x)}} - 1 \right) E_{0e}. \quad (4.8)$$

716 We can then average these estimates at multiple points to get one final estimate.
 717 This method was later rejected in favor of the circle and lines fit described in
 718 Section 4.2. **Add some figures.**

¹For the general formula see https://en.wikipedia.org/wiki/Curvature#Graph_of_a_function

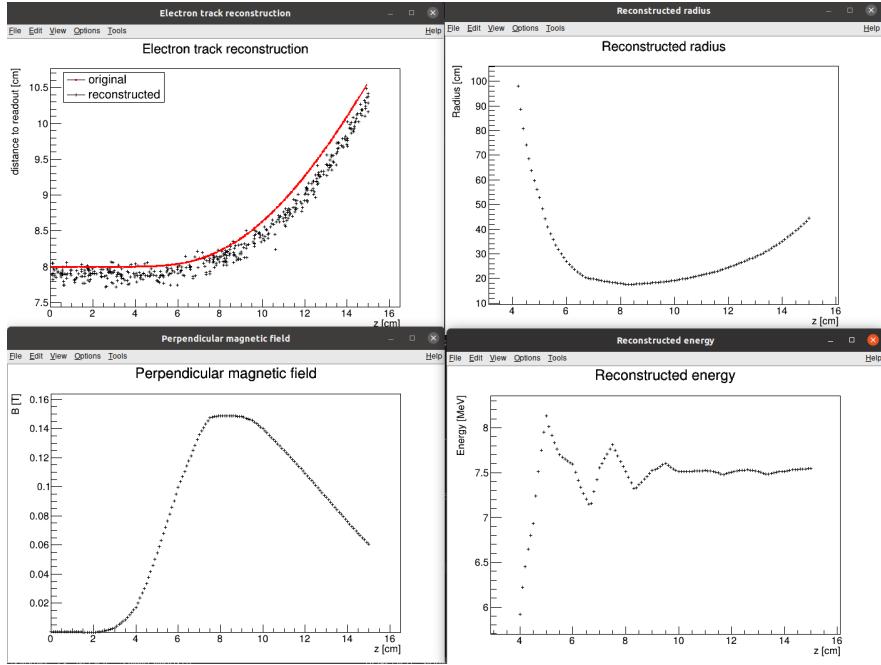


Figure 4.2: First attempt at a track reconstruction using only the drift velocity. Spline energy reconstruction attempt. Swap for better image(s) – subfigure environment, correct coordinates.

719 4.2 Circle and Lines Fit

720 Another way to estimate the particle's kinetic energy is to fit its trajectory with
 721 a circular arc with lines attached smoothly. This shape of trajectory corresponds
 722 to a movement of a charged particle through a homogeneous magnetic field per-
 723 pendicular to the particle's momentum and limited to a certain volume. In gen-
 724 eral, the shape of such a trajectory in a non-perpendicularly oriented field is
 725 a spiral. In our case, this component is negligible since the field is approxi-
 726 mately toroidal and the particle motion is nearly perpendicular to it. At first, we tested
 727 a 2D version of this fit, then we adapted it to 3D.

728 Our field is not homogeneous, it is therefore not entirely clear what value of
 729 magnetic field should be used along with the fitted radius (using equations 4.7
 730 and 4.8) to get the best estimate for the kinetic energy. Since we only use this
 731 method as the first iteration of the particle's energy that we later refine, an op-
 732 timal solution of this problem is not required. Instead, we tested two options:
 733 taking the value of the field in the middle of the fitted circular arc and taking
 734 the average field along it. We haven't really tried to plot this for multiple tracks,
 735 but these estimates are saved somewhere and could be plotted.

736 4.2.1 Two-dimensional fit

737 In the 2D case, the fitted function used for the electron track² described in Sec-
 738 tion 4.1 is defined as follows: Maybe describe this track that we used at the be-
 739 ginning somewhere earlier (section microscopic simulations → Testing track?) so
 740 that it is easier to refer to it in multiple sections. It is not part of the early GitHub

²Electron tracks bend towards negative z , we need to use the upper part of the circle

741 commits, so maybe it won't be possible to create exact replicas of the images,
 742 but they should be at least very similar.

$$z(x) = \begin{cases} a_1x + b_1 & x < x_1 \\ z_0 + \sqrt{r^2 - (x - x_0)^2} & x_1 \leq x \leq x_2, \\ a_2x + b_2 & x > x_2 \end{cases} \quad (4.9)$$

743 where $a_{1,2}$ and $b_{1,2}$ are the parameters of the lines, (x_0, z_0) is the center of the cir-
 744 cle, r is its radius, and $(x_{1,2}, z_{1,2})$ are the coordinates of the function's nodes.
 745 That means we have 9 parameters ($z_{1,2}$ are not used in the function) along with
 746 2 continuity conditions and 2 smoothness conditions. For the fit, we use the co-
 747 ordinates of the nodes and the radius of the circle, which gives us 5 independent
 748 parameters (only the radius has to be larger than half of the distance between
 749 nodes). The continuity conditions (combined with the relations for $z_{1,2}$) are as
 750 follows:

$$z_{1,2} = a_{1,2}x_{1,2} + b_{1,2} = z_0 - \sqrt{r^2 - (x_{1,2} - x_0)^2}. \quad (4.10)$$

751 The smoothness conditions are as follows:

$$a_{1,2} = \frac{x_0 - x_{1,2}}{\sqrt{r^2 - (x_{1,2} - x_0)^2}}. \quad (4.11)$$

752 Equation 4.10 gives us the values of $b_{1,2}$

$$b_{1,2} = z_{1,2} - a_{1,2}x_{1,2}. \quad (4.12)$$

753 For the coordinates of the center of the circle, we can use the fact that the center
 754 has to lie on the axis of its chord. In other words, there is a value of a parameter t
 755 such that, using the parametric equation of the axis

$$\begin{pmatrix} x_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} \frac{x_1+x_2}{2} \\ \frac{z_1+z_2}{2} \end{pmatrix} + t \begin{pmatrix} \frac{z_2-z_1}{2} \\ \frac{x_1-x_2}{2} \end{pmatrix}. \quad (4.13)$$

756 At the same time, the center has to be in a distance of r from the nodes:

$$(x_1 - x_0)^2 + (z_1 - z_0)^2 = r^2, \quad (4.14)$$

$$\left(\frac{x_1 - x_2}{2} + \frac{z_1 - z_2}{2} t \right)^2 + \left(\frac{z_1 - z_2}{2} + \frac{x_2 - x_1}{2} t \right)^2 = r^2, \quad (4.15)$$

$$\left(\left(\frac{x_2 - x_1}{2} \right)^2 + \left(\frac{z_2 - z_1}{2} \right)^2 \right) t^2 + \left(\frac{x_2 - x_1}{2} \right)^2 + \left(\frac{z_2 - z_1}{2} \right)^2 - r^2 = 0. \quad (4.16)$$

757 Since our electron track bends towards negative z and $x_2 > x_1$, we only care
 758 about the solution with $t > 0$

$$t = \sqrt{\frac{r^2}{\left(\frac{x_2 - x_1}{2} \right)^2 + \left(\frac{z_2 - z_1}{2} \right)^2} - 1}, \quad (4.17)$$

759

$$x_0 = \frac{x_1 + x_2}{2} + \frac{z_2 - z_1}{2} \sqrt{\frac{r^2}{\left(\frac{x_2 - x_1}{2} \right)^2 + \left(\frac{z_2 - z_1}{2} \right)^2} - 1}, \quad (4.18)$$

$$z_0 = \frac{z_1 + z_2}{2} - \frac{x_2 - x_1}{2} \sqrt{\frac{r^2}{\left(\frac{x_2 - x_1}{2} \right)^2 + \left(\frac{z_2 - z_1}{2} \right)^2} - 1}. \quad (4.19)$$

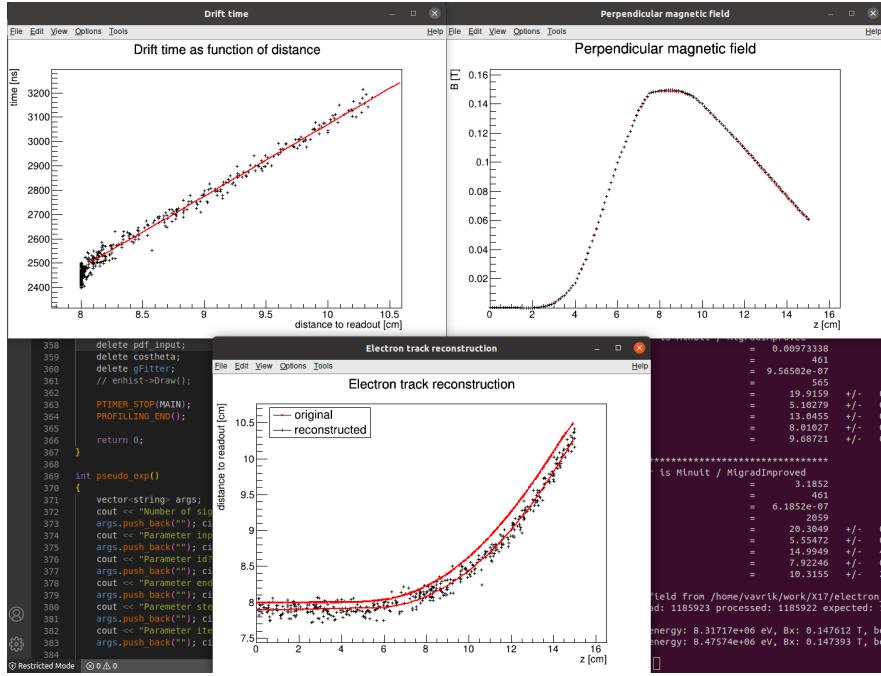


Figure 4.3: First attempt at a track reconstruction using only the drift velocity. Circle and Lines Fit in 2D. Swap for better image, correct coordinates.

760 The function defined in Equation 4.9 along with equations 4.11, 4.12, 4.18 and 4.19
 761 derived using the continuity and smoothness conditions (combined with the re-
 762 lations for $z_{1,2}$) fully define our fitted function with parameters $r, x_{1,2}, z_{1,2}$. Some
 763 pictures of the fit on the tested track. Results of the fit. Again, the actual fit
 764 uses $8-z$. Use GeoGebra schematics to generate a picture of 2D geometry.

765 Tested on a Runge-Kutta sample, and with microscopic tracks + map sim-
 766 ulation. Preliminary 2D version (done) and complete 3D version. Geometry of
 767 the fit with its derivation.

768 4.2.2 Three-dimensional fit

769 Explain the geometry and least square method used for the 3D fit.

770 4.3 Runge-Kutta Fit

771 Single parameter fit with 4th order Runge-Kutta simulated track. Future testing
 772 with microscopic simulations and map simulation. Derivation of the geometry
 773 (least squares).

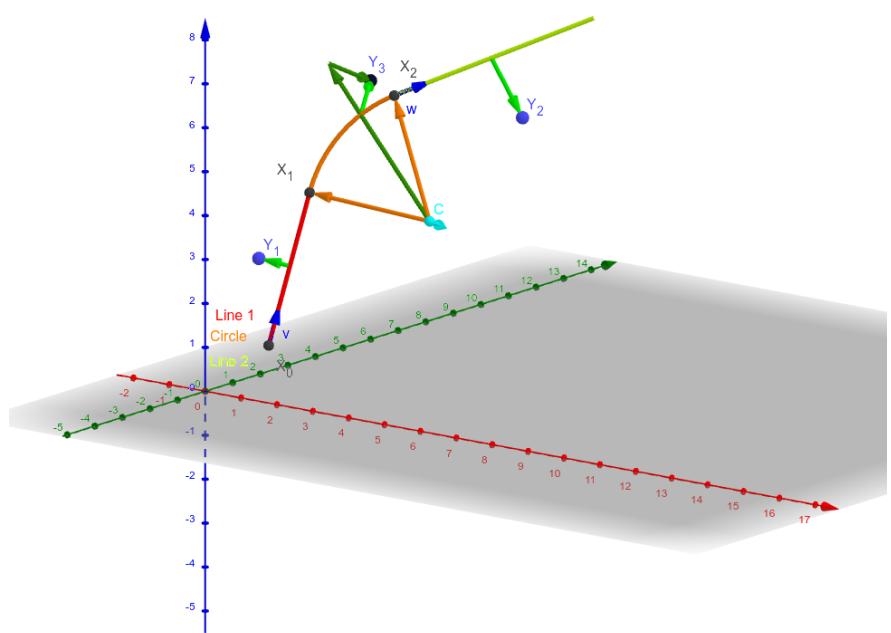


Figure 4.4: Circle and Lines Fit 3D geometry. Swap for better image.

Conclusion

774 Here or at the end of each section. Something about the future of this work?

776 Notes

777 General notes about the thesis:

- 778 • Check that all of the classes and other code are marked the same way in
779 the text. I used italics somewhere, could use different font for this instead.
- 780 • Check unbreakable space in front of articles. Remove excessive article usage
781 with proper nouns.
- 782 • Currently using margins for single-sided printing (bigger on the left side).
- 783 • Check that present tense is used
- 784 • Active vs passive voice usage
- 785 • American English quotation marks ("") instead of British English ('').
- 786 • Some of the overfull hbox warnings might change if duplex printing is used
787 (they generate black rectangles on the edge of the page), leaving them be
788 for now
- 789 • Check nobreakdash usage (is it always necessary)
- 790 • Check capitalized references (e.g., Figure, Section, Equation)
- 791 • Check $\backslash(\dots\backslash)$ math mode instead of $\$...$$. (actually unlike $\backslash[\dots\backslash]$ math mode,
792 there is apparently no real benefit to this clumsy syntax)
- 793 • Use siunitx package to ensure correct formatting.
- 794 • Check other stuff that's written in the MFF UK template. Apparently it
795 has since been updated and there are some differences (check for them).
- 796 • Check correct subscripts in equation (italics vs no italics)
- 797 • Consistent bold marking of points/vectors
- 798 • Correct footnotes (capital letters, etc.).

799 Random notes:

- 800 • Terminology consistency – ionization/primary/secondary electrons
- 801 • Consistent TPC vs OFTPC acronym usage in the text or individual chap-
802 ters.
- 803 • Only electrons that start and end in the sector closer than 0.5 cm are used
804 for reconstruction (newest version).
- 805 • Attachment, Penning transfer and secondary ionization not considered in
806 the microscopic simulation.

- 807 • Suspicious artifacts of trilinear interpolation in Figure 1.5. **Fixed – integers instead of doubles in the implementation, influenced reconstruction SIGNIFICANTLY (but not simulation).**
- 808
- 809

810

Future

811 Things planned for the future:

- 812 • Testing the reconstruction algorithm by measuring real particles with a known energy distribution.
- 814 • The **Fast Simulation with Ionization Electron Map** is planned for the future. It will use the HEED program [27] to simulate the primary particle and the Ionization Electron Map (see Section 3.2) to simulate the drift of secondary electrons. It should be significantly faster than the Microscopic Simulation but offer comparable precision since it will rely on an already simulated drift map. (Primary track simulated in HEED. Readout parameters by interpolating the map. Diffusion from the map for randomization.)
- 816
- 817
- 818
- 819
- 820
- 821 • Account for GEM, delta electrons, ...
- 822 • Likelihood approach instead of least squares (if it improves the reconstruction significantly), we should at least use a better method than taking the center of the TPC bin.
- 823
- 824
- 825 • More detailed electric field simulation (if needed, GEM will have more complex field)
- 826

827

Likelihood - inverse map

828 If we wanted to further improve this procedure, taking into account the whole map \mathcal{M} , we could make an "inverse map" from \mathcal{R} to distributions on \mathcal{D} . We could achieve this by taking the normalized probability density of an electron with initial coordinates (x, y, z) having readout coordinates (x', y', t) . If we fix (x', y', t) , we get an unnormalized probability density $f(x, y, z) = \mathcal{M}_{(x,y,z)}(x', y', t)$ (assuming that all initial coordinates are a priori equally likely). This could potentially improve the discrete reconstruction if we take the mean value of this probability density across the pad and time bin

830

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832

833

834

835

$$f_{\text{pad, bin}}(x, y, z) = \frac{1}{A_{\text{pad}} \Delta t_{\text{bin}}} \int_{\text{pad, bin}} \mathcal{M}_{(x,y,z)}(x', y', t) dx' dy' dt \quad (4.20)$$

836 and using it for a likelihood fit instead of using least squares. This still assumes
837 that all initial coordinates are equally likely which is clearly not the case for
838 a primary particle track. In the future, we could even use the fast track simulation
839 with the map (should be possible to make around 1000 tracks per minute per core
840 with current settings), create a big set of tracks with reasonable parameters and
841 use these to get an approximation of the probability distribution of the detector
842 response. Some approximations would be necessary when interpreting the data to
843 decrease the degrees of freedom of this distribution (we would have to pick a set of
844 parameters and assume that some of them are independent). This could give us

845 an idea about the best achievable resolution (how significantly will the detector
846 response differ for a given change in energy). If the difference is significant, we
847 could try to further improve the likelihood fit.

Bibliography

- [1] Garfield++. <https://garfieldpp.web.cern.ch/garfieldpp/>. Accessed: 2023-05-18.
- [2] Rene Brun and Fons Rademakers. Root — An Object Oriented Data Analysis Framework. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 389(1–2):81–86, Apr 1997. Proceedings AIHENP’96 Workshop, Lausanne, Sep. 1996, See also <https://root.cern/>, Paper published in the Linux Journal, Issue 51, July 1998.
- [3] About MetaCentrum. <https://metavo.metacentrum.cz/en/about/index.html>, 2023. Accessed: 2024-11-27.
- [4] M. E. Rose. Internal Pair Formation. *Phys. Rev.*, 76:678–681, Sep 1949.
- [5] R. Essig, J. A. Jaros, W. Wester, P. Hansson Adrian, S. Andreas, T. Averett, O. Baker, B. Batell, M. Battaglieri, J. Beacham, T. Beranek, J. D. Bjorken, F. Bossi, J. R. Boyce, G. D. Cates, A. Celentano, A. S. Chou, R. Cowan, F. Curciarello, H. Davoudiasl, P. deNiverville, R. De Vita, A. Denig, R. Dharmapalan, B. Dongwi, B. Döbrich, B. Echenard, D. Espriu, S. Fegan, P. Fisher, G. B. Franklin, A. Gasparian, Y. Gershtein, M. Graham, P. W. Graham, A. Haas, A. Hatzikoutelis, M. Holtrop, I. Irastorza, E. Izaguirre, J. Jaeckel, Y. Kahn, N. Kalantarians, M. Kohl, G. Krnjaic, V. Kubarovský, H-S. Lee, A. Lindner, A. Lobanov, W. J. Marciano, D. J. E. Marsh, T. Maruyama, D. McKeen, H. Merkel, K. Moffeit, P. Monaghan, G. Mueller, T. K. Nelson, G. R. Neil, M. Oriunno, Z. Pavlovic, S. K. Phillips, M. J. Pivovaroff, R. Poltis, M. Pospelov, S. Rajendran, J. Redondo, A. Ringwald, A. Ritz, J. Ruz, K. Saenboonruang, P. Schuster, M. Shinn, T. R. Slatyer, J. H. Steffen, S. Stepanyan, D. B. Tanner, J. Thaler, M. E. Tobar, N. Toro, A. Upadye, R. Van de Water, B. Vlahovic, J. K. Vogel, D. Walker, A. Weltman, B. Wojtsekhowski, S. Zhang, and K. Zioutas. Dark Sectors and New, Light, Weakly-Coupled Particles, 2013.
- [6] F.W.N. de Boer, O. Fröhlich, K.E. Stiebing, K. Bethge, H. Bokemeyer, A. Balandia, A. Buda, R. van Dantzig, Th.W. Elze, H. Folger, J. van Klinken, K.A. Müller, K. Stelzer, P. Thee, and M. Waldschmidt. A deviation in internal pair conversion. *Physics Letters B*, 388(2):235–240, 1996.
- [7] F W N de Boer, R van Dantzig, J van Klinken, K Bethge, H Bokemeyer, A Buda, K A Müller, and K E Stiebing. Excess in nuclear pairs near 9 MeV/ invariant mass. *Journal of Physics G: Nuclear and Particle Physics*, 23(11):L85, nov 1997.
- [8] F W N de Boer, K Bethge, H Bokemeyer, R van Dantzig, J van Klinken, V Mironov, K A Müller, and K E Stiebing. Further search for a neutral boson with a mass around 9 MeV/c². *Journal of Physics G: Nuclear and Particle Physics*, 27(4):L29, apr 2001.

- 889 [9] Attila Vitéz, A. Krasznahorkay, J. Gulyas, Margit Csatlos, Lorant Csige,
 890 Zoltan Gacsi, Barna Nyakó, F. Boer, T. Ketel, and J. Klinken. Anomalous
 891 Internal Pair Creation in ${}^8\text{Be}$ as a Signature of the Decay of a New Particle.
 892 *Acta Physica Polonica B - ACTA PHYS POL B*, 39:483, 02 2008.
- 893 [10] A. Krasznahorkay, J. Gulyas, Margit Csatlos, Attila Vitéz, T. Tornyi,
 894 L. Stuhl, Lorant Csige, Zoltan Gacsi, A. J. Krasznahorkay, M. Hunyadi,
 895 and T.J. Ketel. Searching for a light neutral axial-vector boson in isoscalar
 896 nuclear transitions. *Frascati Physics Series*, 56:86, 10 2012.
- 897 [11] A. J. Krasznahorkay, M. Csatlós, L. Csige, Z. Gácsi, J. Gulyás, M. Hunyadi,
 898 I. Kuti, B. M. Nyakó, L. Stuhl, J. Timár, T. G. Tornyi, Zs. Vajta, T. J. Ketel,
 899 and A. Krasznahorkay. Observation of Anomalous Internal Pair Creation
 900 in ${}^8\text{Be}$: A Possible Indication of a Light, Neutral Boson. *Physical Review
 901 Letters*, 116(4), January 2016.
- 902 [12] D.R. Tilley, J.H. Kelley, J.L. Godwin, D.J. Millener, J.E. Purcell, C.G. Sheu,
 903 and H.R. Weller. Energy levels of light nuclei $A=8,9,10$. *Nuclear Physics A*,
 904 745(3):155–362, 2004.
- 905 [13] N. J. Sas, A. J. Krasznahorkay, M. Csatlós, J. Gulyás, B. Kertész, A. Krasznahorkay,
 906 J. Molnár, I. Rajta, J. Timár, I. Vajda, and M. N. Harakeh. Observation
 907 of the X17 anomaly in the ${}^7\text{Li}(p,e^+e^-){}^8\text{Be}$ direct proton-capture
 908 reaction, 2022.
- 909 [14] A. J. Krasznahorkay, M. Csatlós, L. Csige, J. Gulyás, A. Krasznahorkay,
 910 B. M. Nyakó, I. Rajta, J. Timár, I. Vajda, and N. J. Sas. New anomaly
 911 observed in ${}^4\text{He}$ supports the existence of the hypothetical X17 particle.
 912 *Physical Review C*, 104(4), October 2021.
- 913 [15] D.R. Tilley, H.R. Weller, and G.M. Hale. Energy levels of light nuclei $A =$
 914 4. *Nuclear Physics A*, 541(1):1–104, 1992.
- 915 [16] A. J. Krasznahorkay, A. Krasznahorkay, M. Begala, M. Csatlós, L. Csige,
 916 J. Gulyás, A. Krakó, J. Timár, I. Rajta, I. Vajda, and N. J. Sas. New
 917 anomaly observed in ${}^{12}\text{C}$ supports the existence and the vector character of
 918 the hypothetical X17 boson. *Phys. Rev. C*, 106:L061601, Dec 2022.
- 919 [17] F. Ajzenberg-Selove. Energy levels of light nuclei $A = 11,12$. *Nuclear Physics
 920 A*, 506(1):1–158, 1990.
- 921 [18] Péter Kálmán and Tamás Keszthelyi. Anomalous internal pair creation. *The
 922 European Physical Journal A*, 56, 08 2020.
- 923 [19] A. Aleksejevs, S. Barkanova, Yu. G. Kolomensky, and B. Sheff. A Standard
 924 Model Explanation for the "ATOMKI Anomaly", 2021.
- 925 [20] Jonathan L. Feng, Bartosz Fornal, Iftah Galon, Susan Gardner, Jordan
 926 Smolinsky, Tim M. P. Tait, and Philip Tanedo. Protophobic Fifth-Force
 927 Interpretation of the Observed Anomaly in ${}^8\text{Be}$ Nuclear Transitions. *Phys.
 928 Rev. Lett.*, 117:071803, Aug 2016.

- [21] Tran The Anh, Tran Dinh Trong, Attila J. Krasznahorkay, Attila Krasznahorkay, József Molnár, Zoltán Pintye, Nguyen Ai Viet, Nguyen The Nghia, Do Thi Khanh Linh, Bui Thi Hoa, Le Xuan Chung, and Nguyen Tuan Anh. Checking the ${}^8\text{Be}$ Anomaly with a Two-Arm Electron Positron Pair Spectrometer. *Universe*, 10(4):168, April 2024.
- [22] Kh. U. Abraamyan, Ch. Austin, M. I. Baznat, K. K. Gudima, M. A. Kozhin, S. G. Reznikov, and A. S. Sorin. Observation of structures at ~ 17 and ~ 38 MeV/c^2 in the $\gamma\gamma$ invariant mass spectra in pC, dC, and dCu collisions at p_{lab} of a few GeV/c per nucleon, 2023.
- [23] The MEG II collaboration, K. Afanaciev, A. M. Baldini, S. Ban, H. Bemmansour, G. Boca, P. W. Cattaneo, G. Cavoto, F. Cei, M. Chiappini, A. Corvaglia, G. Dal Maso, A. De Bari, M. De Gerone, L. Ferrari Barusso, M. Francesconi, L. Galli, G. Gallucci, F. Gatti, L. Gerritzen, F. Grancagnolo, E. G. Grandoni, M. Grassi, D. N. Grigoriev, M. Hildebrandt, F. Ignatov, F. Ikeda, T. Iwamoto, S. Karpov, P. R. Kettle, N. Khomutov, A. Kolesnikov, N. Kravchuk, V. Krylov, N. Kuchinskiy, F. Leonetti, W. Li, V. Malyshev, A. Matsushita, M. Meucci, S. Mihara, W. Molzon, T. Mori, D. Nicolò, H. Nishiguchi, A. Ochi, W. Ootani, A. Oya, D. Palo, M. Panareo, A. Papa, V. Pettinacci, A. Popov, F. Renga, S. Ritt, M. Rossella, A. Rozhdestvensky, S. Scarpellini, P. Schwendimann, G. Signorelli, M. Takahashi, Y. Uchiyama, A. Venturini, B. Vitali, C. Voena, K. Yamamoto, R. Yokota, and T. Yonemoto. Search for the X17 particle in ${}^7\text{Li}(p, e^+e^-){}^8\text{Be}$ processes with the MEG II detector, 2024.
- [24] Stewart C. Loken, H. Aihara, M. Alston-Garnjost, D. H. Badtke, J. A. Bakken, A. Barbaro-Galtieri, A. V. Barnes, B. A. Barnett, B. Blumenfeld, A. Bross, C. D. Buchanan, W. C. Carithers, O. Chamberlain, J. Chiba, C-Y. Chien, A. R. Clark, O. I. Dahl, C. T. Day, P. Delpierre, K. A. Derby, P. H. Eberhard, D. L. Fancher, H. Fujii, T. Fujii, B. Gabioud, J. W. Gary, W. Gorn, N. J. Hadley, J. M. Hauptman, B. Heck, H. Hilke, J. E. Huth, J. Hylen, H. Iwasaki, T. Kamae, R. W. Kenney, L. T. Kerth, R. Koda, R. R. Kofler, K. K. Kwong, J. G. Layter, C. S. Lindsey, S. C. Loken, X-Q. Lu, G. R. Lynch, L. Madansky, R. J. Madaras, R. Majka, J. Mallet, P. S. Martin, K. Maruyama, J. N. Marx, J. A. J. Matthews, S. O. Melnikoff, W. Moses, P. Nemethy, D. R. Nygren, P. J. Oddone, D. Park, A. Pevsner, M. Pripstein, P. R. Robrish, M. T. Ronan, R. R. Ross, F. R. Rouse, G. Shapiro, M. D. Shapiro, B. C. Shen, W. E. Slater, M. L. Stevenson, D. H. Stork, H. K. Ticho, N. Toge, M. Urban, G. J. Van Dalen, R. van Tyen, H. Videau, M. Wayne, W. A. Wenzel, R. F. vanDaalen Wetters, M. Yamauchi, M. E. Zeller, and W-M. Zhang. Performance of the signal processing system for the time projection chamber. *IEEE Transactions on Nuclear Science*, 30(1):162–166, 1983.
- [25] Misbah Uddin Ahmed. Simulation of the electron and ion movement through a 4-GEM stack, 2021. Institut für Kernphysik.
- [26] Cmglee. Trilinear interpolation visualisation, 2025. Licensed under

973 CC BY-SA 3.0 ([https://creativecommons.org/licenses/by-sa/3.0/
974 legalcode](https://creativecommons.org/licenses/by-sa/3.0/legalcode)). Accessed: 26-March-2025.

975 [27] I.B. Smirnov. Modeling of ionization produced by fast charged particles in
976 gases. *Nuclear Instruments and Methods in Physics Research Section A: Ac-
977 celerators, Spectrometers, Detectors and Associated Equipment*, 554(1):474–
978 493, 2005.

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1043 **List of Tables**

¹⁰⁴⁴ List of Abbreviations

- ¹⁰⁴⁵ **GEM** Gas Electron Multiplier
- ¹⁰⁴⁶ **HEED** High Energy Electro-Dynamics
- ¹⁰⁴⁷ **IEAP CTU** Institute of Experimental and Applied Physics, Czech Technical
¹⁰⁴⁸ University in Prague
- ¹⁰⁴⁹ **IPC** Internal Pair Creation
- ¹⁰⁵⁰ **EPC** External Pair Creation
- ¹⁰⁵¹ **Micromegas** MICRO-MEsh GAseous Structure
- ¹⁰⁵² **MWPC** Multi-Wire Proportional Chamber
- ¹⁰⁵³ **OFTPC** Orthogonal Fields TPC
- ¹⁰⁵⁴ **TPC** Time Projection Chamber
- ¹⁰⁵⁵ **ToA** time-of-arrival
- ¹⁰⁵⁶ **ToT** time-over-threshold
- ¹⁰⁵⁷ **Tpx3** Timepix 3