ARTICLE IN PRESS

Journal of Process Control xxx (2009) xxx–xxx

Contents lists available at ScienceDirect

Journal of Process Control

journal homepage: www.elsevier.com/locate/jprocont

Control of an anaerobic digester through normal form of fold bifurcation

A. Rincon, F. Angulo *, G. Olivar

Department of Electrical and Electronics Engineering and Computer Science, Universidad Nacional de Colombia, Sede Manizales, CeiBA Complexity, Manizales, Colombia

ARTICLE INFO

Article history:
Received 17 September 2008
Received in revised form 15 April 2009
Accepted 23 April 2009
Available online xxxx

Keywords:
Adaptive control
Anaerobic digester
Bifurcations
Feedback linearization
Normal form

ABSTRACT

Nonlinear dynamics is ubiquitous in engineering systems. As some parameters are varied bifurcations arise in the state variables. Generically, when one parameter changes, Hopf and fold bifurcations are found. Other ones can also be present due to special systems characteristics, such as symmetries. Knowing in advance the significant bifurcation scenario, a novel approach to control can be considered. We compute the normal form corresponding to such a bifurcation and we take this model as the nominal model of the plant. Then we design a nonlinear control which takes advantage of the precise bifurcation scenario. This general method is applied, in this paper, to an anaerobic digester. We will control the process with an adaptive controller.

Specifically, we want to compare with the case that the nominal plant is considered as a linear model, such as it is typical in adaptive control techniques. Our proposed method has more benefits in signal control effort, faster convergence rate and low error.

This paper shows how the combination of appropriated nonlinear dynamic techniques such as bifurcations and normal forms, and nonlinear control, can give rise to an improvement of the traditional methodology.

1. Introduction

Nonlinear dynamics has been a hot topic in the last decades, and it is still a very active research area. In the last years we have seen many applications of nonlinear dynamics to science and engineering systems, such as power electronics, biological systems, traffic flow, and so on. The main characteristics from nonlinear dynamics regard to existence of limit cycles, quasiperiodicity or chaos in the state space; coexistence of several attractors with their basins of attraction, and bifurcations as some parameters are varied, to name but a few [1]. Different mathematical tools for analyzing the dynamics, corresponding to nonlinear phenomena, have been described in the literature, such as the reduction to the center manifold, or the normal form theory [2]. But still there is not much cross-fertilization between nonlinear dynamics and control, and only a few papers combine tools from both research fields.

In this paper, we propose a control strategy, partially with the aim to contribute to reduce the gap between these areas. We make specific use of the normal form theory from nonlinear dynamics, and adaptive control from nonlinear control [3]. Both techniques are well-known in the corresponding research fields, but rarely have been put together appropriately. Thus our strategy can be described by the following steps: first, we obtain a nonlinear model for the system; second, we perform bifurcation analysis in order to know which are the nonlinear characteristics of the phenomena that we want to study; third, we compute the corresponding nonlinear normal form for the bifurcation scenario; and forth, we design a nonlinear control taking advantage of the knowledge of the bifurcation scenario. We want to emphasize that this strategy can be applied to many engineering systems, but the details depend very much on the specific system dynamics and the nonlinear control which is chosen. Thus, generically, if we vary a parameter of a nonlinear dynamical system, Hopf and fold bifurcations will occur, and each one has its own normal form [2]. Also, different specific nonlinear controllers can be chosen. For example, in the application which we describe in the following sections, we are interested in the nonlinear phenomena mainly caused by a fold bifurcation, and the nonlinear control we chose is an adaptive control. Explicitly showing the details for each possible bifurcation and each nonlinear control in a general system would be a lengthy and exhaustive task, and no doubt, it would not fit into the standards of this journal. Instead, our aim is to clearly show the steps of our strategy and apply it to a standard problem in chemical engineering processes, such as a wastewater treatment plant. Moreover, we want to emphasize specifically the advantage of considering the nonlinear normal form as the model of the plant, instead of the linear model, which is usually the case in standard control [3].

* Corresponding author. Tel.: +57 6 8879400; fax: +57 6 8879498.
E-mail addresses: arincons@unal.edu.co (A. Rincon), fangulog@unal.edu.co (F. Angulo), golivart@unal.edu.co (G. Olivar).

0959-1524/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jprocont.2009.04.006

Specifically, our strategy can be described by the following steps:

- Obtain a nonlinear model for the system.
- Perform bifurcation analysis close (locally) to the phenomenon that we want to study.
- Compute the corresponding nonlinear normal form for the bifurcation scenario.
- Design a nonlinear controller taking advantage of the knowledge of the bifurcation scenario.

Thus, assume that we have a system of nonlinear differential equations in the state space

$$\dot{x} = f(x, \gamma)$$

where $f = (f_1, \ldots, f_n)$ is a nonlinear smooth vector field, $x = (x_1, \ldots, x_n)$ is the state vector, and $\gamma = (\gamma_1, \ldots, \gamma_m)$ is a parameter vector.

Assume also that after performing several state space simulations, we detect that the phenomenon which we want to study occurs for some concrete values of the parameters

$$\gamma^0 = (\gamma_1^0, \ldots, \gamma_m^0)$$

and that one of the significant parameters, related to the phenomenon, is γ_1 (we can consider γ_1 without losing generality).

Then, varying γ_1 in a certain interval range $[\gamma_1^1, \gamma_1^2]$ including γ_1^0, we will see different qualitative system behavior, that mathematically corresponds to a bifurcation. So, we perform bifurcation analysis in the range $[\gamma_1^1, \gamma_1^2]$, taking γ_1 as the bifurcation parameter. Generically, we will observe a Hopf or a fold bifurcation, although other non-generic bifurcations can also be found if the system, for example, has some sort of symmetries [2].

Depending on the bifurcation scenario, we consider the corresponding normal form. The normal form contains (locally) all the dynamics of the original system, but its expression is considerably simplified. For example, for a two-dimensional system, if we have a Hopf bifurcation (birth of oscillations from an equilibrium point), we can take the normal form

$$\begin{cases} \dot{y}_1 = \beta y_1 - y_2 \pm (y_1^2 + y_2^2) y_1 \\ \dot{y}_2 = y_1 + \beta y_2 \pm (y_1^2 + y_2^2) y_2 \end{cases}$$

where y_1, y_2 are the new state variables, and β is the new parameter, after some corresponding transformations. The sign $+$ or $-$ depends also on the original system.

Instead, if we have a fold bifurcation scenario (annihilation of a stable and an unstable equilibrium points), in a one-dimensional system, the normal form is

$$\dot{y} = \beta \pm y^2$$

where y is the new state variable, and β is the new parameter, after the corresponding transformations. As before, the sign $+$ or $-$ depends also on the original system.

In the case of n-dimensional systems, very similar expressions can be obtained, as it will be observed with detail in our application to a wastewater treatment plant.

The final step regards to the nonlinear controller chosen. The basic fact, that we want to emphasize, and that we think that makes our strategy advantageous, is to take the normal form as the nominal system model for the plant, since it includes, through the bifurcation, the significant ingredient of the nonlinear phenomenon we study.

Once our strategy has been mathematically formalized, we will apply it, with the corresponding details, to a wastewater treatment plant. First, we will obtain the nonlinear differential equations which describe the plant. Then, we will perform bifurcation analysis close to the significant phenomenon (in this case, washout). State space simulations have been reported in the literature [11,12], and we will skip them in this paper. Once the bifurcation scenario is detected and classified (in this case, a fold bifurcation), we will compute the corresponding normal form. Finally, we will take the fold normal form as the nominal model of the plant in order to design an adaptive control.

Regarding our application, it is worth to note that in the last two decades, more rigorous environmental rules have been imposed for the toxic matter released in industrial and urban effluents. This situation has increased the study and application of optimization and automatic control in the wastewater treatment plants (WWTPs) [4–9]. The main objective of any biologic WWTP is to decompose the organic matter of an urban or industrial effluent, such that the chemical oxygen demand (COD) is reduced below a specific value or percentage imposed by the security and environmental rules.

Nevertheless, the application of anaerobic treatment plants in industrial scale is limited, because they are easily destabilized under variation of the process operating conditions and they need expertise knowledge to be operated [10]. However, these problems can be overcome by using on-line monitoring and control systems, so that an optimal performance is maintained, specially the efficiency of pollutant removal [6,7,9,10].

The remaining of this paper is organized as follows. In Section 2, the anaerobic digester and the corresponding mass balance model are briefly described, leading to the nonlinear system of differential equations. Nonlinear behavior is studied including equilibria, transcritical and fold bifurcations, and normal forms. In Section 3 the nominal models are proposed and the designed estimation mechanism is based on recursive least squares (RLS). The control laws using adaptive techniques and input–output linearization are stated. In Section 4 the results obtained by means of simulation are shown. Finally, in Section 5 some conclusions and future work are presented.

2. Nonlinear dynamics

The basic phenomena behind the digester failure (destabilization) consist of an imbalance between acidogenesis and methanogenesis, specially in volatile fatty acids (VFAs) production and

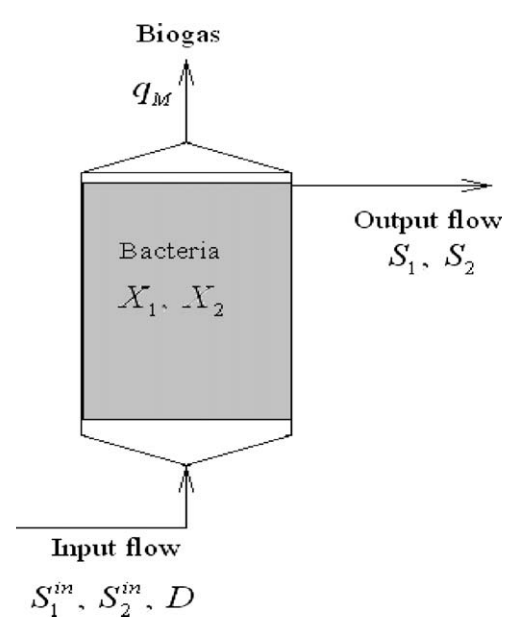

Fig. 1. Sketch of the anaerobic digestion reactor.

Please cite this article in press as: A. Rincon et al., Control of an anaerobic digester through normal form of fold bifurcation, J. Process Contr. (2009), doi:10.1016/j.jprocont.2009.04.006

Table 1
Nomenclature.

Parameter	Description
D	Dilution rate (day^{-1})
X_1	Concentration of acidogenic biomass (g/L)
X_2	Concentration of methanogenic biomass (g/L)
S_1	Concentration of COD (g/L)
S_2	Concentration of VFA (mmol/L)
S_1^{in}	Concentration of COD in the inlet flow (g/L), 5.8 g/L
S_2^{in}	Concentration of VFA in the inlet flow (mmol/L), 52 mmol/L
α	Fraction of the biomass which is not retained in the digester, 0.5
μ_1	Acidogenic biomass growth rate (day^{-1})
μ_2	Methanogenic biomass growth rate (day^{-1})
$\mu_{\max}$	Maximum acidogenic bacteria growth rate (day^{-1}), 1.2 day^{-1}
μ_0	Maximum methanogenic bacteria growth rate (day^{-1}), 0.74 day^{-1}
k_1	Yield coefficient for substrate degradation, 10.53
k_2	Yield coefficient for VFA production (mmol/g), 28.6 mmol/g
k_3	Yield coefficient for VFA consumption (mmol/g), 1074 mmol/g
K_{S1}	Half saturation constant (g/L), 7.1 g/L
K_{S2}	Half saturation constant (mmol/l), 9.28 mmol/L
K_I	Inhibition constant associated to S_2 ((mmol/L)$^{1/2}$) 16 (mmol/L)$^{1/2}$

consumption rates, which are the most important intermediates in the reaction mechanism. The VFAs are substrate for methanogenic bacteria. Large concentration of VFAs decreases methanogenic bacteria growth rate – phenomenon called inhibition, so that acids consumption rate is decreased and concentration of acids is increased – digester acidification – [11,13]. The above imbalance can be caused by several factors: poor mixing, low residence time, high inlet solids concentration, toxic substances, or temperature shocks. Thus VFAs are recognized as important indicators of process imbalance and should be maintained below an inhibitory level (control parameter) [14,15]. When anaerobic digestion is under normal operation the concentration of VFAs is below a threshold, and when VFA is above this threshold digester failure occurs [16]. In [15] the VFA concentration is selected as the controlled output. Destabilization can give rise to washout, where the bacteria disappears.

In Fig. 1 a sketch of the anaerobic digestion reactor is presented and in Table 1 the terms concerning to the dynamical model are shown. The dynamical behavior of the anaerobic digester is described by the mass balance model [9], which has been widely used:

$$\begin{aligned}
\dot{X}_1 &= (\mu_1(S_1) - \alpha D)X_1\\
\dot{X}_2 &= (\mu_2(S_2) - \alpha D)X_2\\
\dot{S}_1 &= (S_1^{in} - S_1)D - k_1\mu_1(S_1)X_1\\
\dot{S}_2 &= (S_2^{in} - S_2)D + k_2\mu_1(S_1)X_1 - k_3\mu_2(S_2)X_2
\end{aligned} \tag{1}$$

where

$$\mu_1(S_1) = \mu_{\max 1}\frac{S_1}{K_{S1} + S_1}$$

and

$$\mu_2(S_2) = \mu_0\frac{S_2}{K_{S2} + S_2 + \left(\frac{S_2}{K_I}\right)^2}$$

The state variables of the model are X_1, X_2, S_1, S_2 which correspond to the concentration of acidogenic bacteria, concentration of methanogenic bacteria, concentration of COD and concentration of VFA, respectively. D is the dilution rate, defined as the ratio of the influent flowrate to the volume occupied by the liquid inside the anaerobic digester. So, the variation of the dilution rate is equivalent to the variation of the influent flowrate.

2.1. Equilibrium points analysis

The analysis of the existence and stability of equilibrium points in the state space has been reported in [11]. In [11] the authors divide the analysis into two subsystems: acidogenesis and methanogenesis. This is done through a simplification hypothesis, where they introduce $\widetilde{S}_{2in}$ as a worst-case upper bound of the total concentration of VFA in the reactor, after the acidogenesis phase. This allows the authors to consider the full 4-dimensional system as two independent 2-dimensional systems. In [12] we followed a different approach without the simplification hypothesis, through bifurcation analysis. This lead to similar results. Here, in our paper, we compute analytically the equilibrium points, we give the conditions for being physically meaningful (they must be positive), and we perform bifurcation analysis, also analytically. Finally in order to apply our control strategy to a concrete case, with fixed parameters, we take from [12] the main bifurcation numerical results that are interesting for this paper.

Taking into account the feasible values of the state variables X_1, X_2, S_1, S_2, the reachable space is given by the intersection of the following subspaces: $\{X_1 \geqslant 0\}, \{X_2 \geqslant 0\}, \{S_1 \geqslant 0\}$ and $\{S_2 \geqslant 0\}$. The normal operation of the system is given by $X_1^{eq} > 0$, $X_2^{eq} > 0$, $S_1^{eq} > 0$, $S_2^{eq} > 0$ where the superscript means the equilibrium values. The washout of acidogenic and methanogenic bacteria can occur for different conditions according to the dynamic analysis, with subspaces $\{X_1 = 0,\ X_2 > 0,\ S_1 > 0,\ S_2 > 0\}$, and $\{X_1 > 0,\ X_2 = 0,\ S_1 > 0,\ S_2 > 0\}$, respectively. Also, washout of both acidogenic and methanogenic bacteria can occur, which correspond to the subspace $\{X_1 = 0,\ X_2 = 0,\ S_1 > 0,\ S_2 > 0\}$.

For computing the equilibrium points in the 4-dimensional state space, we equal to zero the vector field in (1). Thus we obtain the following analytical expressions for the possible six equilibrium points, whose existence finally depend on the system parameters.

Equilibrium 1: (no washout)

$$X_1^{eq} = \frac{(S_1^{in} - S_1^{eq})D}{k_1\mu_1^{eq}},\quad X_2^{eq} = \frac{(S_2^{in} - S_2^{eq}) + (k_2/k_1)(S_1^{in} - S_1^{eq})}{k_3\mu_2^{eq}}D,$$

$$S_1^{eq} = \frac{\alpha D K_{S1}}{\mu_{\max 1} - \alpha D},\quad S_2^{eq} = \frac{-b - \sqrt{b^2 - 4ac}}{2a},$$

where $a = \alpha D, b = K_I^2(\alpha D - \mu_0), c = K_I^2\alpha D K_{S2}, \mu_1^{eq} = \mu_1(S_1^{eq})$ and $\mu_2^{eq} = \mu_2(S_2^{eq})$.

Equilibrium 2: (no washout)

$$X_1^{eq} = \frac{(S_1^{in} - S_1^{eq})D}{k_1\mu_1^{eq}},\quad X_2^{eq} = \frac{(S_2^{in} - S_2^{eq}) + (k_2/k_1)(S_1^{in} - S_1^{eq})}{k_3\mu_2^{eq}}D,$$

$$S_1^{eq} = \frac{\alpha D K_{S1}}{\mu_{\max 1} - \alpha D},\quad S_2^{eq} = \frac{-b + \sqrt{b^2 - 4ac}}{2a},$$

Equilibria 1 and 2 exist and they are physically meaningful (positive) if the straightforward conditions are satisfied:

$$S_1^{eq} < S_1^{in},\quad S_2^{in} - S_2^{eq} + \frac{k_2}{k_1}(S_1^{in} - S_1^{eq}) > 0,$$

$$D < \frac{\mu_{\max 1}}{\alpha},\quad D < \frac{\mu_0}{\alpha},\quad b^2 - 4ac > 0.$$

Equilibrium 3: (washout of acidogenic bacteria)

$$X_1^{eq} = 0,\quad X_2^{eq} = \frac{D(S_2^{in} - S_2^{eq})}{k_3\mu_2^{eq}},\quad S_1^{eq} = S_1^{in},\quad S_2^{eq} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}.$$

Equilibrium 4: (washout of acidogenic bacteria)

$$X_1^{eq} = 0,\quad X_2^{eq} = \frac{D(S_2^{in} - S_2^{eq})}{k_3\mu_2^{eq}},\quad S_1^{eq} = S_1^{in},\quad S_2^{eq} = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

Please cite this article in press as: A. Rincon et al., Control of an anaerobic digester through normal form of fold bifurcation, J. Process Contr. (2009), doi:10.1016/j.jprocont.2009.04.006

Equilibria 3 and 4 exist and they are physically meaningful if the straightforward conditions are satisfied:

$$S_2^{eq} < S_2^{in}, \quad D < \frac{\mu_0}{\alpha}, \quad b^2 - 4ac > 0.$$

It is worth to note that when the corresponding expressions are substituted into $b^2 - 4ac > 0$, it leads to

$$\alpha^2 (K_I^2 - 4K_{S2}) D^2 - 2\mu_0 \alpha K_I^2 D + K_I^2 \mu_0^2 > 0. \qquad (2)$$

Since we will be later interested in the variation of D, we perform further algebraic operations. It is easy to check that the inequality above is equivalent to the following ones, depending on three different cases:

Case I: $K_I^2 = 4K_{S2}$

The inequality (2) is equivalent to

$$D < \frac{\mu_0}{2\alpha}.$$

Case II: $K_I^2 > 4K_{S2}$

The inequality (2) is equivalent to

$$0 < D < D_1 \quad \text{or} \quad D > D_2,$$

where

$$D_1 = K_I \frac{\mu_0}{\alpha} \frac{1}{K_I + 2\sqrt{K_{S2}}}, \quad D_2 = K_I \frac{\mu_0}{\alpha} \frac{1}{K_I - 2\sqrt{K_{S2}}}.$$

Case III: $K_I^2 < 4K_{S2}$

The inequality (2) is equivalent to

$$0 < D < D_3,$$

where

$$D_3 = K_I \frac{\mu_0}{\alpha} \frac{1}{K_I + 2\sqrt{K_{S2}}}.$$

Equilibrium 5: (washout of methanogenic bacteria)

$$X_1^{eq} = \frac{D(S_1^{in} - S_1^{eq})}{k_1 \mu_1^{eq}}, \quad X_2^{eq} = 0, \quad S_1^{eq} = \frac{\alpha D K_{S1}}{\mu_{\max 1} - \alpha D},$$
$$S_2^{eq} = S_2^{in} + \frac{k_2}{D} \mu_1^{eq} X_1^{eq}.$$

Equilibria 5 exists and it is physically meaningful if the straightforward conditions are satisfied:

$$S_1^{eq} < S_1^{in}, \quad D < \frac{\mu_{\max 1}}{\alpha}.$$

Equilibrium 6: (washout of both acidogenic and methanogenic bacteria)

$$X_1^{eq} = 0, \quad X_2^{eq} = 0, \quad S_1^{eq} = S_1^{in}, \quad S_2^{eq} = S_2^{in}.$$

This equilibrium always exists and it is physically meaningful.

In order to analyze the stability of the equilibrium points, one must compute the jacobian (3). Thus the general expression is easily calculated after some linear algebra,

$$J = \begin{bmatrix} J_{11} & 0 & J_{13} & 0 \\ 0 & J_{22} & 0 & J_{24} \\ J_{31} & 0 & J_{33} & 0 \\ J_{41} & J_{42} & J_{43} & J_{44} \end{bmatrix} \qquad (3)$$

where

$$J_{11} = \mu_1^{eq} - \alpha D, \quad J_{13} = \mu_1' X_1^{eq}, \quad J_{22} = \mu_2^{eq} - \alpha D, \quad J_{24} = \mu_2' X_2^{eq},$$
$$J_{31} = -k_1 \mu_1^{eq}, \quad J_{33} = -D - k_1 \mu_1' X_1^{eq}, \quad J_{41} = k_2 \mu_1^{eq}, \quad J_{42} = -k_3 \mu_2^{eq},$$
$$J_{43} = k_2 \mu_1' X_1^{eq}, \quad J_{44} = -D - k_3 \mu_2' X_2^{eq},$$

and

$$\mu_1' = \mu_{\max 1} \frac{K_{S1}}{(K_{S1} + S_1^{eq})^2}, \qquad \mu_2' = \mu_0 \frac{K_{S2} - \left(\frac{S_2^{eq}}{K_I}\right)^2}{\left(K_{S2} + S_2^{eq} + \left(\frac{S_2^{eq}}{K_I}\right)^2\right)^2}$$

are the derivative of $\mu_1(S_1)$ with regards to S_1 at the point S_1^{eq}, and the derivative of $\mu_2(S_2)$ with regards to S_2 at the point S_2^{eq}, respectively.

After further algebraic operations is also easy to check that the jacobian eigenvalues have the following expressions:

$$\lambda_1 = \frac{t_1}{2} + \sqrt{\left(\frac{t_1}{2}\right)^2 - d_1}, \quad \lambda_2 = \frac{t_1}{2} - \sqrt{\left(\frac{t_1}{2}\right)^2 - d_1},$$
$$\lambda_3 = \frac{t_2}{2} + \sqrt{\left(\frac{t_2}{2}\right)^2 - d_2}, \quad \lambda_4 = \frac{t_2}{2} - \sqrt{\left(\frac{t_2}{2}\right)^2 - d_2},$$

where

$$t_1 = J_{11} + J_{33}, \quad t_2 = J_{22} + J_{44}, \quad d_1 = J_{11}J_{33} - J_{13}J_{31},$$
$$d_2 = J_{22}J_{44} - J_{24}J_{42}.$$

Thus, the stability of the equilibrium points is easy to determine from the expressions above.

2.1.1. Discussion

Detailed analysis about these equilibria was performed in [11,12]. Note that depending on the parameter values some of the equilibria exist or not. The expressions for the six equilibrium points are general, but the analysis must be done for realistic parameter values. We have used the parameter values reported in [9].

The behavior of the system is analyzed in three regions, depending on the D value in the interval $[0\ 1.44]$ day^{-1}, as in [9,12].

- For D < 1.07116 day^{-1} the system has six equilibrium points:
  - There are 2 stable nodes. One of them is reachable and corresponds to normal operation conditions. The other one is not reachable and it corresponds to $X_1^{eq} \geqslant 0, X_2^{eq} < 0, S_1^{eq} \geqslant 0$ and $S_2^{eq} \geqslant 0$.
  - There are 4 saddles characterized by washout phenomenon of acidogenic bacteria $(X_1^{eq} = 0)$ or methanogenic bacteria $(X_2^{eq} = 0)$ (three are reachable and one is not).
- For 1.07116 day^{-1} < D < 1.07185 day^{-1} there are six equilibrium points, as in the previous case. Now there are two reachable stable nodes and four saddles (three are always reachable and the other one becomes reachable as D increases and passes through 1.07123). One of the stable nodes corresponds to normal operation of the system and the other one corresponds to washout condition of methanogenic bacteria. Therefore the washout of methanogenic bacteria becomes a stable equilibrium point.
- Just after D = 1.07185 day^{-1} there are only two equilibrium points: a reachable stable node, corresponding to washout condition of methanogenic bacteria, and a reachable saddle corresponding to washout of both acidogenic and methanogenic bacteria. When D passes through 1.0796 day^{-1}, both equilibria meet, and the corresponding to washout of only acidogenic bacteria becomes non-reachable. Thus after D = 1.0796 day^{-1}, until the end of the interval, at D = 1.44 day^{-1}, there is only one equilibrium point, which is reachable and stable, and corresponds to washout of both acidogenic and methanogenic bacteria.

2.2. Bifurcations

Using bifurcation theory we will proceed to complete the dynamical analysis. A bifurcation is a qualitative change of system properties under quantitative parameter variation, and it can be characterized by a change in the nature, either stability or number, of the equilibrium points. The presence of a local bifurcation and its classification are defined by the jacobian eigenvalues at the equilibrium points and the genericity conditions. The point where one of the eigenvalues has real part equal to zero is known as a singularity point, and the corresponding value of the parameter is known as the critical parameter value [2].

Taking advantage of the analytical expression for the jacobian, bifurcation analysis can also be performed analytically. For a fold bifurcation it is necessary that one eigenvalue $\lambda = 0$, while for a Hopf bifurcation it is necessary that we have two eigenvalues with zero real part and non-zero imaginary part.

Thus taking into account the expressions for the jacobian eigenvalues, the necessary conditions for these two kinds of bifurcations are:

- Fold bifurcation: $d_1 = 0$ ($\Rightarrow \lambda_2 = 0$) or $d_2 = 0$ ($\Rightarrow \lambda_4 = 0$).
- Hopf bifurcation: $t_1 = 0$ and $d_1 > 0$, or $t_2 = 0$ and $d_2 > 0$.

For our case, regarding the fold bifurcation, $d_1 = 0$ leads to

$$\mu_1^{eq} - \alpha D - \alpha k_1 \mu_1' X_1^{eq} = 0$$

while $d_2 = 0$ leads to

$$\mu_2^{eq} - \alpha D - \alpha k_3 \mu_2' X_2^{eq} = 0.$$

Regarding the Hopf bifurcation, $t_1 = 0$ leads to

$$\mu_1^{eq} - (\alpha + 1)D - k_1 \mu_1' X_1^{eq} = 0,$$

and $d_1 > 0$ leads to

$$D + k_1 \mu_1' X_1^{eq}(1 - \alpha) < 0.$$

But this last inequality cannot be satisfied since $0 < \alpha < 1$ and all the other terms are positive.

On the other hand, $t_2 = 0$ leads to

$$\mu_2^{eq} - (\alpha + 1)D - k_3 \mu_2' X_2^{eq} = 0,$$

and $d_2 > 0$ leads to

$$D + k_3 \mu_2' X_2^{eq}(1 - \alpha) < 0.$$

In this case, we can have solutions for this inequality since μ_2' can be negative.

Thus, analytical conditions for fold and Hopf bifurcations have been stated.

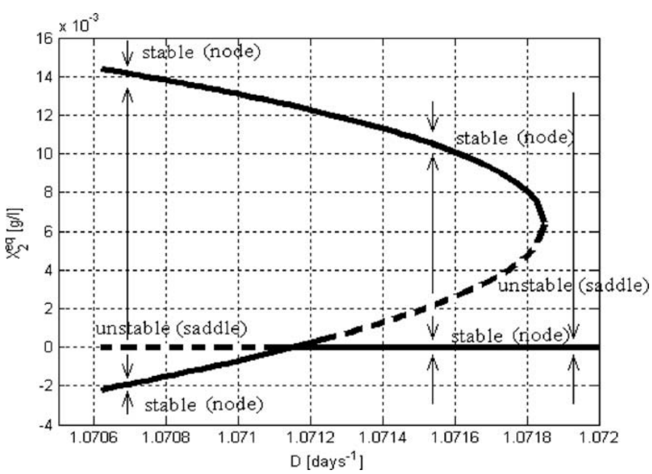

Fig. 2. Bifurcation diagram for the state X_2. For $D < 1.07116$ the upper and lower branches of the parabola correspond to a reachable and an unreachable stable nodes. The trivial branch $X_2^{eq} = 0$ corresponds to a saddle. It is worth to note that even if negative values are mathematically stable, they are physically impossible.

Since bifurcation analysis allows us to let one (or more) parameters free, we will take the dilution rate D as the most convenient parameter to be varied, and thus it will be the bifurcation parameter. Taking into account the chosen values for the remaining parameters, the bifurcation diagram is computed varying D. The bifurcation diagram shows the equilibrium branches for each state. According again to the results in [9,12], D will be varied on the interval $[0, 1.44]$ day^{-1}. The other parameters are kept fixed.

Since equilibria 3 and 4 are always saddles, in the following analysis we will not consider them. Figs. 2 and 3 show bifurcation diagrams for the state X_2 and S_2 respectively, using D as bifurcation parameter. These diagrams exhibit a parabola only for $D < 1.07185$ day^{-1}. The continuous solid line stands for the branch of stable equilibrium points and the dashed line is used for the branch of unstable equilibrium points. Only the equilibria concerned to washout and fold bifurcations are shown. The bifurcation analysis indicates that the washout of either acidogenic or methanogenic bacteria occurs for different conditions. Bifurcation diagrams for

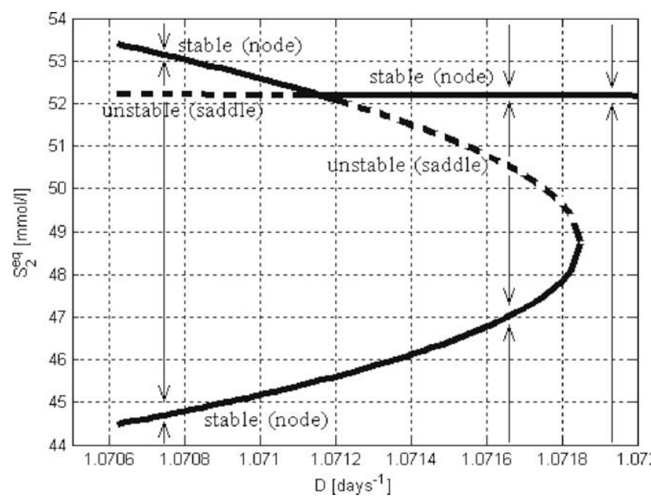

Fig. 3. Bifurcation diagram for the state S_2. For $D < 1.07116$ the upper and lower branches of the parabola correspond to a reachable and an unreachable stable nodes. The trivial branch $S_2^{eq} = S_2^{in} = 52$ mmol/L corresponds to a saddle.

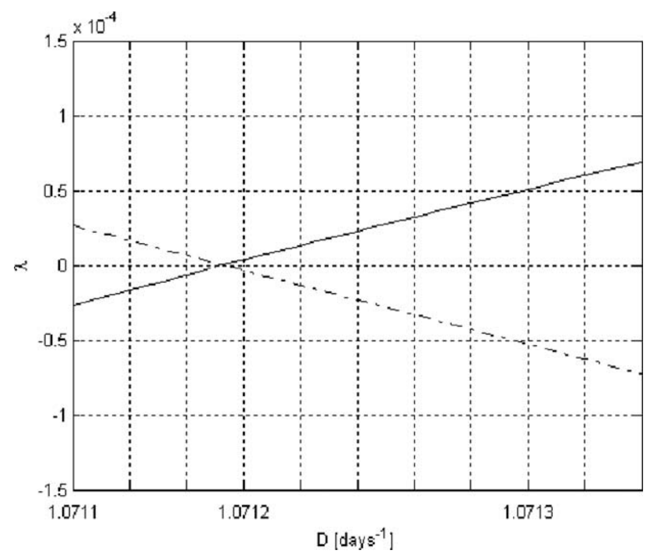

Fig. 4. Behavior of the eigenvalues when a transcritical bifurcation appears. The continuous and dashed lines correspond to eigenvalue curves of the saddle and the unreachable stable node (after the bifurcation, they change to a reachable stable node and a saddle).

Please cite this article in press as: A. Rincon et al., Control of an anaerobic digester through normal form of fold bifurcation, J. Process Contr. (2009), doi:10.1016/j.jprocont.2009.04.006

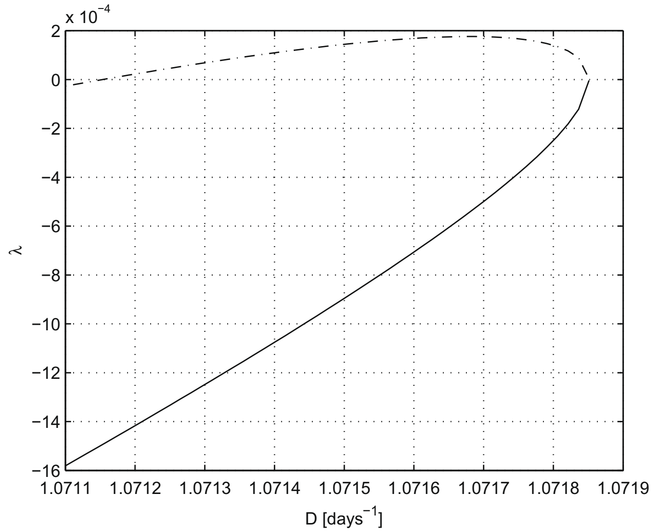

Fig. 5. Behavior of the eigenvalues when a fold bifurcation appears. The continuous and dashed lines correspond to eigenvalue curves of the reachable stable node and the saddle (after the bifurcation, both equilibria are annihilated).

X_1 and S_1 are not shown since we are particularly concerned with the limitation of S_2. Further analysis, presented in the following, shows that the methanogenic bacteria washout is associated to a fold bifurcation.

From Figs. 4 and 5 it can be observed that the eigenvalue curves cross the zero line indicating the presence of bifurcations. Only the eigenvalues of the equilibria related to bifurcations are shown. Note that one eigenvalue crosses by zero twice. The first one occurs for $D = 1.07116\ \text{day}^{-1}$ (first bifurcation corresponding to transcritical bifurcation) and the second one for $D = 1.071851\ \text{day}^{-1}$ (second bifurcation corresponding to fold bifurcation, which agrees with the analytical expressions previously computed).

In the following, we perform a detailed analysis of the bifurcation points.

2.2.1. Transcritical bifurcation

In this subsection we describe the transcritical bifurcation that occurs in the system. For $D < 1.07116\ \text{day}^{-1}$ (parameter value where the transcritical bifurcation occurs) the bifurcation diagram displays three branches. One of them corresponds to the reachable stable node, another one corresponds to a saddle (unstable) with $X_2^{eq} = 0$, and the last one corresponds to the unreachable (with $X_2^{eq} < 0$) stable node (see Fig. 2). Thus for $D < 1.07116\ \text{day}^{-1}$ there is only one reachable stable equilibrium point corresponding to normal operation.

For $D < 1.07116\ \text{day}^{-1}$ three of the four eigenvalues associated to the saddle (with $X_2^{eq} = 0$), are negative while the other eigenvalue is positive. The eigenvalues of the non-reachable stable node are all negative. For $D = 1.07116\ \text{day}^{-1}$ the non-reachable stable node collides with the saddle, through a transcritical bifurcation (defined by the collision of two equilibrium points which interchange the sign of two eigenvalues). This occurs when the equilibrium points coincide at

$$x^{eq} = [0.0146, 0.0, 5.7232, 52.2]$$

and for the critical parameter value

$$D^{transcritical} = 1.07116.$$

(see Fig. 2).

At the bifurcation point ($D = 1.07116\ \text{day}^{-1}$) the positive eigenvalue of the saddle becomes zero and for bigger values of D the eigenvalue becomes negative (see Fig. 4). On the other hand, one eigenvalue of the stable node becomes zero at the bifurcation point and for bigger values of D the eigenvalue becomes positive. Thus, in this collision, the stability characteristics are interchanged, in such a way that the saddle becomes a stable node and the stable node becomes a saddle. With further variation in D, this saddle can be continued until the point $X_2^{eq} = 0.00631368$ g/L (see Fig. 2).

For $1.07116\text{day}^{-1} < D < 1.07185\ \text{day}^{-1}$ the bifurcation diagram has still three branches: two stable equilibrium points, each one with its own basin of attraction, and an (unstable) saddle (see Fig. 2). It is known that the border between the basins is given by the stable manifold of the saddle [2].

2.2.2. Fold bifurcation

For $D \in (1.07116\ 1.07185)\ \text{day}^{-1}$ the bifurcation diagram shows three branches (see Figs. 2 and 3). One of the branches, plotted with a dashed line, corresponds to the (unstable) saddle that appeared after the transcritical bifurcation. The two other branches, plotted with continuous lines, correspond to two reachable stable nodes. One of them is associated to the anaerobic digester normal operation and the other one is associated to abnormal operation, washout of methanogenic bacteria (see Fig. 2). For $D = 1.07185\ \text{day}^{-1}$ the branch of the stable node corresponding to normal operation collides with the branch corresponding to the saddle.

After the collision both equilibria disappear. This collision happens for

$$x^{eq} = [0.0133, 0.00631368, 5.73, 48.75]$$

and the critical parameter value is

$$D^{fold} = 1.07185.$$

Just before the bifurcation, only one of the four eigenvalues of the saddle is positive and all the eigenvalues of the node are negative. At the bifurcation point where both equilibrium points collide, the positive eigenvalue of the saddle and one negative eigenvalue of the stable node become zero as it is shown in Fig. 5. This bifurcation is known as a fold bifurcation.

After $D = 1.07185\ \text{day}^{-1}$ the bifurcation diagram (see Fig. 2) only shows one stable equilibrium point corresponding to washout of methanogenic bacteria. For anaerobic digestion processes it is important to avoid that the system falls in this non-operative equilibrium point. For values $D \in (1.07116, 1.07185)\ \text{day}^{-1}$ it is possible that some disturbances in the system lead it towards washout (since the disturbances can send the variables to the basin of attraction of the washout equilibrium point).

2.2.3. Discussion

With all the information that was obtained we can conclude that washout can be present if the dilution parameter is driven further the critical point corresponding to the fold bifurcation. Due to the presence of a transcritical bifurcation (where the unstable equilibrium point branch coming from the fold bifurcation and the washout equilibrium point interchange stability) the equilibrium branch corresponding to washout can be reached. Thus the transition from normal operation to washout is due to the combination of a fold and a trancritical bifurcation.

Nevertheless, in order to prevent washout from a normal operation point, since the fold bifurcation has only a local (not global) character, only the fold bifurcation is really significant. If we keep the operation point on the stable branch of the fold bifurcation, washout will be avoided. However, much care must be taken close to the critical point of the fold bifurcation, even on the stable branch. As we get closer to the critical fold bifurcation point, the basin of attraction of the stable operation point is dramatically reduced, and even small perturbations can lead the system to washout. Bifurcation analysis shows the characteristics of the biomass washout and indicates the value of the dilution rate that allows its prevention [8,17–19]. In [11], a security criterion is developed for an anaerobic digestion reactor, which determines whether the process is in a normal or dangerous operating mode. The risk criterion is based on the distance -critical overloading- between acidification and working equilibrium, and the distance -overloading tolerance- between the saddle separatrix and working equilibrium. The basin of attraction of the washout equilibrium branch and the normal operation point are essential for determining the potential risk, and the change in the amplitude of the basins as the equilibrium branches get close to the fold bifurcation point are crucial for the process evolution.

Thus, since the most important bifurcation is the fold, we will base our approach on the normal form of the fold bifurcation. This will be the model of the nominal plant which we will consider for the control step in our strategy.

2.3. Normal form of a generic fold bifurcation

A bifurcation is generic if the system satisfies some conditions at the bifurcation point, called the generic conditions, which are different for each bifurcation type. This implies that if the bifurcation satisfies some general characteristics, then the system can be transformed into a representative form describing that bifurcation. This form is called the topological normal form. A system possessing one critical eigenvalue and satisfying the genericity conditions is locally topologically equivalent to the normal form for that bifurcation.

Bifurcations in n-dimensional systems occur in *essentially* the same way as in low dimensional systems (1 or 2-dimensional). In particular, a typical fold bifurcation (called saddle-node bifurcation too) can be perfectly defined by a 1-dimensional dynamical system in the following way.

Let us consider a 1-dimensional system given by

$$\dot{x} = f(x, \gamma), \quad x \in \mathbb{R}, \quad \gamma \in \mathbb{R} \tag{4}$$

with smooth f. Assume that this system has an equilibrium point at $x = 0$ for $\gamma = 0$, and it satisfies all generic conditions for exhibiting a fold bifurcation [2]. Then, this system is topologically equivalent, close to the origin, to the system:

$$\dot{\eta} = \beta \pm \eta^2 + O(\eta^3) \tag{5}$$

where the sign $+$ or $-$ depends on the original system.

Now, if we apply the center manifold theorem, we can decompose any smooth n-dimensional system having a non-hyperbolic equilibrium point $x_0 = 0$ with n_+, n_0, n_- eigenvalues with $Re\lambda > 0, Re\lambda = 0$ and $Re\lambda < 0$, respectively, in the following way:

$$\begin{aligned} \dot{\xi} &= f_1(\xi) = G\xi + g(\xi, V(\xi)) \\ \dot{\psi} &= f_2(\psi) \end{aligned} \tag{6}$$

where $\xi \in \mathbb{R}^{n_0}$ and $\psi \in \mathbb{R}^{n_- + n_+}$. G is a constant matrix, and $g(\xi, V(\xi))$ is a nonlinear term. This transformation is valid only close to the origin. Then, taking into account the center manifold theorem, using the normal form of the fold bifurcation and moving the coordinates to the origin, we obtain

$$\dot{y} = u + b_2 y^2 \tag{7}$$

with $y = S_2 - S_2^{eq} = S_2 - 48.75$ mmol/l and $u = D - D^c = D -$ 1.071851 day^{-1} (see [2] for the details concerning the complete procedure). In the following we will deal with the variables moved to the origin.

Thus, we will use this model as the one for the plant, and we will compute the parameters of the identifier for controlling the system via adaptive control. These results will be compared with those obtained by using a linear model as the plant for the system, which is a standard procedure.

3. Adaptive system for the anaerobic reactor

The design of control systems for biological processes and for WWTPs, simultaneously with modeling and observer design, has been extensively studied in the last two decades. The control objective in a WWTP is to keep appropriate conditions (temperature and pH) and guarantee that the COD in the output of the WWTP is below the environmental limit [7]. Specially COD is difficult to measure, and the input flow concentration has a highly fluctuating immeasurable disturbance [4,6,7,20–22]. The difficult monitoring can be tackled by state observers and the model uncertainty by adaptive systems [7,15,20,23].

Feedback linearization has been widely used for the control of reactors, since it takes into account nonlinear behavior [8,17,18,24,25]. The main drawback of input output linearization control techniques is the need of full model knowledge, for the exact cancellation of nonlinearities; but in presence of model uncertainties -parameter and structural uncertainties- the control performance is degraded. This problem may be solved by means of self-tuning adaptive control, which estimates a plant nominal model based on the measurement of the input and output signals [24,26]. Adaptive controllers can be very efficient with regards to model uncertainties, achieving good regulation. Several strategies for parameter estimation and control law design have been developed [7,20,27].

With the aims to prevent system failure, the VFA concentration (S_2) is chosen as controlled variable, similarly to [15]. The designed controller must handle perturbations in the input flow concentration, variations in parameters, changes in the reference values and uncertainty in the value of D. The control input is the dilution rate D.

Adaptive control is used in order to deal with model uncertainty. Two structures of nominal models are considered to describe the dynamical behavior of the anaerobic digester (1). Namely, a nonlinear model based on the structure of the normal form of fold bifurcation and a linear first order plant. For each one the control law and estimation mechanism are designed. The parameters of nominal models are on-line estimated by recursive least squares (RLS) according to [28]. The input–output linearization described in [3] and pole placement methods described in [28] are applied to design the controller, using the nonlinear and the linear models, respectively.

We want to emphasize that our aim in this paper is not to compare with all classical techniques, but with those which use adaptive control, in order to check the advantage of the bifurcation normal form.

3.1. Adaptive controller based on normal form of fold bifurcation

Since we want to test and use the normal forms taken from bifurcation theory and to apply them to control theory, we think that a good model for the system is the normal form of fold bifurcation shown in Eq. (7). The values $y^{eq} := S_2^{eq}$ and D^c are the ones for which a fold bifurcation is exhibited. These values are taken from the bifurcation diagram and for this particular case they are $y^{eq} = 48.75$ and $D^c = 1.07185$. With the aim to adjust the model for the plant, the differences between y^{eq} and the output of the system $y := S_2$, and $D - D^c$, are fed to the estimator of the parameter in order to compute the value of the estimated parameter $\hat{b}_2$ on line.

Now, using standard results of feedback linearization theory we design u in such a way that the nonlinearities are cancelled and the tracking error converges to zero. Regarding the plant nominal

Please cite this article in press as: A. Rincon et al., Control of an anaerobic digester through normal form of fold bifurcation, J. Process Contr. (2009), doi:10.1016/j.jprocont.2009.04.006

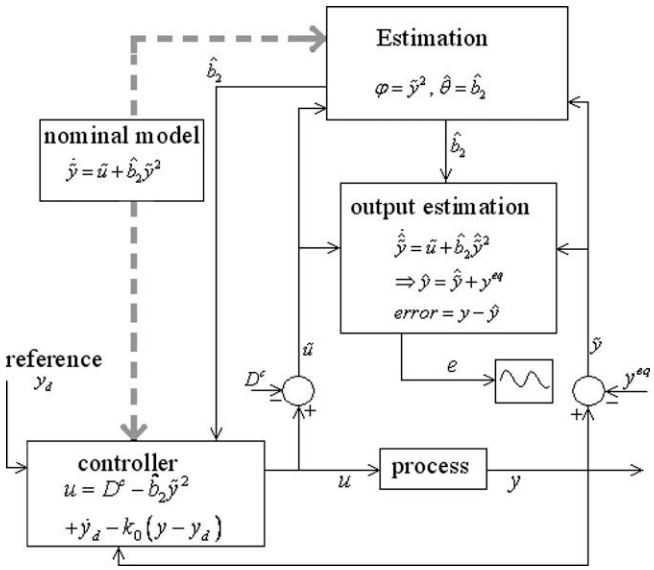

Fig. 6. Schematic diagram of the developed controller based on normal forms.

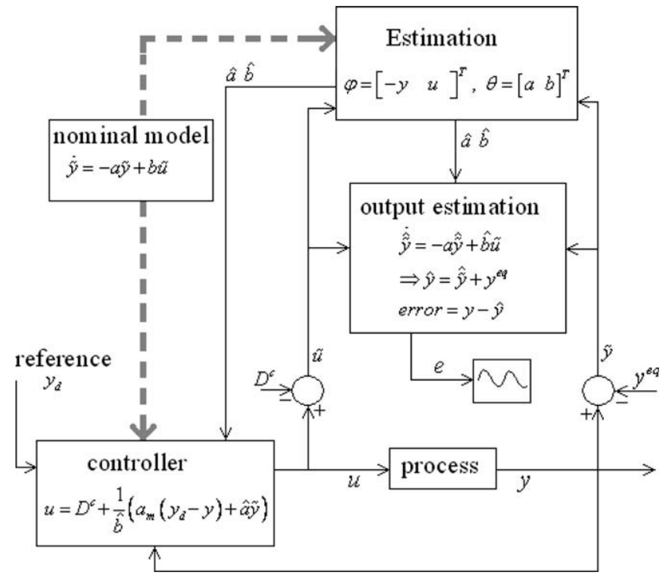

Fig. 7. Schematic diagram of the developed controller based on the linear plant.

model (7) and the estimated parameter, the control law using feedback linearization is [3]

$$u = -\hat{b}_2 y^2 + \dot{y}_d - k_0(y - y_d) \tag{8}$$

where y_d is the reference or desired output and $\hat{b}_2$ is the estimated parameter. The constant k_0 must be positive for guaranteeing an exponentially stable error dynamics. In this way the signal control is calculated using the information of the bifurcation point, set point and output, and estimating the only parameter associated to the normal form (b_2). A schematic diagram of the plant with the controller is shown in Fig. 6.

Here, it is assumed that b_2 is a slowly time-varying parameter which will be estimated with the estimation mechanism. The estimated parameter $\hat{b}_2$ gives the estimated output $\hat{y}$ through Eq. (9).

$$\dot{\hat{y}} = u + \hat{b}_2\hat{y}^2 \tag{9}$$

A satisfactory parameter estimation gives an estimated output close to process output, quantified through the output estimation errors. Since system (9) is linear with regards to $\hat{b}_2$, then the RLS algorithm can be used. For the identifying regressor, parameter vectors and output estimation error we followed the development in [28].

3.2. Adaptive controller based on linear model

The linear controller is based on the minimum degree pole placement. The analysis of step response (not included in this work) shows that the system behaves as a first order linear plant in a wide operating region. Then, we assume that the system can be described by the following differential equation and transfer function:

$$\begin{aligned} \dot{y} &= -ay + bu \\ G(s) &= \frac{B}{A} = \frac{b}{s+a} \end{aligned} \tag{10}$$

In a similar way as the previous case this controller needs $y^{eq} := S_2$ and $D^c := D$ which have now a different meaning. As we will use a linear model, the values of D^c and y^{eq} are provided by linearization around the set point. The chosen values are $D^c = 0.5$ and $y^{eq} = 4.78$.

For computing an adaptive controller, the procedure for pole placement is followed [28]. The main idea of minimum degree pole placement is to place the values of the closed-loop system poles at desired values such that the controlled system follows the desired output noted as y_d (reference). The linear controller is given by:

$$Ru = Ty_d - Sy \tag{11}$$

where u, y_d and y are the control input, reference signal and output signal, respectively. R, S and T are the polynomials of the controller, which must be determined. The desired output reference signal is the output of the reference model which can be computed from

$$G_m(s) = \frac{y_m(s)}{y_d(s)} = \frac{a_m}{s + a_m} \tag{12}$$

where y_m is the output of the reference model and it must be equal to the desired output. Then, the output of the system follows the reference signal if the closed-loop transfer function is equal to the reference model transfer function,

$$\frac{BT}{AR + BS} := \frac{a_m}{s + a_m} \tag{13}$$

A schematic diagram of the control and plant is shown in Fig. 7.

Taking into account the causality conditions, solving the diophantine equation and replacing it in Eq. (11) we obtain the following controller (see [28] for the complete procedure):

$$u = \frac{1}{b}(a_m(y_d - y) + ay) \tag{14}$$

Then, using the estimator of the parameters the control law is given by:

$$u = \frac{1}{\hat{b}}(a_m(y_d - y) + \hat{a}y) \tag{15}$$

where $\hat{a}$ and $\hat{b}$ are obtained with the RLS algorithm.

4. Results

We check two controlled systems by simulation. The first one with the anaerobic digester equations given by (1) using the nominal model obtained in (7) and the controller defined by (8). The second one with the anaerobic digester equations given by (1) using the nominal model obtained in (10) and the controller defined by (15).

The algorithms have an open-loop routine followed by a closed-loop one, both with on-line estimation. The initial reference value is $S_2 = 5\ g/L$. The input in the open-loop routine, included for training purpose, is given by a sinusoidal signal $D = 0.6 \sin 2\pi/13t$ and it is run for the first 150 s. The output is obtained from the anaerobic digester described by (1), and the parameters in the nominal models are estimated with the RLS method.

In the closed-loop routine only the controller with on-line parameter estimation is working.

With the aim to test the advantages of the our new designed controller (based on the normal form) we simulate several types of disturbances in both systems. From a practical point of view, it is possible that the dilution rate D exhibits some uncertainties. Thus we simulate them assuming a 10% error between the real value and the desired value obtained from the controller algorithm. These disturbances allow us to discuss about the controlled system robustness. Some of them have been reported in the literature [9] and the other have been added considering a 10% error in the parameter values and 10% uncertainty in the dilution rate D. Moreover in order to analyze the controlled variable (S_2) we introduce a severe change in S_2^{in}. We do that since probably this is the main parameter that may produce a critical change to S_2.

- At $t = 215$ days the concentration in S_1^{in} is changed in the following way [9]:
  $S_1^{in} = 9.5$ g/L for $t \in [215, 237)$ days
  $S_1^{in} = 15$ g/L for $t \in [237, 240)$ days
  $S_1^{in} = 10$ g/L for $t \in [240, 247)$ days
  $S_1^{in} = 5$ g/L for $t \in [247, 254)$ days

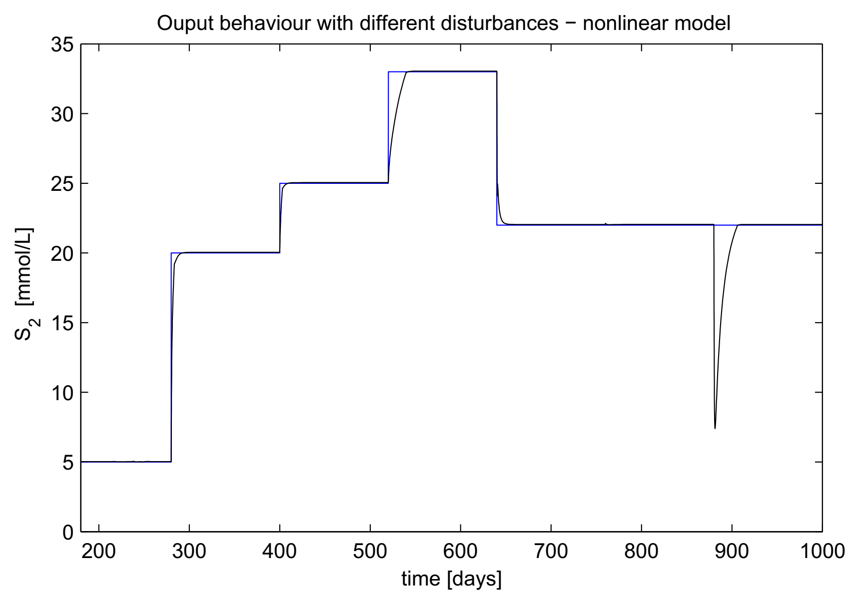

Fig. 8. Behavior of the output using the normal form as the model.

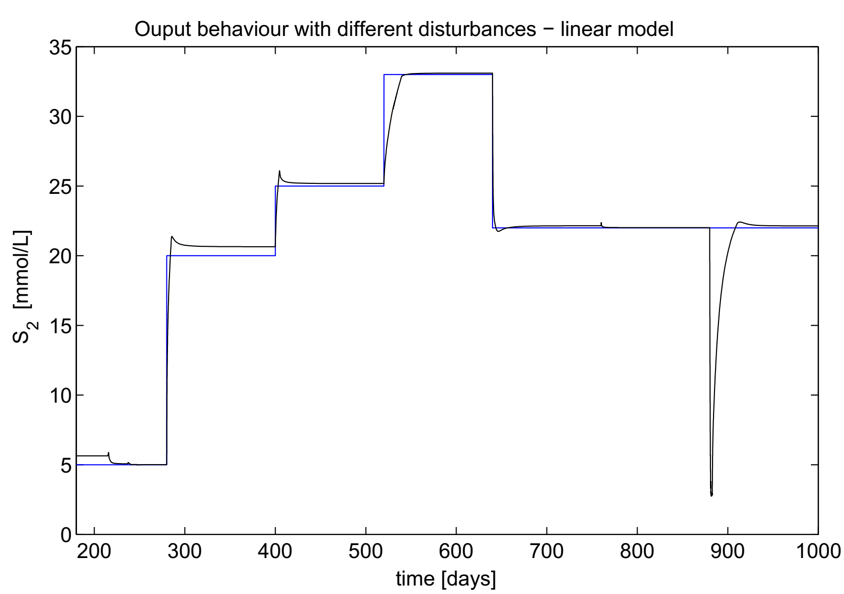

Fig. 9. Behavior of the output using a linear plant as the model.

Please cite this article in press as: A. Rincon et al., Control of an anaerobic digester through normal form of fold bifurcation, J. Process Contr. (2009), doi:10.1016/j.jprocont.2009.04.006

- At $t = 280$ days the reference value is changed in the following way
  $y_d = 20$ g/L for $t \in [280, 400)$ days
  $y_d = 25$ g/L for $t \in [400, 520)$ days
  $y_d = 33$ g/L for $t \in [520, 640)$ days
  $y_d = 22$ g/L for $t \in [640, 760)$ days
- At $t = 760$ days the parameters are varied. Some of them are 10% changed and the other ones are -10% changed.
- At $t = 880$ days the concentration in S_2^{in} is changed to $S_2^{in} = 25$ mmol/L.

Specifically, we changed the set points closer to the bifurcation value and we simulated a change in the concentration of VFA in the inlet flow. With the results (shown in Figs. 8–11) we can conclude that the adaptive control technique based on the normal form has better behavior than the adaptive control based on the linear model. This is true for all set points and parameter values, even if they are far or close to the bifurcaction point, and it can be observed for both variables COD and VFA.

In both cases, fast tracking error convergence to a constant value was achieved. However the controller based on the normal form is faster and it has lower error in steady state than the one based on the linear model.

The controller based on the normal form retains the main dynamical behaviors and it shows better performance even in the presence of the uncertainty in the dilution rate D. The linear model based on linearization around the desired set point was unable to handle the uncertainty in the dilution rate. Hence, the adaptive control based on the linear model showed more steady state error than the adaptive normal form model.

For determining which controller has better performance, we calculate the error criteria IAE (Integral of the Absolute Value of

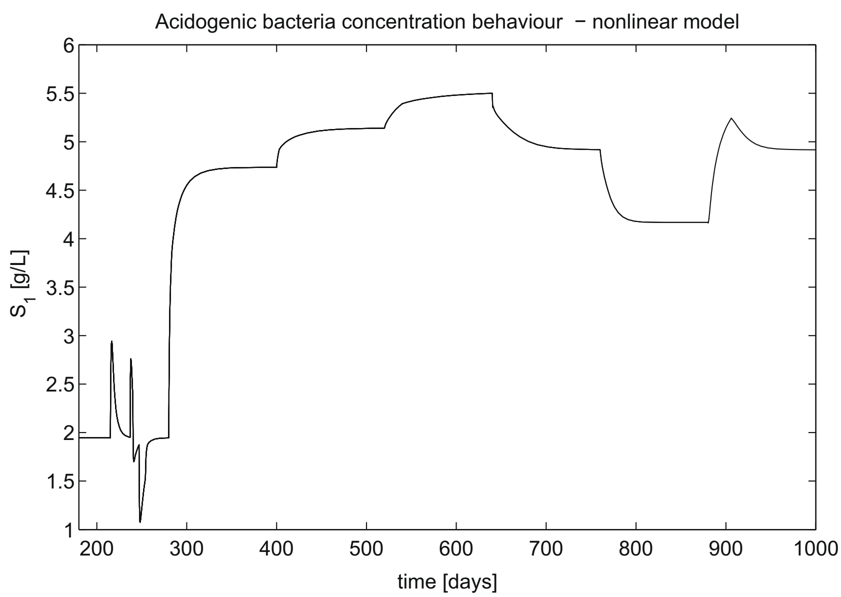

Fig. 10. Behavior of COD using the normal form as the model.

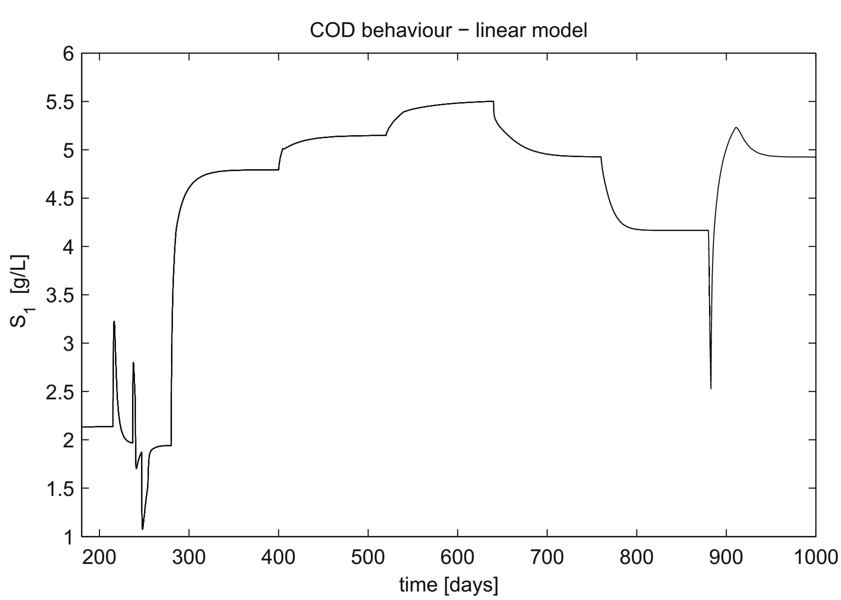

Fig. 11. Behavior of COD using a linear plant as the model.

Please cite this article in press as: A. Rincon et al., Control of an anaerobic digester through normal form of fold bifurcation, J. Process Contr. (2009), doi:10.1016/j.jprocont.2009.04.006

Table 2
Error criteria.

Model	IAE	ITAE	ISE	ITSE
Nonlinear model	240.11	0.2893	629.72	0.7587
Linear model	423.88	0.5107	651.63	0.7851

the Error), ITAE (Integral of the Time-Weighted Absolute Value of the Error), ISE (Integral of the Square of the Error) and ITSE (Integral of the Time-Weighted Square of the Error). The results are shown in Table 2. As it can be seen, in each case, the best performance was always obtained with the controller based on the normal form.

With regards to the control effort, our controller (see Fig. 12) exhibited better performance, with less oscillations and less saturation values than the linear adaptive model (see Fig. 13).

In closed-loop operation the parameters of the linear plant converged to

$$\hat{a} = 0.0015 \qquad \hat{b} = 0.00227.$$

For the nonlinear plant the estimated parameter was $\hat{b_2} = 0.0002911$.

Since the controlled system is high dimensional and the real process is more complex than the normal form [28], the stability of the feedback system is established through the use of bifurcation diagrams. Now the new bifurcation parameter is y_d. For each value of $y_d \in [4.8, 48]$ the controlled system has only one stable

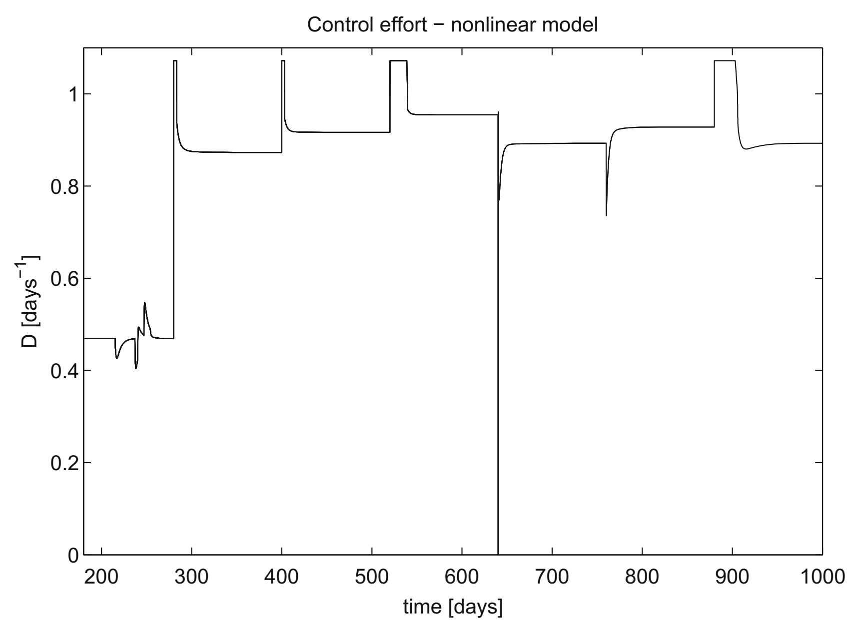

Fig. 12. Control effort using the normal form as the model.

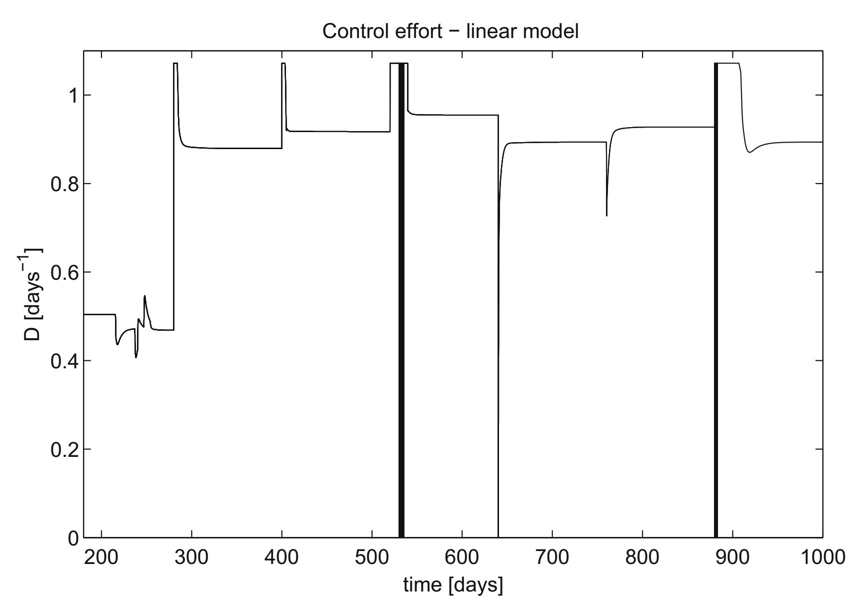

Fig. 13. Control effort using a linear plant as the model.

Please cite this article in press as: A. Rincon et al., Control of an anaerobic digester through normal form of fold bifurcation, J. Process Contr. (2009), doi:10.1016/j.jprocont.2009.04.006

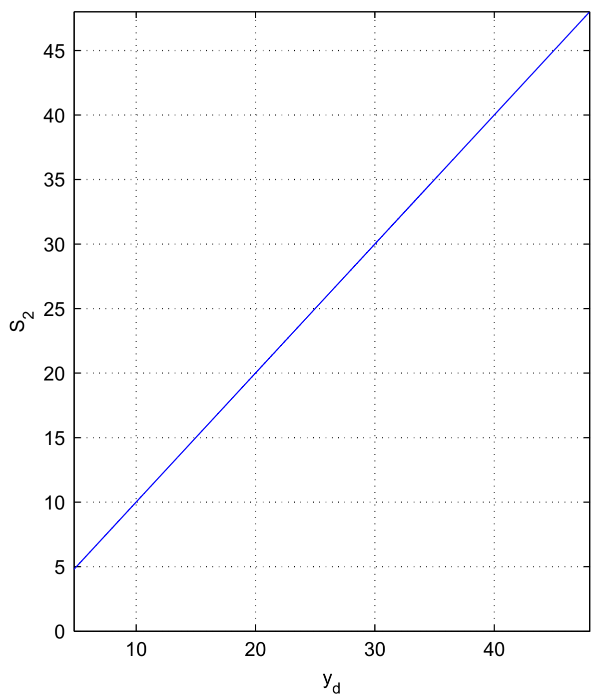

Fig. 14. Bifurcation diagram for the state S_2 using the adaptive controller based on the normal form. The bifurcation parameter is y_d.

equilibrium point. When we have no uncertainties in the dilution rate D, Fig. 14 shows that the equilibrium point is equal to the desired value (bifurcation parameter y_d).

5. Conclusions and further research

The open-loop dynamic analysis showed that the washout condition occurs after a combination of fold and transcritical bifurcations and it can be prevented by limiting the D value. Nevertheless this conclusion can only be stated for the open-loop system, since after the controller is applied, the condition for washout, stability and operation ranges have to be computed again.

The developed adaptive control using a normal form as a model of the system and feedback linearization for canceling the nonlinearities exhibited a very good behavior in presence of a wide variety of disturbances. We included changes in the set point, in the parameters, uncertainty in the value of D, and a severe perturbation in parameter S_2^{in}. On the contrary, the results obtained using a linear model for describing the plant and pole placement as control strategy did not have so good performance. In fact, the controller was unable to handle the uncertainty in D, when it was considered simultaneously with other perturbations. From the obtained results it can be stated that the nominal model given by (7), based on the fold normal form, describes the plant properly. Moreover, taking into account the considered disturbances it can be concluded that the normal form can be used in a very large part of the state space. Probably this is the main reason why the controller based on the normal form is better than the one based on the linear plant. The nonlinear model improves the behavior of the system output and it is more robust than the widely used linear model.

We computed the equilibrium points of the controlled system based on the normal form and we concluded through bifurcation diagrams that there was only one stable equilibrium point for each value of the reference (see Fig. 14).

It is worth to note that despite that the controlled variable is S_2 the new controller proposed helps to keep low S_1, as it can be seen in Fig. 10.

Further simulations (not included in this paper) have shown that the critical value of the dilution rate D for the fold bifurcation increases when the input flow COD concentration S_1^{in} increases too. This shows that washout can be caused by diluted wastewater, as already stated in [6].

Future research topics are oriented to develop a general methodology for generating nominal models based on normal forms, in such a way that they can be used in adaptive control theory. Hence, the number of estimated parameters can be reduced significatively. Since the normal form includes a nonlinear part, the nominal model can retain much better the system dynamics than the linear one.

Acknowledgements

The authors acknowledge financial support provided by DIMA, National University of Colombia, (Resolution/project No. VR-2185, CDP No. 1650). They also acknowledge the CRM (Centre de Recerca Matematica, UAB, Barcelone) for its kind welcome and hosting, where this paper was finally updated.

References

[1] E. Ott, Chaos in Dynamical Systems, Cambridge University Press, 1993.
[2] Y.A. Kuznetsov, Elements of Applied Bifurcation Theory, third ed., Springer-Verlag, New York, 1995.
[3] J-J.E. Slotine, W. Li, Applied Nonlinear Control, Prentice Hall Englewood Cliffs, New Jersey, 1991.
[4] V. Alcaraz-González, J. Harmand, A. Rapaport, J.P. Steyer, V. González-Álvarez, C. Pelayo-Ortiz, Software sensors for highly uncertain WWTPs: a new approach based on interval observers, Water Research 36 (2002) 2515–2524.
[5] O.A. Zanabria, Modelagem, identificação e controle de sistemas de tratamento de lodo ativado com remoção de nitrogênio, Ph.D., Thesis, Escola Politécnica da Universidade de São Paulo, Brazil, 2002 (in portuguese).
[6] L. Mailleret, O. Bernard, J.P. Steyer, Robust regulation of anaerobic digestion processes, Water Science and Technology 48 (2003) 87–94.
[7] N. Hilgert, J. Harmand, J.P. Steyer, J.P. Vila, Nonparametric identification and adaptive control of an anaerobic fluidized bed digester, Control Engineering Practice 8 (2000) 367–376.
[8] H.O. Méndez-Acosta, D.U. Campos-Delgado, R. Femat, V. González-Álvarez, A robust feedforward/feedback control for an anaerobic digester, Computers and Chemical Engineering 29 (2005) 1613–1623.
[9] O. Bernard, Z. Hadj-Sadok, D. Dochain, A. Genovesi, J.P. Steyer, Dynamical model development and parameter identification for an anaerobic wastewater treatment process, Biotechnology and Bioengineering 75 (2001) 424–438.
[10] M. Estaben, M. Polit, J.P. Steyer, Fuzzy control for an anaerobic digester, Control Engineering Practice 5 (1997) 1303–1310.
[11] J. Hess, O. Bernard, Design and study of a risk management criterion for an unstable anaerobic wastewater treatment process, Journal of Process Control 18 (2008) 71–79.
[12] A. Rincón, F. Angulo, G. Olivar, Análisis y control de un biorreactor anaerobio de lecho fijo de flujo ascendente, DYNA, vol. 76, no. 157, March 2009, pp. 123–132 (in Spanish).
[13] A.D. Torre, G. Stephanopoulos, Simulation study of anaerobic digestion control, Biotechnology and Bioengineering 28 (1986) 1138–1153.
[14] P.F. Pind, I. Angelidaki, B.K. Ahring, Dynamics of the anaerobic process: effects of volatile fatty acids, Biotechnology and Bioengineering 82 (2003) 791–801.
[15] J. Seok, Hybrid adaptive optimal control of anaerobic fluidized bed bioreactor for the de-icing waste treatment, Journal of Biotechnology 102 (2003) 165–175.
[16] V.A. Vavilin, L.V. Lokshina, Modeling of volatile fatty acids degradation kinetics and evaluation of microorganism activity, Bioresource Technology 57 (1996) 69–80.
[17] Y. Zhang, A.M. Zamamiri, M.A. Henson, M.A. Hjortso, Cell population models for bifurcation analysis and nonlinear control of continuous yeast bioreactors, Journal of Process Control 12 (2002) 721–734.
[18] I. Simeonov, I. Queinnec, Linearizing control of the anaerobic digestion with addition of acetate (control of the anaerobic digestion), Control Engineering Practice 14 (2006) 799–810.
[19] N.V. Mantzaris, P. Daoutidis, Cell population balance modelling and control in continuous bioreactors, Journal of Process Control 14 (2004) 775–784.
[20] L. Mailleret, O. Bernard, J.P. Steyer, Nonlinear adaptive control for bioreactors with unknown kinetics, Automatica 40 (2004) 1379–1385.

Please cite this article in press as: A. Rincon et al., Control of an anaerobic digester through normal form of fold bifurcation, J. Process Contr. (2009), doi:10.1016/j.jprocont.2009.04.006

[21] C. Cadet, S. Béteau, C. Hernández, Multicriteria control strategy for cost/quality compromise in wastewater treatment plants, Control Engineering Practice 12 (2004) 335–347.

[22] O. Bernard, J-L Gouzé, Closed loop observers bundle for uncertain biotechnological models, Journal of Process Control 14 (2004) 765–774.

[23] I.Y. Smets, J.E. Claes, E.J. November, G.P. Bastin, J.F. Van Impe, Optimal adaptive control of biochemical reactors: past, present and future, Journal of Process Control 14 (2004) 795–805.

[24] R. Femat, J. Álvarez-Ramírez, M. Rosales-Torres, Robust asymptotic linearization via uncertainty estimation: regulation of temperature in a fluidized bed reactor, Computers and Chemical Engineering 23 (1999) 697–708.

[25] E. Elisante, G.P. Rangaiah, S. Palanki, Robust controller synthesis for multivariable nonlinear systems with unmeasured disturbances, Chemical Engineering Science 59 (2004) 977–986.

[26] C.T. Chen, C.S. Dai, Robust controller design for a class of nonlinear uncertain chemical processes, Journal of Process Control 11 (2001) 469–482.

[27] N.I. Marcos, M. Guay, D. Dochain, Output feedback adaptive extremum seeking control of a continuous tank biorreactor with Monod's kinetics, Journal of Process Control 14 (2004) 807–818.

[28] K.J. Astrom, B. Wittenmark, Adaptive Control, Addison-Wesley Publishing Company Inc., Reading, MA, 1995.