RAJIV GANDHI INSTITUTE OF PETROLEUM TECHNOLOGY

Jais, Amethi, Uttar Pradesh

ASSIGNMENT - 1

Basic Relationships between Pixels

Pixel Connectivity and Distance Measures

Digital Image Processing

Student Name:	Shivam Vashishtha
Roll Number:	21CS2020
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Instructor:	Dr. Pallabi Saikia
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1 Question 1: Basic Terminologies (5 Marks)

1.1 Neighbors of a Pixel

A pixel p at coordinates (x, y) in a digital image has several types of neighbors based on spatial proximity:

1.1.1 4-Neighbors ($\mathbf{N_4(p)}$)

The four pixels that are horizontally and vertically adjacent to p:

- $(x + 1, y)$ - Right neighbor
- $(x - 1, y)$ - Left neighbor
- $(x, y + 1)$ - Bottom neighbor
- $(x, y - 1)$ - Top neighbor

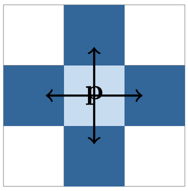
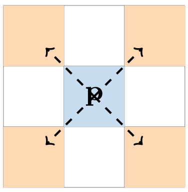


figure4-Neighbors of pixel p (in blue)

1.1.2 Diagonal Neighbors ($\mathbf{N_D(p)}$)

The four pixels that are diagonally adjacent to p:

- $(x + 1, y + 1)$ - Bottom-right diagonal
- $(x + 1, y - 1)$ - Top-right diagonal
- $(x - 1, y + 1)$ - Bottom-left diagonal
- $(x - 1, y - 1)$ - Top-left diagonal



figureDiagonal Neighbors of pixel p (in orange)

1.1.3 8-Neighbors ($\mathbf{N_8(p)}$)

The combination of 4-neighbors and diagonal neighbors:

$$N_8(p) = N_4(p) \cup N_D(p)$$

This includes all eight pixels surrounding p in a 3×3 neighborhood.

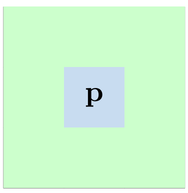


figure8-Neighbors of pixel p (all 8 surrounding pixels in green)

1.2 Adjacency, Connectivity, Regions, and Boundaries

1.2.1 Adjacency

Two pixels are **adjacent** if they are neighbors and satisfy some criterion of similarity (e.g., same intensity value or range). Three types of adjacency exist:

1. **4-Adjacency:** Two pixels p and q with values from set V are 4-adjacent if $q \in N_4(p)$
2. **8-Adjacency:** Two pixels p and q with values from set V are 8-adjacent if $q \in N_8(p)$
3. **m-Adjacency (Mixed Adjacency):** Two pixels p and q with values from set V are m-adjacent if:
   - $q \in N_4(p)$, OR
   - $q \in N_D(p)$ AND $N_4(p) \cap N_4(q)$ has no pixels with values from V

Purpose of m-Adjacency: Eliminates the multiple path problem that can occur with 8-adjacency while maintaining better connectivity than 4-adjacency.

1.2.2 Connectivity

A pixel p is **connected** to pixel q if there exists a path from p to q consisting of pixels with values from a specified set V, where all pixels along the path are adjacent.

Connected Component: A set of pixels that are all connected to each other, forming a maximal region.

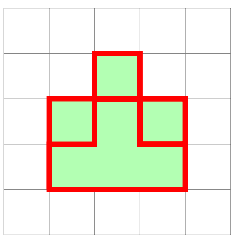
1.2.3 Regions

A **region** R is a connected subset of an image. It has two important properties:

- All pixels in R satisfy some predicate $P(R)$ (e.g., same intensity range)
- R is a connected set

1.2.4 Boundaries

The **boundary** (border or contour) of a region R is the set of pixels in R that have at least one neighbor not in R.


figureRegion (green) with its boundary (red outline)

1.3 Distance Measures

Distance measures quantify the separation between pixels. For pixels p, q, and z at coordinates (x, y), (s, t), and (u, v) respectively, a distance function D must satisfy:

1. $D(p, q) \geq 0$ (non-negativity); $D(p, q) = 0$ iff $p = q$
2. $D(p, q) = D(q, p)$ (symmetry)
3. $D(p, z) \leq D(p, q) + D(q, z)$ (triangle inequality)

1.3.1 Euclidean Distance ($\mathbf{L_2}$)

The straight-line distance between two pixels:

$$D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$$

Example: Distance between $(0,0)$ and $(3,4)$:

$$D_e = \sqrt{(0-3)^2 + (0-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

1.3.2 City-Block Distance (Manhattan, $\mathbf{L_1}$)

The sum of horizontal and vertical distances:

$$D_4(p,q) = |x-s| + |y-t|$$

Example: Distance between $(0,0)$ and $(3,4)$:

$$D_4 = |0-3| + |0-4| = 3+4 = 7$$

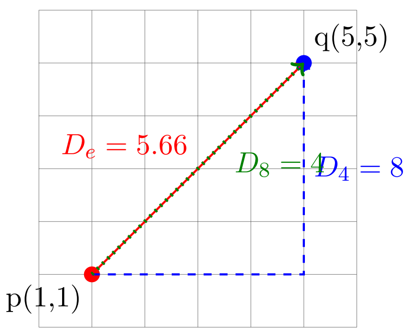
1.3.3 Chessboard Distance ($\mathbf{L_\infty}$)

The maximum of horizontal and vertical distances:

$$D_8(p,q) = \max(|x-s|, |y-t|)$$

Example: Distance between $(0,0)$ and $(3,4)$:

$$D_8 = \max(|0-3|, |0-4|) = \max(3,4) = 4$$



figureComparison of distance measures from p(1,1) to q(5,5)

2 Question 2: Connectivity Analysis (5 Marks)

Consider the two image subsets S_1 and S_2 with $V = \{1\}$:

S_1					S_2				
0	0	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	1	1	0	0	0
0	0	1	1	1	0	0	1	1	1
0	0	1	1	1	0	0	1	1	1

2.1 Analysis with Different Connectivity Types

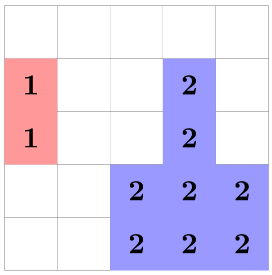
2.1.1 (a) 4-Connected Components

In 4-connectivity, two pixels are connected only if they share a horizontal or vertical edge.

For S_1:

- **Component 1:** Pixels at positions (1,0) and (2,0) - These two pixels are 4-connected vertically
- **Component 2:** Pixels at (1,3), (2,3), (3,2), (3,3), (3,4), (4,2), (4,3), (4,4) form one large connected region

Number of 4-connected components in S_1: 2



S_1 with 4-connectivity (2 components)

For S_2:

- **Component 1:** Pixel at (0,2)
- **Component 2:** Pixel at (0,3)
- **Component 3:** Pixels at (1,1), (2,0), (2,1) form one connected region
- **Component 4:** Pixels at (3,2), (3,3), (3,4), (4,2), (4,3), (4,4) form one connected region
- **Component 5:** Pixel at (1,4)

Number of 4-connected components in S_2: 5

		1	1	
	2			3
	2	2		
		4	4	4
		4	4	4

S_2 with 4-connectivity (5 components)

2.1.2 (b) 8-Connected Components

In 8-connectivity, two pixels are connected if they share an edge or a corner (diagonal neighbors included).

For S_1:

All pixels with value 1 are now connected through 8-connectivity (including diagonal connections).

Number of 8-connected components in S_1: 1

1			1	
1			1	
		1	1	1
		1	1	1

S_1 with 8-connectivity (1 component)

For S_2:

With 8-connectivity, several previously separate components merge through diagonal connections:

- Component 1: All pixels with value 1 are connected (the entire set forms one component through diagonal connections)

Number of 8-connected components in S_2: 1

		1	1	
	1			1
	1	1		
		1	1	1
		1	1	1

S_2 with 8-connectivity (1 component)

2.1.3 (c) m-Connected Components

m-connectivity (mixed connectivity) uses 4-connectivity when possible, but allows diagonal connections only when no 4-connected path exists between common neighbors.

For S_1:
Following m-connectivity rules:

- Pixels (1,0) and (2,0) are m-connected (4-neighbors)
- Pixel (1,3) and pixel (0,1) share no common 4-neighbors, so they cannot connect diagonally
- The bottom-right cluster remains connected through 4-connectivity

Number of m-connected components in S_1: 2
For S_2:
Applying m-connectivity:

- Component 1: Pixels (2,0), (2,1), (1,1) through 4-connectivity and allowed diagonal
- Component 2: Pixels (0,2), (1,2) are m-connected
- Component 3: Pixel (1,4)
- Component 4: Bottom cluster (3,2) through (4,4) remains connected

Number of m-connected components in S_2: 2
(Note: m-connectivity allows the top components and middle components to connect through controlled diagonal connections)

2.2 Summary Table

Image Subset	4-Connected	8-Connected	m-Connected
S_1	2	1	2
S_2	5	1	2

Table 1: Number of connected components for different connectivity types

2.3 Are S_1 and S_2 Adjacent?

Answer: No, S_1 and S_2 are NOT adjacent.

Reasoning: For two sets of pixels to be adjacent, at least one pixel from S_1 must be a neighbor (4-neighbor or 8-neighbor) of at least one pixel from S_2. Since S_1 and S_2 are separate image regions shown side-by-side in the problem statement, they occupy different spatial locations in the image space and share no common boundaries. Therefore, no pixel from S_1 can be adjacent to any pixel from S_2.

If the question intended to ask whether the two images contain pixels that would be adjacent if overlaid or placed next to each other, we would need specific spatial coordinates to determine adjacency.

3 Conclusion

This assignment covered fundamental concepts in digital image processing related to pixel relationships:

- **Pixel Neighborhoods:** Understanding 4-neighbors, 8-neighbors, and diagonal neighbors is essential for defining spatial relationships in images.
- **Connectivity:** Different connectivity types (4, 8, and m) provide various ways to define connected regions, each with specific applications:

  - 4-connectivity: Simple, avoids multiple paths, but may fragment regions
  - 8-connectivity: More inclusive, better captures perceptual regions, but may create unwanted connections
  - m-connectivity: Balances both approaches, eliminating ambiguous paths

- **Distance Measures:** Three primary metrics (Euclidean, City-Block, Chessboard) serve different purposes in image analysis, from morphological operations to pattern recognition.

- **Practical Application:** The connectivity analysis demonstrated how the choice of connectivity type significantly affects region segmentation and component counting, which are crucial for object detection and image analysis tasks.