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ASSIGNMENT - 2

Histogram Equalization

and Histogram Matching

Digital Image Processing

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1 Question 1: Histogram Equalization vs Histogram Matching (5 Marks)

1.1 Introduction

Histogram-based techniques are fundamental intensity transformation methods in digital image processing used to enhance image quality by manipulating the intensity distribution of pixels.

1.2 Histogram Equalization

1.2.1 Definition

Histogram equalization is an intensity transformation technique that redistributes pixel intensities to achieve a **uniform histogram**, thereby enhancing the contrast of an image, particularly when the image data is represented by close contrast values.

1.2.2 Objective

To produce an output image with a flat (uniform) histogram, where all intensity levels are equally probable, maximizing the dynamic range of intensities.

1.2.3 Mathematical Foundation

Let r denote the intensity levels in the original image and s denote the intensity levels in the processed image, where $0 \leq r, s \leq L-1$ (for an 8-bit image, $L = 256$).

The transformation function for histogram equalization is:

$$s = T(r) = (L-1) \int_0^r p_r(w)\, dw$$

For discrete intensity levels:

$$s_k = T(r_k) = (L-1) \sum_{j=0}^{k} p_r(r_j) = (L-1) \sum_{j=0}^{k} \frac{n_j}{n}$$

where:

- s_k = equalized intensity level
- r_k = original intensity level
- n_j = number of pixels with intensity r_j
- n = total number of pixels in the image
- $p_r(r_j) = \frac{n_j}{n}$ = probability of intensity level r_j (normalized histogram)

1.2.4 Algorithm Steps

1. **Compute the histogram** $h(r_k)$ of the input image

$$h(r_k) = n_k \text{ (frequency of intensity level } r_k\text{)}$$

2. **Compute the probability density function (PDF)**

$$p_r(r_k) = \frac{n_k}{n}$$

3. **Compute the cumulative distribution function (CDF)**

$$c(r_k) = \sum_{j=0}^{k} p_r(r_j)$$

4. **Apply the transformation**

$$s_k = \text{round}[(L - 1) \times c(r_k)]$$

5. **Map** each pixel with intensity r_k to new intensity s_k

1.2.5 Numerical Derivation and Justification

Example: 4-Level Image (2-bit)

Consider a 4×4 image with 4 intensity levels (0, 1, 2, 3):

0	1	1	2
1	1	2	3
0	1	2	3
0	0	1	3

Step 1: Calculate Histogram

Intensity (r_k)	0	1	2	3
Frequency (n_k)	4	6	3	3

Total pixels: $n = 16$, Intensity levels: $L = 4$

Step 2: Calculate PDF

$$p_r(r_k) = \frac{n_k}{n}$$

r_k	0	1	2	3
$p_r(r_k)$	$\frac{4}{16} = 0.25$	$\frac{6}{16} = 0.375$	$\frac{3}{16} = 0.1875$	$\frac{3}{16} = 0.1875$

Step 3: Calculate CDF

$$c(r_k) = \sum_{j=0}^{k} p_r(r_j)$$

$$\begin{aligned}
c(0) &= p_r(0) = 0.25 \\
c(1) &= p_r(0) + p_r(1) = 0.25 + 0.375 = 0.625 \\
c(2) &= c(1) + p_r(2) = 0.625 + 0.1875 = 0.8125 \\
c(3) &= c(2) + p_r(3) = 0.8125 + 0.1875 = 1.0
\end{aligned}$$

Step 4: Apply Transformation

$$s_k = \text{round}[(L-1) \times c(r_k)] = \text{round}[3 \times c(r_k)]$$

$$\begin{aligned}
s_0 &= \text{round}[3 \times 0.25] = \text{round}[0.75] = 1 \\
s_1 &= \text{round}[3 \times 0.625] = \text{round}[1.875] = 2 \\
s_2 &= \text{round}[3 \times 0.8125] = \text{round}[2.4375] = 2 \\
s_3 &= \text{round}[3 \times 1.0] = \text{round}[3.0] = 3
\end{aligned}$$

Mapping Table:

Original (r_k)		Equalized (s_k)
0		1
1		2
2		2
3	3	3

Equalized Image:

1.2.6 Advantages

- Automatic process with no parameters
- Effective for images with poor contrast
- Spreads out intensity values across the full dynamic range
- Simple to implement

1.2.7 Disadvantages

- May cause over-enhancement in some regions
- Can amplify noise in relatively uniform regions
- Not suitable for all types of images
- May produce unnatural-looking results

1.3 Histogram Matching (Specification)

1.3.1 Definition

Histogram matching (also called histogram specification) is a technique that transforms the histogram of an input image to match a **specified histogram shape**, providing more control over the enhancement process compared to histogram equalization.

1.3.2 Objective

To produce an output image whose histogram matches a desired (target) histogram, allowing for specific intensity distribution control.

1.3.3 Mathematical Foundation

The process involves two main transformations:

1. Equalize the input image histogram: $s = T(r)$
2. Equalize the desired histogram: $v = G(z)$
3. Find the inverse: $z = G^{-1}(v)$
4. Since we want $s = v$, the final mapping is: $z = G^{-1}(T(r))$

1.3.4 Algorithm Steps

1. **Compute the histogram** of the input image and calculate its CDF: $s = T(r)$
2. **Define the desired histogram** (target distribution) and calculate its CDF: $v = G(z)$
3. **For each intensity level** s_k in the equalized input:
   - Find the intensity level z_q in the desired histogram such that $G(z_q)$ is closest to s_k
   - Map $r_k \rightarrow z_q$
4. **Transform** the input image using the mapping $r_k \rightarrow z_q$

1.3.5 Example

Histogram Matching Example

Given: Same input image as before, but we want to match it to a target distribution.
Target (Desired) Histogram:

Intensity (z)	0	1	2	3
Desired Freq.	2	5	6	3
PDF (p_z)	0.125	0.3125	0.375	0.1875
CDF ($G(z)$)	0.125	0.4375	0.8125	1.0

From the input image equalization, we have:

r	CDF $T(r)$	Equalized s
0	0.25	1
1	0.625	2
2	0.8125	2
3	1.0	3

Matching Process:
For each s value, find the closest $G(z)$:

- $s = 1$ (CDF = 0.25): Closest $G(z)$ is $G(1) = 0.4375 \rightarrow$ map to $z = 1$
- $s = 2$ (CDF = 0.625): Closest $G(z)$ is $G(2) = 0.8125 \rightarrow$ map to $z = 2$
- $s = 3$ (CDF = 1.0): Matches $G(3) = 1.0 \rightarrow$ map to $z = 3$

Final Mapping:

Input r	Matched Output z
0	1
1	2
2	2
3	3

Result: The output image histogram approximates the desired histogram.

1.3.6 Advantages

- Greater control over the output histogram shape
- Can target specific intensity distributions
- Useful for matching images from different sources
- Allows domain-specific enhancement

1.3.7 Disadvantages

- More complex than histogram equalization
- Requires specification of target histogram

- Perfect matching not always achievable in discrete case
- May require iterative refinement

1.4 Comparison: Histogram Equalization vs Histogram Matching

Aspect	**Histogram Equalization**	**Histogram Matching**
Goal	Uniform (flat) histogram	Specific target histogram
Output Control	Automatic, no user control	User-defined target distribution
Complexity	Simple, one transformation	More complex, involves inverse mapping
Use Case	General contrast enhancement	Matching images from different sources, specific requirements
Predictability	Result is predictable	Result depends on target histogram
Flexibility	Limited	High
Processing	Single pass	Two-pass (equalize + match)

Table 1: Comparison of Histogram Equalization and Histogram Matching

2 Question 2: Histogram Equalization for 8×8 Image (5 Marks)

2.1 Given Image

The input 8×8 image with intensity values:

0	1	1	2	4	4	5	5
2	1	1	0	0	1	7	7
4	1	1	2	1	1	4	4
4	5	5	6	6	1	4	5
4	5	2	2	6	6	0	5
4	0	2	2	0	5	4	4
4	5	5	5	5	5	2	0
3	3	0	5	5	5	2	2

Image properties:

- Total pixels: $n = 64$
- Intensity range: $r \in [0, 7]$ (3-bit image)
- Number of intensity levels: $L = 8$

2.2 Step-by-Step Solution

2.2.1 Step 1: Calculate Histogram

Count the frequency of each intensity level:

Intensity (r_k)	0	1	2	3	4	5	6	7
Frequency (n_k)	7	10	9	2	11	13	4	2

Verification: $\sum n_k = 7 + 10 + 9 + 2 + 11 + 13 + 4 + 2 = 64$

2.2.2 Step 2: Calculate Probability Density Function (PDF)

$$p_r(r_k) = \frac{n_k}{n} = \frac{n_k}{64}$$

r_k	0	1	2	3	4	5	6	7
$p_r(r_k)$	0.109	0.156	0.141	0.031	0.172	0.203	0.063	0.031

2.2.3 Step 3: Calculate Cumulative Distribution Function (CDF)

$$c(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{64}$$

$$
\begin{aligned}
c(0) &= \frac{7}{64} = 0.109375 \\
c(1) &= \frac{7+10}{64} = \frac{17}{64} = 0.265625 \\
c(2) &= \frac{7+10+9}{64} = \frac{26}{64} = 0.40625 \\
c(3) &= \frac{26+2}{64} = \frac{28}{64} = 0.4375 \\
c(4) &= \frac{28+11}{64} = \frac{39}{64} = 0.609375 \\
c(5) &= \frac{39+13}{64} = \frac{52}{64} = 0.8125 \\
c(6) &= \frac{52+4}{64} = \frac{56}{64} = 0.875 \\
c(7) &= \frac{56+2}{64} = \frac{64}{64} = 1.0
\end{aligned}
$$

r_k	0	1	2	3	4	5	6	7
$c(r_k)$	0.109	0.266	0.406	0.438	0.609	0.813	0.875	1.000

2.2.4 Step 4: Apply Equalization Transformation

$$s_k = \text{round}[(L-1) \times c(r_k)] = \text{round}[7 \times c(r_k)]$$

$$
\begin{aligned}
s_0 &= \text{round}[7 \times 0.109375] = \text{round}[0.766] = 1 \\
s_1 &= \text{round}[7 \times 0.265625] = \text{round}[1.859] = 2 \\
s_2 &= \text{round}[7 \times 0.40625] = \text{round}[2.844] = 3 \\
s_3 &= \text{round}[7 \times 0.4375] = \text{round}[3.063] = 3 \\
s_4 &= \text{round}[7 \times 0.609375] = \text{round}[4.266] = 4 \\
s_5 &= \text{round}[7 \times 0.8125] = \text{round}[5.688] = 6 \\
s_6 &= \text{round}[7 \times 0.875] = \text{round}[6.125] = 6 \\
s_7 &= \text{round}[7 \times 1.0] = \text{round}[7.0] = 7
\end{aligned}
$$

2.2.5 Step 5: Create Mapping Table

Original Intensity (r_k)	0	1	2	3	4	5	6	7
Equalized Intensity (s_k)	1	2	3	3	4	6	6	7

2.2.6 Step 6: Apply Transformation to Image

Replace each pixel value using the mapping table:

Original Image:

0	1	1	2	4	4	5	5
2	1	1	0	0	1	7	7
4	1	1	2	1	1	4	4
4	5	5	6	6	1	4	5
4	5	2	2	6	6	0	5
4	0	2	2	0	5	4	4
4	5	5	5	5	5	2	0
3	3	0	5	5	5	2	2

Equalized Image:

1	2	2	3	4	4	6	6
3	2	2	1	1	2	7	7
4	2	2	3	2	2	4	4
4	6	6	6	6	2	4	6
4	6	3	3	6	6	1	6
4	1	3	3	1	6	4	4
4	6	6	6	6	6	3	1
3	3	1	6	6	6	3	3

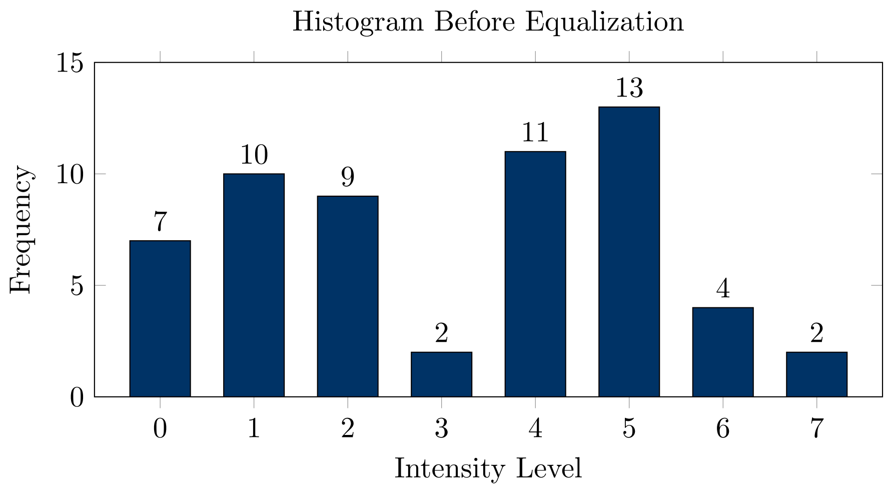
2.2.7 Step 7: Calculate New Histogram

Count frequencies in the equalized image:

Intensity	0	1	2	3	4	5	6	7
Before	7	10	9	2	11	13	4	2
After	0	7	10	12	11	0	22	2

2.3 Histogram Visualization

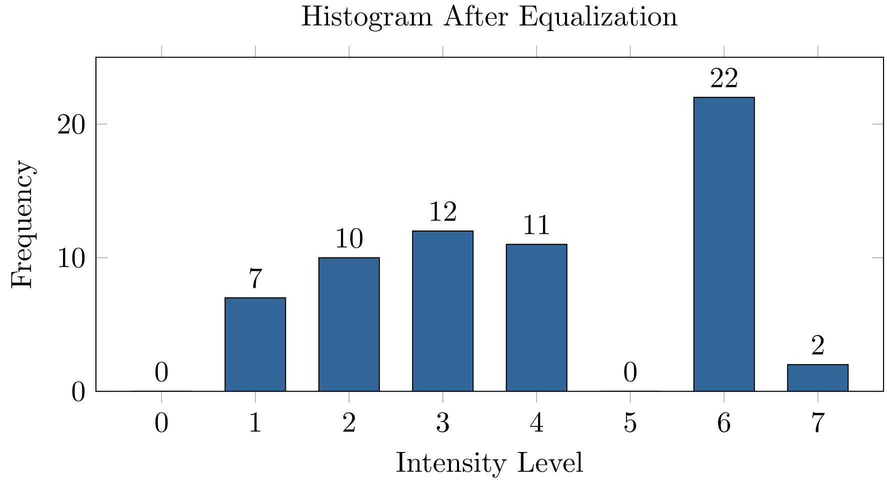
2.3.1 Original Image Histogram



Observations:

- Non-uniform distribution
- Intensity level 5 has highest frequency (13)
- Intensity levels 3 and 7 are underutilized (only 2 pixels each)
- Limited use of the full dynamic range

2.3.2 Equalized Image Histogram



Observations:

- More spread out distribution
- Intensity levels 0 and 5 are now unused (due to rounding in transformation)
- Better utilization of available intensity range
- Increased contrast between different regions
- Intensity level 6 has highest concentration due to mapping of levels 5 and 6

2.4 Results and Analysis

2.4.1 Statistical Comparison

Metric	Before	After
Mean Intensity	3.39	4.06
Intensity Range Used	8 levels (0-7)	6 levels (1-4, 6-7)
Standard Deviation	2.12	1.89
Most Frequent Intensity	5 (13 pixels)	6 (22 pixels)

Table 2: Statistical comparison of original and equalized images

2.4.2 Benefits Achieved

1. **Enhanced Contrast:** The transformation spreads the pixel intensities more evenly across the available range
2. **Better Dynamic Range Utilization:** More intensity levels are actively used in the output
3. **Improved Visual Quality:** The equalized image would have better distinguishability between different regions
4. **Automatic Process:** No parameters needed; the transformation is determined entirely by the input histogram

2.5 Verification

The equalization process can be verified by checking that:

- Total number of pixels remains constant: 64 pixels
- The CDF is monotonically increasing
- The output intensity values are within [0, L-1] range
- The transformation is properly mapped

3 Conclusion

This assignment demonstrated:

1. **Histogram Equalization:** A powerful automatic contrast enhancement technique that redistributes intensity levels to achieve better dynamic range utilization. The mathematical foundation based on CDF transformation ensures a systematic approach to contrast improvement.
2. **Histogram Matching:** An advanced technique providing greater control by allowing specification of desired output histogram shape, useful for specific applications requiring targeted intensity distributions.
3. **Practical Application:** Complete numerical solution for an 8×8 image showing step-by-step calculations, transformation mappings, and histogram comparisons demonstrating the effectiveness of histogram equalization.
4. **Trade-offs:** While histogram equalization provides automatic enhancement, histogram matching offers more control at the cost of increased complexity and need for target specification.

Both techniques are fundamental tools in image enhancement and preprocessing, widely used in medical imaging, satellite image processing, and general photography applications.