ST 311 Practice Exam 2

Multiple Choice: For each, circle one answer only.

1. Some researchers wish to study the effect of cross-fertilization on the height of plant A. They know that the average height of self-fertilized A plants is 10 inches. Also, they expect that cross-fertilized plants would grow taller than self-fertilized plants. To make a decision, they randomly selected 100 cross-fertilized A plants. They find the sample mean height of these plants to be 12 inches. What would be the appropriate null and alternative hypotheses?

   ○ H_0: μ = 10 versus H_A: μ = 12

   ○ H_0: μ = 10 versus H_A: μ > 10

   ○ H_0: μ = 10 versus H_A: μ < 10

   ○ H_0: μ = 12 versus H_A: μ > 12

2. Which of the following is true about t critical values (confidence coefficients) of confidence intervals for means?

   ○ The value of t gets smaller as the degrees of freedom increase.

   ○ The value of t remains unchanged as the degrees of freedom change.

   ○ The value of t gets smaller if we use a larger confidence level.

   ○ None of the above.

3. To test the claim that students tend to perform better, on average, in Chemistry than in Physics, 27 students who are enrolled in both classes are selected and their final exam scores are recorded. Which of the following is NOT necessary to perform this test?

   ○ The populations of the differences must be normal or approximately normal.

   ○ The samples are random.

   ○ The samples are dependent.

   ○ np≥10 and nq≥10

4. Which of the following is **NOT TRUE** about a confidence interval (CI)?

   ○ We construct a CI when we are interested in some population parameter but we only have a sample data.

   ○ We construct a CI around the population parameter.

   ○ The CI changes from sample to sample.

   ○ The CI is a statement about what values are plausible for the population parameter.

5. A random sample of 10 loan providers is selected. For each provider, the interest rate on a $10,000 personal loan this year and the rate from last year are recorded and compared. A test of H0:μ=0 vs HA:μ≠0 is conducted and a p-value of 0.0432 is reported. Assuming interest rates are approximately normal, what conclusion can be made at the 5% significance level?

   ○ There is not sufficient evidence to suggest that the interest rate is, on average, the same as it was a year ago.

   ○ There is sufficient evidence to suggest that the mean interest rate has changed in the last year.

   ○ There is sufficient evidence to suggest that mean interest rate on personal loans is different from 0%.

   ○ There is not sufficient evidence to suggest that the interest rate of these providers is, on average, the same as it was a year ago.

6. Emmett is a researcher interested in investigating if the average income of graduating students has increased in the last 10 years. It is known that 10 years ago, the average graduating income of students was $45,000. What kind of hypothesis test would Emmett want to run?

   ○ A one-sided test of proportions.

   ○ A one-sided test of means.

   ○ A two-sided test of proportions.

   ○ A two-sided test of means.

7. The deterioration of pipeline networks across the country is a growing concern. One rehabilitation option proposed is to thread a liner through existing pipe. Wishing to know whether fusing liner to the pipes increases tensile strength, measurements were taken for 10 unfused liners and 8 fused liners. A 95% confidence interval for $\mu_{No\ fusion} - \mu_{fusion}$ was found to be $(-488, 38)$. We can conclude:

   ○ There is no significant difference in tensile strength between fused and unfused liners because 0 is in the interval.

   ○ There is a significant difference in tensile strength between fused and unfused liners because 0 is in the interval.

   ○ The null hypothesis is true.

   ○ There is not enough evidence to suggest mean tensile strength in fused liners is greater than unfused liners because 0 is in the interval.

   ○ There is enough evidence to suggest mean tensile strength in fused liners is greater than unfused liners because 0 is in the interval.

8. A researcher conducted a study and his results were statistically significant at 1% significance level. Which of the following is **FALSE** about this study?

   ○ The p-value calculated was smaller than 0.05.

   ○ It is unlikely that these results were due to chance.

   ○ The probability that the researcher would observe more extreme test results under the alternative hypothesis is less than 0.01.

   ○ The researcher would still reject the null hypothesis at 10% significance level.

9. A researcher wishes to test $H_0: \mu \geq 21 \ vs \ H_A: \mu < 21$. Which of the following possible sample results gives the most evidence to support H_a (i.e., reject H_0)? Assume the samples are all the same size.

   ○ $\bar{x}$ =19, s=4

   ○ $\bar{x}$ =17, s=7

   ○ $\bar{x}$ =23, s=3

   ○ $\bar{x}$ =18, s=6

10. A critical value tells us...

    ○ The probability of the sample statistic occurring.

    ○ The probability of committing a Type I Error.

    ○ The number of standard deviations/errors from the mean a sample statistic must be to be considered significant/unusual.

    ○ The number of standard deviations/errors from the mean a sample statistic lies, under the null distribution.

11. A researcher wants to determine if a new test can detect a disease correctly in the early stage. After collecting a random and normally distributed sample, he constructs a 85% confidence interval for the proportion of correct results from the new test and he gets an interval of (0.87, 0.93). Which of the following is the correct interpretation of the confidence **level** in this situation?

    ○ There is an 85% probability that the true proportion of correct results from the new test is between 87% and 93%.

    ○ 85% of the confidence intervals constructed capture the true proportion of correct results from the new test.

    ○ 85% of the confidence intervals constructed capture the sample proportion of correct results from the new test.

    ○ There is a 15% probability of detecting the disease while the patient is actually healthy.

12. The main goal of statistical inference (i.e. using confidence intervals and hypothesis tests) is to:

○ Disprove incorrect beliefs about sample statistics using population parameters.

○ ==Find evidence and estimates for the plausible values of population parameters using sample statistics.==

○ Use the sample statistic to prove whether or not the null hypothesis is true.

○ Carefully manipulate confidence/significance levels in order to achieve statistically significant results.

13. Two random samples, each consisting of 6 rats, were exposed to different environment. One sample of rats was held in a normal environment at 26℃ and the other sample was held in a cold 5℃ environment. Blood pressures were measured for the rats. Which type of test should be done to answer the question: Is there evidence that rats in a cold environment have a higher mean bold pressure than in a normal environment?

○ A t test for the mean

○ A z-test for the proportion

○ A paired t-test for the mean difference

○ ==A t test for the difference in means==

○ A z test for the difference in proportions

14. James wants to know the true proportion of residents in Wake county that have received flu shots this year. He decides to compute a confidence interval, but first he needs to calculate the margin of error. Which of the following is NOT TRUE about the margin of error?

○ The margin of error accounts for random sampling variability.

○ ==The margin of error needs to be larger in order to account for sampling bias.==

○ The margin of error depends on the confidence level and sample size.

○ The margin of error determines how wide his confidence interval would be.

15. Which of the following statements are true about p-values?

○ If p-value is greater than the significance level, we accept the null hypothesis.

○ If p-value is less than the significance level, we fail to reject the null hypothesis.

○ ==P-value is the proportion of (i.e. probability under) the null distribution that is more extreme than the observed sample value.==

○ P-value is the proportion of (i.e. probability under) the alternative distribution that is more extreme than the observed sample value.

Short Answer: Show all your work to receive credit.

16. One method used to improve focus is called time-blocking – splitting up your activities into manageable chunks. In a study consisting of 85 randomly selected adults who work remotely, 52 found time-blocking to be effective in improving focus during their work day.

    a. On the distribution below, shade the area that represents a 95% confidence level and label the critical values "CV".
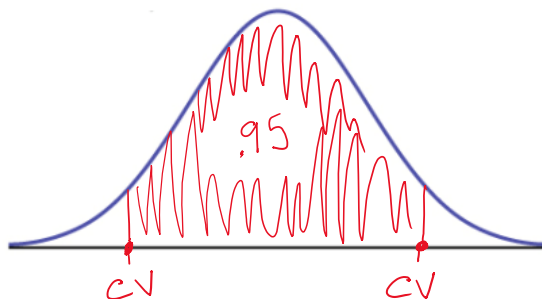
    b. Assume the critical value from above is 1.8. Using this value, find a 95% confidence interval for the proportion of all adult remote workers for whom time-blocking improves focus. *Show all work, including checking assumptions. State your final interval to 4 decimal places.*

$$\hat{p} = \frac{52}{85} = 0.6118, \hat{q} = 0.3882$$
$$n\hat{p} = 52, n\hat{q} = 33, both > 10$$
And the sample is random

$$Assume\ z = 1.8$$

$$95\%\ CI\ for\ p: \hat{p} \pm z * \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= 0.6118 \pm 1.8 * \sqrt{\frac{0.6118 * 0.3882}{85}}$$

$$= 0.6118 \pm 0.1036$$
$$= (0.5167, 0.7069)$$

    c. A separate study indicated that 65% of remote workers benefitted from daily meditation. Can we conclude that mediation works better than time-blocking? Explain your answer using complete sentences and your work from the part above.

    Since 65% falls within the interval estimate for time-blocking found above, it is plausible that the proportion of those who benefit from time-blocking is the same (or slightly higher, since the upper limit is 0.7069). We <u>cannot</u> conclude that mediation works better (has a higher proportion) than time-blocking.

17. In a random sample of 28 American adults, it was found that the average amount of money spent on Halloween candy was $42.40, with a standard deviation of $7.32. Assume the distribution of money spent is approximately Normal. **Construct and interpret** a 90% confidence interval for the mean amount of money American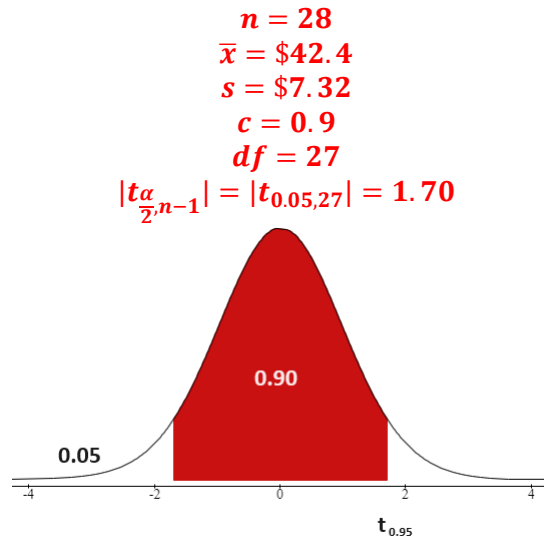 adults spend on Halloween candy in a year. *Show all work.* *Choose from the following critical values for this problem:*

$$|z_{0.10}| = 1.28, |z_{0.05}| = 1.645, |z_{0.025}| = 1.96$$
$$|t_{0.10,27}| = 1.31, |t_{0.05,27}| = 1.70, |t_{0.025,27}| = 2.05$$

Round your final answer using 2 decimals.

Confidence Interval for Mean

$$n = 28$$
$$\overline{x} = \$42.4$$
$$s = \$7.32$$
$$c = 0.9$$
$$df = 27$$
$$|t_{\frac{\alpha}{2},n-1}| = |t_{0.05,27}| = 1.70$$

0.90

0.05

$t_{0.95}$

$$MOE = t^* * SE$$

$$= 1.70 * \frac{7.32}{\sqrt{28}}$$

$$= 1.70 * 1.383$$

$$MOE = 2.35$$

$$\textbf{\textit{Confidence Interval}} \ = \overline{x} \pm MOE = 42.4 \pm 2.35 = (40.05, 44.75)$$

We are 90% confident that the true mean of the money spent on Halloween candy by all American adults in a year is between $40.05 and $44.75.

18. In June of 2021, 55% of Americans were living paycheck-to-paycheck. In a random sample of 250 Americans in September of 2022, 152 reported to be living paycheck-to-paycheck. Test the hypothesis that the proportion of Americans living paycheck-to-paycheck has increased since June 2021. Use α=0.05.

   a. In the space below, state the hypotheses, check conditions, and find the test statistic.

$$H_0: p \leq 0.55 \; ; \; H_A: p > 0.55$$

$$\alpha = 0.05$$

$$n = 250$$

Conditions:

- random sample

- $n * p = 250(0.55) = 137.5 > 10$

- $n * q = 250(0.45) = 112.5 > 10$

$$\hat{p} = \frac{152}{250} = 0.608$$

$$z^* = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.608 - 0.55}{\sqrt{\frac{0.55(1-0.55)}{250}}} = \frac{0.058}{0.0315} = 1.843$$

   b. Write a probability statement for the p-value.

$$P(Z > 1.843)$$

   c. Assume the probability (p-value) is 0.04. State the decision for this test.

We reject the null hypothesis since the p-value is less than the significance level (0.05).

   d. Based on your test above, which statement is correct?

   ○ We have sufficient evidence to conclude that the proportion of Americans living paycheck-to-paycheck has increased since June 2021.

   ○ We do not have sufficient evidence to conclude that the proportion of Americans living paycheck-to-paycheck has increased since June 2021.

   ○ We have sufficient evidence to conclude that the proportion of Americans living paycheck-to-paycheck has *not* increased since June 2021.

   ○ We do not have sufficient evidence to conclude that the proportion of Americans living paycheck-to-paycheck has *not* increased since June 2021.

19. In the June 2007 issue, Consumer Reports also examined the relative merits of top-loading and front-loading washing machines, testing samples of several different brands of each type. Suppose the study tested the null hypothesis that top- and front-loading machines don't differ in their mean costs.

a. What are the hypotheses?

Let μ_{top}= the mean cost of top loading machines,

and μ_{front} = the mean cost of front loading machines.

$$H_0: \mu_{top} - \mu_{front} = 0$$

$$H_A: \mu_{top} - \mu_{front} \neq 0$$

b. Suppose the test had a P-value of 0.32. At $\alpha = 0.05$, what decision would this lead to?

The p-value is much larger than 0.05, so we would Fail to Reject the Null hypothesis.

c. If that conclusion is wrong, which type of error was made?

If we FTR the null, and that was an error, then the null was false. This is a Type II Error.

d. Would a 95% confidence interval for $\mu_{top} - \mu_{front}$ contain 0? Explain.

Since we failed to reject the null that the difference equals 0, then 0 is a plausible value for the difference. Therefore, it would be contained in the interval.