Actividad 2

Determina la solución de las siguientes ecuaciones diferenciales:

1.- $y' = \frac{x^2}{y} \rightarrow \frac{dy}{dx} = \frac{x^2}{y} \rightarrow y\,dy = x^2 dx$

$\int y\,dy = \int x^2 dx$

$\therefore \frac{y^2}{2} = \frac{x^3}{3} + C$ Sol. gral.

2- $y' = \cos^2 x \cos^2 2y \rightarrow \frac{dy}{dx} = \cos^2 x \cos^2 2y \rightarrow \frac{dy}{\cos^2 2y} = \cos^2 x\, dx$

Haciendo la integración en ambas partes:

$\int \frac{1}{\cos^2 2y} dy = \int \cos^2 x\, dx$

Para $\int \frac{dy}{\cos^2 2y} =$ $\quad u = 2y,\ du = 2dy$ $\rightarrow \frac{1}{2}\int \frac{1}{\cos^2 u} du \rightarrow \frac{1}{2}\int \sec^2 u\, du = \frac{\tan u}{2} = \frac{\tan 2y}{2}$

Para $\int \cos^2 x\, dx =$ $\quad \cos^2\theta = \frac{1+\cos 2\theta}{2} \rightarrow \int \frac{1+\cos 2x\, dx}{2} = \int \frac{1}{2} dx + \int \frac{\cos 2x}{2} dx$

$u = 2x,\ du = 2dx$ $\quad \frac{1}{2}\cdot\frac{1}{2}\int \cos u\, du = \frac{1}{4}\operatorname{sen} 2x \quad \therefore \int \cos^2 x\, dx = \frac{1}{2}x + \frac{1}{4}\operatorname{sen} 2x$

Entonces:

$\int \frac{1}{\cos^2 2y} dy = \int \cos^2 x\, dx \rightarrow \frac{\tan 2y}{2} = \frac{1}{2}x + \frac{1}{4}\operatorname{sen} 2x + C$ Sol. gral.

3- $\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y} \rightarrow y + e^y\, dy = x - e^{-x} dx$

$\int y + e^y\, dy = \int x - e^{-x} dx = \frac{y^2}{2} + e^y = \frac{x^2}{2} + e^{-x} + C$ Sol. gral

4- $x\,dx + y e^{-x} dy = 0 \quad y(0) = 1$

$y e^{-x} dy = -x\,dx$

$y\,dy = \frac{-x}{e^{-x}} dx \rightarrow \int y\,dy = -\int x e^x dx$

$\frac{y^2}{2} = -xe^x + e^x + C$ Sol. gral

Para $\int -xe^x dx$

$-xe^x + e^x + C_1$

$-x$	e^x
-1	e^x
0	e^x

Para sol. particular $y(0) = 1$

$\frac{1}{2} = -0e^0 + e^0 + C$

$\frac{1}{2} = 1 + C \rightarrow C = -\frac{1}{2}$

$\therefore \frac{y^2}{2} = -xe^x + e^x - \frac{1}{2}$ Sol. particular

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Determina la solución de las siguientes ecuaciones diferenciales homogéneas:

$$\frac{dy}{dx} = -\frac{4x+3y}{2x+y}$$

Primero se comprueba que son homogéneas:

$$\frac{dy}{dx} = -\frac{4\cancel{\frac{x}{x}} + 3\frac{y}{x}}{2\cancel{\frac{x}{x}} + \frac{y}{x}} \quad \therefore \text{Es homogénea}$$

$$v = \frac{y}{x} \rightarrow y = vx \rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = -\frac{4+3v}{2+v} \rightarrow x\frac{dv}{dx} = -\frac{4+3v}{2+v} - \frac{v}{1} \rightarrow x\,dv = \frac{-(4+3v+2v+v^2)}{2+v}dx$$

$$\rightarrow \left(\frac{2+v}{-(4+5v+v^2)}\right)dv = \frac{dx}{x} \rightarrow -\int \frac{2+v}{v^2+5v+4}dv = \int \frac{dx}{x} \quad \dots (1)$$

o Para $-\int \frac{2+v\ dv}{v^2+5x+4} = \int \frac{2+v}{(v+4)(v+1)}dv = \frac{A}{v+4} + \frac{B}{v+1}$

$$2+v = A(v+1) + B(v+4)$$

① $v=-1 \rightarrow 2-1 = A(-1+1) + B(-1+4) \rightarrow 1 = 0+3B \rightarrow \frac{1}{3} = B$

② $v=-4 \rightarrow 2-4 = A(-4+1) + B(-4+4) \rightarrow -2 = -3A + 0 \rightarrow \frac{-2}{-3} = A \rightarrow A = \frac{2}{3}$

Entonces:

$$-\int \frac{2+v\ dv}{v^2+5x+4} = -\int \frac{2+v\ dv}{(v+4)(v+1)} = -\left(\frac{2}{3}\int \frac{dv}{v+4} + \frac{1}{3}\int \frac{dv}{v+1}\right)$$

$$= -\frac{2}{3}\ln|v+4| - \frac{1}{3}\ln|v+1|$$

Regresando a (1) tenemos:

$$-\int \frac{2+v\ dv}{v^2+5v+4} = \int \frac{dx}{x} \rightarrow -\frac{2}{3}\ln|v+4| - \frac{1}{3}\ln|v+1| = \ln|x| + c$$

Regresando a la variable original:

$$-\frac{2}{3}\ln\left|\frac{y}{x}+4\right| - \frac{1}{3}\ln\left|\frac{y}{x}+1\right| = \ln x + c$$

Sol. gral.

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2) $\frac{dy}{dx} = \frac{y-x}{x} \rightarrow \frac{dy}{dx} = \frac{\frac{y}{x} - \cancel{\frac{x}{x}}}{\cancel{\frac{x}{x}}} \rightarrow \frac{dy}{dx} = \frac{y}{x} - 1$ ∴ Es homogénea

Sustituimos: $v = \frac{y}{x} \rightarrow y = vx \rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = v - 1 \rightarrow x\frac{dv}{dx} = -1 \rightarrow dv = -\frac{1}{x}dx$

Integramos:

$\int dv = -\int \frac{dx}{x} \rightarrow \frac{y^2}{2} = -\ln x + c$ Sol. gral

3) $\frac{dy}{dx} = \frac{2y-x}{2x-y} \rightarrow \frac{dy}{dx} = \frac{2\frac{y}{x} - \cancel{\frac{x}{x}}}{2\cancel{\frac{x}{x}} - \frac{y}{x}} \rightarrow \frac{dy}{dx} = \frac{2\frac{y}{x} - 1}{2 - \frac{y}{x}}$ ∴ Es homogénea

Sustituimos: $v = \frac{y}{x} \rightarrow y = vx \rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{2v-1}{2-v} \rightarrow x\frac{dv}{dx} = \frac{2v-1}{2-v} - v \rightarrow x\frac{dv}{dx} = \frac{2v-1-2v+v^2}{2-v}$

$x\frac{dv}{dx} = \left(\frac{-1+v^2}{2-v}\right) \rightarrow \frac{2-v}{-1+v^2}dv = \frac{dx}{x} \rightarrow \int \frac{2-v}{-1+v^2}dv = \int \frac{dx}{x}$

Para: $\int \frac{2-v}{-1+v^2}dv = \int \frac{2-v \, dv}{(v-1)(v+1)} = \frac{A}{v-1} + \frac{B}{v+1} \rightarrow 2-v = A(v+1) + B(v-1)$

$v=-1 \rightarrow 2+1 = A\cancel{(-1+1)}^{0} + B(-1-1) \rightarrow 3 = -2B$ $\quad B = \frac{-3}{2}$

$v=1 \rightarrow 2-1 = A(1+1) + \cancel{B(1-1)}^{0} \rightarrow 1 = 2A$ $\quad A = \frac{1}{2}$

$\therefore \int \frac{2-A\,dv}{(v-1)(v+1)} = \frac{1}{2}\int\frac{dv}{v-1} - \frac{3}{2}\int\frac{dv}{v+1}$

$\frac{1}{2}\ln|v-1| - \frac{3}{2}\ln|v+1|$

Entonces: $\int \frac{2-v}{-1+v^2}dv = \int \frac{dx}{x} \rightarrow \frac{1}{2}\ln|v-1| - \frac{3}{2}\ln|v+1| = \ln|x| + c$

Regresando a la variable original:

$$\frac{1}{2}\ln\left|\frac{y}{x} - 1\right| - \frac{3}{2}\ln\left|\frac{y}{x} + 1\right| = \ln|x| + c$$ Sol. gral.

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4) $\frac{dy}{dx} = \frac{x+y}{x-y}$ $\quad \frac{dy}{dx} = \frac{\frac{x}{x} + \frac{y}{x}}{\frac{x}{x} - \frac{y}{x}}$ $\therefore$ Es homogénea

$\frac{dy}{dx} = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$ $\quad v = \frac{y}{x} \rightarrow y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

Sustituyendo:

$v + x\frac{dv}{dx} = \frac{1+v}{1-v} \rightarrow x\frac{dv}{dx} = \frac{1+v}{1-v} - v \rightarrow x\frac{dv}{dx} = \frac{1+\cancel{v}-\cancel{v}+v^2}{1-v}$

$x\frac{dv}{dx} = \frac{v^2+1}{1-v} \rightarrow \int \frac{1-v}{v^2+1}dv = \int \frac{dx}{x}$

Para $\int \frac{v-1}{v^2+1}dv = \int \frac{v\,dv}{v^2+1} - \int \frac{1}{v^2+1}dv$ $\quad u = v^2+1$, $du = 2v\,dv$

$\frac{1}{2}\int \frac{du}{u} - \int \frac{1}{v^2+1}dv = \frac{1}{2}\ln|v^2+1| - \tan^{-1}v$

Regresando:

$\int \frac{1-v}{v^2+1}dv = \int \frac{dx}{x}$

$\frac{1}{2}\ln|v^2+1| - \tan^{-1}v = \ln|x| + c$