Calcular 1. $\int \frac{\sqrt{9-x^2}}{x^2}\,dx$ $\quad x = 3\,\text{sen}\,\theta$

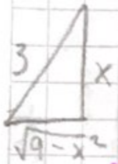
$dx = 3\cos\theta\, d\theta$

$$= \int \frac{\sqrt{9-x^2}}{x^2}\,dx = \int \frac{\sqrt{9-9\,\text{sen}^2\theta}}{9\,\text{sen}^2\theta}\,3\cos\theta\, d\theta$$

$$= 3\int \frac{\sqrt{9(1-\text{sen}^2\theta)}}{9\,\text{sen}^2\theta}\cos\theta\, d\theta = \frac{3}{9}\int \frac{3\sqrt{\cos^2\theta}}{\text{sen}^2\theta}\cos\theta\, d\theta = \int \frac{\cos^2\theta}{\text{sen}^2\theta}\, d\theta$$

$$= \int \cot^2\theta\, d\theta = -$$

Regresando a la variable original:



$\text{Sen}\,\theta = \frac{x}{3} = \frac{Co}{H}$ $\quad \theta = \text{Sen}^{-1}\frac{x}{3}$

$\text{Cot} = \frac{Ca}{Co} = \frac{\sqrt{9-x^2}}{x}$

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = -\text{Sen}^{-1}\frac{x}{3} - \frac{\sqrt{9-x^2}}{x} + C$$

2. $\int \sqrt{x^2+5}\,dx =$ $\quad x = \sqrt{5}\tan\theta \quad dx = \sqrt{5}\sec^2\theta\, d\theta$

usando $\sec^2\theta = \tan^2\theta + 1$

$$= \int\sqrt{(\sqrt{5}\tan\theta)^2+5}\,dx = \int\sqrt{5(\tan^2\theta+1)}\cdot\sqrt{5}\sec^2\theta\, d\theta = \int\sqrt{5}\cdot\sqrt{5}\sqrt{\sec^2\theta}\cdot\sec^2\theta\, d\theta = 5\int\sec^3\theta\, d\theta$$

$$= 5\left(\frac{x}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right) = 5\left(\frac{\sec^2\theta\,\text{sen}\,\theta}{2} + \frac{1}{2}\ln|\tan\theta+\sec\theta|\right)$$

Regresando a la variable original: $\tan\theta = \frac{x}{\sqrt{5}} = \frac{Co}{Ca}$ $\quad \theta = \tan^{-1}\frac{x}{\sqrt{5}}$

$\text{Sec} = \frac{Co}{H} = \frac{x}{\sqrt{5+x^2}}$

$\text{Sen} = \frac{Co}{Ca} = \frac{x}{\sqrt{5}}$

$$\int\sqrt{x^2+5}\,dx = 5\left(\frac{\frac{x}{\sqrt{5+x^2}}\cdot\frac{x}{\sqrt{5}}}{2} + \frac{1}{2}\ln\left|\frac{x}{\sqrt{5+x^2}} + \frac{x}{\sqrt{5}}\right|\right)$$

$$\int\sqrt{x^2+5}\,dx = 5\left(\frac{x}{10}\sqrt{5+x^2} + \frac{1}{2}\ln\left|\frac{x}{\sqrt{5}} + \sqrt{\frac{1}{5}(5+x^2)}\right|\right) + C$$

$3.\ \int \frac{1}{x^3\sqrt{x^2-9}}\,dx =$ $\quad x=3\sec\theta \quad dx = 3\sec\theta\tan\theta\,d\theta$

usando $\sec^2\theta - 1 = \tan^2\theta$

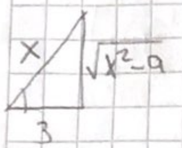
$$= \int \frac{1}{27\sec^3\theta\sqrt{9(\sec^2\theta-1)}}\, 3\sec\theta\tan\theta\,d\theta = \frac{3}{27}\int\frac{1}{\sec^3\theta\, 3\sqrt{\tan^2\theta}}\sec\theta\tan\theta\,d\theta$$

$$=\frac{1}{27}\int\frac{\sec\theta\tan\theta}{\sec^3\theta\tan\theta} = \frac{1}{27}\int\frac{1}{\sec^2\theta}\,d\theta = \frac{1}{27}\int\cos^2\theta\, d\theta = \frac{1}{27}\left(\frac12\left(\theta+\frac12 \operatorname{sen} 2\theta\right)\right) = \frac{1}{54}\left(\theta+\frac12\operatorname{sen}2\theta\right)$$

$\operatorname{sen} 2A = \operatorname{sen} A\cos A + \operatorname{sen} A\cos A$
$= 2\operatorname{sen} A\cos A$

$$= \frac{1}{54}\left(\theta + \frac12\, 2\operatorname{sen}\theta\cos\theta\right)$$

Regresando a la variable original: $\sec\theta = \frac{x}{3} = \frac{Hi}{Ca} \quad \theta = \sec^{-1}\frac{x}{3}$



$\operatorname{sen} = \frac{Co}{Hi} = \frac{\sqrt{x^2-9}}{x}$

$\cos = \frac{Ca}{Hi} = \frac{3}{x}$

$$\int\frac{1}{x^3\sqrt{x^2-9}}\,dx = \frac{1}{54}\left(\sec^{-1}\frac{x}{3}+\frac12\, 2\,\frac{\sqrt{x^2-9}}{x}\cdot\frac{3}{x}\right)$$

$$= \frac{1}{54}\left(\sec^{-1}\frac{x}{3}+\frac{3\sqrt{x^2-9}}{x^2}\right)+C$$

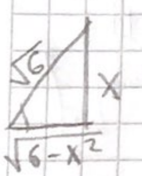
$4.\ \int\frac{1}{(6-x^2)^{3/2}}\,dx = \int\frac{1}{(6-x^2)\sqrt{(6-x^2)}}\,dx =$ $\quad x=\sqrt6\operatorname{sen}\theta \quad dx=\sqrt6\cos\theta\, d\theta$

usando $1-\operatorname{sen}^2\theta=\cos^2\theta$

$$=\int\frac{\sqrt6\cos\theta}{6-(\sqrt6\operatorname{sen}\theta)^2\sqrt{6-(\sqrt6\operatorname{sen}\theta)^2}}\,d\theta = \int\frac{\sqrt6\cos\theta}{6(1-\operatorname{sen}^2\theta)\sqrt{6(1-\operatorname{sen}^2\theta)}}\,d\theta =$$

$$=\frac16\int\frac{\cos\theta\, d\theta}{\cos^2\theta\sqrt{\cos^2\theta}} = \frac16\int\frac{1}{\cos^2}\,d\theta = \frac16\int\sec^2\theta\, d\theta = \frac16(\tan\theta)+C$$

Regresando a la variable original: $\operatorname{sen}\theta = \frac{x}{\sqrt6} = \frac{Co}{H} \quad \theta = \operatorname{sen}^{-1}\frac{x}{\sqrt6}$



$\tan = \frac{Co}{Ca} = \frac{x}{\sqrt{6-x^2}}$

$$\int\frac{1}{\sqrt{(6-x^2)^3}}\,dx = \frac16\left(\frac{x}{\sqrt{6-x^2}}\right) = \frac{x}{6\sqrt{6-x^2}}+C$$