UNIVERSITE DE GENEVE
Faculté des sciences

Section de Mathématiques

Statistical Machine Learning I - 2025-2026
TP 2

Exercise 1

Let $X_1, X_2, \cdots, X_n$ be a sample from the Uniform distribution on $[0, \theta]$. Perform a Monte-Carlo simulation to investiage empirically the asymptotic distribution of $\hat{\alpha}_1 = 2\bar{X}$ and $\hat{\alpha}_2 = \max(X_1, \ldots, X_n)$ considered in class. Show with a qq-plot that

$$\sqrt{n}(\hat{\alpha}_1 - \theta) \xrightarrow{d} \mathcal{N}\left(0, \frac{\theta^2}{3}\right), \quad n(\theta - \hat{\alpha}_2) \xrightarrow{d} \text{Exp}\left(\tfrac{1}{\theta}\right).$$

Exercise 2

For a given vector of data $\mathbf{y} = (y_1, \ldots, y_n)$ and a given level $\alpha \in (0, 1)$, program a function that performs the Student t-test to test

$$H_0 : \mu = 0$$

against

$$\mu \neq 0,$$

where $\mu = \mathbb{E}(Y)$.

Empirically evaluate the power of the test as a function of μ for a given n. Then redo the experiment with a larger n.

Instead of simulating Gaussian observations, simulate data mean mean μ but with heavier tails, e.g., Student with 4 degrees of freedom. Yet employ the same Student t-test to test H_0. What happens to the power curve? Is the level of the test still the desired α?