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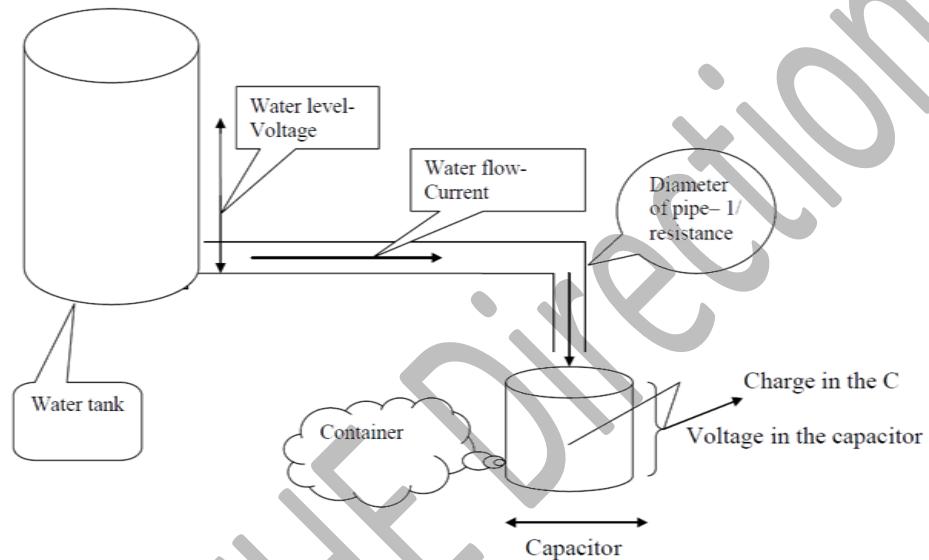
Special Thanks to –

- 1) Introduction to Circuit Analysis - Boylestad
- 2) Fundamental of Electric Circuit - Sadiku
- 3) Multiple Choice Questions - R. K. Kanodia

CHAPTER I

BASIC

Analogy between Electrical and Physical parameters: We are taken the example of water system and relate electrical parameter with them.



Physical parameters	Electrical parameters
Water level	Voltage
Water flow	Current
Diameter of pipe line	Reciprocal of resistance
Tap	Switch
Diameter of container	Capacitor
Water level in the container	Voltage in the capacitor
Quantity of water	Charge
Filling bucket	Charging the capacitor

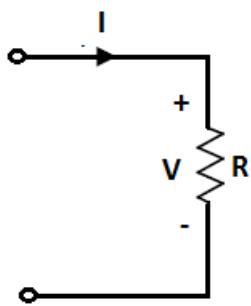
Circuit: The connection of active and passive elements in close loop forms the circuit.

Network: The interconnection of circuits called as network. eg. If one city or town is said to be circuit then state will be the network. As interconnection of cities form a state.

Passive elements: The elements, they are always absorb the power is called as passive elements eg. Resistance, inductor and capacitor (R, L, C).

Resistance: It is ability of element that opposes or resists the flow of electrons (i.e. flow of current) and makes it necessary to apply a voltage to cause current to flow. It is denoted by R and its unit is ohm (Ω).

Current through any material is directly proportional to voltage across it and that proportionality Constant is one by resistance or voltage across any element is directly proportional to current through that element and proportionality Constant is resistance. It is called as ohm's law.



Ohm law:

$$V = RI \dots\dots\dots\dots \text{Ohm's Law}$$

Power dissipation across the resistance is:

$$P_R = VI = I^2 R = \frac{V^2}{R} \quad \text{Watt}$$

Energy absorb by resistance (E_R):

$$E_R = \int_{-\infty}^t P_R dt \quad \text{Joule}$$

Resistance is time invariant, bilateral, Linear and passive element.

Time independent element: If the value of the element is independent of time then that element is called as time independent element.

Bilateral element: The element doesn't having polarity called as bilateral element.

Passive element: The element unable to generate or amplify a power is called as passive element.

Linear element: The element follow linearity property is called as linear element.

Note: In all neutral atoms the number of electrons is equal to the number of protons. For the hydrogen atom, the radius of the smallest orbit followed by the electron is about 5×10^{-11} m.

11 m. The radii of the proton, neutron, and electron are all of the order of magnitude of 2×10^{-15} m.

Electrical resistivity: It is also known as **resistivity** or **volume resistivity**. It is property of material to oppose the flow of electric current. A low resistivity indicates a material that readily allows the movement of electric charge. Resistivity is commonly represented by the Greek letter ρ (rho). The SI unit of electrical resistivity is the **ohm·metre** ($\Omega\text{-m}$). It is given as below

$$\rho = R \frac{A}{\ell}$$

Where,

'R' is the electrical resistance of a uniform specimen of the material (measured in ohms, Ω)

' ℓ ' is the length of the piece of material (measured in meters, m)

'A' is the cross-sectional area of the specimen (measured in square meters, m^2)

Electrical conductivity: It is also called as **specific conductance**. It is the reciprocal of electrical resistivity, and measures a material's ability to conduct an electric current. It is commonly represented by the Greek letter σ (sigma). Its SI unit is **siemens per metre** ($\text{S}\cdot\text{m}^{-1}$)

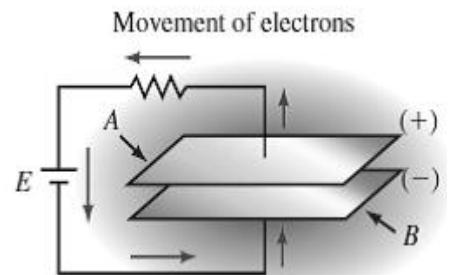
Note: Power dissipated in the resistor is always positive and independent of current direction

Capacitor: It is a circuit component designed to store electrical charge. If you connect a dc voltage source to a capacitor, the capacitor will "charge" to the voltage of the source. If you then disconnect the source, the capacitor will remain charged, i.e., its voltage will remain constant at the value to which it had risen while connected to the source (assuming no leakage). Because of this tendency to hold voltage, **a capacitor opposes sudden changes in voltage**. It is this characteristic that gives capacitors their unique properties.

CAPACITANCE: It is the electrical property of capacitors: it is a measure of how much charge a capacitor can hold.

Working of capacitor:

The plates of the capacitor are metal plates; they contain huge numbers of free electrons. In their normal state, however, they are uncharged, that is, there is no excess or deficiency of electrons on either plate. If a dc source is now connected, electrons are pulled from the top plate by the positive potential of the battery and the same number deposited on the bottom plate. This leaves the top plate with a deficiency of electrons (i.e., positive charge) and the bottom plate with an excess



(i.e., negative charge). In this state, the capacitor is said to be **charged**. If the amount of charge transferred during this process is Q coulombs, we say that the capacitor has a charge of Q .

The capacitor will remain charged, even if source is not present. Because of this, we say that a capacitor can store charge. Practically Capacitors with little leakage can hold their charge for a considerable time.

Charge: The amount of charge Q that a capacitor can store depends on the applied voltage. Experiments show that for a given capacitor, charge (Q) is proportional to voltage. Let the constant of proportionality be C . Then,

$$Q = CV \text{ Coulomb}$$

C : Capacitance parameter of conductance.

Current to the capacitance:

$$i(t) = \frac{dQ}{dt} = \frac{d(CV(t))}{dt} = \frac{CV'(t)}{dt} \text{ Ampere}$$

Voltage across capacitance:

$$v(t) = \frac{1}{C} \int_{-\infty}^t i(t) dt \text{ Volt}$$

Power absorb by the capacitor is $P(t)$ and its unit is watt:

$$P(t) = i(t) v(t) \text{ Watt}$$

Total energy across capacitor is $E_{total}(t)$:

$$E_{total}(t) = \int_{-\infty}^t P(t) dt = \int_{-\infty}^t i(t) v(t) dt \text{ joule}$$

Instantaneous energy across capacitor is $E_{instantaneous}(t)$:

$$E_{instantaneous}(t) = \frac{CV^2}{2} \text{ J}$$

Total energy absorb by capacitor upto any time 't' = Instantaneous energy across capacitor from time $t=-\infty$ to time 't'

Range: Most practical capacitors range in size from pico farads (pF or $10^{-12} F$) to microfarads (μF or $10^{-6} F$).

The direction of the field is defined as the direction of force on a positive charge. Field lines never cross, and the density of the lines indicates the strength of the field; i.e., the more dense the lines, the stronger the field.

The strength of an electric field, also called its **electric field intensity**, is the force per unit charge that the field exerts on a small, positive test charge.

INDUCTOR: It is property of material to resist the change in flow of current through that material.

When current is flow through the conductor (coil) the magnetic flux will produce across the conductor (coil) and which is directly proportional to the current flow the conductor and these proportionality constant is inductor.

$$\text{i.e. } \Psi \propto I_L \text{ (also } \Psi = n\phi)$$

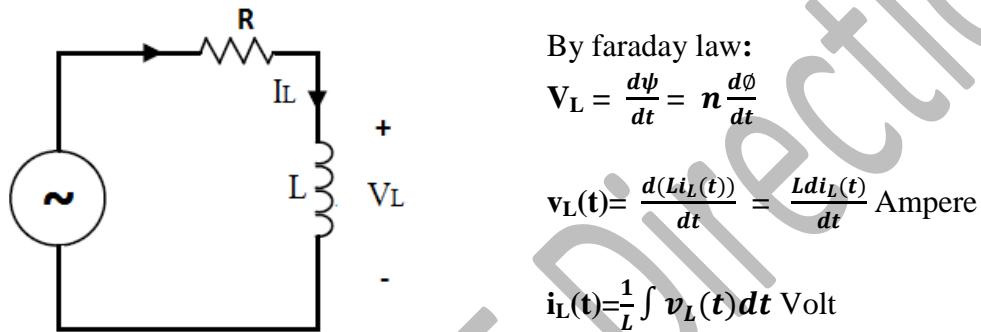
$$\Psi = LI_L$$

Where, Ψ : Total magnetic flux generate across the inductor (L)

I_L : Current flowing through the inductor (L)

Φ : Magnetic flux generated by one turn of inductor (coil)

n : Total number of turn present in inductor



Power absorb by the inductor is $P_L(t)$ and its unit is watt:

$$P_L(t) = i_L(t) v_L(t) \text{ Watt}$$

Total energy absorb by inductor is $E_{total}(t)$:

$$E_{total}(t) = \int_{-\infty}^t P_L(t) dt = \int_{-\infty}^t i_L(t) v_L(t) dt \text{ Joule}$$

Instantaneous energy across inductor is $E_{instantaneous}(t)$:

$$E_{instantaneous}(t) = \frac{Li_L^2}{2} \text{ Joule}$$

Total energy absorb by inductor upto any time 't' = Instantaneous energy across inductor from time $t=-\infty$ to time 't'

Sources:

Voltage:

This voltage is due entirely to the separation of positive and negative charges. When charges are detached from one body and transferred to another, a potential difference or voltage results between them. A familiar example is the voltage that develops when you walk across a carpet.

A constant voltage is called a **DC** voltage. And a voltage that varies sinusoidally with time is called an **AC voltage**.

Note:

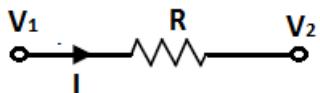
- Power **supplied** by voltage source is positive if current flows from negative to positive within the terminal.
- Power **absorbed** by voltage source is positive if current flows from positive to negative within the terminal.

Current:

Electric current results from the movement of electric charge. The **SI** unit of current (I) is the **ampere** with unit symbol **A**. If a steady flow of 1 C of charge passes a given point in a conductor in 1 s, the resulting current is **1 A**.

$$\text{Current (I)} = \frac{\text{charge (Q)}}{\text{time (sec)}}$$

Current has an associated direction. By convention the direction of current flow is in the direction of positive charge movement and opposite the direction of negative charge movement. The current is flow from higher potential to lower potential.



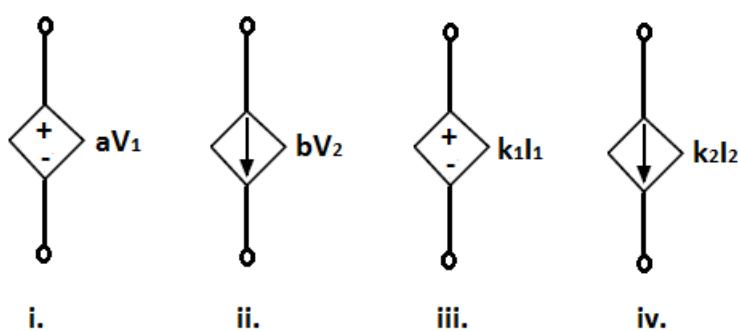
$$I = \frac{V_1 - V_2}{R}$$

Dependant sources:

An independent current source provides a certain current, and an independent voltage source provides a certain voltage, both independently of any other voltage or current. In contrast, a dependent source (also called a controlled source) provides a voltage or current that depends on a voltage or current elsewhere in a circuit.

There are four types of dependent sources:

- i. a voltage-controlled voltage source
- ii. a voltage-controlled current source
- iii. a current-controlled voltage source
- iv. a current-controlled current source



Dependent sources are rarely separate physical components. But they are important because they occur in models of electronic components such as operational amplifiers and transistors.

Power (P): is the rate of doing work or the energy flow rate. When a charge of dq coulombs is moved from point A to point B with a potential difference of v volts, the energy supplied to the charge will be $v dq$ joule [J]. If this movement takes place in dt seconds, the power supplied to the charge will be $v dq/dt$ watts [W]. Because dq/dt is the charge flow rate i.e. current i , the power supplied to the charge can be written as:

$$P = vi = v^2/R = i^2R \text{ Watt}$$

Energy (E): It is integration of power from $-\infty$ to present time t

$$E = \int_{-\infty}^t P(t) dt \quad \text{Joule}$$

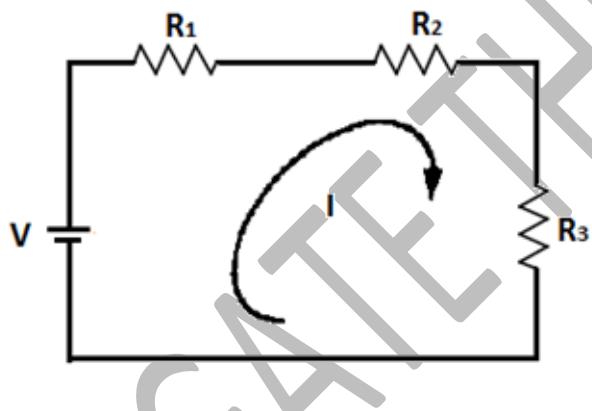
SERIES CIRCUITS:

In circuit if all elements are connected in series then circuit is called as series circuit.

In the circuit two elements are in series if-

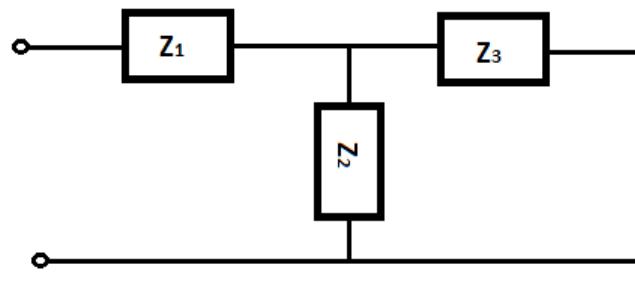
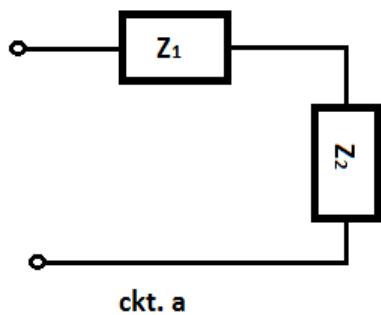
1. They have only one terminal in common (i.e., one lead of one is connected to only one lead of the other).
2. The common point between the two elements is not connected to another current-carrying element.

i.e The current is the same through series elements.



In this circuit R_1 , R_2 , R_3 are connected in series because same current is flowing to this element also the voltage source is also in series with R_1 , R_2 , R_3 i.e. all the element in the circuit are in series and so the this circuit is a series circuit.

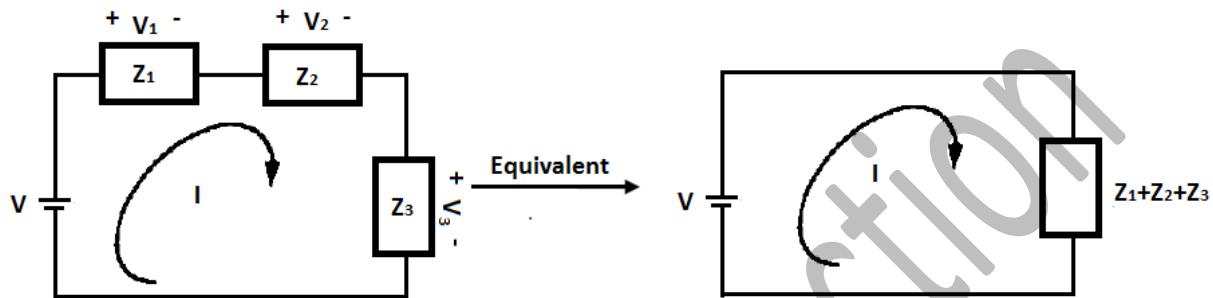
Exp:



- a. In **circuit a** Z_1 and Z_2 are in series (as current in Z_1 and Z_2 is same).
 b. In **circuit b** Z_1 is not in series with Z_2 (as current in Z_1 and Z_2 is not same because of presence of another current carrying Z_3 at common point between Z_1 and Z_2)

Impedance in series:

In the circuit given below $-V + IZ_1 + IZ_2 + IZ_3 = 0$ i.e. $-V + I(Z_1 + Z_2 + Z_3) = 0$. So in the above circuit we can replace the Z_1, Z_2 and Z_3 by $Z_1 + Z_2 + Z_3$ i.e.



$$Z_{eq} = Z_1 + Z_2 + Z_3$$

The total impedance of a series circuit is the sum of individual impedance i.e. if N impedance connected in series then equivalent impedance will be:

$$Z_{eq} = Z_1 + Z_2 + Z_3 + \dots + Z_N$$

For Resistance: In series connection

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_N \quad (\text{Science here } Z=R)$$

For Inductance: In series connection

$$L_{eq} = L_1 + L_2 + L_3 + \dots + L_N \quad (\text{Science here } Z=j\omega L)$$

For Capacitor: In series connection

$$1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3 + \dots + 1/C_N \quad (\text{Science here } Z=1/(j\omega C))$$

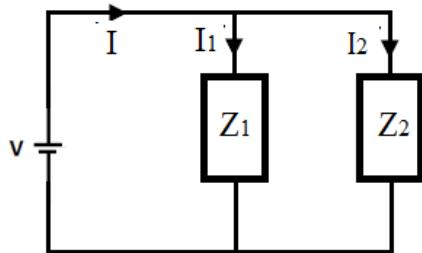
PARALLEL CIRCUITS:

In circuit if all elements are connected in parallel then circuit is called as parallel circuit.

In the circuit two elements are in parallel if

1. They have both the terminal in common.
 i.e The Voltage is the same across parallel elements.

Impedance in parallel connection:



$$\begin{aligned}I &= I_1 + I_2 \\V/Z_{eq} &= V/Z_1 + V/Z_2 \\1/Z_{eq} &= 1/Z_1 + 1/Z_2\end{aligned}$$

In general:

$$1/Z_{eq} = 1/Z_1 + 1/Z_2 + 1/Z_3 + \dots + 1/Z_N$$

For Resistance: In parallel connection

$$1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3 + \dots + 1/R_N$$

(Science here $Z=R$)

For Inductance: In parallel connection

$$1/L_{eq} = 1/L_1 + 1/L_2 + 1/L_3 + \dots + 1/L_N$$

(Science here $Z=jwL$)

For Capacitor: In parallel connection

$$C_{eq} = C_1 + C_2 + C_3 + \dots + C_N$$

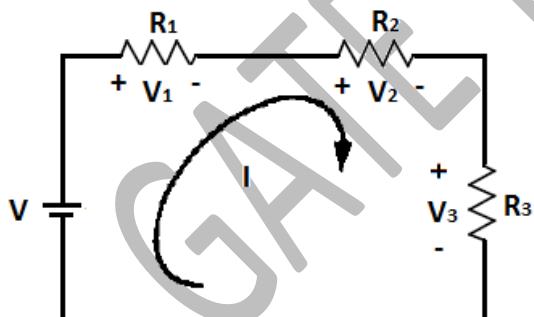
(Science here $Z=1/(jwC)$)

KVL: The summation of voltage rises and voltage drops around a closed loop is equal to zero.

$$\sum V = 0 \text{ For any close loop}$$

KVL is work on the principle the conservation of energy in every loop of lump electric circuit.

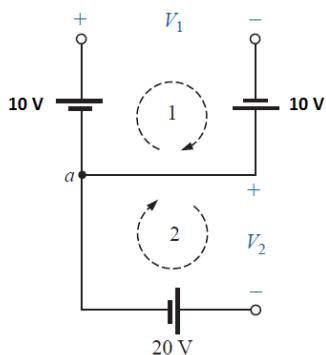
By solving equation we will get,



$$-V + V_1 + V_2 + V_3 = 0 \text{ i.e. } -V + IR_1 + IR_2 + IR_3 = 0$$

Where: IR_1 , IR_2 and IR_3 are the voltage drop across R_1 , R_2 and R_3 respectively (By ohm law)

Example1. In the following circuit find the V_1 and V_2



Apply 1st loop (by KVL):

$$-10 - 10 + V_1 = 0$$

$$V_1 = 20V$$

Apply 2nd loop (by KVL): $20 + V_2 = 0$

$$V_2 = -20V$$

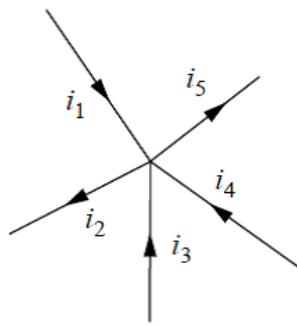
KCL: Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.

A node is a connection point between two or more branches. On a circuit diagram a node is sometimes indicated by a dot that may be a solder point in the actual circuit. The node also includes all wires connected to the point. In other words, it includes all points at the same potential. If a short circuit connects two nodes, these two nodes are equivalent; in fact they are just a single node, even if two dots are shown.

$$\sum_{n=1}^N i_n = 0$$

KCL is work on the principle the conservation of charge at any node of lump electric circuit

Example 1.



By KCL:
 $-i_1 + i_2 - i_3 - i_4 + i_5 = 0$
 $i_1 + i_2 + i_3 = i_2 + i_5$

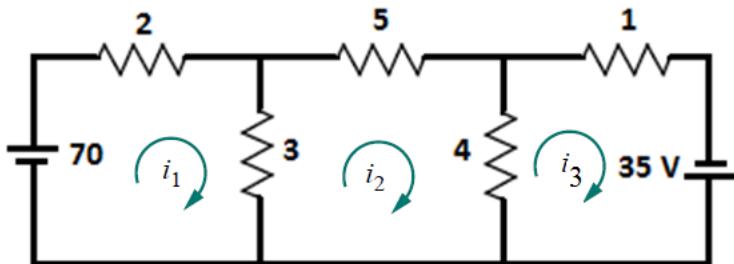
Mesh analysis:

STEPS:

1. Arbitrarily assign a clockwise current to each interior independent closed loop in the network. (Although the assigned current may be in any direction, a clockwise direction is used to make later work simpler).
2. Using the assigned loop currents, indicate the voltage polarities across all resistors in the circuit. For a resistor which is common to two loops, the polarities of the voltage drop due to each loop current should be indicated on the appropriate side of the component.
3. Applying Kirchhoff's voltage law, write the loop equations for each loop in the network. Do not forget that resistors which are common to two loops will have two voltage drops, one due to each loop.
4. Solve the resultant simultaneous linear equations.
5. Branch currents are determined by algebraically combining the loop currents which are common to the branch.

Point to remember

- To find voltage at any node, start at same node and go towards ground in a shortest path preferably following **KVL** sign conventions.
- To find V_{AB} , start at A and go towards B following KVL sign conventions.
- V_{AB} : Voltage at A with respect to B = $V_A - V_B$.
- V_A : Voltage at A with respect ground by default.

Example 1.

$$\text{For } 1^{\text{st}} \text{ loop: } 5i_1 - 3i_2 = 70$$

$$\text{For } 2^{\text{nd}} \text{ loop: } -3i_1 + 12i_2 - 4i_3 = 0$$

$$\text{For } 3^{\text{rd}} \text{ loop: } -4i_2 + 5i_3 = 35$$

By solving above equations:

$$i_1 = 20\text{A}$$

$$i_2 = 10\text{A}$$

$$i_3 = 15\text{A}$$

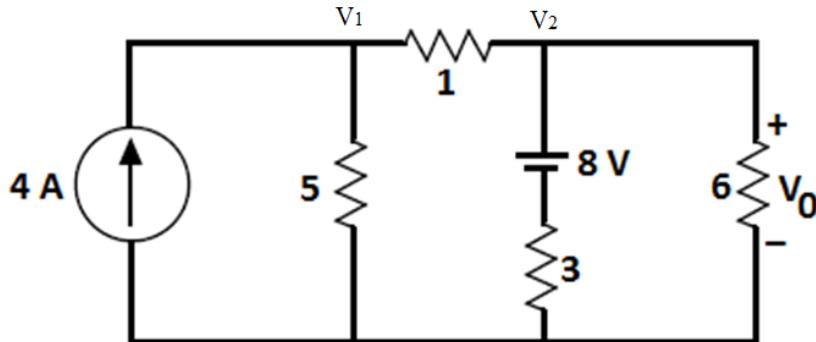
Nodal analysis:

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations.

Given a circuit with n nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps.

1. Select a node as the reference node. Assign voltages v_1, v_2, \dots, v_{n-1} to the remaining $n - 1$ nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the $n - 1$ non reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
3. Solve the resulting simultaneous equations to obtain the unknown node voltages.

Example 1.



Nodal analysis at node 1(V₁)

$$V_1 \left[\frac{1}{5} + 1 \right] - \frac{V_2}{1} = 4$$

Nodal analysis at node 2(V₂)

$$-\frac{V_1}{1} + V_2 \left[\frac{1}{1} + \frac{1}{3} + \frac{1}{6} \right] - \frac{8}{3} = 0$$

By solving the above equations

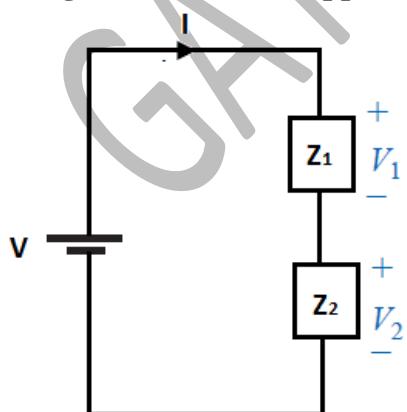
$$V_1 = \frac{65}{6} \text{ V and } V_2 = 9 \text{ V}$$

$$V_0 = V_2 = 9 \text{ V}$$

VOLTAGE DIVIDER RULE:

In series circuit the voltage across the impedance elements will divide as the magnitude of the impedance levels.

Voltage division rule is applicable to only series circuit.



Apply KVL:

$$V = I(Z_1 + Z_2) ; I = \frac{V}{Z_1 + Z_2}$$

By ohm law:

$$V_1 = IZ_1 = \frac{VZ_1}{Z_1 + Z_2}$$

Similarly,

$$V_2 = IZ_2 = \frac{VZ_2}{Z_1 + Z_2}$$

I. For resistance:

In above circuit $Z_1 = R_1$ and $Z_2 = R_2$

$$V_1 = \frac{VR_1}{R_1+R_2}$$

Similarly,

$$V_2 = \frac{VR_2}{R_1+R_2}$$

II. For inductance:

In above circuit $Z_1=jwL_1$ and $Z_2=jwL_2$

$$V_1 = \frac{VL_1}{L_1+L_2}$$

Similarly,

$$V_2 = \frac{VL_2}{L_1+L_2}$$

III. For capacitor:

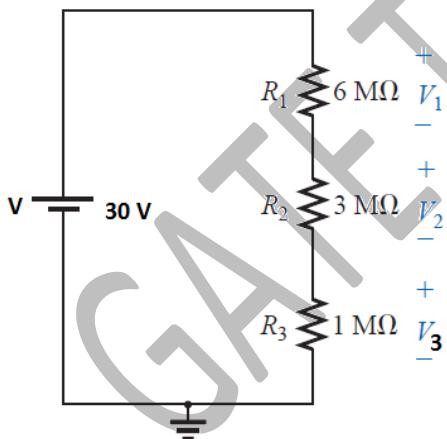
In above circuit $Z_1=1/jwC_1$ and $Z_2=1/jwC_2$

$$V_1 = \frac{VC_2}{C_1+C_2}$$

Similarly,

$$V_2 = \frac{VC_1}{C_1+C_2}$$

Example 1. Find V_1 , V_2 and V_3



$$V_1 = \frac{VR_1}{R_1+R_2+R_3} = 18 \text{ V}$$

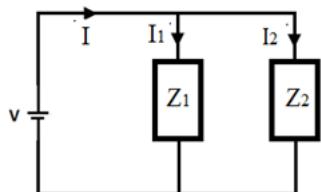
$$V_2 = \frac{VR_2}{R_1+R_2+R_3} = 9 \text{ V}$$

$$V_3 = \frac{VR_3}{R_1+R_2+R_3} = 3 \text{ V}$$

CURRENT DIVIDER RULE:

In parallel circuit **current will divide**

1. For two parallel elements of equal value, the current will divide equally.
2. For parallel elements with different values, the smaller the resistance, the greater the share of input current.
3. For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values



$$I_1 = \frac{IZ_2}{Z_1 + Z_2} \text{ and } I_2 = \frac{IZ_1}{Z_1 + Z_2}$$

For resistor:

$$I_1 = \frac{IR_2}{R_1 + R_2} \text{ and } I_2 = \frac{IR_1}{R_1 + R_2}$$

For inductor:

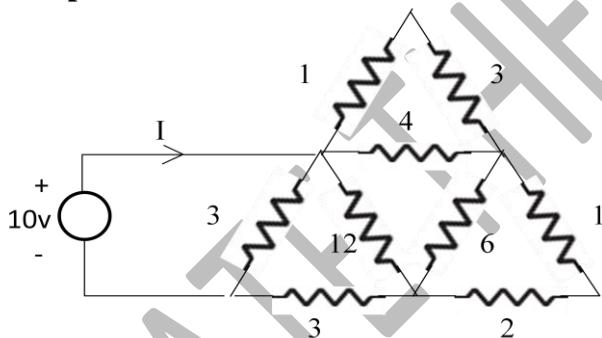
$$I_1 = \frac{IL_2}{L_1 + L_2} \text{ and } I_2 = \frac{IL_1}{L_1 + L_2}$$

For Capacitor:

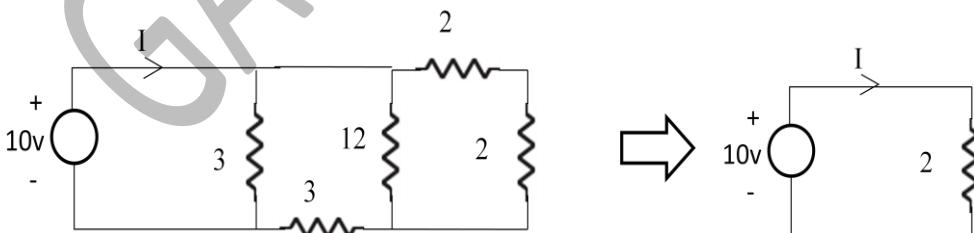
$$I_1 = \frac{IC_1}{C_1 + C_2} \text{ and } I_2 = \frac{IC_2}{C_1 + C_2}$$

Some solved example:

Example 1: Find I

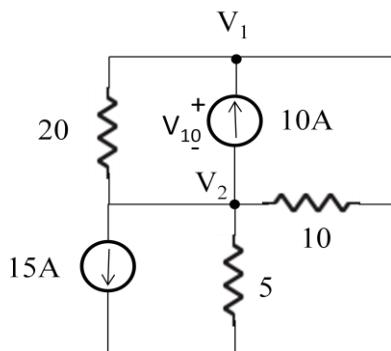


Solution: Reduce it as-



$$I = 10/2 = 5A$$

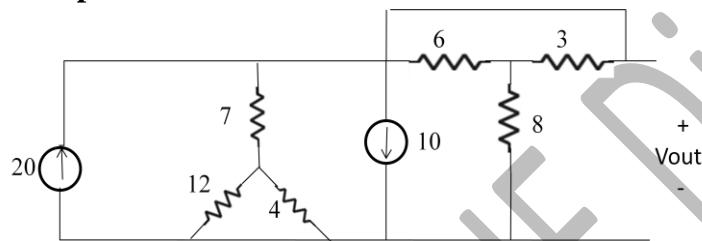
Example 2: find voltage across 10A source.

**Solution:**

$$V_1 \left[\frac{1}{20} + \frac{1}{10} + \frac{1}{5} \right] - 10 - 15 = 0$$

$$V_1 = 71.42$$

$$V_{10} = V_1 - V_2 = 71.42 - 0 = 71.42 \text{ V}$$

Example 3: Find V_{out} 

$$\text{Ans: } V_{out} = 50 \text{ V}$$

Problems on Energy calculation across capacitor and inductor:**Important formulae:**

$$\text{Energy store (in capacitor):- } E = \frac{1}{2} CV^2$$

$$\text{Energy store (in inductor):- } E = \frac{1}{2} LI^2$$

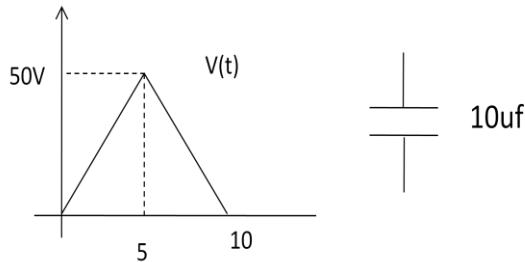
Energy store: Capacitor (C) and Inductor (L)**Energy dissipate:** Resistant (R)

$$\text{Energy Absorb} = \text{Energy store} + \text{Energy dissipate}$$

Practical capacitor and inductor absorb the energy. In which ideal capacitor and inductor store the energy and internal resistor dissipate the energy.

Example 1: The voltage across the capacitor is given below, Find out the energy store by ideal capacitor upto

- i) 5 sec ii) 6 sec iii) 10 sec

**Solution:**

$$\begin{aligned} V(t) &= 10t && \text{For } t = 0 \text{ to } 5 \text{ sec,} \\ V(t) &= 100 - 10t && \text{For } t = 5 \text{ to } 10 \text{ sec,} \end{aligned}$$

i) Energy upto 1st 5 sec:**By direct formula:**

$$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 10 \times 10^{-6} \times 50^2$$

$$E|_{t=5} = \frac{1}{80} J$$

By integration of power:

$$E = \int_0^5 CV \frac{dv}{dt} dt = \int_0^5 10 \times 10^{-6} \times 10t \times 10 = 10^{-3} \left[\frac{t^2}{2} \right]_0^5 = \frac{25}{2} \times 10^{-3}$$

$$E|_{t=5} = \frac{1}{80} J$$

ii) upto $t=6$ sec

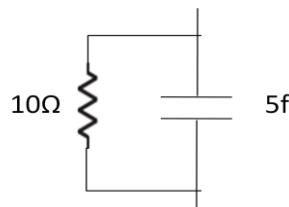
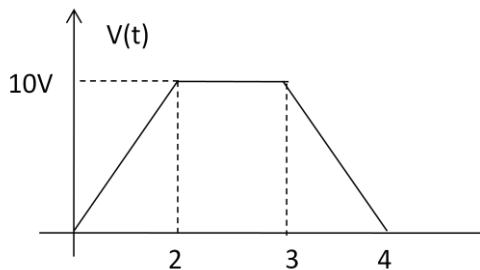
$$\begin{aligned} E|_{t=6} &= C \left[\int_0^5 100t + \int_5^6 100(t-10) \right] dt \\ &= \frac{1}{125} J \end{aligned}$$

iii) upto $t=10$ sec

$$\begin{aligned} E|_{t=10} &= \int_0^5 10 \times 10^{-6} \times 10t \times 10 + \int_5^{10} CV \frac{dv}{dt} dt \\ &= C \left[\int_0^5 100t + \int_5^{10} 100(t-10) \right] dt \\ &= 0 J \end{aligned}$$

Example 2: The voltage across the capacitor is given below, Find out the

- Energy stored by capacitor upto $t=4$ sec
- Energy dissipated by internal resistance across practical capacitor upto $t=4$ sec
- Total energy absorbed by practical capacitor upto $t=4$ sec

**Solution:**

$$v(t) = 5t \quad 0 \leq t \leq 2$$

$$v(t) = 10 \quad 2 \leq t \leq 3$$

$$v(t) = -10(t-4) \quad 3 \leq t \leq 4$$

→ Energy stored across capacitor upto $t=4$ sec

$$E|_{t=4} = \int_0^2 C(5t) \times 5 + \int_2^3 0 + \int_3^4 C[-10(t-4)](-10)dt = 0 \text{ J}$$

Or Energy stored across capacitor upto $t=4$ sec

$$E|_{t=4} = \frac{1}{2}CV(4)^2 = 0 \text{ J} \quad \{ \text{since } V(4) = 0 \text{ V} \}$$

→ Energy Dissipation in Resistance upto $t=4$ sec

$$\begin{aligned} E_R| &= E_R|_{t=2} + E_R|_{t=3} + E_R|_{t=4} \\ &= \int_0^2 \frac{(5t)^2}{10} dt + \int_2^3 \frac{(10)^2}{10} dt + \int_3^4 \frac{(-10(t-4))^2}{10} dt \\ &= \frac{25}{10} \left[\frac{t^3}{3} \right]_0^2 + 10t|_2^3 + \frac{100}{10} \left[\frac{t^3}{3} - 8 \frac{t^2}{2} + 16t \right]|_3^4 \\ &= 20 \text{ J} \end{aligned}$$

Total energy absorb by practical capacitor = Energy store by capacitor + Energy dissipated by resistor

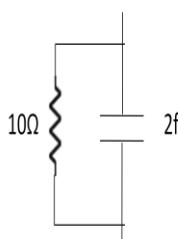
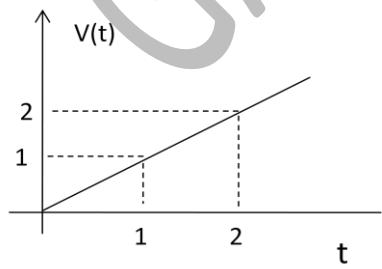
Total energy absorb by practical capacitor (upto $t=4$ sec) = $0+20 = 20 \text{ J}$

Example 3: The voltage across the capacitor is given below, Find out the

i. Energy store by capacitor upto $t=2$ sec

ii. Energy dissipated by internal resistance across practical capacitor upto $t=2$ sec

iii. Total energy absorb by practical capacitor upto $t=2$ sec



$$V(t) = t \quad \text{for } t \geq 0$$

$$E_R = \int (v^2(t)/R) dt$$

Energy dissipated in resistor upto any time 't'

$$E_R = \int_0^t \frac{t^2}{10} dt = \frac{t^3}{30} \Big|_0^t$$

At t=2,

$$E_R = \frac{t^3}{30} \Big|_0^2 = \frac{8}{30} J$$

Total energy store by practical capacitor upto any time 't'

$$E_C = 1/2 CV^2 = \frac{Ct^2}{2}$$

At t=2,

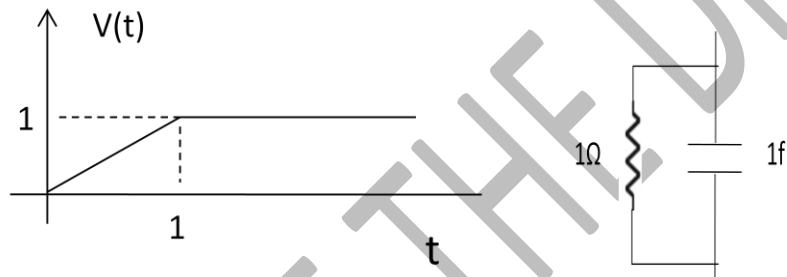
$$E_{c(store)} = \frac{Ct^2}{2} = 4J$$

Total energy absorb by practical capacitor upto t=2 sec

$$E_{absorb} = \frac{8}{30} + 4 = \frac{128}{30} J$$

Example 4: The voltage across the capacitor is given below, Find out the

- Energy store by capacitor upto t=1 sec
- Energy dissipated by internal resistance across practical capacitor upto t=1 sec
- Total energy absorb by practical capacitor upto t=1 sec



Energy dissipated in resistor upto t=1 sec

At t=1,

$$E_R = \int_0^1 t^2 dt = \frac{t^3}{3} \Big|_0^1 = 1/3$$

Total energy store by practical capacitor upto t=1 sec

$$E_C = 1/2 CV^2 = \frac{Ct^2}{2}$$

At t=1,

$$E_{c(store)} = \frac{Ct^2}{2} = 0.5 J$$

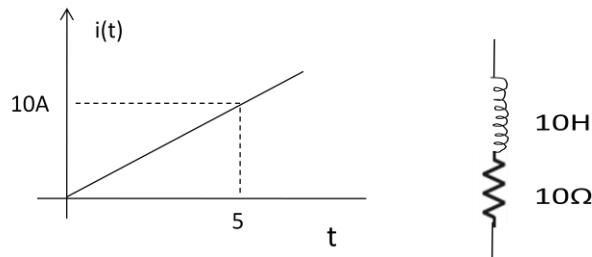
Total energy absorb by practical capacitor upto t=1 sec

$$E_{absorb} = \frac{1}{3} + 0.5 = \frac{5}{6} J$$

Inductor:

→ Energy store in inductor

Example 1: Find energy absorb upto time t?



$$i(t) = 2t$$

Energy dissipated in resistor upto any time t

$$E_R = \int_0^t i^2(t)R dt$$

$$E_R = \frac{40t^3}{3} \Big|_0^t$$

Total energy store by practical inductor upto any time t

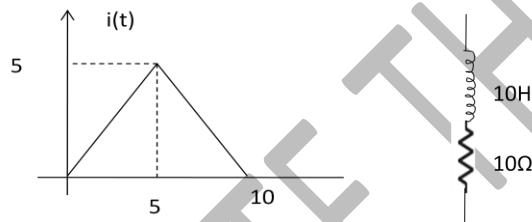
$$E_L = \frac{1}{2} L i^2(t)$$

$$= 40 \frac{t^2}{2} \Big|_0^t$$

Total energy absorb by practical inductor upto any time t

$$E_{\text{absorb}} = E_R + E_L = \frac{40t^3}{3} \Big|_0^t + 40 \frac{t^2}{2} \Big|_0^t$$

Example 2: Find energy absorb upto $t=10$ sec



Solution:

$$i(t) = t \quad 0 < t < 5$$

$$i(t) = 10 - t \quad 5 < t < 10$$

Energy store upto $t=10$ sec

$$E_L = \frac{1}{2} L i^2(t)$$

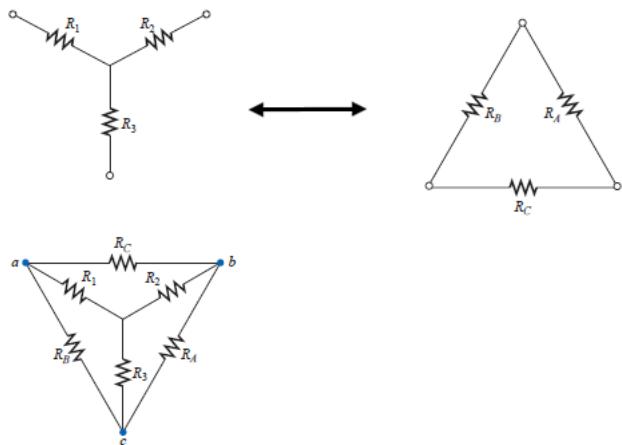
$$E_{L|t=10} = 0 \text{ J}$$

$$E_R = \int_0^5 t^2 R dt + \int_5^{10} (t-10)^2 R dt \\ = 2500/3 \text{ J}$$

Total energy absorb

$$E_{\text{absorb}}|_{t=10} = E_{L|t=10} + E_R|_{t=10} = 0 + 2500/3$$

$$E_{\text{absorb}}|_{t=10} = 2500/3 \text{ J}$$

Delta to star and star to delta conversion:**Star to delta conversion:**

$$R_A = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_B = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_C = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Delta to star conversion:

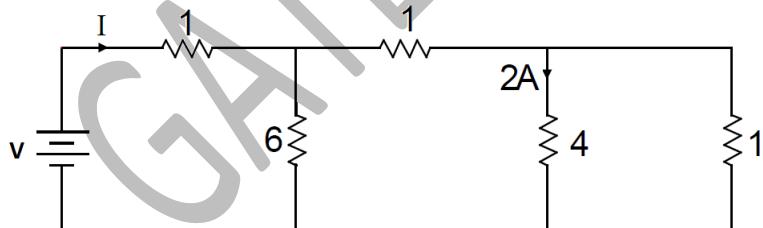
$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Problems:

1. Find I and V respectively



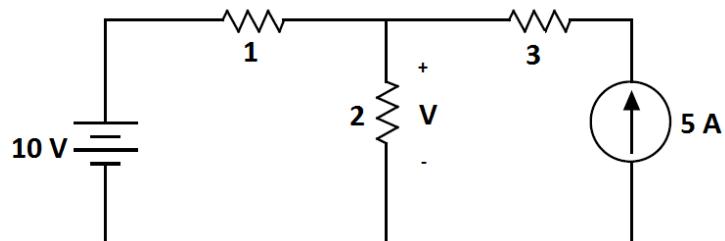
A) 13 A, 91 V

B) 13 A, 31 V

C) 10 A, 10 V

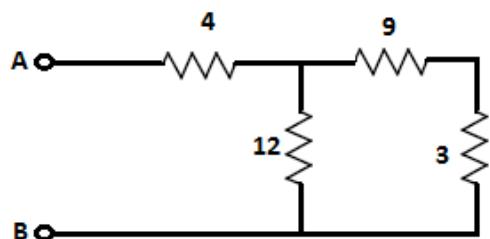
D) Can't determine

2. Find V



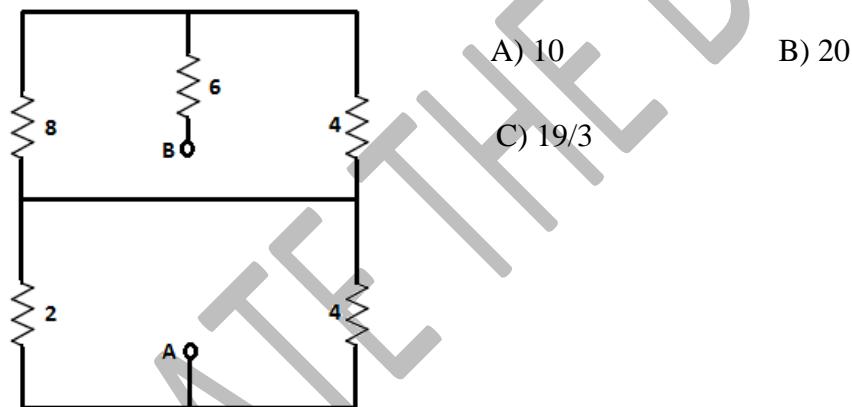
- A) 5 V B) 10 V C) 15 V D) 20 V

3. Find R_{AB} (in ohm)



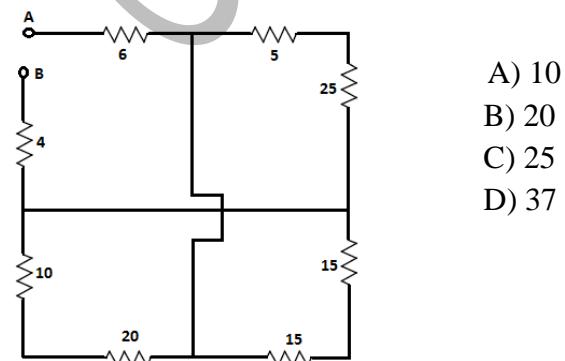
- A) 10 B) 4 C) 16 D) 12

4. Find the R_{AB} (in ohm)



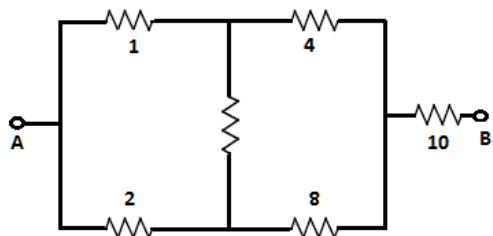
- A) 10 B) 20 C) $19/3$ D) none

5. Find R_{AB} (in ohm)



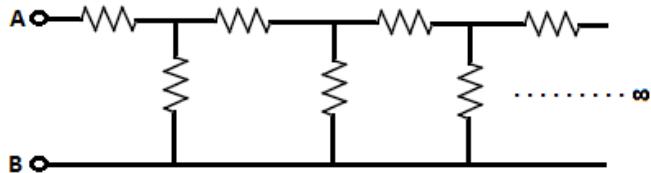
- A) 10 B) 20 C) 25 D) 37

6. Find R_{AB} (in ohm)



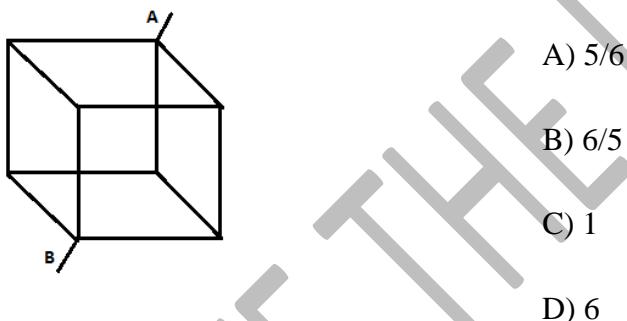
- A) 40
- B) $40/3$
- C) $20/3$
- D) Data insufficient

7. Find R_{AB} (in ohm) if all the resistance has value of 1 ohm



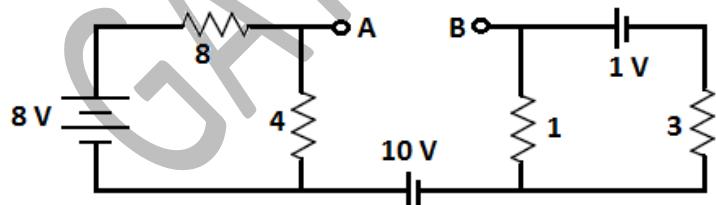
- A) 1
- B) 1.6
- C) 1.9
- D) 1.5

8. Find R_{AB} (in ohm) if all the branch one resistance with value of 1 ohm.



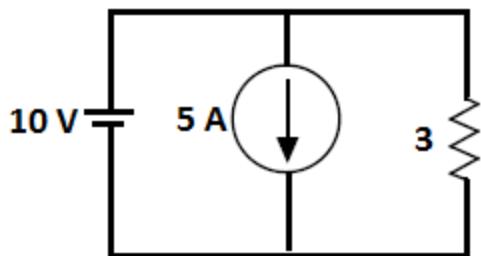
- A) $5/6$
- B) $6/5$
- C) 1
- D) 6

9. Find V_{AB}



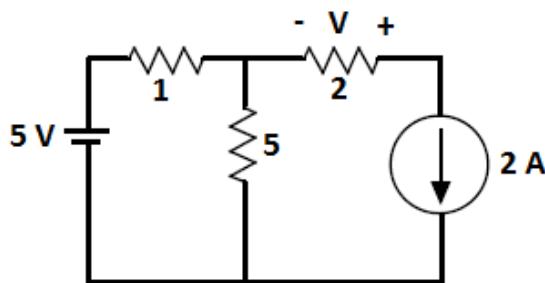
- A) 10 V
- B) 13 V
- C) 13.3 V
- D) 12.41 V

10. Current through the voltage source and also find out the power deliver by voltage source respectively



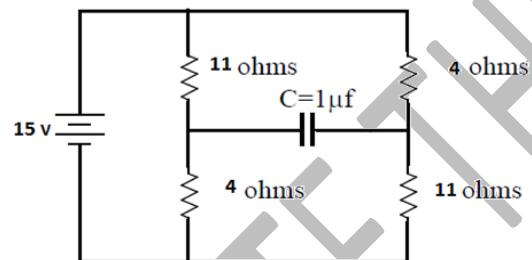
- A) 5 A, 50 W B) 10/3 A, 100/3 W C) 10 A, 100 W D) 25/3 A, 250/3 W

11. Find V



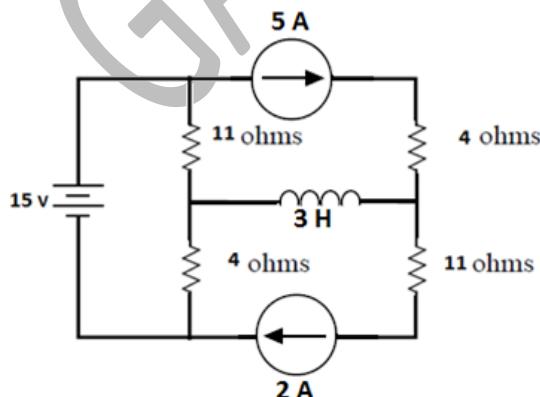
- A) -4 V B) 4 V C) -2 V D) 10/7 V

12. Find the energy absorb across capacitor is:



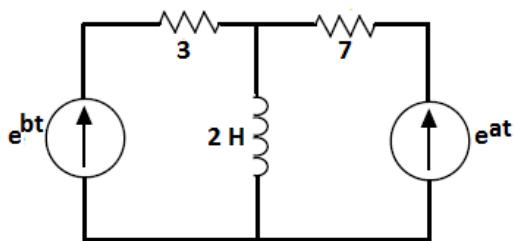
- A) 24.5 μJ B) 25 μJ C) 49 μJ D) None

13. Find the energy absorb across inductor is:



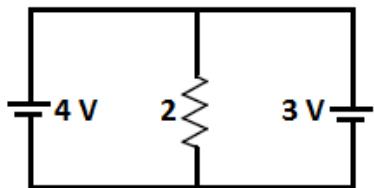
- A) 27 J B) 13.5 J C) 73.5 J D) None

14. Find the voltage across 2 H inductor



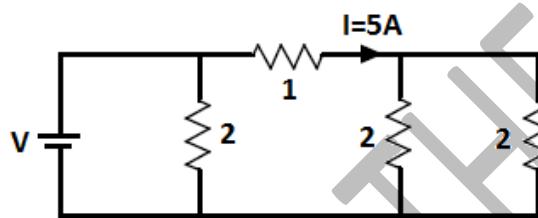
- A) $(be^{bt}+ae^{at})$ V B) $2(be^{bt}+ae^{at})$ V C) $(3be^{bt}+7ae^{at})$ V D) $(3e^{bt}+7e^{at})$ V

15. Find the voltage across 2 ohm resistors



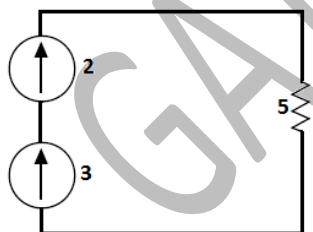
- A) 4 V B) 3 V C) Violation of KCL D) Violation of KVL

16. Find V



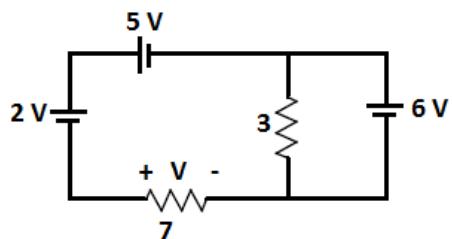
- A) 10 V B) 13 V C) 5 V D) 15 V

17. Find current through 5 ohm



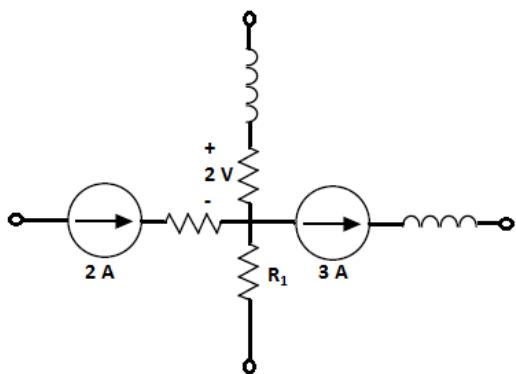
- A) Violation of KVL B) Violation of KCL C) 5A D) Both A and B

18. Find the voltage V



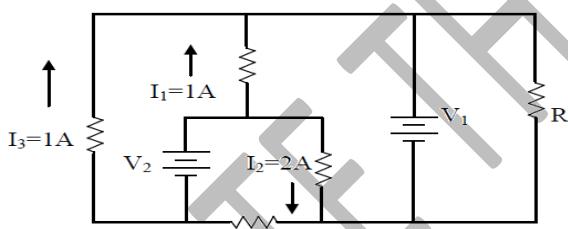
- A) 13 V B) 7 V C) 9 V D) 5 V

19. Find the voltage across R_1 . As all resistances are 1 ohm and all inductance is 1 H.



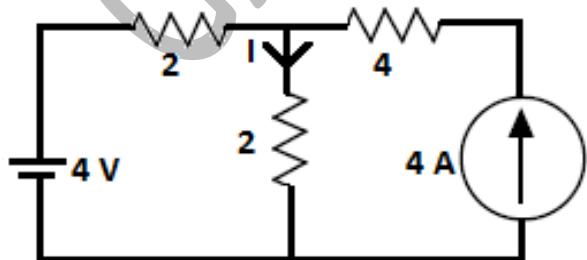
- A) 1 V B) 2 V C) 3 V D) 4 V

20. Find I_4



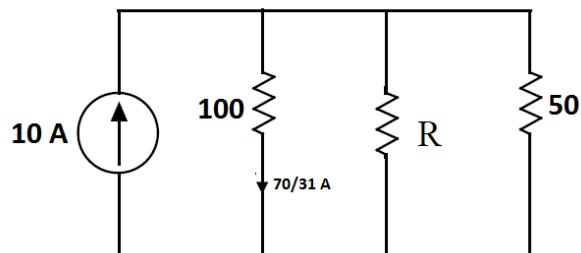
- A) 2 A B) -4 A C) 4 A D) 9/2 A

21. Find I



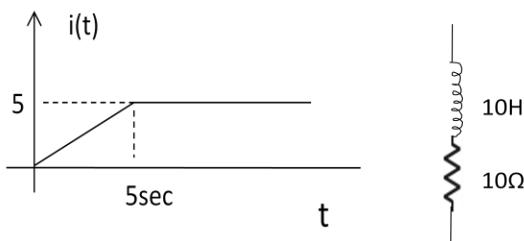
- A) 3 A B) -3 A C) 2 A D) 4 A

22. Find R (in ohm)



- A) 60 B) 70 C) 75 D) 80

23. Find energy Stored by inductor upto $t=\infty$?



- A) ∞ B) 0 C) 125 J D) 250 J

24. The nodal method of circuit analysis is based on

- A) KVL & OHM's law B) KCL & OHM's law
C) KCL, KVL & OHM's law D)

25. The mesh method of circuit analysis is based on

- A) KVL & OHM's law B) KCL & OHM's law
C) KCL & KVL D) KCL, KVL & OHM's law

26. A delta connection contains three impedances of 90Ω each. The impedances of equivalent star connection will be

- A) 270Ω each B) 90Ω each C) 30Ω each D) 180Ω each

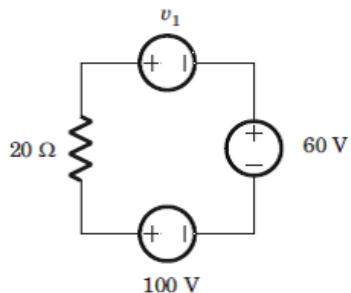
Answers:

1. B	2. B	3. A	4. A	5. B	6. B	7. B	8. A	9. D
10. D	11. A	12. A	13. B	14. B	15. D	16. A	17. B	18. C
19. A	20. B	21. B	22. B	23. C	24. B	25. A	26. C	

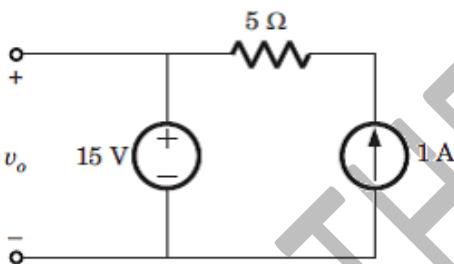
Additional Problems on Basic Concept

1. A lightning bolt carrying 15,000 A lasts for 100 μ s. If the lightning strikes an airplane flying at 2 km, the charge deposited on the plane is

- A) $13.33 \mu\text{C}$ B) 75 C C) $1500 \mu\text{C}$ D) 1.5 C
2. If 120 C of charge passes through an electric conductor in 60 sec , the current in the conductor is
 A) 0.5 A B) 2 A C) 3.33 mA D) 0.3 mA
3. The energy required to move 100 coulomb through 4 V is
 A) 250 mJ B) 400 J C) 25 J D) 400 mJ
4. In the circuit, a charge of 600 C is delivered to the 100 V source in a 1 minute . The value of V_1 must be

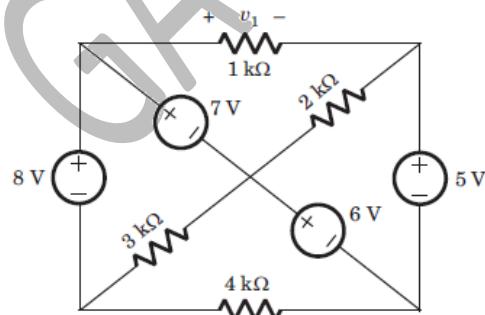


- A) 240 V B) 120 V C) 60 V D) 30 V
5. For the circuit, the value of voltage V_0 is

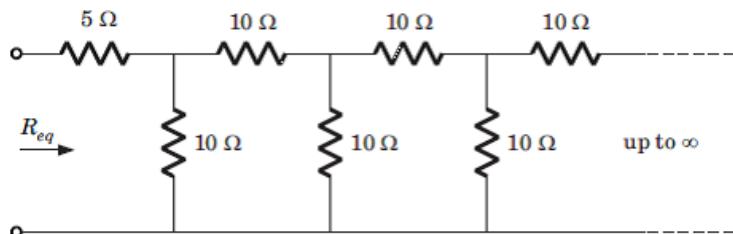


- A) 10 V B) 15 V C) 20 V D) None
6. Twelve 1Ω resistor are used as edge to form a cube. The resistance between two diagonally opposite corner of the cube is
 A) $5/6 \Omega$ B) $6/5 \Omega$ C) 5Ω D) 6Ω

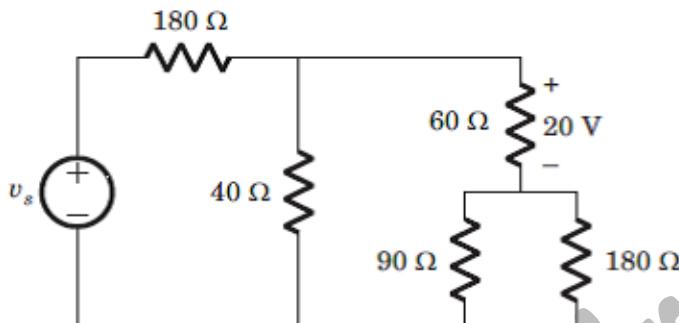
7. Find V_1



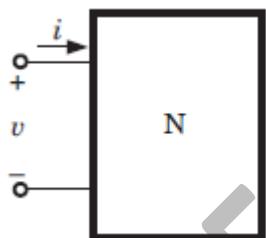
- A) -11 V B) 5 V C) 8 V D) 18 V
8. Find R_{eq}



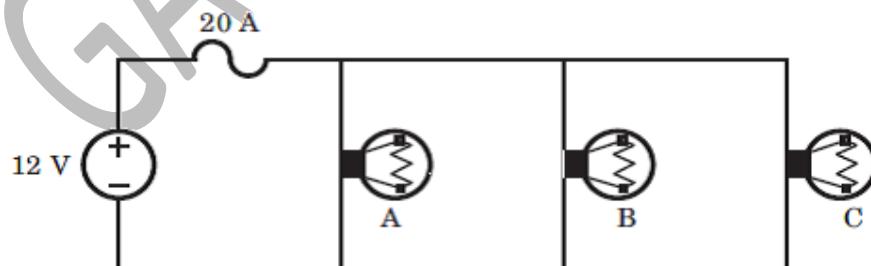
- A) 11.86Ω B) 10Ω C) 25Ω D) 11.18Ω
 9. Find V_s



- A) $320V$ B) $280V$ C) $240V$ D) $200V$
 10. Let $i(t) = 3te^{-100t}$ A and $v(t) = 0.6(0.01-t)e^{-100t}$ V for the network. The power being absorbed by the network element at $t = 5$ ms is



- A) $18.4 \mu W$ B) $9.2 \mu W$ C) $16.6 \mu W$ D) $8.3 \mu W$
 11. In the circuit, bulb A uses 36 W when light, bulb B uses 24 W when light, and bulb C uses 14.4 W when light. The additional A bulbs in parallel to this circuit, that would be required to blow the fuse is

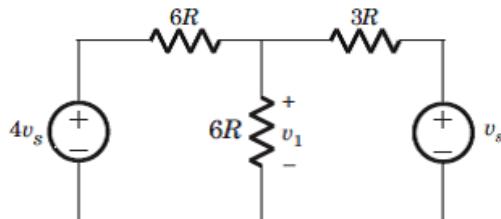


- A) 4 B) 5 C) 6 D) 7

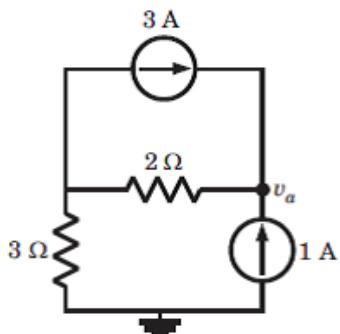
Answers to additional Problems:

1. D	2. B	3. B	4. A	5. B	6. A	7. C	8. D	9. B
10. C	11. B							

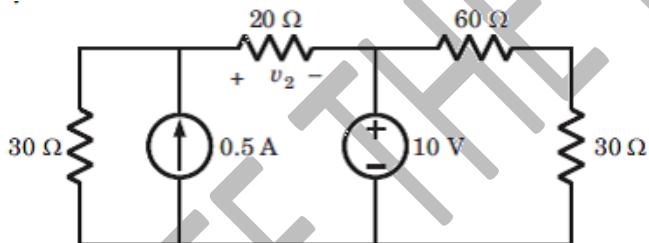
Problems on Methods of analysis

1. Find V_1 

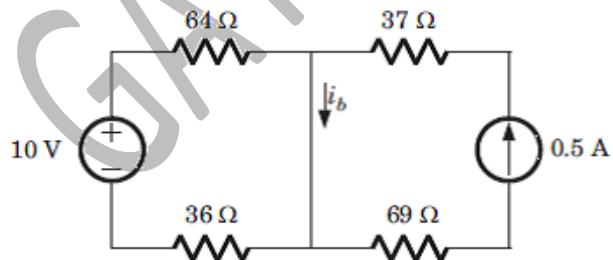
- A) 0.4 V_s B) 1.5 V_s C) 0.67 V_s D) 2.5 V_s

2. Find V_a 

- A) -11V B) 11 V C) 3V D) -3V

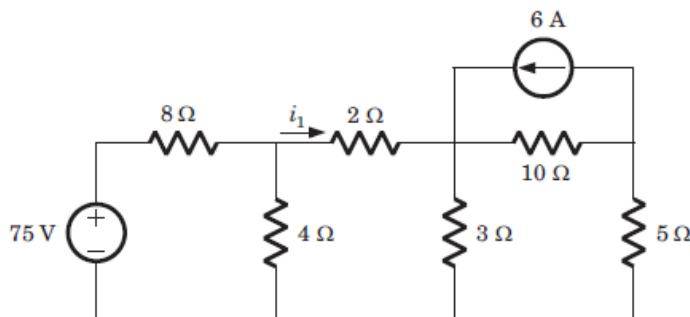
3. Find V_2 

- A) 0.5V B) 1.0V C) 1.5V D) 2.0V

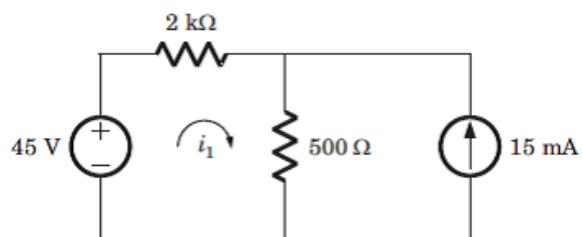
4. Find i_b 

- A) 0.6A B) 0.5A C) 0.4A D) 0.3A

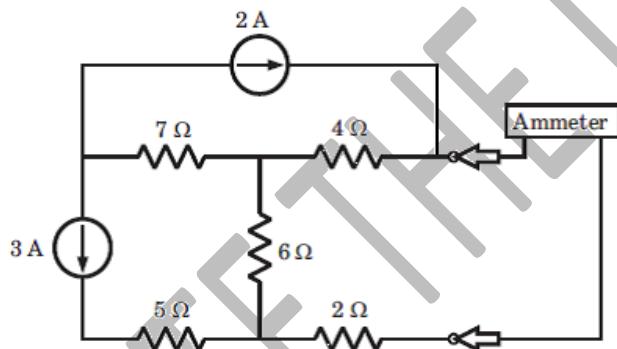
5. Find i_1



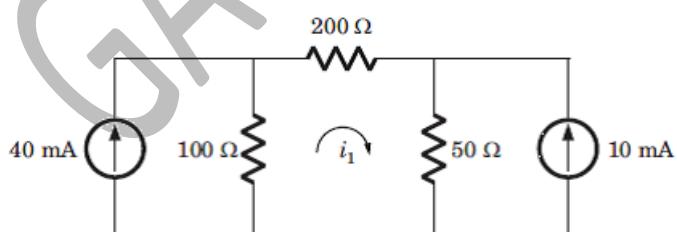
6. Find i_1
- A) 3.3A B) 2.1A C) 1.7A D) 1.1A



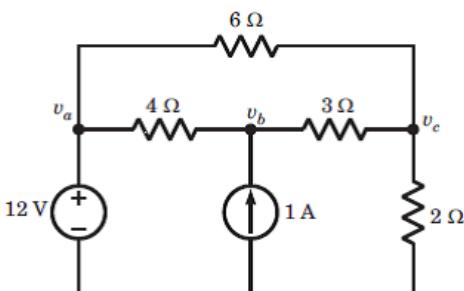
- A) 20mA B) 15mA C) 10mA D) 5mA
7. The value of the current measured by the ammeter in the figure is



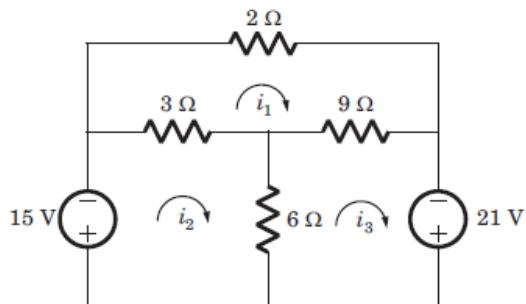
- A) $\frac{2}{3}$ A B) $\frac{5}{3}$ A C) $-\frac{5}{6}$ A D) $\frac{2}{9}$
8. Find i_1



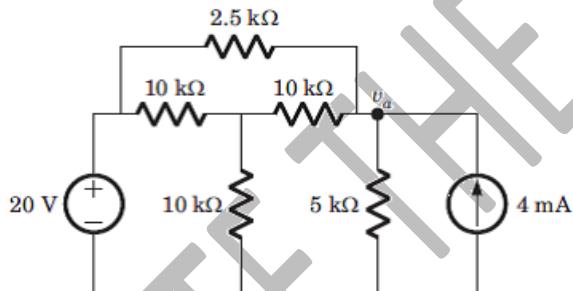
- A) 10mA B) -10mA C) 0.4mA D) -0.4mA
9. The value of node voltage are $V_a = 12V$, $V_b = 9.88V$ and $V_c = 5.29 V$. The power supplied by the voltage source is



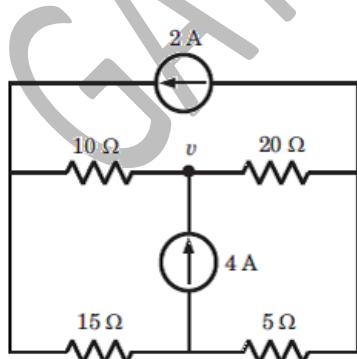
10. $i_1, i_2, i_3 = ?$
- A) 19.8W B) 27.3W C) 46.9W D) 54.6W



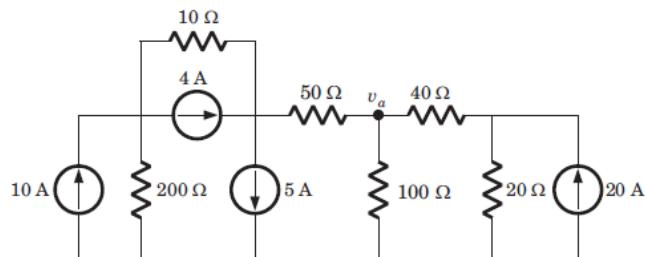
11. Find V_a
- A) 3A, 2A and 4A B) 3A, 3A and 8A C) 1A, 3A and 4A D) 1A, 2A and 8A



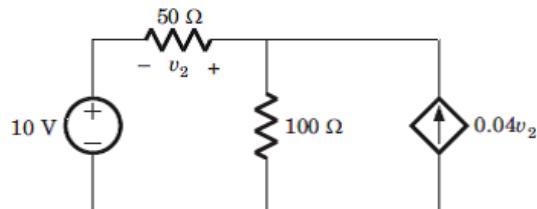
12. Find V
- A) 26V B) 19V C) 13V D) 18V



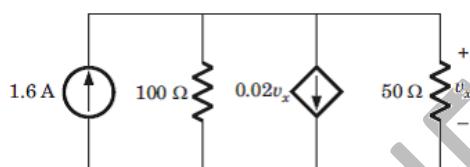
13. Find V_a
- A) 60V B) -60V C) 30V D) -30V



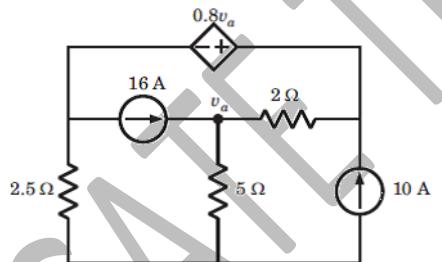
14. Find V_2
- A) 342V B) 171 V C) 198V D) 396 V



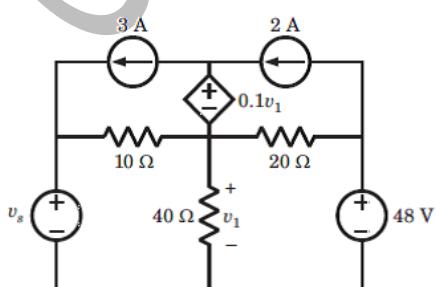
15. Find V_x
- A) 5V B) 75V C) 3V D) 10V



16. Find V_a
- A) 32V B) -32V C) 12V D) -12V

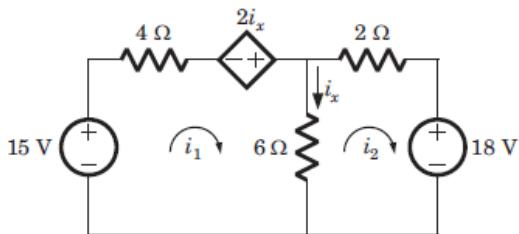


17. For the following circuit, find the value of V_s , that will result in $V_1=0$
- A) 25.91V B) -25.91V C) 51.82V D) -51.82V



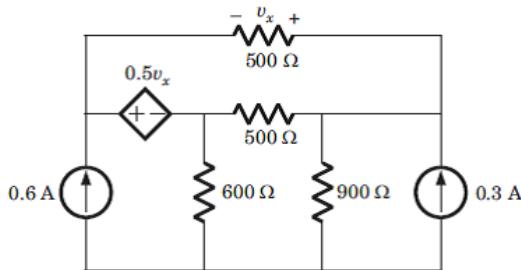
- A) 28V B) -28V C) 14V D) -14V

18. Find i_1 & i_2



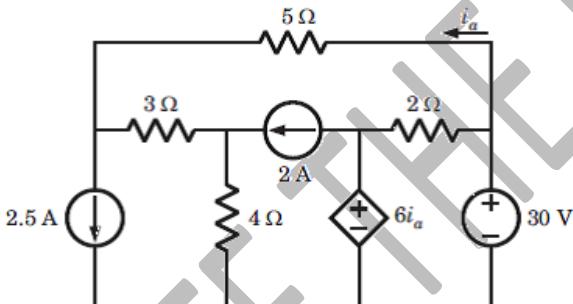
- A) 2.6A, 1.4A B) 2.6A, -1.4A C) 1.6A, 1.35A D) 1.2A, -1.35A

19. Find V_x



- A) 9V B) -9V C) -10V D) 10V

20. The power being dissipated in the 12Ω resistor in the following circuit is



- A) 76.4W B) 305.6W C) 52.5W D) 210 W

Answers:

1. B	2. C	3. D	4. A	5. B	6. B	7. C	8. A	9. A
10. A	11. B	12. A	13. A	14. D	15. A	16. C	17. D	18. D
19. C	20. C							

CHAPTER II

THEOREMS

Theorems:

To handle the complexity of electric network, we use some theorems to simplify circuit analysis like superposition theorem, thevenin and norton theorem etc.

Since some of the theorems are applicable to linear circuits, we first discuss the concept of circuit linearity. In addition to circuit theorems, we discuss the concepts of, source transformation, and maximum power transfer.

Superposition theorem:

The superposition theorem is used to determine the current through and the voltage across any resistor or branch in a network in presence of more than one independent source.

The superposition theorem states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

The superposition theorem is applicable to only linear circuit. It is useful to simplify the circuit has more than one independent source.

1. We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every **voltage source by 0 V i.e. short circuit** and every **current source by 0 A i.e. an open circuit**.
2. Dependent sources are left intact because they are controlled by circuit variables.

NOTE: Superposition theorem is not applicable to power calculation because superposition is based on linearity and power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition theorem and then calculate power dissipation as:

$$P = I^2R = V^2/R$$

Linearity property: This property is a combination of both the **homogeneity property** and the **additivity property**.

The **homogeneity property** is state that if the input (i.e. excitation) is multiplied by a constant, then the output (i.e. response) is also multiplied by the same constant.

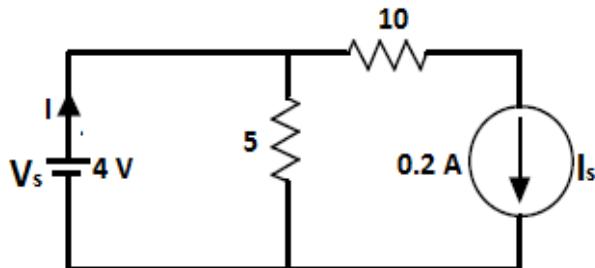
Ex. $y(t)=ax(t)$;

If input $x(t)$ is multiply by k then corresponding response also multiply by same factor k so $a[kx(t)] = ky(t)$

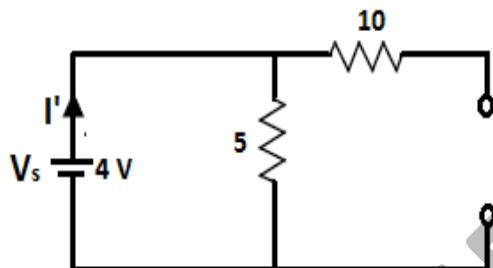
The **additivity property** state that the response to a sum of inputs is the sum of the responses to each input applied separately.

if $V_1=RI_1$ and $V_2=RI_2$ then,
 $R(I_1+I_2)=V_1+V_2$

Example1. Find the current I

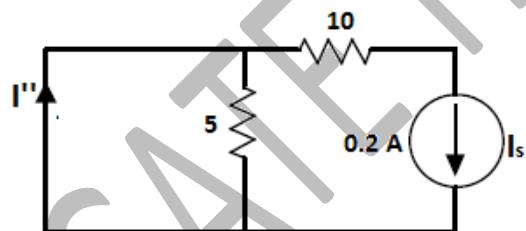


STEP 1: Take only one independent source (V_s) and make other independent sources as zero ($I_s=0$ A so open circuit current source) and calculate I'



$$I' = \frac{V_s}{5} = \frac{4}{5} = 0.8 \text{ A}$$

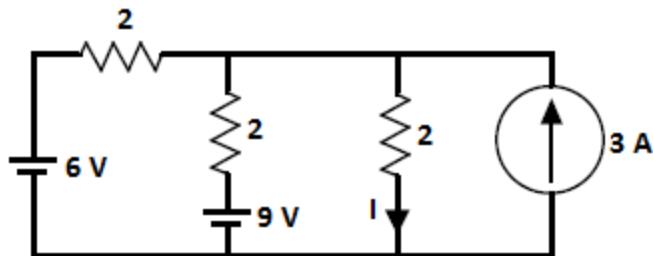
STEP 2: Take current source (I_s) and make other source as zero ($V_s=0$ V so short circuit voltage source) and calculate I''



$$I'' = I_s = 0.2 \text{ A}$$

STEP 3: Total response is algebraic sum of all individual responses
 $I = I' + I'' = 0.8 + 0.2 = 1 \text{ A}$

Example 2. Find I



(ans: $I=3.5 \text{ A}$)

Because of only 6V (voltage source) $\rightarrow I'=1 \text{ A}$

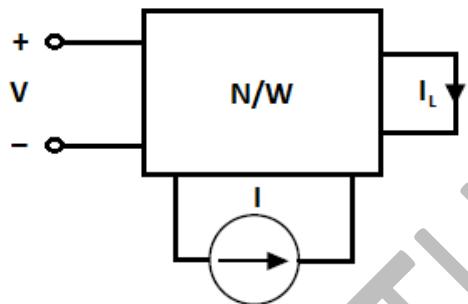
Because of only 9V (voltage source) $\rightarrow I''=1.5 \text{ A}$

Because of only 3A (current source) $\rightarrow I'''=1 \text{ A}$

So total current I:

$$I=I'+I''+I'''=3.5 \text{ A}$$

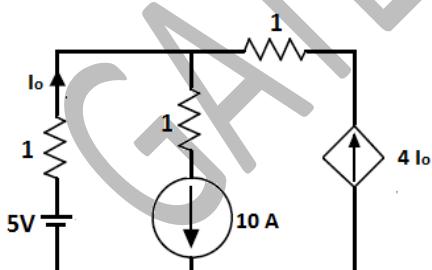
Example 3. Find load current I_L , If n/w is linear and load current due to only Voltage source V is 5A and due to only current source I is 2 A



(Ans: 7A)

$$I_L = I_L' + I_L'' = 5 + 2 = 7 \text{ A}$$

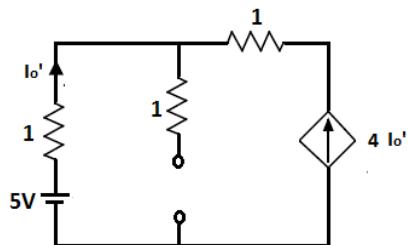
Example:4. Find i_o



The circuit involves a dependent source, which must be left intact. We let

$$I_o = I_o' + I_o''$$

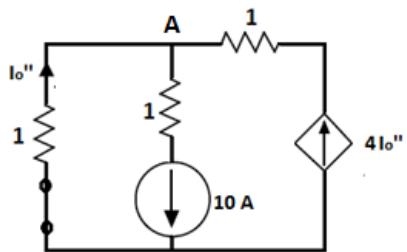
where I_o' and I_o'' are due to the 5 V voltage source and 10 A current source respectively. To obtain I_o' , we turn off the current source 10 A (i.e. open circuit current source) as



$$I_o' = -4I_o'$$

$$5I_o' = 0; \text{ Therefore } I_o' = 0$$

To obtain I_o'' , we turn off the voltage source 5V (i.e. short circuit voltage source) as



$$\text{Apply KCL at node A, } I_o'' + 4I_o'' - 10 = 0$$

$$I_o'' = 2 \text{ A.}$$

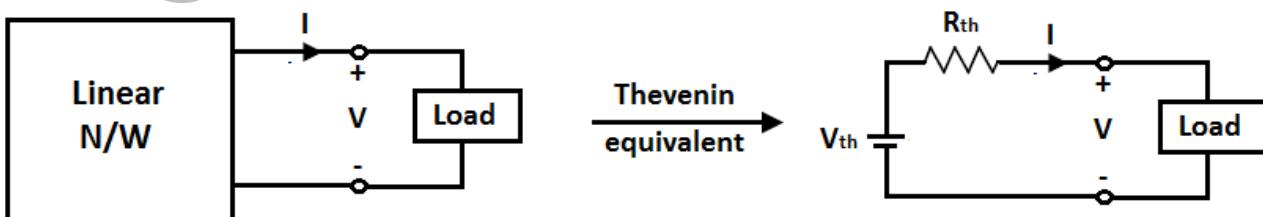
$$\text{Therefore, } I_o = I_o' + I_o'' = 0 + 2 = 2 \text{ A}$$

Note:

- When frequency of the sources are same either DC or AC use superposition theorem to find current and voltage but not power. However when AC sources are there it takes more time to find current or voltage, hence it is recommended not to use the same.
- When **frequencies of the sources are different then mostly use superposition theorem** to find current, voltage.

Thevenin's theorem:

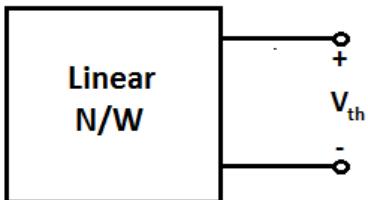
Thevenin's theorem states that any linear two terminals circuit can be replaced by equivalent circuit containing voltage source V_{th} with resistance R_{th} in series. Whereas V_{th} is the open-circuit voltage at the terminals and R_{th} is the input or equivalent resistance at the terminals when the independent sources are off (i.e. independent current sources open circuit and independent voltage sources short circuit)



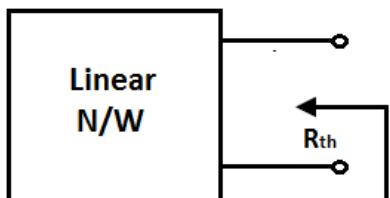
Step to find out the V_{th} and R_{th}

Case 1. Network contains only independent sources along with passive element:

A. Calculate V_{th} : V_{th} is the open-circuit voltage at the terminals



B. Calculate R_{th} : R_{th} is resistance look at terminal with the independent sources are off (i.e. independent current sources open circuit and independent voltage sources short circuit)



All the source make zero i.e.
Voltage source short and
current source open

Case 2. Network contains both independent sources and dependant sources along with passive element:

A. Calculate V_{th} : Same as previous

B. Calculate R_{th} : If network contain dependant source then firstly calculate I_{sc} (short circuit current) by short circuit the terminals as



The thevenin resistance R_{th} is, $R_{th} = \frac{V_{th}}{I_{sc}}$

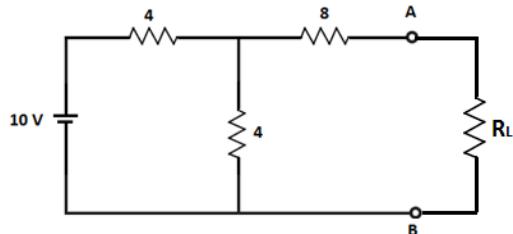
Case 3. Network contains only dependant sources along with passive element:

A. Calculate V_{th} : $V_{th} = 0 \text{ V}$

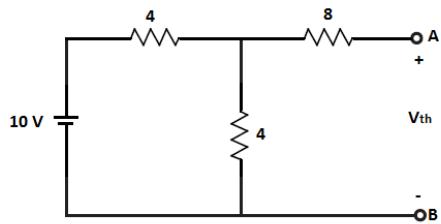
B. Calculate R_{th} : By Applying any know voltage V' across open terminal and then calculate current through voltage source as I'

$$R_{th} = \frac{V'}{I'}$$

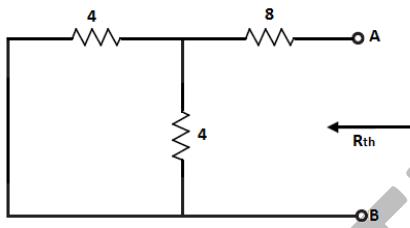
Example 1. Find R_{th} and V_{th} across terminal A-B and also find the current through load resistance R_L if $R_L=5$ ohm.



As circuit contain only independent source so it is a problem of case 1 and V_{th} is the open circuit voltage across terminal A-B

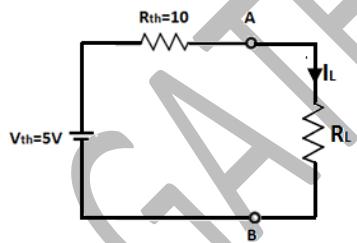


$$V_{th} = \frac{10 \times 4}{4+4} = 5V \text{ (by voltage division)}$$



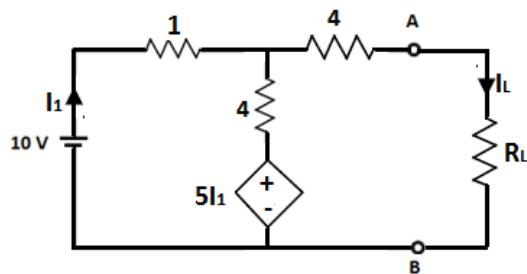
$$R_{th}=8+4||4 = 10 \text{ ohm}$$

The thevenin's equivalent circuit as:

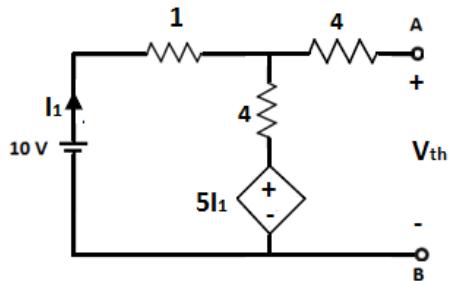


$$\text{Current through } R_L \text{ is } I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{5}{10 + 5} = 1/3 \text{ A}$$

Example 2. Find R_{th} and V_{th} across terminal A-B and also find the current through load resistance R_L if $R_L=5$ ohm.



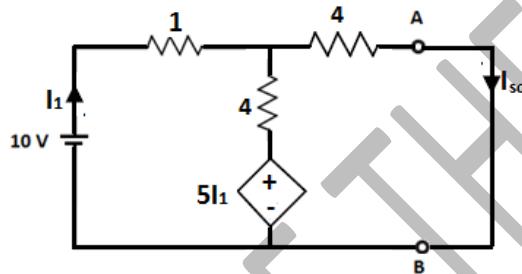
To contain both dependant and independent source so calculate V_{th} as:



$$V_{th} = 9 \text{ V}$$

For R_{th} : Firstly calculate the I_{sc} science it contain both dependant and independent source.

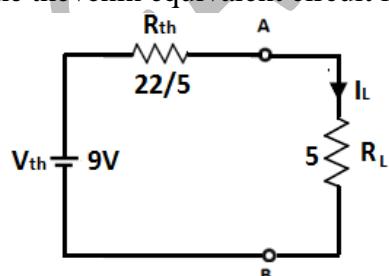
I_{sc} calculate by short circuit the terminals A-B as:



$$I_{sc} = 45/22$$

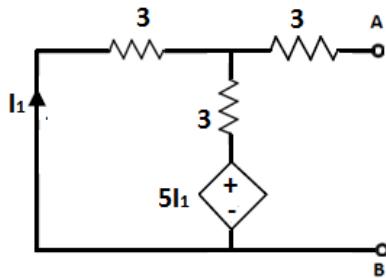
$$R_{th} = \frac{V_{th}}{I_{sc}} = 22/5 \text{ ohm}$$

The thevenin equivalent circuit is



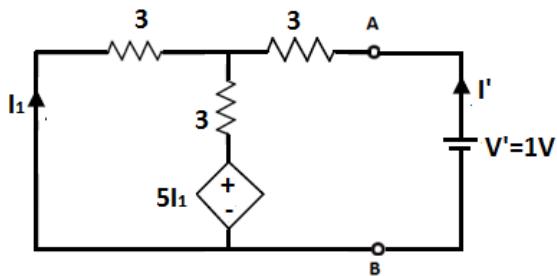
$$\text{Current through } R_L \text{ is } I_L = \frac{V_{th}}{R_{th} + R_L} = 45/47 \text{ A}$$

Example 3. Calculate the thevenin equivalent across terminal A-B



The circuit contains only dependant sources so $V_{th}=0\ V$

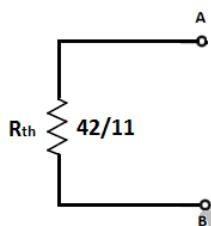
For calculation of R_{th} : Apply any known voltage source ($V'=1V$) across terminal A-B and then find the current through terminal A-B (I')



$$I' = 11/42\ A$$

$$R_{th} = \frac{V'}{I'} = \frac{1}{11/42} = 42/11\ \Omega$$

Thevenin equivalent circuit is



Norton's theorem:

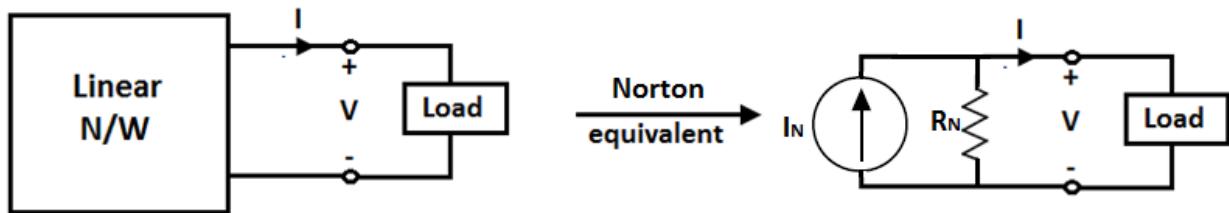
Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit having a current source I_N parallel with a resistor R_N , where I_N is the short-circuit current through the terminals and R_N is equivalent resistance at the terminals when the independent sources are off.

We can find the R_N same as we find the R_{th}

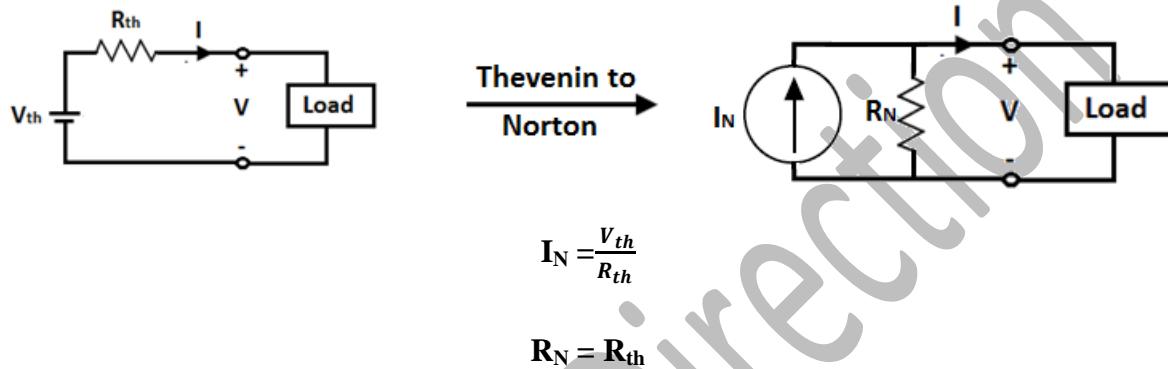
$$R_N = R_{th}$$

And we can find out I_N same as I_{sc}

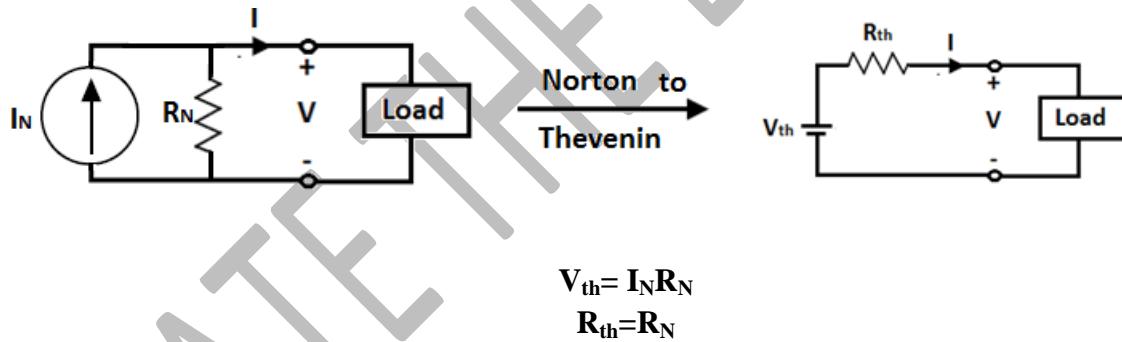
$$I_N = I_{sc}$$



We can convert thevenin's equivalent circuit into Norton equivalent by source transform as:



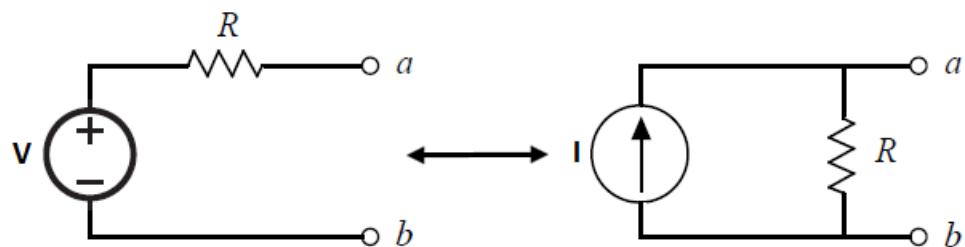
Similarly we can convert Norton equivalent into thevenin's equivalent circuit by source transform as:



Source Transform:

The source transform is used to simplify the circuit. It reduces the number of loops and number of the nodes. It is applicable to only practical sources (For ideal sources the source transform can't be applied). The source transform is also applicable to non linear circuit (i.e. It is applicable to circuit having diode). It is also applicable to dependent sources.

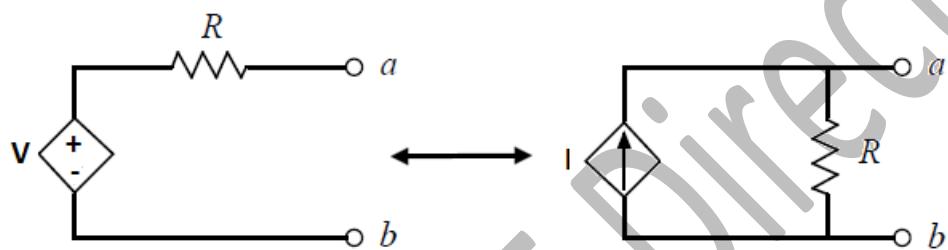
In the source transform we will convert the practical voltage source into practical current source and vice versa as shown below:



A source transformation is the process of replacing a voltage source V in series with a resistor R by a current source I in parallel with a resistor R , or vice versa.

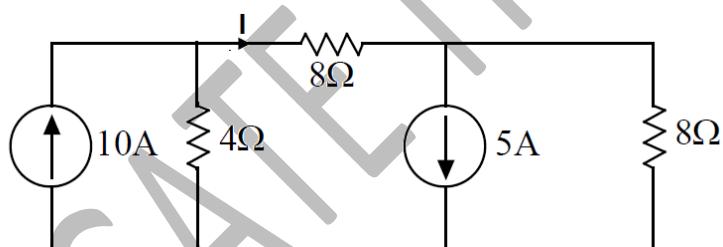
$$I = \frac{V}{R} \text{ or } V = IR$$

It is also applicable to dependant source as:

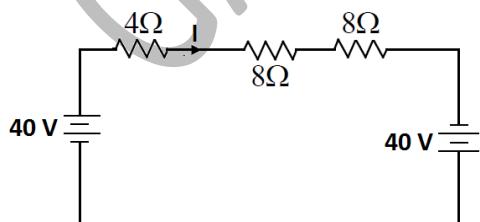


The dependent voltage source in series with a resistor can be transformed to a dependent current source in parallel with the resistor or vice versa.

Example: Find I



Solution: Convert the current sources (10A and 5A) into voltage sources as:

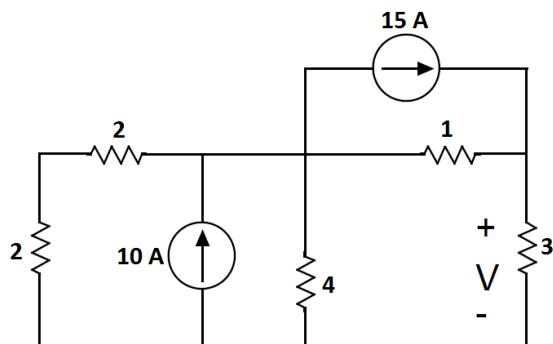


By source transform, we can reduce the three loops into one loop

Apply KVL: $-40 + 4I + 8I + 8I - 40 = 0$

Therefore $I = 4A$

Example: Find V



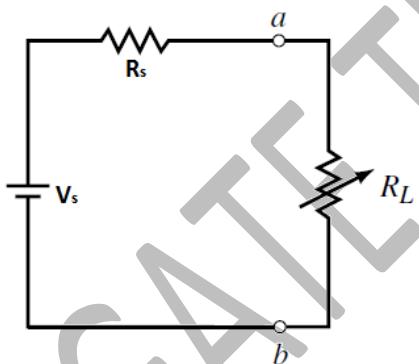
(Ans: 17.5 V)

Maximum power transfer:

In many applications (such as areas like communications), we have to provide maximum power to a load. In this case we have to minimize power losses in the process of transmission and distribution so most of the power will transfer to load.

The Maximum power transfer theorem is **applicable to circuit having variable load**. (If load is **not variable** then choose minimum internal resistance so maximum current will flow through the load and we will get maximum power across **fix load**).

Case I: Variable pure resistive load



In this case a load resistance will receive maximum power from a circuit when the resistance of the load is exactly the same as the source resistance R_s .

Condition for maximum power transfer:

$$R_L = R_s$$

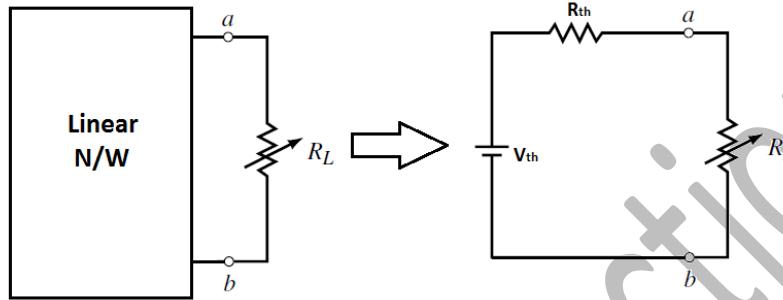
The power delivered to the load is

$$P_L = \frac{V_s^2}{4R_L}$$

The efficiency of power transfer to load resistance is (η):

$$\% \eta = \frac{\text{power transfer to load}}{\text{total power}} = \frac{\frac{V_s^2}{4R_L}}{\frac{V_s^2}{(R_L + R_S)}} \times 100 = 50\%$$

Note: If circuit is not in the form as given above then at first convert the circuit in above form by using thevenin theorem as:



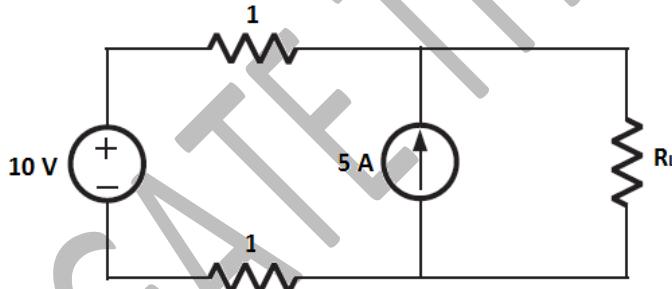
Condition for maximum power transfer is

$$R_L = R_{th}$$

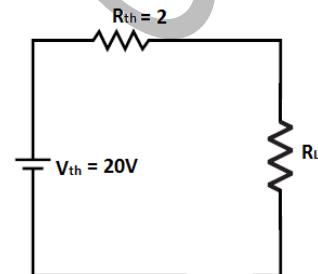
The maximum power delivered to the load is

$$P_L = \frac{V_{th}^2}{4R_L}$$

Example 1. Find the R_L so maximum power will transfer to load (R_L) also find the maximum power delivered across the load.



Firstly we have to convert the given circuit in thevenin equivalent form as:



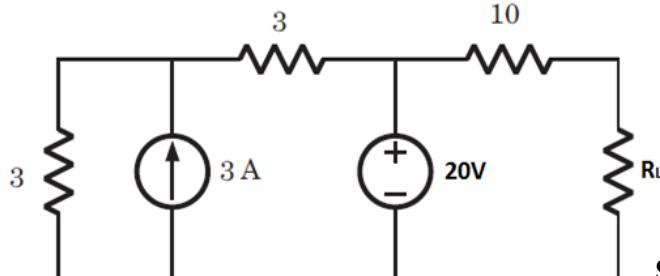
So R_L for maximum power transfer is

$$R_L = R_{th} = 2 \text{ Ohm}$$

The maximum power delivered to the load is

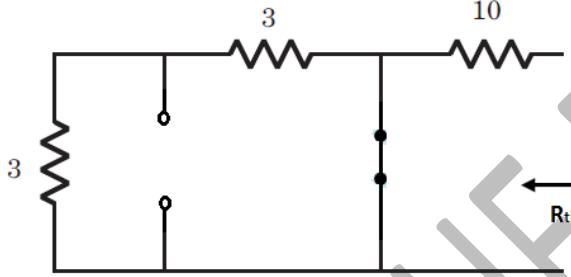
$$P_L = \frac{V_{th}^2}{4R_L} = 50 \text{ watt}$$

Example 2: Find load resistance value (in ohm) in following circuit so maximum power will transfer to load.



- A) 10
- B) 60/16
- C) 6
- D) none

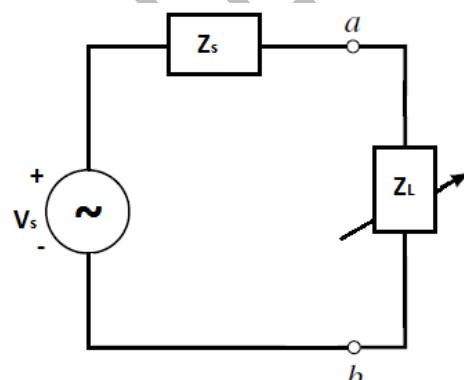
Solution: In this example we have to find out the only R_{th} , as $R_L = R_{th}$ for maximum power transfer



$$R_{th} = 10 \text{ Ohm}$$

$$\text{Therefore } R_L = R_{th} = 10 \text{ Ohm}$$

Case II: Variable Load impedance (resistance and reactance):



$$Z_s = R_s + jX_s \text{ and } Z_L = R_L + jX_L$$

$$\text{Power transfer to load is } P_L = |I_L|^2 R_L$$

Where I_L is load current as:

$$I_L = \frac{V_s}{(R_s + R_L) + j(X_s + X_L)}$$

$$|I_L| = \frac{V_s}{\sqrt{(R_s + R_L)^2 + (X_s + X_L)^2}}$$

$$P_L = \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

A) Both R_L and X_L is variable:

Condition for maximum power transfer is:

$$R_L = R_s \text{ and } X_L = -X_s$$

i.e. $Z_L = Z_s^*$ (Load resistor is complex conjugate of source resistance)

Maximum power transfer to load is

$$P_L = \frac{V_s^2 R_s}{4R_L}$$

B) Only X_L is variable (R_L is constant):

Condition for maximum power transfer is:

$$X_L = -X_s$$

Maximum power transfer to load is:

$$P_L = \frac{V_s^2 R_L}{(R_s + R_L)^2}$$

C) Only R_L is variable

i. X_L is constant:

Condition for maximum power transfer is:

$$R_L = \sqrt{R_s^2 + (X_s + X_L)^2}$$

Maximum power transfer to load is:

$$P_L = \frac{V_s^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2}$$

ii. X_L is zero:

Condition for maximum power transfer is:

$$R_L = \sqrt{R_s^2 + X_s^2}$$

Maximum power transfer to load is:

$$P_L = \frac{V_s^2 R_L}{(R_s + R_L)^2 + X_s^2}$$

Reciprocity theorem

It can only be used with single source circuits. The reciprocity theorem is applicable to only **Linear, passive and bilateral** network. The presence of any dependant source make n/w active so reciprocity theorem is **not applicable** to n/w having dependant sources. The theorem states the following:

Voltage source:

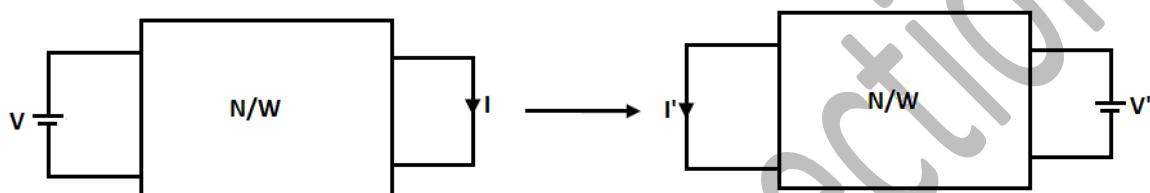
A **voltage** source (V) causing a current I in any branch of a circuit. The voltage source may remove from the original location and placed into that branch having the current I. The voltage

source in the new location will produce a current in the original source location which is exactly equal to the originally calculated current, I .

When applying the reciprocity theorem for a voltage source, the following steps must be followed:

1. The voltage source is replaced by a short circuit in the original location.
2. The polarity of the source in the new location is such that the current direction in that branch remains unchanged.

By application of reciprocity and homogeneity:



If N/W is same with Linear, passive and bilateral then:

$$\frac{V}{I} = \frac{V'}{I'}$$

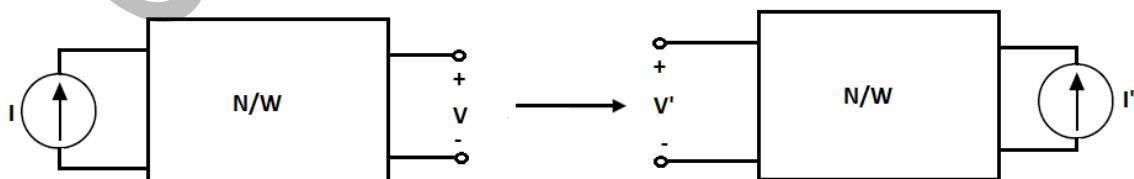
Current source:

A **current source** causing a voltage V at any node of a circuit may be removed from the original location and connected to that node. The current source in the new location will produce a voltage in the original source location which is exactly equal to the originally calculated voltage, V .

When applying the reciprocity theorem for a current source, the following conditions must be met:

1. The current source is replaced by an open circuit in the original location.
2. The direction of the source in the new location is such that the polarity of the voltage at the node to which the current source is now connected remains unchanged.

By application of reciprocity and homogeneity:



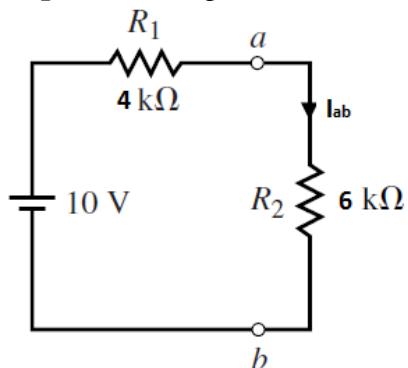
If N/W is same with Linear, passive and bilateral then:

$$\frac{I}{V} = \frac{I'}{V'}$$

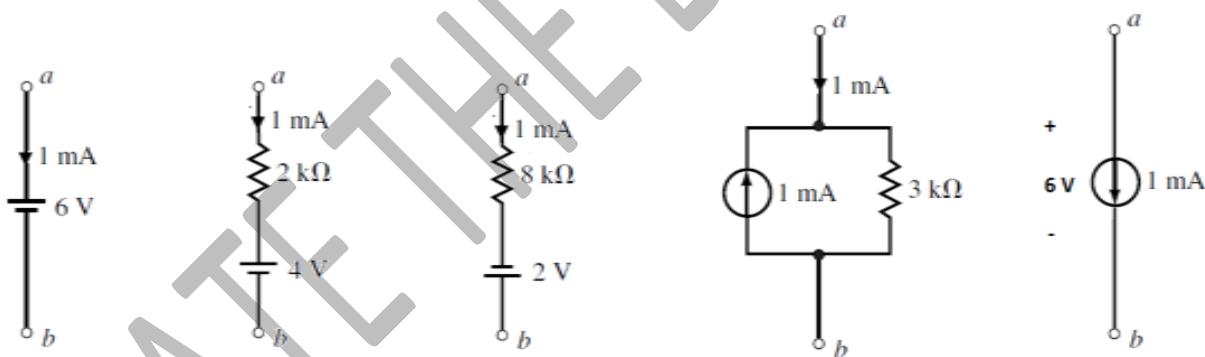
Substitution theorem:

The **substitution theorem** states as any branch within a circuit may be replaced by an equivalent branch, provided the replacement branch has the same current through it and voltage across it as the original branch.

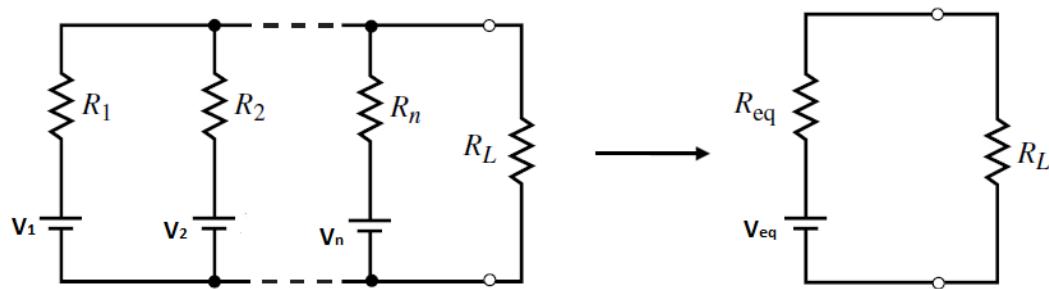
Example 1. In the given circuit the a-b can be replace by many way as



In above circuit the load consist of current of 1mA and voltage across a-b is 6V so we can replace that branch a-b by any another branch having same current (1 mA) flowing through it and same voltage (6 V) across terminal a-b. So we can replace the above branch by any of following branch a-b.

**Millman theorem**

It is used to simplify circuits having several parallel voltage. It is more simplified form of source transform for several parallel voltage connected in circuit.



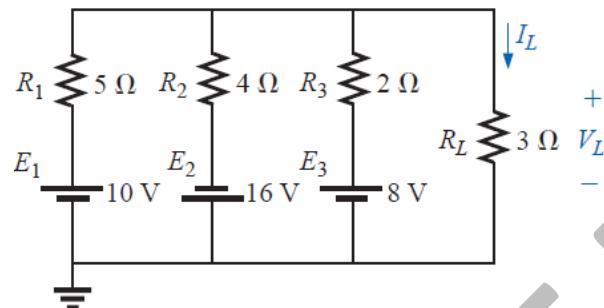
Where: R_{eq} and V_{eq} are the millman equivalent resistance and voltage respectively.

$$R_{eq} = \frac{1}{G_1 + G_2 + \dots + G_n}$$

$$V_{eq} = \frac{V_1 G_1 + V_2 G_2 + \dots + V_n G_n}{G_1 + G_2 + \dots + G_n}$$

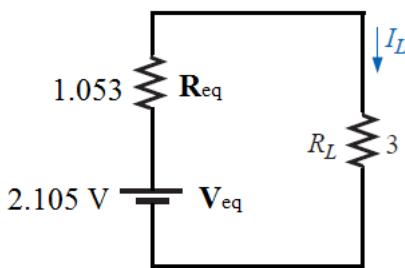
$$\text{Where } G_1 = \frac{1}{R_1}, G_2 = \frac{1}{R_2}, \dots, G_n = \frac{1}{R_n}$$

Example: find out the I_L



$$V_{eq} = \frac{10 \times \frac{1}{5} + 16 \times \frac{1}{4} + 8 \times \frac{1}{2}}{\frac{1}{5} + \frac{1}{4} + \frac{1}{2}} = 40/19 = 2.105 \text{ V}$$

$$Req = \frac{1}{\frac{1}{5} + \frac{1}{4} + \frac{1}{2}} = 20/19 = 1.053 \Omega$$



The Millman equivalent circuit is

$$I_L = \frac{V_{eq}}{Req + R_L} = 0.519 \text{ A}$$

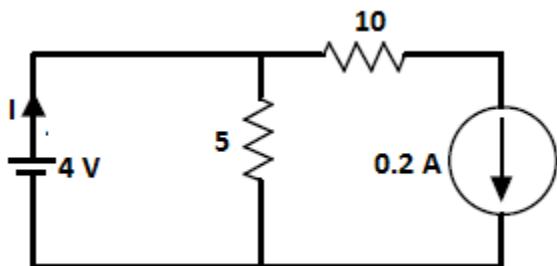
$$V_L = I_L R_L = 1.558 \text{ V}$$

Tellegen theorem

The algebraic sum of power at any instance in any circuit is zero i.e. total power deliver by element in the circuit is equal to total power absorb by circuit.

- If current enter in positive terminal of element then it will be absorb the power and if current enter in negative terminal of the element then it will deliver the power.
- All the passive elements are absorb the power since current is always inter in positive element in passive element.
- The active element maybe absorb or deliver the power depend on current i.e. is it enter in positive terminal or negative terminal of the active element.

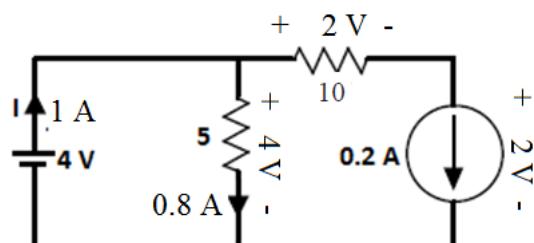
Example: Verify the tellegen theorem



Solution: In above problem $I = 4/5 + 0.2 = 1 \text{ A}$

Current through 5 ohm resistance is $= 4/5 = 0.8 \text{ A}$

Voltage across the 0.2 A current source is $= 2 \text{ V}$



Power across 4 V voltage source is $P_{4V} = 4 \times 1 = 4 \text{ Watt}$ (Deliver as current enter in negative terminal)

Power across 0.2 A current source is $P_{0.2A} = 0.2 \times 2 = 0.4 \text{ Watt}$ (absorb as current enter in positive terminal)

Power across 5 ohm resistance is $P_{5\Omega} = 4 \times 0.8 = 3.2 \text{ Watt}$ (absorb as current enter in positive terminal)

Power across 10 ohm resistance is $P_{10\Omega} = 2 \times 0.2 = 0.4 \text{ Watt}$ (absorb as current enter in positive terminal)

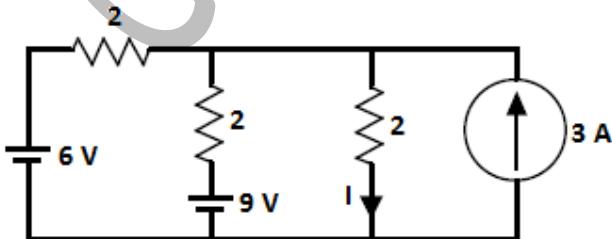
Total power deliver = 4 W

Total power absorb = $0.4 + 3.2 + 0.4 = 4 \text{ W}$

i.e. **Total power deliver = Total power absorb**

Problems:

1. Find I



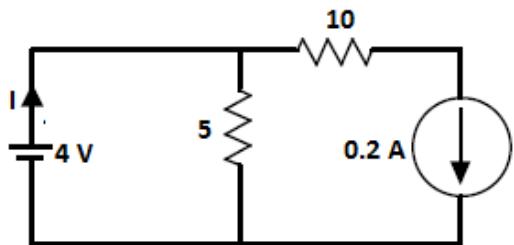
A) 3.5 A

B) -3 A

C) 0.5 A

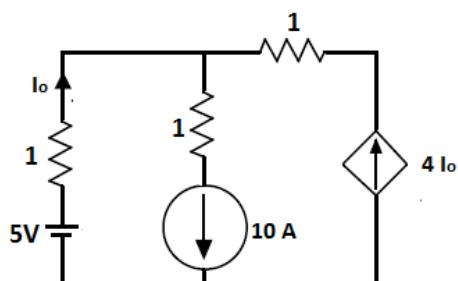
D) 3 A

2. Find I



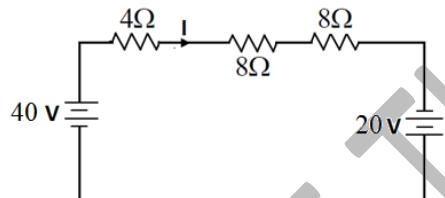
- A) 0.6 A B) 1 A C) 0.2 A D) 0.8 A

3. Find I_o



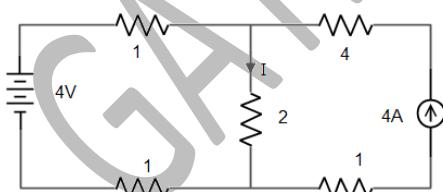
- A) 2 A B) -2 A C) 10 A D) 5 A

4. The power across 40 V voltage source is



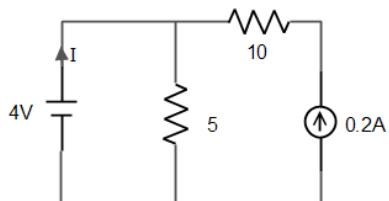
- A) Absorb 120 W B) deliver 120 W C) Absorb 160 W D) deliver 160 W

5. Find I



- A) 3A B) 1.5 A C) -1.5 A D) 0 A

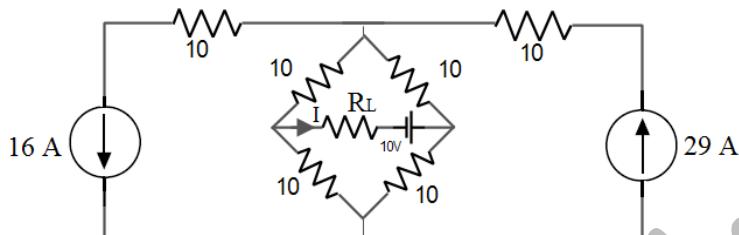
6. Find I.



- A) 0.8 A B) 0.2 A C) -0.2 A D) 1A

Link question(7-8)

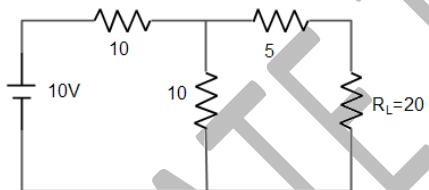
The network is given below as:



7. What will be value of R_L so maximum power will transfer through the load resistance R_L
 A) $10/3$ B) 5 C) 10 D) any value

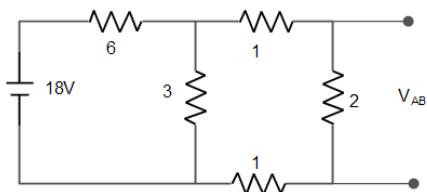
8. What will be the load current I for R_L as found above
 A) 1 A B) 0.5 A C) 1.54 A D) 1.24 A

9. Find V_{TH} , R_{TH} and I_L .



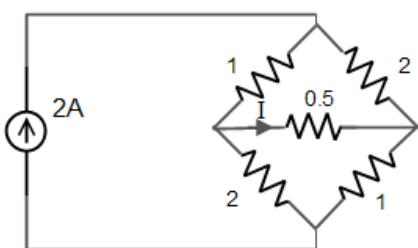
- A) $V_{TH}=5$, $R_{TH}=10\Omega$, $I_L=1/6$ A
 C) $V_{TH}=5$, $R_{TH}=10\Omega$, $I_L=1/6$ A
 B) $V_{TH}=10$, $R_{TH}=10\Omega$, $I_L=1/6$ A
 D) None

10. Find V_{AB} .



- A) 1 V B) 3 V C) 2 V D) 18 V

11. Find I.



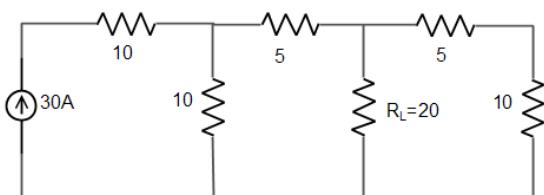
A) 0.5 A

B) 1 A

C) 1.5 V

D) 0 A

12. Find I_L .



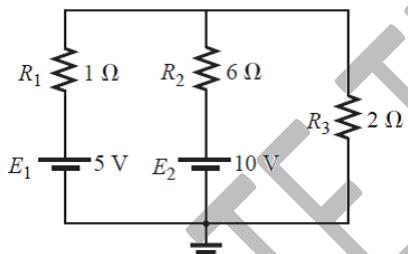
A) 5 A

B) 5.85 A

C) 5.45 A

D) 6.25 A

13. Find the current through the 2 ohm resistor



A) 1 A

B) 2 A

C) 3 A

D) 1.5 A

14. To calculate Thevenin's equivalent value in a circuit

- A) all independent voltage sources are opened and all independent current sources are short circuited.
- B) both voltage and current sources are open circuited
- C) all voltage and current sources are shorted.
- D) all voltage sources are shorted while current sources are opened.

15. A 26 dBm output in watts equals to

A) 2.4W

B) 0.26W

C) 0.156W

D) 0.4W

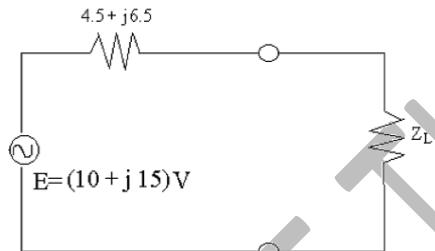
16. Millman's theorem is applicable during determination of

- A) Load current in a network of generators and impedances with two output terminals.
 B) Load conditions for maximum power transfer.
 C) Dual of a network.
 D) Load current in a network with more than one voltage source.
17. An attenuator is a
 A) R's network B) RL network C) RC network D) LC network.

18. Compensation theorem is applicable to
 A) non-linear networks B) linear networks.
 C) linear and non-linear networks D) None of the above

19. Millman theorem yields
 A) equivalent resistance of the circuit
 B) equivalent voltage source
 C) equivalent voltage OR current source
 D) value of current in milli amperes input to a circuit from a voltage source

20. In the circuit shown, maximum power will be transferred when



- A) $Z_L = (4.5 + j 6.5)\Omega$ B) $Z_L = (4.5 - j 6.5)\Omega$ C) $Z_L = (6.5 + j 4.5)\Omega$ D) $Z_L = (6.5 - j 4.5)\Omega$

21. Superposition theorem is not valid for
 A) Voltage responses B) Current responses
 C) Power responses D) Phase responses

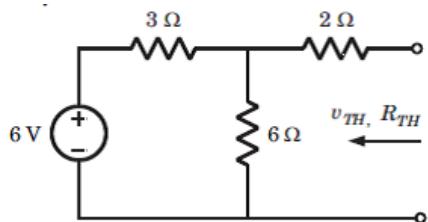
22. Norton's equivalent form in any complex impedance circuit consists of
 A) an equivalent current source in parallel with an equivalent resistance.
 B) an equivalent voltage source in series with an equivalent resistance.
 C) an equivalent current source in parallel with equivalent impedance.
 D) an equivalent voltage source in series with equivalent impedance.

Answers:

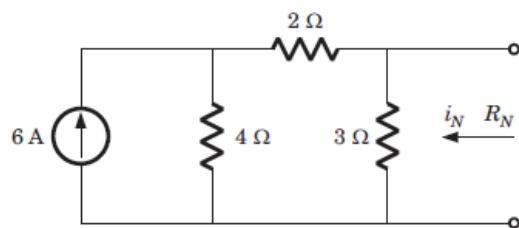
1. A	2. B	3. A	4. B	5. A	6. D	7. C	8. B	9. A
10. C	11. A	12. C	13. B	14. D	15. D	16. D	17. A	18. C
19. C	20. B	21. C	22. C					

Additional Problems :

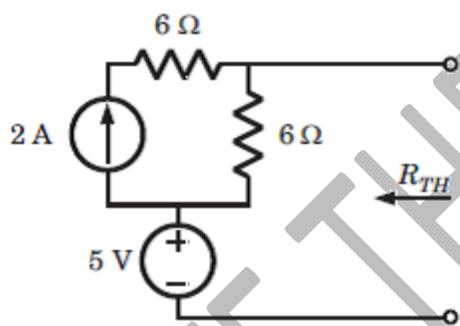
- 1) R_{TH} and V_{TH} for below circuit is respectively



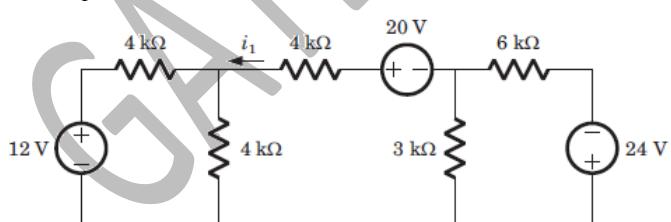
- A) $2\Omega, 4V$ B) $4\Omega, 4V$ C) $4\Omega, 5V$ D) $2\Omega, 5V$
 2) Find i_N, R_N



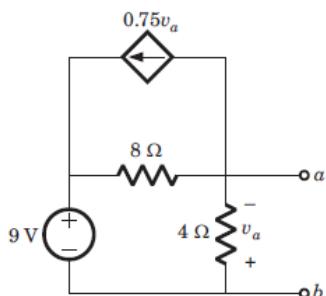
- A) $4A, 3\Omega$ B) $2A, 6\Omega$ C) $2A, 9\Omega$ D) $4A, 2\Omega$
 3) Find R_{TH}



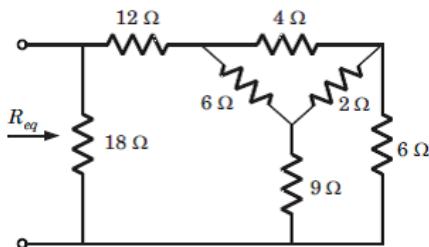
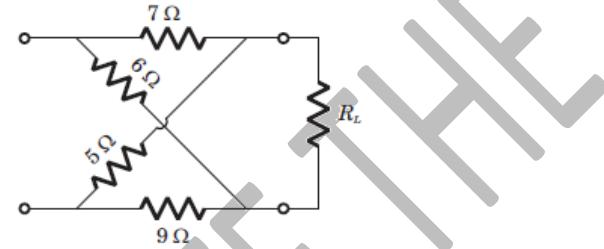
- A) 3Ω B) 12Ω C) 6Ω D) ∞
 4) Find i_1



- A) $3mA$ B) $0.75mA$ C) $2mA$ D) $1.75mA$
 5) A practical DC current source provides 20 kW to a 50Ω load and 20 kW to a 200Ω load. The maximum power, that can be drawn from it, is
 A) 22.5 kW B) 45 kW C) 30.3 kW D) 40 kW
 6) Consider a 24 V battery of internal resistance $r = 4\Omega$ connected to a variable resistance RL . The rate of heat dissipated in the resistor is maximum when the current drawn from the battery is i . The current drawn from the battery will be $i/2$ when RL is equal to

A) 2Ω B) 4Ω C) 8Ω D) 12Ω 7) The value of R_{TH} at terminal ab isA) -3Ω B) $-9/8\Omega$ C) $-8/3\Omega$

D) none of these

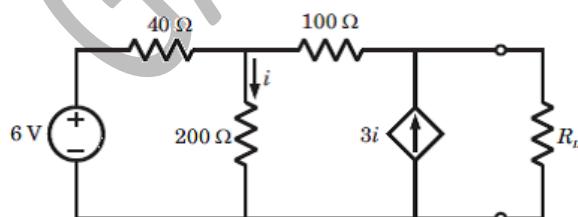
8) $R_{eq}=?$ A) 18Ω B) $72/13\Omega$ C) $36/13\Omega$ D) 9Ω 9) In the lattice network the value of R_L for the maximum power transfer to it isA) 6.67Ω B) 9Ω C) 6.52Ω D) 8Ω 10) A battery has a short-circuit current of 30 A and an open circuit voltage of 24 V. If the battery is connected to an electric bulb of resistance 2Ω , the power dissipated by the bulb is

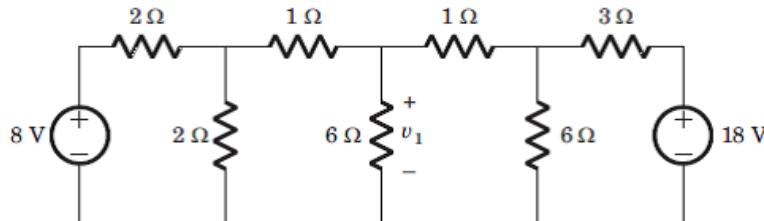
A) 80 W

B) 1800 W

C) 112.5 W

D) 228 W

11) In the circuit , the RL will absorb maximum power if RL is equal toA) $400/3\Omega$ B) $2/9\text{ K}\Omega$ C) $800/3\Omega$ D) $4/9\text{ K}\Omega$ 12) $V_1=?$



- A) 6V B) 7V C) 8V D) 10V

13) Superposition theorem can be applied only to circuits having

- A) resistive elements B) passive elements
C) non-linear elements D) linear bilateral elements

14) While calculating R_{th} in Thevenin's theorem and Norton equivalent

- A) all independent sources are made dead
B) only current sources are made dead
C) only voltage sources are made dead
D) all voltage and current sources are made dead

15) A DC voltmeter with a sensitivity of 20 kΩ/V is used to find the Thevenin equivalent of a linear network. Reading on two scales are as follows:

- (a) 0 - 10 V scale : 4 V
(b) 0 - 15 V scale : 5 V

The Thevenin voltage and the Thevenin resistance of the network is

- A) 16/3 V, 200/3 kΩ B) 32/3 V, 1/15 MΩ C) 18 V, 2/15 MΩ D) 36 V, 200/3 kΩ

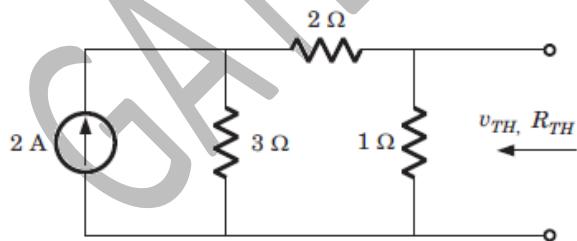
16) The following result were obtained from measurements taken between the two terminal of a resistive network

Terminal Voltage	12 V	0 V
Terminal Current	0 A	1.5 A

The thevenin resistance of the network is

- A) 16Ω B) 8Ω C) 0 D) ∞

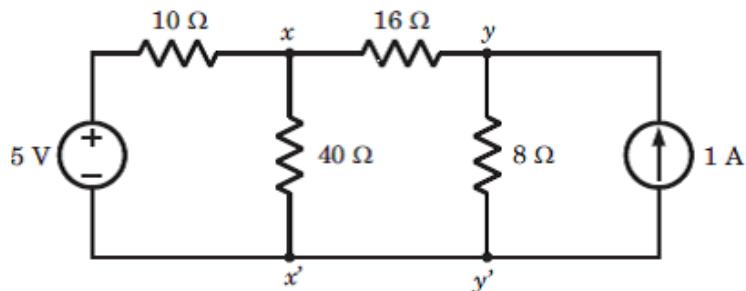
17) Find V_{th} & R_{th}



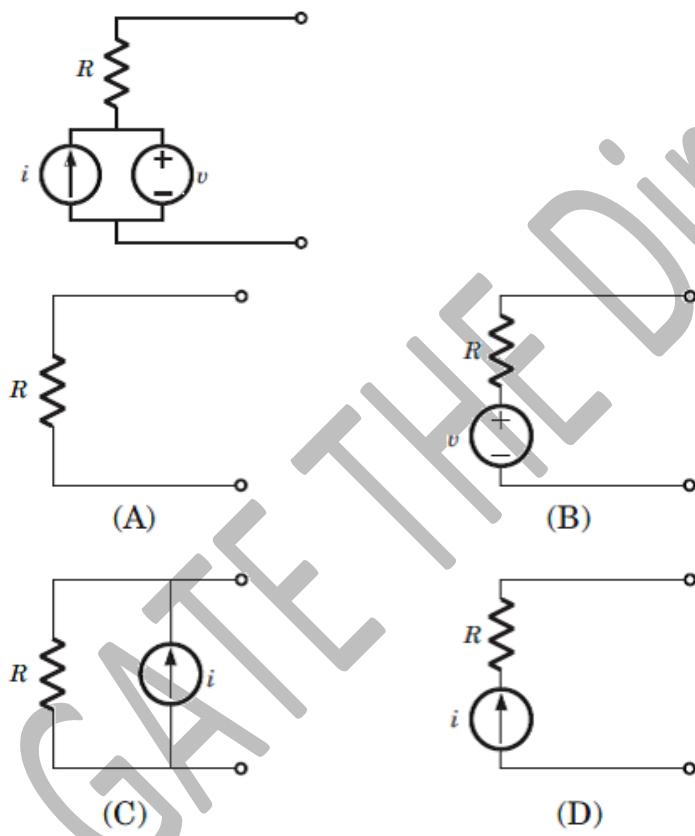
- A) -2V, 6/5 Ω B) 2V, 5/6 Ω C) 1V, 5/6 Ω D) -1V, 6/5 Ω

Statement for Que.18–19:

For the given circuit find the Thevenin equivalent as given in question.



- 18) As viewed from terminal x and x' is
 A) 8 V, 6Ω B) 5 V, 6 Ω C) 5 V, 32 Ω D) 8 V, 32 Ω
- 19) As viewed from terminal y and y' is
 A) 8 V, 32 Ω B) 4 V, 32 Ω C) 5 V, 6 Ω D) 7 V, 6 Ω
- 20) A simple equivalent circuit of the 2 terminal network shown in figure is



Answers:

1. B	2. D	3. C	4. B	5. A	6. D	7. C	8. D	9. C
10. C	11. C	12. A	13. D	14. A	15. A	16. B	17. C	18. B
19. D	20. B							

CHAPTER III

TRANSIENT & STEADY STATE ANALYSIS

TRANSIENT ANALYSIS

RC and LR and RLC network

First we convert time domain into Laplace domain. We convert any network from one domain to another domain for just simplification of calculation.

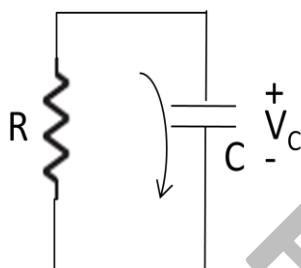
→ R,L,C circuit has transient response and steady state response. The transient response is due to presence of storing element (L, C) in network.

→ RC and RL are first order network and RLC n/w is second order n/w.

Circuit may be excited by two ways-

- 1) **By initial condition:-** If capacitor or inductor are initially charged then in absence of any independent source these capacitor and inductor energy cause flow of current and dissipates energy across resistor.

Source free RC circuit:



$$\begin{aligned}
 I(s) \left(\frac{1}{sC} + I(s) R \right) &= 0 \\
 V_c + RC \left(\frac{dV_c}{dt} \right) &= 0 \\
 RC \frac{dV_c}{dt} &= -V_c \\
 \int_{V_c(0)}^{V_c(t)} \frac{1}{V_c} dV_c &= -\frac{1}{RC} \int_0^t dt \\
 \ln[V_c(t)] - \ln[V_c(0)] &= -t/RC \\
 V_c(t) &= V_c(0) e^{-t/RC}
 \end{aligned}$$

- 2) **By direct method:-**

$$V_c(t) = V_f(t) + [V_i(t) - V_f(t)] e^{-t/\tau}$$

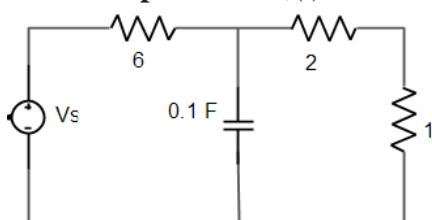
If input supply is zero (i.e. $V_i(t)=0$) then,

$$V_c(t) = V_i(t) e^{-t/RC}$$

The response is exponentially decayed. This response is due to initially stored energy and not due to any source so it is called natural response of circuit.

$$\begin{aligned}
 V_c(t) &= V_c(\infty) + [V_c(0) - V_c(\infty)] e^{-t/\tau} \\
 V_c(t) &= V_c(0) e^{-t/\tau}
 \end{aligned}$$

For Example: find $V_c(t)$ if initially $V_c(0)=10V$ for following circuit.



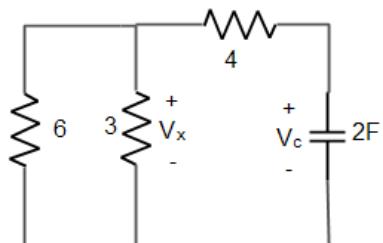
Solution: $R=R_{\text{th}}$ from terminal of capacitor.

$$\tau = (6 \parallel 3) 0.1 = 0.2 \text{ Sec}$$

$$V_c(t) = 10 e^{-t/0.2}$$

For Example: find $V_c(t)$ if initially $V_c(0)=20V$ for following circuit.

Solution: $R_{eq}=R_{th}=6\Omega$, $C=2F$



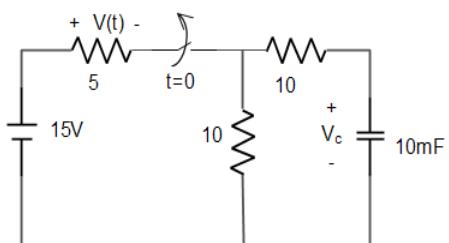
$$\tau = R_{eq} \cdot C = 6 \times 2 = 12 \text{ Sec}$$

$$V_c(t) = 20e^{-t/12} \text{ V}$$

$$V_x(t) = (2 \times 20e^{-t/12}) / (2+4)$$

$$V_x(t) = (40/6)e^{-t/12}$$

For example: Circuit closed for long time & open at $t=0$



$$R_{eq} = 20, C = 10mF$$

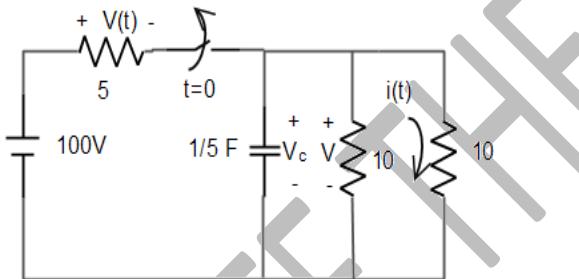
$$\tau = R_{eq} \cdot C = 20 \times 10 \times 10^{-3} = 0.2 \text{ Sec}$$

$$V_c(0) = 10V$$

$$V_c(t) = 10 e^{-t/0.2}$$

$$V(t) = 0$$

For example: Circuit closed for long time & open at $t=0$



$$R_{eq} = R_{th} = 5 \Omega, C = 1/5 \text{ F}$$

$$\text{Req. } C = \tau = 1$$

$$V_c(0) = 50 \text{ V}$$

$$V_c(t) = 50 e^{-t}$$

The Source free RL circuit

$$V_L + V_R = 0$$

$$R i(t) + L (d i(t)/dt) = 0$$

$$L (d i(t)/dt) = - R i(t) \rightarrow L d i(t) = - R i(t) dt$$

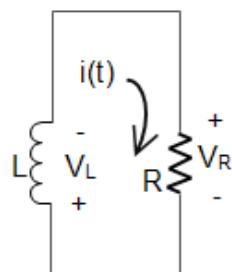
$$L \int_{i(0)}^{i(t)} \frac{1}{i(t)} d i(t) = -R \int_0^t dt$$

$$\log_e i(t) - \log_e i(0) = -R t/L$$

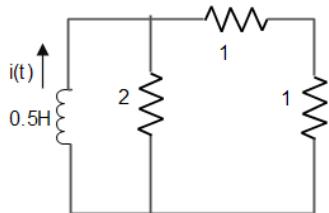
$$\log_e [i(t)/i(0)] = -R t/L$$

$$i(t) = i(0) e^{-Rt/L}$$

$$i(t) = i(\infty) + [i(0) - i(\infty)] e^{-t/\tau} \quad \{\text{as } \tau = L/R\}$$

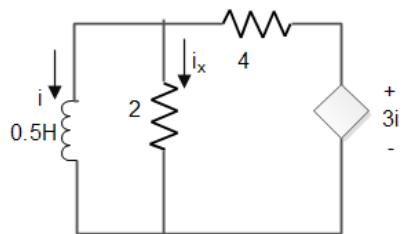


For example: If initially current through inductor is $i_L(0)=1$ A, find the current through inductor for $t > 0$



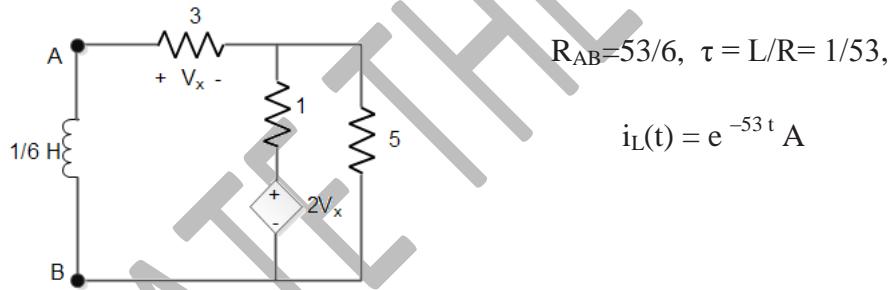
$$\tau = L/R_{\text{th}} = 0.5, \\ i(t) = 1 \times e^{-t/0.5} = e^{-2t}$$

For example: If initially current through inductor is $i_L(0)=10$ A, find the current $i_x(t)$ for $t > 0$



$$R_{\text{TH}}=1/3, L=0.5, L/R_{\text{TH}}=1.5, \\ i_L(t) = 10 e^{-t/1.5} \text{ A} \\ V_L(t) = L di/dt = (0.5 \times 10 e^{-t/1.5}) / (-1.5) \\ V_L(t) = (-10/3) e^{-t/1.5} \\ V_L(t) = V_{R(2)}(t) = 2 i_x(t) \\ i_x(t) = (-10/6) e^{-t/1.5}$$

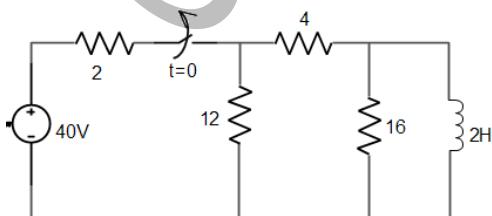
For example: If initially current through inductor is $i_L(0)=1$ A, find the current through inductor for $t > 0$



$$R_{AB}=53/6, \tau = L/R = 1/53,$$

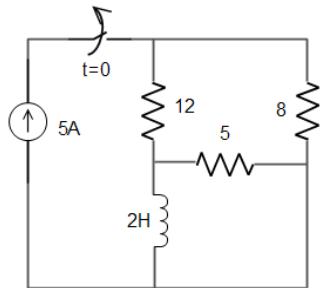
$$i_L(t) = e^{-53t} \text{ A}$$

For example: If initially current through inductor is $i_L(0)=6$ A, find the current through inductor for $t > 0$



$$i_L(0)=6 \text{ A} \\ \text{Req } = R_{\text{TH}}=8\Omega, L=2\text{H} \\ T = L/R = 1/4, \\ i_L(t) = 6 e^{-4t}$$

For example: If initially current through inductor is $i_L(0)=2$ A, find the current through inductor for $t > 0$

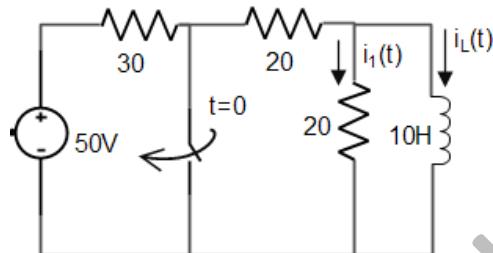


$$i_L(0) = 2 \text{ A}, R_{TH} = 4\Omega, L = 2\text{H},$$

$$\tau = L/R = 2/4 = 1/2,$$

$$i(t) = 2 e^{-2t}$$

For example: Find $i_L(t)$ & $i_1(t)$ if $i_L(0)=1$ A



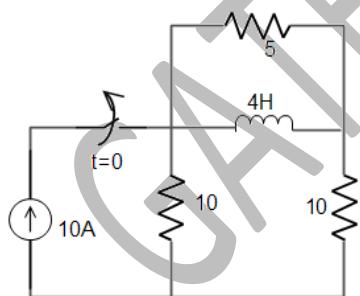
$$i_L(0) = 1 \text{ A}$$

$$R_{eq} = 20\parallel 20 = 10 \Omega, L = 10\text{H}, \tau = 1,$$

$$i_L(t) = e^{-t},$$

$$i_1(t) = (-1/2) e^{-t}$$

For example: If initially current through inductor is $i_L(0)=5$ A, find the current through inductor for $t > 0$



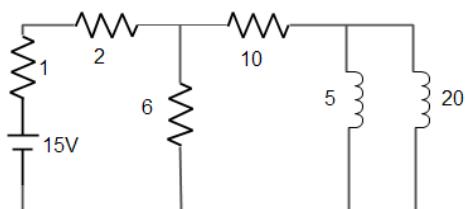
$$i_L(0) = 5 \text{ A},$$

$$R_{TH} = 20\parallel 5 = 4\Omega,$$

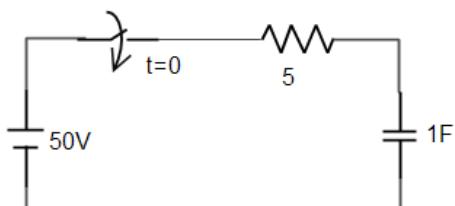
$$\tau = L/R_{TH} = 1,$$

$$i_L(t) = 5 e^{-t}$$

For example: Find the time constant.



$$R_{eq} = 12, L_{eq} = 4, \tau = L_{eq}/R_{eq} = 1/3 \text{ sec}$$

With Source:**For example:** Find $V_c(t)$ for $t > 0$ 

$$V_c(0^+) = 0, V_c(\infty) = 50V$$

$$V_c(t) = 50(1 - e^{-t/5})$$

For example: Find $V_c(t)$ at $t = 2$ sec and 4 sec

$$V_c(0^-) = (10/3)(1/[2^{-1} + 6^{-1} + 3^{-1}])$$

$$V_c(0^-) = 10/3$$

$$\tau_1 = 1 \text{ sec}$$

$$V_c(t) = V_c(0^-) e^{-t} \quad \rightarrow 0 < t < 2$$

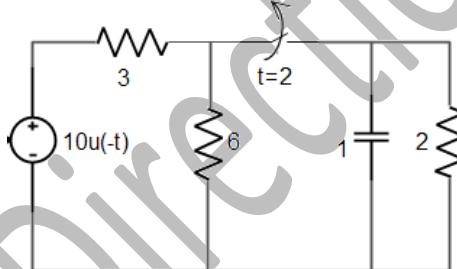
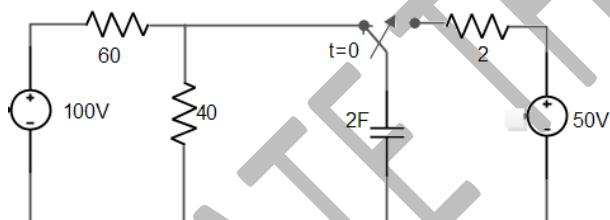
$$V_c(2) = 0.4511 \text{ V}$$

$$V_c(4) = V_c(2) e^{-(t-2)/\tau_2} \quad \rightarrow 2 < t < \infty$$

$$\tau_2 = 2 \text{ sec}$$

$$V_c(4) = 0.4511 e^{-2/2}$$

$$V_c(4) = 0.166 \text{ V}$$

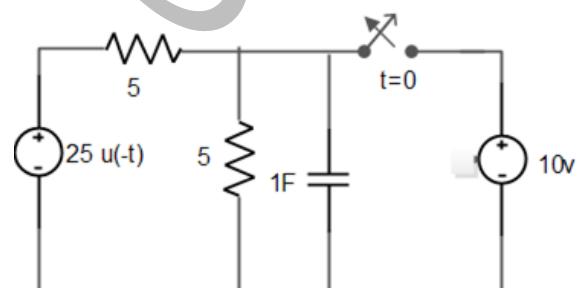
**For Example:** Find $V_c(t)$ for $t > 0$ 

$$\tau = 2 \times 2 = 4 \quad t > 0$$

$$V_c(0^-) = 40 \text{ V}$$

$$V_c(\infty) = 50 \text{ V}$$

$$V_c(t) = 50 + (40 - 50)e^{-t/4}$$

For example: Find $V_c(t)$ for $t > 0$ 

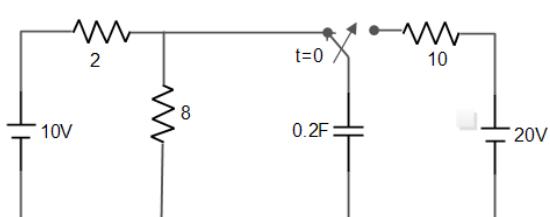
$$V_c(0) = 10 \text{ V}$$

$$V_c(\infty) = 0 \text{ V}$$

$$\tau = 2.5$$

$$V_c(t) = 10e^{-t/2.5}$$

For example: Find $V_c(t)$ for $t > 0$



$$V_c(0^-) = V_c(0^+) = 8V$$

$$V_c(\infty) = 20V$$

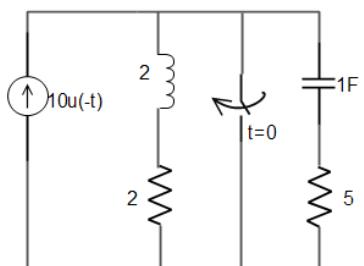
$$\tau = 10 \times 0.2 = 2 \text{ sec}$$

$$V_c(t) = 20 + (8 - 20)e^{-t/2}$$

$$V_c(t) = 20 - 12e^{-t/2}$$

RLC:

For example: Find $V_c(t)$ and $i_L(t)$ for $t > 0$



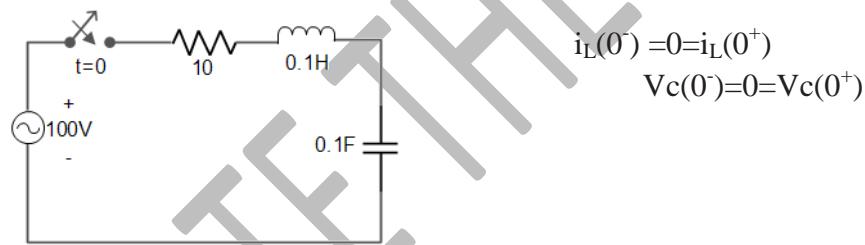
$$i_L(0^-) = 10A, \tau_L = 1$$

$$V_c(0^-) = 20V, \tau_R = 5$$

$$i_L(t) = 10e^{-t}$$

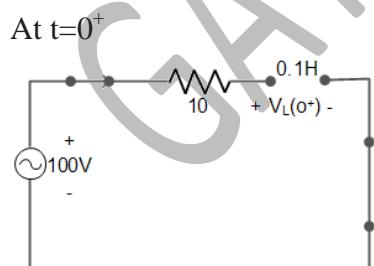
$$V_c(t) = 20e^{-t/5}$$

For example: Find $dV_c(0^+)/dt$ & $di_L(0^+)/dt$



$$i_L(0^-) = 0 = i_L(0^+)$$

$$V_c(0^-) = 0 = V_c(0^+)$$



$$V_L(0^-) = 0V$$

$$V_C(0^-) = 0V$$

$$V_L(0^+) = 100V$$

$$V_C(0^+) = 0V$$

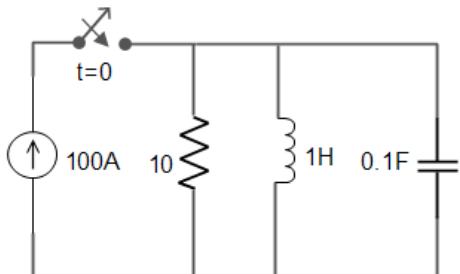
$$V_L(\infty) = 0V$$

$$V_C(\infty) = 100V$$

$$dV_c(0^+)/dt = i_c(0^+)/C = 0 A/F$$

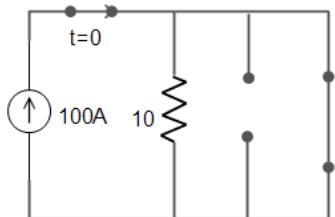
$$di_L(0^+)/dt = V_L(0^+)/L = 100/0.1 = 1000 V/H$$

For example: Find dV_c/dt & di_L/dt at $t=0^+$



At $t = 0^- \rightarrow i_L(0^-) = 0 \text{ & } V_c(0^-) = 0$

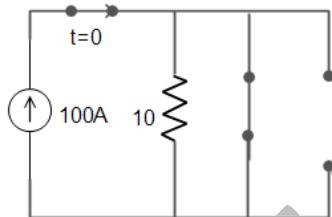
at $t = 0^+$



At $t = 0^+ \rightarrow i_L(0^+) = 0, V_c(0^+) = 0 \text{ & } i_C(0^+) = 100A, V_L(0^+) = 0V$

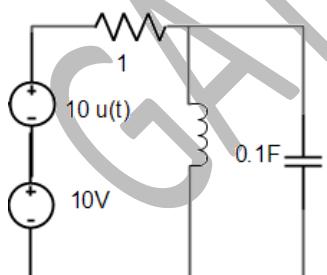
$$\begin{aligned} dV_c(0^+)/dt &= i_c(0^+)/C = 100/0.1 = 1000 \text{ A/F} \\ di_L(0^+)/dt &= V_L(0^+)/L = 0 \end{aligned}$$

At $t = \infty$



$\rightarrow i_L(\infty) = 100A \text{ & } V_c(\infty) = 0V$

For example: Find dV_c/dt at $t = 0^+$

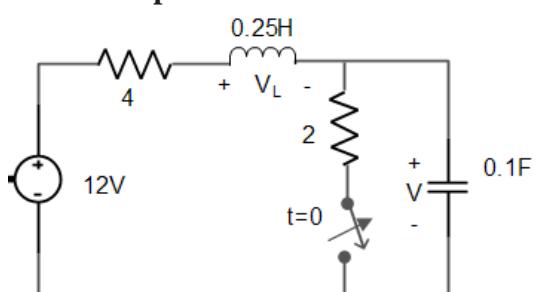


At $t = 0^- \rightarrow i_L(0^-) = 10A \text{ & } V_c(0^-) = 0V$

At $t = 0^+ \rightarrow i_L(0^+) = 10A, V_c(0^+) = 0 \text{ & } i_C(0^+) = 20A, V_L(0^+) = 0V$

$$\frac{dV_c}{dt} \Big|_{t=0^+} = \frac{i_c(0^+)}{C} = \frac{20}{0.1} = 200V/s$$

For example:



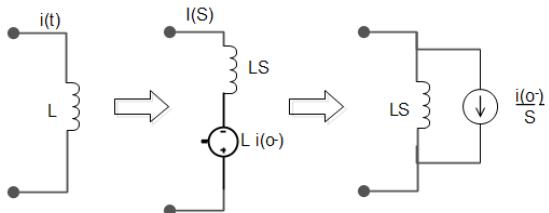
At $t = 0^- \rightarrow i_L(0^-) = 2A = i_L(0^+) \text{ & } V_c(0^-) = 4 = V_c(0^+)$

At $t = 0^+ \rightarrow i_L(0^+) = 2A, V_c(0^+) = 0V$

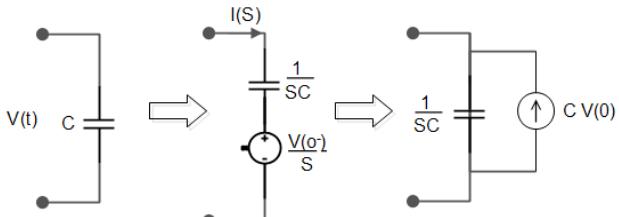
At $t = \infty \rightarrow i_L(\infty) = 0A \text{ & } V_c(\infty) = 12V$

S-Domain:

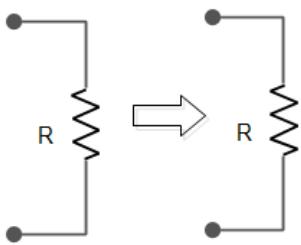
- i) Equivalent for inductor



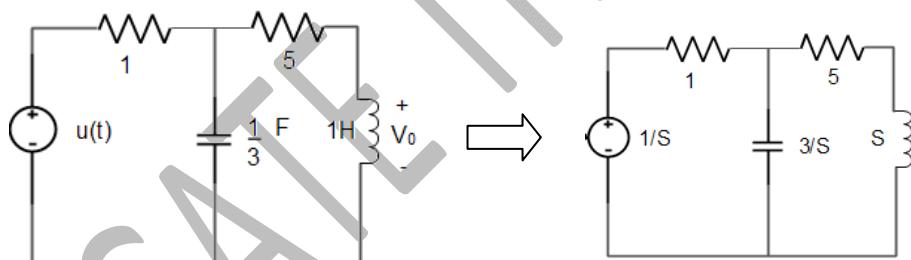
- ii) Equivalent for capacitor



- iii) Equivalent for resistance



For example: Find $V_0(t)$



$$\frac{1}{s} = \left(1 + \frac{3}{s}\right) I_1(s) - \frac{3}{s} I_2(s) \quad \dots \dots \dots (1)$$

$$-\frac{3}{s} I_1(s) + \left(\frac{3}{s} + s + 5\right) I_2(s) = 0 \quad \dots \dots \dots (2)$$

From equation (1)&(2)

$$I_1(s) = \frac{s}{3} \left(\frac{3}{s} + s + 5\right) I_2(s)$$

$$I_1 = \frac{1}{3} (s^2 + 8s + 18) I_2$$

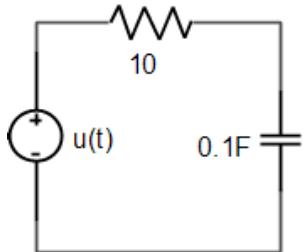
$$I_2 = \frac{3}{s^3 + 8s^2 + 18s}$$

$$V_0(S) = SI_2(S) = \frac{3}{s^2 + 8s + 18}$$

$$V_0(S) = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{(S+4)^2 + (\sqrt{2})^2}$$

$$V_0(t) = \frac{3}{\sqrt{2}} e^{-4t} \sin \sqrt{2} t$$

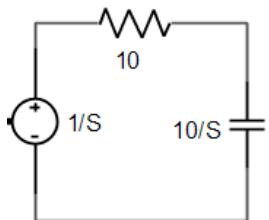
For example:



$$V_C(0)=0, V_C(\infty)=1, \tau=1$$

$$V_C(t)=1-e^{-t} \text{ for } t > 0$$

By laplace transform,



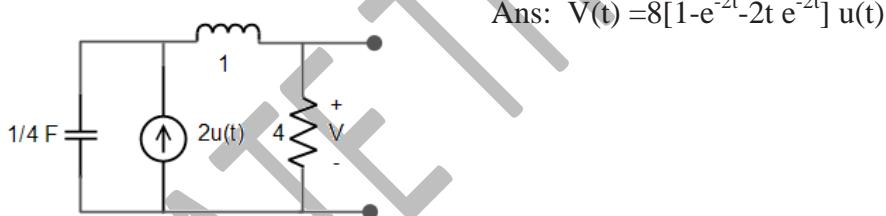
$$-\frac{1}{s} + \left(10 + \frac{10}{s}\right) I(s) = 0$$

$$I(s) = \frac{1}{10s+10}$$

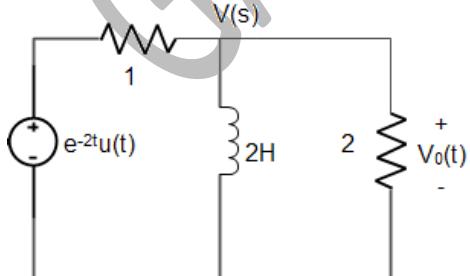
$$V_C(s) = \frac{10}{s} I(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{(s+1)} = u(t) - e^{-t} u(t)$$

$$V_C(t) = (1-e^{-t}) \text{ for } t > 0$$

Example:



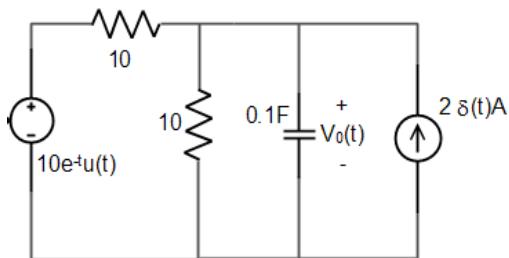
Example:



Hint: Solve by nodal Analysis

$$\text{Ans: } V_0(t) = \frac{4}{5} e^{-2t} - \frac{2}{15} e^{-t/3}$$

Example: Find $V_0(t)$ for $t > 0$



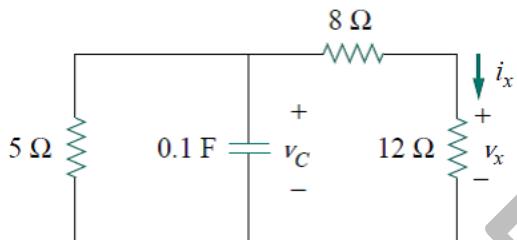
Hint: Solve by nodal Analysis

$$\text{Ans: } V_0(t) = 10[e^{-t} + e^{-2t}]u(t)$$

Problems:

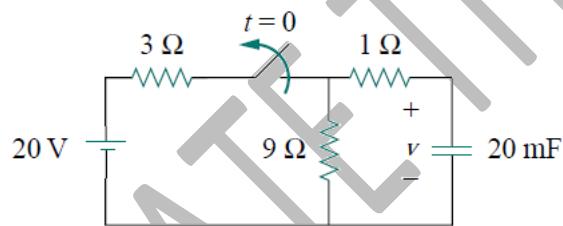
1. An RLC series circuit is said to be inductive if
 A) $wL > 1/wC$ B) $wL < 1/wC$ C) $wL > wC$ D) $wL = wC$

2. If $v_C(0) = 15 \text{ V}$ Then find i_x for $t > 0$.



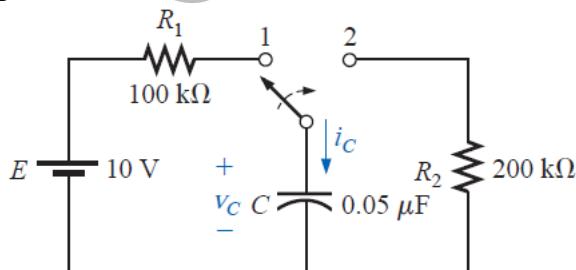
- A) $0.5e^{-2.5t}$
 C) $0.75e^{-2.5t}$
 B) $0.25e^{-2.5t}$
 D) $1.e^{-2.5t}$

3. The switch in the circuit given below has been closed for a long time, and it is opened at $t = 0$. Find $v(t)$ for $t \geq 0$.



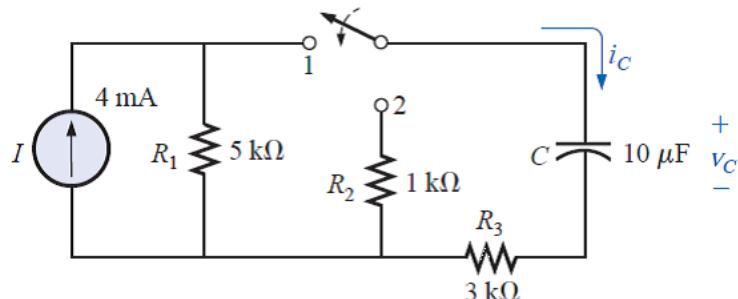
- A) $20e^{-5t}$
 B) $15e^{-5t}$
 C) $15e^{-0.2t}$
 D) $20e^{-0.2t}$

4. The switch in the circuit given below has been at position 1 for a long time, and it is goes to position 2 at $t = 0$. Find $v_c(t)$ for $t \geq 0$



- A) $10e^{-0.01t}$
 B) $10e^{-100t}$
 C) $10e^{-200t}$
 D) $6.67e^{-200t}$

5. The switch in the circuit given below has been at position 1 for a long time, and it is goes to position 2 at $t = 0$. Find $v_c(t)$ for $t \geq 0$

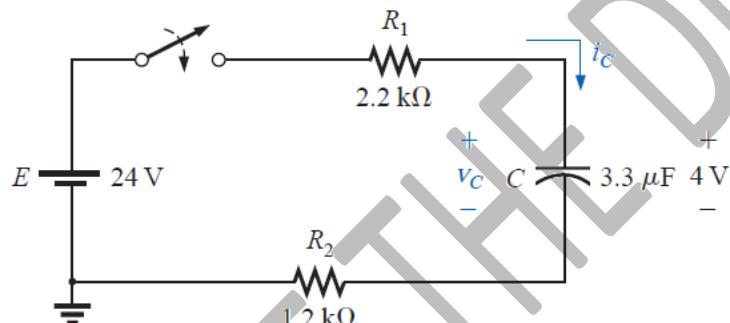


- A) $20e^{-0.04t}$ B) $10e^{-25t}$ C) 0 D) $20e^{-25t}$

6. In above problem find out $i_c(t)$ (in mA) for $t \geq 0$

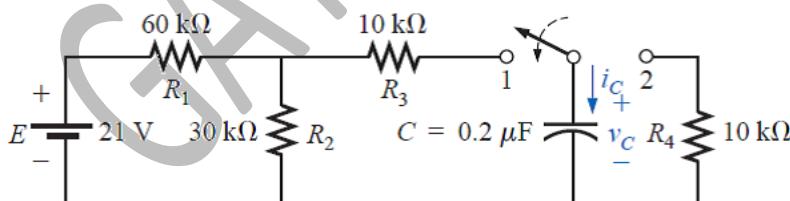
- A) $5e^{-25t}$ B) $-5e^{-25t}$ C) $-20e^{-25t}$ D) $-25e^{-25t}$

7. Find out the voltage across the capacitor if switch is close at $t=0$ sec. The capacitor has an initial voltage of 24 V.



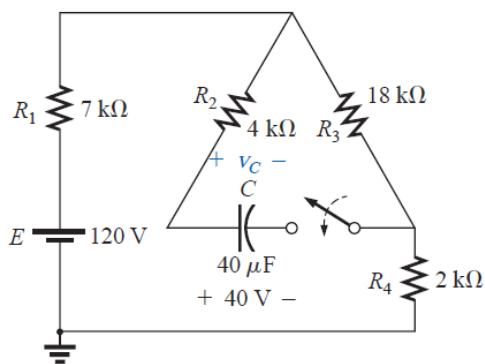
- A) $24(1-0.011t)e^{-89t}$ B) 0 C) $24e^{-89t}$ D) 24

8. Find out the time constant for $t > 0$. If switch is at position 1 for $t > 0$



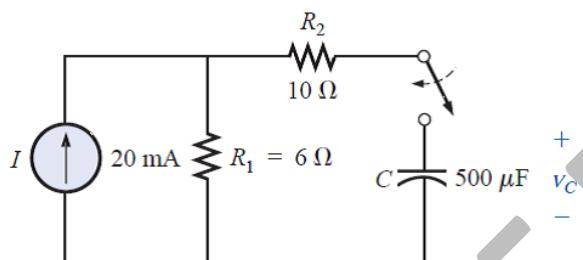
- A) 2 ms B) 4 ms C) 6 ms D) 8 ms

9. The capacitor is initially charged to 40 V. Find the mathematical expression for $v_C(t)$ for $t > 0$ if the switch is close at $t > 0$



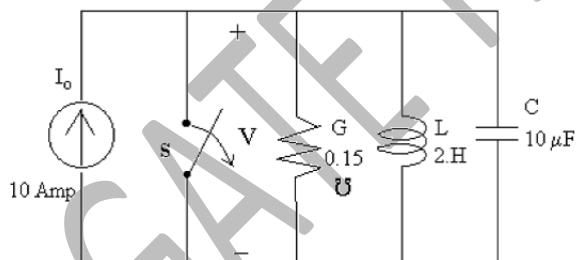
- A) $80(1-1e^{-2.5t})$ B) $80(1-0.5e^{-2.5t})$ C) $80(1+1e^{-2.5t})$ D) $80(1+0.5e^{-2.5t})$

10. The capacitor is initially discharged. Find the mathematical expression for $v_C(t)$ for $t > 0$ if the switch is close at $t > 0$



- A) $0.12(1-1e^{-125t})$ B) $0.2(1-1e^{-125t})$ C) $0.12 e^{-125t}$ D) $0.2 e^{-125t}$

11. In the given circuit switch S is opened at time $t=0$, then find the $\frac{dv(0^+)}{dt}$



- A) 10^5 volt / sec B) 0 volt / sec C) 10^6 volt / sec D) 1 volt / sec

Answers:

1. A	2. C	3. B	4. B	5. D	6. B	7. D	8. C	9. B	10. A
11. C	12.	13.	14.	15.	16.	17.	18.	19.	20.

Additional Problems on RLC circuit:

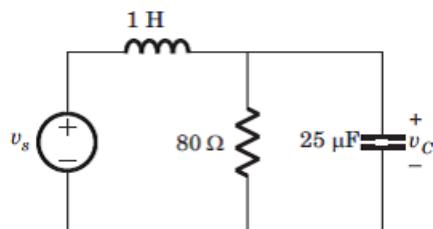
1. The natural response of an RLC circuit is described by the differential equations

$$\frac{d^2v}{dt^2} + 2 \frac{dv}{dt} + v = 0, \quad v(0)=10, \quad \frac{dv(0)}{dt} = 0. \quad \text{The } v(t) \text{ is}$$

A) $10(1+t)e^{-t} \text{ V}$ B) $10(1-t)e^{-t} \text{ V}$ C) $10e^{-t} \text{ V}$ D) $10t e^{-t} \text{ V}$

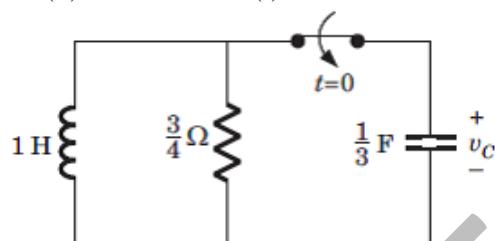
2. In the circuit $v_s=0$ for $t > 0$. The initial condition are $v(0) = 6 \text{ V}$ and $dv(0)/dt = -3000 \text{ V/s}$.

The $v(t)$ for $t > 0$ is



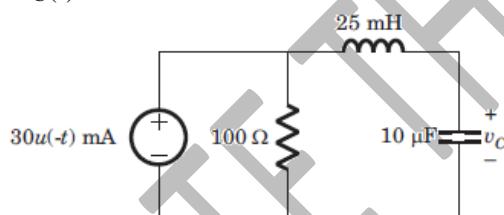
A) $-2 e^{-100t} + 8 e^{-400t} \text{ V}$ B) $6 e^{-100t} + 8 e^{-400t} \text{ V}$ C) $6 e^{-100t} - 8 e^{-400t} \text{ V}$ D) none of the above

3. The circuit shown has been open for a long time before closing at $t = 0$. The initial condition is $v(0) = 2 \text{ V}$. The $v(t)$ for $t > 0$ is



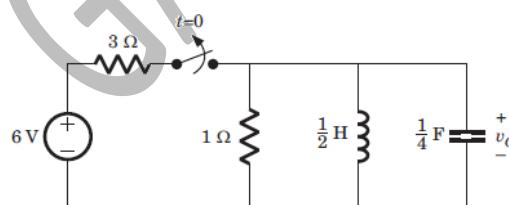
A) $5 e^{-t} - 7 e^{-3t} \text{ V}$ B) $7 e^{-t} - 5 e^{-3t} \text{ V}$ C) $-e^{-t} + 3 e^{-3t} \text{ V}$ D) $3 e^{-t} - e^{-3t} \text{ V}$

4. $V_C(t) = ?$ for $t > 0$



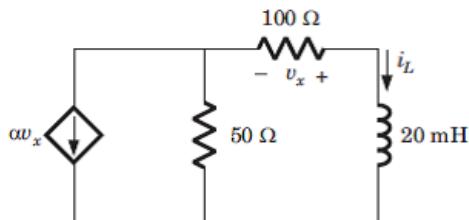
A) $4 e^{-1000t} - e^{-2000t} \text{ V}$ B) $(3+6000t)e^{-2000t} \text{ V}$ C) $2e^{-1000t} + e^{-2000t} \text{ V}$ D) $(3-6000t)e^{-2000t} \text{ V}$

5. The switch of the circuit shown in figure is opened at $t = 0$ after long time. The $v(t)$, for $t > 0$ is



A) $4e^{-2t} \sin 2t \text{ V}$ B) $-4e^{-2t} \sin 2t \text{ V}$ C) $4e^{-2t} \cos 2t \text{ V}$ D) $-4e^{-2t} \cos 2t \text{ V}$

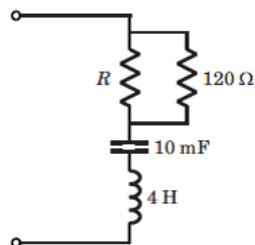
6. The forced response for the capacitor voltage $V_f(t)$ is



- A) $0.2t + 1.17 \times 10^{-3} V$ B) $0.2t - 1.17 \times 10^{-3} V$ C) $1.17 \times 10^{-3}t - 0.2 V$ D) $1.17 \times 10^{-3}t + 0.2 V$
7. For a RLC series circuit $R = 20 \Omega$, $L = 0.6 \text{ H}$, the value of C will be
[CD =critically damped, OD =over damped, UD =under damped].

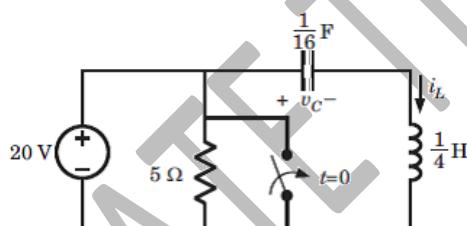
CD	OD	UD
A) $C = 6 \text{ mF}$	$C > 6 \text{ mF}$	$C < 6 \text{ mF}$
B) $C = 6 \text{ mF}$	$C < 6 \text{ mF}$	$C > 6 \text{ mF}$
C) $C > 6 \text{ mF}$	$C = 6 \text{ mF}$	$C < 6 \text{ mF}$
D) $C < 6 \text{ mF}$	$C = 6 \text{ mF}$	$C > 6 \text{ mF}$

8. The circuit shown in figure is critically damped. The value of R is

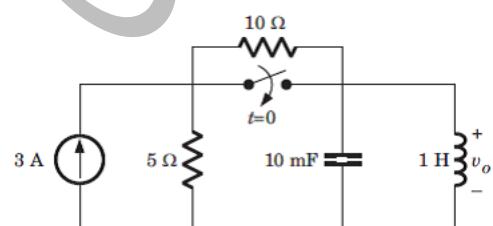


- A) 40Ω B) 60Ω
C) 120Ω D) 180Ω

9. In the circuit of figure the switch is closed at $t = 0$ after long time. The current $i(t)$ for $t > 0$ is



- A) $-10 \sin 8t \text{ A}$ B) $10 \sin 8t \text{ A}$ C) $-10 \cos 8t \text{ A}$ D) $10 \cos 8t \text{ A}$
10. In the circuit of figure a steady state has been established before switch closed. The $v_0(t)$ for $t > 0$ is

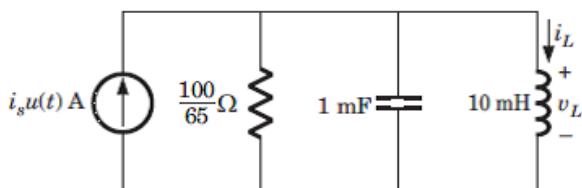


- A) $100t e^{-10t} \text{ V}$ B) $200t e^{-10t} \text{ V}$ C) $400t e^{-50t} \text{ V}$ D) $800t e^{-50t} \text{ V}$
11. Parameters for a RLC circuits are $R=2\Omega$, $L=1 \text{ H}$, $C=1 \text{ F}$. if these are connected in series first and then in parallel. The system response for both the circuits will be

- A) under damped, undamped B) critically damped, over damped
 C) critically damped, under damped D) under damped, critically damped

Statement for Q. 12, 13 & 14:

In the circuit shown in figure all initial condition are zero.



12. If $i_s(t) = 1 \text{ A}$, then the inductor current $i_L(t)$ is
 A) 1 A B) $t \text{ A}$ C) $(t + 1) \text{ A}$ D) 0 A
13. If $i_s(t) = 0.5 \text{ A}$, then $i_L(t)$ is
 A) $0.5t + 3.25 \times 10^{-3} \text{ A}$ B) $2t - 3250 \text{ A}$ C) $0.5t - 0.25 \times 10^{-3} \text{ A}$ D) $2t + 3250 \text{ A}$
14. If $i_s(t) = 2e^{-250t} \text{ A}$ then $i_L(t)$ is
 A) $(4000/3) t e^{-250t} \text{ A}$ B) $(4000/3) e^{-250t} \text{ A}$ C) $(200/7) e^{-250t} \text{ A}$ D) $(200/7) t e^{-250t} \text{ A}$

Answers :

1. A	2. A	3. C	4. B	5. B	6. B	7. A	8. B	9. A
10. B	11. C	12. A	13. A	14. B				

CHAPTER IV

AC ANALYSIS

AC Analysis

$$V(t) = A \sin(\omega t + \theta)$$

$\rightarrow V_m = A$ (amplitude)

$\rightarrow \text{Phase } \Phi = \theta$

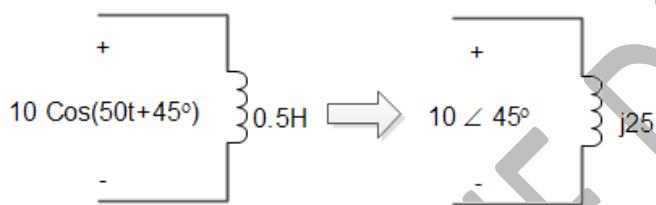
$\rightarrow \text{Angular Frequency} = \omega$

$R \rightarrow R$

$L \rightarrow j\omega L$

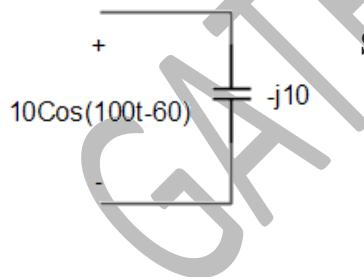
$C \rightarrow 1/j\omega C$

For Example: Suppose $10\cos(50t+45^\circ)$ is connected across 0.5 H inductor. Find steady state current.



$$\text{Solution: } I = \frac{10\angle 45^\circ}{25\angle 90^\circ} = 0.4\angle -45^\circ, \quad i(t) = 0.4 \cos(50t - 45^\circ)$$

For Example: Suppose $10\sin(100t+30)$ is connected across 1mF capacitor. Find out current through capacitor in steady state.



$$\begin{aligned} \text{Solution: } V(t) &= 10\sin(100t+30) = 10\cos(100t - 60) = 10\angle -60 \\ Z &= -j10 = 10\angle -90 \\ I &= \frac{10\angle -60}{10\angle -90} = 1\angle 30 \\ i(t) &= \cos(100t+30) \end{aligned}$$

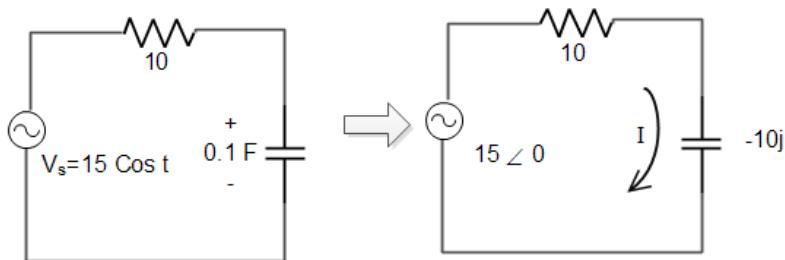
Impedance:

$$Z = R + jX$$

X:- '+ve' inductive

X:- '-ve' capacitive

For example:



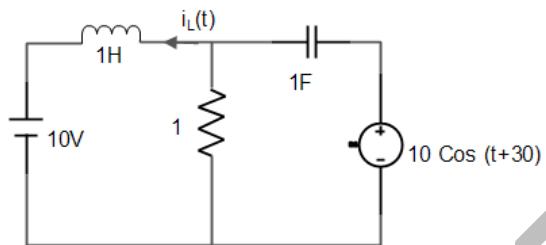
$$\text{Solution: } I = \frac{15\angle 0}{10 - 10j} = \frac{15\angle 0}{14.142\angle -45}$$

$$I = 1.06 \cos(t+45)$$

$$\text{Voltage } V_c = ZI = 1.06\angle 45 \times 10\angle -90$$

$$V_c = 10.6\angle -45$$

For example: Find $i_L(t)$



Solution: By Superposition,

For only 10 V (supply)

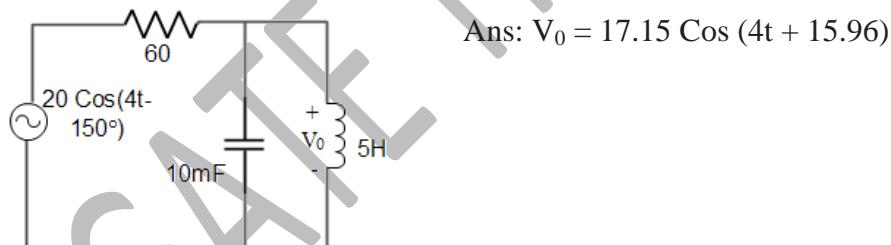
$$\rightarrow i'_L(t) = -10$$

For only $10 \cos(t+30)$

$$\rightarrow i''_L(t) = 10 \cos(t+30)$$

$$i_L = -10 + 10 \cos(t+30)$$

For example: Find V_0

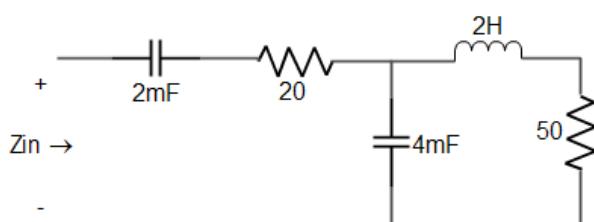


$$\text{Ans: } V_0 = 17.15 \cos(4t + 15.96)$$

Phasor:

Sinusoidal Signal easily expressed in phasors.

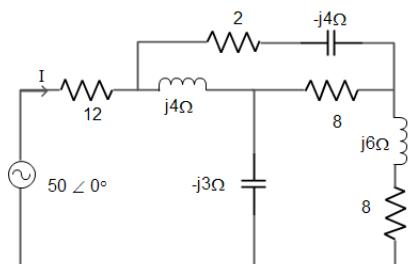
For example:



At $\omega = 10 \text{ rad/s}$

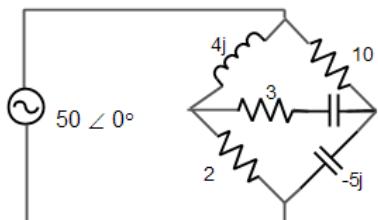
$$\text{Ans: } -32.38 - j73.76 \Omega$$

For example:



Ans: $I = 3.666 \angle -4.20$

For example:



Ans: $I = 12.041 \angle 8.366$ A

For example: Compute the V_1 and V_2 in the circuit.

Solution: $V_2 = 31.41 \angle -87.18^\circ$ V

$$V_1 = V_2 + 10 \angle 45^\circ = 25.78 \angle -70.48^\circ$$

By nodal analysis at super node.

$$\frac{V_1}{-j3} + \frac{V_2}{j6} + \frac{V_2}{12} = 36$$

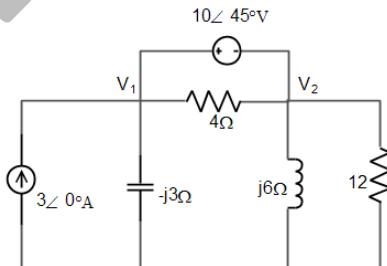
$$4j V_1 + (1-2j) V_2 = 36 \quad \dots \dots \dots (1)$$

$$V_1 - V_2 = 10 \angle 45^\circ \quad \dots \dots \dots (2)$$

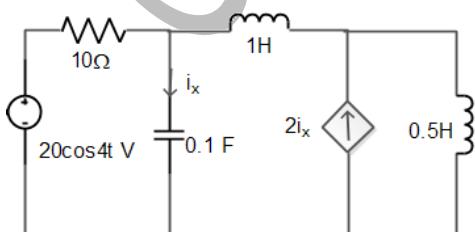
From equation (1) & (2) \rightarrow

$$40 \angle 135^\circ + 4j V_2 + (1-2j)V_2 = 36$$

$$V_2 = 31.4 \angle -87.18^\circ \text{ & } V_1 = 25.78 \angle -70.48^\circ$$



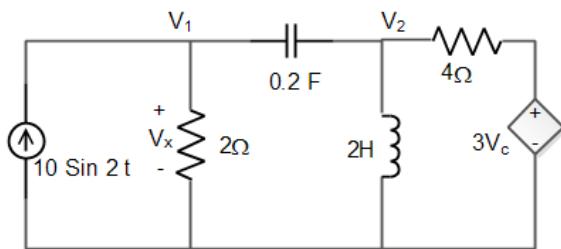
For example:



[Hint: Apply Nodal Analysis]

Ans: $i_x(t) = 7.59 \cos(4t + 108.43)$

For example:



Solution: $V_1 = 11.34 \angle -30^\circ$
 $V_2 = 33.07 \angle -32.87^\circ$

Transient Analysis:

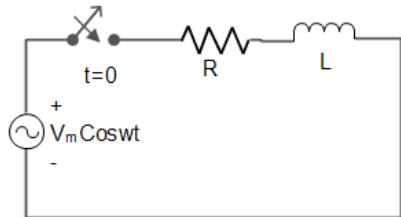
Transient analysis is due to energy store element.

As RL and RC circuit form a first order differential equation.

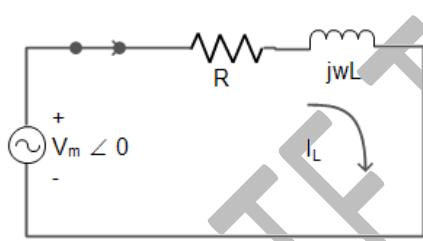
$$C.S = C.F + P.I = K e^{-t/\tau} + i_{ss}$$

response

For example: For R-L Circuit



Solution: For i_{ss}



$$i_L(t) = i_L(\text{transient}) + i_L(\text{S.S})$$

$$i_L(t) = K e^{-t/\tau} + i_{ss}$$

$$\text{As } I_{ss} = \frac{V_m \angle 0}{R + j\omega L} = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}}$$

$$i_L(t) = \frac{V_m \angle \theta - \tan^{-1}(WL/R)}{\sqrt{R^2 + (\omega L)^2}} + K e^{-t/\tau}$$

$$\text{at } t=0, i_L(0^-) = i_L(0^+) = 0$$

$$k = -\frac{V_m \angle \theta - \tan^{-1}(WL/R)}{\sqrt{R^2 + (\omega L)^2}}$$

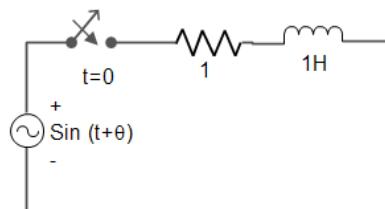
$$k = -\frac{V_m \cos[\theta - \tan^{-1}(WL/R)]}{\sqrt{R^2 + (\omega L)^2}}$$

For removing transient state $k=0$.

$$\text{It is possible if } \theta - \tan^{-1}\left(\frac{WL}{R}\right) = \pm \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} + \tan^{-1}\left(\frac{WL}{R}\right) \dots \dots \text{condition for transient free response once for cosine excitation.}$$

For example: Find out ' θ ' so that response will transient free.



$$\text{Ans: } \theta = \tan^{-1}(WL/R) = 45^\circ$$

For example: For the above problem if we used $\cos(t+\theta)$ then find out ' θ '

$$\text{Ans: } \theta = (\pi/2) + (\pi/4) = 3\pi/4$$

For example: For the above problem if switch is close at $t=100\text{ms}$, then find out ' θ '.

$$\text{Ans: } wt_0 + \theta = \tan^{-1}(WL/R) \rightarrow \theta = (\pi/4) - 100 = 0.685 \text{ rad} = 39.27^\circ$$

Power Calculation

$$\text{Let } V(t) = V_m \cos(wt + \theta_1)$$

$$i(t) = i_m \cos(wt + \theta_2)$$

Case I:

I. Active Power:

$$P = V_{\text{rms}} I_{\text{rms}} \cos \theta \text{ watt}$$

θ is angle between V and I .

II. Reactive Power:

$$Q = V_{\text{rms}} I_{\text{rms}} \sin \theta \text{ (VAR)}$$

III. Complete Power:

$$S = V_{\text{rms}} I_{\text{rms}} \text{ (VA)}$$

$$S = \sqrt{P^2 + Q^2}$$

For example: $V(t) = 100 \cos(10t+100^\circ)$ and $i(t) = 50 \sin(10t+190^\circ)$, then find V_{rms} , I_{rms} , Power factor nature of network.

$$\text{Solution: } i(t) = 50 \cos(10t + 190 - 90) = 50 \cos(10t + 100)$$

$$\theta = 100 - 100 = 0$$

$$\cos \theta = 1$$

Current and voltage are in phase. So that circuit is purely resistive nature.

For example: $V(t) = V_m \cos(wt + 80)$ and $i(t) = I_m \cos(10t + 20)$

$$\text{Solution: } \theta = 80 - 20 = 60$$

$$P.F = \cos 60 = 1/2$$

Current lag by 60° , so circuit is inductive nature.

Case II:

$$Z = R \pm jX$$

$$I^2_{\text{rms}} Z = I^2_{\text{rms}} R + j I^2_{\text{rms}} X$$

$$S = P \pm jQ$$

$$\angle S = \pm \tan^{-1}(Q/P)$$

Case III:

$$Y = G + jB$$

$$V^2_{\text{rms}} Y = V^2_{\text{rms}} G \mp j V^2_{\text{rms}} B$$

$$S = P \mp Q$$

For Example: $V(t) = V_m \cos(\omega t + \theta)$ and $i(t) = I_m \sin(\omega t + \theta)$

Solution: $i(t) = I_m \cos(\omega t + \theta - 90^\circ)$

Here current is lag by voltage by 90° . So it is purely inductive circuit.

$$\text{Average Power} = P = (1/2) V_m I_m \cos 90^\circ = 0$$

$$\text{Reactive power} = (1/2) V_m I_m \sin 90^\circ = V_m I_m / 2$$

For example: Find out active power across $Z = 10 + 15j$, if voltage across it is $V = 100 \angle 0$

Solution: $I = 5.547 \angle -56.3^\circ$

$$P = (1/2) V_m I_m \cos \theta$$

$$P = 307.3/2 \text{ W}$$

For example: If $I = 10 \angle 30$ and $Z = 20 \angle 30$, then Find P_{avg}

Solution: $V = 200 \angle 60$

$$P_{avg} = V_m I_m \cos \theta = 866.025 \text{ W}$$

For example: $V(t) = V_m \cos(\omega t + 100^\circ)$ and $i(t) = I_m \sin(\omega t + 100^\circ)$

Solution: $i(t) = I_m \cos(\omega t + 10^\circ)$, $\theta = 90^\circ$

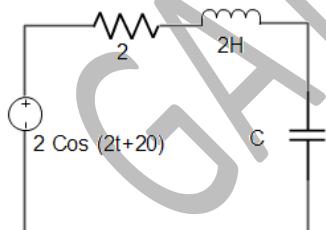
It is purely inductive circuit. So power dissipate (i.e. active power) is equal to zero and total delivered power is stored.

For example: If $I(t) = 10 + 15i$ and $Z = 1 + 1i$. Find out P.F.

$$Z = 1 + 1i = \sqrt{2} \angle 45^\circ$$

$$\text{P.F.} = \cos 45^\circ = 1/\sqrt{2}$$

For Example: Find 'c' for P.F=1.

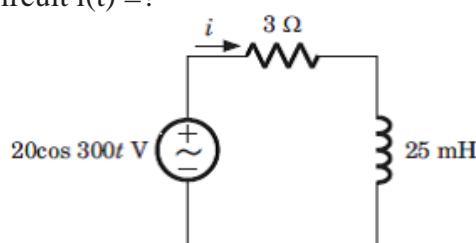


$$X_C = X_L \rightarrow \frac{1}{2 \times C} = 2 \times 2$$

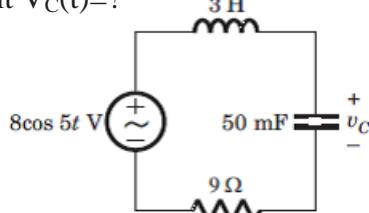
$$C = 1/8 \text{ F}$$

Problems :

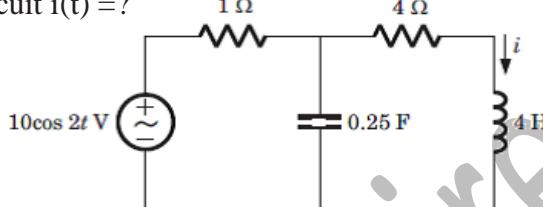
1. For following circuit $i(t) = ?$



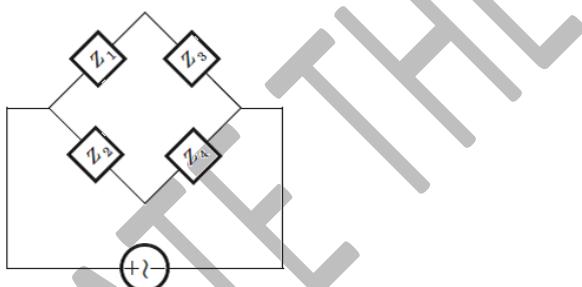
- A) $20 \cos(300t+68.2^\circ)$ A B) $20 \cos(300t-68.2^\circ)$ A
 C) $2.48 \cos(300t+68.2^\circ)$ A D) $2.48 \cos(300t-68.2^\circ)$ A
 2. For following circuit $V_C(t)=?$



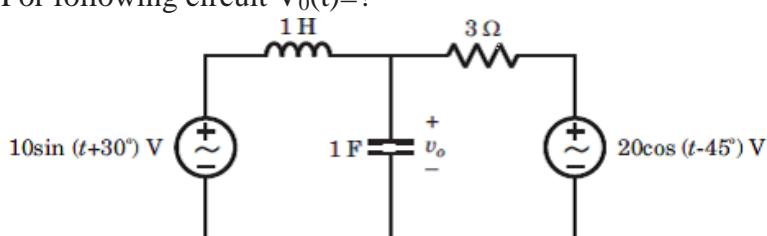
- A) $2.25 \cos(5t+150^\circ)$ V B) $2.25 \cos(5t-150^\circ)$ V
 C) $2.25 \cos(5t+140.71^\circ)$ V D) $2.25 \cos(5t-140.71^\circ)$ V
 3. For following circuit $i(t)=?$



- A) $2 \sin(2t+5.77^\circ)$ A B) $\cos(2t-84.23^\circ)$ A
 C) $2 \sin(2t-5.77^\circ)$ A D) $\cos(2t+84.23^\circ)$ A
 4. In the bridge shown in figure, $Z_1 = 300 \Omega$, $Z_2 = (300 - j600) \Omega$, $Z_3 = (200+j100) \Omega$. The Z_4 at balance is



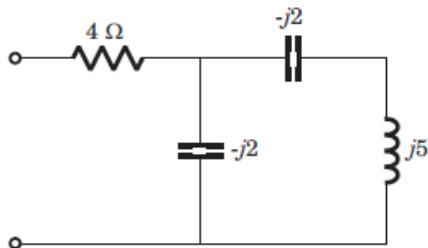
- A) $400+j300\Omega$ B) $400-j300\Omega$ C) $j100\Omega$ D) $-j900\Omega$
 5. In a two element series circuit, the applied voltage and the resulting current are $v(t)=60+66\sin(10^3 t)$ V, $i(t)=2.3\sin(10^3 t + 68.3^\circ)$ A. The nature of the elements would be
 A) R - C B) L - C C) R - L D) R - R
 6. For following circuit $V_o(t)=?$



- A) $31.5 \cos(t+112^\circ)$ V B) $43.2 \cos(t+23^\circ)$ V

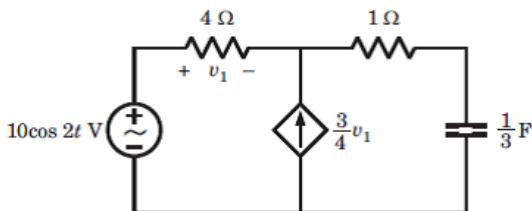
- C) $31.5 \cos(t - 112^\circ)$ V D) $43.2 \cos(t - 23^\circ)$ V

7. In the following circuit power factor is



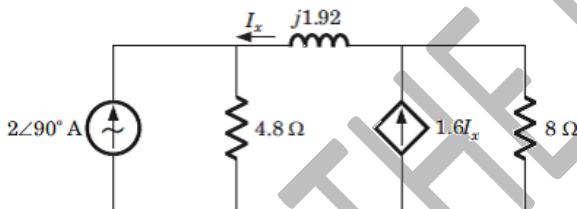
- A) 56.31 (capacitive) B) 56.31 (inductive) C) 0.555 (capacitive) D) 0.555 (inductive)

8. The power factor seen by the voltage source is



- A) 0.8 (leading) B) 0.8 (lagging) C) 36.9 (leading) D) 39.6 (lagging)

9. The average power supplied by the dependent source is



- A) 96 W B) -96 W C) 92 W D) -192 W

10. A relay coil is connected to a 210 V, 50 Hz supply. If it has resistance of 30Ω and an inductance of 0.5H , the apparent power is

- A) 30 VA B) 275.6 VA C) 157 VA D) 187 VA

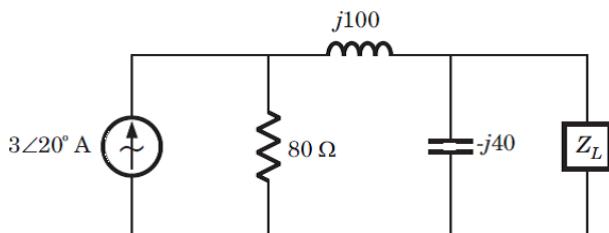
11. The magnitude of the complex power supplied by a 3-phase balanced Y-Y system is 3600 VA. The line voltage is $208\text{ V}_{\text{rms}}$. If the line impedance is negligible and the power factor angle of the load is 25° , the load impedance is

- A) $5.07 + j10.88\Omega$ B) $10.88 + j5.07\Omega$ C) $43.2 + j14.6\Omega$ D) $14.6 + j43.2\Omega$

12. A three-phase circuit has two parallel balanced Δ loads, one of the 6Ω resistor and one of 12Ω resistors. The magnitude of the total line current, when the line-to-line voltage is $480\text{ V}_{\text{rms}}$, is

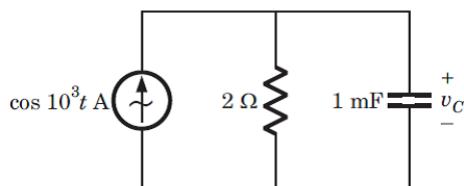
- A) $120\text{ A}_{\text{rms}}$ B) $360\text{ A}_{\text{rms}}$ C) $208\text{ A}_{\text{rms}}$ D) $470\text{ A}_{\text{rms}}$

13. The value of the load impedance, that would absorbs the maximum average power is



- A) $12.8 - j49.6 \Omega$ B) $12.8 + j49.6 \Omega$ C) $33.9 - j86.3 \Omega$ D) $33.9 + j86.3 \Omega$

14. $V_c(t) = ?$

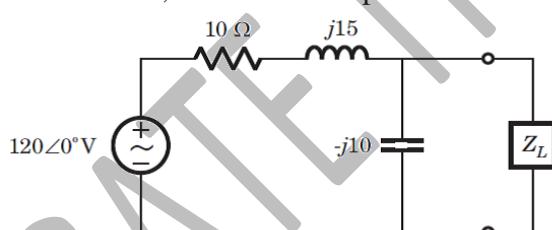


- A) $0.89 \cos(10^3 t - 63.43^\circ)$ B) $0.89 \cos(10^3 t + 63.43^\circ)$
 C) $0.45 \cos(10^3 t + 26.57^\circ)$ D) $0.45 \cos(10^3 t - 26.57^\circ)$

15. Determine the complete power for the values $P=269W$, $Q=150$ VAR (Capacitive)
 A) $150-j269$ VA B) $150+j269$ VA C) $269-j150$ VA D) $269+j150$ VA

16. Determine the complete power for the values $Q=2000$ VAR, $\text{pf}=0.9$ (leading)
 A) $4129.8-j2000$ VA B) $2000+j4129.8$ VA C) $2000-j4129.8$ VA D) $4129.8-j2000$ VA
 17. A balanced three-phase Y-connected load has one phase voltage $V_c=277\angle 45^\circ$ V. The phase sequence is abc. The line to line voltage V_{AB} is
 A) $480\angle 45^\circ$ V B) $480\angle -45^\circ$ V C) $339\angle 45^\circ$ V D) $339\angle -45^\circ$

18. In the circuit, the maximum power absorbed by Z_L is



- A) 180W B) 90W C) 140W D) 700W

19. To a highly inductive circuit, a small capacitance is added in series. The angle between voltage and current will

- A) decrease B) increase
 C) remain nearly the same D) become indeterminate

Answers:

1. D	2. D	3. B	4. B	5. A	6. C	7. D	8. A	9. A
10. B	11. B	12. C	13. B	14. A	15. C	16. D	17. B	18. A
19. A								

CHAPTER V

TWO PORT NETWORK

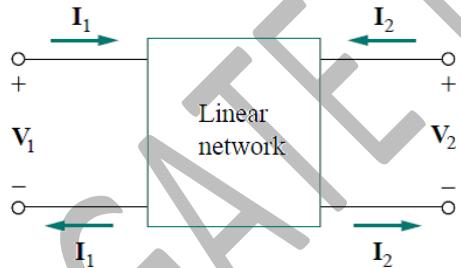
TWO PORT NETWORK

A pair of terminals through which a current may enter or leave a network is known as a port. In other word we can say that the pair of terminal from which power can be delivering or dissipating is called as port. Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one port networks as shown below.



- The current entering one terminal leaves through the other terminal so that the net current entering the port equals zero

A two-port network is an electrical network with two separate ports generally one for input and other for output.



Thus, a two-port network has two terminal pairs acting as access points. The current entering one terminal of a pair leaves the other terminal in the pair. Three-terminal devices such as transistors can be configured into two port networks by taking one terminal common for both input and output.

To analyze a two-port network requires that we relate the terminal quantities V_1 , V_2 , I_1 , and I_2 as show in above fig, out of which two are independent and two are dependant parameters so we can make six (${}^4C_2=6$) different relation between 2 dependant variable with 2 independent variables. The various terms that relate these voltages and currents are called parameters so there are six different type of parameter.

The parameter of the two port network:

1. Impedance parameters (z- parameter)
2. Admittance parameters (y-parameter)
3. Hybrid parameters (h-parameter)
4. Inverse hybrid parameters (g- parameter)
5. Transmission parameters (T parameter)
6. Inverse transmission parameters (T' parameter)

1. Impedance parameters (z- parameter):

Impedance and admittance parameters are commonly used in the synthesis of filters. They are also useful in the design and analysis of impedance-matching networks and power distribution networks.

In the Z-parameter the terminal voltages can be related to the terminal currents as

$$\mathbf{V}_1 = \mathbf{Z}_{11}\mathbf{I}_1 + \mathbf{Z}_{12}\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{Z}_{21}\mathbf{I}_1 + \mathbf{Z}_{22}\mathbf{I}_2$$

Where the **Z** terms are called the impedance parameters, or simply z parameters, and have units of ohms.

$$\begin{aligned} \mathbf{z}_{11} &= \left. \frac{\mathbf{V}_1}{\mathbf{I}_1} \right|_{\mathbf{I}_2=0}, & \mathbf{z}_{12} &= \left. \frac{\mathbf{V}_1}{\mathbf{I}_2} \right|_{\mathbf{I}_1=0} \\ \mathbf{z}_{21} &= \left. \frac{\mathbf{V}_2}{\mathbf{I}_1} \right|_{\mathbf{I}_2=0}, & \mathbf{z}_{22} &= \left. \frac{\mathbf{V}_2}{\mathbf{I}_2} \right|_{\mathbf{I}_1=0} \end{aligned}$$

Since the z parameters are obtained by open-circuiting the input or output port, they are also called the open-circuit impedance parameters. Specifically,

\mathbf{z}_{11} = Open-circuit input impedance

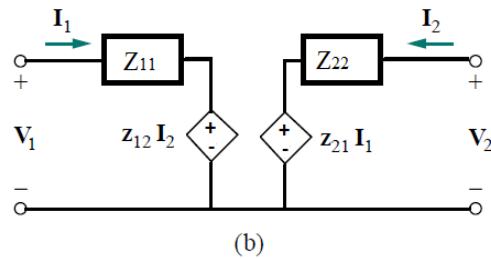
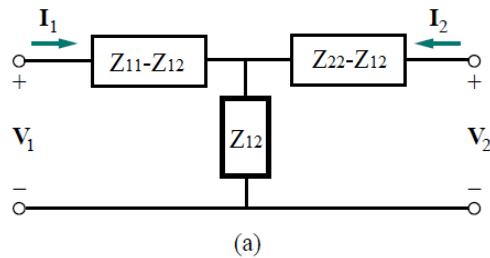
\mathbf{z}_{12} = Open-circuit transfer impedance from port 1 to port 2

\mathbf{z}_{21} = Open-circuit transfer impedance from port 2 to port 1

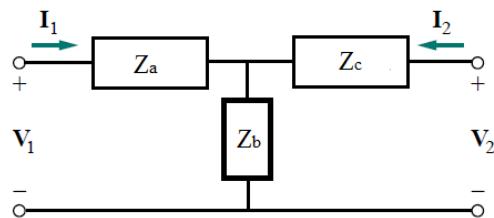
\mathbf{z}_{22} = Open-circuit output impedance

- \mathbf{z}_{11} and \mathbf{z}_{22} are called driving-point impedances, while \mathbf{z}_{21} and \mathbf{z}_{12} are called transfer impedances
- $\mathbf{z}_{11} = \mathbf{z}_{22}$, the two-port network is said to be symmetrical
- $\mathbf{z}_{12} = \mathbf{z}_{21}$, the two-port is said to be reciprocal (Any two-port that is made entirely of resistors, capacitors, and inductors i.e. by using linear element must be reciprocal).

For a reciprocal network, the T-equivalent circuit in **Fig.a** can be used. If the network is not reciprocal, a more general equivalent network is shown in **Fig.b**

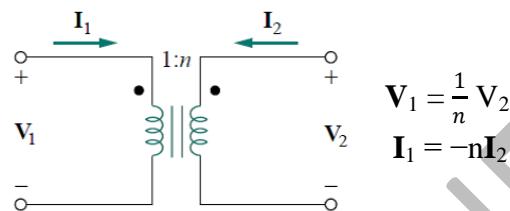


For linear T-network:



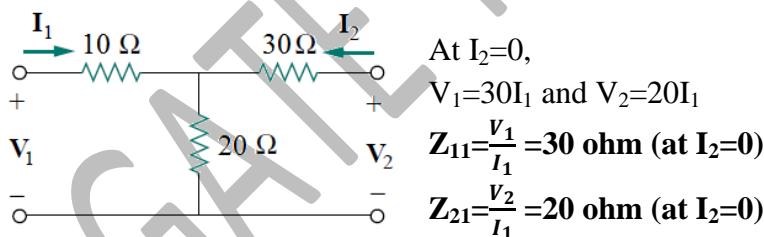
$$\begin{aligned} Z_{11} &= Z_a + Z_b \\ Z_{12} &= Z_{21} = Z_b \\ Z_{22} &= Z_c + Z_b \end{aligned}$$

For some two-port networks, the z parameters do not exist for example, consider the ideal transformer.



It is observed that it's impossible to express the voltages in terms of the currents, and vice versa. However, it does have hybrid parameters.

Example 1. Find the Z-parameter



At $I_1=0$,
 $V_1=20I_2$, and $V_2=50I_2$

$$Z_{12} = \frac{V_1}{I_2} = 20 \text{ ohm (at } I_1=0\text{)}$$

$$Z_{22} = \frac{V_2}{I_2} = 50 \text{ ohm (at } I_1=0\text{)}$$

Second method:

As it is T network,

$$Z_{11} = Z_a + Z_b = 10 + 20 = 30 \text{ ohm}$$

$$Z_{12} = Z_{21} = Z_b = 20 = 20 \text{ ohm}$$

$$Z_{22} = Z_c + Z_b = 30 + 20 = 50 \text{ ohm}$$

2. Admittance parameters (y-parameter):

In the Y-parameter the terminal currents can be related to the terminal voltages as:

$$\mathbf{I}_1 = \mathbf{y}_{11}\mathbf{V}_1 + \mathbf{y}_{12}\mathbf{V}_2$$

$$\mathbf{I}_2 = \mathbf{y}_{21}\mathbf{V}_1 + \mathbf{y}_{22}\mathbf{V}_2$$

The \mathbf{y} terms are known as the admittance parameters (y parameters) and have units of siemens. The values of the parameters will be determined by setting $\mathbf{V}_1 = 0$ (input port short-circuited) or $\mathbf{V}_2 = 0$ (output port short-circuited). Since the y parameters are obtained by short-circuiting the input or output port, they are also called the short-circuit admittance parameters.

$$\begin{aligned} \mathbf{y}_{11} &= \left. \frac{\mathbf{I}_1}{\mathbf{V}_1} \right|_{\mathbf{V}_2=0}, & \mathbf{y}_{12} &= \left. \frac{\mathbf{I}_1}{\mathbf{V}_2} \right|_{\mathbf{V}_1=0} \\ \mathbf{y}_{21} &= \left. \frac{\mathbf{I}_2}{\mathbf{V}_1} \right|_{\mathbf{V}_2=0}, & \mathbf{y}_{22} &= \left. \frac{\mathbf{I}_2}{\mathbf{V}_2} \right|_{\mathbf{V}_1=0} \end{aligned}$$

\mathbf{y}_{11} = Short-circuit input admittance

\mathbf{y}_{12} = Short-circuit transfer admittance from port 2 to port 1

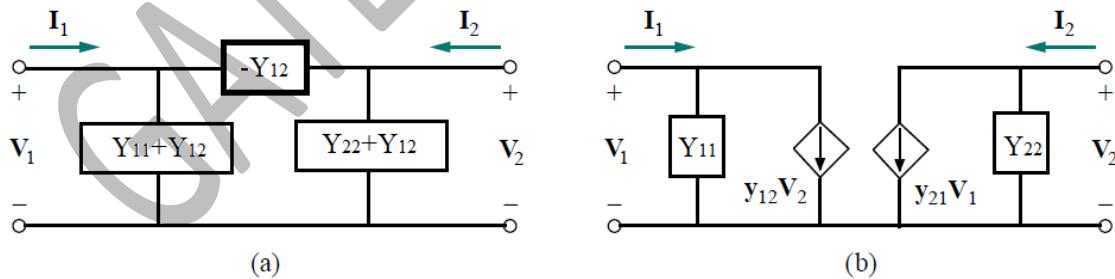
\mathbf{y}_{21} = Short-circuit transfer admittance from port 1 to port 2

\mathbf{y}_{22} = Short-circuit output admittance

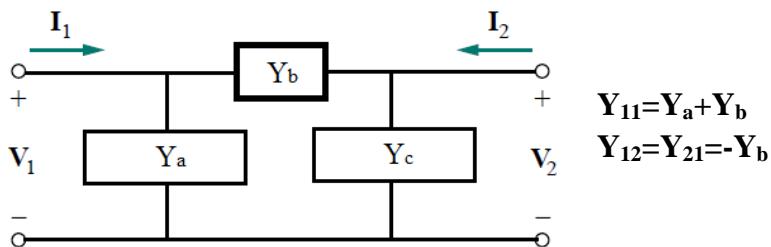
- \mathbf{Y}_{11} and \mathbf{Y}_{22} are called driving-point admittance, while \mathbf{Y}_{21} and \mathbf{Y}_{12} are called transfer impedances
- $\mathbf{Y}_{11} = \mathbf{Y}_{22}$, the two-port network is said to be symmetrical
- $\mathbf{Y}_{12} = \mathbf{Y}_{21}$, the two-port is said to be reciprocal (Any two-port that is made entirely of resistors, capacitors, and inductors i.e. by using linear element must be reciprocal).

A reciprocal network ($\mathbf{y}_{12} = \mathbf{y}_{21}$) can be modeled by the π -equivalent circuit as shown in Fig. a.

If the network is not reciprocal, a more general equivalent network is shown in Fig. b.



For linear π -network:



$$\mathbf{Y}_{22} = \mathbf{Y}_c + \mathbf{Y}_b$$

3. Hybrid parameters (h-parameter)

The z and y parameters of a two-port network do not always exist. So there is a need for developing another set of parameters. In hybrid parameter the dependant variable are V_1 and I_2 and independent variable are I_1 and V_2 .

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

The **h** terms are known as the hybrid parameters (h parameters) because they are a hybrid combination of ratios. They are very useful for describing electronic devices such as transistors.

$$h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0}, \quad h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0}$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0}, \quad h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0}$$

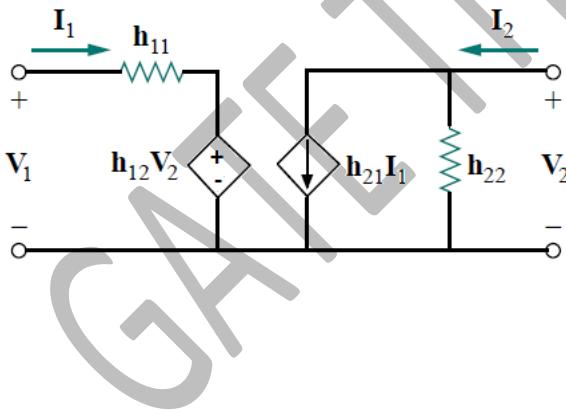
h_{11} = Short-circuit input impedance

h_{12} = Open-circuit reverse voltage gain

h_{21} = Short-circuit forward current gain

h_{22} = Open-circuit output admittance

General equivalent network is of h parameter is shown below:



4. Inverse hybrid parameters (g- parameter):

In inverse hybrid parameter the dependant variable are I_1 and V_2 and independent variable are V_1 and I_2 .

$$I_1 = g_{11}V_1 + g_{12}I_2$$

$$V_2 = g_{21}V_1 + g_{22}I_2$$

The **g** terms are known as the inverse hybrid parameters (g parameters)

$$g_{11} = \left. \frac{I_1}{V_1} \right|_{I_2=0}, \quad g_{12} = \left. \frac{I_1}{I_2} \right|_{V_1=0}$$

$$g_{21} = \left. \frac{V_2}{V_1} \right|_{I_2=0}, \quad g_{22} = \left. \frac{V_2}{I_2} \right|_{V_1=0}$$

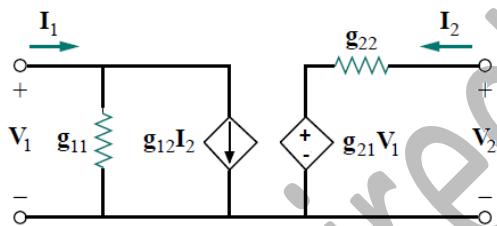
g_{11} = Open-circuit input admittance

g_{12} = Short-circuit reverse current gain

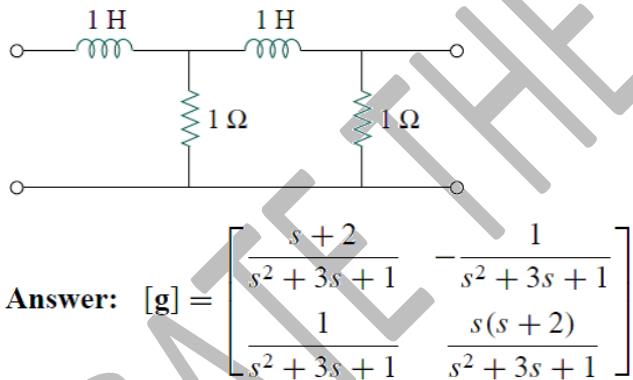
g_{21} = Open-circuit forward voltage gain

g_{22} = Short-circuit output impedance

General equivalent network is of h parameter is shown below:



Example: find out the g-parameter of following circuit



Answer: $[g] = \begin{bmatrix} \frac{s+2}{s^2 + 3s + 1} & -\frac{1}{s^2 + 3s + 1} \\ \frac{1}{s^2 + 3s + 1} & \frac{s(s+2)}{s^2 + 3s + 1} \end{bmatrix}$

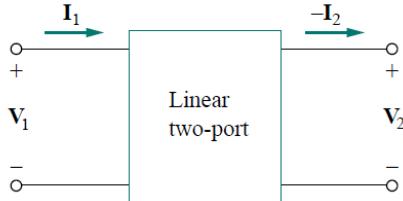
5. Transmission parameters (T parameter)

Transmission parameters are set of parameters that relate the variables at the input port to those at the output port.

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

In computing the transmission parameters, $-\mathbf{I}_2$ is used rather than \mathbf{I}_2 , because the current is considered to be leaving the network, as shown below:



I_2 opposed to entering the network. This is done merely for conventional reasons; when we cascade two-ports (output to input), it is most logical to think of I_2 as leaving the two-port. The two-port parameters are used to determine how a circuit transmits voltage and current from a source to a load. They are useful in the analysis of transmission lines (such as cable and fiber) because they express sending-end variables (V_1 and I_1) in terms of the receiving-end variables (V_2 and $-I_2$). For this reason, they are called transmission parameters. They are also known as **ABCD** parameters. They are used in the design of telephone systems, microwave networks, and radars. Since the transmission parameters provide a direct relationship between input and output variables, they are very useful in cascaded networks.

The transmission parameters are determined as

$$\begin{aligned} A &= \left. \frac{V_1}{V_2} \right|_{I_2=0}, & B &= -\left. \frac{V_1}{I_2} \right|_{V_2=0} \\ C &= \left. \frac{I_1}{V_2} \right|_{I_2=0}, & D &= -\left. \frac{I_1}{I_2} \right|_{V_2=0} \end{aligned}$$

A = Open-circuit voltage ratio

B = Negative short-circuit transfer impedance

C = Open-circuit transfer admittance

D = Negative short-circuit current ratio

A and **D** are dimensionless, **B** is in ohms, and **C** is in siemens

6. Inverse transmission parameters (T' parameter)

Inverse transmission parameters expressing the variables at the output port in terms of the variables at the input port.

$$V_2 = A'V_1 - B'I_1$$

$$I_2 = CV_1 - DI_1$$

The parameters **A'**, **B'**, **C'**, and **D'** are called the inverse transmission parameters.

$$A' = \frac{V_2}{V_1} \Big|_{I_1=0}, \quad B' = -\frac{V_2}{I_1} \Big|_{V_1=0}$$

$$C' = \frac{I_2}{V_1} \Big|_{I_1=0}, \quad D' = -\frac{I_2}{I_1} \Big|_{V_1=0}$$

A' = Open-circuit voltage gain

B' = Negative short-circuit transfer impedance

C' = Open-circuit transfer admittance

D' = Negative short-circuit current gain

Parameters	Dependant variable	Independent variable	Relation	Condition for 1.reciprocity 2.symmetry
Z- parameter	V ₁ , V ₂	I ₁ , I ₂	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	1. Z ₁₂ =Z ₂₁ 2. Z ₁₁ =Z ₂₂
Y- parameter	I ₁ , I ₂	V ₁ , V ₂	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$	1. Y ₁₂ =Y ₂₁ 2. Y ₁₁ =Y ₂₂
h- parameter	V ₁ , I ₂	I ₁ , V ₂	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$	1. h ₁₂ =-h ₂₁ 2. h ₁₁ h ₂₂ -h ₁₂ h ₂₁ =1
g- parameter	I ₁ , V ₂	V ₁ , I ₂	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$	1. g ₁₂ =-g ₂₁ 2. g ₁₁ g ₂₂ -g ₁₂ g ₂₁ =1
T- parameter	V ₁ , I ₁	V ₂ , I ₂	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$	1. AD-BC=1 2. A=D
T'- parameter	V ₂ , I ₂	V ₁ , I ₁	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$	1. A'D'-B'C'=1 2. A'=D'

Relationships between parameters:

Since the six sets of parameters relate the same input and output terminal variables of the same two-port network, they should be interrelated. If two sets of parameters exist, we can relate one set to the other set.

Example. Given the z parameters, let us obtain the y parameters

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Therefore $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ Also $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

i.e. $[Z]^{-1} = [Y]$

Similarly $[h]^{-1} = [g]$

But $[T]^{-1} \neq [T']$

	z		y		h		g		T		t	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$	$\frac{d}{c}$	$\frac{1}{c}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta_g}{g_{11}}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{\Delta_t}{c}$	$\frac{a}{c}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta_g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$	$\frac{a}{b}$	$-\frac{1}{b}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$	$\frac{1}{B}$	$\frac{A}{B}$	$-\frac{\Delta_t}{b}$	$\frac{d}{b}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta_g}$	$\frac{g_{12}}{\Delta_g}$	$\frac{B}{D}$	$\frac{\Delta_T}{D}$	$\frac{b}{a}$	$\frac{1}{a}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{g_{21}}{\Delta_g}$	$\frac{g_{11}}{\Delta_g}$	$-\frac{1}{D}$	$\frac{C}{D}$	$\frac{\Delta_t}{a}$	$\frac{c}{a}$
g	$\frac{1}{z_{11}}$	$-\frac{z_{12}}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta_h}$	$-\frac{h_{12}}{\Delta_h}$	$\frac{g_{11}}{g_{11}}$	g_{12}	$\frac{C}{A}$	$-\frac{\Delta_T}{A}$	$\frac{c}{d}$	$-\frac{1}{d}$
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta_z}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$-\frac{h_{21}}{\Delta_h}$	$\frac{h_{11}}{\Delta_h}$	g_{21}	g_{22}	$\frac{1}{A}$	$\frac{B}{A}$	$\frac{\Delta_t}{d}$	$-\frac{b}{d}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{h_{21}}{h_{21}}$	$\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$	A	B	$\frac{d}{\Delta_t}$	$\frac{b}{\Delta_t}$
	$\frac{1}{z_{22}}$	$-\frac{\Delta_y}{z_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{y_{12}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta_g}{g_{21}}$	C	D	$\frac{c}{\Delta_t}$	$\frac{a}{\Delta_t}$
t	$\frac{z_{22}}{z_{12}}$	$\frac{\Delta_z}{z_{12}}$	$-\frac{y_{11}}{y_{12}}$	$-\frac{1}{y_{12}}$	$-\frac{h_{11}}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$-\frac{\Delta_g}{g_{12}}$	$-\frac{g_{22}}{g_{12}}$	$\frac{D}{\Delta_T}$	$\frac{B}{\Delta_T}$	a	b
	$\frac{1}{z_{12}}$	$\frac{z_{01}}{z_{12}}$	$-\frac{\Delta_y}{y_{12}}$	$-\frac{y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta_h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$	$-\frac{1}{g_{12}}$	$\frac{C}{\Delta_T}$	$\frac{A}{\Delta_T}$	c	d

$$\Delta_z = z_{11}z_{22} - z_{12}z_{21}, \quad \Delta_h = h_{11}h_{22} - h_{12}h_{21}, \quad \Delta_T = AD - BC$$

$$\Delta_y = y_{11}y_{22} - y_{12}y_{21}, \quad \Delta_g = g_{11}g_{22} - g_{12}g_{21}, \quad \Delta_t = ad - bc$$

Here Inverse transmission parameter taken as a,b,c,d

Example: Find [z] and [g] of a two-port network if

$$[T] = \begin{bmatrix} 10 & 1.5 \Omega \\ 2 S & 4 \end{bmatrix}$$

Answer:

$$[z] = \begin{bmatrix} 5 & 18.5 \\ 0.5 & 2 \end{bmatrix} \Omega, \quad [g] = \begin{bmatrix} 0.2 S & -3.7 \\ 0.1 & 0.15 \Omega \end{bmatrix}$$

INTERCONNECTION OF NETWORKS:

A large, complex network may be divided into sub networks so it will be simplify the analysis and design. The sub networks are modeled as two-port networks and interconnect them to form the original network. The interconnection can be in series, in parallel, or in cascade. Generally,

- When the networks are in series, their individual z parameters add up to give the z parameters of the larger network.
- When network are in parallel, their individual y parameters add up to give the y parameters the larger network.
- When network are in cascaded, their individual transmission parameters can be multiplied together to get the transmission parameters of the larger network.

1. Two sub network are connected in Series:

For network N_a ,

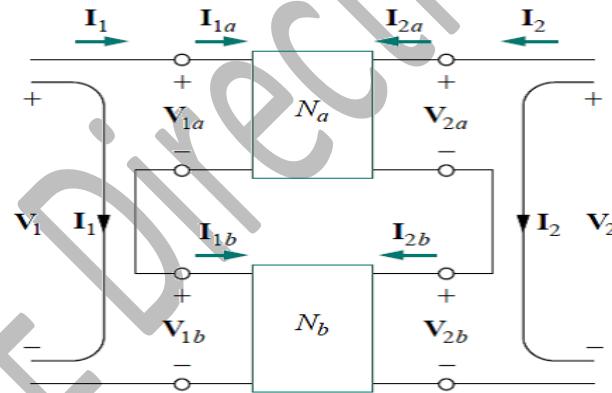
$$\mathbf{V}_{1a} = \mathbf{z}_{11a}\mathbf{I}_{1a} + \mathbf{z}_{12a}\mathbf{I}_{2a}$$

$$\mathbf{V}_{2a} = \mathbf{z}_{21a}\mathbf{I}_{1a} + \mathbf{z}_{22a}\mathbf{I}_{2a}$$

For network N_b ,

$$\mathbf{V}_{1b} = \mathbf{z}_{11b}\mathbf{I}_{1b} + \mathbf{z}_{12b}\mathbf{I}_{2b}$$

$$\mathbf{V}_{2b} = \mathbf{z}_{21b}\mathbf{I}_{1b} + \mathbf{z}_{22b}\mathbf{I}_{2b}$$



From circuit:

$$\mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b}, \mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b}$$

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b} = (\mathbf{z}_{11a} + \mathbf{z}_{11b})\mathbf{I}_1 + (\mathbf{z}_{12a} + \mathbf{z}_{12b})\mathbf{I}_2$$

$$\mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b} = (\mathbf{z}_{21a} + \mathbf{z}_{21b})\mathbf{I}_1 + (\mathbf{z}_{22a} + \mathbf{z}_{22b})\mathbf{I}_2$$

Therefore over all z-parameter as:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11a} + Z_{11b} & Z_{12a} + Z_{12b} \\ Z_{21a} + Z_{21b} & Z_{22a} + Z_{22b} \end{bmatrix}$$

i.e. $[Z] = [z_a] + [z_b]$

So that the z parameters for the overall network are the sum of the z parameters for the individual networks connected in series.

2.Two sub network are connected in parallel:

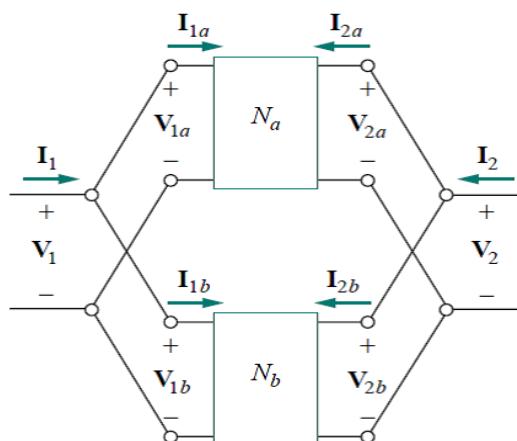
For network N_a ,

$$\mathbf{I}_{1a} = \mathbf{y}_{11a}\mathbf{V}_{1a} + \mathbf{y}_{12a}\mathbf{V}_{2a}$$

$$\mathbf{I}_{2a} = \mathbf{y}_{21a}\mathbf{V}_{1a} + \mathbf{y}_{22a}\mathbf{V}_{2a}$$

For network N_b ,

$$\mathbf{I}_{1b} = \mathbf{y}_{11b}\mathbf{V}_{1b} + \mathbf{y}_{12b}\mathbf{V}_{2b}$$



$$\mathbf{I}_{2a} = \mathbf{y}_{21b}\mathbf{V}_{1b} + \mathbf{y}_{22b}\mathbf{V}_{2b}$$

From above block diagram:

$$\mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b}; \mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b} \text{ and } \mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b}; \mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b}$$

$$\mathbf{I}_1 = (\mathbf{y}_{11a} + \mathbf{y}_{11b})\mathbf{V}_1 + (\mathbf{y}_{12a} + \mathbf{y}_{12b})\mathbf{V}_2$$

$$\mathbf{I}_2 = (\mathbf{y}_{21a} + \mathbf{y}_{21b})\mathbf{V}_1 + (\mathbf{y}_{22a} + \mathbf{y}_{22b})\mathbf{V}_2$$

The overall y-parameter as:

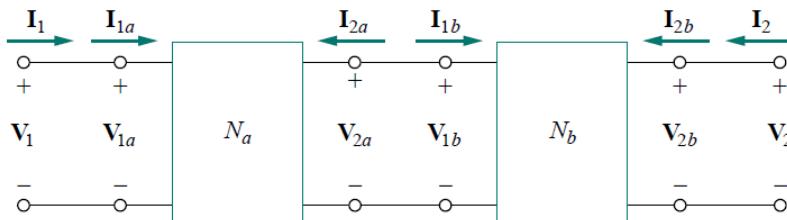
$$\begin{bmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_{11a} + \mathbf{y}_{11b} & \mathbf{y}_{12a} + \mathbf{y}_{12b} \\ \mathbf{y}_{21a} + \mathbf{y}_{21b} & \mathbf{y}_{22a} + \mathbf{y}_{22b} \end{bmatrix}$$

i.e. $[\mathbf{y}] = [\mathbf{y}_a] + [\mathbf{y}_b]$

So that the y parameters for the overall network are the sum of the y parameters for the individual networks connected in parallel.

3.Two sub network are connected in cascade:

Two networks are said to be cascaded when the output of one is the input of the other.



For network N_a ,

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

For network N_b ,

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

Also, $\mathbf{V}_1 = \mathbf{V}_{1a}$, and $\mathbf{I}_1 = \mathbf{I}_{1a}$

$\mathbf{V}_2 = \mathbf{V}_{2a}$, and $\mathbf{I}_2 = \mathbf{I}_{2a}$

$\mathbf{V}_{2a} = \mathbf{V}_{1b}$, and $\mathbf{I}_{2a} = -\mathbf{I}_{1b}$

From above equations

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix}$$

$$[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b]$$

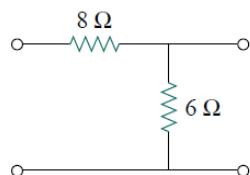
Thus, the transmission parameters for the overall network are the product of the transmission parameters for the individual transmission parameters connected in cascade.

Notes:

- As $[Z]=[Y]^{-1}$, so that if $|Z|=0$ then for that network we cannot able to define Y-parameter similarly if $|Y|=0$, then for that network we can't able to define Z-parameter.
- As $[h]=[g]^{-1}$, so that if $|h|=0$ then for that network we cannot able to define g-parameter similarly if $|g|=0$, then for that network we can't able to define h-parameter.

Problems:

1. Find the z-parameter of following



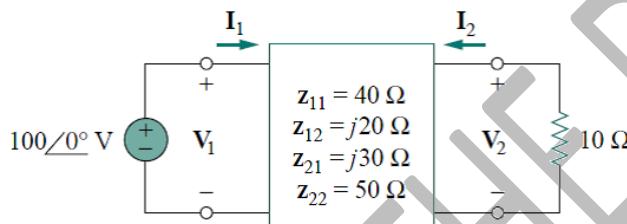
A) $\begin{bmatrix} 8 & 6 \\ 6 & 6 \end{bmatrix}$

B) $\begin{bmatrix} 14 & 6 \\ 6 & 6 \end{bmatrix}$

C) $\begin{bmatrix} 8 & 6 \\ 6 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 14 & 6 \\ 6 & 0 \end{bmatrix}$

2. Find I_1 and I_2 respectively



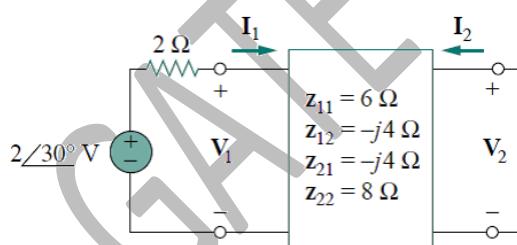
A) $2 < 0 \text{ A}, 1 < -90 \text{ A}$

C) $2 < -90 \text{ A}, 1 < -90 \text{ A}$

B) $2 < 0 \text{ A}, 1 < 90 \text{ A}$

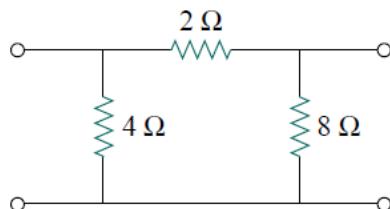
D) $2 < 90 \text{ A}, 1 < -90 \text{ A}$

3. Calculate I_1 and I_2 in the given two-port network



A) $2 < 20 \text{ A}, 1 < -60 \text{ A}$ B) $1 < 20 \text{ A}, 1 < -60 \text{ A}$ C) $1 < 20 \text{ A}, 2 < -60 \text{ A}$ D) $2 < -20 \text{ A}, 1 < -60 \text{ A}$

4. Obtain the y parameters for the π network given below



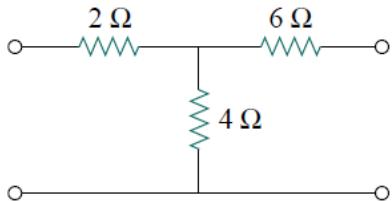
A) $\begin{bmatrix} 6 & -2 \\ -2 & 10 \end{bmatrix}$

B) $\begin{bmatrix} 0.75 & -0.5 \\ -0.5 & 0.625 \end{bmatrix}$

C) $\begin{bmatrix} 6 & 2 \\ 2 & 10 \end{bmatrix}$

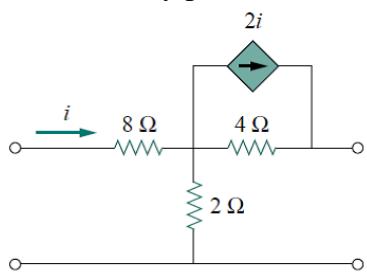
D) $\begin{bmatrix} 0.75 & 0.5 \\ 0.5 & 0.625 \end{bmatrix}$

5. Obtain the y parameters for the T network shown below



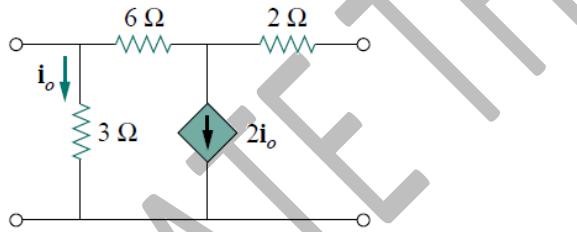
A) $\begin{bmatrix} 0.227 & -0.09 \\ -0.09 & 0.136 \end{bmatrix}$ B) $\begin{bmatrix} 0.227 & 0.09 \\ 0.09 & 0.136 \end{bmatrix}$ C) $\begin{bmatrix} -0.227 & -0.09 \\ -0.09 & -0.136 \end{bmatrix}$ D) $\begin{bmatrix} 0.25 & -0.1 \\ -0.1 & 0.2 \end{bmatrix}$

6. Obtain the y parameters for the T network shown below



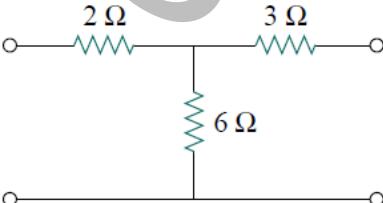
A) $\begin{bmatrix} 0.25 & -0.05 \\ -0.05 & 0.25 \end{bmatrix}$ B) $\begin{bmatrix} 0.15 & -0.05 \\ -0.05 & 0.25 \end{bmatrix}$ C) $\begin{bmatrix} 0.15 & -0.05 \\ -0.25 & 0.25 \end{bmatrix}$ D) $\begin{bmatrix} 0.25 & -0.5 \\ -0.5 & 0.625 \end{bmatrix}$

7. Obtain the y parameters for the T network shown below



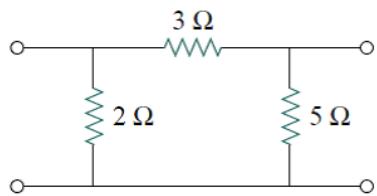
A) $\begin{bmatrix} 0.625 & -0.125 \\ -0.375 & 0.125 \end{bmatrix}$ B) $\begin{bmatrix} 0.625 & -0.125 \\ -0.125 & 0.125 \end{bmatrix}$ C) $\begin{bmatrix} 0.625 & -0.125 \\ 0.375 & 0.125 \end{bmatrix}$ D) $\begin{bmatrix} 0.625 & 0.125 \\ 0.375 & 0.125 \end{bmatrix}$

8. Find the hybrid parameters of the following circuit



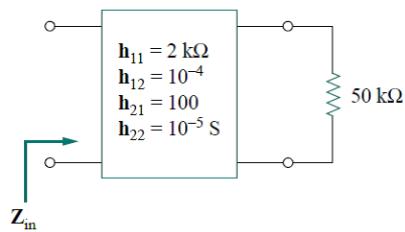
A) $\begin{bmatrix} 4 & 2/3 \\ -2/3 & 1/9 \end{bmatrix}$ B) $\begin{bmatrix} 4 & 2 \\ -3 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 4 & -2/3 \\ -2/3 & 1/9 \end{bmatrix}$ D) $\begin{bmatrix} 4 & -2 \\ -2 & 1/9 \end{bmatrix}$

9. Find the hybrid parameters of the following circuit



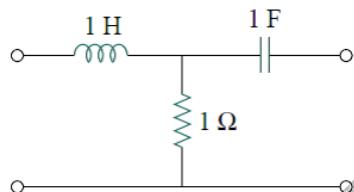
- A) $\begin{bmatrix} 1.2 & 0.4 \\ -0.4 & 0.4 \end{bmatrix}$ B) $\begin{bmatrix} 0.4 & 0.4 \\ -0.4 & 0.4 \end{bmatrix}$ C) $\begin{bmatrix} 1.2 & -0.4 \\ -0.4 & 0.4 \end{bmatrix}$ D) $\begin{bmatrix} 0.4 & -0.4 \\ -0.4 & 0.4 \end{bmatrix}$

10. Find the impedance at the input port of the circuit (in ohm)



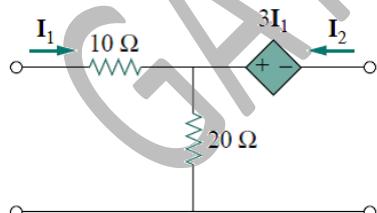
- A) 1667 B) 2000 C) 1000 D) 1500

11. Find the hybrid parameters of the following circuit



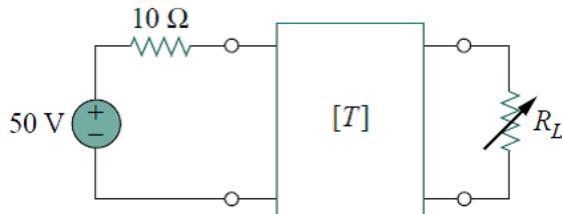
- A) $\begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+1} \\ \frac{1}{s+1} & \frac{s^2+s+1}{s(s+1)} \end{bmatrix}$ B) $\begin{bmatrix} 1 & -\frac{1}{s+1} \\ \frac{1}{s+1} & \frac{s^2+s+1}{s(s+1)} \end{bmatrix}$ C) $\begin{bmatrix} \frac{1}{s+1} & -\frac{1}{s+1} \\ \frac{1}{s+1} & \frac{s^2+s+1}{s(s+1)} \end{bmatrix}$ D) $\begin{bmatrix} -1 & -\frac{1}{s+1} \\ \frac{1}{s+1} & \frac{s^2+s+1}{s(s+1)} \end{bmatrix}$

12. Find the transmission parameters for the two-port network given below



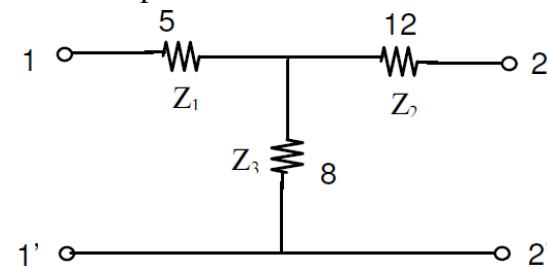
- A) $\begin{bmatrix} 30/17 & 1/17 \\ 260/17 & 20/17 \end{bmatrix}$ B) $\begin{bmatrix} 30/17 & 260/17 \\ 1/17 & 20/17 \end{bmatrix}$
 C) $\begin{bmatrix} 30/17 & -260/17 \\ 1/17 & 20/17 \end{bmatrix}$ D) $\begin{bmatrix} 30/17 & 260/17 \\ 1/17 & 1/17 \end{bmatrix}$

13. The ABCD parameters of the two-port network is given as $\begin{bmatrix} 4 & 20 \\ 0.1 & 2 \end{bmatrix}$. The output port is connected to a variable load for maximum power transfer. Find R_L



- A) 7 B) 8 C) 9 D) 10

14. The z-parameters of the shown T-network are given by



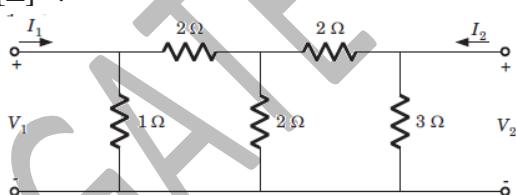
- A) 5, 8, 12, 0 B) 13, 8, 8, 20 C) 8, 20, 13, 12 D) 5, 8, 8, 12

Answers:

1. B	2. A	3. A	4. B	5. A	6. C	7. C	8. A	9. A
10. A	11. C	12. B	13. B	14. B				

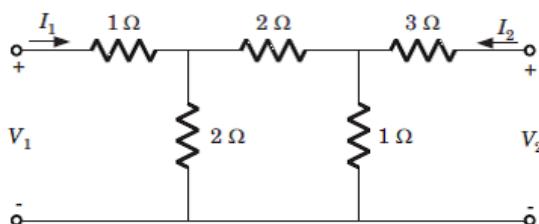
Additional Problems :

1) $[Z] = ?$



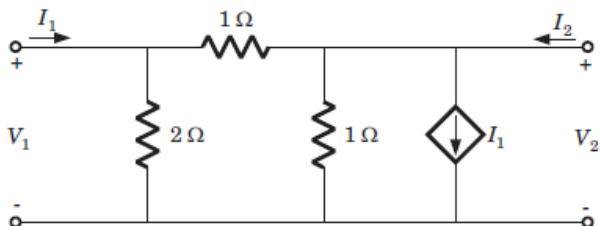
- A) $\begin{bmatrix} 21/16 & 1/8 \\ 1/8 & 7/12 \end{bmatrix}$ B) $\begin{bmatrix} 7/9 & 1/6 \\ 1/6 & 7/4 \end{bmatrix}$ C) $\begin{bmatrix} 21/16 & -1/8 \\ -1/8 & 7/12 \end{bmatrix}$ D) none

2) $[y] = ?$

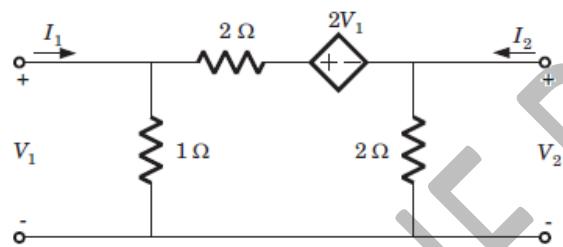


A) $\begin{bmatrix} 11/41 & 2/41 \\ 2/41 & 19/41 \end{bmatrix}$
 C) $\begin{bmatrix} 19/41 & 2/41 \\ 2/41 & 11/41 \end{bmatrix}$

B) $\begin{bmatrix} 11/41 & -2/41 \\ -2/41 & 19/41 \end{bmatrix}$
 D) $\begin{bmatrix} 19/41 & -2/41 \\ -2/41 & 11/41 \end{bmatrix}$

3) $[y]=?$ 

A) $\begin{bmatrix} 1/2 & 1 \\ 3/2 & -1 \end{bmatrix}$ B) $\begin{bmatrix} 3/2 & -1 \\ 1/2 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1/2 & 1/2 \\ -1/4 & 3/4 \end{bmatrix}$ D) $\begin{bmatrix} -1/4 & 3/4 \\ 1/2 & 1/2 \end{bmatrix}$

4) $[Z]=?$ 

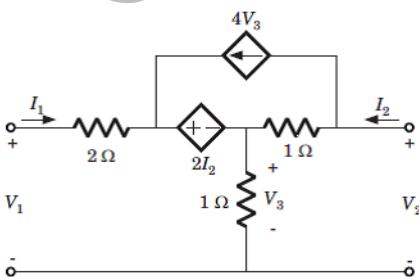
A) $\begin{bmatrix} 4/3 & 2/3 \\ -2/3 & 2/3 \end{bmatrix}$ B) $\begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1 \end{bmatrix}$ C) $\begin{bmatrix} -2/3 & 2/3 \\ 4/3 & 2/3 \end{bmatrix}$ D) $\begin{bmatrix} 1/2 & 1 \\ 1/2 & -1/2 \end{bmatrix}$

5) A two port is described by $V_1=I_1+2V_2$, $I_2=-2I_1+0.4V_2$, $[h]=?$

A) $\begin{bmatrix} 3 & -6 \\ 4 & -4 \end{bmatrix}$ B) $\begin{bmatrix} 4 & -2 \\ -2 & 4.4 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 2 \\ -2 & 0.4 \end{bmatrix}$ D) $\begin{bmatrix} 11 & 5 \\ 5 & 2.5 \end{bmatrix}$

6) A two port is described by $V_1=I_1+2V_2$, $I_2=-2I_1+0.4V_2$, $[T]=?$

A) $\begin{bmatrix} 2.2 & 0.5 \\ 0.2 & 0.5 \end{bmatrix}$ B) $\begin{bmatrix} 2.2 & -0.5 \\ 0.2 & -0.5 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 2 \\ -2 & 0.4 \end{bmatrix}$ D) $\begin{bmatrix} 1 & -2 \\ -2 & -0.4 \end{bmatrix}$

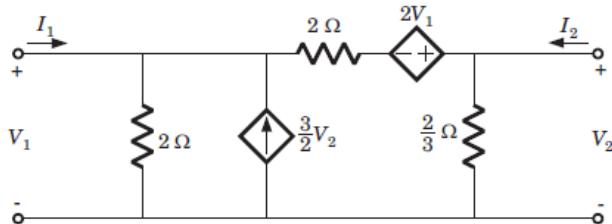
7) $[Z]=?$ 

A) $\begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix}$

B) $\begin{bmatrix} -3 & -2 \\ 3 & 3 \end{bmatrix}$

C) $\begin{bmatrix} 3 & 3 \\ 3 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 3 \\ -3 & -2 \end{bmatrix}$

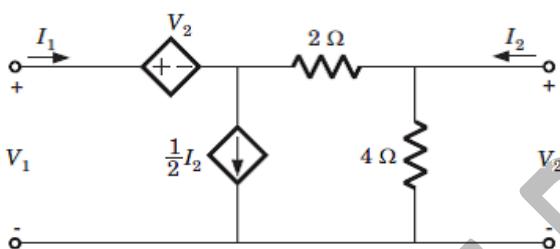
8) $[Z] = ?$ 

A) $\begin{bmatrix} 2 & 2 \\ 3/2 & 2 \end{bmatrix}$

B) $\begin{bmatrix} -2 & 3/2 \\ 2 & -2 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 3/2 \\ 2 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 2 & -2 \\ -3/2 & 2 \end{bmatrix}$

9) $[h] = ?$ 

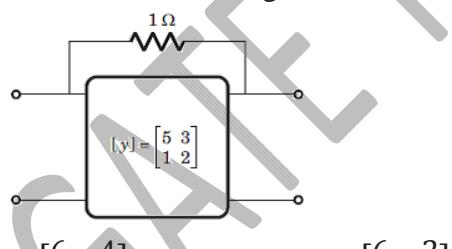
A) $\begin{bmatrix} 4 & 3/2 \\ -2 & 1/2 \end{bmatrix}$

B) $\begin{bmatrix} -2 & 1/2 \\ 4 & 3/2 \end{bmatrix}$

C) $\begin{bmatrix} 4 & -3/2 \\ 2 & 1/2 \end{bmatrix}$

D) $\begin{bmatrix} 2 & 1/2 \\ 4 & -3/2 \end{bmatrix}$

10) The y-parameters of a 2-port network are $[y] = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} S$. A resistor of 1 ohm is connected across as shown in figure. The new y-parameter would be



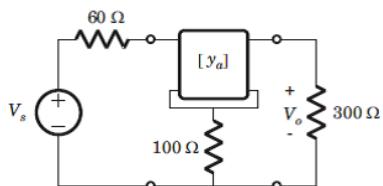
A) $\begin{bmatrix} 6 & 4 \\ 2 & 3 \end{bmatrix} S$

B) $\begin{bmatrix} 6 & 2 \\ 0 & 3 \end{bmatrix} S$

C) $\begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix} S$

D) $\begin{bmatrix} 4 & 4 \\ 2 & 1 \end{bmatrix} S$

11) For the 2-port of figure, $[y] = \begin{bmatrix} 2 & 0 \\ 0 & 10 \end{bmatrix} mS$



The value of V_o/V_s is

A) 3/32

B) 1/16

C) 2/33

D) 1/17

12) A 2-port network is driven by a source $V_S = 100$ V in series with 5Ω , and terminated in a 25Ω resistor. The impedance parameters are $[z] = \begin{bmatrix} 20 & 2 \\ 40 & 10 \end{bmatrix} \Omega$.

The thevenin's equivalent circuit presented to the 25Ω resistor is

- A) 80 V, 2.8Ω B) 160 V, 6.8Ω C) 100 V, 2.4Ω D) 120 V, 6.4Ω

13) For a 2-port symmetrical bilateral network, if transmission parameters $A = 3$ and $B = 1\Omega$, the value of parameter C is

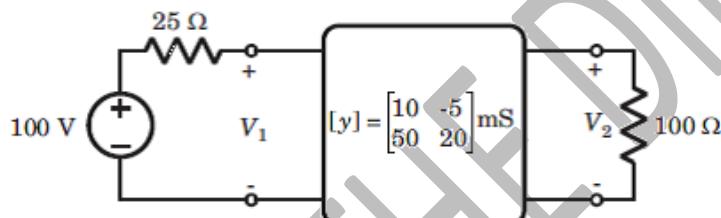
- A) 3 B) 8 S C) 8 D) 9

14) A 2-port resistive network satisfy the condition $A=D=3/2$ $B = 4/3 C$.

The z_{11} of the network is

- A) $4/3$ B) $3/4$ C) $2/3$ D) $3/2$

15) Find V_1 & V_2



- A) -68.6 V, 114.3 V B) 68.6 V, -114.3 V C) 114.3 V, -68.6 V D) -114.3 V, 68.6 V

Answers:

1. D	2. D	3. B	4. A	5. C	6. B	7. D	8. A	9. A
10. B	11. A	12. B	13. B	14. A	15. B			

CHAPTER VI

GRAPH THEORY

Graph Theory

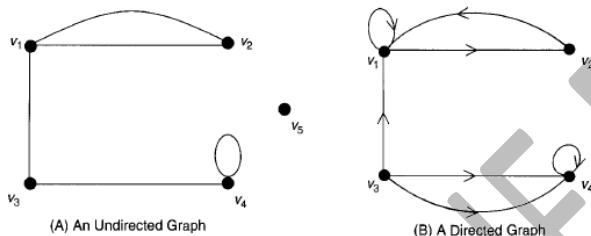
A graph $G = (N, E)$ consists of n -node & b -edges having two sets: a finite set $N(a) = (n_1, n_2 \dots n_n)$ of elements called nodes and a finite set $E = (e_1, e_2 \dots e_b)$ of elements called edges.

Simple Graph: The graph in which at most one edge is available between any two nodes is called as simple graph.

Complete Graph: The graph in which one edge is present between every pair of nodes is called as complete graph.

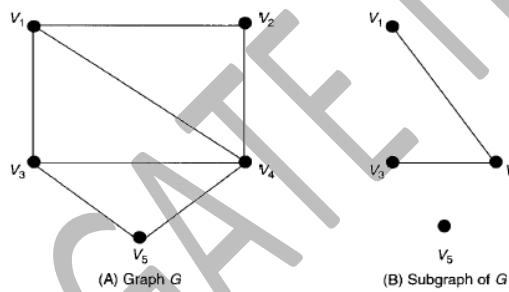
$$B = [n(n-1)]/2$$

Directed Graph: If the edges of G are identified with ordered pairs of nodes or direction of graph is provided, then G is called a directed or an oriented graph; otherwise, it is called an undirected graph.

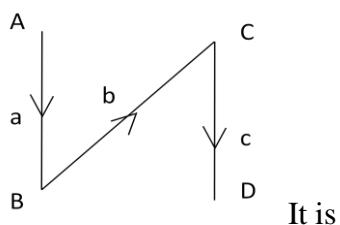


Sub Graph: It contain some of the branches of original graph.

$G = (N, E)$, & $G_1 = (N_1, E_1)$, then G_1 will be sub graph of G if (N_1, E_1) are sub set of (N, E) .



Matrix:

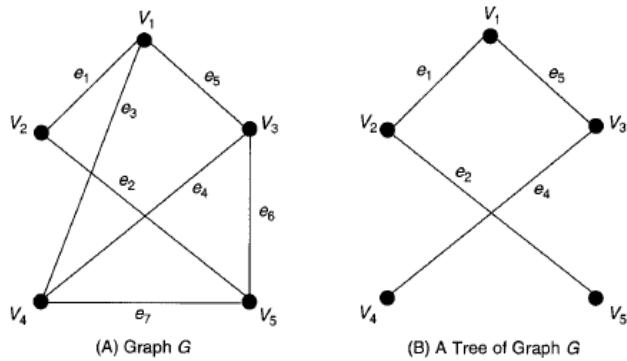


It is

	A	B	C
A	1	0	0
B	-1	1	0
C	0	-1	1
D	0	0	-1

Tree:
sub graph which contain all

the nodes and not any close loop.



Number of possible different tree = $n^{(n-2)}$

Link = graph – tree

$$L = b - (n-1)$$

Incidence Matrix (A_I):

The Incidence matrix will get by applying KCL at all the node.

Rank of Incidence matrix is = rank of graph = $(n-1)$

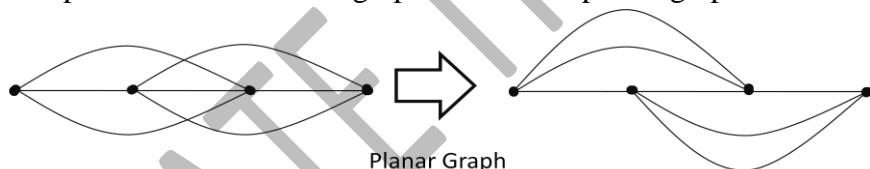
In Incidence matrix all nodes are write in row and all branch write in column. So Incidence matrix is rectangular matrix of $(n \times b)$.

Rank of Incidence matrix is $(n-1)$. As addition of every column is zero. So we can reduce incidence matrix by removing any row.

Number of possible tree = $|A_r A_r^T|$, as $A_r \rightarrow$ reduce Incidence matrix.

Planar Graph:

If we draw graph on 2-D plane then the graph in which no two edges of graph never intersect except out node. Then that graph is called as planar graph.



For simplification of any network we need all branch currents & all node voltages even if we know either all branch voltage or all branch current then we can find out other things.

For finding all branch currents, we required at least minimum branch currents that should known these are loop current.

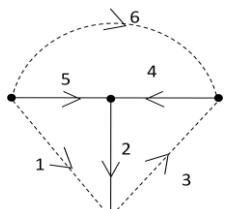
If we know loop currents then we can calculate all branch currents just by apply KCL.

Tie Set Matrix (fundamental loop matrix) B_f :

As we can calculate all branch voltage by applying KVL i.e. loop. The minimum number of loop required to solved any given network is number of link. The addition of any link make a close loop and we will get one KVL equation. Similarly for '1' link we have '1' number of different loop equations. These loop equations are called as Tie Set and corresponding to these equation is called as tie set matrix or fundamental loop matrix. It has $(b-n+1)$ number of row (i.e. number of link) & 'b' number of column (i.e. number of branches).

Method:

- 1) 1st select any tree
- 2) Add one link at a time. One link make one loop and assign loop current in direction of link.
- 3) For 'l' link we get 'l' different loop. By solving all of these loops, we will get all branch voltages.

Example:

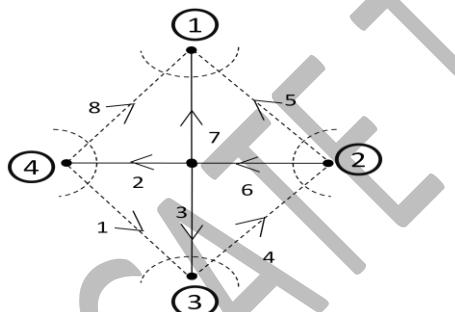
Link / Branch

$$\begin{array}{ll} & \begin{matrix} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \end{matrix} \\ f_1 & \left[\begin{matrix} 1 & -1 & 0 & 0 & -1 & 0 \end{matrix} \right]_{l \times b} \\ f_2 & \left[\begin{matrix} 0 & 1 & 1 & 1 & 0 & 0 \end{matrix} \right]_{l \times b} \\ f_3 & \left[\begin{matrix} 0 & 0 & 0 & 1 & -1 & 1 \end{matrix} \right]_{l \times b} \end{array} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}_{b \times l}$$

Every tree gives one tie set matrix. So total number of possible tie set matrix for given graph is number of possible tree. i.e. $|A_r A_r^T|$.

Cut Set Matrix:

It is minimum number of branches required to remove. So that graph will divide into two part. Suppose 'G' is connected graph & we have to divide it into two part.



Number of twig/ branches

<i>cut-set</i>	1	2	3	4	5	6	7	8
(1,2,8)	2	$\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$						
(1,3,4)	3	$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$						
(4,5,6)	6	$\begin{bmatrix} 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 \end{bmatrix}$						
(5,7,8)	7	$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}_{(n-1) \times b}$						

Fundamental Cut Set matrix:

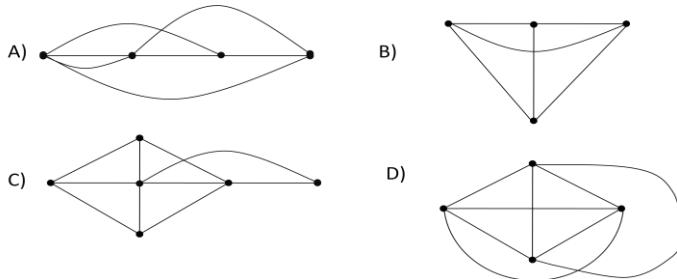
The twigs are connected to each and node. So if we make voltage of all the twig is zero, then voltage drop across each and every branch is zero. i.e. we can calculate the current in each and

every branch by applying KCL at end of every twig. So we have $(n-1)$ different KCL equations these are sufficient to calculate the current through each & every branch at network.

- The number of fundamental cut set matrix possible is equal to number of trees
- The number of fundamental tie set matrix possible is equal to number of trees

Problems:

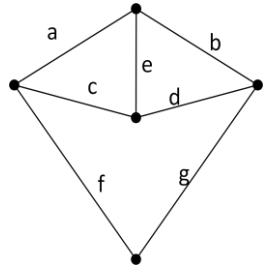
1) Which of the following is non planner?



2) The complete graph has 4 nodes. Find the number of branches.

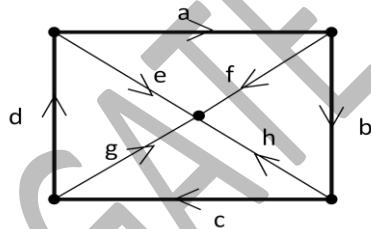
- A) 4 B) 6 C) 8 D) 16

3) Which of the following is the tree?



- A) (b, d, c, f) B) (a, b, c, d) C) (a, c, e, g, f) D) none

4) Find the twig of following network?

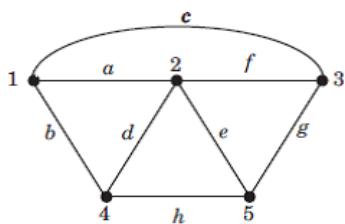


- (1) a, c, h (2) a, c, g (3) a, c, g, f (4) a, d, c, h, f
 A) 1 & 2 B) only 3 C) only 4 D) 1, 2, & 3

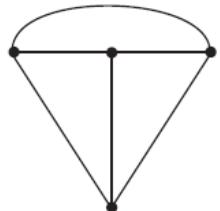
5) A graph of an electrical network has 4 nodes and 7 branches. The number of links l, with respect to the chosen tree, would be

- A) 2 B) 3 C) 4 D) 5

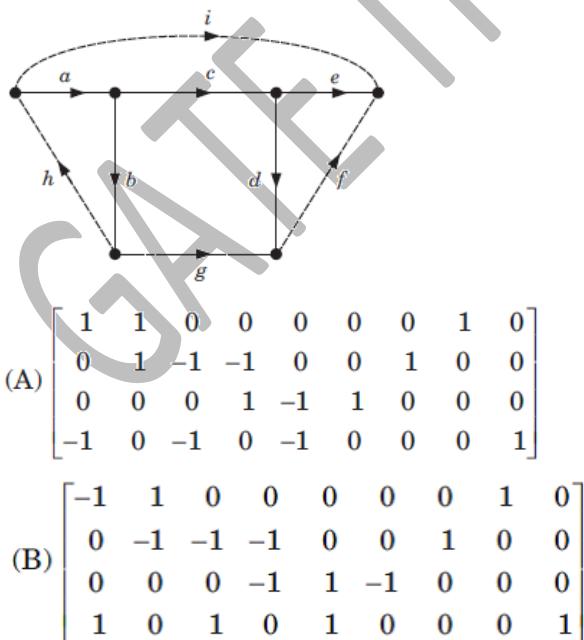
6) A tree of the graph shown in figure is



- A) a, d, e, h B) a, c, f, h C) a, f, h, g D) a, e, f, g
 7) The graph of a network is shown in figure. The number of possible tree are



- A) 8 B) 12 C) 16 D) 20
 8) If the number of branch in a network is b, the number of nodes is n and the number of dependent loop is l, then the number of independent node equations will be
 A) n B) b - 1 C) b - n + 1 D) n - 1
 9) A network has 8 nodes and 5 independent loops. The number of branches in the network is
 A) 11 B) 12 C) 8 D) 6
 10) A branch has 6 nodes and 9 branches. The independent loops are
 A) 3 B) 4 C) 5 D) 6
 11) In the graph shown in figure solid lines are twigs and dotted lines are link. The fundamental loop matrix is



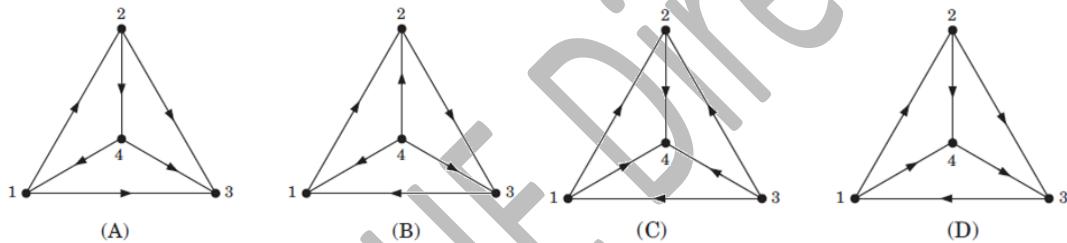
$$(C) \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(D) \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

12) The incidence matrix of a graph is as given below

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

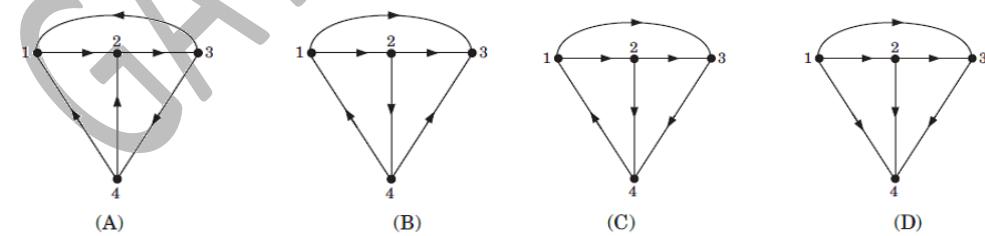
The graph is



13) The incidence matrix of a graph is as given below

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$$

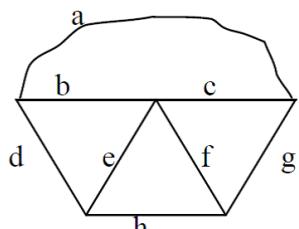
The graph is



14) Tie-set is a dual of

- A) KVL B) Cut set C) Spanning sub graph D) None

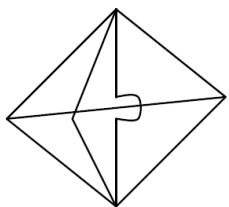
15) Identify which of the following is not a tree of the graph shown



- 16) This graph is called as
 A) begh B) defg

C) abfg D) aegh

- 16) This graph is called as

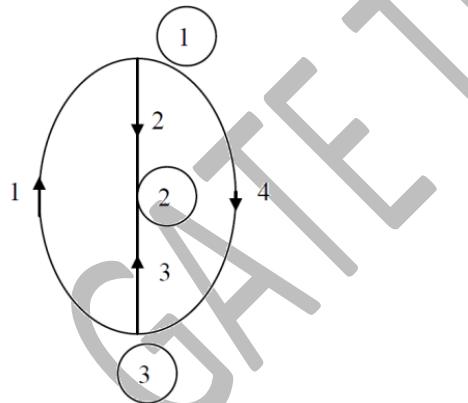


- 17) Edge of co-tree is
 A) chord B) Twig C) branch D) none

- 18) Another name of tree
 A) Complete graph B) spanning sub graph C) twig D) none
- 19) The number of chords in a graph with b number of branches and n number of nodes is
 A) $b-n+1$ B) $b+n-1$ C) $b+n$ D) $b-n$

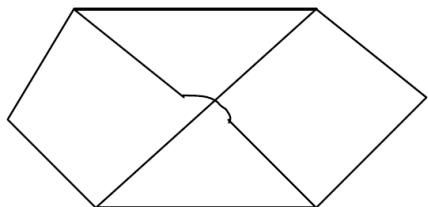
- 20) The number of edges in a complete graph of n vertices is
 A) $n(n - 1)$ B) $n(n-1) / 2$ C) n D) $n-1$

- 21) For the graph shown in fig. The number of possible trees is



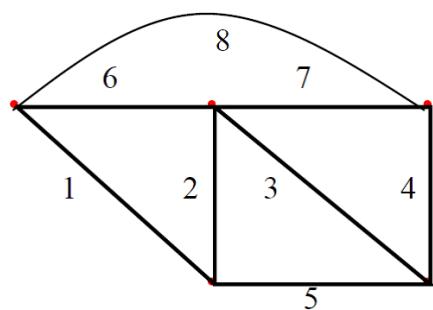
- A) 6 B) 5 C) 4 D) 3

- 22) Identify the graph



- A) Non planner B) planner C) spanning D) complete graph

23) Match the following, the tree branch 1,2,3 and 8 of the graph shown in



List A

- A) Twig
- B) Link
- C) Fundamental cutset
- D) Fundamental loop

List B

- 1) 4,5,6,7
- 2) 1,2,3,8
- 3) 1,2,3,4,8
- 4) 4,7,8

	A	B	C	D
A)	2	1	4	3
B)	3	2	1	4
C)	1	4	3	2
D)	3	4	1	2

Answers:

1. D	2. B	3. A	4. B	5. C	6. C	7. C	8. D	9. B
10. B	11. A	12. D	13. C	14. B	15. C	16. A	17. A	18. B
19. A	20. B	21. B	22. B	23. A				

CHAPTER VII

MUTUAL COUPLING

Mutual inductance: When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is **known as mutual inductance**.

Mutual inductance is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).

When two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be **magnetically coupled**. The **transformer** is an electrical device designed on the basis of the **concept of magnetic coupling**.

M_{12} and M_{21} are equal, that is, $M_{12} = M_{21} = M$

Two coils are said to be mutually coupled if the magnetic flux φ emanating from one passes through the other. The mutual inductance between the two coils is given by

$$M = k \sqrt{L_1 L_2}$$

Where k is the coupling coefficient, $0 < k < 1$.

$0 < k < 0.5$ called loosely coupled

$0.5 < k < 1$ called tightly coupled

$k=1$ perfectly coupled

NOTE:

- Mutual coupling only exists when the inductors or coils are in **close proximity**, and the circuits are driven by time-varying sources. We recall that inductors act like short circuits to dc.
- Although mutual inductance M is always a positive quantity, the mutual voltage $M \frac{di}{dt}$ may be negative or positive.
- Capacitor opposes sudden changes of voltage and inductor opposes sudden changes of current.
- Inductor (capacitor) absorb energy from the active source and that absorb energy is stored by inductor (capacitor). It will supply that stored energy back to circuit (dissipate through the resistance) in the absence of the active source.

Polarity of mutually induced voltage:

The choice of the correct polarity for $M \frac{di}{dt}$ is made by examining the orientation or particular way in which both coils are physically wound and applying Lenz's law in conjunction with the right-hand rule.

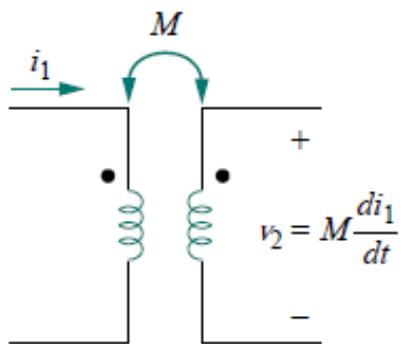


Figure 1

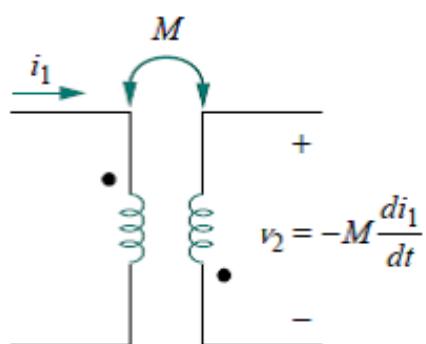


Figure 2

If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil (as shown in figure 1 and figure 2)

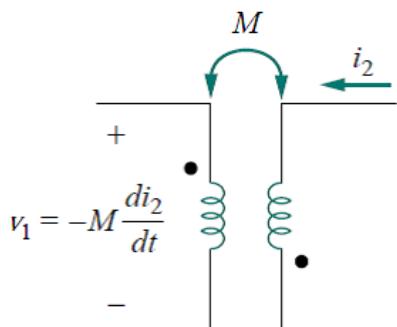


Figure 3

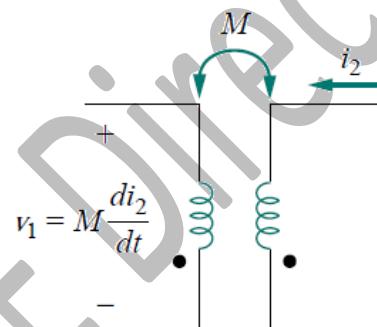
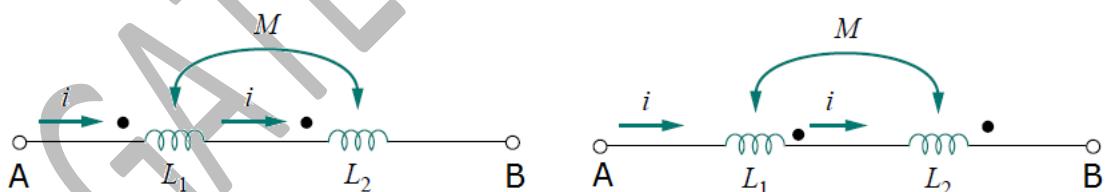


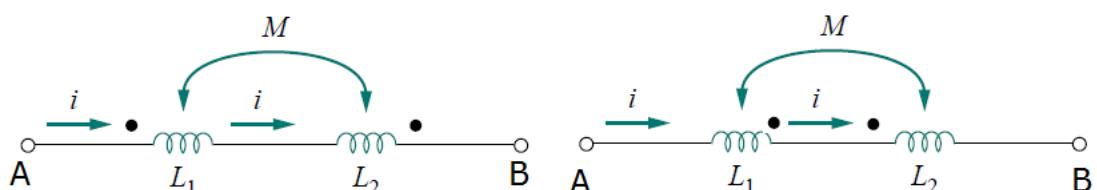
Figure 4

If a current leaves the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is negative at the dotted terminal of the second coil (as shown in fig. 3 and 4).

Inductors series connections:

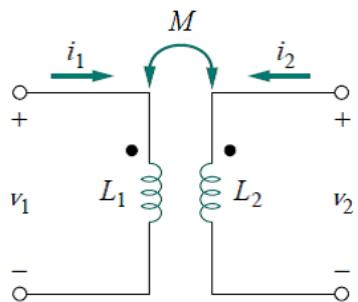


$$L_{AB} = L_1 + L_2 + 2M$$



$$L_{AB} = L_1 + L_2 - 2M$$

Energy in a coupled circuit:



$$E = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm Mi_1 i_2$$

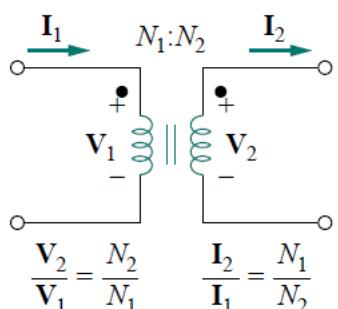
In this equation $Mi_1 i_2$ is positive if both coil currents entered (or leave) the dotted terminals. If one current enters one dotted terminal while the other current leaves the other dotted terminal, the mutual voltage is negative, so that the mutual energy $Mi_1 i_2$ is also negative.

Relationship between current, voltage and turn ratio in Transformers:

A **step-down transformer** is one whose secondary voltage is less than its primary voltage.

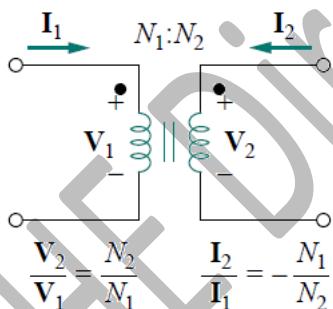
A **step-up transformer** is one whose secondary voltage is greater than its primary voltage.

Turn ratio is N_2/N_1



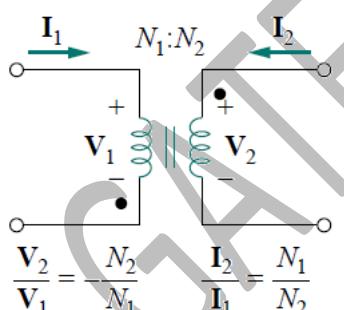
$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

(a)



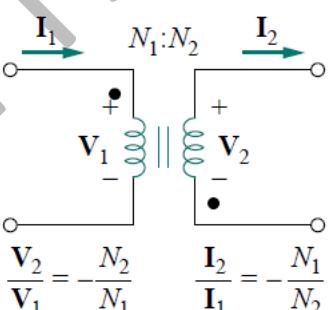
$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

(b)



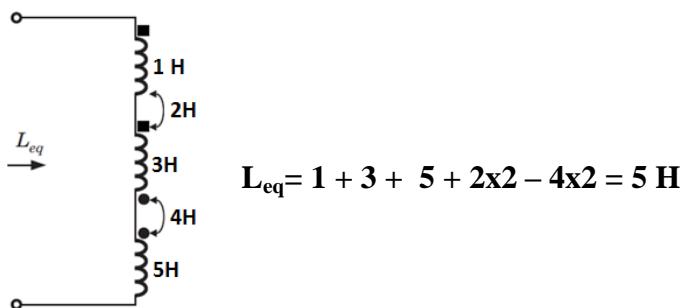
$$\frac{V_2}{V_1} = -\frac{N_2}{N_1}$$

(c)



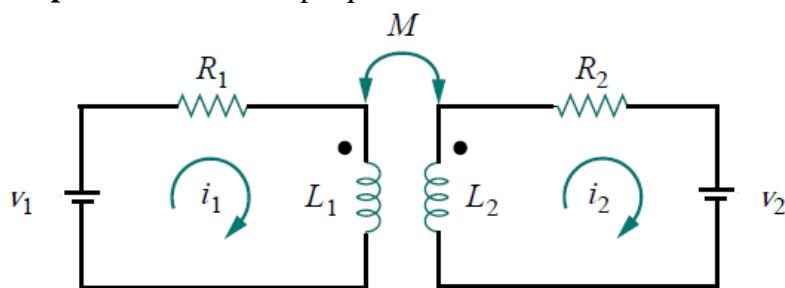
$$\frac{V_2}{V_1} = -\frac{N_2}{N_1}$$

(d)

Example 1. Find the L_{eq} 

$$L_{eq} = 1 + 3 + 5 + 2 \times 2 - 4 \times 2 = 5 \text{ H}$$

Example 2. Write the loop equations



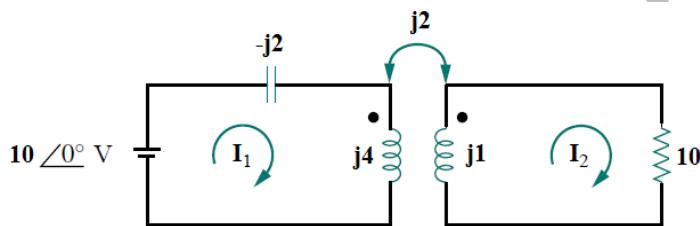
$$\text{For 1}^{\text{st}} \text{ loop: } R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{d(-i_2)}{dt} = V_1$$

(As for 1st loop current i_1 is enter at dot and also- i_2 is enter in dot in second loop so we take $-i_2$)

$$\text{For 2}^{\text{nd}} \text{ loop: } R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{d(i_1)}{dt} = -V_2$$

(As for 2nd loop current i_2 is leave at dot and also - i_1 is leave in dot in second loop so we take $-i_1$)

Example 3.



$$1^{\text{st}} \text{ loop: } (-j2+j4)I_1 - j2I_2 = 10 \quad \text{i.e. } j2I_1 - j2I_2 = 10$$

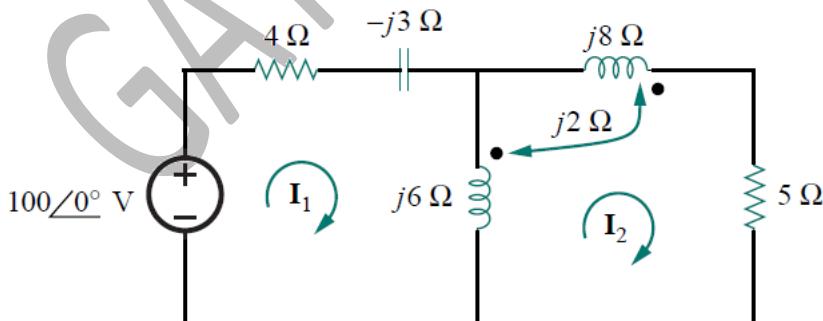
$$2^{\text{nd}} \text{ loop: } -j2I_1 + (10+j1) = 0$$

By solving equation we will get,

$$I_1 = 5 < -78.57^\circ \text{ A}$$

$$I_2 = 0.995 < 5.71^\circ \text{ A}$$

Example 4.



$$1^{\text{st}} \text{ loop: } (4-j3+j6)I_1 - j6I_2 - j2I_2 = 100$$

$$\text{i.e. } (4+j3) - j2I_2 = 100$$

$$2^{\text{nd}} \text{ loop: } -j6I_1 + (5+j8+j6)I_2 + j2(I_2 - I_1) + j2(I_2) = 0$$

$$-j8I_1 + (5+j18)I_2 = 0$$

By solving equation we will get,

$$I_1=20.3 < 3.5 \text{ A}$$

$$I_2=8.693 < 19.02 \text{ A}$$

Examples:

1. The coefficient of coupling for two coils having $L_1 = 4 \text{ H}$, $L_2 = 9 \text{ H}$, $M = 4 \text{ H}$ is:

- A) 0.667 B) 0.75 C) 1.5 D) 9

2. A transformer is used in stepping down or stepping up:

- A) dc voltages B) ac voltages C) both dc and ac voltages D) None

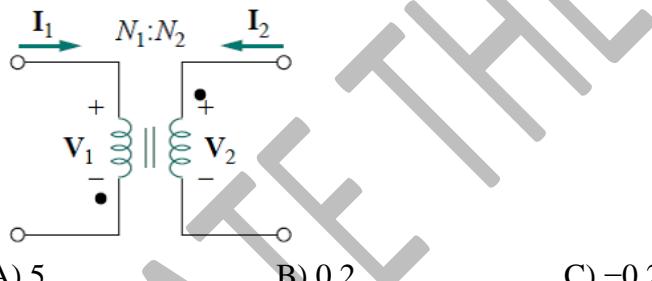
3. In order to match a source with internal impedance of 500Ω to a 15Ω load, what is needed is:

- A) step-up linear transformer B) step-down linear transformer
C) step-up ideal transformer D) step-down ideal transformer

4. Which of these transformers can be used as an isolation device?

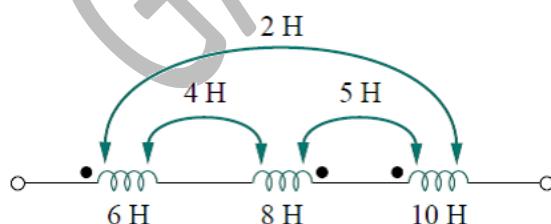
- A) linear transformer B) ideal transformer C) autotransformer D) all of the above

5. The ideal transformer has $N_2/N_1 = 5$. The ratio V_2/V_1 is:



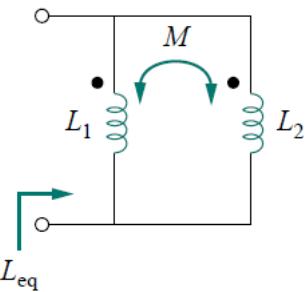
- A) 5 B) 0.2 C) -0.2 D) -5

6. For the three coupled coils calculate the total inductance.



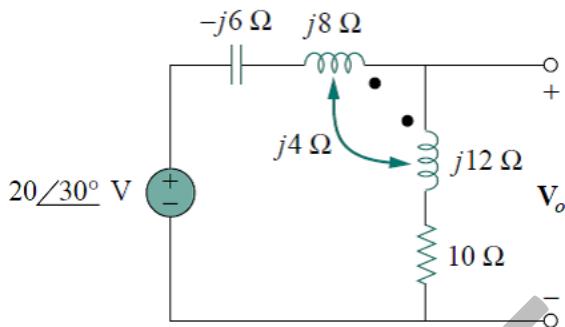
- A) 8 H B) 10 H C) 12 H D) 14 H

7. Find L_{eq} in the following circuit



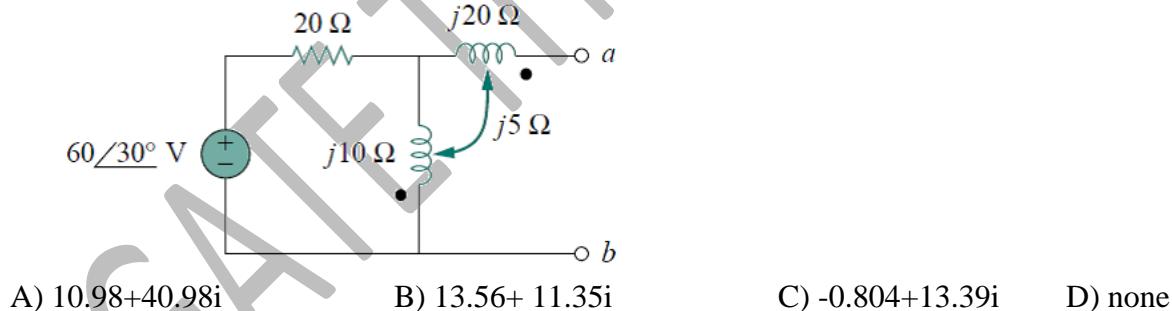
- A) $\frac{L_1 L_2 - M^2}{L_1 L_2 - 2M^2}$ B) $\frac{L_1 L_2 + M^2}{L_1 L_2 - 2M^2}$ C) $\frac{L_1 L_2 - M^2}{L_1 L_2 + 2M^2}$ D) $\frac{L_1 L_2 + M^2}{L_1 L_2 + 2M^2}$

8. Find \$V_o\$ in the following circuit



- A) \$17.378+13.43i\$ B) \$16.35 + 7.88i\$ C) \$17.49 + 20.29i\$ D) none

9. Find the thevenin equivalent voltage for the circuit show below at terminals \$a-b\$.

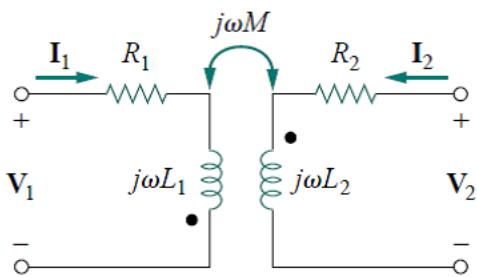


- A) \$10.98+40.98i\$ B) \$13.56 + 11.35i\$ C) \$-0.804+13.39i\$ D) none

10. Two coils connected in series-aiding fashion have a total inductance of 300 mH. When connected in a series-opposing configuration, the coils have a total inductance of 200 mH. If the inductance of one coil (\$L_1\$) is four times the other, what is the coupling coefficient?

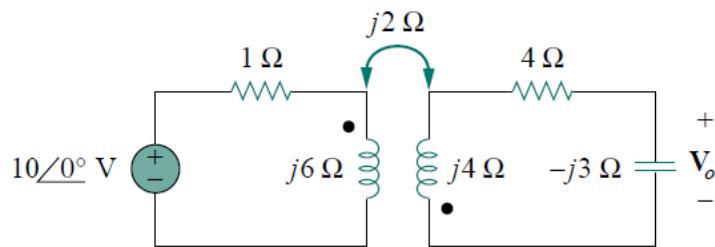
- A) 1.25 B) 0.5 C) 0.25 D) 0.723

11. Determine \$V_2\$ in terms of \$I_1\$ and \$I_2\$ in the following circuit



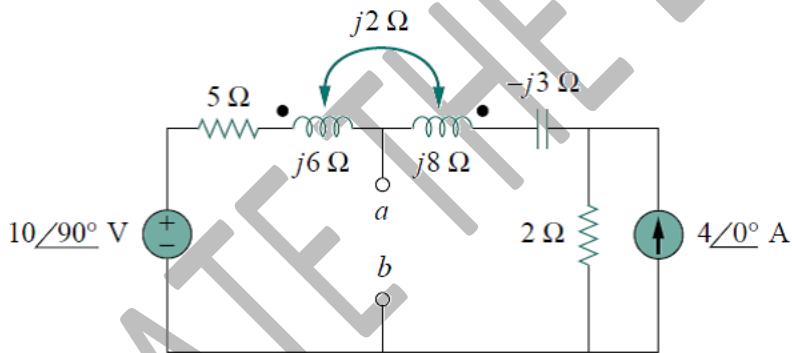
- A) $(R_2 + j\omega L_2)I_2$
 B) $(R_2 + j\omega L_2)I_2 + j\omega M I_1$
 C) $(R_2 + j\omega L_2)I_2 + j\omega M I_2$
 D) $(R_2 + j\omega L_2)I_2 - j\omega M I_1$

12. Find the V_o in the following circuit



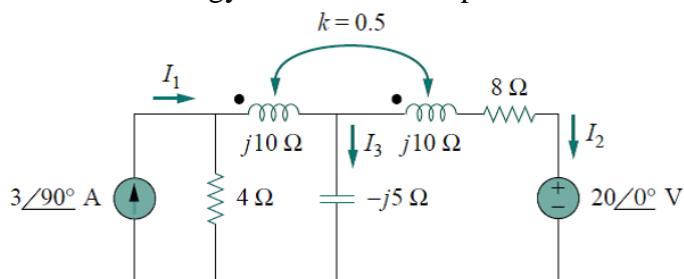
- A) $10.34 < 36.23$
 B) $3.45 < 39.63$
 C) $2.392 < 94.57$
 D) $4.23 < 67.43$

13. Obtain the Thevenin equivalent circuit for the circuit show below at terminals $a-b$.



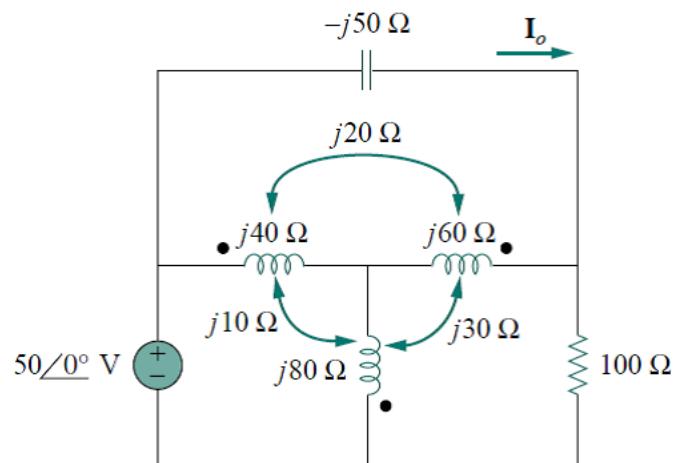
- A) $V_{Th} = 5.349 < 34.114 \text{ V}$, $Z_{Th} = 2.33 < 50 \Omega$
 B) $V_{Th} = 3.27 < 39.65 \text{ V}$, $Z_{Th} = 2.33 < 50 \Omega$
 C) $V_{Th} = 2.36 < 34.35 \text{ V}$, $Z_{Th} = 2.33 < 50 \Omega$
 D) $V_{Th} = 6.27 < 78.67 \text{ V}$, $Z_{Th} = 2.33 < 50 \Omega$

14. Find the energy stored in the coupled coils at $t = 2 \text{ ms}$. Take $\omega = 1000 \text{ rad/s}$.



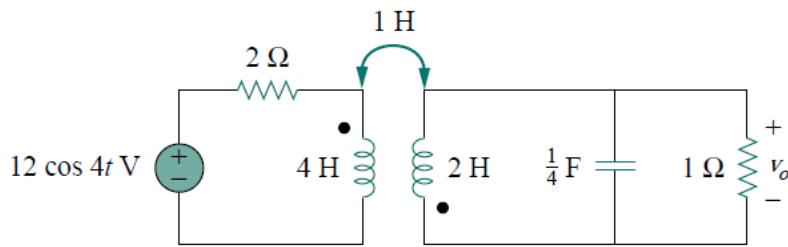
- A) 43.67 mJ B) 36.56 mJ C) 59.45 mJ D) 25.57 mJ

15. Find current I_o in the following circuit



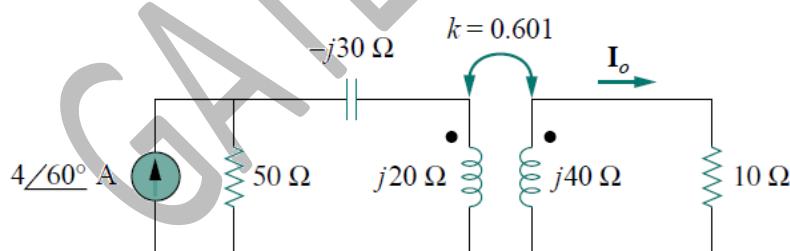
- A) $1.24 < 142.3$ A B) $2.57 < 23.45$ A C) $3.2 < -175.2$ A D) $6.78 < 34.1$ A

16. Determine the energy stored in the coupled inductors at $t = 2$ s.



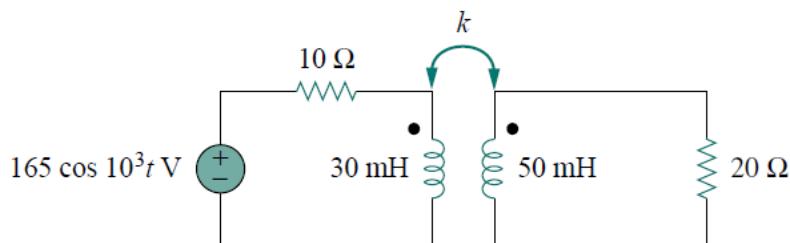
- A) 2.354 J B) 0.35 J C) 1.168 J D) 3.567 J

17. Find I_o in the following circuit



- A) $3.56 < 56.3$ A B) $3.755 < -36.34$ A C) $4.56 < -56.2$ A D) $3.67 < 23.7$ A

18. In the circuit, find the value of the coupling coefficient k that will make the 10Ω resistor dissipate 320 W. For this value of k , find the energy stored in the coupled coils at $t = 1.5$ s.

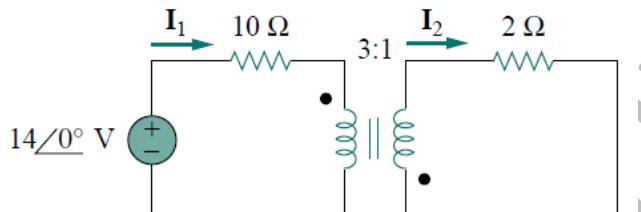


- A) 0.984, 130.5 mJ B) 0.567, 116.5 mJ C) 0.462, 78.7 mJ D) 0.785, 240.5 mJ

19. A 4-kVA, 2300/230-V rms transformer has an equivalent impedance of $2 < 10 \Omega$ on the primary side. If the transformer is connected to a load with 0.6 power factor leading, calculate the input impedance.

- A) $4.56 < 56.7 \text{ k } \Omega$ B) $1.324 < -53.05 \text{ k } \Omega$ C) $2.45 < 67.5 \text{ k } \Omega$ D) $5.68 < 34.4 \text{ k } \Omega$

20. Determine I_2 in the following circuit



- A) 1.5 A B) 0.5 A C) -1.5 A D) -0.5 A

21. An ideal filter should have

- A) Zero attenuation in the pass band.
- B) Zero attenuation in the attenuation band.
- C) Infinite attenuation in the pass band.
- D) Infinite attenuation in the attenuation band.

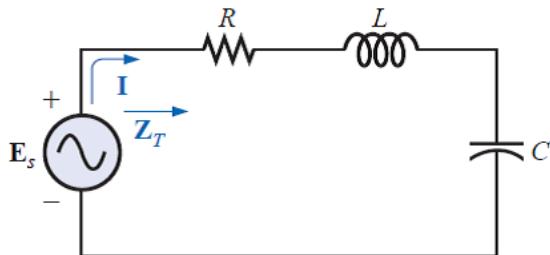
1. A	2. B	3. D	4. B	5. D	6. B	7. A	8. A	9. C	10. C
11. D	12. C	13. A	14. A	15. C	16. C	17. B	18. A	19. B	20. C
21. D	22.	23.	24.	25.	26.	27.	28.	29.	30.

CHAPTER VIII

RESONANCE CIRCUIT

There are two types of resonant circuits: series and parallel resonant circuits

SERIES RESONANCE CIRCUIT:



The total impedance of this network at any frequency is determined as
 $Z_T = R + j X_L - j X_C = R + j (X_L - X_C)$

At resonance: The circuit is resistive in nature

$$Z_T = R \text{ (minimum)}$$

$$\text{i.e. } (X_L - X_C) = 0 \Rightarrow X_L = X_C$$

$$w_s L = 1/w_s C$$

$$w_s = \frac{1}{\sqrt{LC}} \text{ and } f_s = \frac{1}{2\pi\sqrt{LC}}$$

where, w_s : resonance frequency in series RLC circuit in rad/sec

f_s : resonance frequency in series RLC circuit in Hz

Quality factor:

$$Q_s = \frac{\text{Store power (either in } C \text{ or in } L)}{\text{Desipated power}}$$

$$Q_s = \frac{I^2 X_L}{I^2 R} = \frac{I^2 X_C}{I^2 R}$$

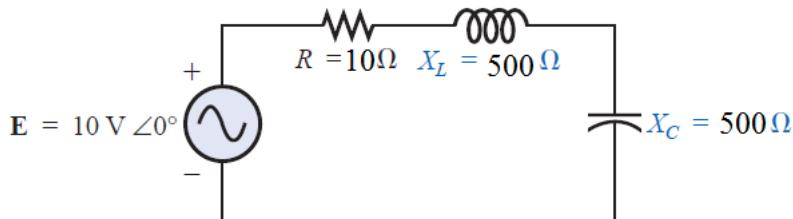
$$Q_s = \frac{X_L}{R} = \frac{X_C}{R}$$

$$Q_s = \frac{wL}{R} = \frac{1}{wCR} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Band width:

$$B.W. = \frac{f_s}{Q_s}$$

Example 1: Find out the quality factor Q and magnitude of voltage drop across resistance



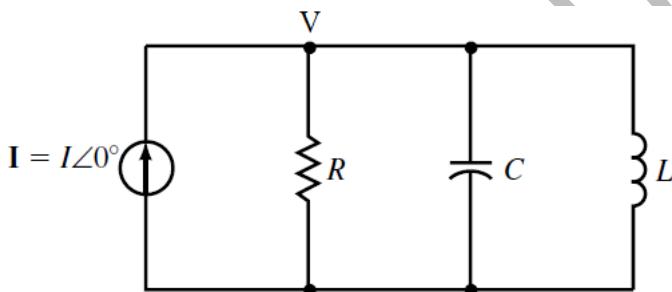
$$Q_s = X_L/R = 500/10 = 50$$

Current through the circuit is $I = E/R = 10/10 = 1 \text{ A}$

$$|V_L| = I^2 X_L = 500 \text{ V}$$

- Current in phase with voltage

PARALLEL RESONANCE CIRCUIT:



Admittance

$$Y(t) = 1/R + 1/j\omega L + j\omega C$$

At resonance:

Circuit is resistive in nature

$$1/j\omega L + j\omega C = 0$$

$$\omega_p = \frac{1}{\sqrt{LC}}$$

$$f_p = \frac{1}{2\pi\sqrt{LC}}$$

Therefore, $Y(t) = 1/R$

Impedance at resonance $Z(t) = R$ (Maximum)

i.e. At resonance the impedance is minimum in series circuit and maximum in parallel circuit.

Quality factor (Q_p):

$$Q_p = \frac{\text{Store power (either in } C \text{ or in } L)}{\text{Desipated power}}$$

$$Q_p = \frac{V^2/X_L}{I^2/R} = \frac{V^2/X_C}{V^2/R}$$

$$Q_p = \frac{R}{X_L} = \frac{R}{X_C}$$

$$Q_p = \frac{R}{wL} = wCR = R\sqrt{\frac{C}{L}}$$

Band width:

$$\text{B.W.} = \frac{f_p}{Q_p}$$

Resonance: A condition established by the application of a particular frequency (the resonant frequency) to a series or parallel $R-L-C$ network. The transfer of power to the system is a maximum, and, for frequencies above and below, the power transfer drops off to significantly lower levels.

Band (cutoff, half-power, corner) frequencies: Frequencies that define the points on the resonance curve that are 0.707 of the peak current or voltage value. In addition, they define the frequencies at which the power transfer to the resonant circuit will be half the maximum power level.

Bandwidth (BW): The range of frequencies between the band, cutoff, or half-power frequencies.

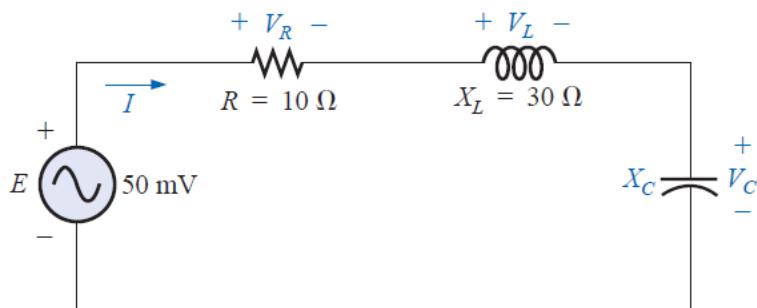
Quality factor (Q): A ratio that provides an immediate indication of the sharpness of the peak of a resonance curve. The higher the Q , the sharper the peak and the more quickly it drops off to the right and left of the resonant frequency.

Selectivity: A characteristic of resonant networks directly related to the bandwidth of the resonant system. High selectivity is associated with small bandwidth (high Q 's), and low selectivity with larger bandwidths (low Q 's).

Example:

1. Find the resonant f_s for the series circuit with the following parameters: $R = 10\Omega$, $L = 2.5 \text{ mH}$, $C = 8 \text{ uF}$
- A) 7.071 kHz B) 50 MHz C) 1.125 kHz D) 7.95 MHz

2. Find the magnitude of V_c at resonance frequency

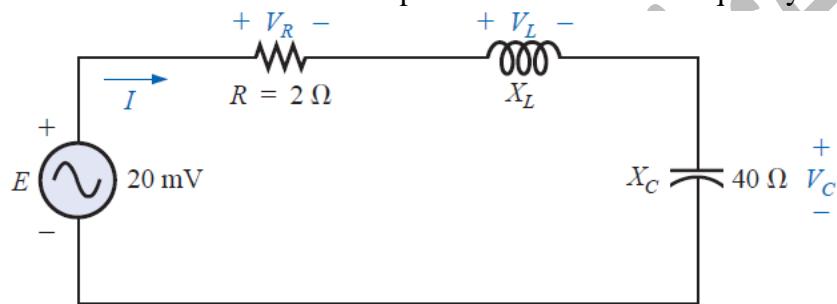


- A) 0.15 V B) 0.05 V C) 0.0167 V D) 0.5 V

3. In the above problem find the I and quality factor respectively at resonance

- A) 1.25 mA, 3 B) 5 mA, 3 C) 0.71 mA, 1.5 D) 0.71 mA, 3

4. Find the bandwidth of the response if the resonant frequency is 5 kHz.

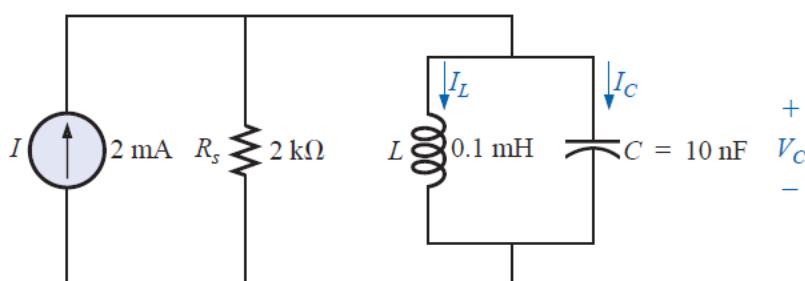


- A) 125 Hz B) 250 Hz C) 500 Hz D) 2500 Hz

5. In above problem, what are the low and high cutoff frequencies if the resonant frequency is 5 kHz?

- A) 4.5 kHz and 5 kHz B) 4.75 kHz and 5.25 kHz
C) 4.875 kHz and 5.125 kHz D) none

6. Find the resonance frequency for the following circuit



- A) $1 \mu\text{rad/sec}$ B) 1 Mrad/sec C) 1 Trad/sec D) 1 Grad/sec

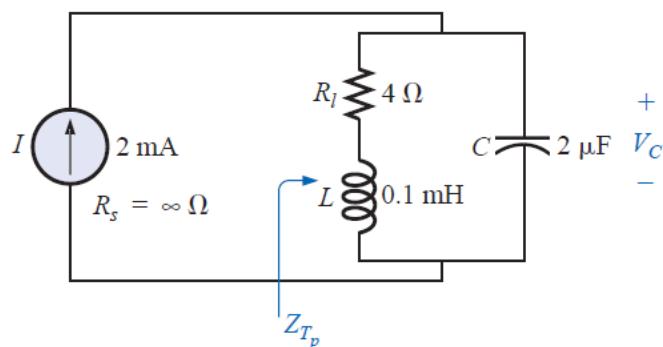
7. In the above problem, determine the currents I_L and I_C at resonance.

- A) 2 mA B) 40 mA C) 200 mA D) 100 mA

8. In the above problem, find the voltage V_C at resonance.

- A) 0.2 V B) 400 V C) 4 V D) 100 V

9. For the parallel resonant network as show below find resonance frequency f_s .



- A) 11,253.95 Hz B) 9,245.45 Hz C) 15,234.83 Hz D) none

10. In the above problem determine Q_L using $f = f_s$. Can the approximate approach be applied?

- A) 1.77 B) 3.76 C) 5.58 D) 7.45

11. Higher the value of Q of a series circuit

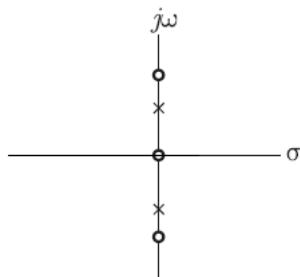
- A) Sharper is its resonance
B) Greater is its bandwidth.
C) Broader is its resonant curve.
D) Narrower is its bandwidth

1. C	2. A	3. B	4. B	5. C	6. B	7. B	8. C	9. A	10. A
11. D									

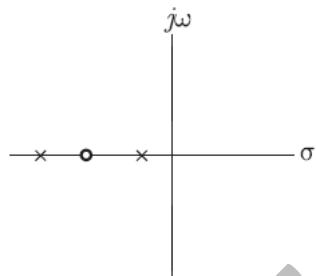
CHAPTER IX

DRIVING POINT IMPEDANCE & FILTERS

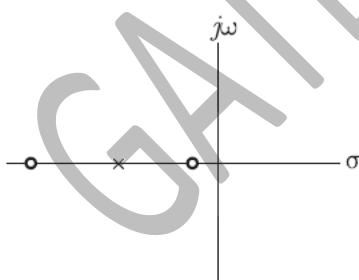
LC Impedance: In driving point LC impedance function, the poles and zeros are alternately lies on imaginary axis ($j\omega$ -axis)



RC Impedance: In driving point RC impedance function, the poles and zeros are alternately lies on '-ve' real axis with nearer to origin is pole (pole can be lies at origin)



RL Impedance: In driving point RL impedance function, the poles and zeros are alternately lies on '-ve' real axis with nearer to origin is zero (zero can be lies at origin)

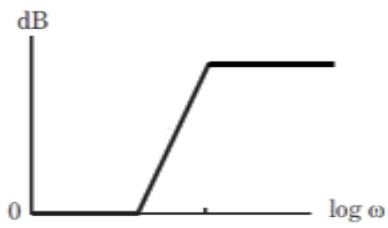


RLC Impedance: In the RLC impedance function, the pole and zero are complex conjugate pairs symmetrical about '-ve' real axis.

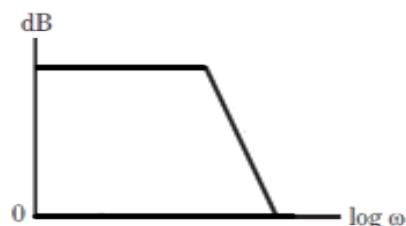
Note:

Immittance function mean either impedance or admittance function.

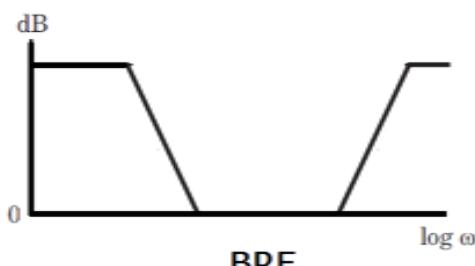
RC impedance function is same as RL admittance function and vice versa.

Filters:

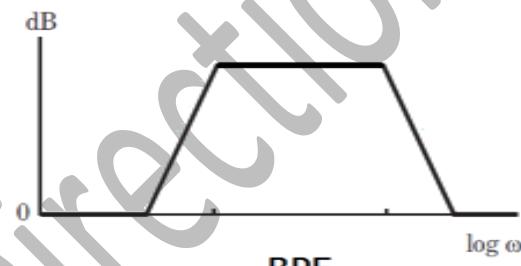
HPF



LPF



BRF

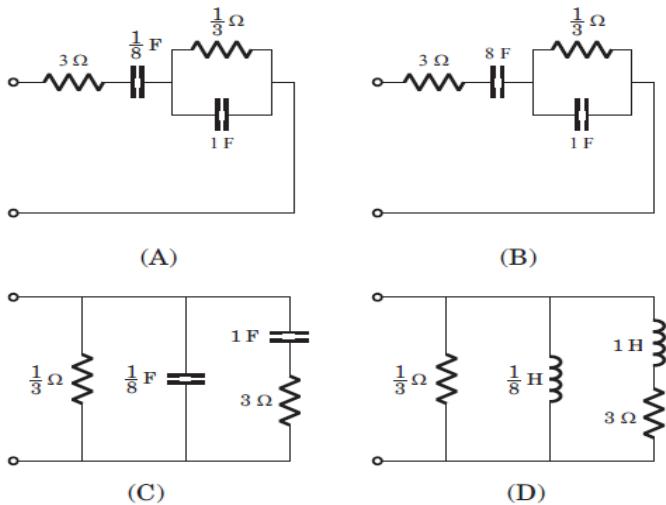


BPF

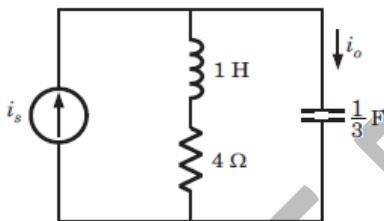
Filter	$H(s) _{w=0}$	$H(s) _{w=\infty}$
HPF	Zero	Finite
LPF	Finite	Zero
BRF	Finite	Finite
BPF	Zero	Zero

Example:

- The network function $\frac{s^2+10s+24}{s^2+8s+15}$ represent a
 A) RC admittance B) RL impedance C) LC impedance D) None
- The network function $\frac{s(s+4)}{(s+1)(s+2)(s+3)}$ represents an
 A) RC impedance B) RL impedance C) LC impedance D) None
- The network function $\frac{s(3s+8)}{(s+1)(s+3)}$ represents an
 A) RL admittance B) RC impedance C) RC admittance D) None
- The network function $\frac{(s+1)(s+4)}{s(s+2)(s+5)}$ is a _____ function.
 A) RL impedance B) RC impedance C) LC impedance D) Above all
- A impedance function is given as $Z(s) = \frac{3(s+2)(s+4)}{s(s+3)}$. The network for this function is

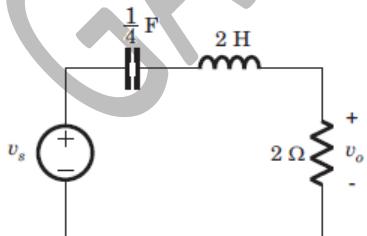


Statement for Que 6,7 & 8



For circuit shown:

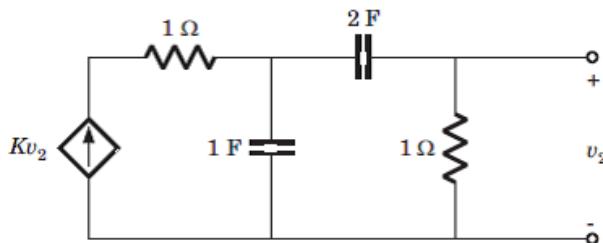
6. The current ratio transfer function I_0/I_s
 - A) $\frac{S(S+4)}{S^2+3S+4}$
 - B) $\frac{S(S+4)}{(S+1)(S+3)}$
 - C) $\frac{S^2+3S+4}{S(S+4)}$
 - D) $\frac{(S+1)(S+3)}{S(S+4)}$
7. The response is
 - A) Over damped
 - B) Under damped
 - C) Critically damped
 - D) can't be determined
8. If input i_s is $2u(t)$ A, the output current i_0 is
 - A) $(2e^{-t}-3te^{-3t}) u(t)$ A
 - B) $(3te^{-t}-e^{-3t}) u(t)$ A
 - C) $(3e^{-t}-e^{-3t}) u(t)$ A
 - D) $(e^{-3t}-3e^{-t}) u(t)$ A
9. In the network, all initial conditions are zero. The damping exhibited by the network is



- A) Over damped
- B) Under damped
- C) Critically damped
- D) value of voltage is requires
10. The voltage response of a network to a unit step input is $V_0(S) = \frac{10}{S(S^2+8S+16)}$. The response is

- A) under damped B) over damped
 C) critically damped D) can't be determined

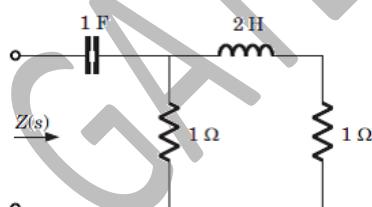
11. The current response of a network to a unit step input is $I_0 = \frac{10(s+2)}{s^2(s+11s+30)}$. The response is
 A) Under damped B) Over damped C) Critically damped D) None of the above
12. A circuit has input $V_{in}(t) = \cos 2t u(t)$ V and output $i_0(t) = 2 \sin 2t u(t)$ A. The circuit had no internal stored energy at $t = 0$. The admittance transfer function is
 A) $2/s$ B) $s/2$ C) s D) $1/s$
13. The network shown in the figure is stable if



- A) $K \geq (5/2)$ B) $K \leq (5/2)$ C) $K \geq (2/5)$ D) $K \leq (2/5)$
14. The voltage across $200 \mu\text{F}$ capacitor is given by $V_c(S) = \frac{2S+6}{S(S+3)}$
 The steady state voltage across capacitor is
 A) 6 V B) 0 V C) ∞ D) 2 V
15. The transformed voltage across the $60 \mu\text{F}$ capacitor is given by
 $V_s = \frac{20S+6}{(10S+3)(S+4)}$. The initial current through capacitor is
 A) 0.12 mA B) -0.12 mA C) 0.48 mA D) -0.48 mA

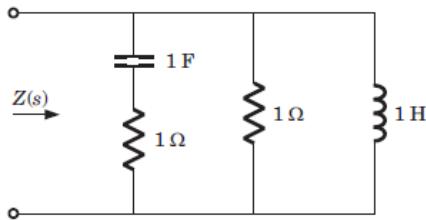
16. The current through an 4 H inductor is given by
 $I_L(S) = \frac{10}{S(S+2)}$. The initial voltage across inductor is
 A) 40 V B) 20 V C) 10 V D) 5 V

17. In the following circuit the value of $Z(S)$ is



- A) $\frac{s^2+1.5s+1}{s(s+1)}$ B) $\frac{s^2+3s+1}{s(s+1)}$ C) $\frac{2s^2+3s+2}{s(s+1)}$ D) $\frac{2s^2+3s+1}{2s(s+1)}$

18. In the following circuit the value of $Z(S)$ is



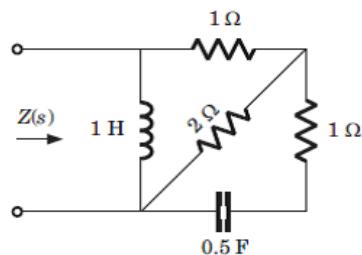
A) $\frac{s^2+s+1}{s(s+1)}$

B) $\frac{2s^2+s+1}{s(s+1)}$

C) $\frac{s(s+1)}{2s^2+s+1}$

D) $\frac{s(s+1)}{s^2+s+1}$

19. In the following circuit the value of $Z(S)$ is



A) $\frac{3s^2+8s+7}{s(5s+6)}$

B) $\frac{s(5s+6)}{3s^2+8s+7}$

C) $\frac{3s^2+7s+6}{s(5s+6)}$

D) $\frac{s(5s+6)}{3s^2+7s+6}$

20. A unit step current of 1 A is applied to a network whose driving point impedance is

$Z(S) = \frac{V(S)}{I(S)} = \frac{(S+3)}{(S+2)^2}$. The steady state and initial values of the voltage developed across the source would be respectively

A) $\frac{3}{4}$ V, 1V

B) $\frac{1}{4}$ V, $\frac{3}{4}$ V

C) $\frac{3}{4}$ V, 0V

D) 1 V, $\frac{3}{4}$ V

21. The immittance function is given as $I(s) = \frac{(s+2)(s+7)}{(s+5)(s+10)}$

A) RC admittance

B) RC impedance

C) LR admittance

D) LC impedance

22. A notch filter is a

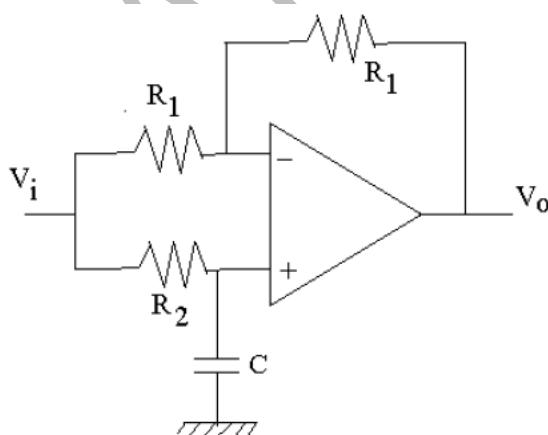
A) Wide band pass filter

B) Narrow band pass filter

C) Wide band reject filter

D) Narrow band reject filter

23. The circuit shown below represent a



A) Low pass filter B) High pass filter C) Band pass filter D) Band reject filter

24. All pass filter

- A) passes whole of the audio band.
- B) passes whole of the radio band.
- C) passes all frequencies with very low attenuation.
- D) passes all frequencies without attenuation but phase is changed.

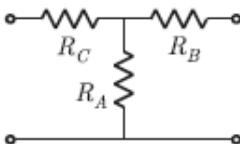
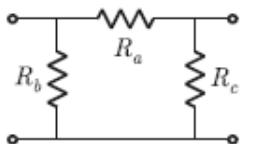
Answers:

1. D	2. D	3. C	4. B	5. A	6. B	7. A	8. C	9. B
10. C	11. B	12. A	13. B	14. D	15. D	16. A	17. A	18. C
19. D	20. C	21. A	22. D	23. A	24. D			

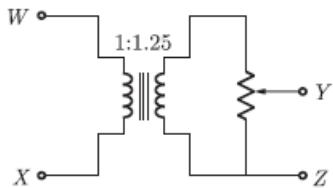
Previous Year Gate Questions

GATE 2013

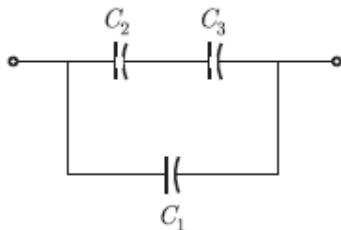
- 1) Consider a delta connection of resistors and its equivalent star connection as shown below. If all elements of the delta connection are scaled by a factor k , $k > 0$, the elements of the corresponding star equivalent will be scaled by a factor of



- (A) k^2 (B) k (C) $1/k$ (D) \sqrt{k}
- 2) The transfer function $\frac{V_2(s)}{V_1(s)}$ of the circuit shown below is
- A circuit diagram showing two parallel branches. The left branch contains a capacitor of $100 \mu\text{F}$ in series with the input voltage $V_1(s)$. The right branch contains a resistor of $10 \text{ k}\Omega$ in series with the output voltage $V_2(s)$. Both branches are connected to a common ground node.
- (A) $\frac{0.5s+1}{s+1}$ (B) $\frac{3s+6}{s+2}$ (C) $\frac{s+2}{s+1}$ (D) $\frac{s+1}{s+2}$
- 3) A source $v_s(t) = V \cos 100\pi t$ has an internal impedance of $(4 + j3)\Omega$. If a purely resistive load connected to this source has to extract the maximum power out of the source, its value in Ω should be
- (A) 3 (B) 4 (C) 5 (D) 7
- 4) In the circuit shown below, if the source voltage $V_S = 100 < 53.13^\circ \text{V}$ then the Thevenin's equivalent voltage in Volts as seen by the load resistance R_L is
- A circuit diagram showing a source voltage V_S of $100 < 53.13^\circ \text{V}$. The circuit consists of a 3Ω resistor in series with the source, followed by a $j4 \Omega$ capacitor. This is followed by a dependent current source $j40I_2$. To the right of this is a $10V_{L1}$ voltage source in series with a $j6 \Omega$ capacitor. This is followed by another dependent current source $10V_{L1}$. Finally, there is a 5Ω resistor in series with the load $R_L = 10 \Omega$.
- (A) $100 < 90^\circ$ (B) $800 < 0^\circ$ (C) $800 < 90^\circ$ (D) $100 < 60^\circ$
- 5) The following arrangement consists of an ideal transformer and an attenuator which attenuates by a factor of 0.8. An ac voltage $V_{WX1} = 100 \text{ V}$ is applied across WX to get an open circuit voltage V_{YZ1} across YZ. Next, an ac voltage $V_{YZ2} = 100 \text{ V}$ is applied across YZ to get an open circuit voltage V_{WX2} across WX. Then, V_{YZ1}/V_{WX1} , V_{WX2}/V_{YZ2} are respectively,



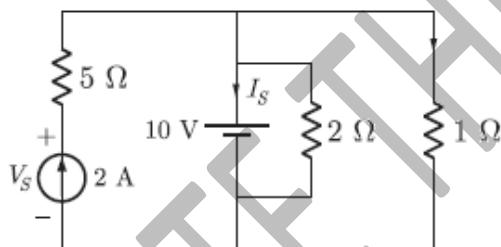
- (A) 125/100 and 80/100 (B) 100/100 and 80/100
 (C) 100/100 and 100/100 (D) 80/100 and 80/100
- 6) Three capacitors C_1 , C_2 and C_3 whose values are 10 mF, 5 mF, and 2 mF respectively, have breakdown voltages of 10 V, 5 V and 2 V respectively. For the interconnection shown below, the maximum safe voltage in Volts that can be applied across the combination, and the corresponding total charge in μC stored in the effective capacitance across the terminals are respectively,



- (A) 2.8 and 36 (B) 7 and 119 (C) 2.8 and 32 (D) 7 and 80

Common Data For Q. 7 and 8:

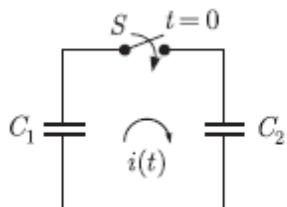
Consider the following figure



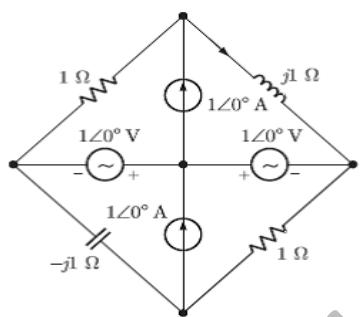
- 7) The current I_S in Amps in the voltage source, and voltage V_S in Volts across the current source respectively, are
 (A) 13, -20 (B) 8, -10 (C) -8, 20 (D) -13, 20
- 8) The current in the 1Ω resistor in Amps is
 (A) 2 (B) 3.33 (C) 10 (D) 12
- 9) Two magnetically uncoupled inductive coils have Q factors q_1 and q_2 at the chosen operating frequency. Their respective resistances are R_1 and R_2 . When connected in series, their effective Q factor at the same operating frequency is
 (A) $q_1 + q_2$ (B) $(1/q_1) + (1/q_2)$
 (C) $(q_1 R_1 + q_2 R_2)/(R_1 + R_2)$ (D) $(q_1 R_2 + q_2 R_1)/(R_1 + R_2)$

GATE 2012

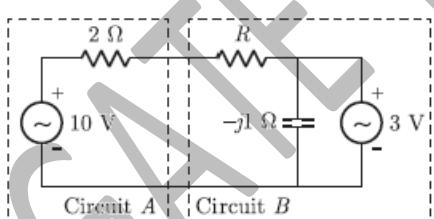
- 10) In the following figure, C_1 and C_2 are ideal capacitors. C_1 has been charged to 12 V before the ideal switch S is closed at $t = 0$. The current $i(t)$ for all t is



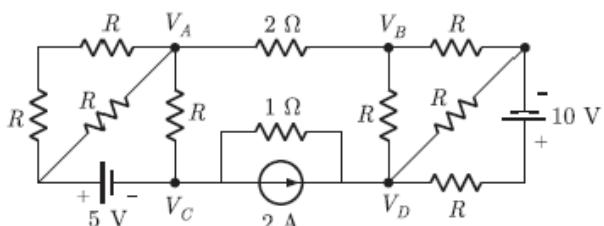
- (A) zero
 (B) a step function
 (C) an exponentially decaying function
 (D) an impulse function
- 11) The average power delivered to an impedance $(4 - j3)\Omega$ by a current $5 \cos(100\pi t + 100)A$ is
 (A) 44.2W
 (B) 50W
 (C) 62.5W
 (D) 125W
- 12) In the circuit shown below, the current through the inductor is



- (A) $\frac{2}{1+j} A$
 (B) $\frac{-1}{1+j} A$
 (C) $\frac{1}{1+j} A$
 (D) 0 A
- 13) Assuming both the voltage sources are in phase, the value of R for which maximum power is transferred from circuit A to circuit B is



- (A) 0.8Ω
 (B) 1.4Ω
 (C) 2Ω
 (D) 2.8Ω
- 14) If $V_A - V_B = 6 V$ then $V_C - V_D$ is



- (A) -5 V
 (B) 2 V
 (C) 3 V
 (D) 6 V

Common Data For Q. 15 and 16:

With 10 V dc connected at port A in the linear nonreciprocal two port network shown below, the following were observed:

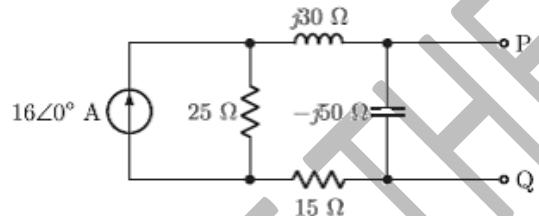
- (i) 1Ω connected at port B draws a current of 3A
- (ii) 2.5Ω connected at port B draws a current of 2A



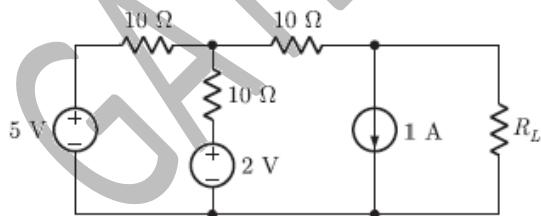
- 15) With 10 V dc connected at port A, the current drawn by 7Ω connected at port B is
 (A) $3/7$ A (B) $5/7$ A (C) 1A (D) $9/7$ A
- 16) For the same network, with 6 V dc connected at port A, 1Ω connected at port B draws $7/3$ A.
 If 8 V dc is connected to port A, the open circuit voltage at port B is
 (A) 6 V (B) 7 V (C) 8 V (D) 9 V

GATE 2011

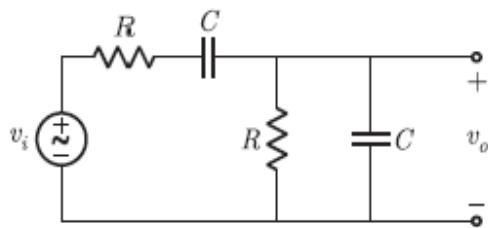
- 17) In the circuit shown below, the Norton equivalent current in amperes with respect to the terminals P and Q is



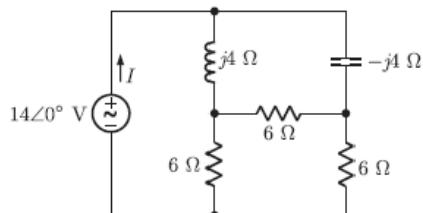
- (A) $6.4 - j 4.8$ (B) $6.56 - j 7.87$ (C) $10 + j0$ (D) $16 + j0$
- 18) In the circuit shown below, the value of R_L such that the power transferred to R_L is maximum is



- (A) 5Ω (B) 10Ω (C) 15Ω (D) 20Ω
- 19) The circuit shown below is driven by a sinusoidal input $V_i = V_p \cos(t/RC)$. The steady state output v_o is

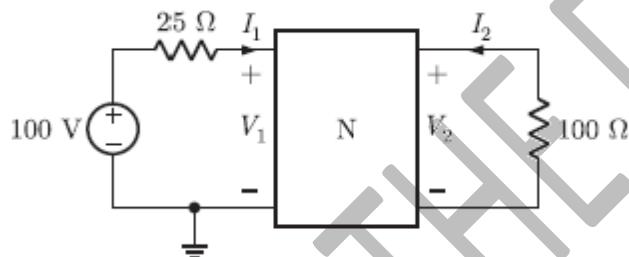


- (A) $(V_p/3)\cos(t/RC)$ (B) $(V_p/3)\sin(t/RC)$ (C) $(V_p/2)\cos(t/RC)$ (D) $(V_p/2)\sin(t/RC)$
 20) In the circuit shown below, the current I is equal to

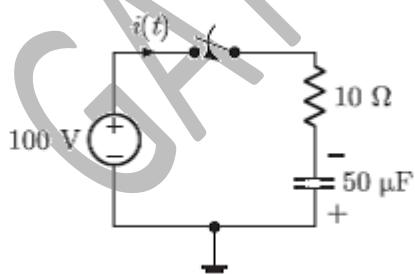


- (A) $1.4 < 0^\circ \text{A}$ (B) $2.0 < 0^\circ \text{A}$ (C) $2.8 < 0^\circ \text{A}$ (D) $3.2 < 0^\circ \text{A}$
 21) In the circuit shown below, the network N is described by the following Y matrix:

$$Y = \begin{bmatrix} 0.1S & -0.01S \\ 0.01S & 0.1S \end{bmatrix} \text{ the voltage gain } \frac{V_2}{V_1} \text{ is}$$



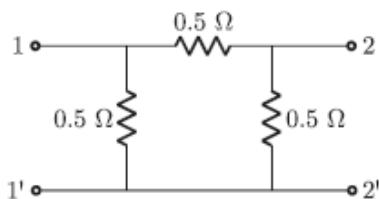
- (A) $1/90$ (B) $-1/90$ (C) $-1/99$ (D) $-1/11$
 22) In the circuit shown below, the initial charge on the capacitor is 2.5 mC , with the voltage polarity as indicated. The switch is closed at time $t = 0$. The current $i(t)$ at a time t after the switch is closed is



- (A) $i(t) = 15 \exp(-2 \times 10^3 t) \text{ A}$
 (B) $i(t) = 5 \exp(-2 \times 10^3 t) \text{ A}$
 (C) $i(t) = 10 \exp(-2 \times 10^3 t) \text{ A}$
 (D) $i(t) = 5 \exp(-2 \times 10^3 t) \text{ A}$

GATE 2010

23) For the two-port network shown below, the short-circuit admittance parameter matrix is

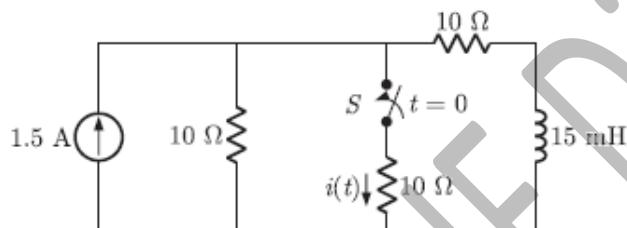


- (A) $\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \text{ S}$ (B) $\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} \text{ S}$ (C) $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \text{ S}$ (D) $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \text{ S}$

24) For parallel RLC circuit, which one of the following statements is NOT correct ?

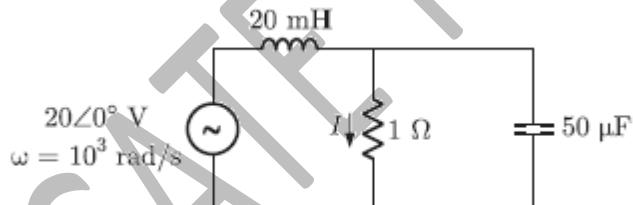
- (A) The bandwidth of the circuit decreases if R is increased
 (B) The bandwidth of the circuit remains same if L is increased
 (C) At resonance, input impedance is a real quantity
 (D) At resonance, the magnitude of input impedance attains its minimum value.

25) In the circuit shown, the switch S is open for a long time and is closed at $t = 0$. The current $i(t)$ for $t \geq 0+$ is



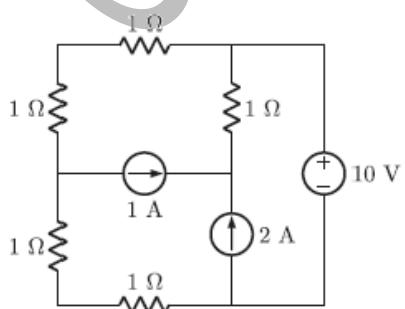
- (A) $i(t) = 0.5 - 0.125e^{-1000t} \text{ A}$
 (B) $i(t) = 1.5 - 0.125e^{-1000t} \text{ A}$
 (C) $i(t) = 0.5 - 0.5e^{-1000t} \text{ A}$
 (D) $i(t) = 0.375e^{-1000t} \text{ A}$

26) The current I in the circuit shown is



- (A) $-j1\text{A}$ (B) $j1\text{A}$ (C) 0 A (D) 20 A

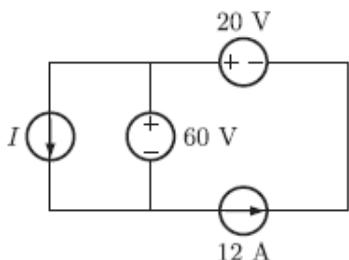
27) In the circuit shown, the power supplied by the voltage source is



- (A) 0 W (B) 5 W
 (C) 10 W (D) 100 W

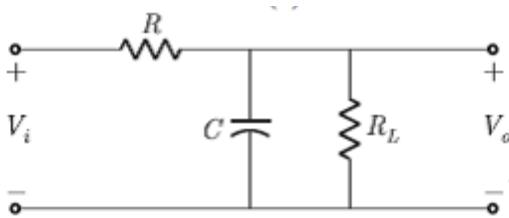
GATE 2009

- 28) In the interconnection of ideal sources shown in the figure, it is known that the 60 V source is absorbing power.



Which of the following can be the value of the current source I ?

- (A) 10 A (B) 13 A (C) 15 A (D) 18 A
 29) If the transfer function of the following network is $\frac{V_0(S)}{V_1(S)} = \frac{1}{2+sCR}$

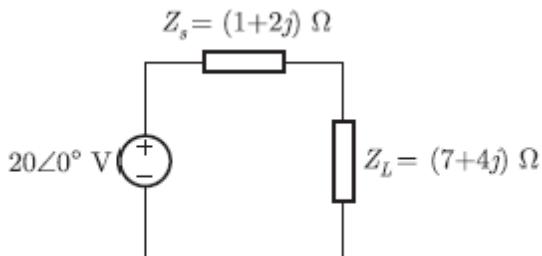


The value of the load resistance R_L is

- (A) $R/4$ (B) $R/2$ (C) R (D) $2R$
 30) A fully charged mobile phone with a 12 V battery is good for a 10 minute talk-time. Assume that, during the talk-time the battery delivers a constant current of 2 A and its voltage drops linearly from 12 V to 10 V as shown in the figure. How much energy does the battery deliver during this talk-time?

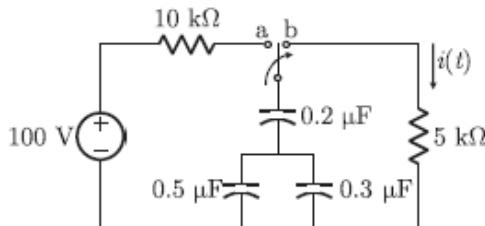


- (A) 220 J (B) 12 kJ (C) 13.2 kJ (D) 14.4 J
 31) An AC source of RMS voltage 20 V with internal impedance $Z_s = (1 + 2j)\Omega$ feeds a load of impedance $Z_L = (7 + 4j)\Omega$ in the figure below. The reactive power consumed by the load is



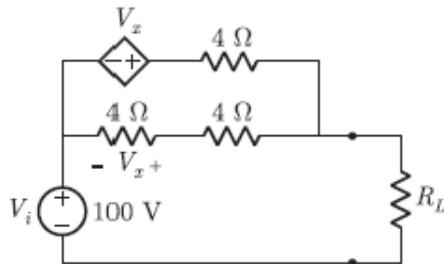
- (A) 8 VAR (B) 16 VAR (C) 28 VAR (D) 32 VAR

32) The switch in the circuit shown was on position a for a long time, and is move to position b at time $t = 0$. The current $i(t)$ for $t > 0$ is given by



- (A) $0.2e^{-125t} u(t)$ mA (B) $20e^{-1250t} u(t)$ mA (C) $0.2e^{-1250t} u(t)$ mA (D) $20e^{-1000t} u(t)$ mA

33) In the circuit shown, what value of R_L maximizes the power delivered to R_L ?



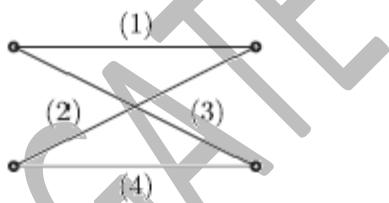
- (A) 2.4Ω (B) $\frac{8}{3} \Omega$ (C) 4Ω (D) 6Ω

34) The time domain behavior of an RL circuit is represented by $L \frac{di}{dt} + R_i = V_0 (1 + Be^{-Rt/L} \sin t)$ $u(t)$.

For an initial current of $i(0) = \frac{V_0}{R}$, the steady state value of the current is given by

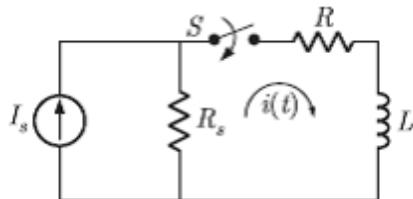
- (A) $i(t) \rightarrow \frac{V_0}{R}$ (B) $i(t) \rightarrow \frac{2V_0}{R}$ (C) $i(t) \rightarrow \frac{V_0}{R} (1 + B)$ (D) $i(t) \rightarrow \frac{2V_0}{R} (1 + B)$

35) In the following graph, the number of trees (P) and the number of cut-set (Q) are



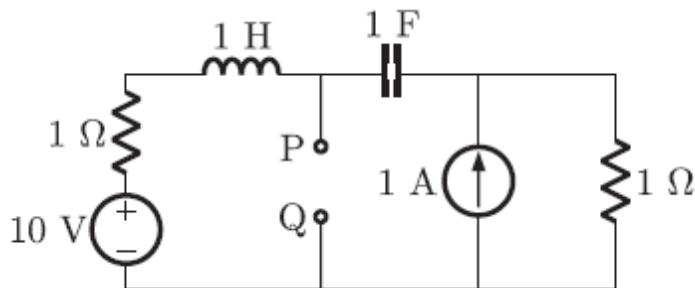
- (A) $P = 2, Q = 2$ (B) $P = 2, Q = 6$ (C) $P = 4, Q = 6$ (D) $P = 4, Q = 10$

36) In the following circuit, the switch S is closed at $t = 0$. The rate of change of current $\frac{di}{dt}(0^+)$ is given by



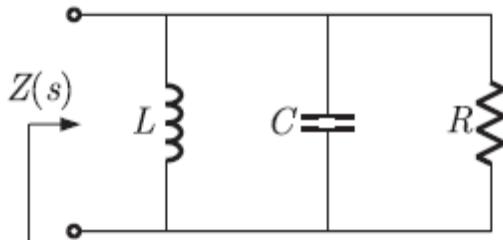
- (A) 0 (B) $\frac{R_S I_S}{L}$ (C) $\frac{(R+R_S)I_S}{L}$ (D) ∞

37) The Thevenin equivalent impedance Z_{th} between the nodes P and Q in the following circuit is



- (A) 1 (B) $1 + s + \frac{1}{s}$ (C) $2 + s + \frac{1}{s}$ (D) $\frac{s^2 + s + 1}{s^2 + 2s + 1}$

38) The driving point impedance of the following network $Z(s) = \frac{0.2s}{s^2 + 0.1s + 2}$



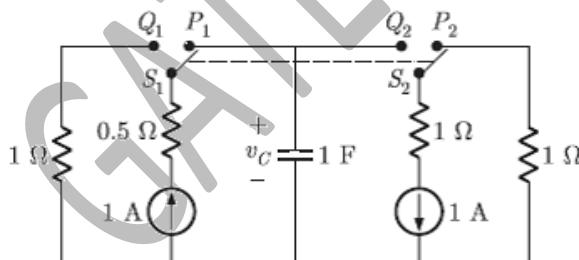
The component values are

- (A) $L = 5 \text{ H}, R = 0.5 \Omega, C = 0.1 \text{ F}$ (B) $L = 0.1 \text{ H}, R = 0.5 \Omega, C = 5 \text{ F}$
 (C) $L = 5 \text{ H}, R = 2 \Omega, C = 0.1 \text{ F}$ (D) $L = 0.1 \text{ H}, R = 2 \Omega, C = 5 \text{ F}$

39) The circuit shown in the figure is used to charge the capacitor C alternately from two current sources as indicated. The switches S₁ and S₂ are mechanically coupled and connected as follows:

For $2nT \leq t \leq (2n+1)T$, ($n = 0, 1, 2, \dots$) S₁ to P₁ and S₂ to P₂

For $(2n+1)T \leq t \leq (2n+2)T$, ($n = 0, 1, 2, \dots$) S₁ to Q₁ and S₂ to Q₂

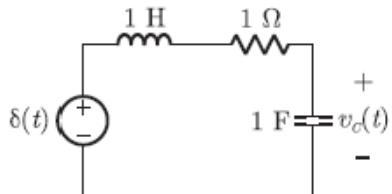


Assume that the capacitor has zero initial charge. Given that $u(t)$ is a unit step function, the voltage $v_C(t)$ across the capacitor is given by

- (A) $\sum_{n=1}^{\infty} (-1)^2 t u(t - nT)$
 (B) $u(t) + 2 \sum_{n=1}^{\infty} (-1)^2 t u(t - nT)$
 (C) $t u(t) + 2 \sum_{n=1}^{\infty} (-1)^2 t u(t - nT) (t - nT)$
 (D) $\sum_{n=1}^{\infty} [0.5 - e^{-(t-2nT)} + 0.5e^{-(t-2nT)} - T]$

Common Data For Q. 40 & 41 :

The following series RLC circuit with zero initial conditions is excited by a unit impulse function $\delta(t)$.



40) For $t > 0$, the output voltage $V_C(t)$ is

(A) $\frac{2}{\sqrt{3}} \left(e^{-\frac{1}{2}t} - e^{\frac{\sqrt{3}}{2}t} \right)$

(B) $\frac{2}{\sqrt{3}} \left(t e^{-\frac{1}{2}t} \right)$

(C) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos \left(\frac{\sqrt{3}}{2}t \right)$

(D) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin \left(\frac{\sqrt{3}}{2}t \right)$

41) For $t > 0$, the voltage across the resistor is

(A) $\frac{1}{\sqrt{3}} \left(e^{\frac{\sqrt{3}}{2}t} - e^{-\frac{1}{2}t} \right)$

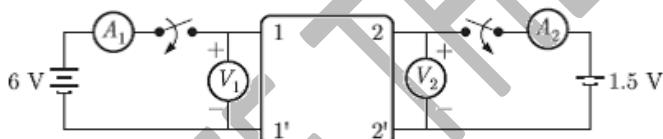
(B) $e^{-\frac{1}{2}t} \left[\cos \left(\frac{\sqrt{3}}{2}t \right) - \frac{1}{\sqrt{3}} \sin \left(\frac{\sqrt{3}}{2}t \right) \right]$

(C) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \sin \left(\frac{\sqrt{3}}{2}t \right)$

(D) $\frac{2}{\sqrt{3}} e^{-\frac{1}{2}t} \cos \left(\frac{\sqrt{3}}{2}t \right)$

Statement for linked Answers Questions 42 & 43:

A two-port network shown below is excited by external DC source. The voltage and the current are measured with voltmeters V_1, V_2 and ammeters A_1, A_2 (all assumed to be ideal), as indicated



Under following conditions, the readings obtained are:

- (1) S_1 - open, S_2 - closed $A_1 = 0, V_1 = 4.5 \text{ V}, V_2 = 1.5 \text{ V}, A_2 = 1 \text{ A}$
 (2) S_1 - open, S_2 - closed $A_1 = 4 \text{ A}, V_1 = 6 \text{ V}, V_2 = 0$

42) The z-parameter matrix for this network is

(A) $\begin{bmatrix} 1.5 & 1.5 \\ 4.5 & 1.5 \end{bmatrix}$

(B) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 4.5 \end{bmatrix}$

(C) $\begin{bmatrix} 1.5 & 4.5 \\ 1.5 & 1.5 \end{bmatrix}$

(D) $\begin{bmatrix} 4.5 & 1.5 \\ 1.5 & 4.5 \end{bmatrix}$

43) The h-parameter matrix for this network is

(A) $\begin{bmatrix} -3 & 3 \\ -1 & 0.67 \end{bmatrix}$

(B) $\begin{bmatrix} -3 & -1 \\ 3 & 0.67 \end{bmatrix}$

(C) $\begin{bmatrix} 3 & 3 \\ 1 & 0.67 \end{bmatrix}$

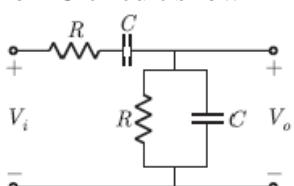
(D) $\begin{bmatrix} 3 & 1 \\ -3 & -0.67 \end{bmatrix}$

GATE 2007

44) An independent voltage source in series with an impedance $Z_s = R_s + jX_s$ delivers a maximum average power to a load impedance Z_L when

- (A) $Z_L = R_s + jX_s$ (B) $Z_L = R_s$ (C) $Z_L = jX_s$ (D) $Z_L = R_s - jX_s$

45) The RC circuit shown in the figure is



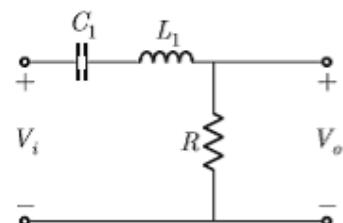
- (A) a low-pass filter

- (C) a band-pass filter

- (B) a high-pass filter

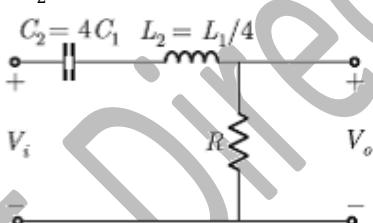
- (D) a band-reject filter

46) Two series resonant filters are as shown in the figure. Let the 3-dB bandwidth of Filter 1 be B_1 and that of Filter 2 be B_2 . the value $\frac{B_1}{B_2}$ is



- (A) 4

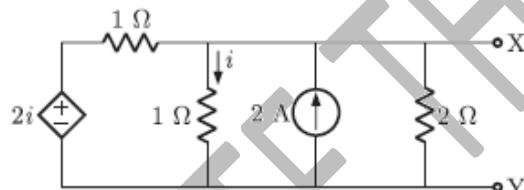
- (B) 1



- (C) 1/2

- (D) 1/4

47) For the circuit shown in the figure, the Thevenin voltage and resistance looking into X - Y are



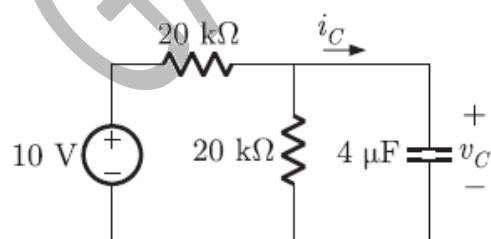
- (A) $\frac{4}{3}$ V, 2 Ω

- (B) 4 V, $\frac{2}{3}$ Ω

- (C) $\frac{4}{3}$ V, $\frac{2}{3}$ Ω

- (D) 4 V, 2 Ω

48) In the circuit shown, V_C is 0 volts at $t = 0$ sec. For $t > 0$, the capacitor current $i_C(t)$, where t is in seconds is given by



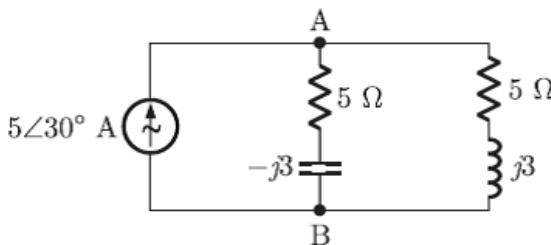
- (A) $0.50 \exp(-25t)$ mA

- (C) $0.50 \exp(-12.5t)$ mA

- (B) $0.25 \exp(-25t)$ mA

- (D) $0.25 \exp(-6.25t)$ mA

49) In the ac network shown in the figure, the phasor voltage V_{AB} (in Volts) is



- (A) 0 (B) $5 < 30^\circ$ (C) $12.5 < 30^\circ$ (D) $17 < 30^\circ$

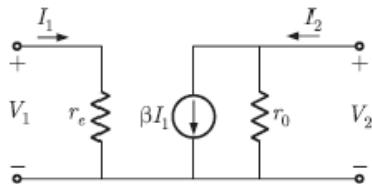
GATE 2006

50) A two-port network is represented by ABCD parameters given by $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

If port-2 is terminated by R_L , the input impedance seen at port-1 is given by

- (A) $\frac{A+BR_L}{C+DR_L}$ (B) $\frac{AR_L+C}{BR_L+D}$ (C) $\frac{DR_L+A}{BR_L+C}$ (D) $\frac{B+AR_L}{D+CR_L}$

51) In the two port network shown in the figure below, Z_{12} and Z_{21} are respectively

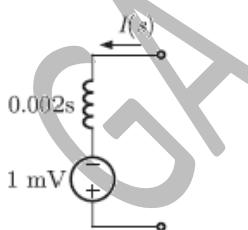


- (A) r_e and βr_0 (B) 0 and $-\beta r_0$ (C) 0 and βr_0 (D) r_e and $-\beta r_0$

52) The first and the last critical frequencies (singularities) of a driving point impedance function of a passive network having two kinds of elements are a pole and a zero respectively. The above property will be satisfied by

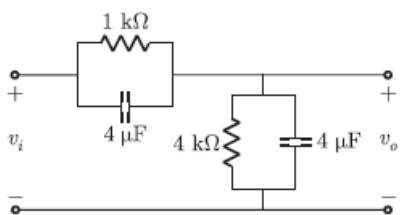
- (A) RL network only (B) RC network only
 (C) LC network only (D) RC as well as RL networks

53) A 2 mH inductor with some initial current can be represented as shown below, where s is the Laplace Transform variable. The value of initial current is



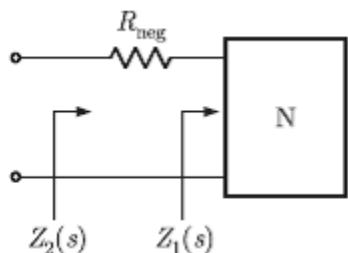
- (A) 0.5 A (B) 2.0 A (C) 1.0 A (D) 0.0 A

54) In the figure shown below, assume that all the capacitors are initially uncharged. If $v_i(t) = 10u(t)$ Volts, $v_o(t)$ is given by



- (A) $8e^{-t/0.004}$ Volts (B) $8(1 - e^{-t/0.004})$ Volts
 (C) $8u(t)$ Volts (D) 8 Volts

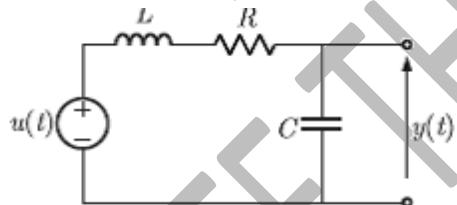
55) A negative resistance R_{neg} is connected to a passive network N having driving point impedance as shown below. For $Z_2(s)$ to be positive real,



- (A) $|R_{\text{neg}}| \leq R_e Z_1(jw), \forall w$ (B) $|R_{\text{neg}}| \leq Z_1(jw), \forall w$
 (C) $|R_{\text{neg}}| \leq \text{Im } Z_1(jw), \forall w$ (D) $|R_{\text{neg}}| \leq \angle Z_1(jw), \forall w$

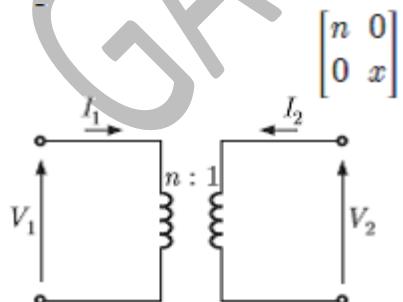
GATE 2005

56) The condition on R,L and C such that the step response $y(t)$ in the figure has no oscillations, is



- (A) $R \geq \frac{1}{2} \sqrt{\frac{L}{C}}$ (B) $R \geq \sqrt{\frac{L}{C}}$ (C) $R \geq 2 \sqrt{\frac{L}{C}}$ (D) $R \geq \sqrt{\frac{1}{LC}}$

57) The ABCD parameters of an ideal n:1 transformer shown in the figure are



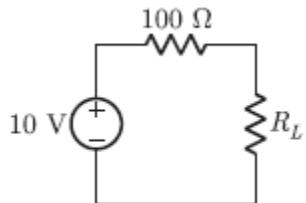
The value of x will be

- (A) n (B) $\frac{1}{n}$ (C) n^2 (D) $\frac{1}{n^2}$

58) In a series RLC circuit, $R = 2 \text{ k}\Omega$, $L = 1 \text{ H}$, and $C = \frac{1}{400} \mu\text{F}$ The resonant frequency is

- (A) $2 \times 10^4 \text{ Hz}$ (B) $\frac{1}{\pi} \times 10^4 \text{ Hz}$ (C) 10^4 Hz (D) $2\pi \times 10^4 \text{ Hz}$

59) The maximum power that can be transferred to the load resistor R_L from the voltage source in the figure is

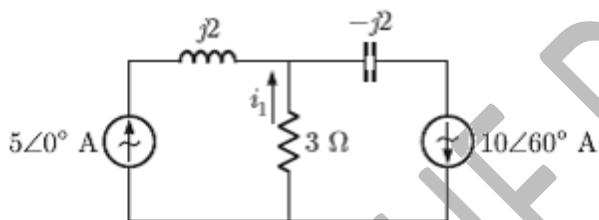


- (A) 1 W (B) 10 W (C) 0.25 W (D) 0.5 W

60) The first and the last critical frequency of an RC -driving point impedance function must respectively be

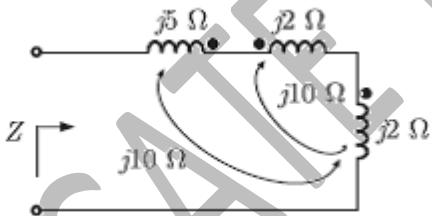
- (A) a zero and a pole (B) a zero and a zero (C) a pole and a pole (D) a pole and a zero

61) For the circuit shown in the figure, the instantaneous current $i_1(t)$ is



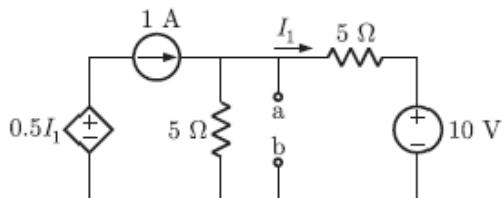
- (A) $\frac{10\sqrt{3}}{2} < 90^\circ \text{ A}$ (B) $\frac{10\sqrt{3}}{2} < -90^\circ \text{ A}$ (C) $5 < 60^\circ \text{ A}$ (D) $5 < -60^\circ \text{ A}$

62) Impedance Z as shown in the given figure is



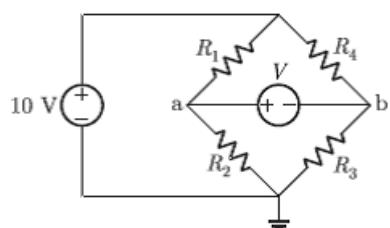
- (A) $j29 \Omega$ (B) $j9 \Omega$ (C) $j19 \Omega$ (D) $j39 \Omega$

63) For the circuit shown in the figure, Thevenin's voltage and Thevenin's equivalent resistance at terminals a - b is

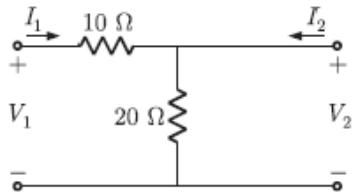


- (A) 5 V and 2Ω (B) 7.5 V and 2.5Ω (C) 4 V and 2Ω (D) 3 V and 2.5Ω

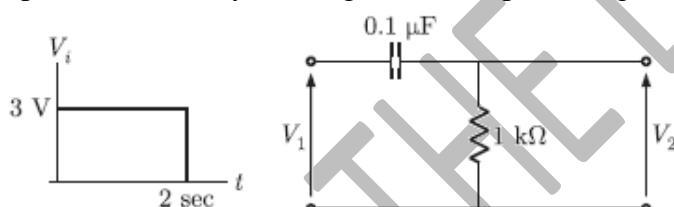
- 64) If $R_1 = R_2 = R_4 = R$ and $R_3 = 1.1R$ in the bridge circuit shown in the figure, then the reading in the ideal voltmeter connected between a and b is



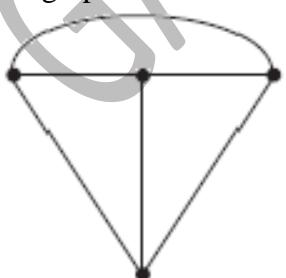
- (A) 0.238 V (B) 0.138 V (C) -0.238 V (D) 1 V
- 65) The h parameters of the circuit shown in the figure are

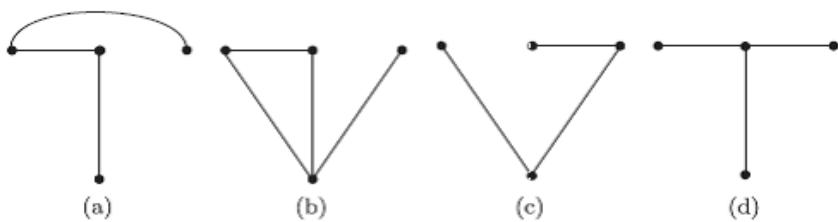


- (A) $\begin{bmatrix} 0.1 & 0.1 \\ -0.1 & 0.3 \end{bmatrix}$ (B) $\begin{bmatrix} 10 & -1 \\ 1 & 0.05 \end{bmatrix}$ (C) $\begin{bmatrix} 30 & 20 \\ 20 & 20 \end{bmatrix}$ (D) $\begin{bmatrix} 10 & 1 \\ -1 & 0.05 \end{bmatrix}$
- 66) A square pulse of 3 volts amplitude is applied to C-R circuit shown in the figure. The capacitor is initially uncharged. The output voltage V_2 at time $t = 2$ sec is



- (A) 3 V (B) -3 V (C) 4 V (D) -4 V
- GATE 2004**
- 67) Consider the network graph shown in the figure. Which one of the following is NOT a 'tree' of this graph?





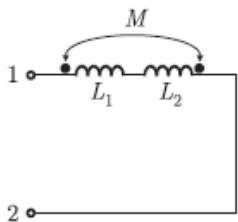
(A) a

(B) b

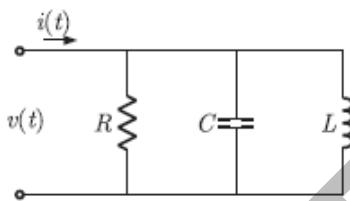
(C) c

(D) d

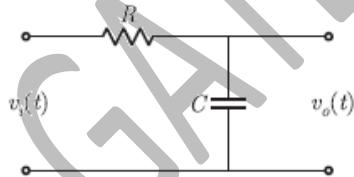
- 68) The equivalent inductance measured between the terminals 1 and 2 for the circuit shown in the figure is

(A) $L_1 + L_2 + M$ (B) $L_1 + L_2 - M$ (C) $L_1 + L_2 + 2M$ (D) $L_1 + L_2 - 2M$

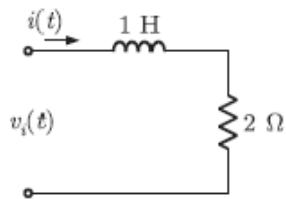
- 69) The circuit shown in the figure, with $R = \frac{1}{3} \Omega$, $L = \frac{1}{4} H$ and $C = 3 F$ has input voltage $v(t) = \sin 2t$. The resulting current $i(t)$ is

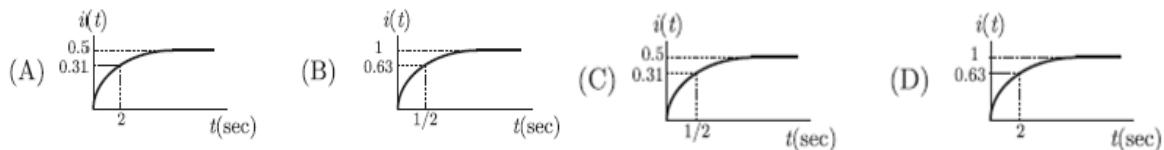
(A) $5 \sin(2t + 53.1^\circ)$ (C) $25 \sin(2t + 53.1^\circ)$ (B) $5 \sin(2t - 53.1^\circ)$ (D) $25 \sin(2t - 53.1^\circ)$

- 70) For the circuit shown in the figure, the time constant $RC = 1 \text{ ms}$. The input voltage is $v_i(t) = \sqrt{2} \sin 10^3 t$. The output voltage $v_o(t)$ is equal to

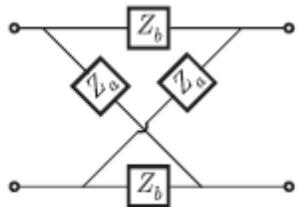
(A) $\sin(10^3 t - 45^\circ)$ (B) $\sin(10^3 t + 45^\circ)$ (C) $\sin(10^3 t - 53^\circ)$ (D) $\sin(10^3 t + 53^\circ)$

- 71) For the R - L circuit shown in the figure, the input voltage $v_i(t) = u(t)$. The current $i(t)$ is



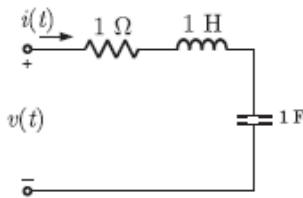


- 72) For the lattice shown in the figure, $Z_a = j2 \Omega$ and $Z_b = 2 \Omega$. The values of the open circuit impedance parameters $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ are



- (A) $\begin{bmatrix} 1-j & 1+j \\ 1+j & 1+j \end{bmatrix}$ (B) $\begin{bmatrix} 1-j & 1+j \\ -1+j & 1-j \end{bmatrix}$
(C) $\begin{bmatrix} 1-j & 1+j \\ 1-j & 1-j \end{bmatrix}$ (D) $\begin{bmatrix} 1-j & -1+j \\ -1+j & 1+j \end{bmatrix}$

- 73) The circuit shown in the figure has initial current $i_L(0^-) = 1$ A through the inductor and an initial voltage $V_C(0^-) = -1$ V across the capacitor. For input $v(t) = u(t)$, the Laplace transform of the current $i(t)$ for $t \geq 0$ is

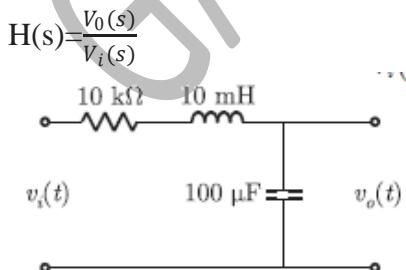


- (A) $\frac{s}{s^2+s+1}$ (B) $\frac{s+2}{s^2+s+1}$ (C) $\frac{s-2}{s^2+s+1}$ (D) $\frac{1}{s^2+s+1}$

- 74) The transfer function $H(s) = \frac{V_o(s)}{V_i(s)}$ of an RLC circuit is given by $H(s) = \frac{10^6}{s^2 + 20s + 10^6}$. The Quality factor (Q-factor) of this circuit is

- (A) 25 (B) 50 (C) 100 (D) 5000

- 75) For the circuit shown in the figure, the initial conditions are zero. Its transfer function



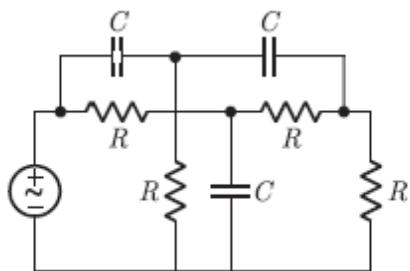
- (A) $\frac{1}{s^2 + 10^6 s + 10^6}$ (B) $\frac{10^6}{s^2 + 10^3 s + 10^6}$
(C) $\frac{10^3}{s^2 + 10^3 s + 10^6}$ (D) $\frac{10^6}{s^2 + 10^6 s + 10^6}$

76) Consider the following statements S1 and S2

- S1 : At the resonant frequency the impedance of a series RLC circuit is zero.
 S2 : In a parallel GLC circuit, increasing the conductance G results in increase in its Q factor.
 Which one of the following is correct?
- (A) S1 is FALSE and S2 is TRUE
 - (B) Both S1 and S2 are TRUE
 - (C) S1 is TRUE and S2 is FALSE
 - (D) Both S1 and S2 are FALSE

GATE 2003

77) The minimum number of equations required to analyze the circuit shown in the figure is



- (A) 3 (B) 4 (C) 6 (D) 7

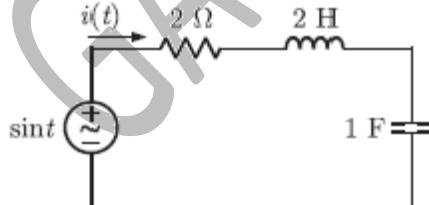
78) A source of angular frequency 1 rad/sec has a source impedance consisting of 1 Ω resistance in series with 1 H inductance. The load that will obtain the maximum power transfer is

- (A) 1 Ω resistance
- (B) 1 Ω resistance in parallel with 1 H inductance
- (C) 1 Ω resistance in series with 1 F capacitor
- (D) 1 Ω resistance in parallel with 1 F capacitor

79) A series RLC circuit has a resonance frequency of 1 kHz and a quality factor $Q = 100$. If each of R, L and C is doubled from its original value, the new Q of the circuit is

- (A) 25 (B) 50 (C) 100 (D) 200

80) The differential equation for the current $i(t)$ in the circuit of the figure is



- (A) $2 \frac{d^2i}{dt^2} + 2 \frac{di}{dt} + i(t) = \sin(t)$ (B) $\frac{d^2i}{dt^2} + 2 \frac{di}{dt} + 2i(t) = \cos(t)$
 (C) $2 \frac{d^2i}{dt^2} + 2 \frac{di}{dt} + i(t) = \cos(t)$ (D) $\frac{d^2i}{dt^2} + 2 \frac{di}{dt} + 2i(t) = \sin(t)$

81) Twelve 1 Ω resistances are used as edges to form a cube. The resistance between two diagonally opposite corners of the cube is

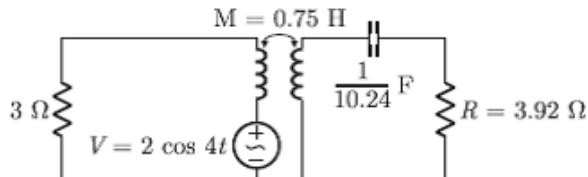
(A) $\frac{5}{6} \Omega$

(B) 1Ω

(C) $\frac{6}{5} \Omega$

(D) $\frac{3}{2} \Omega$

- 82) The current flowing through the resistance R in the circuit in the figure has the form $P \cos 4t$ where P is



(A) $(0.18 + j0.72)$

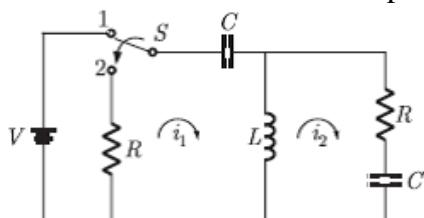
(B) $(0.46 + j1.90)$

(C) $-(0.18 + j1.90)$

(D) $-(0.192 + j0.144)$

The circuit for Q. 83 & 84 is given below.

Assume that the switch S is in position 1 for a long time and thrown to position 2 at $t = 0$.



- 83) At $t = 0^+$, the current i_1 is

(A) $\frac{-V}{2R}$

(B) $\frac{V}{R}$

(C) $\frac{-V}{4R}$

(D) zero

- 84) $I_1(s)$ and $I_2(s)$ are the Laplace transforms of $i_1(t)$ and $i_2(t)$ respectively. The equations for the loop currents $I_1(s)$ and $I_2(s)$ for the circuit shown in the figure, after the switch is brought from position 1 to position 2 at $t = 0$, are

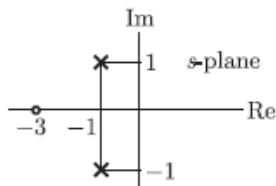
$$(A) \begin{bmatrix} R + Ls + \frac{1}{Cs} & -Ls \\ -Ls & R + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{V}{s} \\ 0 \end{bmatrix}$$

$$(B) \begin{bmatrix} R + Ls + \frac{1}{Cs} & -Ls \\ -Ls & R + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -\frac{V}{s} \\ 0 \end{bmatrix}$$

$$(C) \begin{bmatrix} R + Ls + \frac{1}{Cs} & -Ls \\ -Ls & R + Ls + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} -\frac{V}{s} \\ 0 \end{bmatrix}$$

$$(D) \begin{bmatrix} R + Ls + \frac{1}{Cs} & -Cs \\ -Ls & R + Ls + \frac{1}{Cs} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{V}{s} \\ 0 \end{bmatrix}$$

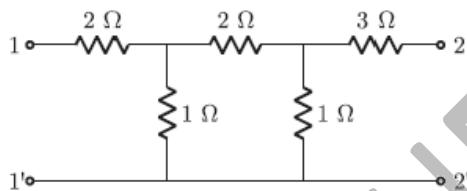
- 85) The driving point impedance $Z(s)$ of a network has the pole-zero locations as shown in the figure. If $Z(0) = 3$, then $Z(s)$ is



- (A) $\frac{3(s+3)}{s^2+2s+3}$ (B) $\frac{2(s+3)}{s^2+2s+3}$ (C) $\frac{3(s+3)}{s^2+2s+2}$ (D) $\frac{2(s-3)}{s^2-2s-3}$

- 86) An input voltage $v(t) = 10\sqrt{2} \cos(t + 10^\circ) + 10\sqrt{5} \cos(2t + 10^\circ)$ V is applied to a series combination of resistance $R = 1\Omega$ and an inductance $L = 1$ H. The resulting steady-state current $i(t)$ in ampere is
- (A) $10 \cos(t + 55^\circ) + 10 \cos(2t + 10^\circ + \tan^{-1}2)$
 (B) $10 \cos(t + 55^\circ) + 10\sqrt{\frac{3}{2}} \cos(2t + 55^\circ)$
 (C) $10 \cos(t - 35^\circ) + 10 \cos(2t + 10^\circ - \tan^{-1}2)$
 (D) $10 \cos(t - 35^\circ) + 10\sqrt{\frac{3}{2}} \cos(2t + 35^\circ)$

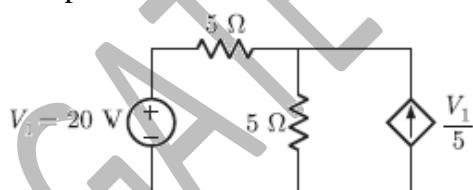
- 87) The impedance parameters z_{11} and z_{12} of the two-port network in the figure are



- (A) $z_{11} = 2.75 \Omega$ and $z_{12} = 0.25 \Omega$
 (B) $z_{11} = 3 \Omega$ and $z_{12} = 0.5 \Omega$
 (C) $z_{11} = 3 \Omega$ and $z_{12} = 0.25 \Omega$
 (D) $z_{11} = 2.25 \Omega$ and $z_{12} = 0.5 \Omega$

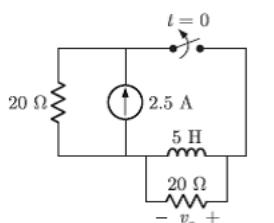
GATE 2002

- 88) The dependent current source shown in the figure



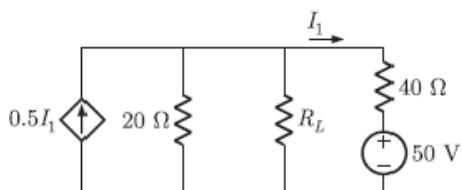
- (A) delivers 80 W (B) absorbs 80 W (C) delivers 40 W (D) absorbs 40 W

- 89) In figure, switch was closed for a long time before opening at $t = 0$. The voltage v_x at $t = 0^+$ is



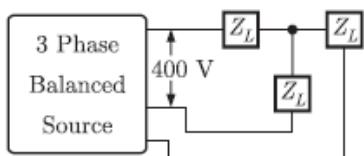
- (A) 25 V (B) 50 V (C) -50 V (D) 0 V

90) In the network of the fig, the maximum power is delivered to R_L if its value is



- (A) 16Ω (B) $\frac{40}{3}\Omega$ (C) 60Ω (D) 20Ω

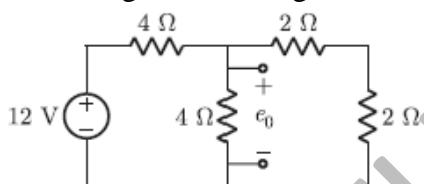
91) If the 3-phase balanced source in the figure delivers 1500 W at a leading power factor 0.844 then the value of Z_L (in ohm) is approximately



- (A) $90 < 32.44^\circ$ (B) $80 < 32.44^\circ$ (C) $80 < -32.44^\circ$ (D) $90 < -32.44^\circ$

GATE 2001

92) The Voltage e_0 in the figure is

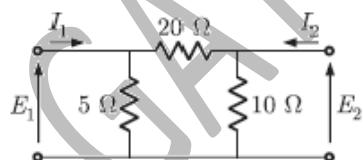


- (A) 2 V (B) $4/3$ V (C) 4 V (D) 8 V

93) If each branch of Delta circuit has impedance $\sqrt{3} Z$, then each branch of the equivalent Wye circuit has impedance

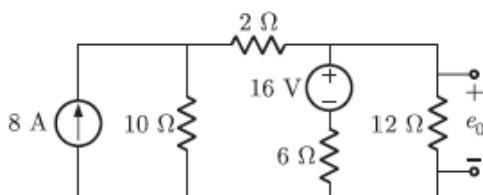
- (A) $\frac{Z}{\sqrt{3}}$ (B) $3Z$ (C) $3\sqrt{3} Z$ (D) $\frac{Z}{3}$

94) The admittance parameter Y_{12} in the 2-port network in Figure is



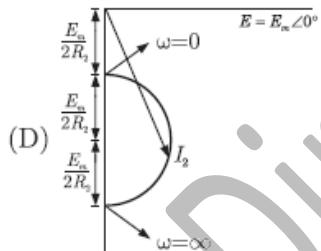
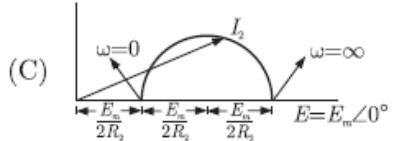
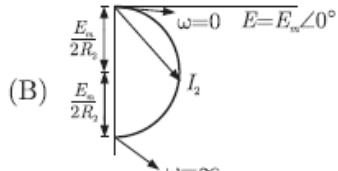
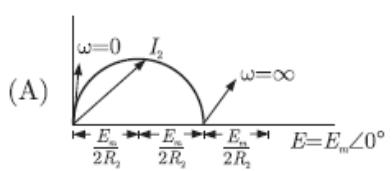
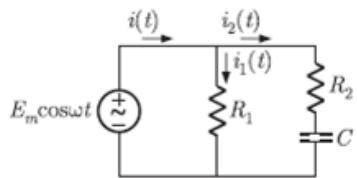
- (A) -0.02 mho (B) 0.1 mho (C) -0.05 mho (D) 0.05 mho

95) The Voltage e_0 in the figure is

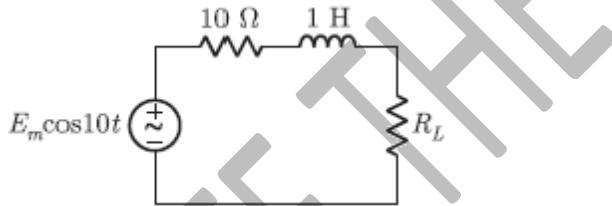


- (A) 48 V (B) 24 V (C) 36 V (D) 28 V

- 96) When the angular frequency ω in the figure is varied 0 to ∞ , the locus of the current phasor I_2 is given by

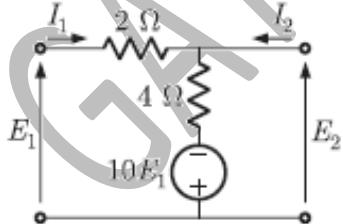


- 97) In the figure, the value of the load resistor R_L which maximizes the power delivered to it is



- (A) 14.14 Ω (B) 10 Ω (C) 200 Ω (D) 28.28 Ω

- 98) The z parameters z_{11} and z_{21} for the 2-port network in the figure are

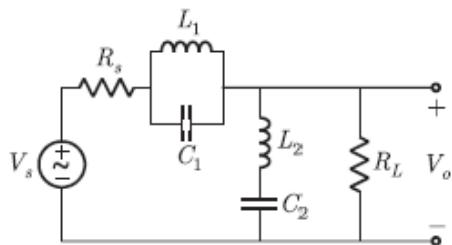


- (A) $Z_{11} = \frac{6}{11} \Omega$; $Z_{21} = \frac{16}{11} \Omega$
 (C) $Z_{11} = \frac{6}{11} \Omega$; $Z_{21} = -\frac{16}{11} \Omega$

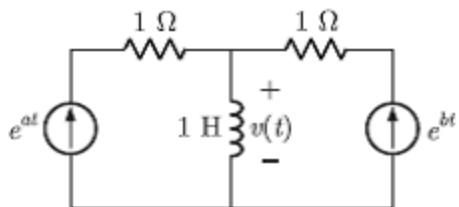
- (B) $Z_{11} = \frac{6}{11} \Omega$; $Z_{21} = \frac{4}{11} \Omega$
 (D) $Z_{11} = \frac{4}{11} \Omega$; $Z_{21} = \frac{4}{11} \Omega$

GATE 2000

- 99) The circuit of the figure represents a

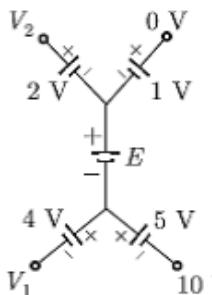


- (A) Low pass filter
 (C) band pass filter
 100) In the circuit of the figure, the voltage $v(t)$ is

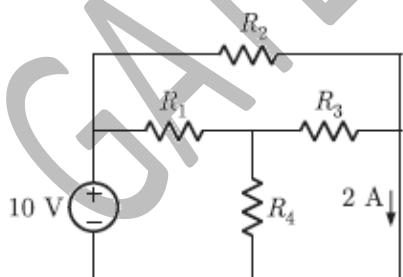


- (B) High pass filter
 (D) band reject filter

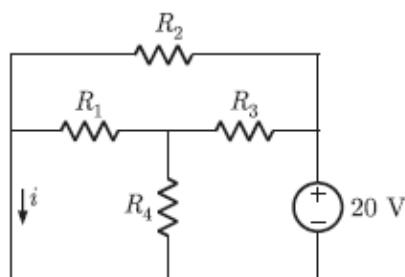
- (A) $e^{at} - e^{bt}$
 (B) $e^{at} + e^{bt}$
 (C) $ae^{at} - be^{bt}$
 (D) $ae^{at} + be^{bt}$
- 101) In the circuit of the figure, the value of the voltage source E is



- (A) -16 V
 (B) 4 V
 (C) -6 V
 (D) 16 V
- 102) Use the data of the figure (a). The current i in the circuit of the figure (b)



- (a)
 (A) -2 A
 (B) 2 A



- (b)
 (C) -4 A
 (D) 4 A

ANSWERS:

1. B	2. D	3. C	4. C	5. C	6. C	7. C	8. D	9. C	10. D
11. B	12. C	13. A	14. A	15. C	16. B	17. A	18. C	19. A	20. B
21. D	22. A	23. A	24. D	25. A	26. A	27. A	28. A	29. C	30. C
31. B	32. B	33. C	34. A	35. C	36. B	37. A	38. D	39. C	40. D
41. B	42. C	43. A	44. D	45. C	46. D	47. D	48. A	49. D	50. D
51. B	52. B	53. A	54. B	55. A	56. C	57. B	58. B	59. C	60. C
61. A	62. B	63. B	64. C	65. D	66. B	67. B	68. D	69. A	70. A
71. C	72. D	73. B	74. B	75. D	76. D	77. B	78. C	79. B	80. C
81. A	82. --	83. A	84. C	85. B	86. C	87. A	88. A	89. C	90. A
91. D	92. C	93. A	94. C	95. D	96. A	97. A	98. C	99. D	100. D
101. A	102. C								