W2	Learning Area	Basic Calculus	Grade Level	Grade 12
	Quarter	Quarter 4	Date	May, 2021

I. LESSON TITLE	***Antiderivatives of Algebraic Functions***
II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)	The learner should be able to: (a) illustrate an antiderivative of polynomial, radical, exponent, and trigonometric function, (b) compute the general antiderivative of polynomial, radical, exponent, and trigonometric function, and (c) apply in real-life situation.
III. CONTENT/CORE CONTENT	

IV. LEARNING PHASES AND LEARNING ACTIVITIES

I. Introduction (Time Frame: 30 minutes)

In some previous lesson we learned how to find derivatives of different functions. Now we will do the inverse differentiation. We call this process antidifferentiation. Given a function f, can we find the function F whose derivatives is f? A function F is an antiderivatives of the function f on an interval I if F'(x)=f(x) for every value of x in I.
Let's Study first some terminologies and symbol before we start.

Terminologies and Notations:

- ***Antidifferentation*** is the process of finding the antiderivative.
- The symbol $\int$, also called the ***integral sign***, denotes the operation of antidifferentiation.
- The function f is called the ***integrand***.
- If F is an antiderivative of f, we write $\int f(x)\,dx = F(x) + C$.
- The symbols $\int$ and dx go hand-in-hand and dx helps us identify the variable of integration.
- The expression $F(x)+C$ is called the ***general antiderivative*** of f. Meanwhile, each antiderivative of f is called a ***particular antiderivative*** of f.

Activity 1:
Try to find the derivatives of the following functions before we continue.

a. $F(x) = x^3 + 2x^2 + x$
b. $F(x) = x^3 + x^2 + x$
c. $F(x) = 3x^3 + 3x - 1$
d. $F(x) = 2x^3 - 3x^2 + 1$
e. $F(x) = 5x^2 - 2x - 3$

D. Development (Time Frame: 30 minutes)
Let's Study first some terminologies and symbol before we continue.

The process of antidifferentiation is just the inverse process of finding the derivatives of functions. We have shown in the previous lesson that a function can have a family of antiderivatives.
Let us recall the following differentiation formulas.

(a) $D_x(x) = 1$.

(b) $D_x(x^n) = nx^{n-1}$, where n is any real number.

(c) $D_x[a(f(x))] = aD_x[f(x)]$.

(d) $D_x[f(x) \pm g(x)] = D_x[f(x)] \pm D_x[g(x)]$.

The above formulas lead the following theorem which are used in obtaining the antiderivatives of functions. We apply them to integrate polynomial, rational functions and radical functions.

IV. LEARNING PHASES AND LEARNING ACTIVITIES

Theorems on Antidifferentiation

(a) $\int dx = x + C$

(b) If n is any real number and n≠-1, then $\int x^n dx = \frac{x^n+1}{n+1} + C$.

(c) If a is any constant and f is a function, then $\int af(x)dx = a\int f(x)dx$.

(d) If f and g are functions defined on the same interval, $\int[f(x) \pm g(x)]dx = \int f(x)dx \neq \int g(x)dx$.

Example 1: Determine the derivatives of $\int 3dx$

Solution:

Using a and c theorem, we have $\int 3dx = 3\int dx = 3x + C$

Example 2: Determine the derivatives of $\int x^6 dx$

Solution:

Using b of the theorem, we have $\int x^6 dx = \frac{x^{6+1}}{6+1} + C = \frac{x^7}{7} + C$

Theorems on Integrals Yielding the Exponential and Logarithmic Functions

(a) $\int e^x dx = e^x + C$

(b) $\int a^x dx = \frac{a^x}{\ln a} + C. Here, a > 0\ with\ a \neq 1.$

(c) $\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$

Example 3: Find the integral of $\int(e^x + 2^x)dx$

Solution:

Using a and b theorem, we have $\int e^x + 2^x)dx = \int(e^x)dx + \int(2^x)dx = e^x + \frac{2^x}{\ln 2} + C$

Example 4: Find the integral of $\int 3^{x+1}dx$

Solution:

Using b of the theorem, we have $\int 3^{x+1}dx = \int(3^x)(3^1)dx = 3\int(3^x)dx = 3\frac{3^x}{\ln 3} + C$

Theorems on Antiderivatives of Trigonometric Functions

(a) $\int \sin x\, dx = -\cos x + C$

(b) $\int \cos x\, dx = \sin x + C$

(c) $\int sec^2 x\, dx = \tan x + C$

(d) $\int csc^2 x\, dx = -\cot x + C$

(e) $\int \sec x \tan x\, dx = \sec x + C$

(f) $\int \csc x \cot x\, dx = -\csc x + c$

Example 5: Determine the antiderivatives of $\int(\cos x - \sin x)dx$

Solution:

Using a and b of the theorem, we have $\int(\cos x - \sin x)dx = \int \cos x\, dx - \int \sin x\, dx = \sin x - (-\cos x) + C = sinx + cosx + C$

Example 6: Determine the antiderivatives of $\int \frac{\sin x}{cos^2 x} dx$

Solution:

$$\int \frac{\sin x}{cos^2 x} dx = \int \frac{\sin x}{\cos x} \frac{1}{\cos x} dx = \int \tan x \sec x\, dx = \sec x + C$$

Try the following functions.

Activity 2:

Determine the following antiderivatives.

a. $\int \frac{1}{x^6} dx$

b. $\int 4\sqrt{u}du$

c. $\int(12x^2 + 2x)dx$

d. $\int t(2t - 3\sqrt{t})dt$

e. $\int \frac{x^2+1}{x^2} dx$

f. $\int 5^{x+1}dx$

g. $\int \frac{2}{x} dx$

h. $\int(\sin u + u)du$

i. $\int \frac{\cos x}{sin\ ^2x} dx$

j. $\int \frac{\cos x}{sin^2 x} dx$

f. $\int 5^{x+1}dx$

g. $\int \frac{2}{x} dx$

h. $\int(\sin u + u)du$

i. $\int \frac{\cos x}{sin\ ^2x} dx$

j. $\int \frac{\cos x}{sin^2 x} dx$

IV. LEARNING PHASES AND LEARNING ACTIVITIES

E. Engagement (Time Frame: 60 minutes)

Some of our common mistakes in antidifferentiation is distributing the integral sign over a product or a quotient. Please reiterate that

$$\int f(x)g(x)dx \neq \int f(x)dx \bullet \int g(x)dx \; and \; \int \frac{f(x)}{g(x)}dx \neq \frac{\int f(x)dx}{\int g(x)dx}$$

In antidifferentiation, it is better to rewrite a product or a quotient into a sum or difference. This technique was in some items in the examples above.

Apply all of these to the activity below.

Activity 3:

1. $\int x^2 dx$
2. $\int \sqrt[3]{x^2} dx$
3. $\int (x^3 + x^2) dx$
4. $\int \sqrt[4]{x^4} dx$
5. $\int x^{-2} dx$
6. $\int tan^2 x\, dx$
7. $\int (1 - \cos u) du$
8. $\int cot^2 x\, dx$
9. $\int 3x^{-1} dx$
10. $\int 2(5^x) dx$

A. Assimilation (Time Frame: 10 minutes)

Since we already presented $\int \sin x\, dx$ and $\int \cos x\, dx$, we can now do the following:

1. $\int \tan x\, dx =$ ___________
2. $\int \cot x\, dx =$ ___________
3. $\int \sec x\, dx =$ ___________
4. $\int \csc x\, dx =$ ___________

We will take more integrals yielding the exponential and logarithmic functions since we discussed integration by substitution. Let's try to evaluate the following:

5. $\int e^{2x} dx =$ ___________
6. $\int 2^{4x} dx =$ ___________
7. $\int \frac{1}{2x-1} dx =$ ___________

Let's also find the antiderivatives of the following:

8. $\int (13x^2 + 3x) dx =$ ___________
9. $\int x^5 dx =$ ___________
10. $\int \frac{x^3+1}{x^3} dx =$ ___________

V. ASSESSMENT (Time Frame: ________)

(Learning Activity Sheets for Enrichment, Remediation, or Assessment to be given on Weeks 3 and 6)

Determine the following antiderivatives of the following:

1. $\int \sqrt[3]{x} dx$
2. $\int (u^2 + u + 1) du$
3. $\int (\frac{1}{4}v^4 + v^2 + v) dv$
4. $\int \frac{w^3+w^2+w}{w^3} dw$
5. $\int 1000 dx$
6. $\int \frac{3}{x} dx$
7. $\int 5x^{-1} dx$
8. $\int (2e^x + 7^x) dx$
9. $\int \frac{1}{\sec y \tan y} dy$
10. $\int \frac{1}{sin^2 x cos^2 x} dx$

VI. REFLECTION (Time Frame: ________)

- Communicate your personal assessment as indicated in the Learner's Assessment Card.

Personal Assessment on Learner's Level of Performance

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below:

☆ - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/ lesson.

✓ - I was able to do/perform the task. It was quite challenging, but it still helped me in understanding the target content/lesson.

? – I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

Learning Task	LP	Learning Task	LP	Learning Task	LP	Learning Task	LP
Number 1		Number 3		Number 5		Number 7	
Number 2		Number 4		Number 6		Number 8	

VII. REFERENCES	TEACHING_GUIDE_FOR_SENIOR_HIGH_SCHOOL_Basic_Calculus_CORE_SUBJECT Commission_on_Higher_Education_in_collaboration_with_the_Philippine_Normal_University

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