| W1 | Learning Area | Basic Calculus | Grade Level | Grade 11 |
|---|---|---|---|---|
| | Quarter | Quarter 4 | Date | |

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| **I. LESSON TITLE** | Antiderivatives of a functions. |
| **II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)** | 1. Illustrate the antiderivative of a function;<br>2. Compute the general antiderivative of polynomial functions;<br>3. Compute the general antiderivative of root functions; |
| **III. CONTENT/CORE CONTENT** | |

**IV. LEARNING PHASES AND LEARNING ACTIVITIES**

**I.** Introduction (Time Frame: 20 mins)

In the previous discussions, we learned how to find the derivatives of different functions. Now, we will introduce the inverse of differentiation. We shall call this process antidifferentiation. A natural question then arises:

A function F is an antiderivative of the function f on an interval I if :
F0(x) = f(x) for every value of x in I.

As previously discussed, the process of antidifferentiation is just the inverse process of finding the derivatives of functions. We have shown in the previous lesson that a function can have a family of antiderivatives.

We will look at antiderivatives of different types of functions. Particularly, we will find the antiderivatives of polynomial functions, rational functions and radical functions.

**Terminologies and Notations:**

- ***Antidifffferentation*** is the process of finding the antiderivative.
- The symbol $\int$, also called the ***integral sign***, denotes the operation of antidifferentiation.
- The function f is called the ***integrand***.
- If F is an antiderivative of f, we write $\int \quad f(x)dx = F(x) + C.$
- The symbols $\int$ and dx go hand-in-hand and dx helps us identify the variable of integration.
- The expression F(x)+C is called the general antiderivative of f. Meanwhile,

each antiderivative of f is called a particular antiderivative of f.

**D.** Development (Time Frame: 60 mins.)

We will now give examples of antiderivatives of functions.

**EXAMPLE 1:**

(a) An antiderivative of *f*(x) = 12x2+2x is *F*(x)=4x3+x2. As we can see, the derivative of *F* is given by *F0*(x) = $12x^2 + 2x$ = *f*(x).

(b) An antiderivative of *g*(x) = cos x is *G*(x) = sin x because *G0*(x) = cos x = *g*(x).

**Remark 1:** The antiderivative *F* of a function *f* is **not** unique.

**EXAMPLE 2:**

(a) Other antiderivatives of *f*(x) = $12x^2 + 2x$ are *F*1(x)=$4x^3 + x^2 - 1$ and $F_2$(x) = $4x^3 + x^2 + 1$. In fact, any function of the form *F*(x)=$4x^3 + x^2 + C$, where C ∈ R is an antiderivative of *f*(x). Observe that $F_0$(x) = $12x^2 + 2x + 0 = 12x^2 + 2x$ = *f*(x).

(b) Other antiderivatives of *g*(x) = cos x are $G_1$(x) = sin x + $\pi$ and $G_2$(x) = sin x - 1. In fact, any function *G*(x) = sin x + *C*, where C∈ R is an antiderivative of *g*(x).

**Theorem 10.** *If F is an antiderivative of f on an interval I, then every antiderivative of f on I is given by F*(x) *+ C, where C is an arbitrary constant.*

**Remark 2:** Using the theorem above, we can conclude that if *F*1 and *F*2 are antiderivatives of *f*, then *F*2 (x) = *F*1 (x) + *C*. That is, *F*1 and *F*2 differ only by a constant.

Let us recall first the following differentiation formulas:

(a) *Dx*(x)=1.

(b) $D_x(x^n) = nx^{n-1}$, where $n$ is any real number.

(c) $D_x[a(f(x))] = aD_x[f(x)]$.

(d) $D_x[f(x) \pm g(x)] = D_x[f(x)] \pm D_x[g(x)]$.

The above formulas lead to the following theorem which are used in obtaining the antiderivatives of functions. We apply them to integrate polynomials, rational functions and radical functions.

**Theorem 11. (Theorems on Antidifferentiation)**

(a) $\int dx = x + C$

(b) *If n is any real number and n ≠ 1 then* $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

(c) *If a is any constant and f is a function, then* $\int af(x)\,dx = a\int f(x)dx$

(d) *If f and g are functions defined on the same interval* $\int [f(x) + g(x)] = \int f(x)dx + \int g(x)dx$

**E.** Engagement (Time Frame: 60 mins.)

**Learning Activity 1:**

Determine the general antiderivatives of the following functions:

1. $f(x)=6x^5$ - x4 + $2x^2$ - 3

2. $f(x) = -3$

3. $g(x) = x^3-3x^2 + 3x- 1$

4. $h(x) = \sin x + \cos x- \sec^2 x$

**Learning Activity 2:**

Matching type: Match the functions in Column A with their corresponding antiderivatives in Column B.

| Column A | Column B |
|---|---|
| a. $f(x)=3x^2 + 2x + 1$ | a. $F(x)=3x^3 - x$ |
| b. $f(x)=9x^2 -1$ | b. $F(x) = x^3 + x^2 + x$ |
| c. $f(x) = x^2 - 2$ | c. $F(x) = 2x^2 - \frac{1}{3}x^3$ |
| d. $f(x)=(x + 1)(x ! 1)$ | d. $F(x) = 2x^2 + \frac{1}{3}x^3$ |
| e. $f(x) = x(4 - x)$ | e. $F(x) = \frac{1}{3}x^3 - 2x + 1$ |
| f. $f(x) = x(x - 4)$ | d. $F(x) = \frac{1}{3}x^3 - x + 1$ |

**Learning Activity 3:**

Determine the following antiderivatives:

**1.** $\int x^2 dx$

**2.** $\int \quad x^3 + 2x^2 dx$

**3.** $\int \quad (\frac{3}{4}x^4 + 3x^2 + 1)dx$

**4.** $\int \quad x^{-2}dx$

**5.** $\int \quad (x^{-3} + x^{-2} + x^{-1})dx$

**6.** $\int \quad x^{-100}dx$

**7.** $\int \quad (3x^{-2} + x + 2)dx$

**8.** $\int \quad (\frac{4}{x^2} + 1\ \frac{2}{8x^3})\ \mathrm{dx}$

**9.** $\int \quad (\sqrt{x^2} - 7\sqrt[6]{x^5} + 17\sqrt[3]{x^{10}}\ )dx$

**10.** $\int \quad (\sqrt{x}\ [x^2 - \frac{1}{4x}1)dx$

**A.** Assimilation (Time Frame: 5 mins.)

**Antiderivatives** are the opposite of derivatives. An **antiderivative** is a function that reverses what the derivative does. One function has many **antiderivatives**, but they all take the form of a function plus an arbitrary constant. **Antiderivatives** are a key part of indefinite integrals.

**V. ASSESSMENT** (Time Frame: 20 mins)

Compute for the Indefinite Integrals

1. $\int \quad 1000\ dx$
2. $\int \quad (2x^6 - 4x^3 + 7x + 5)\ dx$
3. $(\sqrt[4]{2x^4} + \sqrt[3]{x^2}\ )\mathrm{dx}$
4. $\int \quad (\frac{w^3 + w^2 + w}{w^2})dw$
5. $\int \quad (\frac{1}{1+y^2} + \frac{6}{\sqrt{1-y^2}})\ dx$

**VI. REFLECTION** (Time Frame: __________)

- Communicate your personal assessment as indicated in the Learner's Assessment Card.

**Personal Assessment on Learner's Level of Performance**

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below:

🗀 - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/ lesson.

- I was able to do/perform the task. It was quite challenging, but it still helped me in understanding the target content/lesson.

? – I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

| Learning Task | LP | Learning Task | LP | Learning Task | LP | Learning Task | LP |
|---|---|---|---|---|---|---|---|
| Number 1 | | Number 3 | | Number 5 | | Number 7 | |
| Number 2 | | Number 4 | | Number 6 | | Number 8 | |

| **VII. REFERENCES** | DepEd Learner's Material for Basic Calculus<br>pp. 193-201<br>STEM_BC11-IVa-&b-1 |
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