W3	Learning Area	Basic Calculus	Grade Level	11
	Quarter	4	Date	

I. LESSON TITLE	Antidifferentiation by Substitution and by Table of Integrals
II. MOST ESSENTIAL LEARNING COMPETENCIES (MELCs)	Compute the antiderivative of a function using the substitution rule.
III. CONTENT/CORE CONTENT	The learners demonstrate an understanding of the antiderivatives and Riemann integral

IV. LEARNING PHASES AND LEARNING ACTIVITIES

I. Introduction (Time Frame: 15 mins)

After this lesson, you are expected to:
a) compute the antiderivative of a function using the substitution rule;
b) compute the antiderivative of a function using a table of integrals (including those whose antiderivatives involve logarithmic and inverse trigonometric functions)

Before we begin let us recall about the Theorem 9 (Chain Rule). *Let f be a function differentiable at c and let g be a function differentiable at f(c). Then the composition g o f is differentiable at c and* $D_x(g \circ f)(c) = g'(f(c)) \cdot f'(c)$.

Another way to state the Chain Rule is the following: If *y* is a differentiable function of *u* defined by *y* = *f*(*u*) and *u* is a differentiable function of *x* defined by *u* = *g*(*x*), then *y* is a differentiable function of *x*, and the derivative of *y* with respect to *x* is given by

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

D. Development (Time Frame: 75 mins)
The Integration by Substitution is also known as the "U-Substitution", is a method to find the integral but only if we can put-up in different way. This lesson focuses on the most basic technique - antidifferentiation by substitution which is inverse of the chain rule.

Suppose we are given an integral of the form $\int f(g(x)) * g'(x)dx$. We can transform this into another form by changing the independent variable x to u using the substitution u = g(x). In this case, $\frac{du}{dx}$ = g'(x) dx. Therefore

$$\int f(g(x)) * g'(x)dx = \int f(u)du$$

This change of variable is one of the most important tools available to us. This technique is called ***integration by substitution***. It is often important to guess what will be the useful substitution.

EXAMPLE 1:

Evaluate $\int (x + 4)^5 dx$

Solution:

We let *u* = x + 4.

Now, since *u* = x + 4 it follows that $\frac{du}{dx}$ = 1 and so *du* = *dx*. So, substituting (x + 4) and *dx*, we have

$$\int (x+4)^5 dx = \int (u)^5 du$$

The resulting integral can be evaluated immediately to give $\frac{u^6}{6}$ + C. Recalling that *u* = x+4, we have

$\int (x+4)^5 dx = \int (u)^5 du$

$= \frac{u^6}{6} + C$

$= \frac{(x+4)^6}{6} + C$

Example 2: $\int (x^5+2)^9\, 5x^4 dx$

Solution:
we let *u* = x^5 + 2, then *du* = $5x^4$ *dx*

$\int (x^5+2)^9\, 5x^4 dx = \int (u)^9 du$, where u = $x^5 + 2$

$= \frac{u^{10}}{10} + C$

$= \frac{(x^5+2)^{10}}{10} + C$

Example 3: $\int \frac{z^2}{\sqrt{1+z^3}} dz$

In this example, we let *u* = 1+ z^3 so that $\frac{du}{dz} = 3z^2$. If *u* = 1+ z^3, then we need to express z^2 *dz* in the integrand in terms of *du* or a constant multiple of *du*. From $\frac{du}{dz} = 3z^2$ it follows that *du* = $3z^2$ *dz* and z^2 *dz* = $\frac{1}{3}$ *du. Thus,*

$\int \frac{z^2}{\sqrt{1+z^3}} dz = \int \frac{z^2}{\sqrt{1+z^3}} dz \,.\, z^2 dz$

$= \int \frac{1}{\sqrt{u}} \cdot \frac{1}{3} du$

$= \frac{1}{3}\int u^{-\frac{1}{2}} du$

$= \frac{1}{3}(\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C_1)$

$= \frac{2}{3}u^{\frac{1}{2}} + \frac{C_1}{3}$

$= \frac{2}{3}(1 + z^3)^{1/2} + C$, where $C = \frac{C_1}{3}$

Example 4: Evaluate $\int e^{3x} dx$

Solution:

We let u = 3x. Then du = 3dx. Hence, dx = $\frac{du}{3}$, So

$\int e^{3x}$ dx = $\int e^u \frac{du}{3}$

= $\frac{1}{3}\int e^u\, du$

= $\frac{1}{3}e^u$ + C

= $\frac{1}{3}e^{3x}$ + C

INTEGRALS OF INVERSE CIRCULAR FUNCTIONS

We now present the formulas for integrals yielding the inverse circular functions.

1. $\int \frac{du}{\sqrt{1-u^2}}$ = $\sin^{-1}$ u + C

2. $\int \frac{du}{1+u^2}$ = $\tan^{-1}$ u + C

3. $\int \frac{du}{u\sqrt{u^2-1}}$ = $\sec^{-1}$ u + C

If the constant 1 in these integrals is replaced by some other positive number, one can use the following generalizations:

Let a > 0. Then

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$
$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$
$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

E. Engagement (Time Frame: 80 mins)
LEARNING TASK 1: Determine the antiderivatives of the following functions:

1. f(x) = $(x+1)^{100}$

2. f(x) = $(x^2 + x)^{10}$ (2x + 1)

3. g(x) = $\frac{x^3}{\sqrt{1-x^4}}$

4. h(x) = $\frac{6x^2+2}{\sqrt{x^3+x+1}}$

LEARNING TASK 2: Evaluate the following integrals:

1. $\int \frac{x^2 - x}{1 + 3x^2 - 2x^3}$ dx

2. $\int \frac{\cos x}{1 + 2\sin x}$ dx

3. $\int \frac{x^2 + 2}{x + 1}$ dx

4. $\int \frac{3x^5 - 2x^3 + 5x^2 - 2}{x^3 + 1}$

5. $\int \frac{1}{x + \sqrt{x}}$ dx

A. Assimilation (Time Frame: 40 mins)
Answer more practice exercises on pages 216 item numbers 8-13 of the Basic calculus DepEd Learner's Material for Grade 11.

Summarize what you have learned about Antidifferentiation by Substitution and by Table of Integrals by answering the following questions:

1. What is the other name of Integration by substitution?
2. What are the important tools or techniques that we used in integration by substitution?
3. What are the different formulas of the inverse of the circular functions?

V. ASSESSMENT (Time Frame: 30 mins)

To determine how much you have learned about the Antidifferentiation by Substitution and by table of values of Integral answer page 216 item numbers 21 - 25 in Basic Calculus DepEd Learner's Material for Grade 11.

VI. REFLECTION (Time Frame: 5 mins)

- Communicate your personal assessment as indicated in the Learner's Assessment Card.

Personal Assessment on Learner's Level of Performance

Using the symbols below, choose one which best describes your experience in working on each given task. Draw it in the column for Level of Performance (LP). Be guided by the descriptions below:

🗀 - I was able to do/perform the task without any difficulty. The task helped me in understanding the target content/ lesson.
- I was able to do/perform the task. It was quite challenging, but it still helped me in understanding the target content/lesson.
? – I was not able to do/perform the task. It was extremely difficult. I need additional enrichment activities to be able to do/perform this task.

Learning Task	LP	Learning Task	LP	Learning Task	LP	Learning Task	LP
Number 1		Number 3		Number 5		Number 7	
Number 2		Number 4		Number 6		Number 8	

VII. REFERENCES	*Basic Calculus Learner's Material,* Pasig City; Department of Education, 2016

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