

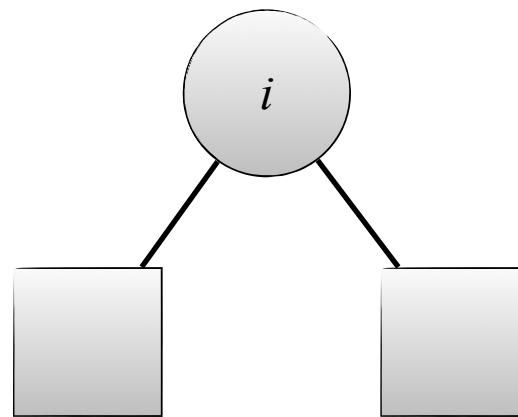
Red-Black Trees

- Balanced binary search tree
- A different representation of 2-3 tree

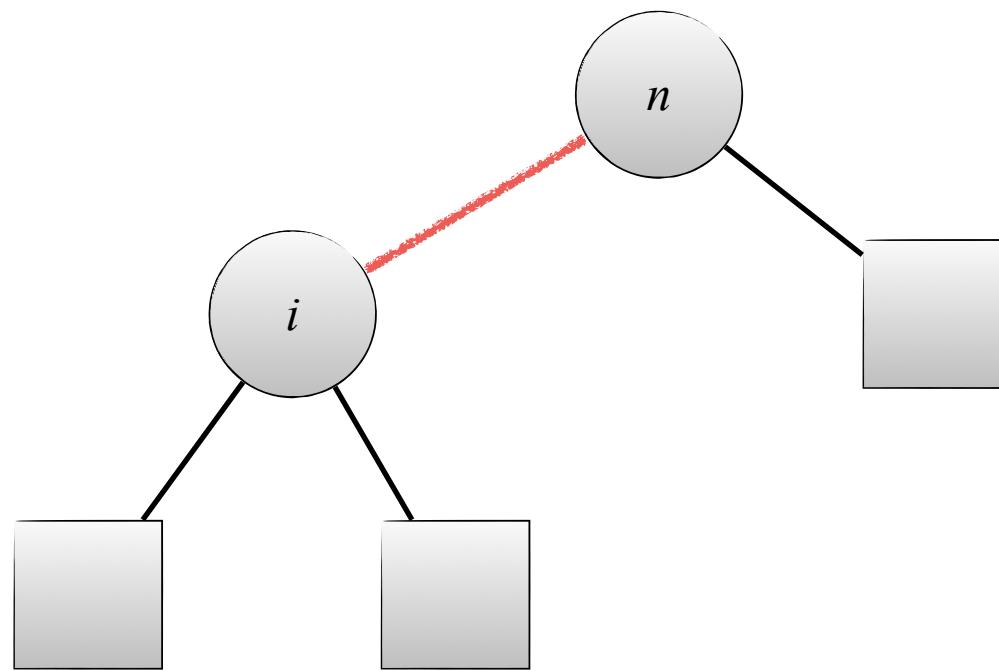
Definition: Red-Black Tree

- A red-black tree is a binary search tree where each link/edge is either red or black. Further
 - All red links lean left
 - No node has two red links connected to it
 - The tree has a perfect balance: every path from the root to a leaf has the same number of black links
 - Links to leaves are black

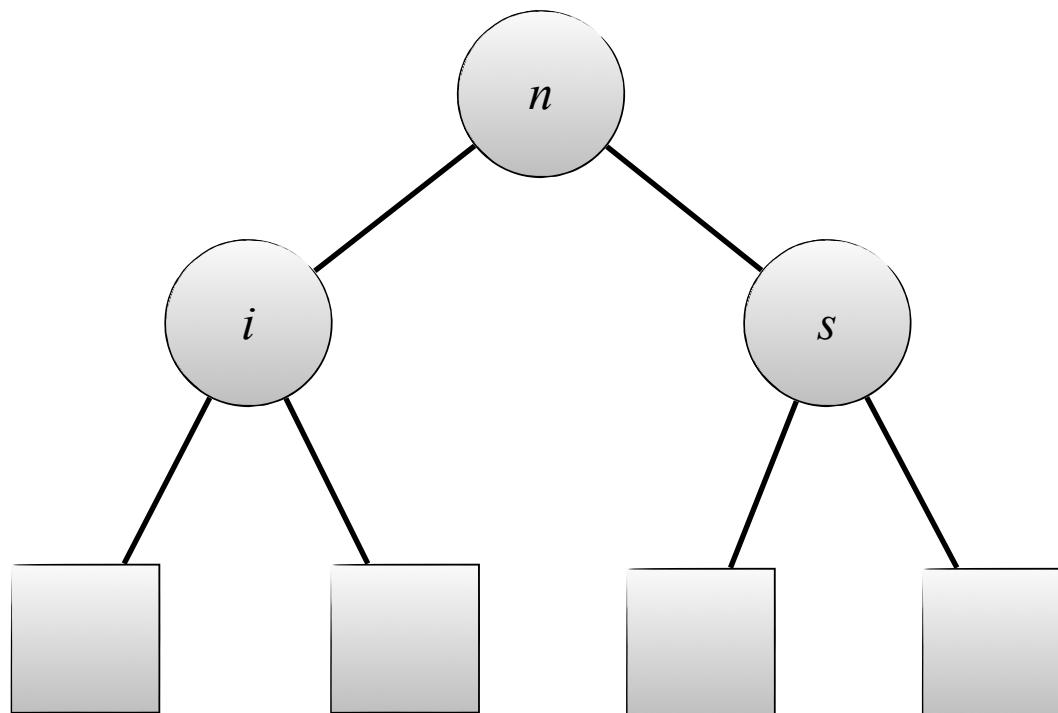
Example: a red-black tree



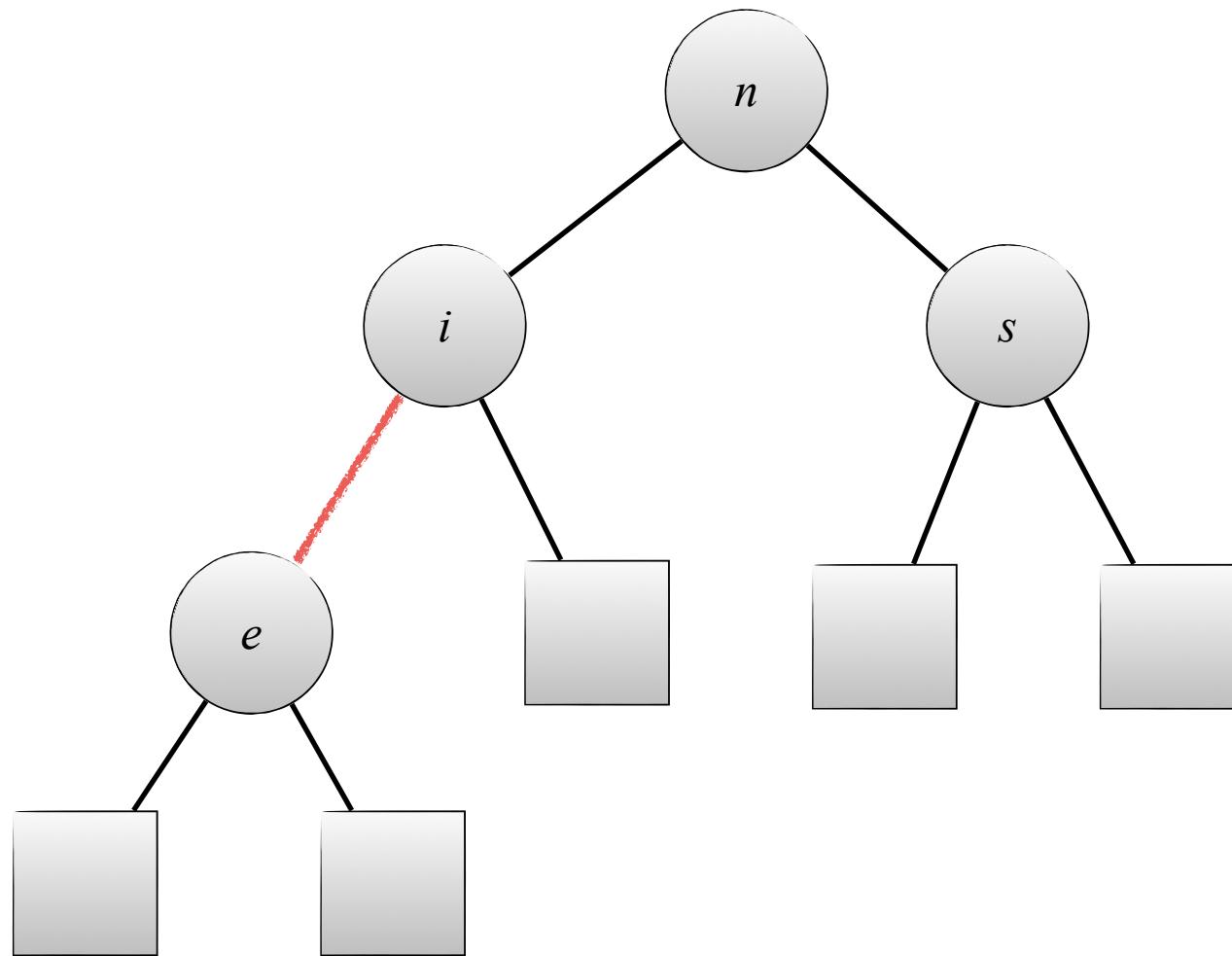
Example: a red-black tree



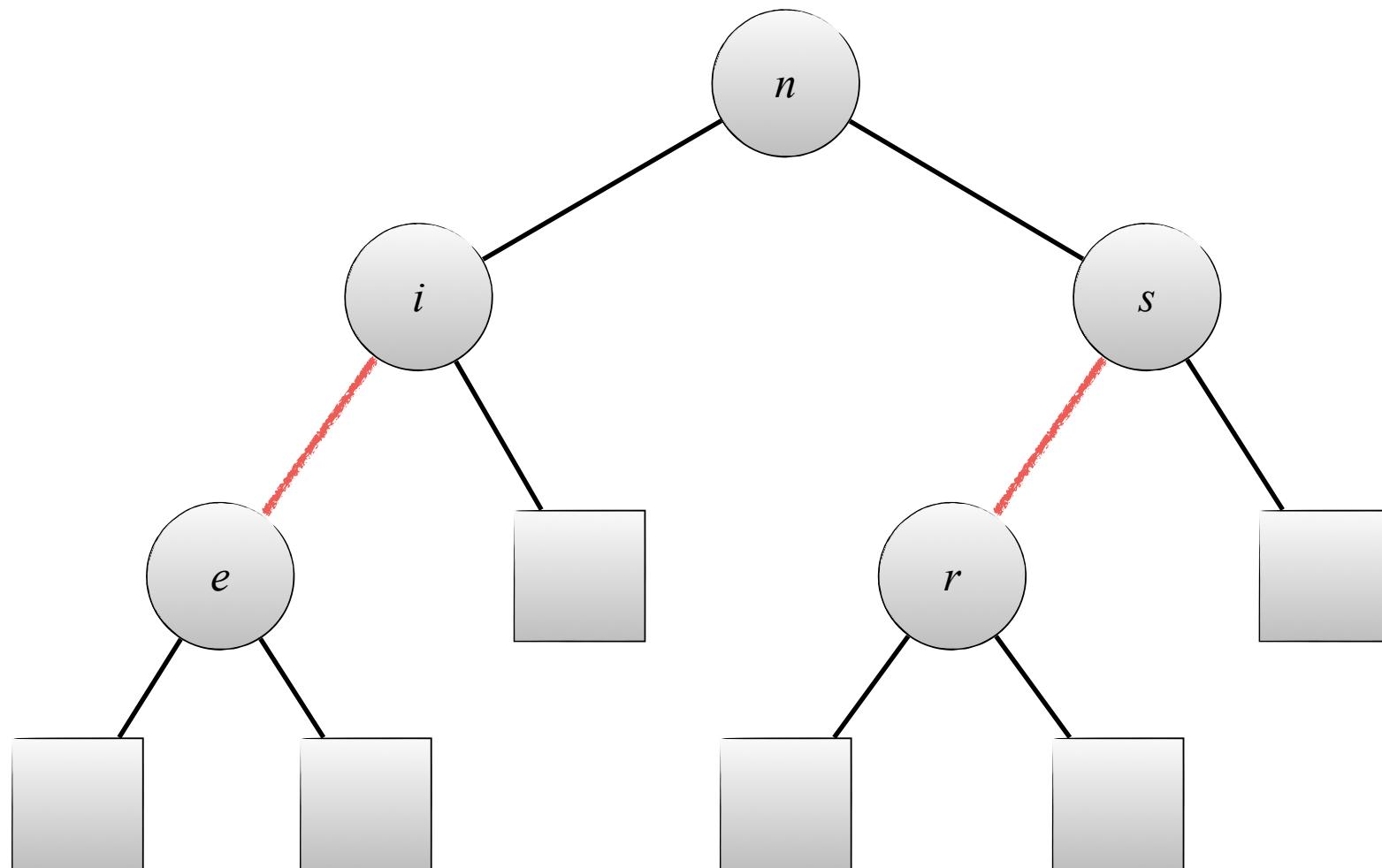
Example: a red-black tree



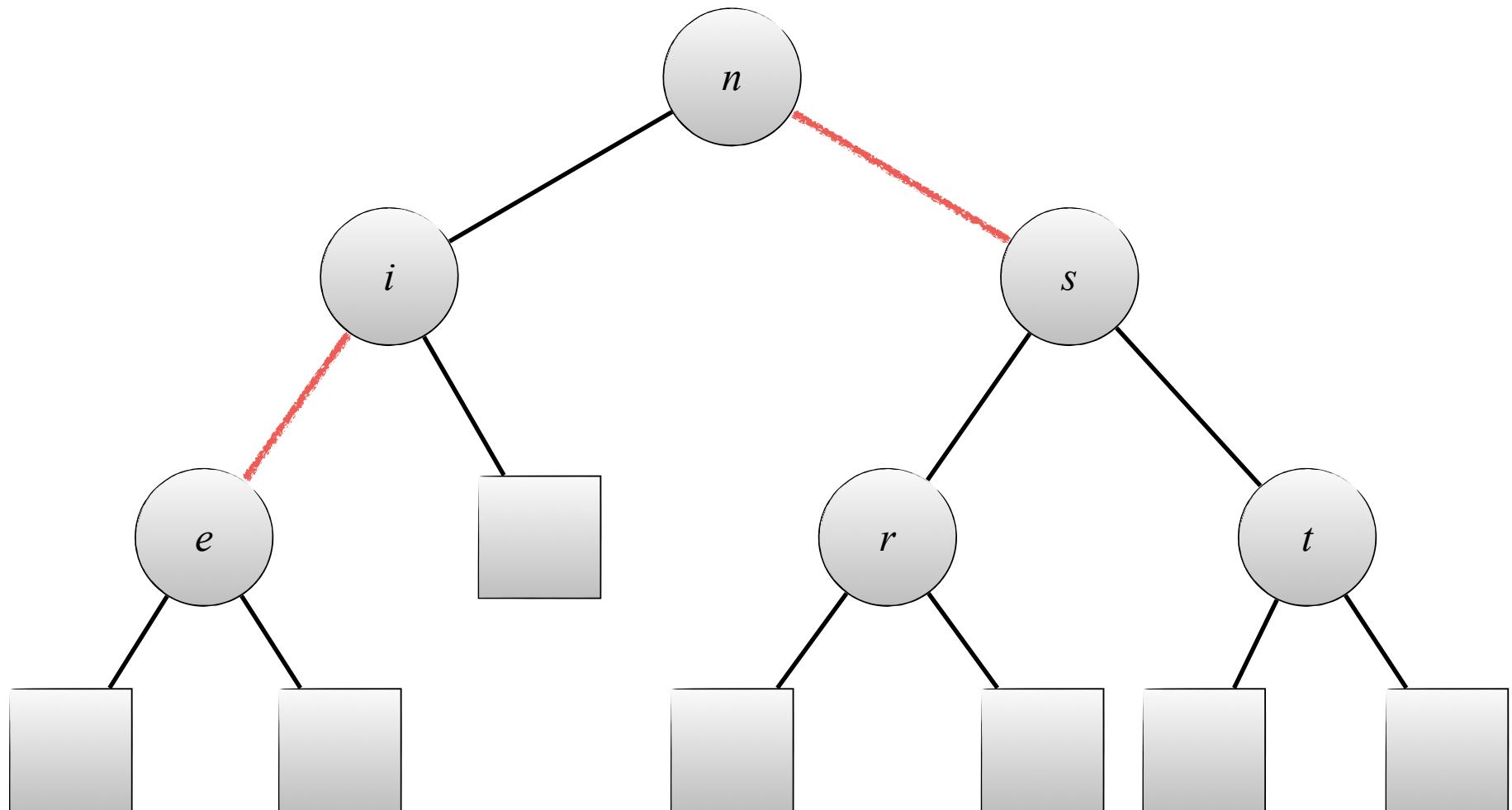
Example: a red-black tree



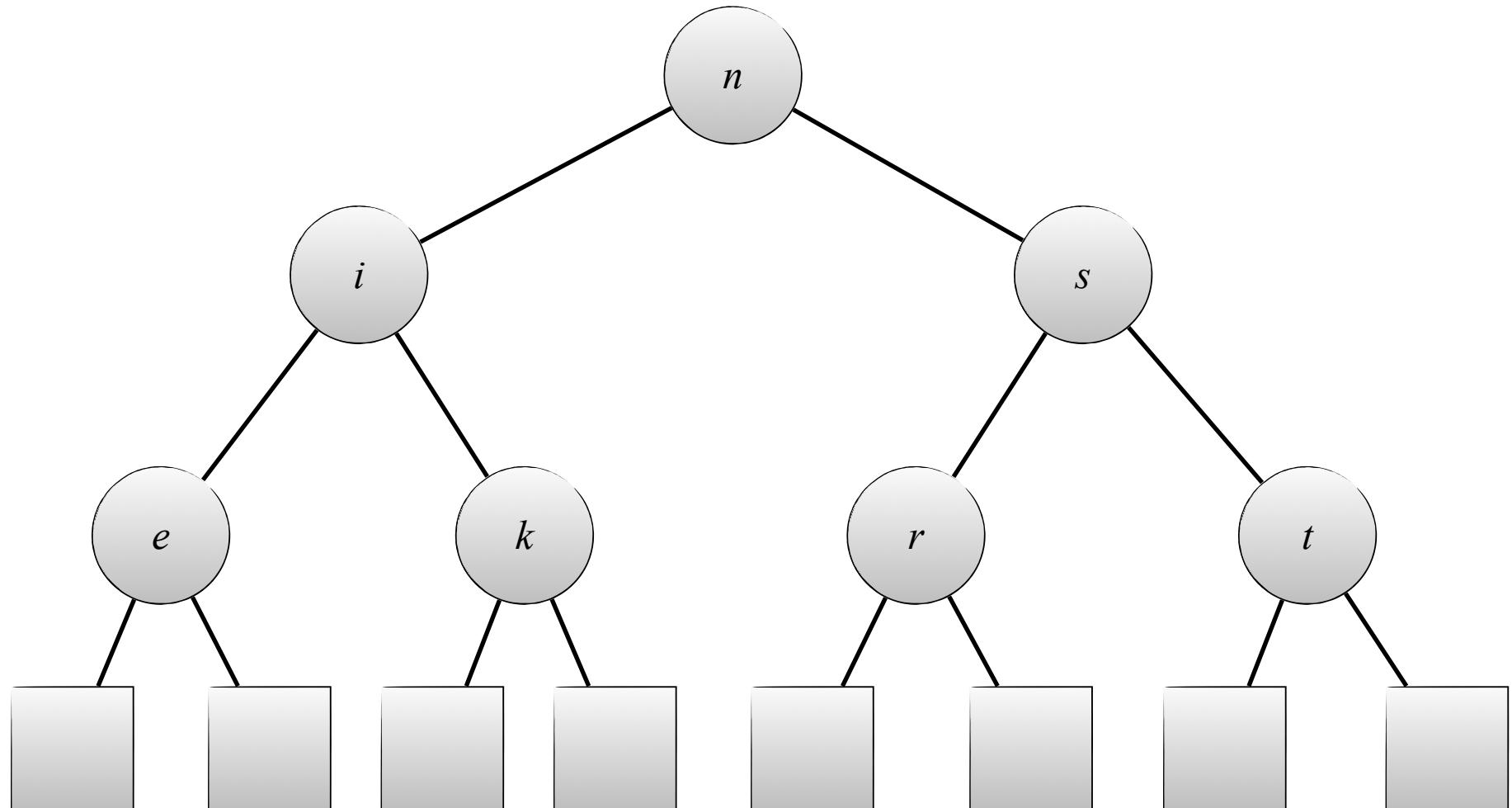
Example: a red-black tree



Example: **not** a red-black tree



Example: a red-black tree



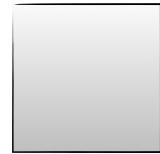
Red-black trees and 2-3 trees

- There is a 1-1 correspondence between red-black BSTs and 2-3 trees.
- To see this, imagine that red links are collapsed: the collapsed nodes correspond to 3-nodes, all others to 2-nodes.

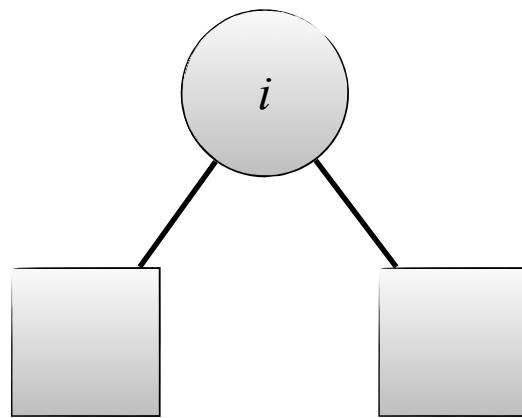
Inserting into a red-black tree

- Insert just as in BST
- but: link/edge to new node is **red**
- rotations and color flipping (depending on case)

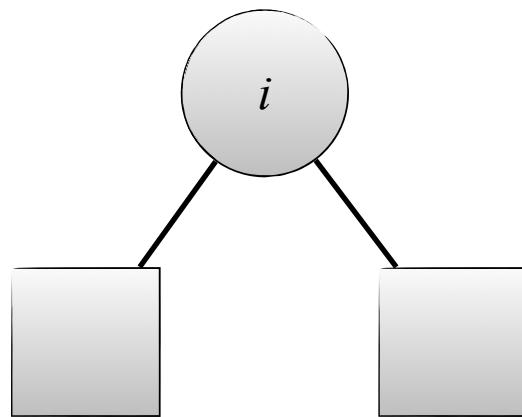
Insert key i into empty tree



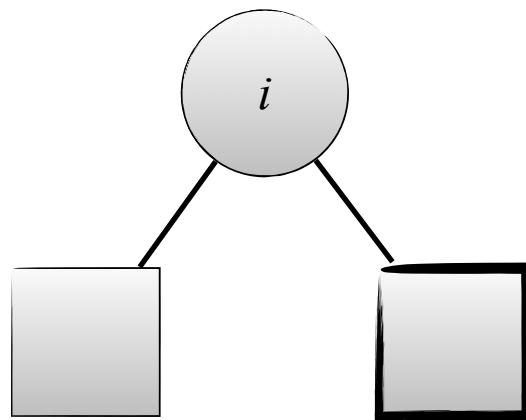
Insert key i into empty tree



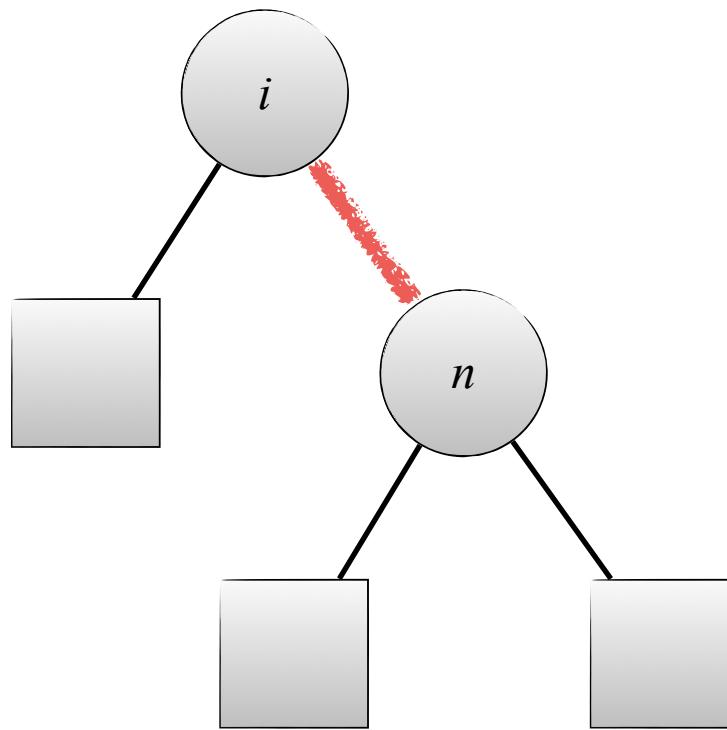
Insert key n



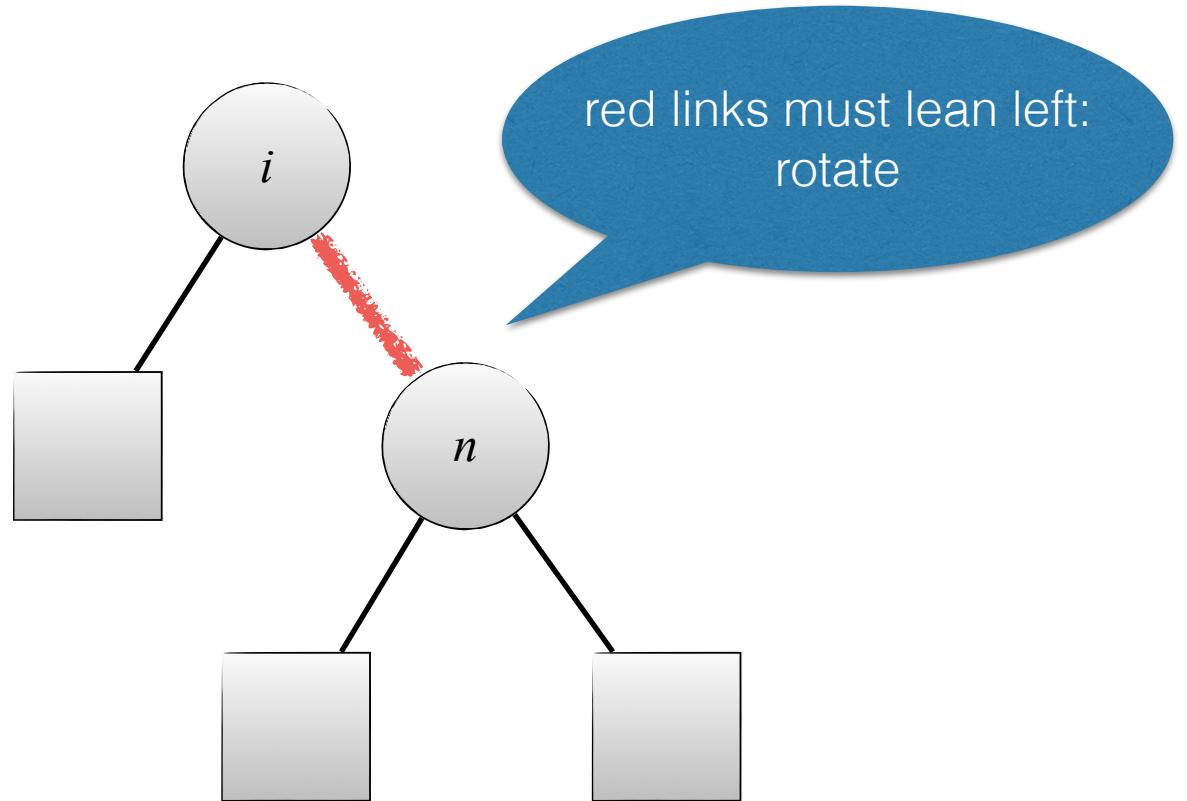
Insert key n



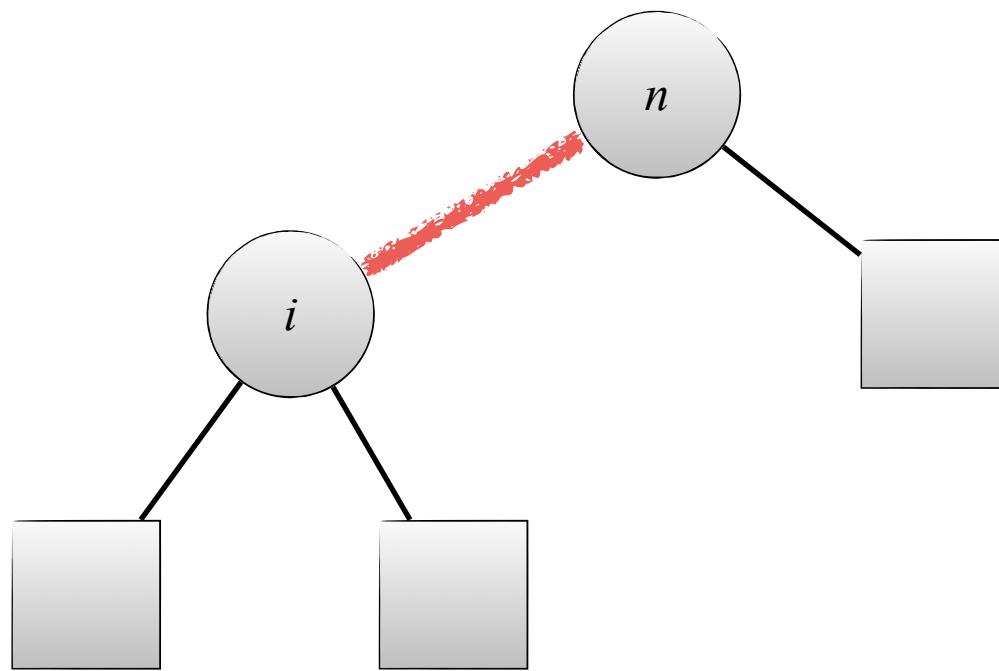
Link to new node is red



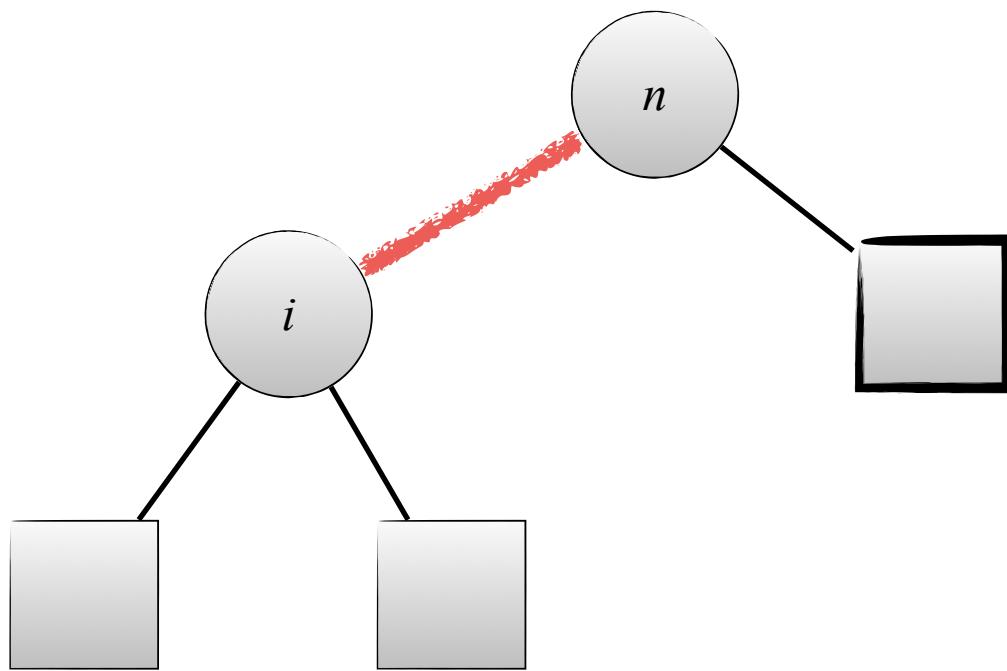
Link to new node is red



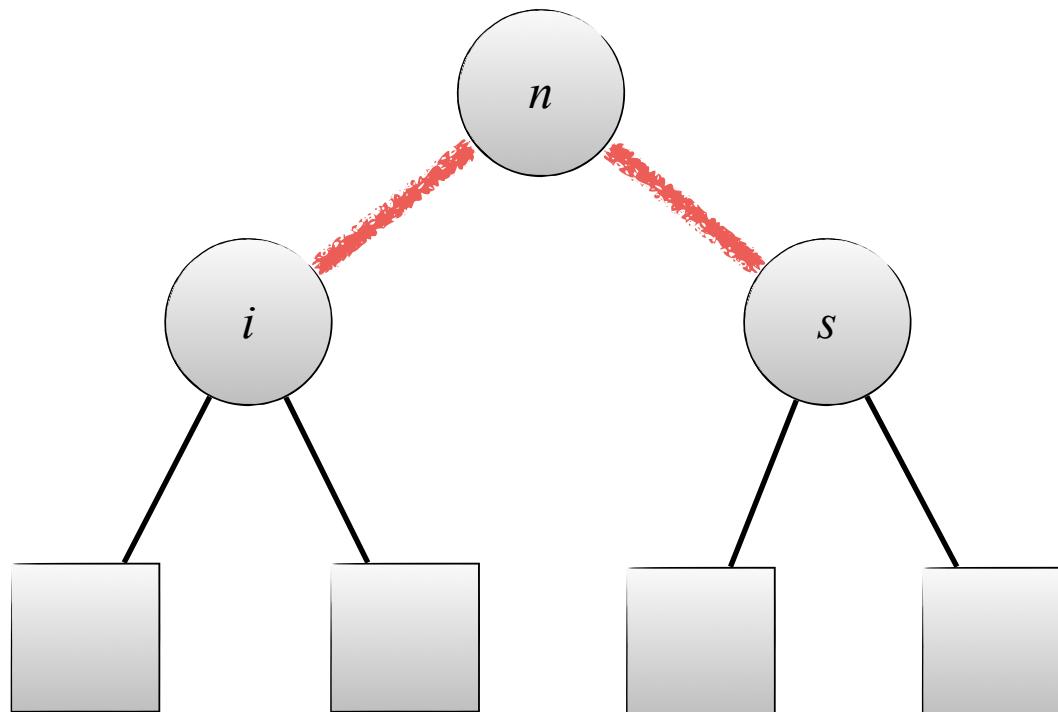
After rotation



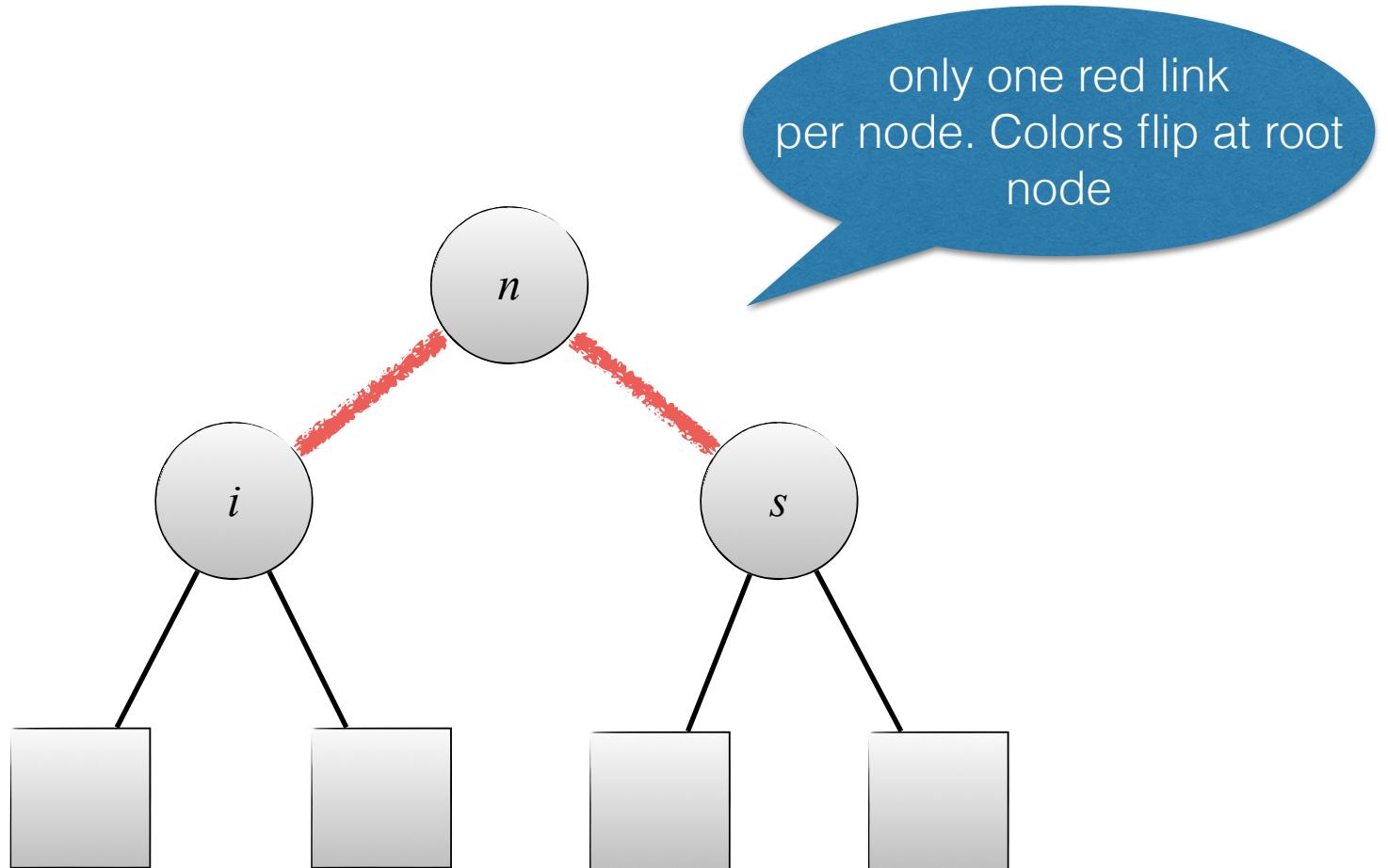
Insert key s



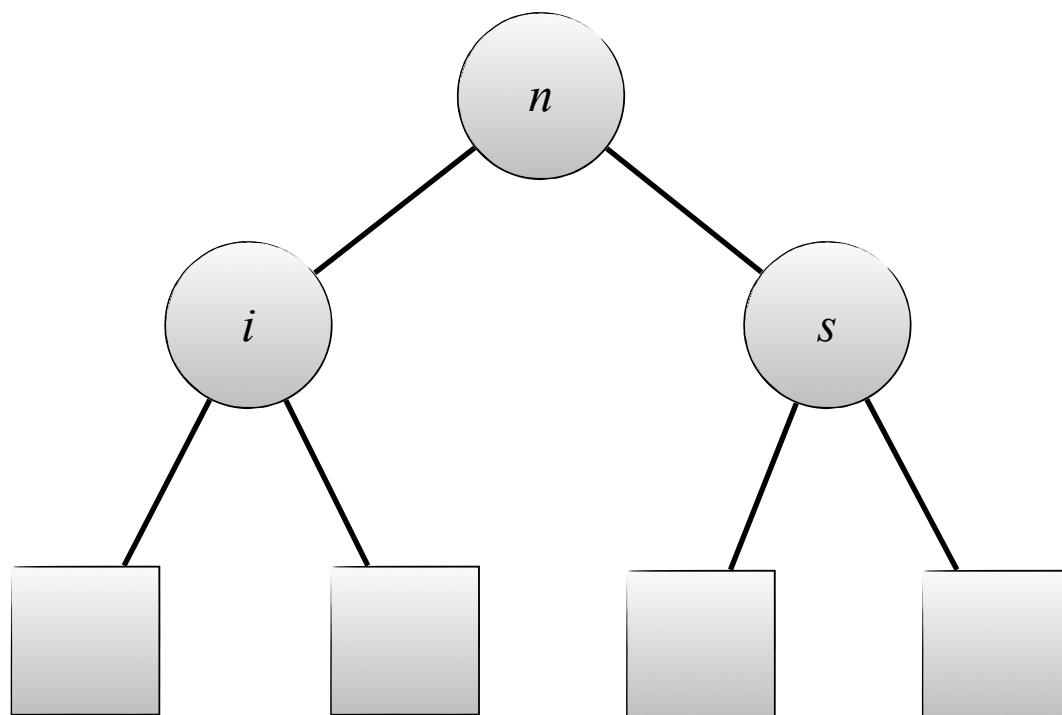
Insert key s , new link is red



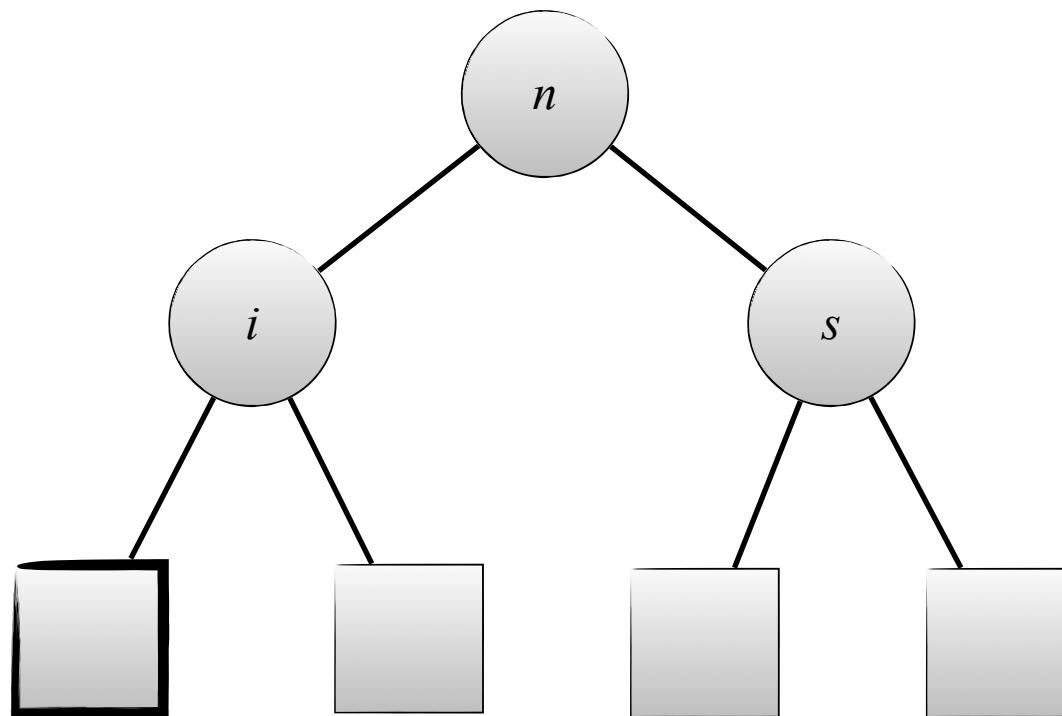
Insert key s , new link is red



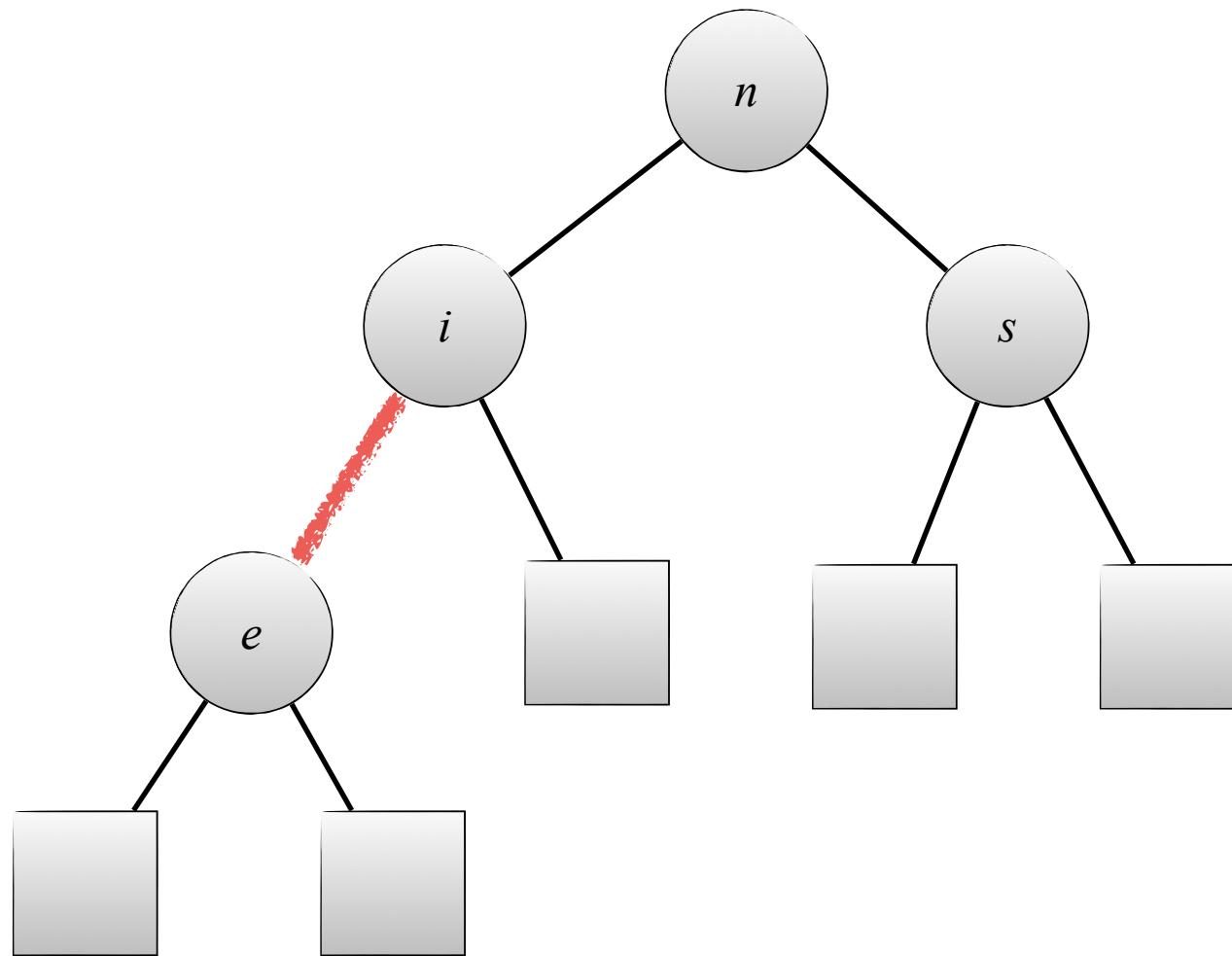
Colors flipped to black



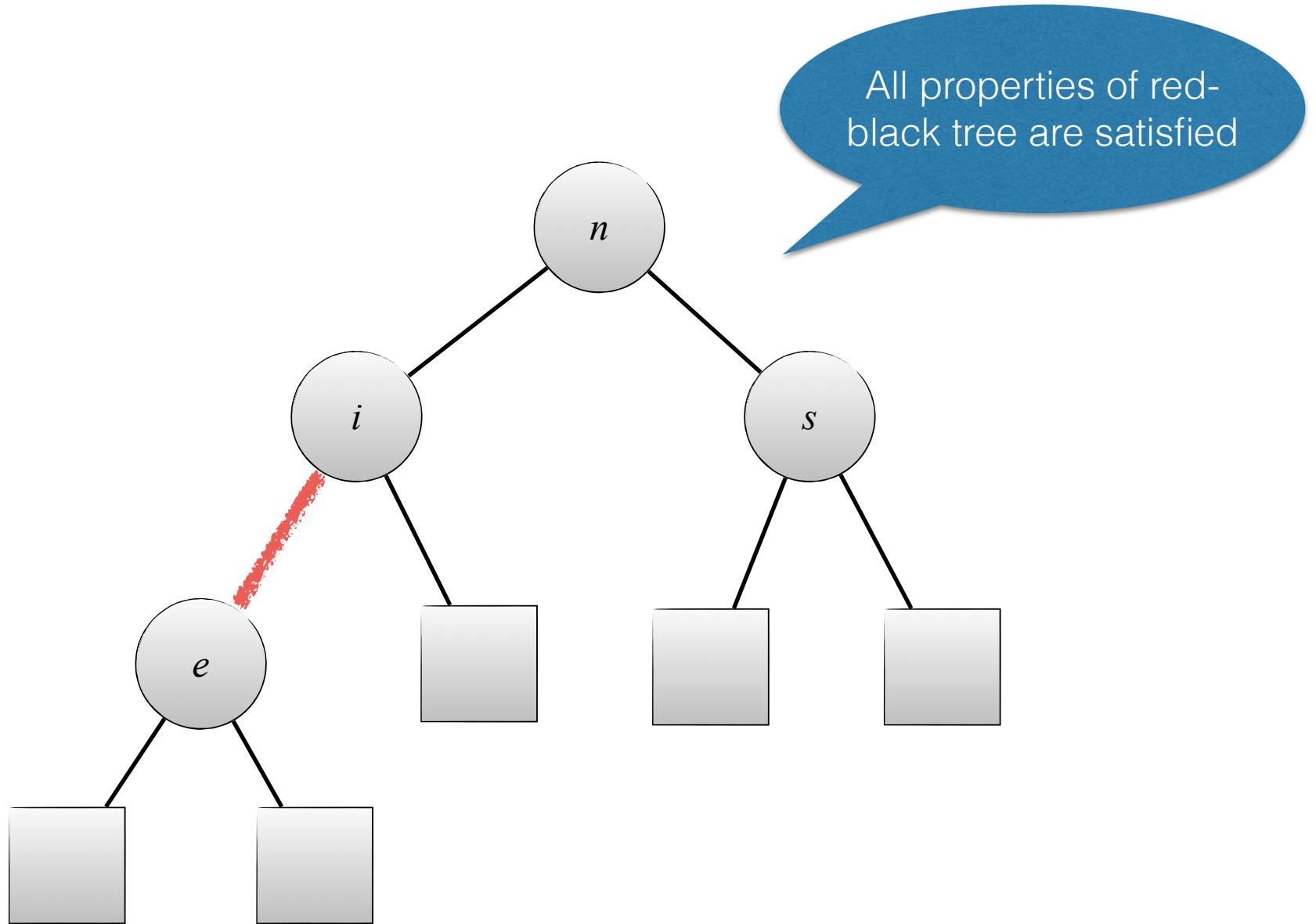
Insert key e



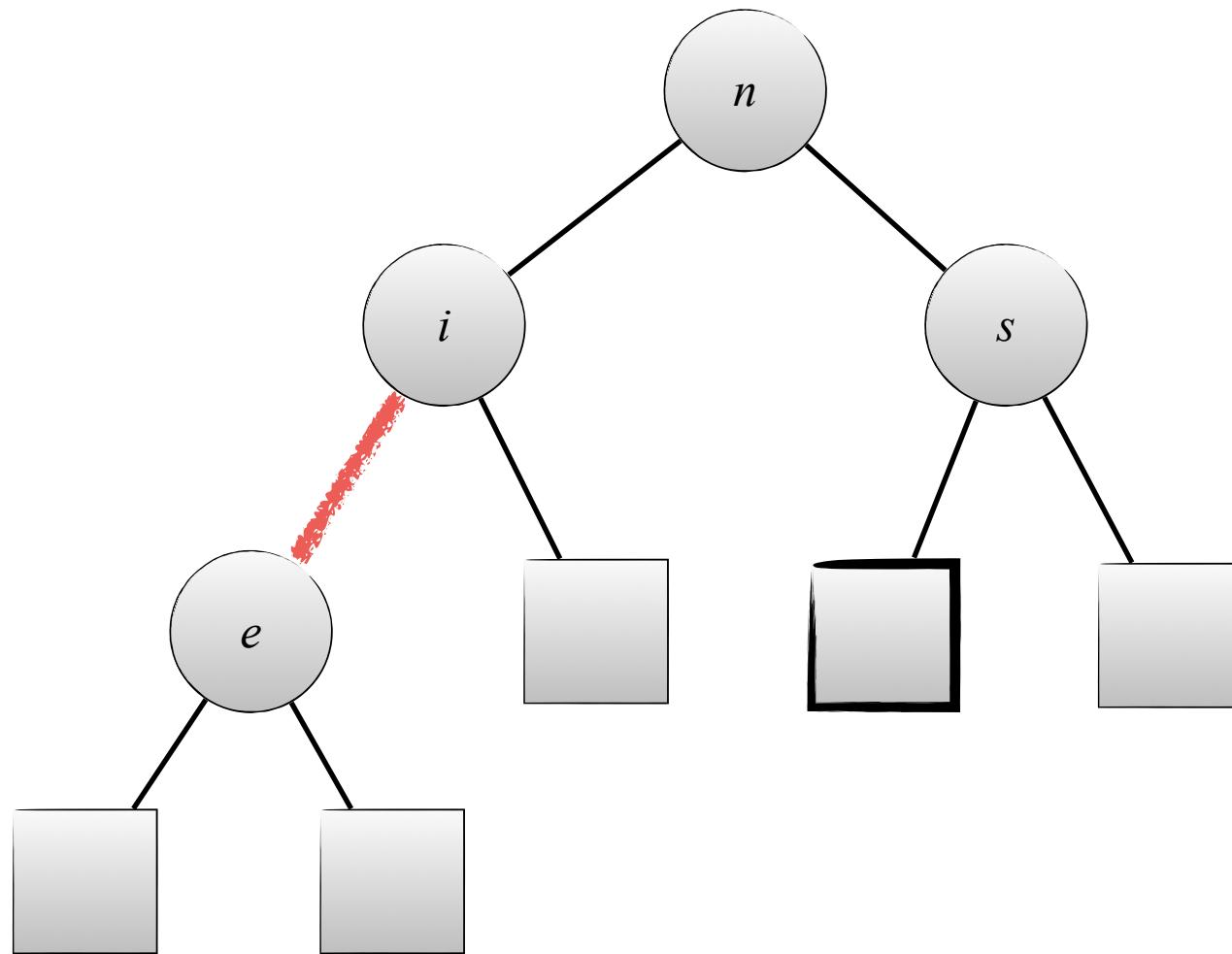
Insert key e , new link is red



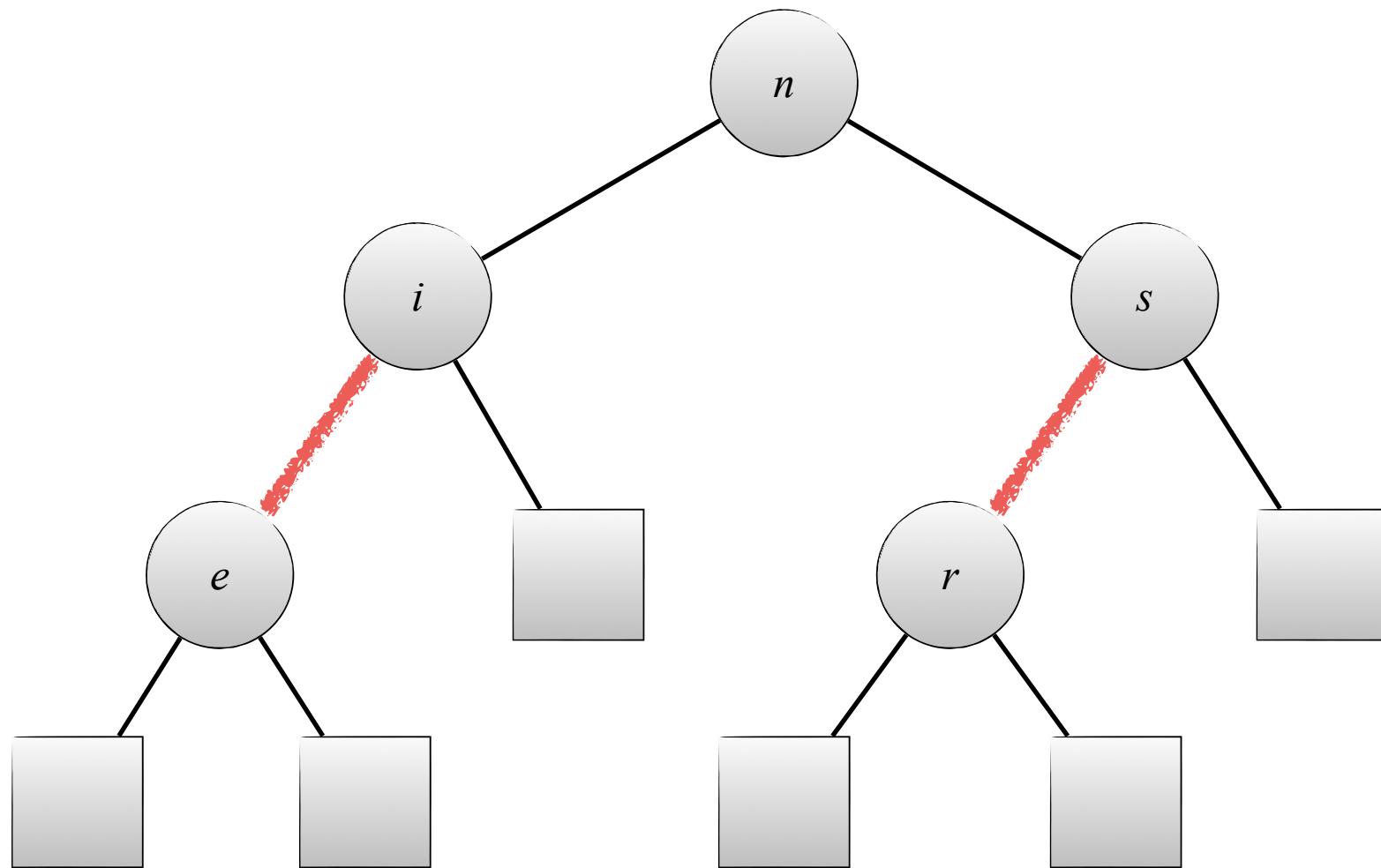
Insert key e , new link is red



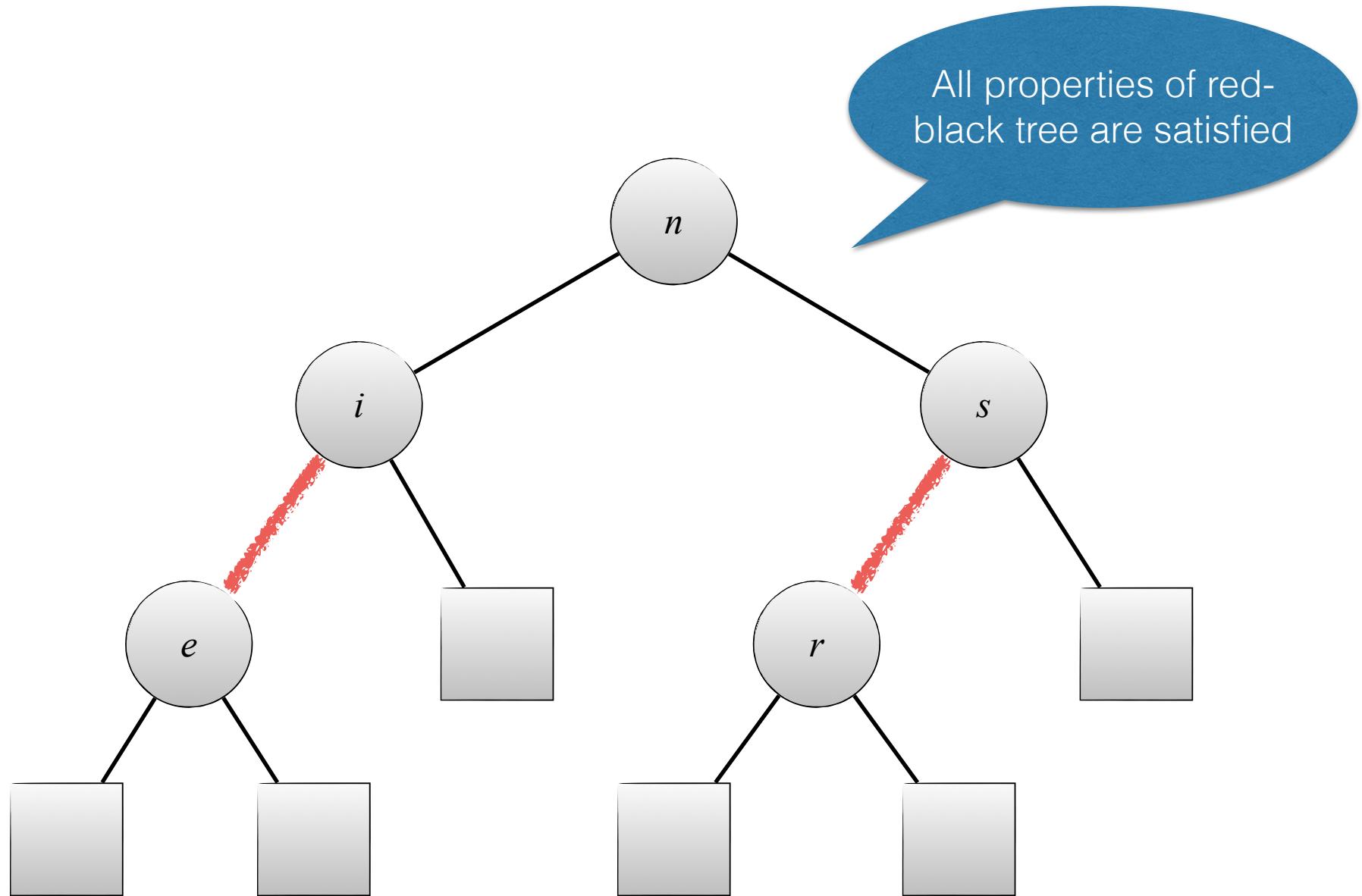
Insert key r



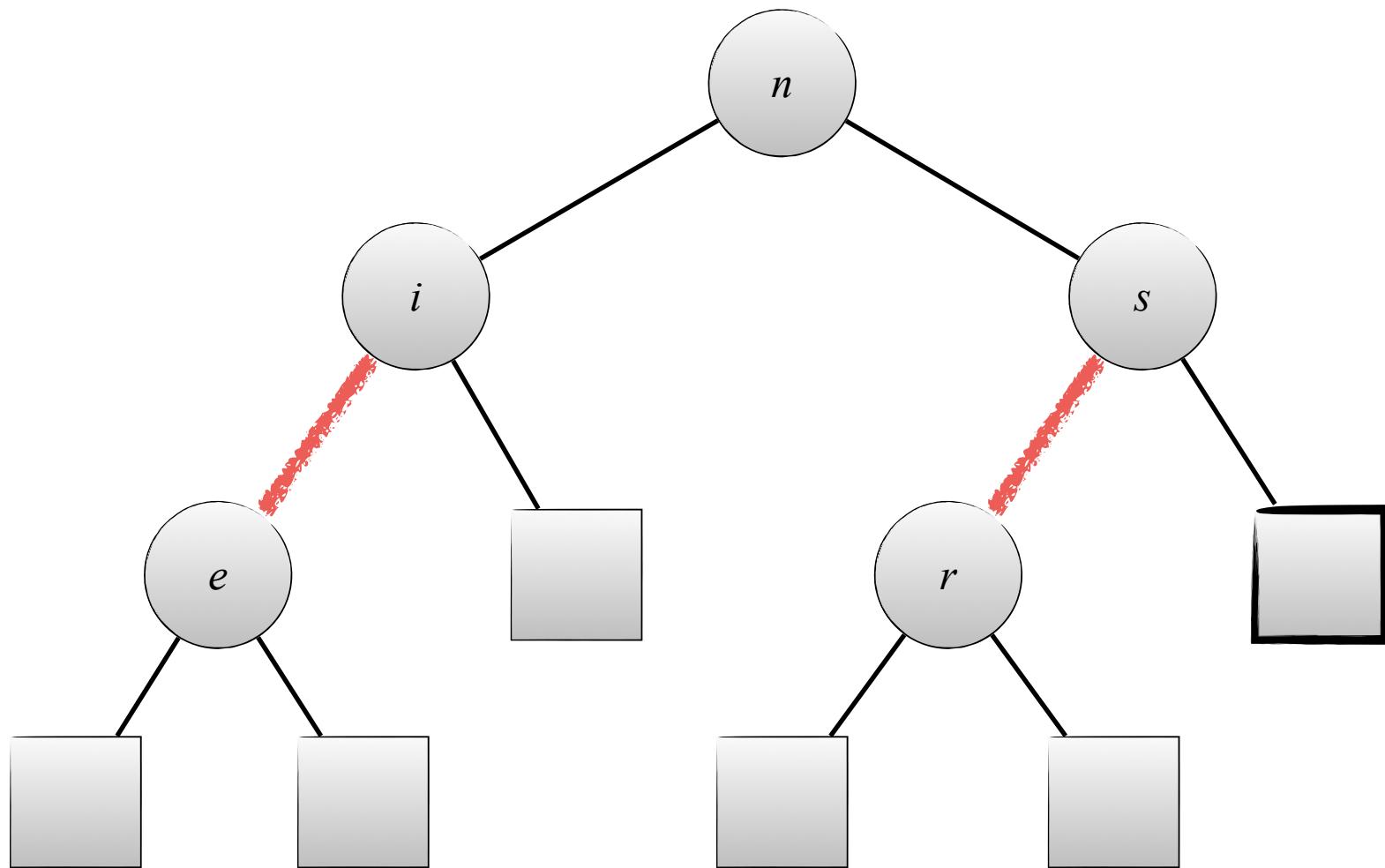
Insert key r , new link is red



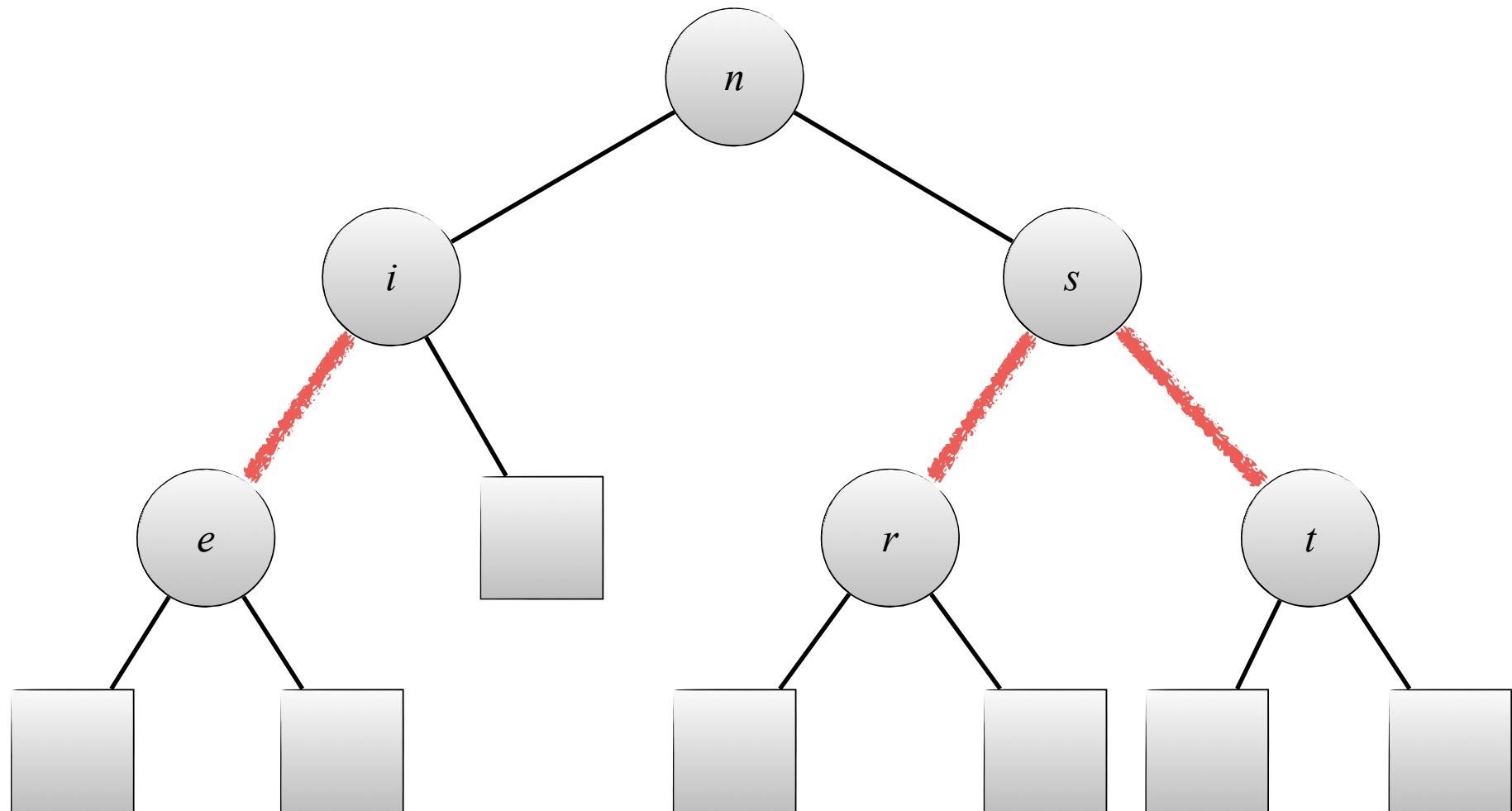
Insert key r , new link is red



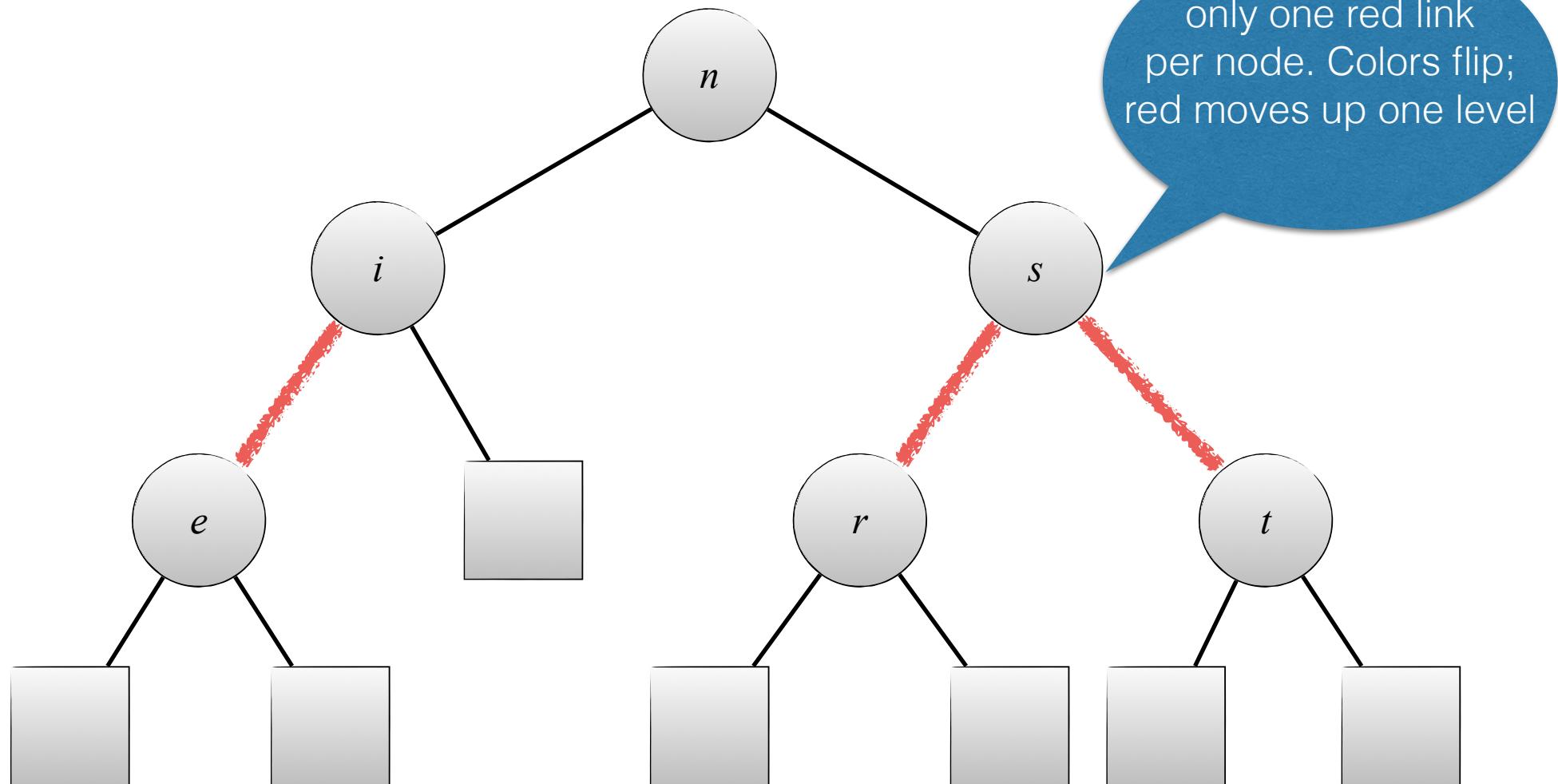
Insert key t



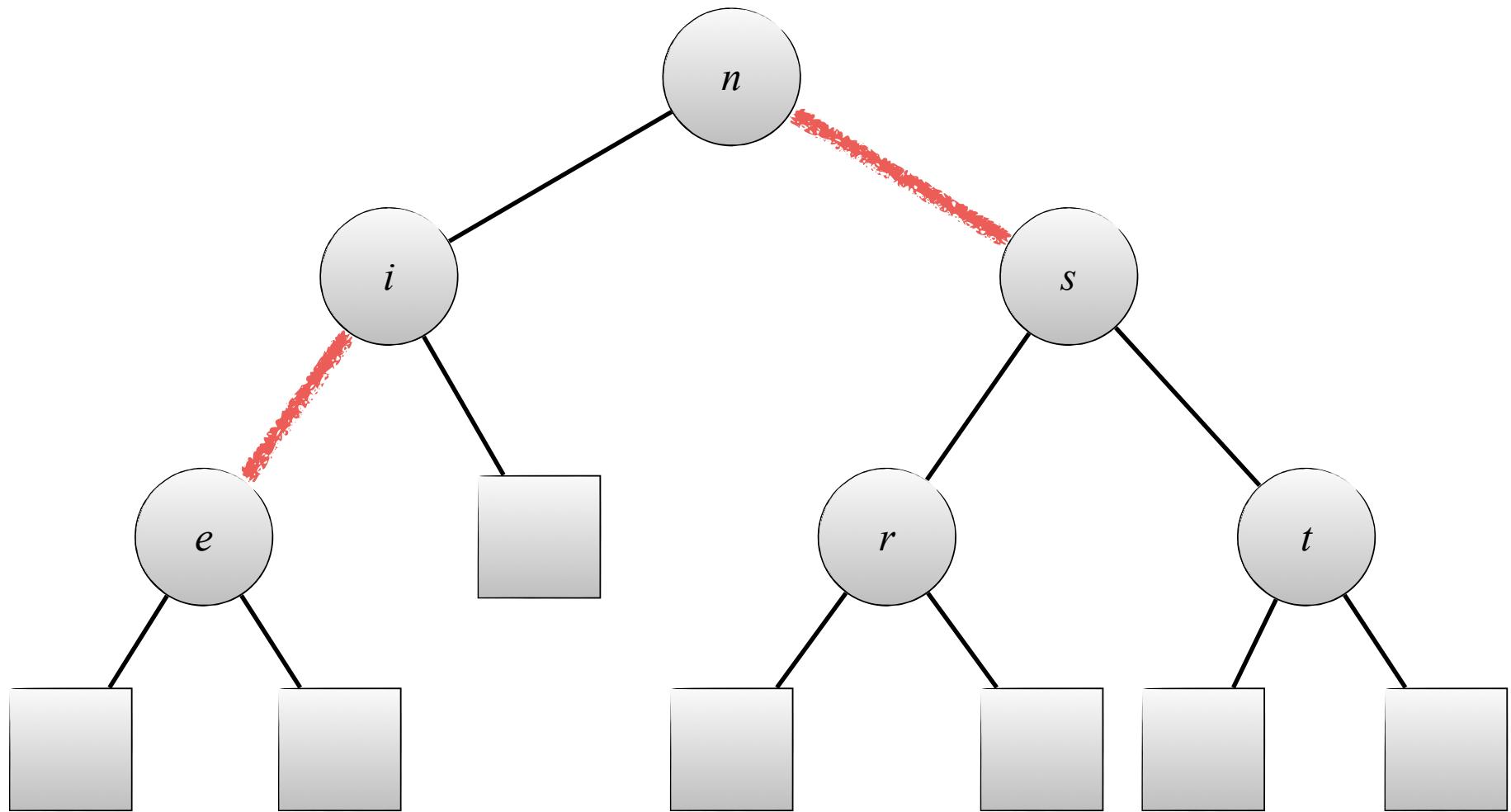
Insert key t , new link is red



Insert key t , new link is red

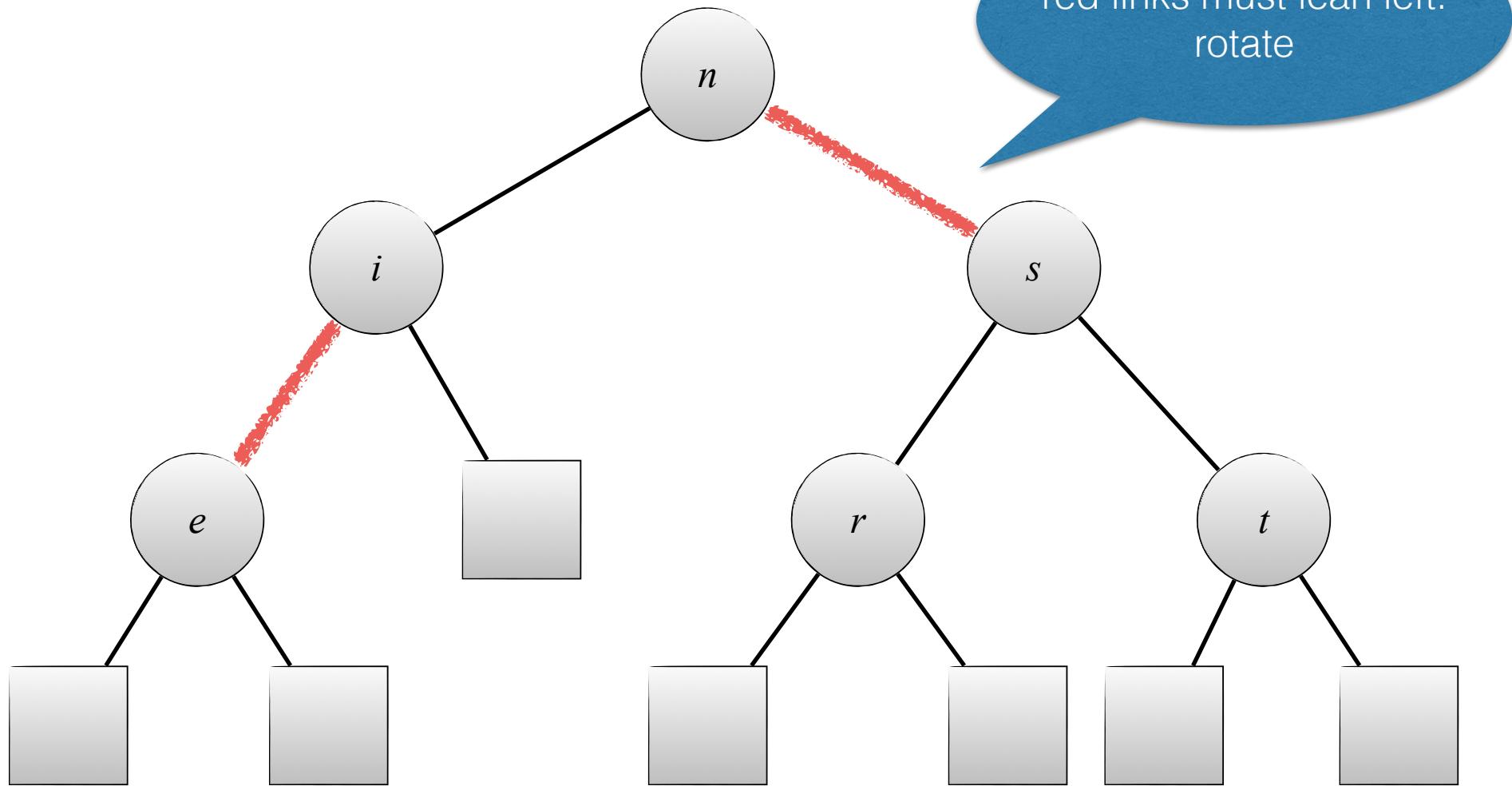


Insert key t

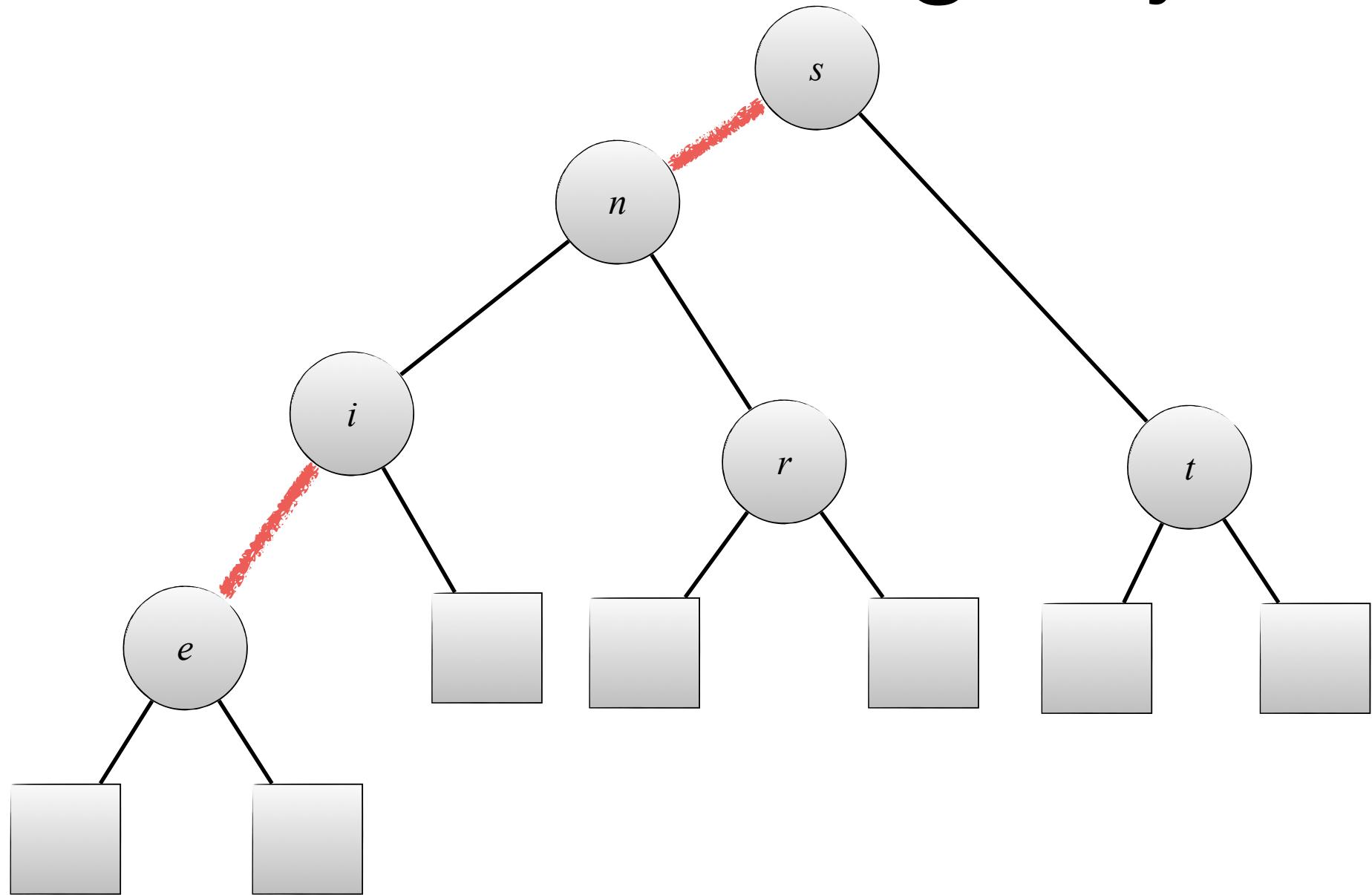


Insert key t

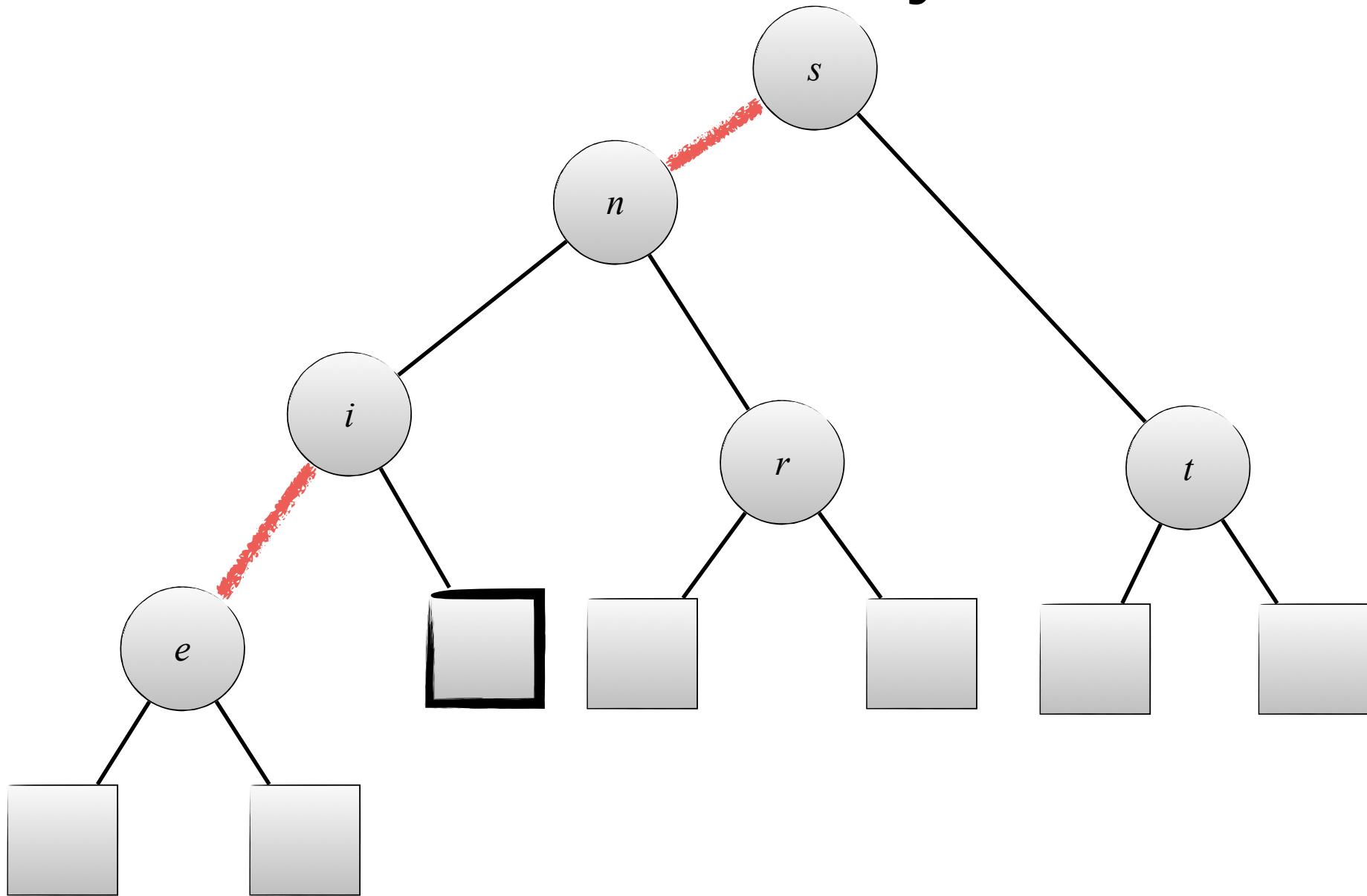
red links must lean left:
rotate



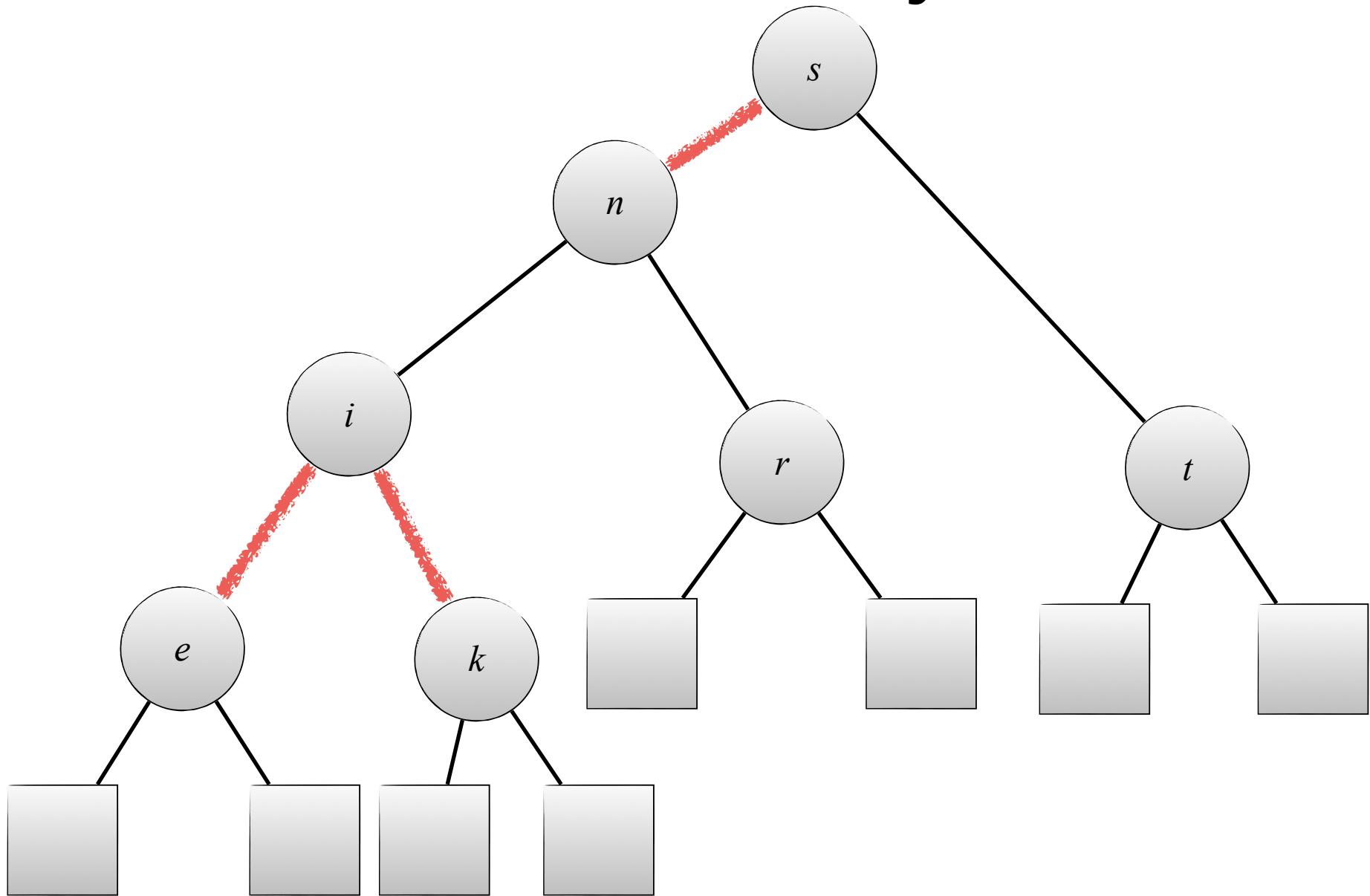
After inserting key t



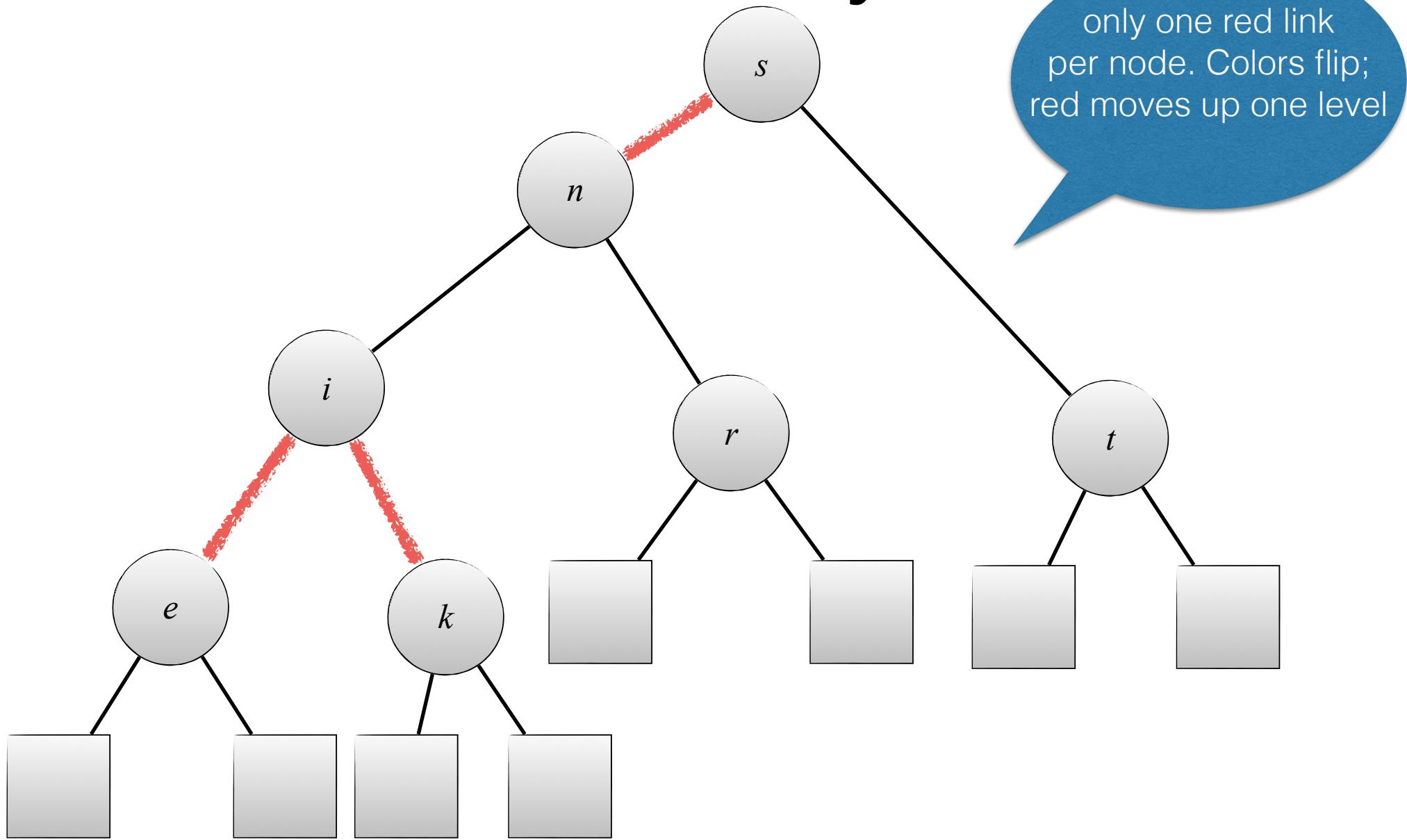
Insert key k



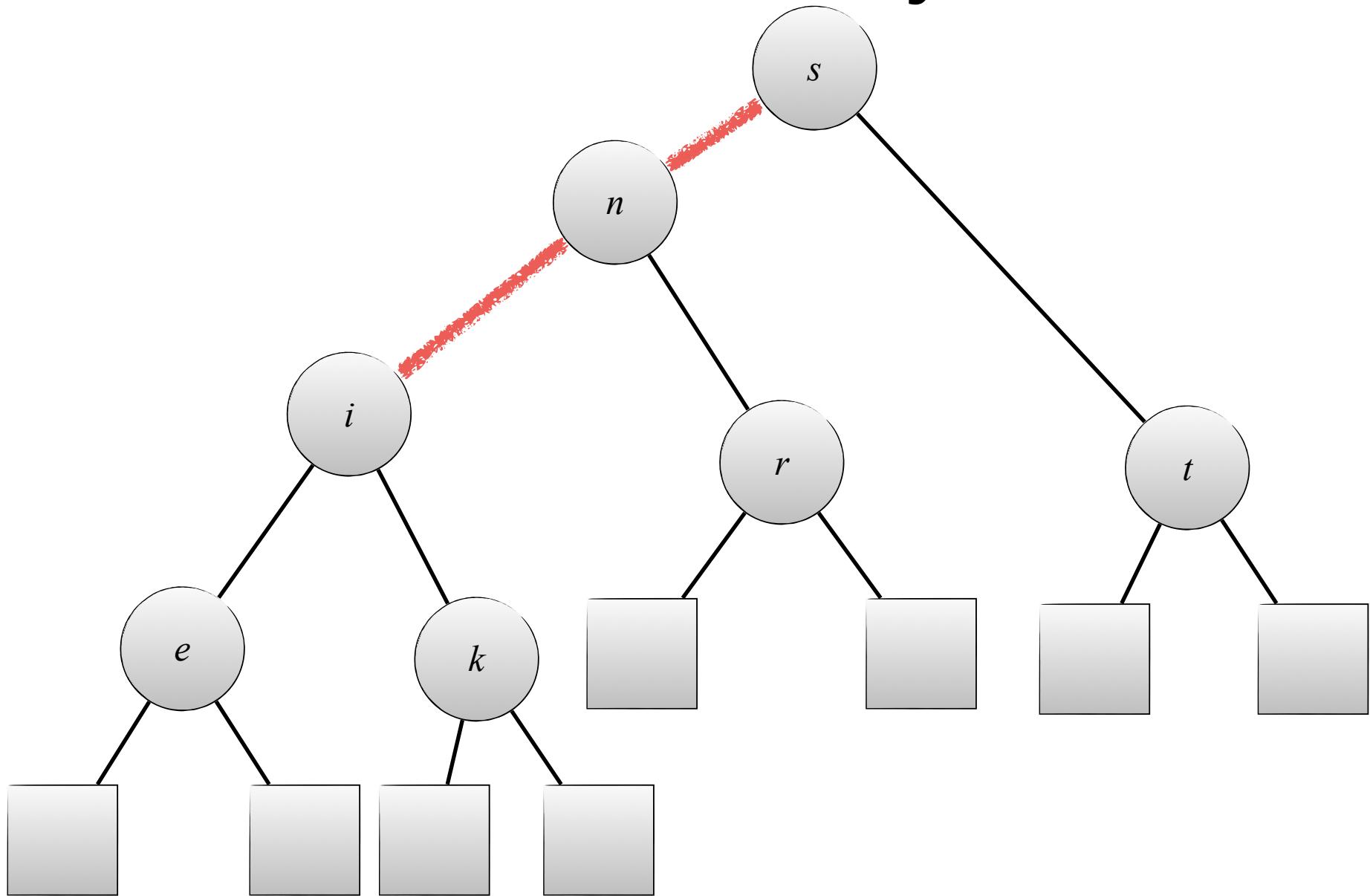
Insert key k



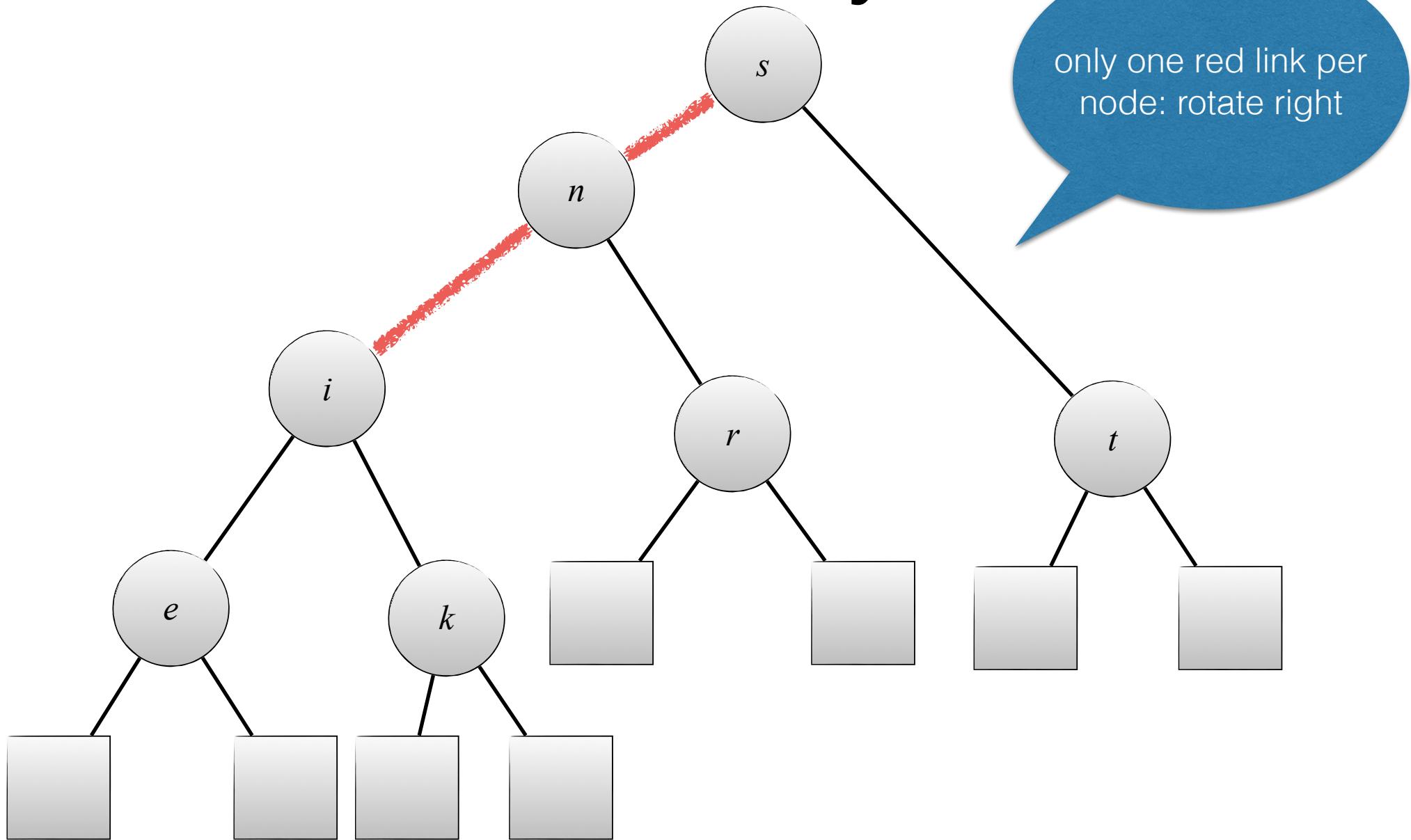
Insert key k



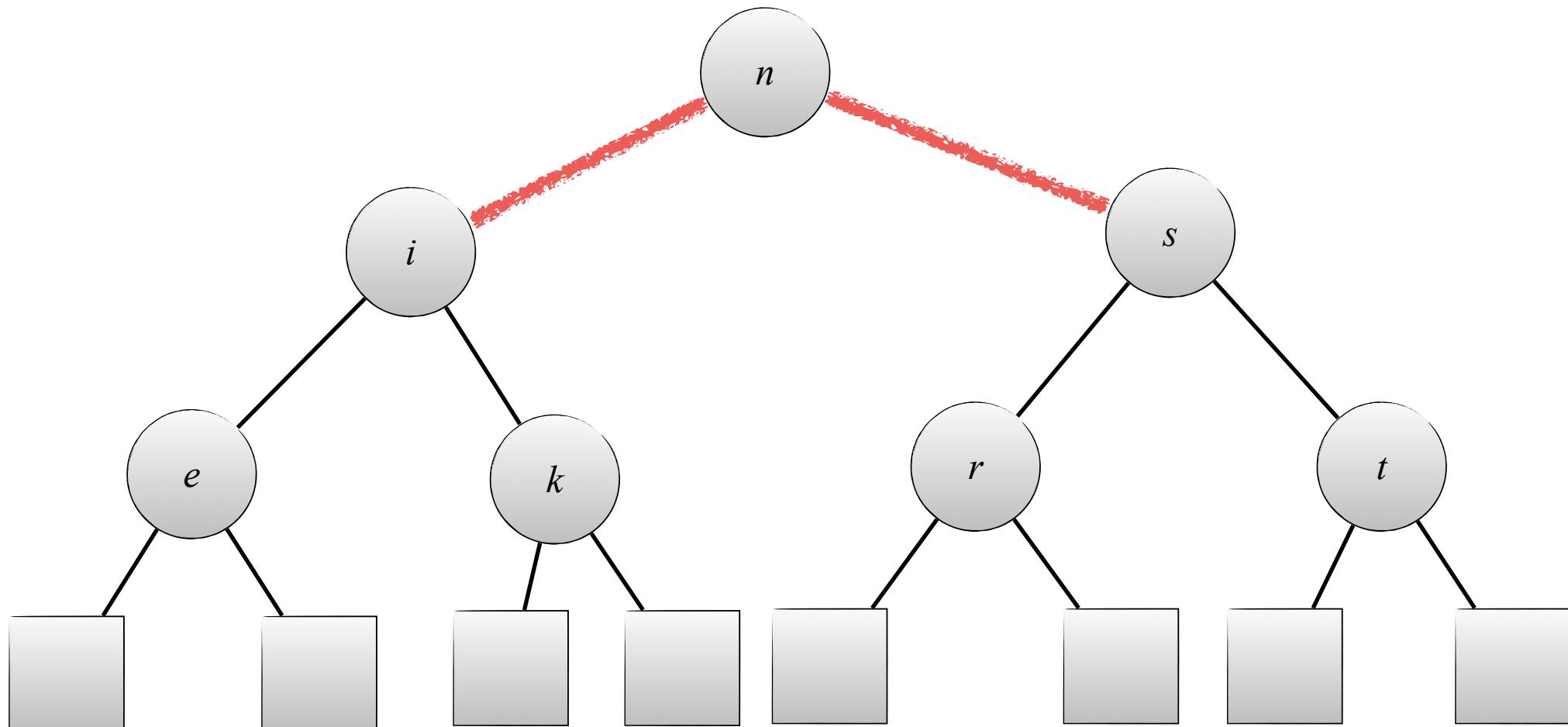
Insert key k



Insert key k

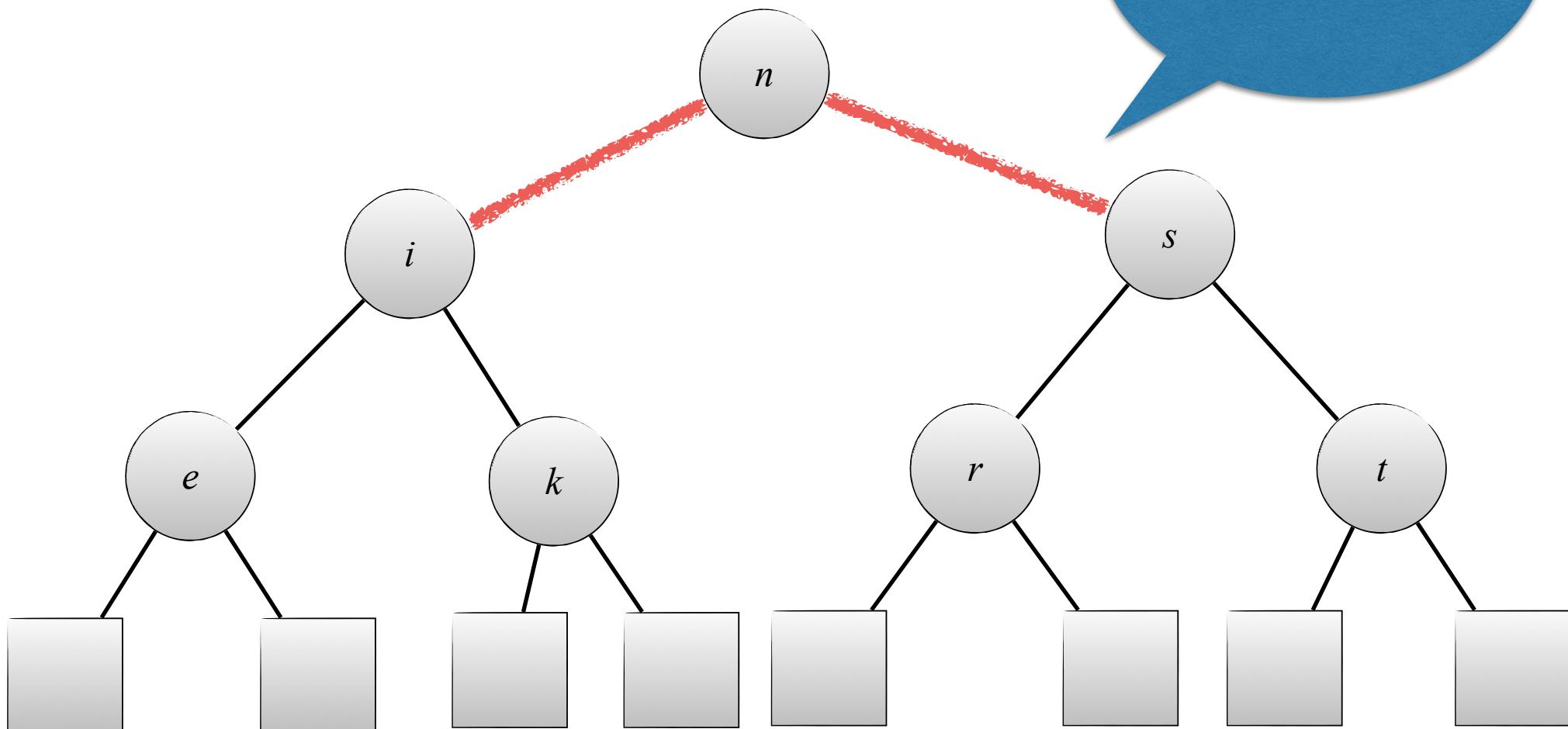


Insert key k

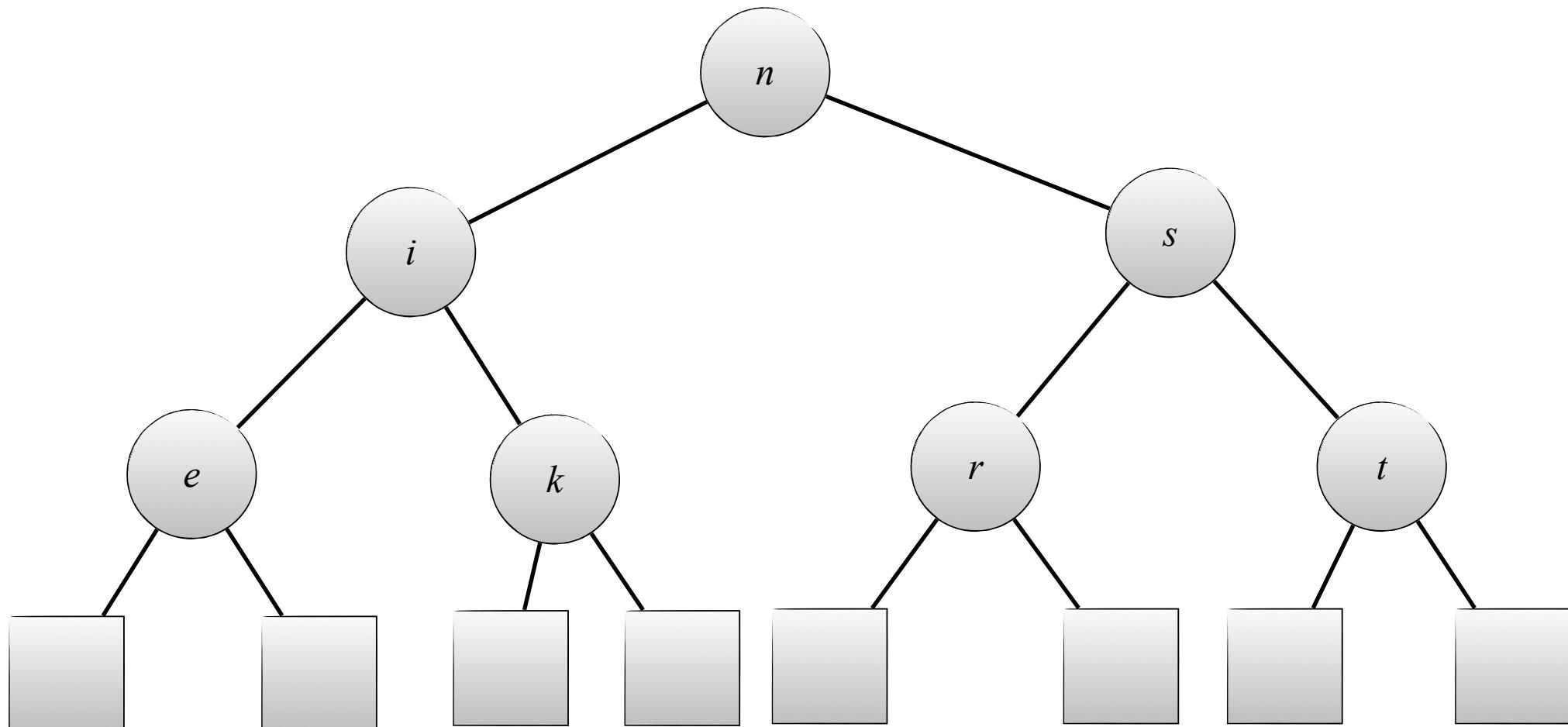


Insert key k

only one red link per node: flip colors



After inserting key k



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