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2018

21st Annual High School Mathematical Contest in Modeling (HiMCM)
Summary Sheet
(Please make this the first page of your electronic Solution Paper.)

Team Control Number: 8880
Problem Chosen: A

It could be troublesome finding rides around theme parks to get ourselves excited and fit, and many tend to rely on rankings online or elsewhere. Nonetheless, most recreational rankings (e.g. roller-coaster ranking in our case) are majorly or solely based on subjective reviews. This may consequently mislead one to a disappointing ride.

As our solution, a totally objective ranking model is first developed to mitigate subjective effects. Reference Point algorithm is conducted for our first model, which main feature is non-subjective. In this model, there are 3 steps in sequence: finding reference points, scoring and ranking.

Meanwhile, the given data set with problem is incomplete, affecting our regard to many potentially high-ranked roller-coasters. Thus, a data revision is performed to fill the gaps. Correlation between quantitative, qualitative and both sorts of criteria are studied in separate ways.

Our team is thereby inspired to rank through an enhanced version of such model, in which considered are subjective and practical factors of a real ranking. Besides, from an amusement park-themed source turns out that a high-ranked roller coaster is the most profitable (intrigues most riders) and mostly with an middle-upper scare level, matching our assumption. After all the research, optimization are respectively performed for the scoring and ranking step, along with compared results before and after it.

Afterwards, critical comparisons and analysis are conducted for our objective rankings with those existing online. For the first model, tremendous difference between its results and rankings online reveals its utopian downside, while the second model turns out more realistic while objective, especially after our modification. This is followed by validation, assuring their robustness under various data.

Finally, as required, we conceptualized a mobile App called RollerRanker for tourists to customize their own ranking algorithm with preferences, determined by either selecting or giving band scores (stars). Individual preferences may be further collected for macro-analysis to update our model while being robust. Pseudocodes and decision trees are introduced to interpret ideas.

For the paper fairly academic-toned, a one-page News-Release is attached after all for everyone.

Key-words:

Multi-objective ranking Roller-coaster Ranking app
Reference Point algorithm Model optimization

Please paste or type a summary of your results on this page. Please remember not to include the name of your school, advisor, or team members on this page.

ROLLER COASTER: RANKING FOR INDIVIDUALITY

Team #8880

Press Release:

Greetings! Didn't you ever have trouble finding rides around theme parks that are not only exciting but also fit your preference? Did you ever investigate some popular ranks of roller coasters but find them worthless at all due to overwhelming subjectivity? If so, we have THE SOLUTION for you!

To start with, we'd like to introduce our algorithms (how we made our own ranking via some tricky math)-the kernel of our solution. In order to generate a ranking that mitigates any human bias, we firstly developed a **Reference Point Algorithm** that automatically finds reference points by distributing data evenly, scores various factors such as speed, height or drop without manually weighting them and ranks total scores of all roller coasters in our database from highest to lowest thereby produces our very own, innovative, and absolutely objective top 10 list.

However, the given data set is incomplete, affecting our regard to many potentially high-ranked roller-coasters. Thus, we utilized a **Data Revision Algorithm** to "fill in the blank". By doing so we are able to predict the missing data accurately by analyzing the correlation between them and those exist. (Just like parents can always guess your precise real ranking of French by gazing at your class ranking and English grade within seconds. But surely our parents are far more accurate than our model.)

Our team is thereby inspired to rank through an enhanced version of such model. We believe that people speak, and those cold numbers can't reflect the true excitement of screaming and yelling on a train faithfully. Therefore, in our **second generation of ranking algorithm** we took some subjective and practical factors into account. After research, optimization is respectively performed for the scoring and ranking step, along with compared results before and after it.

Finally, after we completed as much missing data as possible and poured them into our optimized "ranking machine", a fresh, unparalleled, world-first, masterpiece-level, and absolutely objective(sounds more like a press release……) top 10 list of roller coasters worldwide was generated and followed by validation, assuring its robustness under various data. In our list, "**Hyperion**", "**Schwur des Kärnan**" and "**Flying Aces**" are ranked top 3 all over the world! Roller fanatics all over the world, please take a ride to exam whether they truly deserve it!

Oh, one last thing. As a heartful gift to roller coaster fans, we designed a fabulous mobile App titled "the" **Roller Ranker** for you to customize our ranking with your own preferences. Prefer an old school wooden roller coaster? Wanna take a speedy ride? Or simply like to drop by a random surprise we specially recommended for you? No problem at all within your fingertip! Our app has the "**Rollerpedia**", a showcase exhibiting the ranking result generated from our model and other available rankings; the "**Your List Maker**", where you can pick your private raking list by responding to a few questions and share them with friends freely; and finally, the "**Forum**", a socialized, interactive part for all of you to join and share-post reviews on roller coasters you took, gather experience points by answering to others' questions, browse news about roller coasters, and make some friends sharing a same interest-building a resort for roller coaster fans all over the world, to satisfy all your demands. But more importantly, it is a gate for us to learn your individual preferences for further macro/micro-analysis to update our ranking engine while being robust. The more you use our app, the more we truly understand you and therefore enabling us to provide you customized, timely service and information beyond any compare.

1 Problem Preview

1.1 Foreword

While travelling, if you are always a theme park seeker and a breathtaking-ride lover, roller coaster must be far front in your plan! However, riding experiences do vary, so pretty much would we first search for the best one around. For this, a range of (both official and empirical) ranking system naturally come into our sights. As everybody seems to take such information for granted, seldom could one question or study them to a depth.

Mathematically speaking, each ranking system has its own criterion, based on, probably complicated, algorithm; and simultaneously, it differs in passengers' taste. Then what turns out is rankings you rely on may finally let you DOWN! Conclusively, an individual-based or even customizable ranking generator are intrinsically in everyone's demand. As a problem-solving HiMCM team, we would try our best to look into existing rankings and make one ourselves, plus to develop a concept of an App for all coaster lovers.

1.2 Restatement of Problem

Chosen problem A, our team is bound to study and develop an objective, quantitative, while individual-friendly roller coaster ranking system.

Basically, being quantitative and objective means not to use emotional adjectives even a bit. Our algorithm will be solely dependent on physical properties on roller coasters like speed, duration of rides, age, drop, etc. To establish such an objective ranking algorithm, multiple models are reviewed in advance to extract a combined and optimized way to fit in this scenario.

After setting up our own ranking system, roller coasters in the given datasheet will all be assessed and ranked. Once the ranking obtained, we will make critical comparisons to various online, official rankings to both validate our nascent model and analyze the others.

Then comes the practical part when all those are done. We'll be conceptualizing a mobile App to "customize" rankings according to every user's preference, with a brief news-release conveying our ideas and the science behind them to all readers.

1.3 Assumptions

Different assumptions are respectively proposed for the ranking model and whose optimization, in which assumptions for ranking are mostly simple and objective, while those for optimization are more complicated, as subjective judging towards roller-coasters is involved.

Assumptions are detailly elucidated before each modelling.

1.4 Symbols in text

Chapter 2.1			
x	Input value of a linear equation	$\bar{x}$	Average of x
y	Output value of a linear equation	$\bar{y}$	Average of y
r	PMCC value	$\hat{y}$	Regressive y value
a	Coefficient of x in a linear equation	b	Constant in a linear equation
Chapter 2.3			
r	Number of rows	c	Number of columns
t	freedom degree	χ	Calculated Chi (independence) value
x_i	Observed number	m_i	Expected number
Chapter 3.1			
$\sigma_{j\ sum}$	Sum of ranking score	ω_i	weights
v_{ij}	Criteria involved	i, j	Ordinal number of an item
Models			
σ_i	Ranking of a specific criterion	σ	Overall ranking
$\overline{v}$	Average value of a criterion	v_i	A specific value of a criterion
v_{min}	Minimum of a criterion	v_{max}	Maximum of a criterion
v_i^{bu}	Upper bound of a criterion	v_i^{bl}	Lower bound of a criterion
r_i	Reservation point	a_i	Aspiration point
n	Number of items	ρ	Normalizing coefficient, may be empirical
Chapter 4.3			
u_Y, u_H, u_S, u_L, u_N		User star levels of Year opened, Height, Speed, Length and Number of inversions	
$\alpha, \beta, \gamma, \lambda, \mu$		Anchor points, explained in chapter 3	
Other symbols are previously annotated in chapter 3, models			
Chapter 5			
a_m	model-generated average	a_a	Actual average

2 Data Revision

2.1 Correlations between quantitative criteria

While attempting to fit the given data into our newly-built models, crucial (speed, height, etc.) data loss of some roller coasters appeared to bother us a lot. Though some data was then retrieved online, there was still a loss of around 10 numbers. Eventually, as both a perfection and a modelling-research practice, we then calculated the **Pearson Product-Moment Correlation Coefficient (PMCC or *r*)** between potentially correlative couples of criteria through the formula shown below, and 3 relatively highly-correlated couples are shown as scatter plots in Figures 2.1-2.3.

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2\sum(y-\bar{y})^2}}$$

Where r $\in$ $[-1,1]$. The closer$|r|$ to 1, the stronger linear correlation two variables(x,y) would have and between.

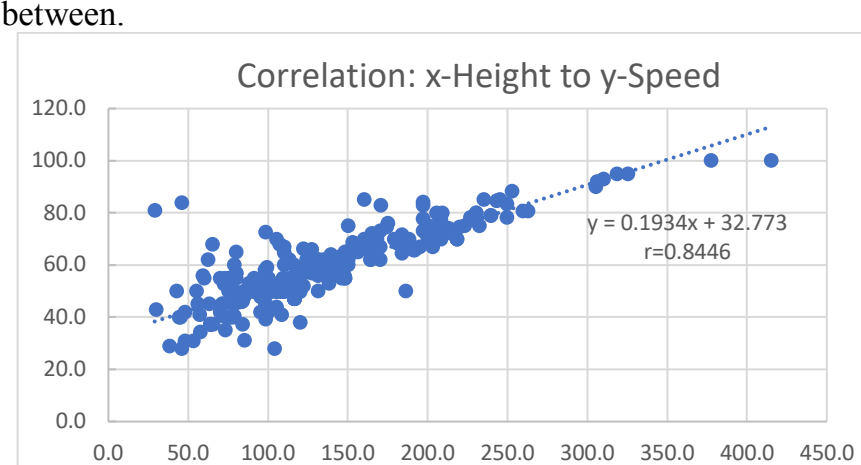


Fig. 2.1 Correlation between height and speed

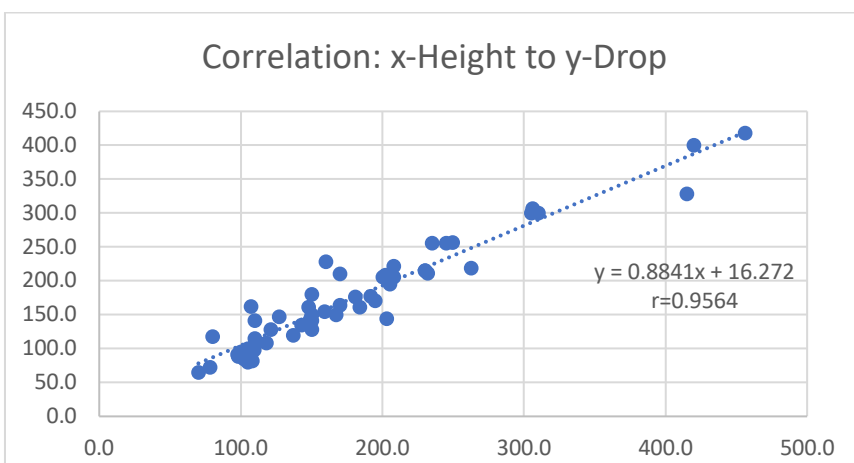


Fig.2.2 Correlation between height and drop*

* Drop of a roller coaster: largest vertical difference in a single coast.

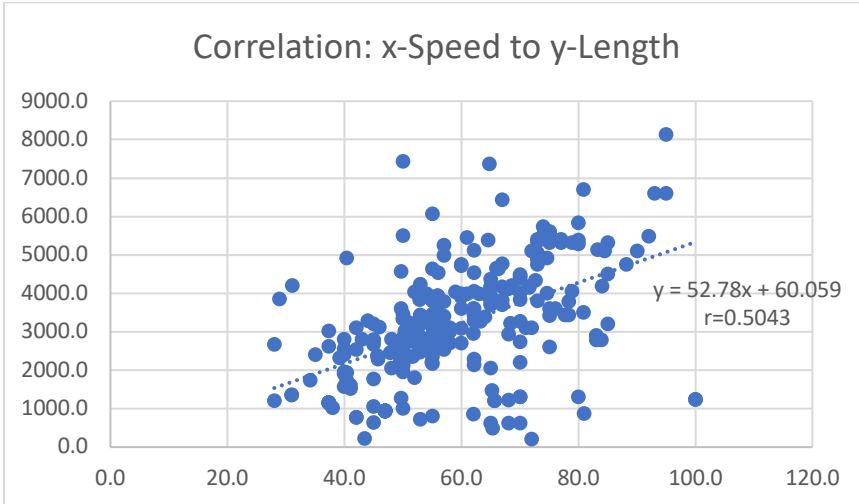


Fig.2.3 Correlation between speed and length

Regarding ***r*** value (referring to how accurate the linear regression is) displayed in charts, r (height, speed) =0.8446, r (height, drop) =0.9564, r (speed, length) =0.5043

The result turns out that there is relatively close linear correlation between height and speed, height and drop, speed and length. Therefore, the missing data can be approximately regained with the least-square regression lines (Formula shown below).

$$\hat{y} = ax + b,$$

where $b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$, a=$\bar{y} - b\bar{x}$

And these are linear equations produced:

$$\hat{y}\ \text{(height)} = 0.1934\times\ \text{(speed)} + 32.773,$$

$$\hat{y}\ \text{(drop)} = 0.8841\times\text{(height)} + 16.272,$$

$$\hat{y}\ \text{(length)} = 52.78\times\ \text{(speed)} + 60.059.$$

2.2 Correlations between quantitative and qualitative criteria

By combining both popularity (subjective, extracted from chart 4.1) and physical (objective) data can we make an analysis of the objective variables' effect on coasters' popularity. PMCC values are calculated between each criterion and the overall ranking, meanwhile the algorithm behind is both applied for revising data and making new rankings.

During our regressive analyze, all PMCC values are intrinsically low and there's a threshold made to eliminate criteria those are far away from "correlative", whose examples are stated below.

Regarding the duration of rides, its PMCC value turned out less than 0.001, (shown in figure 2.4, which indicates an extremely weak correlation. Thus, its correlativity is on the threshold to be ignored.

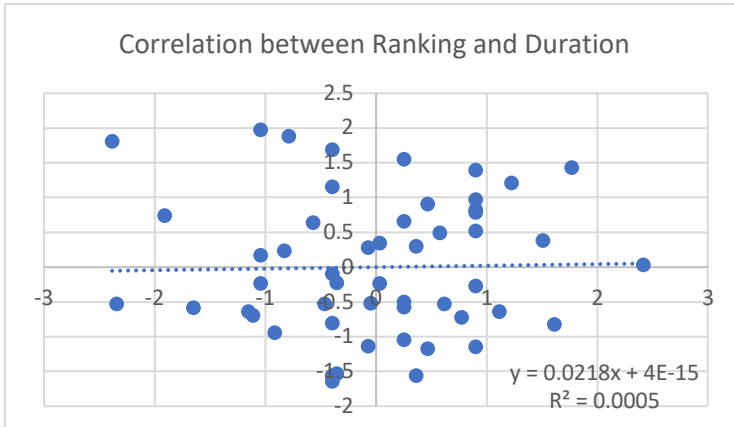


Fig. 2.4 Example of an almost-ignored case: Correlation between Ranking and Duration.

In case of G force and vertical angle, due to the fact that more than 2⁄3 of sites given (218 of G force and 209 of vertical angel) are literally without such data, the sample sizes failed to suffice our analytical requirements to interpolate their major data loss, those 2 criteria are unfortunately ignored.

Here are the PMCC value of each criterion that are decently correlative. (Chart 2.1).

PROPERTIES	**Date Opened**	**Height /ft**	**Speed /mph**	**Length /ft**	**Number of Inversions**	**Drop /feet**
PMCC	-0.1413	0.0590	0.1181	0.0361	-0.1903	0.1308

Chart 2.1 PMCC Values by criteria

Though PMCC values of each variable are relatively low ($|r| < 0.2$), we can yet approximately evaluate whether they are positively or negatively correlated and compare the relative correlation coefficient value. Since PMCC indicates how strong the linear correlation is between two variables, it reveals the effect of one variable to another to some extent, thus is a proper way to determine weight of each criterion.

2.3 Correlations between qualitative criteria

In terms of categorical variables such as type and construction, we conduct two Chi-square tests to test whether the ranking (given from Coasterbuzz.com) is dependent to each variable.

Here are the processes of Chi-test:

1) Chi-test for independence of construction and ranking
   $Hypothesis_0$: The ranking is independent of construction.
   $Hypothesis_A$: The ranking is dependent of construction.
   The matrix of observed values is arrayed as followed:

	In top100	Not in top100
steel	53	197
wood	16	33

Chart 2.2

A matrix of expected values is found to be:

	In top100	Not in top100
steel	57.69	192.31
wood	11.31	37.69

Chart 2.3

Followed by the testing equation:

$$\chi^2 = \sum \frac{(x_i - m_i{}^2)}{m_i}, \mathrm{t} = (\mathrm{r}-1)(\mathrm{c}-1),$$

Where r is number of rows, c is number of columns, x_i (for i= 1, 2…, k= r × c) represents observed values, m_i represents expected values, and t is the freedom degree of χ^2 distribution.

In this case χ^2=3.0274, t=1.

Referred to the table above, 0.05< (P-value) <0.10. Because P-value>0.05, we cannot reject $Hypothesis_0$. The data do NOT provide strong statistical evidence that the ranking is related to coasters' construction.

2) Similarly, Chi-test for independence of type and ranking

$Hypothesis_0$: The ranking is independent of type.

$Hypothesis_A$: The ranking is dependent of type.

The matrix of observed values is arrayed as followed:

	top	Not in top100
sit down	58	179
inverted	8	35
fly	1	4
wing	2	5
Suspended	1	1
stand up	0	2

Chart 2.4

A matrix of expected values is found to be:

	top	Not in top100
sit down	55.86	180.34
inverted	10.14	32.72
fly	1.18	3.80
wing	1.65	5.33
Suspended	0.71	2.28
stand up	0.47	1.52

Chart 2.5

Since the expected number of suspended and stand up coasters are too low, we remove these two variables to meet the condition of Chi-squared test (All expected counts are greater than 1, and at least 80% of the expected counts are greater than 5.)

A matrix of edited expected values is found to be

	top	Not in top100
sit down	56.00	181.00
inverted	10.16	32.84
fly	1.18	3.82
wing	1.65	5.36

Chart 2.6

For data from the matrix, χ^2=0.8262, t=3.

Referred from the Chi-square distribution table (see appendix), (P-value)>0.25. Because P-value>0.05, we cannot reject $Hypothesis_0$. The data do NOT provide strong statistical evidence that the ranking is related to coasters' type.

The Chi-squared tests indicate that the ranking is not statistically related to type or construction, so we can simply remove these two categorical variables from our algorithm.

3 Models

3.1 Prior to modelling: How objective can they be?

As problem required, our goal is to generate an "**objective**" model. Although absolute objectivity is clearly – after Heisenberg proclaimed his well-known uncertainty principle – not attainable; this very concept could be treated as an ultimate ideal worth striving for; therefore, in this paper, "objective ranking" is characterized as an approach to ranking that is as objective as possible. Thus, extended discussion is necessarily made here to confirm why some old-school approaches are not applicable in this scenario, and, why the reference point model, as the foundation of this paper's own algorithm, seems to be the most suitable.

To start with, is the simplest weighted-sum aggregation, as a common approach toward multiple-criteria decision-making. In such models, a weighting coefficient for all $i \in I$, with an additional restriction on the scaling of that $\sum_{i \in I} \omega_i = 1$, is adopted to aggregate all criteria by a weighted sum:

$$\sigma_{j\,sum} = \sum_{i \in I} \omega_i v_{ij}$$

Then, ranking is generated from the previous result sequentially. Despite its convenience and usefulness, there're some major downsides, especially in objective ranking cases, and this couple of reasons shown below is considered the most significant:

1. The linear aggregation of preferences expressed by weighted-sum tends to promote decisions with unbalanced criteria, and, more importantly, introduces subjective bias since it depends heavily on decision maker's/makers' opinion(s) to decide the weights. (except if all criteria have the same importance and thus, of identical weights).
2. The weighted-sum is based on a relatively tacit assumption that a worsening of one criterion's value could be compensated by the increment of another one. Though often encountered in economic or financial circumstances, such character of criteria is not very common in other interdisciplinary applications.

Generally speaking, a linear weighted-sum aggregation is mathematically succinct but a bit naive while presenting typical human preferences that are usually nonlinear; utilizing such an "innocent" approach may sometimes adversely result in unpredictable side-effects. So, from the perspective of this paper, for objective ranking, weighted-sum aggregation is not applicable, except in the simplest case of equal weights.

Therefore, our paper is aimed to develop a relatively more specific, bias-blind nonlinear approximation model, which requires neither long time nor decision makers' preferences (or weights). More specifically, to develop a system that defines objective ranking as independent of no other subjective preferences than some defined criteria with their potential order of priority and be dependent only on the given data. But problems again: how to define the grounds of an objective ranking since almost all conventional aggregation methods focus on rational subjective

aggregation of preferences and thus subjective rankings? In short, neither of these approaches is honestly feasible for our rational objective ranking problem and therefore the only, seemingly hopeful approach appropriate here for rational objective ranking is Reference Point Approach described below for the reason that reference levels needed could be established statistically from the given data set.

3.2 Modelling

Model 1: Reference Point-Scoring Approach for Ranking

<u>Assumptions</u>

1. All criteria involved are of the same weight, namely the same significance.
2. Of all quantitative criteria, the larger a number is, the higher its ranking is.
3. As proven in chapter 2.2, all qualitative criteria (e.g. locality, material, etc.) are way less correlative to overall rankings, meanwhile, those criteria can be concluded as technical advancement of roller-coasters along time. After all, all those qualitative criteria are condensed into a relatively decent criterion---"year opened".

After all pre-modelling analysis, our team developed a ranking system based on what so called, a **Reference Point Model**. The most significant feature of this model is, corresponding to problem requirement, no weights. In our previous researches, we found that literally there hardly is any reasonable explanations of how weights are introduced and determined. On the contrary, in our model, there're only a couple of **normalizing factors**, which won't interfere the actual significance of each ranking factor/criterion.

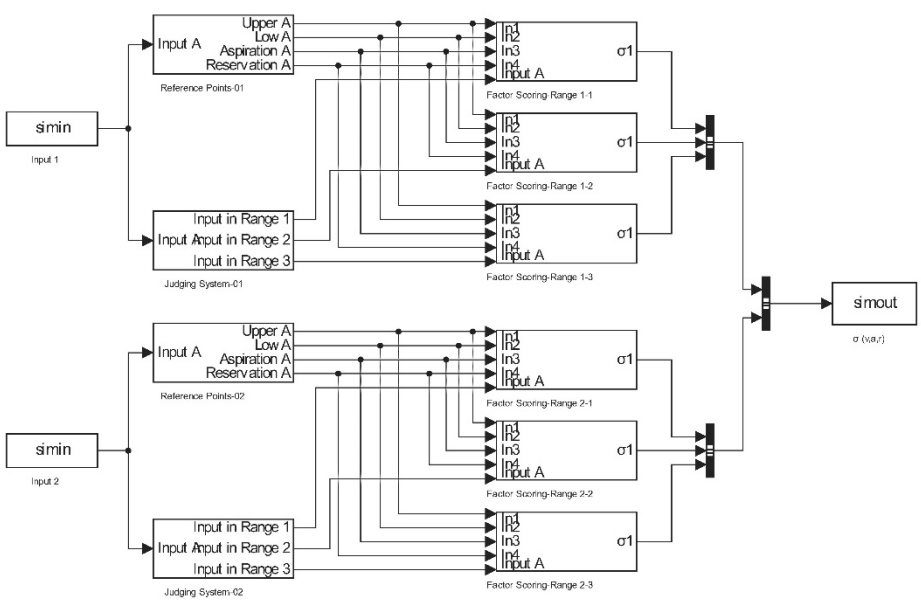


Fig.3.1 Flow chart of Reference Point algorithm.

More specifically speaking, our algorithm can be depicted with the following 3 steps.

The first step is to find out the **Reference Points** among all criteria (must be numerical). There are 2 boundaries-----the upper bound (the maximum) and the lower bound (the minimum) -----and 2 reference points----- called "aspiration" and "reservation" levels, which refer to a mid-upper, expecting and a mid-lower, sustaining average point respectively.

Next, referred to points determined, ranking/σ_i of each criterion can be calculated by a range-based set of equations and the result will be stored in a matrix named the criteria space$\sigma_i(v_i, r_i, a_i)$, moreover, this equation set will monotonically (or in a strictly linear fashion) map all raw ranking scores onto a definite range (e.g. 1-10) as well. <u>The mechanism of step 2 is briefly drawn in figure 3.2.</u>

The last and determining step is to find out the overall ranking/σ of each target per criterion by an average-like formula (why called that is that averages may be adjusted with a uniformly applied coefficient to all items).

<u>All formulas are shown below.</u>

Step 1: Find bounds and points

Group of criteria: Σ or $\sigma_i \in \Sigma$, and their average: $\overline{v} = {}^{1}/_{n} \sum_{i=1}^{n} \sigma_i$

Maximum of criteria: if $v_{max} \in \Sigma$, then $v_{max} = v_i^{bu}$

Minimum of criteria: if $v_{min} \in \Sigma$, then $v_{min} = v_i^{bl}$

Aspiration Level: $a_i = {}^{1}/_{2}\,(v_i^{bu} + \overline{v})$

Reservation Level: $r_i = {}^{1}/_{2}\,(v_i^{bl} + \overline{v})$

Step 2: Criteria Data Transformation: setting up a scoring function, $\sigma_i(v_i, r_i, a_i)$

$$\sigma_i(v_i, r_i, a_i) = \begin{cases} \alpha + \dfrac{\beta - \alpha}{r_i - v_i^{bl}}(v_i - v_i^{b_l}) & v_i^{bl} \le v_i < r_i \\ \beta + \dfrac{(\gamma - \beta)}{\overline{v_i} - r_i}(v_i - r_i) & r_i \le v_i < \overline{v_i} \\ \gamma + \dfrac{(\lambda - \gamma)}{a_i - \overline{v_i}}(v_i - \overline{v_i}) & \overline{v_i} \le v_i < a_i \\ \lambda + \dfrac{(\mu - \lambda)}{v_i^{bu} - a_i}(v_i - a_i) & a_i \le v_i < v_i^{bu} \end{cases}$$

In this formula, α, β, γ, λ, μ are pre-determined anchor points, while, v_i^{bl}, r_i, $\overline{v_i}$, a_i, v_i^{bu} are anchor points of given data, shown below in <u>chart 3.1</u>

Anchor Points	Year/Date Opened	Height (feet)	Speed (mph)	Length (feet)	Number of Inversions	Drop (feet)
v_i^{bl}	1924	28.956	28.0	215	0	27
r_i	1962	82	44	1678	1	81
$\overline{v_i}$	2001	135.43	59.48	3141.67	2	134
a_i	2009	296	104	5637	8	276
v_i^{bu}	2018	456	149.1	8133.2	14	418

Chart 3.1 Anchor points of given data

This is how scoring function works if $\alpha = 0$, $\beta = 3$, $\gamma = 5$, $\lambda = 7$ and $\mu = 10$.

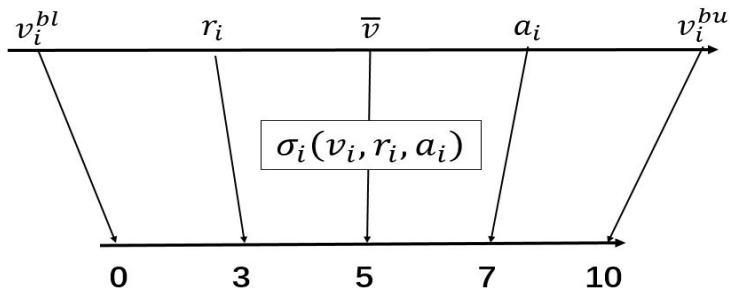


Fig.3.2 How scoring function works in brief. Everything is kept linear-fashioned.

Step 3: Ranking: with ranking function $\sigma(v, r, a)$.

$$\sigma(v, r, a) = \left[\sum_{\sigma_i \in \Sigma} \sigma_i(v_i, r_i, a_i)\right] / i$$

And here come the overall multi-criteria ranking results.
(decided by ranking the outcome number σ from the largest).

Model 2 Optimization of scoring and ranking functions

Assumptions

1. Though rankings of all criteria are set "larger is better" in model 1, subjective rankings are not really based on that (especially height, speed, inversions, etc.). Honestly, most common riders would prefer rides with average characters. Here is an example: for children, or what parents may believe is their limit should be lower than average, while for young adults, that may be somewhere between average and maximum. Moreover, what is remarkable is a negative correlation is found between "number of inversions" and overall ranking, this reveals that people may be frightened off or unbearable while number of inversions increases.
   A more explicit example, considering the height a of roller-coaster and its popularity, most popular and profitable roller-coasters are built as medium-height. An amusement park-themed research has stated the same fact as well, matching our assumption and such property of distribution (shown in fig. 3.3 below).

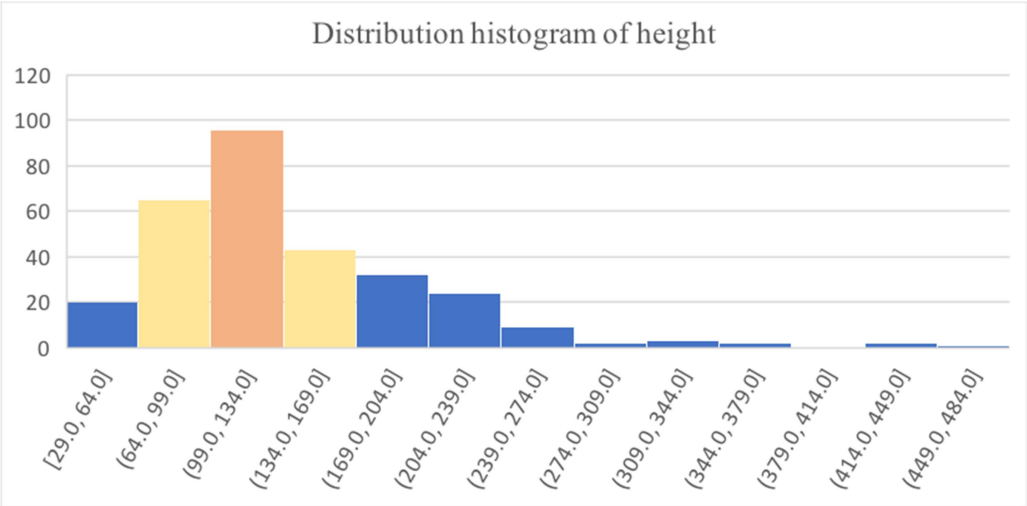


Fig. 3.3 Distribution of roller-coasters by height

2. For different people do have different preferences of criteria, ranking varies with the priority of criteria. By here, deficiency of each roller-coaster will be focused and mitigated to optimize our objective ranking.

Optimized results

(a) Optimization of scoring function

Anchor Points	Year/Date Opened	Height (feet)	Speed (mph)	Length (feet)	Number of Inversions	Drop (feet)
α	0	0	0	0	5	0
β	3	5	5	5	10	5
γ	5	7	7	7	5	7
λ	10	10	10	10	3	10
μ	8	5	5	7	0	5

Chart 3.2 Optimized/normalized anchor points

(b) Correlations before and after optimization, based on assumption 1.

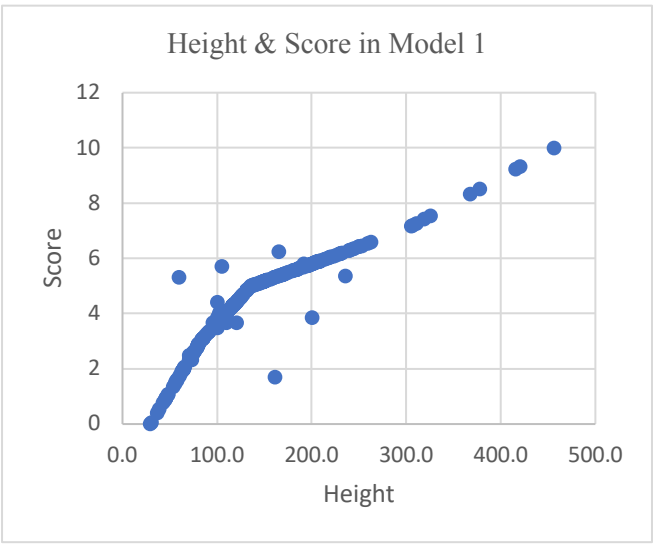


Fig. 3.4a Correlation of height-score, model 1original

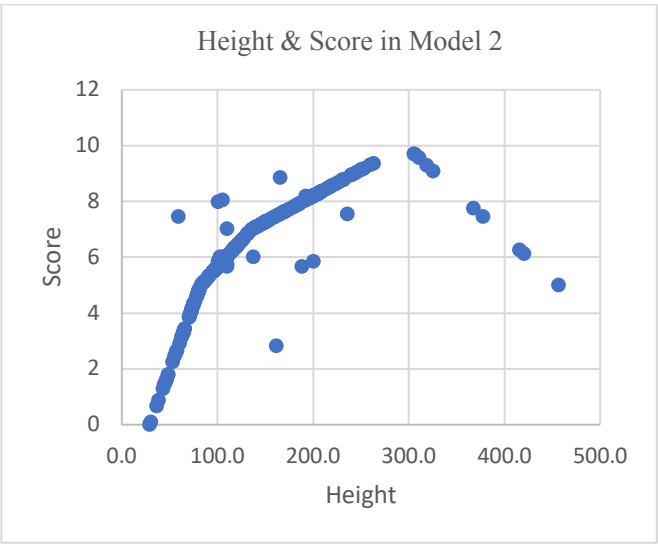


Fig. 3.4b Correlation of height-score, optimized

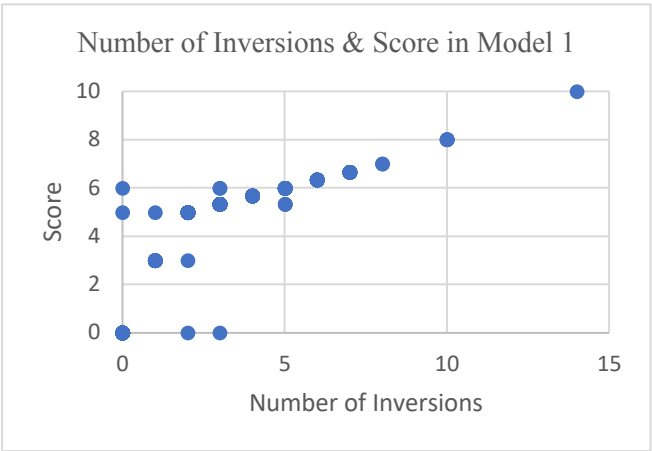


Fig. 3.4c Correlation of #inv-score, model 1original

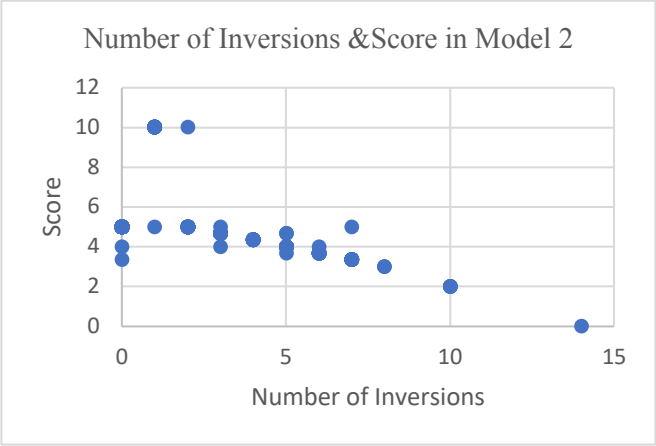


Fig. 3.4d Correlation of #inv-score, optimized

(c) Optimized ranking function based on assumption 2.

$$\sigma(v,r,a) = \min_{\sigma_i \in \Sigma} \sigma_i(v_i, r_i, a_i) + ({}^{\rho}/_{n}) \sum_{\sigma_i \in \Sigma} \sigma_i(v_i, r_i, a_i)$$

4 Consequences

4.1.1 Our Own Top-10 Roller-Coaster Ranking by Model 1

Name	Scores	Ranking by Model 1
Steel Vengeance	6.706	1
Kingda Ka	6.594	2
Fury 325	6.538	3
Soaring Dragon & Dancing Phoenix	6.528	4
Hyperion	6.491	5
Red Force	6.425	6
Hyper Coaster	6.328	7
Valravn	6.300	8
Flash	6.217	9
Banshee	6.187	10

Chart 4.1 Our top-10 list based on ***model 1***

4.1.2 Our Own Top-10 Roller-Coaster Ranking by Model 2

Name	Scores	Ranking by Model 2
Hyperion	11.66126	1
Schwur des Kärnan	11.61597	2
Flying Aces	11.50605	3
Soaring with Dragon	10.71428	4
Iron Rattler	10.45382	5
Hades 360	10.18407	6
Fluch von Novgorod	8.965729	7
Intimidator 305	8.47083	8
Leviathan	8.464955	9
Shambhala	8.402221	10

Chart 4.2 Our top-10 list based on ***model 2***, optimized

4.2 Critical Comparison

Ranking This Week	Ranking Last Week	Name and locality of roller coasters	Score	Number of reviews	Ranking by Model 1	Ranking by Model 2
1	1	Steel Vengeance, Cedar Point	4.94	177	1	89
2	2	Fury 325, Carowinds	4.86	174	3	15
3	3	El Toro, Six Flags Great Adventure	4.83	355	126	36
4	4	Lightning Rod, Dollywood	4.81	83	74	34
5	5	Twisted Timbers, Kings Dominion	4.78	59	———	———
6	6	Millennium Force, Cedar Point	4.77	771	31	26
7	7	Twisted Colossus, Six Flags Magic Mountain	4.76	92	30	49
8	8	Boulder Dash, Lake Compounce	4.73	146	198	87
9	9	Wicked Cyclone, Six Flags New England	4.70	81	64	99
10	10	Voyage, The, Holiday World	4.70	358	98	40
11	11	Maverick, Cedar Point	4.69	675	91	58

Chart 4.3 ***Online*** Subjective Ranking 1 compared with our Model

The ranking method of this method claimed on the webpage can be explained as following: Every week, the website collects its experienced members' reviews of roller coasters. Then it calculates the scores of the coasters which appear in at least half of all records and rank them 1 to 100. Here, in order to compare it with our top 10 list, we extracted the top ten coasters.

According to its sampling and ranking method, this list is based on popularity and customer experience. Subjectivity dominated, it virtually neglects many objective factors. factors like opening dates, ride length and types are nowhere to be observed. On account that it derives from customer ranking, the results are considerably biased with personal taste. Besides, since it only includes about half of existing coasters, it is possible that a coaster with great quality is left out due to other lurking variables, geographical location, weather condition, emergency events, etc. And on top of it, only would the reviews of roller-coaster enthusiasts (one has published more than 10 reviews of different sites) be recognized by this website. Therefore, its list only shows the most popular coaster of one week but isn't for everyone.

Ranking	Speed	Roller Coaster	Amusement Park (Location)	Opened
1st	149.1 mph	Formula Rossa	Ferrari World Abu Dhabi (Abu Dhabi, United Arab Emirates)	11/4/2010
2nd	128 mph	Kingda Ka	Six Flags Great Adventure (Jackson, New Jersey, USA)	5/21/2005
3rd	120 mph	Top Thrill Dragster	Cedar Point (Sandusky, Ohio, USA)	5/4/2003
4th	111.8 mph	Do-Dodonpa Red Force	Fuji-Q Highland (Fujiyoshida, Yamanashi, Japan)	7/15/2016 4/7/2017
5th	100 mph	Superman: Escape from Krypton Tower of Terror II	Ferrari Land (Salou, Tarragona, Spain)	3/19/2011
Coequal 5th	99.4 mph	Ring Racer	Six Flags Magic Mountain (Valencia, California, USA) Dreamworld (Coomera, Queensland, Australia)	11/4/2013
6th	95 mph	Steel Dragon 2000 Fury 325	Nürburgring (Nürburg, Rhineland-Palatinate, Germany)	9/3/2006 3/28/2015
7th	93 mph	Millennium Force	Nagashima Spa Land (Nagashima, Kuwana, Mie, Japan) Carowinds (Charlotte, North Carolina, USA)	5/13/2000
8th	92 mph	Leviathan	Cedar Point (Sandusky, Ohio, USA)	5/6/2012
9th	90 mph	Intimidator 305	Canada's Wonderland (Vaughan, Ontario, Canada)	4/2/2010
10th	88.2 mph	Hyperion	Kings Dominion (Doswell, Virginia, USA)	7/14/2018

Chart 4.4 Ranking by speed from RCDB online database,
Rankings are not assigned to roller coasters that are not currently operating.

This (chart 4.4) a ranking list that only considers the speed of the coasters. Comparing to our list produced through ranking algorithm, it only involves one objective variable. However, a roller coaster cannot be simply evaluated by its speed. As listed in the database, other variables including height, length and number of inversions also play important roles.

However, after comparing these lists with ours, we discovered that both subjective and objective variables affect coaster ranking to a large extent. It is particularly conspicuous in terms of the inconsistency between our list and those two downloaded.

Results (not shown) from our model 1 is far from both 2 rankings online. In detail, it literally only contains 1 top-10 roller-coasters found online.

After all, rankings from our model 2 matches the same 2 rankings online way better, reaching a concerted condition of both subjectivity and objectivity.

4.3 App Conceptualization: Roller-Ranker

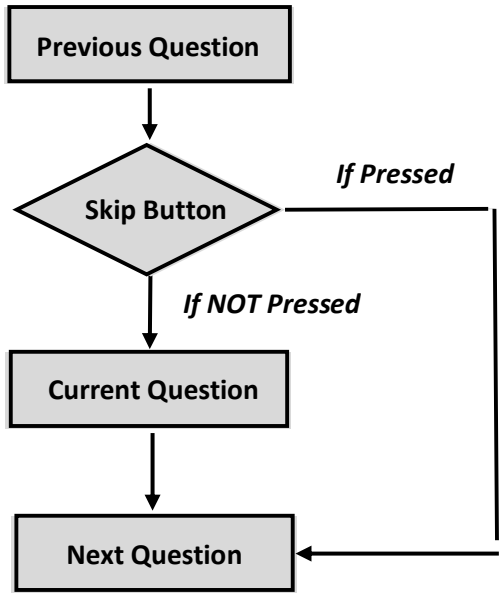
In order to make the algorithms of our conceptual mobile App CoasteRanker friendlier to readers without in-depth knowledge in computer programming, this paper decides to utilize Pseudocode, which resembles written English largely, to better illustrate our main ideas. So, pace yourself, leave the technical details away, and focus on capturing the big picture.

Inputs

Raw Database [DB] (a 2-dimensional array/matrix that contains pre-processed data from the provided data sheet to better cooperate with the program with a size of 9*300);
Popularity Database (a ranking reflecting subjective, emotional assessments based on riders' voting and number of riders voted);
UI01 (Material Preference from users- Steel or Wood or Indifferent?);
UI02 (Inversion Preference- Do you like Inversions or Not or Indifferent?);
UI03 (Location Preference- This data can be either acquired through GPS auto-location on mobile phones or manually edited by users' choice-Where you' like to take the ride?);
UI04 (Riding Mode Preference- What kind of roller coaster you'd like to have a shot? A flying one? A standing one? A suspended one or whatever?);
UI05 (Popularity Ranking- Though all above are objective, clam logics or numbers, as the question requested, we believe that people speak. Therefore, we introduced a ranking-based solely on riders' voting and the number of riders, providing our users another dimension of reference.);

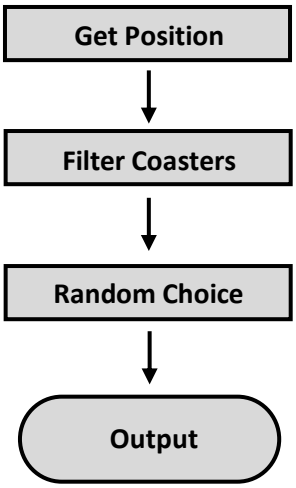
Customized Functions

Skip Button This is a simple Boolean logic function. If the "Skip" button on the GUI is pressed by the user, the value of this function will be stimulated and turn into 0 (false) and therefore skip the current question without adjusting the ranking inherited from last step. As shown below:



Flowchart 4.1

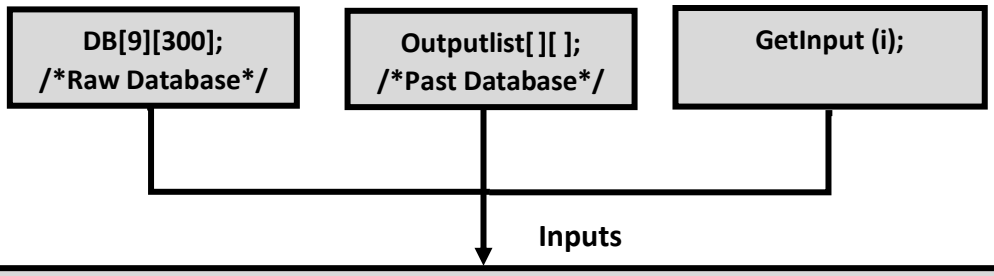
SurpriseMe This is a "Masterpiece" that presents a random roller coaster to the user within 100km radius, as shown below:



Flowchart 4.2

Workflow

And now we may turn to the core part of our mobile app-its stem workflow. In order to develop a user-friendly and use-at-ease mobile app, we decide to use a **decision tree**-based workflow to adjust our existing ranking, as shown below:

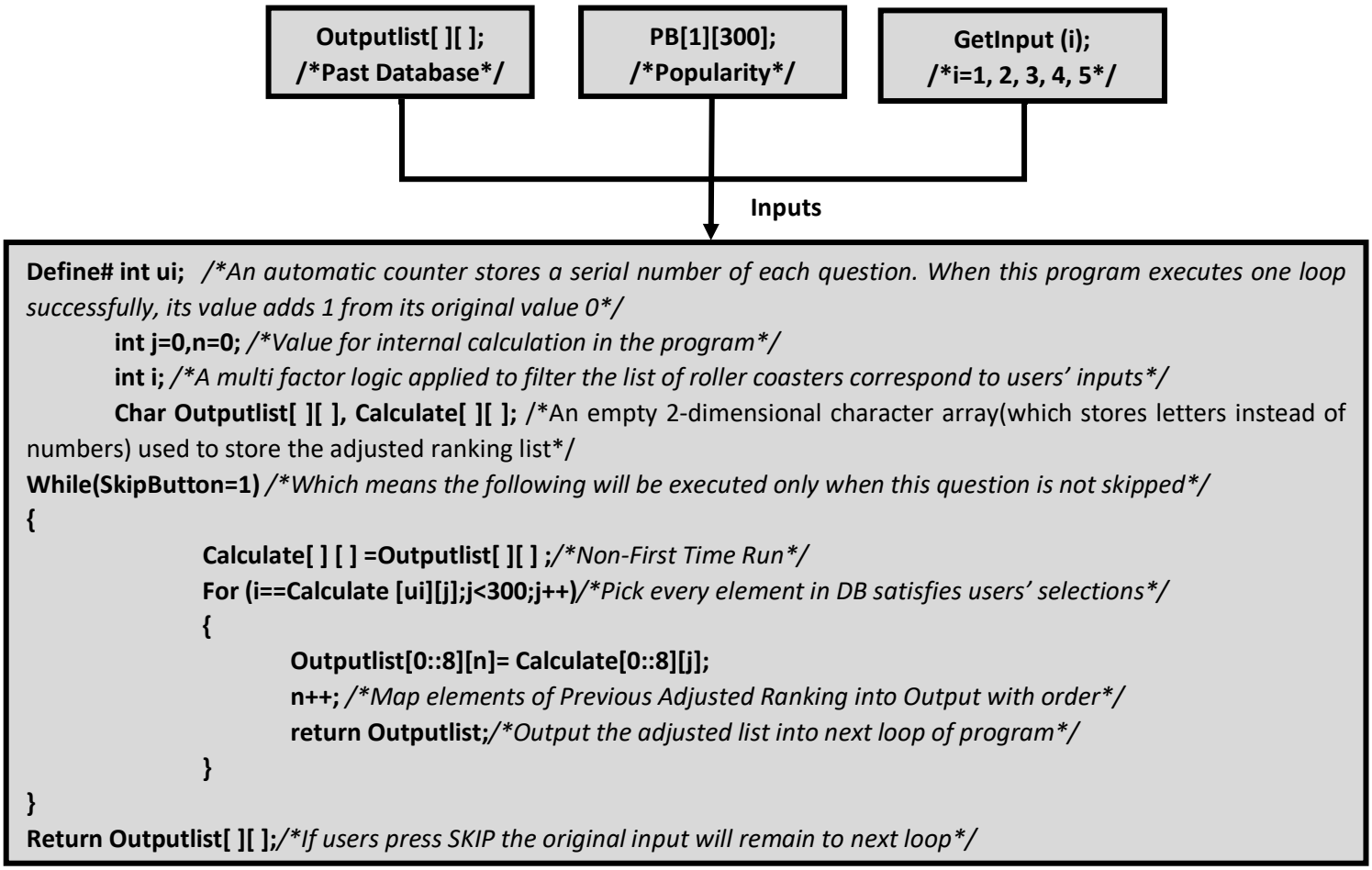



```
Load Char Calculate[ ][ ]=Char DB [9][300]; /*Install the database into the program for later use when this program runs
for the first time*/
Define# int ui;  /*An automatic counter stores a serial number of each question. When this program executes one loop
successfully, its value adds 1 from its original value 0*/
        int j=0,n=0; /*Value for internal calculation in the program*/
        Boolean i; /*A Boolean logic applied to filter the list of roller coasters correspond to users' inputs*/
        Char Outputlist[ ][ ], Calculate[ ][ ]; /*An empty 2-dimensional character array (which stores letters instead of
numbers) used to store the adjusted ranking list*/
While(SkipButton=1) /*Which means the following will be executed only when this question is not skipped*/
{
                Calculate[ ] [ ] =Outputlist[ ][ ] ;/*Non-First Time Run*/
                For (i==Calculate [ui][j];j<300;j++)/*Pick every element in DB satisfies users' selections*/
                {
                        Outputlist[0::8][n]= Calculate[0::8][j];
                        n++; /*Map elements of Previous Adjusted Ranking into Output with order*/
                        return Outputlist;/*Output the adjusted list into next loop of program*/
                }
}
Return Outputlist[ ][ ];/*If users press SKIP the original input will be retained till next loop*/
```

Flowchart 4.3

This process may be then utilized onto UI01; UI02; UI03

Since UI04; UI05 are multi-factor calculations instead of Boolean logics, we applied different algorithm to deal with this difference, as shown below:



Flowchart 4.4

Above are the different calculation programs for different types of layers in our decision tree model. To summarize the structure of our algorithms, we plotted a system workflow.

As depicted below:

(Due to an over-scaled quantity of possible branches of the decision tree, we are only able to partially show it.)

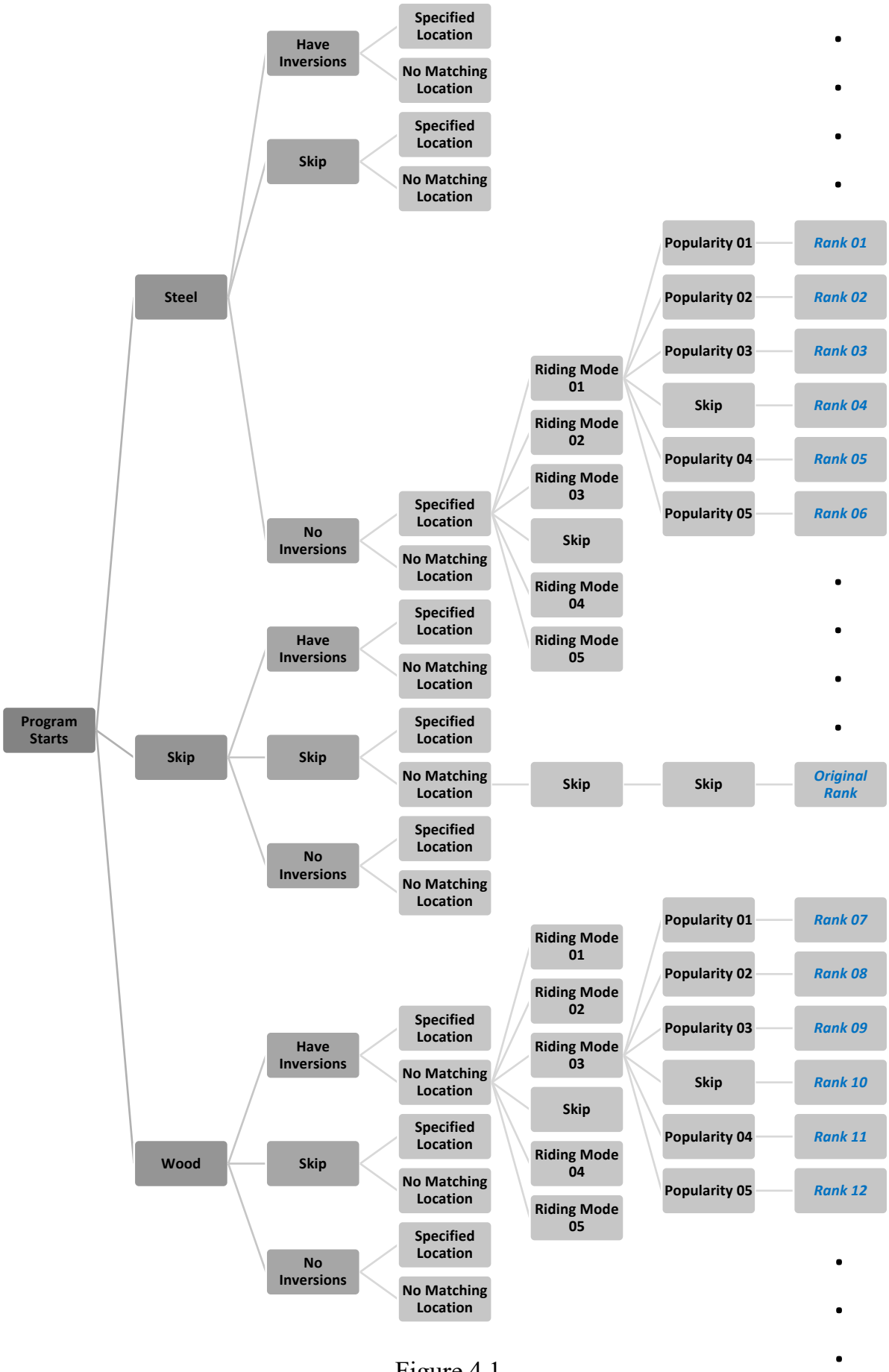

Program Starts
Steel
Skip
Wood
Have Inversions
Skip
No Inversions
Have Inversions
Skip
No Inversions
Have Inversions
Skip
No Inversions
Specified Location
No Matching Location
Specified Location
No Matching Location
Specified Location
No Matching Location
Specified Location
No Matching Location
Specified Location
No Matching Location
Specified Location
No Matching Location
Specified Location
No Matching Location
Specified Location
No Matching Location
Riding Mode 01
Riding Mode 02
Riding Mode 03
Skip
Riding Mode 04
Riding Mode 05
Skip
Riding Mode 01
Riding Mode 02
Riding Mode 03
Skip
Riding Mode 04
Riding Mode 05
Popularity 01
Popularity 02
Popularity 03
Skip
Popularity 04
Popularity 05
Skip
Popularity 01
Popularity 02
Popularity 03
Skip
Popularity 04
Popularity 05
Rank 01
Rank 02
Rank 03
Rank 04
Rank 05
Rank 06
Original Rank
Rank 07
Rank 08
Rank 09
Rank 10
Rank 11
Rank 12

Figure 4.1

In order to make it truly a user-friendly mobile APP, we designed a conceptual UI as depicted below:

Name: Roller Ranker

Icon:

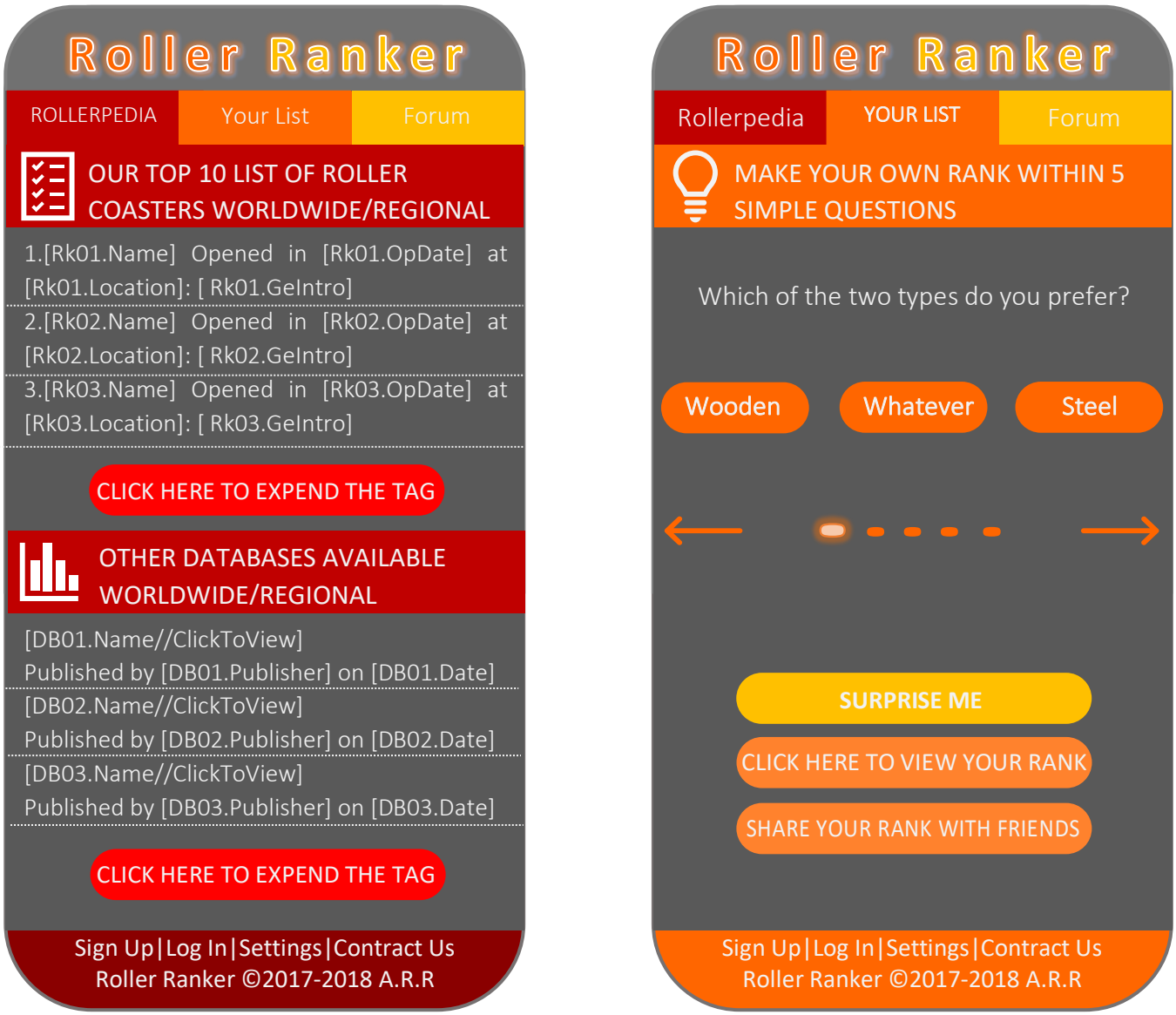


Figure 4.2

In general, each page in this app is composed by three structural components-the top banner, the container and the bottom ribbon- that have functions as following:

1. The top banner: The top banner mainly works as bookmarks guiding users to different pages.
2. The container: The container is a universal utensil for delivering information such as our rank, reference databases, the preference questions or the forum. And in each container there's various universal components-text boxes, buttons, or hyperlinks- cooperate to achieve certain goals of certain pages.
3. The bottom ribbon: The bottom ribbon works mainly as a gateway to essential links irreverent from the focus of pages and a billboard to exhibit some legal information.

And now we will take a closer look at each page.

The first page, named "Rollerpedia", is a showcase exhibiting the ranking result generated from our model and other available ranking databases. In the upper part of the container of first page is a collapsible tag showing general information of our ranking. Users may click the "Click for More" button for more detailed information such as the location on map, opening time, technical performances or other users' comments of our top 10s.Similarly, the bottom part of the container consists a collapsible tag presenting other ranking databases that users may be interested to look into. Moreover, in our hypothesis, ranks on page 1 may be global or regional, responsive to users' choices.

The second page, called "Your List", is where the custom raking list based on users' responds to certain questions in our decision tree generated. On the upper part, users may see five questions with interactive buttons and arrows. As they respond to each of them, the decision tree algorithm behind the interface will take effect to filter the original ranking generated from our model with users' preferences. With three buttons in the bottom, "Surprise Me", "View Personalized Ranking" and "Share with Friends", users can get detailed introduction of or even route to a random roller coaster in our database within users' reach based on their positions, view their own rankings, and share them with friends.

Moreover, in order to enhance users' adhesion to our app, we anticipated a socialized part, the forum. For roller-coaster fans to join and share. In the forum, registered users may post their reviews on roller coasters they took, gather "experience points" by answering to others' questions, browse news about roller coasters and make more friends who share one same interest. Just like TripAdvisor or Airbnb for frequent travelers, the roller ranker community is a resort for roller coaster fevers all over the world.

Further Improvements

These improvements of our stem algorithm are dedicated for a better individualized experience.

In our improved model, users may star from 1-5(u = $(u_Y, u_H, u_S, u_L, u_N)$, $u_Y, u_H, u_S, u_L, u_N \in \{1,2,3,4,5\}$) on 6 different parameters (Opening Date、Height、Speed、Length、Number of Inversions、Drop).The rating(u), which reflects users' preference on these 6 measures, represents various weights of these parameters in our ranking algorithm(see chapter 3.2), i.e., $\omega_i = u_i/5$.

Thus, the ranking function can be written as following:

$$\sigma(v, r, a) = \sum_{\sigma_i \in \Sigma} \omega_i \sigma_i(v_i, r_i, a_i)$$

Moreover, this rating stands for the maximum user-acceptable values of each parameter (scale conversion is applied), which are anchor points (see chapter 3.2) in our algorithm. For instance, a $u_H = 1$ (a one star on height preference) means that this user prefers much lower roller coasters, which, translate into our calculation process means that the highest score on height parameter for this user's ranking belongs to the lowest data in the array. (See chart below)

u_i	α	β	γ	λ	μ
1	10	7	5	3	0
2	5	10	7	3	0
3	0	5	10	5	0
4	0	3	7	10	5
5	0	3	5	7	10

Chart 4.5

Therefore, the anchor matrix of new scoring function is generated from users' stars. For example, if a user has an input of u = (1,2,3,4,5,3), the corresponding anchor matrix runs as following:

Anchor Points	Date Opened	Height (feet)	Speed (mph)	Length (feet)	Number of Inversions	Drop (feet)
α	10	5	0	0	0	0
β	7	10	5	3	3	5
γ	5	7	10	7	5	10
λ	3	3	5	10	7	5
μ	0	0	0	5	10	0

Chart 4.6

Thanks to such a dynamic matrix into our ranking algorithm, the resulting ranking in our mobile app could be as vibrant as 453,496,632 possibilities (Note, if take geo-location into account the quantity may explode into billions!). In this way, we are not only able to exhibit our original, non-biased top 10 ranking, but also capable of providing our users their personal, customized lists to find their "the roller coaster".

5 Model Validation: How correlations affect accuracy

As our team interpolated some lost data through a linear function generated by correlations between multiple couples of criteria, the output rankings WILL consequently be interfered with those correlation coefficients or PMCC values mentioned before.

Demonstrated in chart 5.1, correlation coefficients dramatically vary from pairs to pairs. The accuracy of this model will be affected while different criterion dominates. For instance, in a ranking-customization App, different people will prefer rankings by different properties of a roller-coaster; or in different sets of data, criteria may change, or data may be incomplete.

To validate our predictive model, both model-generated average/a_m and actual average/a_a values are compared in the same ranking algorithm. According to chart 5.2, even some differences seem influential, they are eventually not that disturbing, as all errors are less than 5%, yet complaint with all accuracy concerns.

CRITERIA	**Height /ft**	**Speed/mph**	**Length /ft**
Height (feet)	1.0000	0.9438	0.3683
Speed (mph)	0.9438	1.0000	0.3198
Length (feet)	0.3683	0.3198	1.0000

Chart 5.1: Correlations

VALUES/CRITERIA	**Height /ft**	**Speed/mph**	**Length /ft**
a_a as input	135.3	62.0	3149.9
a_m as input	131.3	42.0	2915.5
Averaged error	-0.05	-0.02	-0.01

Chart 5.2: Errors in validation

(The End)

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[*Source of Chart 4.1 Ranking*]
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[*Source of Chart 4.2 Ranking*]
"Record Holders by speed", rcdb, https://rcdb.com/rhr.htm, accessed November 10, 2018

[*Source of cover page picture*]
"The 10 Craziest Roller Coaster Rides For Thrill Seekers", BRIT+CO, https://www.brit.co/best-roller-coaster-rides/, accessed November 19, 2018

Appendix

(A) Reference table for Chapter 2.3 χ^2-test

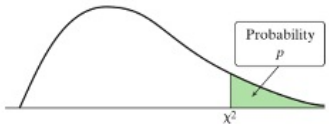

Table entry for p is the point χ^2 with probability p lying to its right.

Table C Chi–square distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

From:

Yates, D. and David Moore and D. Starnes, 2008. *The practice of statistics*, 3rd ed., New York: W.H. Freeman and Company

(B) Data after revision

Name	*Year/Date Opened*	*Height (feet)*	*Speed (mph)*	*Length (feet)*	*Number of Inversions*	*Drop (feet)*
10 Inversion Roller Coaster	2006	98.4	45.0	2788.8	10	103.3
Abismo	2006	151.6	65.2	1476.4	2	150.3
Adrenaline Peak	2018	72.0	45.0	1050.0	3	79.9
Afterburn	1999	113.0	62.0	2956.0	6	116.2
Alpengeist	1997	195.0	67.0	3828.0	6	170.0
Alpina Blitz	2014	108.3	51.6	2358.9	0	112.0
Altair	2014	108.3	52.8	2879.8	10	112.0
American Eagle	1981	127.0	66.0	4650.0	0	147.0
Anaconda	1989	118.1	55.9	3937.0	0	40.0
Apocalypse	2012	100.0	55.0	2900.0	2	90.0
Apocalypse the Ride	2009	95.0	50.1	2877.0	0	87.3
Apollo's Chariot	1999	170.0	73.0	4882.0	0	210.0
Atlantica SuperSplash	2005	98.4	49.7	1279.5	0	103.3
Backlot Stunt Coaster	2005	45.2	40.0	1960.0	0	31.2
Balder	2003	118.1	55.9	3510.5	0	120.7
Bandit	1999	91.2	49.7	3605.7	0	81.7
Banshee	2014	167.0	68.0	4124.0	7	150.0
Bat	1993	78.0	51.0	2352.0	0	85.2
Batman - The Dark Knight	2002	117.8	55.0	2600.0	5	120.4
Batman the Ride	2000	109.3	49.7	2260.5	5	112.9
Batman The Ride-92	1992	100.0	50.0	2700.0	5	104.7
Batman The Ride-93	1993	105.0	50.0	2693.0	5	109.1
Batman The Ride-94	1994	105.0	50.0	2700.0	5	109.1
Batman The Ride-95	1995	105.0	50.0	2693.0	5	109.1
Batman The Ride-99	1999	105.0	50.0	2700.0	5	109.1
Batman: Arkham Asylum	2002	105.0	49.7	2700.2	5	109.1
Batwing	2001	115.0	50.0	3340.0	5	117.9
Beast	1979	110.0	64.8	7359.0	0	141.0
Behemoth	2008	230.0	77.0	5318.0	0	219.6
Big Apple Coaster	1997	203.0	67.0	4777.0	2	144.0
Big Loop	1983	98.4	39.2	2316.3	4	103.3
Big One	1994	213.0	74.0	5497.0	0	205.0
Big Thunder Mountain	1992	72.2	40.4	4921.3	0	39.3
Big Thunder Mountain Railroad	1979	104.0	28.0	2671.0	0	108.2
Bizarro	1999	142.0	61.0	3985.0	7	141.8
Black Mamba	2006	85.3	49.7	2519.7	4	88.6
blue fire Megacoaster	2009	124.7	62.1	3464.5	4	126.5
Blue Hawk	1992	122.0	52.0	2742.0	5	124.1
Blue Streak	1964	78.0	40.0	2558.0	0	72.0

Boardwalk Bullet	2007	96.0	51.0	3236.0	0	92.0
Bocaraca	2005	64.0	37.3	1148.3	2	72.9
Boomerang	1999	116.5	47.0	935.0	3	119.3
Boomerang	2013	116.5	47.0	935.0	3	119.3
Boomerang	2000	116.5	47.0	935.0	3	119.3
Boomerang	1988	116.5	47.0	935.0	3	119.3
Boomerang	1998	116.5	47.0	935.0	3	119.3
Boomerang	1988	116.5	47.0	935.0	3	119.3
Boomerang	1999	116.5	47.0	935.0	3	119.3
Boomerang	1996	116.5	47.0	935.0	3	119.3
Boomerang	2000	116.5	47.0	935.0	3	119.3
Boomerang Coast to Coaster	1998	116.5	47.0	935.0	3	119.3
Boss	2000	122.0	66.3	4631.0	0	150.0
Boulder Dash	2000	110.0	60.0	4725.0	0	115.0
Bullet Coaster	2012	196.8	83.0	2910.1	0	221.2
Cannibal	2015	208.0	70.0	2735.0	4	200.2
Cedar Creek Mine Ride	1969	48.0	42.0	2540.0	0	58.7
Coaster Express	2002	120.0	49.7	4574.0	0	120.0
Coaster Thrill Ride	1935	55.0	50.0	2650.0	0	64.9
Coaster Through the Clouds	2016	242.8	84.5	5105.0	0	255.9
Colorado Adventure	1996	85.3	31.1	4199.5	0	91.7
Colossus	2002	98.4	45.0	2788.8	10	103.3
Comet	1989	78.7	40.4	1935.7	3	85.9
Corkscrew	1976	85.0	48.0	2050.0	3	91.4
Corkscrew	2005	85.0	50.0	1950.0	3	91.4
Crazy Bird	2013	98.4	51.8	1279.5	2	98.0
Crazy Coaster	2013	108.3	52.8	2870.8	10	112.0
Cyclone	1940	80.0	55.0	2800.0	0	80.0
Demon	1976	102.0	50.0	2130.0	4	90.0
Demon	1976	103.7	50.0	2130.0	4	90.0
Desafio	1999	109.3	49.7	2260.5	5	112.9
Desert Race	2007	62.3	62.1	2132.6	0	71.4
Desperado	2005	209.0	80.0	5843.0	0	225.0
Diamondback	2009	230.0	80.0	5282.0	0	215.0
Dinoconda	2012	226.4	78.3	3444.9	3	216.4
Doble Loop	2004	59.1	56.0	2822.0	2	68.5
Do-Dodonpa	2001	160.8	111.8	4081.3	1	158.4
Dragon Khan	1995	148.0	65.0	4165.0	8	161.0
Dragon Mountain	1993	186.0	50.0	5500.0	4	180.7
Dragon's Run	2017	150.0	65.0	3738.0	6	148.9
Eejanaika	2006	249.3	78.3	3782.8	3	236.7
El Toro	2009	80.5	45.6	2379.0	0	87.4
El Toro	2006	181.0	70.0	4400.0	0	176.0
Eurosat Can Can	1989	83.7	37.3	3024.8	0	90.3

Coaster						
Expedition GeForce	2001	173.9	74.6	4002.6	0	170.0
Extreme Rusher	2011	170.6	83.0	2788.8	0	196.8
Fahrenheit	2008	121.0	58.0	2700.0	6	123.2
Firehawk	2007	115.0	50.0	3340.0	5	117.9
Firewhip	2008	109.3	49.7	2260.5	5	112.9
Flash	2016	200.2	71.5	4176.5	2	190.3
Flashback	2000	116.5	47.0	935.0	3	119.3
Flight Deck	1995	109.3	49.7	2260.5	5	112.9
Flight Deck	1993	102.0	50.0	2260.0	3	91.0
Flight of Fear	1996	74.2	54.0	2705.0	4	81.9
Flight of Fear	1996	74.2	54.0	2705.0	4	81.9
Flight of the Phoenix	2006	101.7	49.7	2723.1	8	106.2
Fluch von Novgorod	2009	131.3	62.1	2296.6	1	132.4
Fly the Great Nor'Easter	1995	115.0	55.0	2170.0	5	117.9
Flying Aces	2016	206.7	74.6	4921.3	1	199.0
Force One	2010	72.2	40.4	1738.8	0	80.1
Formula Rossa	2010	170.6	149.1	6561.7	0	167.1
Fujiyama	1996	259.2	80.8	6708.7	0	229.7
Full Throttle	2013	160.0	70.0	2200.0	2	157.7
Furius Baco	2007	46.0	83.9	2788.8	1	56.9
Fury 325	2015	325.0	95.0	6602.0	0	320.0
Gao	1998	131.3	60.9	5457.7	0	132.4
GateKeeper	2013	170.0	67.0	4164.0	6	164.0
Gemini	1978	125.3	60.0	3935.0	0	118.0
GhostRider	1998	118.0	56.0	4533.0	0	108.0
Giant Dipper	1925	73.0	55.0	2600.0	0	80.8
Giant Dipper	1924	70.0	55.0	2640.0	0	65.0
Goliath	2006	200.0	70.0	4480.0	0	175.0
Goliath	2012	191.6	65.6	1204.0	3	177.0
Goliath	2008	105.0	50.0	2693.0	5	80.0
Goliath	2014	165.0	72.0	3100.0	2	180.0
Goliath	2000	235.0	85.0	4500.0	0	255.0
Goudurix	1989	118.0	56.0	3116.8	7	108.0
Great White	1997	108.0	50.0	2562.0	5	81.2
Green Lantern Coaster	2011	108.3	41.0	1601.1	2	112.0
Griffon	2007	205.0	71.0	3108.0	2	197.5
Grizzly	1986	91.0	55.0	3250.0	0	96.7
Hades 360	2005	136.0	60.0	4746.0	1	140.0
Half Pipe	2004	98.4	43.5	229.7	0	103.3
Happy Angel	2017	110.3	54.1	2915.5	6	114.0
HeiBe Fahrt	2004	57.4	34.2	1745.4	0	67.0
Helix	2014	134.5	62.1	4530.8	7	135.2
Hydra the Revenge	2005	95.0	53.0	3198.0	7	105.0
Hyper Coaster	2018	200.2	71.5	4176.5	2	190.3

Hyperion	2018	252.6	88.2	4757.2	1	269.0
Incredible Hulk	1999	110.0	67.0	3700.0	7	104.9
Incredicoaster	2001	120.0	55.0	6072.0	1	108.0
Intimidator	2010	232.0	75.0	5316.0	0	211.0
Intimidator 305	2010	305.0	90.0	5100.0	0	300.0
Invertigo	1999	131.3	50.0	1013.8	3	132.4
Iron Dragon	1987	76.0	40.0	2800.0	0	83.5
Iron Rattler	2013	179.0	70.0	3266.0	1	171.0
iSpeed	2009	180.4	68.4	3215.2	2	175.8
Joker	2016	100.0	53.0	3200.0	3	104.7
Joker	2012	120.0	38.0	1019.0	5	54.0
Journey to Atlantis	2004	95.0	42.0	780.0	0	70.0
Journey to Atlantis	2007	100.0	42.0	780.0	0	104.7
Judge Roy Scream	1980	71.0	45.0	2670.0	0	65.0
Jungle Trailblazer	2016	111.6	54.1	3989.5	0	91.8
Jupiter	1992	138.0	57.0	5249.3	0	138.3
Katapul	1999	137.8	53.0	722.0	1	138.1
Katun	2000	164.1	64.6	3937.0	6	147.7
Kawazemi	2008	101.7	52.8	2477.0	0	98.4
Kingda Ka	2005	456.0	128.0	3118.0	0	418.0
Kong	1998	109.3	49.7	2260.5	5	112.9
Kraken	2000	149.0	65.0	4177.0	7	144.0
Kumba	1993	143.0	60.0	3978.0	7	135.0
Legend	2000	99.0	59.0	4042.0	0	103.8
Leviathan	2012	306.0	92.0	5486.0	0	306.0
Lightning Rod	2016	207.0	73.0	3800.0	0	165.0
Limit	1999	109.3	49.7	2260.5	5	112.9
Magnum XL-200	1989	205.0	72.0	5106.0	0	194.7
Mako	2016	200.0	73.0	4760.0	0	200.0
Mamba	1998	205.0	75.0	5600.0	0	205.0
Mammut	2008	98.4	55.9	2821.5	0	103.3
Manta	2009	140.0	56.0	3359.0	4	113.0
Manta	2012	30.0	43.0	2800.0	0	54.0
Maverick	2007	105.0	70.0	4450.0	2	100.0
Medusa	2000	150.0	65.0	3937.0	7	150.0
Medusa Steel Coaster	2009	98.0	58.0	3000.0	3	102.9
Millennium Force	2000	310.0	93.0	6595.0	0	300.0
Mind Eraser	1997	109.3	49.7	2260.5	5	112.9
Mind Eraser	1995	109.3	49.7	2260.5	5	112.9
Mine Blower	2017	83.0	45.8	2290.0	1	80.5
Monster	2010	131.3	55.9	3937.0	6	132.4
Montana Rusa	1964	110.0	52.8	4000.0	0	113.5
Montana Rusa	2000	70.0	46.3	1250.0	2	78.2
Montana Rusa	2005	36.1	39.8	1197.5	1	48.2
Montezooma's Revenge	1978	148.0	55.0	800.0	1	147.1
Montezum	1999	139.1	64.0	3380.0	0	139.3

Montu	1996	150.0	60.0	3983.0	7	128.0
MP-Xpress	2001	109.3	49.7	2260.5	5	112.9
Mr. Freeze Reverse Blast	1998	218.0	70.0	1300.0	1	209.0
Mr. Freeze Reverse Blast	1998	218.0	70.0	1300.0	1	209.0
Mystic Timbers	2017	109.2	53.0	3265.0	0	98.0
Nemesis	1994	42.6	50.0	2349.0	4	104.0
Nemisis Inferno	2003	95.2	47.8	2460.7	4	100.4
Nessie Superrollercoaster	1980	85.3	49.1	2431.1	1	91.7
New Revolution	1976	113.0	55.0	3457.0	1	116.2
New Texas Giant	2011	153.0	68.6	4200.0	0	147.0
Ninja	1988	60.0	55.0	2800.0	0	69.3
Nitro	2001	230.0	80.0	5394.0	0	215.0
Oblivion	1998	65.0	68.0	1222.0	0	180.0
OCT Thrust SSC1000	2014	196.8	83.0	4440.8	0	221.2
Outlaw Run	2013	107.0	68.0	2937.0	3	162.0
Pandemonium	2008	53.0	31.0	1351.0	0	27.0
Pandemonium	2007	48.0	31.0	1351.0	0	58.7
Patriot	2006	149.0	60.0	3081.0	4	123.0
Phaethon	2007	147.7	55.9	3280.8	6	146.9
Phantom's Revenge	1991	160.0	85.0	3200.0	0	228.0
Phoenix	1985	78.0	45.0	3200.0	0	72.0
Piraten	2008	101.7	52.8	2477.0	0	98.4
Poltergeist	1999	78.8	60.0	2705.0	4	85.9
Prowler	2009	102.3	51.2	3074.0	0	84.9
Pyrenees	1997	147.7	62.1	4048.6	6	146.9
Python in Bamboo Forest	2016	164.1	62.1	5111.0	0	134.5
Quimera	2007	111.0	53.0	3444.0	3	114.4
Racer	1972	88.0	53.0	3415.0	0	94.1
Raging Bull	1999	202.0	73.0	5057.0	0	208.0
RailBlazer	2018	106.0	52.0	1800.0	3	110.0
Ranier Rush	2013	63.0	45.0	1775.0	1	72.0
Raptor	2008	109.3	49.7	2260.5	5	112.9
Raptor	1994	137.0	57.0	3790.0	6	119.0
Raven	1995	80.0	48.0	2800.0	0	87.0
Ravine Flyer II	2008	80.0	57.0	2900.0	0	118.0
Red Force	2017	367.3	111.8	2887.2	0	341.0
Renegade	2007	97.5	51.3	3113.0	0	91.4
Revenge of the Mummy the Ride	2004	44.4	40.0	1906.0	0	55.5
Riddler Revenge	1997	109.3	49.7	2260.5	5	112.9
Riddler's Revenge	1998	156.0	65.0	4370.0	6	146.0
Ride of Steel	1999	208.0	73.0	5400.0	0	205.0
Road Runner Express	1997	73.0	35.0	2400.0	0	67.0

Rock 'n' Roller Coaster	2002	80.0	57.0	3403.0	3	87.0
Rougarou	1996	145.0	60.0	3900.0	4	137.0
Saw - The Ride	2009	100.0	55.0	2362.0	3	104.7
Schwur des Kärnan	2015	239.5	78.9	4051.8	1	219.8
Scream!	2003	150.0	63.0	3985.0	7	141.0
Screamer	1995	90.0	50.0	2600.0	0	80.0
Shambhala	2012	249.3	83.3	5131.3	0	255.9
Shock Wave	1978	116.0	60.0	3600.0	2	118.8
Sidewinder	1995	56.0	45.0	635.0	1	65.8
Silver Bullet	2004	146.0	55.0	3125.0	6	109.0
Silver Star	2002	239.5	78.9	5315.0	0	219.8
Sky Scream	2014	150.0	62.0	863.0	1	148.9
Sky Wheel	2004	151.6	65.3	492.2	2	150.3
Skyrush	2012	200.0	75.0	3600.0	0	193.1
Smiler	2013	72.2	52.8	3838.6	14	98.4
Snow Mountain Flying Dragon	2002	117.8	54.9	2202.8	4	111.8
Soaring Dragon & Dancing Phoenix	2016	196.8	84.0	4192.0	6	190.3
Soaring with Dragon	2016	196.8	77.7	3438.3	1	190.3
Spatiale Experience	2000	65.6	37.3	2624.7	0	74.3
SpeedSnake FREE	1982	64.0	37.3	1148.3	2	72.9
Stampida	1997	84.0	46.0	3127.0	0	90.5
Star Mountain	1993	75.0	40.0	2400.0	2	82.6
Star Wars Hyperspace Mountain: Rebel Mission	1995	105.0	44.0	3281.0	3	109.1
Stealth	2006	205.1	80.0	1312.3	0	197.6
Steel Dragon 2000	2000	318.3	95.0	8133.2	0	306.8
Steel Eel	1999	150.0	65.0	3700.0	0	150.0
Steel Force	1997	200.0	75.0	5600.0	0	205.0
Steel Vengeance	2018	205.0	74.0	5740.0	4	200.0
Steel Venom	2003	185.0	68.0	630.0	0	175.0
Storm Chaser	2016	100.0	52.0	2744.0	3	104.7
Storm Runner	2004	150.0	75.0	2600.0	3	180.0
Stunt Fall	2002	191.0	65.6	1204.0	3	177.0
Super Tornado	1981	64.0	37.3	1148.3	2	72.9
Superman - Ride Of Steel	2000	197.0	73.0	5350.0	0	205.0
Superman - Ultimate Flight	2003	106.0	51.0	2798.0	2	100.0
Superman / la Atracción de Acero	2002	164.1	62.1	3608.9	7	161.4
Superman el Último Escape	2004	219.8	74.6	5577.4	0	205.0
Superman Krypton Coaster	2000	168.0	70.0	4015.0	6	164.8

Superman the Ride	2000	208.0	77.0	5400.0	0	221.0
Superman: Escape from Krypton	1997	415.0	100.0	1235.0	0	328.1
Swarm	2012	127.0	57.0	2543.0	5	128.6
T Express	2008	183.8	64.6	5383.8	0	150.9
Takabisha	2011	141.0	62.1	3280.8	7	140.9
Talon	2001	135.0	58.0	3110.0	4	120.0
Taron	2016	98.4	72.7	4330.8	0	103.3
Tatsu	2006	170.0	62.0	3602.0	4	111.0
Taunusblitz	1999	45.9	28.0	1213.9	0	56.9
Temple of the Night Hawk	1988	38.4	28.9	3851.7	0	50.2
Terminator Salvation: The Coaster	2009	29.0	81.0	876.9	0	41.9
Texas Tornado	1985	80.0	65.0	2050.0	2	80.0
Thunder Dolphin	2003	262.5	80.8	3500.0	0	218.2
Timber Drop	2011	96.0	41.0	2224.0	2	101.1
Timber Wolf	1989	100.0	53.0	4230.0	0	95.0
Timberhawk: Ride of Prey	2003	75.0	50.0	2600.0	0	82.6
Time Traveler	2008	100.0	50.3	3020.0	3	90.0
Titan	2001	245.0	85.0	5312.0	0	255.0
Titan Cascabel	1994	75.0	55.0	2500.0	0	82.6
Tonnerre de Zeus	1997	98.0	52.0	4044.0	0	102.9
Top Thrill Dragster	2003	420.0	120.0	2800.0	0	400.0
Tornado	2001	64.0	37.3	1148.3	2	72.9
Tower of Terror II	1997	377.3	100.0	1235.0	0	328.1
Twisted Colossus	2015	121.0	57.0	4990.0	2	128.0
Twister II	1995	100.0	55.0	4640.0	0	104.7
Ultimate	1991	107.0	50.0	7442.0	0	110.9
V2: Vertical Velocity	2001	150.0	65.0	630.0	0	148.9
Valravn	2016	223.0	75.0	3415.0	3	214.0
Velikolukskiy Myasokombinat	2015	124.0	62.1	3464.0	4	125.9
Velikolukskiy Myasokombinat-2	2017	108.0	52.8	2870.0	10	111.8
Vertical Velocity	2001	185.0	70.0	630.0	0	179.8
Viper	1990	188.0	70.0	3830.0	7	171.0
Viper	1995	100.0	50.0	3458.0	0	80.0
Vortex	1987	148.0	55.0	3800.0	6	138.0
Voyage	2006	159.0	67.0	6442.0	0	154.0
Whirl Wind Looping Coaster	2003	64.0	37.3	1148.0	2	72.9
Whizzer	1976	70.0	42.0	3100.0	0	64.0
Wicked Cyclone	2015	109.0	55.0	3320.0	3	112.6
Wicked Twister	2002	215.0	72.0	215.0	0	206.0
Wild One	1986	98.0	53.0	4000.0	0	88.0

Wild Thing	1996	207.0	74.0	5460.0	0	196.0
Wild Thing	1997	75.0	40.0	1565.0	3	64.0
Wildfire	2016	183.8	71.5	4150.3	3	160.8
Winjas	2002	57.1	41.0	1526.6	0	52.5
Wodan Timbur Coaster	2012	131.3	62.1	3444.9	0	132.4
X2	2002	175.0	76.0	3610.0	2	215.0
Xcelerator	2002	145.3	63.2	3278.6	0	144.7

(C) Matlab ® Program

```
function D= model(A,P,F)
%D is the array for scoring, A is an array contains original data and P, F are arrays
for anchoring original data and scores.

[m,n] = size(A);
for j=1:n
    chuli=A(:,j);
    Ppoint=P(:,j);
    fenpoint=F(:,j);
    for i=1:m
        vbl=Ppoint(1,1);fen1=fenpoint(1,1);
        r=Ppoint(2,1);fen2=fenpoint(2,1);
        ave=Ppoint(3,1);fen3=fenpoint(3,1);
        a=Ppoint(4,1);fen4=fenpoint(4,1);
        vbu=Ppoint(5,1);fen5=fenpoint(5,1);
        if vbl<=chuli(i,1)&&chuli(i,1)<r
            D(i,j)=fen1+((chuli(i,1)-vbl)*(fen2-fen1)/(r-vbl));
        else if r<=chuli(i,1)&&chuli(i,1)<ave
                D(i,j)=fen2+((chuli(i,1)-r)*(fen3-fen2)/(ave-r));
            else if ave<=chuli(i,1)&&chuli(i,1)<a
                    D(i,j)=fen3+((chuli(i,1)-ave)*(fen4-fen3)/(a-ave));
                else
                    D(i,j)=fen4+((chuli(i,1)-a)*(fen5-fen4)/(vbu-a));
                end
            end
        end
    end
end
```

(D) Data of Model 1

Name	*Year/Date Opened*	*Height (feet)*	*Speed (mph)*	*Length (feet)*	*Number of Inversions*	*Drop (feet)*	*score*
10 Inversion Roller Coaster	6.250	3.614	3.129	4.518	8.000	3.842	4.892
Abismo	6.250	5.201	5.257	2.587	5.000	5.230	4.921
Adrenaline Peak	10.000	2.434	3.129	1.712	5.333	2.939	4.258
Afterburn	4.897	4.160	5.113	4.746	6.333	4.328	4.930
Alpengeist	4.795	5.742	5.338	5.550	6.333	5.507	5.544
Alpina Blitz	8.667	3.984	3.982	3.930	0.000	4.170	4.122
Altair	8.667	3.984	4.137	4.642	8.000	4.170	5.600
American Eagle	3.974	4.684	5.293	6.209	0.000	5.183	4.224
Anaconda	4.385	4.351	4.537	5.637	0.000	0.722	3.272
Apocalypse	8.000	3.674	4.421	4.670	5.000	3.340	4.851
Apocalypse the Ride	7.000	3.487	3.788	4.638	0.000	3.238	3.692
Apollo's Chariot	4.897	5.431	5.607	6.395	0.000	6.070	4.733
Atlantica SuperSplash	6.000	3.614	3.736	2.183	0.000	3.842	3.229
Backlot Stunt Coaster	6.000	0.919	2.250	3.385	0.000	0.233	2.131
Balder	5.500	4.351	4.537	5.296	0.000	4.498	4.030
Bandit	4.897	3.344	3.736	5.372	0.000	3.026	3.396
Banshee	8.667	5.393	5.383	5.787	6.667	5.225	6.187
Bat	4.590	2.774	3.904	3.921	0.000	3.158	3.058
Batman - The Dark Knight	5.250	4.340	4.421	4.260	6.000	4.487	4.793
Batman the Ride	4.538	3.674	3.775	4.396	6.000	3.894	4.380
Batman The Ride-92	4.590	3.861	3.775	4.387	6.000	4.060	4.446
Batman The Ride-93	4.641	3.861	3.775	4.396	6.000	4.060	4.456
Batman The Ride-94	4.692	3.861	3.775	4.387	6.000	4.060	4.463
Batman The Ride-95	4.897	3.861	3.775	4.396	6.000	4.060	4.498
Batman The Ride-99	4.949	4.022	3.736	3.796	6.000	4.204	4.451
Batman: Arkham Asylum	5.250	3.861	3.736	4.397	6.000	4.060	4.551
Batwing	5.000	4.235	3.775	5.159	6.000	4.392	4.760
Beast	3.872	4.048	5.239	9.070	0.000	5.099	4.555
Behemoth	6.750	6.178	5.787	6.744	0.000	6.206	5.277
Big Apple Coaster	4.795	5.842	5.338	6.311	5.000	5.141	5.404
Big Loop	4.077	3.614	2.100	3.872	5.667	3.842	3.862
Big One	4.641	5.966	5.652	6.888	0.000	6.000	4.858
Big Thunder Mountain	4.538	2.446	2.325	6.426	0.000	0.683	2.736
Big Thunder Mountain Railroad	3.872	3.824	0.000	4.357	0.000	4.026	2.680
Bizarro	4.897	5.082	5.068	5.676	6.667	5.110	5.417
Black Mamba	6.250	3.124	3.736	4.150	5.667	3.287	4.369
blue fire Megacoaster	7.000	4.598	5.118	5.259	5.667	4.717	5.393
Blue Hawk	4.538	4.497	4.034	4.454	6.000	4.626	4.692
Blue Streak	3.103	2.774	2.250	4.202	0.000	2.500	2.471
Boardwalk Bullet	6.500	3.524	3.904	5.076	0.000	3.415	3.737

Bocaraca	6.000	1.982	1.744	1.914	5.000	2.550	3.198
Boomerang	4.846	4.291	3.388	1.476	5.333	4.445	3.972
Boomerang	4.333	4.291	3.388	1.476	5.333	4.445	3.946
Boomerang	4.333	4.291	3.388	1.476	5.333	4.445	3.878
Boomerang	8.333	4.291	3.388	1.476	5.333	4.445	3.980
Boomerang	4.897	4.291	3.388	1.476	5.333	4.445	3.972
Boomerang	4.949	4.291	3.388	1.476	5.333	4.445	3.980
Boomerang	4.897	4.291	3.388	1.476	5.333	4.445	3.963
Boomerang	4.949	4.291	3.388	1.476	5.333	4.445	4.545
Boomerang	4.744	4.291	3.388	1.476	5.333	4.445	3.878
Boomerang Coast to Coaster	4.846	4.291	3.388	1.476	5.333	4.445	3.963
Boss	4.949	4.497	5.306	6.194	0.000	5.225	4.362
Boulder Dash	4.949	4.048	5.023	6.269	0.000	4.283	4.095
Bullet Coaster	8.000	5.764	6.057	4.684	0.000	6.228	5.122
Cannibal	9.000	5.904	5.473	4.444	5.667	5.932	6.070
Cedar Creek Mine Ride	3.359	1.077	2.625	4.178	0.000	1.761	2.167
Coaster Express	5.250	4.422	3.736	6.148	0.000	4.472	4.005
Coaster Thrill Ride	0.868	1.473	3.775	4.328	0.000	2.106	2.092
Coaster Through the Clouds	9.333	6.337	6.124	6.574	0.000	6.717	5.848
Colorado Adventure	4.744	3.124	0.581	5.848	0.000	3.404	2.950
Colossus	5.250	3.614	3.129	4.518	8.000	3.842	4.725
Comet	4.385	2.813	2.325	3.352	5.333	3.185	3.566
Corkscrew	6.000	3.112	3.775	3.372	5.333	3.392	4.164
Corkscrew	3.718	3.112	3.517	3.508	5.333	3.392	3.764
Crazy Bird	8.333	3.614	4.008	2.183	5.000	3.642	4.463
Crazy Coaster	8.333	3.984	4.137	4.630	8.000	4.170	5.542
Cyclone	1.263	2.887	4.421	4.533	0.000	2.944	2.675
Demon	3.718	3.812	3.775	3.618	5.667	3.340	3.988
Demon	3.718	3.749	3.775	3.618	5.667	3.340	3.978
Desafio	4.897	4.022	3.736	3.796	6.000	4.204	4.443
Desert Race	6.500	1.886	5.118	3.621	0.000	2.467	3.265
Desperado	6.000	5.916	5.922	7.248	0.000	6.282	5.228
Diamondback	7.000	6.178	5.922	6.715	0.000	6.141	5.326
Dinoconda	8.000	6.133	5.845	5.243	5.333	6.161	6.119
Doble Loop	5.000	5.316	7.519	5.753	3.000	5.344	5.322
Do-Dodonpa	5.750	1.705	4.550	4.563	5.000	2.306	3.979
Dragon Khan	4.692	5.157	5.248	5.820	7.000	5.380	5.550
Dragon Mountain	4.590	5.630	3.775	6.890	5.667	5.658	5.368
Dragon's Run	9.667	5.181	5.248	5.478	6.333	5.210	6.186
Eejanaika	6.250	6.418	5.845	5.514	5.333	6.446	5.968
El Toro	7.000	2.915	3.207	3.958	0.000	3.242	3.387
El Toro	6.250	5.568	5.473	6.009	0.000	5.592	4.815
Eurosat Can Can Coaster	4.385	3.064	1.744	4.840	0.000	3.351	2.897
Expedition GeForce	5.000	5.479	5.679	5.690	0.000	5.507	4.559

Extreme Rusher	7.667	5.438	6.057	4.518	0.000	5.885	4.927
Fahrenheit	6.750	4.460	4.809	4.396	6.333	4.592	5.223
Firehawk	6.500	4.235	3.775	5.159	6.000	4.392	5.010
Firewhip	6.750	4.022	3.736	3.796	6.000	4.204	4.751
Flash	9.333	5.807	5.540	5.829	5.000	5.793	6.217
Flashback	4.949	4.291	3.388	1.476	5.333	4.445	3.980
Flight Deck	4.590	3.749	3.775	3.795	5.333	3.377	4.408
Flight Deck	4.692	4.022	3.736	3.796	6.000	4.204	4.103
Flight of Fear	4.744	2.559	4.292	4.403	5.667	3.034	4.116
Flight of Fear	4.744	2.559	4.292	4.403	5.667	3.034	4.116
Flight of the Phoenix	6.250	3.737	3.736	4.428	7.000	3.951	4.850
Fluch von Novgorod	7.000	4.845	5.118	3.845	3.000	4.940	4.791
Fly the Great Nor'Easter	4.692	4.235	4.421	3.672	6.000	4.392	4.569
Flying Aces	9.333	5.888	5.679	6.426	3.000	5.915	6.040
Force One	7.333	2.446	2.325	3.083	0.000	2.950	3.023
Formula Rossa	7.333	5.438	10.000	8.111	0.000	5.466	6.058
Fujiyama	4.744	6.542	5.958	8.288	0.000	6.348	5.313
Full Throttle	8.333	5.306	5.473	3.713	5.000	5.334	5.527
Furius Baco	6.500	0.964	6.097	4.518	3.000	1.661	3.790
Fury 325	9.000	7.544	6.596	8.160	0.000	7.930	6.538
Gao	4.846	4.845	5.064	6.856	0.000	4.940	4.425
GateKeeper	8.333	5.431	5.338	5.819	6.333	5.423	6.113
Gemini	3.821	4.621	5.023	5.636	0.000	4.396	3.916
GhostRider	4.846	4.348	4.550	6.115	0.000	4.019	3.980
Giant Dipper	0.000	2.321	4.421	4.315	0.000	2.111	2.195
Giant Dipper	0.079	2.491	4.421	4.260	0.000	2.989	2.373
Goliath	6.750	3.861	3.775	4.387	6.000	2.944	4.863
Goliath	6.250	5.804	5.473	6.073	0.000	5.577	5.865
Goliath	8.000	5.700	5.275	2.028	5.333	5.606	5.021
Goliath	4.949	6.240	6.146	6.089	0.000	6.704	5.324
Goliath	8.667	5.368	5.562	4.943	5.000	5.648	4.620
Goudurix	4.385	4.348	4.550	4.966	6.667	4.019	4.822
Great White	4.795	3.973	3.775	4.208	6.000	3.008	4.293
Green Lantern Coaster	7.667	3.984	2.438	2.842	5.000	4.170	4.350
Griffon	6.500	5.867	5.518	4.954	5.000	5.894	5.622
Grizzly	4.231	3.337	4.421	5.087	0.000	3.592	3.445
Hades 360	6.000	5.007	5.023	6.286	3.000	5.085	5.067
Half Pipe	5.750	3.614	2.906	0.030	0.000	3.842	2.690
Happy Angel	9.667	4.059	4.305	4.691	6.333	4.245	5.550
HeiBe Fahrt	5.750	1.609	1.163	3.092	0.000	2.222	2.306
Helix	8.667	4.965	5.118	6.113	6.667	5.017	6.091
Hydra the Revenge	6.000	3.487	4.163	5.045	6.667	3.906	4.878
Hyper Coaster	10.000	5.807	5.540	5.829	5.000	5.793	6.328
Hyperion	10.000	6.459	6.290	6.295	3.000	6.901	6.491
Incredible Hulk	4.897	4.048	5.338	5.447	6.667	3.902	5.050
Incredicoaster	5.000	4.422	4.421	7.523	3.000	4.019	4.731

Intimidator	7.333	6.203	5.697	6.743	0.000	6.085	5.343
Intimidator 305	7.333	7.169	6.371	6.570	0.000	7.507	5.825
Invertigo	4.897	4.845	3.775	1.638	5.333	4.940	4.238
Iron Dragon	4.282	2.661	2.250	4.533	0.000	3.094	2.803
Iron Rattler	8.333	5.543	5.473	5.100	3.000	5.521	5.495
iSpeed	7.000	5.560	5.401	5.059	5.000	5.589	5.601
Joker	8.000	4.422	1.875	1.649	6.000	1.500	3.908
Joker	9.333	3.674	4.163	5.047	5.333	3.894	5.241
Journey to Atlantis	6.500	3.674	2.625	1.159	0.000	3.894	2.975
Journey to Atlantis	5.750	3.487	2.625	1.159	0.000	2.389	2.568
Judge Roy Scream	3.923	2.378	3.129	4.355	0.000	2.111	2.649
Jungle Trailblazer	9.333	4.108	4.305	5.680	0.000	3.408	4.472
Jupiter	4.538	5.032	4.680	6.689	0.000	5.061	4.333
Katapul	4.897	5.030	4.163	1.040	3.000	5.058	3.865
Katun	4.949	5.357	5.230	5.637	6.333	5.193	5.450
Kawazemi	6.750	3.737	4.137	4.092	0.000	3.657	3.729
Kingda Ka	6.000	10.000	8.596	4.968	0.000	10.000	6.594
Kong	4.846	4.022	3.736	3.796	6.000	4.204	4.434
Kraken	4.949	5.169	5.248	5.830	6.667	5.141	5.501
Kumba	4.590	5.094	5.023	5.670	6.667	5.014	5.343
Legend	4.949	3.636	4.938	5.722	0.000	3.860	3.851
Leviathan	8.000	7.188	6.461	6.879	0.000	7.634	6.027
Lightning Rod	9.333	5.891	5.607	5.528	0.000	5.437	5.299
Limit	4.897	4.022	3.736	3.796	6.000	4.204	4.443
Magnum XL-200	4.385	5.867	5.562	6.574	0.000	5.855	4.707
Mako	9.333	5.804	5.607	6.297	0.000	5.930	5.495
Mamba	4.846	5.867	5.697	6.970	0.000	6.000	4.897
Mammut	6.750	3.614	4.537	4.563	0.000	3.842	3.884
Manta	7.000	5.057	4.550	5.174	5.667	4.208	5.276
Manta	8.000	0.059	2.813	4.533	0.000	1.500	2.817
Maverick	6.500	3.861	5.473	6.049	5.000	3.717	5.100
Medusa	4.949	5.181	5.248	5.637	6.667	5.225	5.485
Medusa Steel Coaster	7.000	3.599	4.809	4.806	5.333	3.826	4.896
Millennium Force	4.949	7.263	6.506	8.151	0.000	7.507	5.729
Mind Eraser	4.692	4.022	3.736	3.796	6.000	4.204	4.408
Mind Eraser	4.795	4.022	3.736	3.796	6.000	4.204	4.425
Mine Blower	9.667	3.037	3.233	3.836	3.000	2.972	4.291
Monster	7.333	4.845	4.537	5.637	6.333	4.940	5.604
Montana Rusa	3.103	4.048	4.137	5.688	0.000	4.226	3.534
Montana Rusa	4.949	2.321	3.297	2.122	5.000	2.844	3.422
Montana Rusa	6.000	0.404	2.213	2.015	3.000	1.178	2.468
Montezooma's Revenge	3.821	5.157	4.421	1.200	3.000	5.185	3.797
Montezum	4.897	5.046	5.203	5.191	0.000	5.075	4.235
Montu	4.744	5.181	5.023	5.674	6.667	4.774	5.344
MP-Xpress	5.000	4.022	3.736	3.796	6.000	4.204	4.460
Mr. Freeze Reverse Blast	4.846	6.028	5.473	2.225	3.000	6.056	4.605

Mr. Freeze Reverse Blast	4.846	6.028	5.473	2.225	3.000	6.056	4.605
Mystic Timbers	9.667	4.018	4.163	5.099	0.000	3.642	4.431
Nemesis	4.641	0.772	3.775	3.917	5.667	3.868	3.773
Nemisis Inferno	5.500	3.494	3.491	4.070	5.667	3.732	4.326
Nessie Superrollercoaster	3.923	3.124	3.659	4.029	3.000	3.404	3.523
New Revolution	3.718	4.160	4.421	5.253	3.000	4.328	4.147
New Texas Giant	7.667	5.219	5.410	5.848	0.000	5.183	4.888
Ninja	4.333	1.756	4.421	4.533	0.000	2.350	2.899
Nitro	5.000	6.178	5.922	6.805	0.000	6.141	5.008
Oblivion	4.846	2.039	5.383	2.065	0.000	5.648	3.330
OCT Thrust SSC1000	8.667	5.764	6.057	6.041	0.000	6.228	5.460
Outlaw Run	8.333	3.936	5.383	4.720	5.333	5.394	5.517
Pandemonium	6.750	1.360	0.563	2.329	0.000	0.000	1.834
Pandemonium	6.500	1.077	0.563	2.329	0.000	1.761	2.038
Patriot	6.250	5.169	5.023	4.917	5.667	4.585	5.269
Phaethon	6.500	5.153	4.537	5.112	6.333	5.182	5.469
Phantom's Revenge	4.487	5.306	6.146	5.047	0.000	6.324	4.552
Phoenix	4.179	2.774	3.129	5.047	0.000	2.500	2.938
Piraten	6.750	3.737	4.137	4.092	0.000	3.657	3.729
Poltergeist	4.897	2.819	5.023	4.403	5.667	3.185	4.332
Prowler	7.000	3.760	3.930	4.908	0.000	3.147	3.791
Pyrenees	4.795	5.153	5.118	5.727	6.333	5.182	5.385
Python in Bamboo Forest	9.333	5.357	5.118	6.578	0.000	5.007	5.232
Quimera	6.500	4.086	4.163	5.242	5.333	4.260	4.931
Racer	3.513	3.225	4.163	5.219	0.000	3.494	3.269
Raging Bull	4.897	5.829	5.607	6.535	0.000	6.042	4.819
RailBlazer	10.000	3.898	4.034	3.167	5.333	4.094	5.088
Ranier Rush	8.333	1.925	3.129	3.133	3.000	2.500	3.670
Raptor	6.750	4.022	3.736	3.796	6.000	4.204	4.751
Raptor	4.641	5.020	4.680	5.520	6.333	4.434	5.105
Raven	4.692	2.887	3.517	4.533	0.000	3.226	3.143
Ravine Flyer II	6.750	2.887	4.680	4.670	0.000	4.396	3.897
Red Force	9.667	8.337	7.519	4.652	0.000	8.373	6.425
Renegade	6.500	3.580	3.943	4.961	0.000	3.392	3.729
Revenge of the Mummy the Ride	5.750	0.873	2.250	3.312	0.000	1.583	2.295
Riddler Revenge	4.795	4.022	3.736	3.796	6.000	4.204	4.425
Riddler's Revenge	4.846	5.256	5.248	5.985	6.333	5.169	5.473
Ride of Steel	4.897	5.904	5.607	6.810	0.000	6.000	4.870
Road Runner Express	4.795	2.491	1.313	3.987	0.000	2.222	2.468
Rock 'n' Roller Coaster	5.250	2.887	4.680	5.209	5.333	3.226	4.431
Rougarou	4.744	5.119	5.023	5.608	5.667	5.042	5.200
Saw - The Ride	7.000	3.674	4.421	3.935	5.333	3.894	4.710
Schwur des Kärnan	9.000	6.296	5.872	5.729	3.000	6.208	6.018
Scream!	5.500	5.181	5.158	5.676	6.667	5.099	5.547
Screamer	4.692	3.299	3.775	4.260	0.000	2.944	3.162

Shambhala	8.000	6.418	6.070	6.595	0.000	6.717	5.633
Shock Wave	3.821	4.273	5.023	5.367	5.000	4.426	4.652
Sidewinder	4.692	1.530	3.129	0.861	3.000	2.156	2.561
Silver Bullet	5.750	5.132	4.421	4.977	6.333	4.057	5.112
Silver Star	5.250	6.296	5.872	6.742	0.000	6.208	5.062
Sky Scream	8.667	5.181	5.113	1.329	3.000	5.210	4.750
Sky Wheel	5.750	5.201	5.261	0.568	5.000	5.230	4.502
Skyrush	8.000	5.804	5.697	5.367	0.000	5.832	5.117
Smiler	8.333	2.446	4.137	5.559	10.000	3.657	5.689
Snow Mountain Flying Dragon	5.250	4.340	4.408	3.717	5.667	4.162	4.591
Soaring Dragon & Dancing Phoenix	9.333	5.764	6.102	5.842	6.333	5.793	6.528
Soaring with Dragon	9.333	5.764	5.819	5.238	3.000	5.793	5.824
Spatiale Experience	4.949	2.072	1.744	4.294	0.000	2.628	2.614
SpeedSnake FREE	4.026	1.982	1.744	1.914	5.000	2.550	2.869
Stampida	4.795	3.075	3.258	4.980	0.000	3.358	3.244
Star Mountain	4.590	2.604	2.250	3.987	5.000	3.060	3.582
Star Wars Hyperspace Mountain: Rebel Mission	4.692	3.861	3.000	5.112	5.333	4.060	4.343
Stealth	6.250	5.868	5.922	2.250	0.000	5.896	4.364
Steel Dragon 2000	4.949	7.418	6.596	10.000	0.000	7.651	6.102
Steel Eel	4.897	5.181	5.248	5.447	0.000	5.225	4.333
Steel Force	4.795	5.804	5.697	6.970	0.000	6.000	4.878
Steel Vengeance	10.000	5.867	5.652	7.124	5.667	5.930	6.706
Steel Venom	5.500	5.617	5.383	0.851	0.000	5.577	3.821
Storm Chaser	9.333	3.674	4.034	4.457	5.333	3.894	5.121
Storm Runner	5.750	5.181	5.697	4.260	5.333	5.648	5.312
Stunt Fall	5.250	5.692	5.275	2.028	5.333	5.606	4.864
Super Tornado	3.974	1.982	1.744	1.914	5.000	2.550	2.861
Superman - Ride Of Steel	4.949	5.767	5.607	6.770	0.000	6.000	4.849
Superman - Ultimate Flight	5.500	3.898	3.904	4.530	5.000	3.717	4.425
Superman / la Atracción de Acero	5.250	5.357	5.118	5.374	6.667	5.386	5.525
Superman el Último Escape	5.750	6.051	5.679	6.952	0.000	6.000	5.072
Superman Krypton Coaster	4.949	5.406	5.473	5.700	6.333	5.434	5.549
Superman the Ride	4.949	5.904	5.787	6.810	0.000	6.225	4.946
Superman: Escape from Krypton	4.795	9.231	6.820	2.092	0.000	8.101	5.173
Swarm	8.000	4.684	4.680	4.182	6.000	4.796	5.390
T Express	6.750	5.602	5.230	6.797	0.000	5.238	4.936
Takabisha	7.667	5.069	5.118	5.112	6.667	5.097	5.788
Talon	5.000	4.984	4.809	4.957	5.667	4.472	4.981
Taron	9.333	3.614	5.594	5.953	0.000	3.842	4.723

Tatsu	6.250	5.431	5.113	5.369	5.667	4.132	5.327
Taunusblitz	4.897	0.958	0.000	2.048	0.000	1.661	1.594
Temple of the Night Hawk	4.333	0.534	0.169	5.569	0.000	1.289	1.982
Terminator Salvation: The Coaster	7.000	0.002	5.967	1.357	0.000	0.828	2.526
Texas Tornado	4.179	2.887	5.248	3.508	5.000	2.944	3.961
Thunder Dolphin	5.500	6.583	5.958	5.287	0.000	6.186	4.919
Timber Drop	7.667	3.524	2.438	3.746	5.000	3.758	4.355
Timber Wolf	4.385	3.674	4.163	5.872	0.000	3.528	3.604
Timberhawk: Ride of Prey	5.500	2.604	3.775	4.260	0.000	3.060	3.200
Time Traveler	6.750	3.674	3.814	4.834	5.333	3.340	4.624
Titan	5.000	6.365	6.146	6.740	0.000	6.704	5.159
Titan Cascabel	4.641	2.604	4.421	4.123	0.000	3.060	3.142
Tonnerre de Zeus	4.795	3.599	4.034	5.723	0.000	3.826	3.663
Top Thrill Dragster	5.500	9.325	8.064	4.533	0.000	9.620	6.174
Tornado	5.000	1.982	1.744	1.914	5.000	2.550	3.032
Tower of Terror II	4.795	8.524	6.820	2.092	0.000	8.101	5.055
Twisted Colossus	9.000	4.460	4.680	6.481	5.000	4.774	5.732
Twister II	4.692	3.674	4.421	6.201	0.000	3.894	3.814
Ultimate	4.487	3.936	3.775	9.169	0.000	4.128	4.249
V2: Vertical Velocity	5.000	5.181	5.248	0.851	0.000	5.210	3.582
Valravn	9.333	6.091	5.697	5.219	5.333	6.127	6.300
Velikolukskiy Myasokombinat	9.000	4.572	5.118	5.258	5.667	4.694	5.718
Velikolukskiy Myasokombinat-2	9.667	3.973	4.137	4.629	8.000	4.162	5.761
Vertical Velocity	5.000	5.617	5.473	0.851	0.000	5.645	3.764
Viper	4.436	5.655	5.473	5.552	6.667	5.521	3.390
Viper	4.692	3.674	3.775	5.254	0.000	2.944	5.550
Vortex	4.282	5.157	4.421	5.528	6.333	5.056	5.130
Voyage	6.250	5.294	5.338	7.967	0.000	5.282	5.022
Whirl Wind Looping Coaster	5.500	1.982	1.744	1.913	5.000	2.550	3.115
Whizzer	3.718	2.321	2.625	4.943	0.000	2.056	2.610
Wicked Cyclone	9.000	4.011	4.421	5.143	5.333	4.192	5.350
Wicked Twister	5.250	5.991	5.562	0.000	0.000	6.014	3.803
Wild One	4.231	3.599	4.163	5.688	0.000	3.264	3.491
Wild Thing	4.744	5.891	5.652	6.858	0.000	5.873	4.836
Wild Thing	4.795	2.604	2.250	2.768	5.333	2.056	3.301
Wildfire	9.333	5.602	5.540	5.808	5.333	5.377	6.166
Winjas	5.250	1.592	2.438	2.690	0.000	1.417	2.231
Wodan Timbur Coaster	8.000	4.845	5.118	5.243	0.000	4.940	4.691
X2	5.250	5.493	5.742	5.375	5.000	6.141	5.500
Xcelerator	5.250	5.123	5.167	5.110	0.000	5.151	4.300

(E) Data of Model 2, optimized

Name	*Year/Date Opened*	*Heigh t (feet)*	*Speed (mph)*	*Length (feet)*	*Number of Inversions*	*Drop (feet)*	*score*
10 Inversion Roller Coaster	8.125	5.614	5.129	6.518	2.000	5.842	4.215
Abismo	8.125	7.302	7.385	4.311	5.000	7.344	6.942
Adrenaline Peak	8.000	4.057	5.129	2.854	4.667	4.898	4.827
Afterburn	4.897	6.160	7.170	6.746	3.667	6.328	5.998
Alpengeist	4.795	8.113	7.507	7.825	3.667	7.761	6.311
Alpina Blitz	8.889	5.984	5.982	5.930	5.000	6.170	7.530
Altair	8.889	5.984	6.137	6.642	2.000	6.170	4.388
American Eagle	3.974	6.684	7.439	8.813	5.000	7.275	6.587
Anaconda	4.385	6.351	6.537	7.956	5.000	1.204	3.299
Apocalypse	9.333	5.674	6.421	6.670	5.000	5.340	7.563
Apocalypse the Ride	10.000	5.487	5.788	6.638	5.000	5.238	7.543
Apollo's Chariot	4.897	7.646	7.911	9.092	5.000	8.606	7.774
Atlantica SuperSplash	7.500	5.614	5.736	3.638	5.000	5.842	5.860
Backlot Stunt Coaster	7.500	1.531	3.750	5.385	5.000	0.389	1.959
Balder	6.250	6.351	6.537	7.443	5.000	6.498	7.539
Bandit	4.897	5.344	5.736	7.558	5.000	5.026	7.135
Banshee	8.889	7.590	7.574	8.181	3.333	7.338	6.194
Bat	4.590	4.623	5.904	5.921	5.000	5.158	6.670
Batman - The Dark Knight	5.625	6.340	6.421	6.260	4.000	6.487	6.342
Batman the Ride	4.538	5.674	5.775	6.396	4.000	5.894	6.152
Batman The Ride-92	4.590	5.861	5.775	6.387	4.000	6.060	6.178
Batman The Ride-93	4.641	5.861	5.775	6.396	4.000	6.060	6.182
Batman The Ride-94	4.692	5.861	5.775	6.387	4.000	6.060	6.185
Batman The Ride-95	4.897	5.861	5.775	6.396	4.000	6.060	6.199
Batman The Ride-99	4.949	6.022	5.736	5.796	4.000	6.204	6.180
Batman: Arkham Asylum	5.625	5.861	5.736	6.397	4.000	6.060	6.245
Batwing	5.000	6.235	5.775	7.238	4.000	6.392	6.309
Beast	3.872	6.048	7.358	7.930	5.000	7.148	6.362
Behemoth	9.375	8.767	8.181	9.616	5.000	8.808	8.316
Big Apple Coaster	4.795	8.262	7.507	8.966	5.000	7.211	7.578
Big Loop	4.077	5.614	3.500	5.872	4.333	5.842	5.449
Big One	4.641	8.449	7.978	9.832	5.000	8.500	7.601
Big Thunder Mountain	4.538	4.076	3.875	9.140	5.000	1.139	2.990
Big Thunder Mountain Railroad	3.872	5.824	0.000	6.357	5.000	6.026	1.805
Bizarro	4.897	7.123	7.102	8.014	3.333	7.165	5.842
Black Mamba	8.125	5.124	5.736	6.150	4.333	5.287	6.650
blue fire Megacoaster	10.000	6.598	7.177	7.388	4.333	6.717	7.148
Blue Hawk	4.538	6.497	6.034	6.454	4.000	6.626	6.277
Blue Streak	3.103	4.623	3.750	6.202	5.000	4.167	4.892
Boardwalk Bullet	8.750	5.524	5.904	7.113	5.000	5.415	7.514

Bocaraca	7.500	3.303	2.906	3.190	5.000	4.250	4.650
Boomerang	4.897	6.291	5.388	2.461	4.667	6.445	4.471
Boomerang	4.949	6.291	5.388	2.461	4.667	6.445	4.474
Boomerang	9.111	6.291	5.388	2.461	4.667	6.445	4.752
Boomerang	4.333	6.291	5.388	2.461	4.667	6.445	4.433
Boomerang	4.846	6.291	5.388	2.461	4.667	6.445	4.467
Boomerang	4.744	6.291	5.388	2.461	4.667	6.445	4.460
Boomerang	4.897	6.291	5.388	2.461	4.667	6.445	4.471
Boomerang	4.333	6.291	5.388	2.461	4.667	6.445	4.433
Boomerang	4.949	6.291	5.388	2.461	4.667	6.445	4.474
Boomerang Coast to Coaster	4.846	6.291	5.388	2.461	4.667	6.445	4.467
Boss	4.949	6.497	7.460	8.791	5.000	7.338	7.618
Boulder Dash	4.949	6.048	7.035	8.904	5.000	6.283	7.497
Bullet Coaster	9.333	8.147	8.585	6.684	5.000	8.842	8.106
Cannibal	8.667	8.356	7.709	6.444	4.333	8.399	7.261
Cedar Creek Mine Ride	3.359	1.795	4.375	6.178	5.000	2.935	3.371
Coaster Express	5.625	6.422	5.736	8.722	5.000	6.472	7.532
Coaster Thrill Ride	0.868	2.455	5.775	6.328	5.000	3.509	2.464
Coaster Through the Clouds	8.444	9.006	8.686	9.360	5.000	9.575	8.338
Colorado Adventure	4.744	5.124	0.969	8.272	5.000	5.404	2.936
Colossus	5.625	5.614	5.129	6.518	2.000	5.842	4.048
Comet	4.385	4.689	3.875	5.352	4.667	5.185	5.752
Corkscrew	3.718	5.112	5.517	5.508	4.667	5.392	5.712
Corkscrew	7.500	5.112	5.775	5.372	4.667	5.392	6.921
Crazy Bird	9.111	5.614	6.008	3.638	5.000	5.642	5.972
Crazy Coaster	9.111	5.984	6.137	6.630	2.000	6.170	4.402
Cyclone	1.263	4.811	6.421	6.533	5.000	4.907	3.192
Demon	3.718	5.749	5.775	5.618	4.333	5.340	5.753
Demon	3.718	5.812	5.775	5.618	4.333	5.340	5.758
Desafio	4.897	6.022	5.736	5.796	4.000	6.204	6.177
Desert Race	8.750	3.143	7.177	5.621	5.000	4.111	5.397
Desperado	7.500	8.375	8.383	9.752	5.000	8.923	8.195
Diamondback	10.000	8.767	8.383	9.573	5.000	8.711	8.362
Dinoconda	9.333	8.700	8.268	7.365	4.667	8.741	7.805
Doble Loop	5.000	7.474	9.135	8.130	10.000	7.515	8.150
Do-Dodonpa	6.875	2.841	6.550	6.563	5.000	3.843	4.953
Dragon Khan	4.692	7.235	7.372	8.230	3.000	7.570	5.540
Dragon Mountain	4.590	7.945	5.775	9.835	4.333	7.987	7.031
Dragon's Run	8.222	7.272	7.372	7.717	3.667	7.315	6.438
Eejanaika	8.125	9.127	8.268	7.771	4.667	9.170	7.809
El Toro	10.000	4.859	5.207	5.958	5.000	5.242	7.276
El Toro	8.125	7.851	7.709	8.513	5.000	7.887	8.006
Eurosat Can Can Coaster	4.385	5.064	2.906	6.840	5.000	5.351	4.876
Expedition GeForce	5.000	7.719	8.019	8.035	5.000	7.761	7.769

Extreme Rusher	9.556	7.657	8.585	6.518	5.000	8.327	8.043
Fahrenheit	9.375	6.460	6.809	6.396	3.667	6.592	6.287
Firehawk	8.750	6.235	5.775	7.238	4.000	6.392	6.559
Firewhip	9.375	6.022	5.736	5.796	4.000	6.204	6.476
Flash	8.444	8.210	7.810	8.244	5.000	8.189	8.060
Flashback	4.949	6.291	5.388	2.461	4.667	6.445	4.474
Flight Deck	4.590	5.749	5.775	5.795	4.667	5.377	6.720
Flight Deck	4.692	6.022	5.736	5.796	4.000	6.204	6.163
Flight of Fear	4.744	4.265	6.292	6.403	4.333	5.034	6.336
Flight of Fear	4.744	4.265	6.292	6.403	4.333	5.034	6.336
Flight of the Phoenix	8.125	5.737	5.736	6.428	3.000	5.951	5.332
Fluch von Novgorod	10.000	6.845	7.177	5.845	10.000	6.940	8.966
Fly the Great Nor'Easter	4.692	6.235	6.421	5.672	4.000	6.392	6.228
Flying Aces	8.444	8.332	8.019	9.140	10.000	8.373	11.506
Force One	9.778	4.076	3.875	5.083	5.000	4.917	6.057
Formula Rossa	9.778	7.657	5.000	8.889	5.000	7.699	7.935
Fujiyama	4.744	9.312	8.437	8.712	5.000	9.022	7.759
Full Throttle	9.111	7.459	7.709	5.713	5.000	7.501	7.833
Furius Baco	8.750	1.607	8.646	6.518	10.000	2.769	4.159
Fury 325	8.667	9.094	9.394	8.840	5.000	8.451	8.296
Gao	4.846	6.845	7.096	9.784	5.000	6.940	7.547
GateKeeper	9.111	7.646	7.507	8.229	3.667	7.634	6.586
Gemini	3.821	6.621	7.035	7.954	5.000	6.396	6.276
GhostRider	4.846	6.348	6.550	8.673	5.000	6.019	7.342
Giant Dipper	0.079	4.152	6.421	6.260	5.000	4.981	1.872
Giant Dipper	0.000	3.869	6.421	6.315	5.000	3.519	1.675
Goliath	9.375	5.861	5.775	6.387	4.000	4.907	6.420
Goliath	8.125	8.206	7.709	8.609	5.000	7.866	8.034
Goliath	9.333	8.049	7.412	3.380	4.667	7.908	6.097
Goliath	4.949	8.860	8.720	8.633	5.000	9.556	7.997
Goliath	8.889	7.552	7.844	6.943	5.000	7.972	7.947
Goudurix	4.385	6.348	6.550	6.966	3.333	6.019	5.573
Great White	4.795	5.973	5.775	6.208	4.000	5.008	6.117
Green Lantern Coaster	9.556	5.984	4.063	4.737	5.000	6.170	6.430
Griffon	8.750	8.300	7.776	6.954	5.000	8.342	8.008
Grizzly	4.231	5.337	6.421	7.130	5.000	5.592	6.478
Hades 360	7.500	7.011	7.035	8.929	10.000	7.127	10.184
Half Pipe	6.875	5.614	4.844	0.050	5.000	5.842	1.932
Happy Angel	8.222	6.059	6.305	6.691	3.667	6.245	6.146
HeiBe Fahrt	6.875	2.681	1.938	5.092	5.000	3.704	3.623
Helix	8.889	6.965	7.177	8.670	3.333	7.025	6.137
Hydra the Revenge	7.500	5.487	6.163	7.068	3.333	5.906	5.697
Hyper Coaster	8.000	8.210	7.810	8.244	5.000	8.189	8.030
Hyperion	8.000	9.189	8.935	8.942	10.000	9.852	11.661
Incredible Hulk	4.897	6.048	7.507	7.671	3.333	5.902	5.691
Incredicoaster	5.000	6.422	6.421	9.477	10.000	6.019	7.889

Intimidator	9.778	8.804	8.046	9.614	5.000	8.627	8.325
Intimidator 305	9.778	9.719	9.057	9.354	5.000	9.155	8.471
Invertigo	4.897	6.845	5.775	2.730	4.667	6.940	4.854
Iron Dragon	4.282	4.434	3.750	6.533	5.000	5.094	5.690
Iron Rattler	9.111	7.814	7.709	7.149	10.000	7.782	10.454
iSpeed	10.000	7.840	7.601	7.088	5.000	7.883	8.028
Joker	8.444	5.674	6.163	7.070	4.667	5.894	7.194
Joker	9.333	6.422	3.125	2.748	4.000	2.500	4.375
Journey to Atlantis	6.875	5.487	4.375	1.931	5.000	3.981	3.774
Journey to Atlantis	8.750	5.674	4.375	1.931	5.000	5.894	4.039
Judge Roy Scream	3.923	3.963	5.129	6.355	5.000	3.519	5.378
Jungle Trailblazer	8.444	6.108	6.305	8.019	5.000	5.408	7.619
Jupiter	4.538	7.048	6.680	9.534	5.000	7.091	7.198
Katapul	4.897	7.044	6.163	1.733	10.000	7.087	4.194
Katun	4.949	7.536	7.345	7.956	3.667	7.289	6.249
Kawazemi	9.375	5.737	6.137	6.092	5.000	5.657	7.533
Kingda Ka	7.500	5.000	7.339	6.968	5.000	5.000	7.454
Kong	4.846	6.022	5.736	5.796	4.000	6.204	6.174
Kraken	4.949	7.254	7.372	8.245	3.333	7.211	5.891
Kumba	4.590	7.141	7.035	8.005	3.333	7.021	5.808
Legend	4.949	5.636	6.938	8.082	5.000	5.860	7.380
Leviathan	9.333	9.688	9.191	9.818	5.000	8.944	8.465
Lightning Rod	8.444	8.337	7.911	7.791	5.000	7.655	8.009
Limit	4.897	6.022	5.736	5.796	4.000	6.204	6.177
Magnum XL-200	4.385	8.300	7.844	9.362	5.000	8.282	7.263
Mako	8.444	8.206	7.911	8.946	5.000	8.394	8.127
Mamba	4.846	8.300	8.046	9.956	5.000	8.500	7.823
Mammut	9.375	5.614	6.537	6.563	5.000	5.842	7.595
Manta	10.000	7.085	6.550	7.261	4.333	6.208	7.096
Manta	9.333	0.098	4.688	6.533	5.000	2.500	1.975
Maverick	8.750	5.861	7.709	8.573	5.000	5.717	7.774
Medusa	4.949	7.272	7.372	7.956	3.333	7.338	5.881
Medusa Steel Coaster	10.000	5.599	6.809	6.806	4.667	5.826	7.314
Millennium Force	4.949	9.563	9.259	8.849	5.000	9.155	8.067
Mind Eraser	4.795	6.022	5.736	5.796	4.000	6.204	6.170
Mind Eraser	4.692	6.022	5.736	5.796	4.000	6.204	6.163
Mine Blower	8.222	5.037	5.233	5.836	10.000	4.954	7.573
Monster	9.778	6.845	6.537	7.956	3.667	6.940	6.448
Montana Rusa	3.103	6.048	6.137	8.032	5.000	6.226	5.406
Montana Rusa	4.949	3.869	5.297	3.537	5.000	4.741	5.363
Montana Rusa	7.500	0.673	3.688	3.358	10.000	1.963	2.486
Montezooma's Revenge	3.821	7.235	6.421	1.999	10.000	7.277	4.449
Montezum	4.897	7.069	7.305	7.287	5.000	7.112	7.475
Montu	4.744	7.272	7.035	8.011	3.333	6.774	5.811
MP-Xpress	5.000	6.022	5.736	5.796	4.000	6.204	6.184
Mr. Freeze Reverse Blast	4.846	8.543	7.709	3.708	10.000	8.585	6.601

Mr. Freeze Reverse Blast	4.846	8.543	7.709	3.708	10.000	8.585	6.601
Mystic Timbers	8.222	6.018	6.163	7.148	5.000	5.642	7.546
Nemesis	4.641	1.286	5.775	5.917	4.333	5.868	3.141
Nemisis Inferno	6.250	5.494	5.491	6.070	4.333	5.732	6.558
Nessie Superrollercoaster	3.923	5.124	5.659	6.029	10.000	5.404	6.332
New Revolution	3.718	6.160	6.421	7.379	10.000	6.328	6.385
New Texas Giant	9.556	7.328	7.615	8.272	5.000	7.275	8.003
Ninja	4.333	2.926	6.421	6.533	5.000	3.917	4.868
Nitro	5.000	8.767	8.383	9.708	5.000	8.711	8.038
Oblivion	4.846	3.398	7.574	3.442	5.000	7.972	5.546
OCT Thrust SSC1000	8.889	8.147	8.585	8.562	5.000	8.842	8.202
Outlaw Run	9.111	5.936	7.574	6.720	4.667	7.592	7.440
Pandemonium	9.375	2.266	0.938	3.882	5.000	0.000	1.431
Pandemonium	8.750	1.795	0.938	3.882	5.000	2.935	2.491
Patriot	8.125	7.254	7.035	6.917	4.333	6.585	7.017
Phaethon	8.750	7.229	6.537	7.167	3.667	7.273	6.375
Phantom's Revenge	4.487	7.459	8.720	7.070	5.000	8.986	7.269
Phoenix	4.179	4.623	5.129	7.070	5.000	4.167	6.178
Piraten	9.375	5.737	6.137	6.092	5.000	5.657	7.533
Poltergeist	4.897	4.698	7.035	6.403	4.333	5.185	6.503
Prowler	10.000	5.760	5.930	6.908	5.000	5.147	7.583
Pyrenees	4.795	7.229	7.177	8.090	3.667	7.273	6.215
Python in Bamboo Forest	8.444	7.536	7.177	9.368	5.000	7.011	7.969
Quimera	8.750	6.086	6.163	7.363	4.667	6.260	7.286
Racer	3.513	5.225	6.163	7.329	5.000	5.494	5.694
Raging Bull	4.897	8.244	7.911	9.303	5.000	8.563	7.825
RailBlazer	8.000	5.898	6.034	5.167	4.667	6.094	7.057
Ranier Rush	9.111	3.209	5.129	5.133	10.000	4.167	5.659
Raptor	4.641	7.029	6.680	7.779	3.667	6.434	6.082
Raptor	9.375	6.022	5.736	5.796	4.000	6.204	6.476
Raven	4.692	4.811	5.517	6.533	5.000	5.226	6.811
Ravine Flyer II	9.375	4.811	6.680	6.670	5.000	6.396	7.407
Red Force	8.222	7.772	9.135	6.652	5.000	7.711	7.966
Renegade	8.750	5.580	5.943	6.961	5.000	5.392	7.508
Revenge of the Mummy the Ride	6.875	1.456	3.750	5.312	5.000	2.639	3.125
Riddler Revenge	4.795	6.022	5.736	5.796	4.000	6.204	6.170
Riddler's Revenge	4.846	7.384	7.372	8.477	3.667	7.254	6.267
Ride of Steel	4.897	8.356	7.911	9.715	5.000	8.500	7.856
Road Runner Express	4.795	4.152	2.188	5.987	5.000	3.704	3.909
Rock 'n' Roller Coaster	5.625	4.811	6.680	7.314	4.667	5.226	6.955
Rougarou	4.744	7.179	7.035	7.912	4.333	7.063	6.884
Saw - The Ride	10.000	5.674	6.421	5.935	4.667	5.894	7.239
Schwur des Kärnan	8.667	8.944	8.309	8.094	10.000	8.813	11.616
Scream!	6.250	7.272	7.237	8.014	3.333	7.148	5.950
Screamer	4.692	5.299	5.775	6.260	5.000	4.907	6.821

Shambhala	9.333	9.127	8.605	9.392	5.000	9.575	8.402
Shock Wave	3.821	6.273	7.035	7.551	5.000	6.426	6.228
Sidewinder	4.692	2.549	5.129	1.435	10.000	3.593	3.262
Silver Bullet	6.875	7.197	6.421	6.977	3.667	6.057	6.146
Silver Star	5.625	8.944	8.309	9.613	5.000	8.813	8.087
Sky Scream	8.889	7.272	7.170	2.215	10.000	7.315	5.072
Sky Wheel	6.875	7.302	7.392	0.947	5.000	7.344	3.271
Skyrush	9.333	8.206	8.046	7.551	5.000	8.249	8.092
Smiler	9.111	4.076	6.137	7.838	0.000	5.657	2.188
Snow Mountain Flying Dragon	5.625	6.340	6.408	5.717	4.333	6.162	6.639
Soaring Dragon & Dancing Phoenix	8.444	8.147	8.652	8.263	3.667	8.189	6.691
Soaring with Dragon	8.444	8.147	8.228	7.357	10.000	8.189	10.714
Spatiale Experience	4.949	3.454	2.906	6.294	5.000	4.380	4.705
SpeedSnake FREE	4.026	3.303	2.906	3.190	5.000	4.250	4.418
Stampida	4.795	5.075	5.258	6.980	5.000	5.358	6.959
Star Mountain	4.590	4.340	3.750	5.987	5.000	5.060	5.665
Star Wars Hyperspace Mountain: Rebel Mission	4.692	5.861	5.000	7.168	4.667	6.060	6.897
Stealth	8.125	8.302	8.383	3.750	5.000	8.344	6.544
Steel Dragon 2000	4.949	9.303	9.394	7.000	5.000	8.915	7.919
Steel Eel	4.897	7.272	7.372	7.671	5.000	7.338	7.534
Steel Force	4.795	8.206	8.046	9.956	5.000	8.500	7.762
Steel Vengeance	8.000	8.300	7.978	9.876	4.333	8.394	7.459
Steel Venom	6.250	7.926	7.574	1.418	5.000	7.866	3.821
Storm Chaser	8.444	5.674	6.034	6.457	4.667	5.894	7.145
Storm Runner	6.875	7.272	8.046	6.260	4.667	7.972	7.406
Stunt Fall	5.625	8.038	7.412	3.380	4.667	7.908	5.849
Super Tornado	3.974	3.303	2.906	3.190	5.000	4.250	4.414
Superman - Ride Of Steel	4.949	8.150	7.911	9.655	5.000	8.500	7.893
Superman - Ultimate Flight	6.250	5.898	5.904	6.530	5.000	5.717	7.353
Superman / la Atracción de Acero	5.625	7.536	7.177	7.562	3.333	7.579	5.921
Superman el Último Escape	6.875	8.576	8.019	9.928	5.000	8.500	8.127
Superman Krypton Coaster	4.949	7.609	7.709	8.050	3.667	7.651	6.309
Superman the Ride	4.949	8.356	8.181	9.715	5.000	8.838	7.951
Superman: Escape from Krypton	4.795	6.281	9.730	3.486	5.000	8.165	5.983
Swarm	9.333	6.684	6.680	6.182	4.000	6.796	6.645
T Express	9.375	7.904	7.345	9.696	5.000	7.357	8.112
Takabisha	9.556	7.104	7.177	7.167	3.333	7.146	6.099
Talon	5.000	6.984	6.809	6.957	4.333	6.472	6.770
Taron	8.444	5.614	7.891	8.430	5.000	5.842	7.748

Tatsu	8.125	7.646	7.170	7.553	4.333	6.132	7.064
Taunusblitz	4.897	1.597	0.000	3.414	5.000	2.769	1.178
Temple of the Night Hawk	4.333	0.890	0.281	7.854	5.000	2.148	1.648
Terminator Salvation: The Coaster	10.000	0.004	8.450	2.262	5.000	1.380	1.811
Texas Tornado	4.179	4.811	7.372	5.508	5.000	4.907	6.298
Thunder Dolphin	6.250	9.374	8.437	7.431	5.000	8.779	8.018
Timber Drop	9.556	5.524	4.063	5.746	5.000	5.758	6.439
Timber Wolf	4.385	5.674	6.163	8.308	5.000	5.528	6.722
Timberhawk: Ride of Prey	6.250	4.340	5.775	6.260	5.000	5.060	6.519
Time Traveler	9.375	5.674	5.814	6.834	4.667	5.340	7.180
Titan	5.000	9.047	8.720	9.609	5.000	9.556	8.129
Titan Cascabel	4.641	4.340	6.421	6.123	5.000	5.060	6.446
Tonnerre de Zeus	4.795	5.599	6.034	8.085	5.000	5.826	7.151
Top Thrill Dragster	6.250	6.125	8.226	6.533	5.000	5.634	7.518
Tornado	5.000	3.303	2.906	3.190	5.000	4.250	4.483
Tower of Terror II	4.795	7.459	9.730	3.486	5.000	8.165	6.062
Twisted Colossus	8.667	6.460	6.680	9.222	5.000	6.774	7.853
Twister II	4.692	5.674	6.421	8.801	5.000	5.894	7.125
Ultimate	4.487	5.936	5.775	7.831	5.000	6.128	6.831
V2: Vertical Velocity	5.000	7.272	7.372	1.418	5.000	7.315	3.643
Valravn	8.444	8.636	8.046	7.329	4.667	8.690	7.721
Velikolukskiy Myasokombinat	8.667	6.572	7.177	7.388	4.333	6.694	7.055
Velikolukskiy Myasokombinat-2	8.222	5.973	6.137	6.629	2.000	6.162	4.342
Vertical Velocity	5.000	7.926	7.709	1.418	5.000	7.968	3.753
Viper	4.692	5.674	5.775	7.380	5.000	4.907	6.921
Viper	4.436	7.982	7.709	7.828	3.333	7.782	5.938
Vortex	4.282	7.235	6.421	7.791	3.667	7.085	6.099
Voyage	8.125	7.440	7.507	9.033	5.000	7.423	7.968
Whirl Wind Looping Coaster	6.250	3.303	2.906	3.189	5.000	4.250	4.566
Whizzer	3.718	3.869	4.375	6.943	5.000	3.426	5.248
Wicked Cyclone	8.667	6.011	6.421	7.214	4.667	6.192	7.278
Wicked Twister	5.625	8.487	7.844	0.000	5.000	8.521	2.365
Wild One	4.231	5.599	6.163	8.032	5.000	5.264	6.517
Wild Thing	4.744	8.337	7.978	9.787	5.000	8.310	7.687
Wild Thing	4.795	4.340	3.750	4.614	4.667	3.426	5.132
Wildfire	8.444	7.904	7.810	8.213	4.667	7.566	7.640
Winjas	5.625	2.653	4.063	4.483	5.000	2.361	3.973
Wodan Timbur Coaster	9.333	6.845	7.177	7.365	5.000	6.940	7.844
X2	5.625	7.739	8.113	7.563	5.000	8.711	7.850
Xcelerator	5.625	7.184	7.251	7.165	5.000	7.226	7.630

(The End)