QUIZ 1(10 Points)

Choose 2 of the following problems.

(1) Find the smallest number of iterations that will guarantee us a root of the function $f(x) = x^5 - 2x + 1/2$ on the interval $[0, 1]$ accurate to less than 10^{-3}
Proof: First, we are guaranteed a root of the function in the interval because $f(1)f(0) < 0$, and f is continuous. We have an error bound for the distance from p_n to p, where p_n is the nth iterate, and p is the correct value, as follows:

$$|p_n - p| \leq \frac{b-a}{2^n}.$$

In this case, $b - a = 1$, so we just need the smallest n such that $2^{-n} < 1000$, which is $n = 10$.

(2) If $f(h)$ converges to 0 at rate $O(h)$, (that is $f(h) = O(h)$) as $h \to 0$, and $g(h) = O(h)$ as $h \to 0$, show that $f(h)g(h) = O(h^2)$ as $h \to 0$.
Proof: For sufficiently small h, we have that $|f(h)| \leq K_1 h$ and $|g(h)| \leq K_2 h$, and therefore, $|fg| = |f||g| \leq K_1 K_2 h^2$, which implies that $fg = O(h)$.

(3) Suppose p^* must approximate p with relative error at most 10^{-3}. Find the largest interval in which p^* must lie for $p = 150$.

Proof:

$$\frac{|p-p^*|}{|p|} \leq 10^{-3} \to |150-p^*| \leq .15 \to -.15 \leq p^* - 150 \leq .15.$$

We conclude, that $p^* \in [149.85, 150.15]$.