Lec3 : Bisection Method

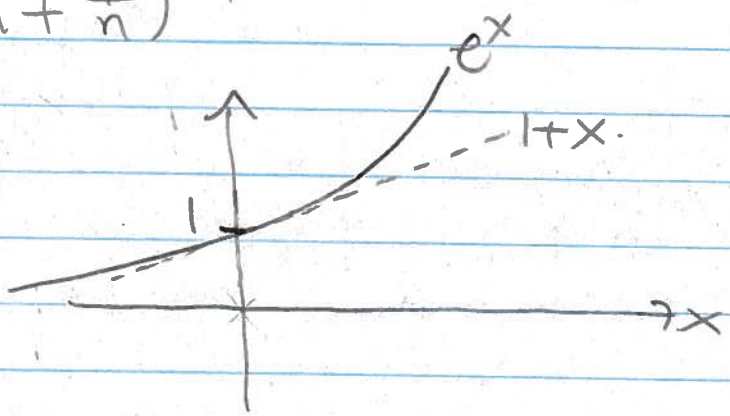
- Rate of convergence.

Ex $\alpha_n = \log \frac{n+1}{n}$.

$\alpha_n \to \log 1 = 0. \quad (= \alpha)$

$\alpha_n = \log\left(1 + \frac{1}{n}\right)$

Consider :



$\Rightarrow \quad 1+x \leq e^x$

$\Rightarrow \quad \log(1+x) \leq x.$

$\Rightarrow \quad \log\left(1+\frac{1}{n}\right) \leq \frac{1}{n}.$

$\Rightarrow \quad \alpha_n \leq \frac{1}{n}.$

Thus, in this example,

$$\beta_n = \frac{1}{n} \to 0$$

and

$$K = 1.$$

(Other choices of K, β_n are possible).

Ex $a_n = \frac{n+1}{n^2} \leq \frac{n+1 \cdot n}{n^2} = \frac{2}{n}$ $\therefore$ $a_n \to a = 0$

i.e. $|a_n - a| \leq K\beta_n$ where $K = 2$, $\beta_n = \frac{1}{n}$.

Ex $f(h) = \frac{\sin(h) - \sin(0)}{h}$

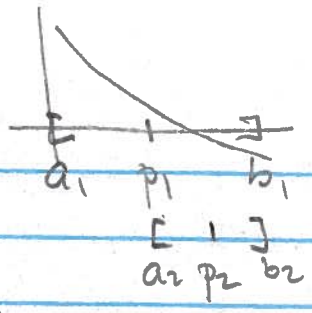
$f(h) \xrightarrow[h \to 0]{} \frac{d}{dh}\sin(h)\Big|_{h=0} = \cos(0) = 1 = L$

$|f(h) - L| = \frac{\sin(h) - \sin(0) - h}{h}$

$= \frac{[h + O(h^3)] - 0 - h}{h}$

$= O(h^2)$

- **Bisection Method** →



→ How to stop iterations?

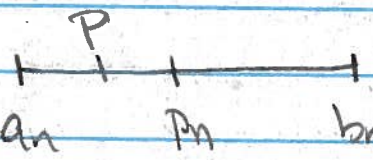
(1) Allow only a max # iterations

(2) Stop if $|f(p_n)| \leq tol$

or $|p_{n+1} - p_n| \leq tol$

$$\left| \frac{p_{n+1} - p_n}{p_n} \right| \leq tol$$

→ Error Analysis:



$\Rightarrow |p_n - p| \leq \frac{1}{2}(b_n - a_n)$