1) Why does Borel resumm. work? Borel regularity (Θ)

a) Watson condition (abstract)

b) "Thimble integrals" (pieces of exponential ints.) homology basis

c) Linear ODEs with reg. sing at 0 and irreg at ∞

2) Which parts of the theory can be described purely in terms of holo. functions & Laplace transforms?

LINEAR ODEs

polynomials, $\deg P = \deg Q + 1$ holo.

$$z\,P\!\left(\frac{\partial}{\partial z}\right) + Q\!\left(\frac{\partial}{\partial z}\right) + R\!\left(\frac{1}{z}\right)$$

holo fn.

"Integrable singularity"

bdry terms vanish

$\mathcal{L}_\zeta$ from α

Ecalle's singularities !

breakthrough: generalization to nonlin. world

1-Gevrey trans-monomial solution

α root of P

$$\frac{\partial}{\partial \zeta}\cdot P(-\zeta) + Q(-\zeta) + R(\partial_\zeta^{-1})$$

$$\begin{tikzcd}
F_\alpha & & \tilde{F}_\alpha \arrow[d, "\mathcal{B}"] \\
f_\alpha = \hat{f}_\alpha \arrow[u] & & \tilde{f}_\alpha \arrow[ll, "\text{sum}"']
\end{tikzcd}$$

$\mathcal{B}_*$

looks like $\zeta^{-\tau_\alpha}\left(\text{holo on } \alpha \to\right)$

$\tau_\alpha \in \mathbb{Q} \cap (-\infty, 1)$

Michéle Loday-Richaud articles on linear ODEs

φ algebraic function $\qquad$ ν is merom. 1-form

α crit. point of φ

$\varphi''(\alpha) \neq 0$

$$F_\alpha(z) = \int_{C_\alpha} e^{-\varphi(t) z} \, \nu \quad \xrightarrow[\text{saddle point.}]{z \to \infty} \quad z^{1/2} \tilde{W}_\alpha(z) =: \tilde{F}_\alpha(z) \in \mathbb{C}[[z^{-1}]]$$

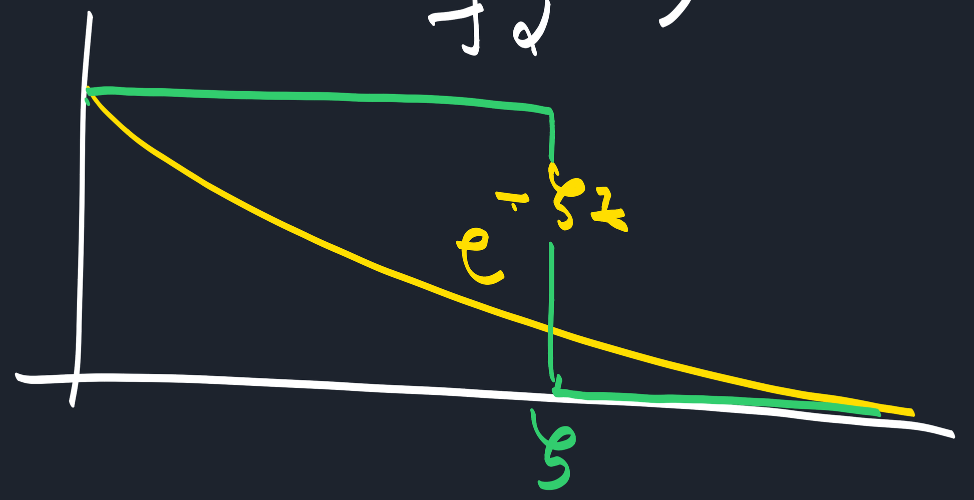
is 1-Gevrey

$$\overset{\zeta = \varphi(t)}{=} \int_\alpha^{+\infty} e^{-\zeta z} \, \underbrace{\varphi^* \nu}$$

$\mathcal{B}$

$$\boxed{\partial^{1/2}_{\zeta,\alpha} \tilde{f}_\alpha(\zeta) = \tilde{W}_\alpha(\zeta)} = \partial^{3/2}_{\zeta,\alpha} \left(\int_{C_\alpha(\zeta)} \nu \right)$$

$$\int_{C_\alpha} \mathrm{step}_\zeta \, \nu$$



$$(\varphi^* \nu)(\zeta) = \hat{f}_\alpha \xleftarrow{\text{sum}} \tilde{f}_\alpha(\zeta)$$

$\mathcal{L}_{\zeta,\alpha}$

Ramis: exponential torus action varies Stokes consts.

example

time domain

freq. domain (Borel plane)

GENERALIZATIONS

1) $\int_e \exp[-z(4u^3-3u)]\,\frac{du}{u^p}$ $\quad p\in\mathbb{Q}$ $\quad \xleftarrow{\mathcal{L}} \quad {}_3F_2(\quad \zeta)$

Chebyshev

2) $\int_\tau \exp[-z\,T_n(u)]\,du$ $\quad \longleftarrow \quad {}_2F_1(\text{nice family}, \zeta)$

dihedral

3) $\int_\tau \exp[-z\,p(u)]\,du$

- p cubic
  - gen'c: equiv. to T_3
  - degen: u^3
- p quartic
- p quintic
- ⋮

values assoc. with spaces of stability conditions for $A_{\deg p}$ quiver reps (?)

4) Schwarz's list of alg. hypgeom fns