Airy function: Kawai+Takei vs. Mariño

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Kawai and Takei want to solve

$$\left[\left(\frac{d}{dx}\right)^2 - \eta^2 x\right]\psi(x,\eta) = 0.$$

They define $\psi_B(x,y)$ as the inverse Laplace transform of $\psi(x,\eta)$ with respect to η.

With $w = x\eta^{2/3}$, the equation above is equivalent to

$$\left[\left(\frac{d}{dw}\right)^2 - w\right]\psi(w\eta^{-2/3},\eta) = 0.$$

Proof: substitute back to get

$$\begin{aligned}
\left[\eta^{-4/3}\left(\frac{d}{dx}\right)^2 - \eta^{2/3}x\right]\psi(x,\eta) &= 0 \\
\left[\eta^{-4/3}\left(\frac{d}{dx}\right)^2 - \eta^{-4/3}\eta^2 x\right]\psi(x,\eta) &= 0 \\
\eta^{-4/3}\left[\left(\frac{d}{dx}\right)^2 - \eta^2 x\right]\psi(x,\eta) &= 0.
\end{aligned}$$

Hence, $\psi(w\eta^{-2/3},\eta) = k(\eta)\,\mathrm{Ai}(w)$ is a solution for any holomorphic function k.