AKQ Poker Optimisation

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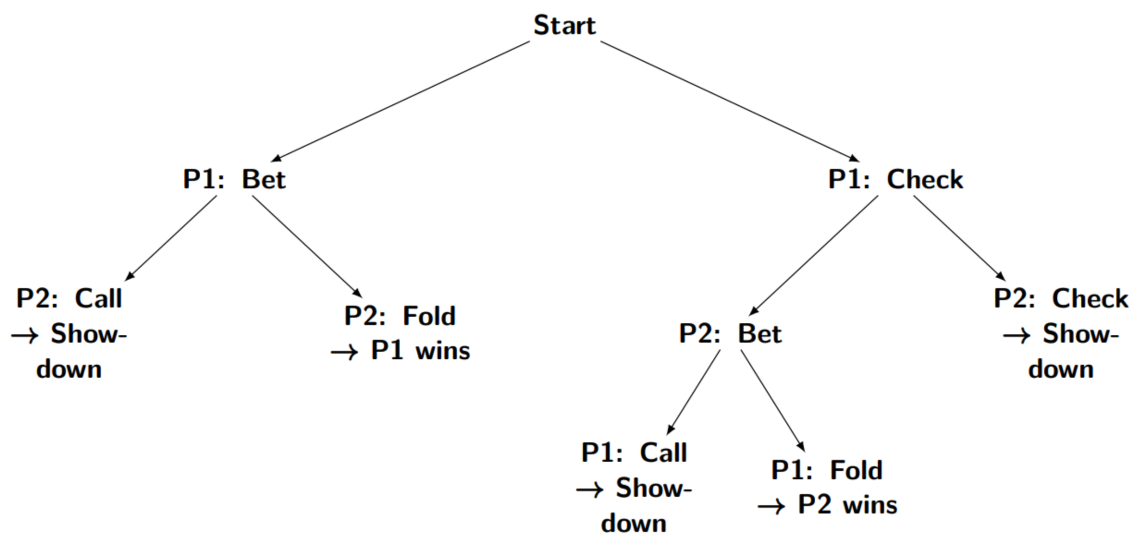
Gameplay and Rules

- This is a two-player sequential game involving players **P1** and **P2**.
- The deck consists of three distinct cards: **Ace (A), King (K), and Queen (Q)**, with ranking: **A > K > Q**.
- Both players contribute an **ante of $1** to the pot and randomly receive one card **different from the opponent**.
- Player 1 starts with a **bet of $1 or check**.
- Player 2 can return with a **raise of $1 or check** against a check leading to showdown.
- Either Player can **return a raise or bet with a call** of $1 leading to showdown or **fold their cards losing ante**.
- At showdown, the player with the **higher rank** card wins the pot

Game Tree and Proceedings



Strategic Analysis

Simple Interactions with one clearly best move:

- **Q should never call** a bet as it always loses a showdown.
- **A should always raise a check** as it never wins less than a check.
- **K should never bet** as the opponent always wins more with an A and folds to lose less with a Q.

Analysis for Player 1 -

- **Ace -** If checks, will only win additional if Q bluffs and a bet will only be called by K thinking P1 is bluffing Q.
- **King -** Should always check (explained above)
- **Queen -** Can either bluff so the opponent pays off our A more with his King. If the opponent calls less with his K, then we get a better Q EV.

Analysis for Player 2 -

- **Ace -** Should always raise or call if the opponent bets as it always wins.
- **King -** If the opponent checked, then check back (explained above). If the opponent has bet, K can call thinking Q has bluffed or fold thinking opponent has A.
- **Queen -** Should never call a raise. On receiving a check, behaves like Player 1's Q.

Bluffing with Q, Why?

- If the opponent plays optimally and knows **we never bet with Q**, they would **never pay off our A** bet and always fold with K when faced with a bet.
- If we bluff with Q, we always **lose an additional $1 against A** but when the opponent has a K, he faces a dilemma. If he folds and we have a Q, they go from **+1 to -1 interaction**, but if he calls and we have an A, they go from **-1 to -2 interaction**.
- As a result, they are forced to **call often with K** instead of playing safe.

Payoff Table

	PLAYER 1		ACE		KING		QUEEN	
PLAYER 2			CALL/ FOLD	CHECK/ BET	CALL/ FOLD	CHECK/ BET	CALL/ FOLD	CHECK/ BET
ACE	BET				-2 / +1		-2 / +1	
	CHECK					-1 / NA		-1 / NA
		CALL/FOLD				-2 / -1		-2 / -1
KING	BET		+2 / +1				-2 / +1	
	CHECK			+1 / NA				-1 / NA
		CALL/FOLD		+2 / -1				-2 / -1
QUEEN	BET		+2 / +1		+2 / +1			
	CHECK			+1 / NA		+1 / NA		
		CALL/FOLD		+2 / -1		+2 / -1		

- All the payoffs written above are **with respect to Player 1.**

Expected Values of Simple Interactions :

- Since we are **not certain about the opponent's actions,** we assume that the opponent plays each of the two actions with a **probability of ½.**
- However, if it is certain that **one player will always lose** in a particular situation, we do not assign a probability of ½. If player 1 calls then player two (with Q) won't bet anytime.
- Same for player 2 (with **A**) won't check .
- Player 1 (with **K**) will always check.
- **Player 1 :**
    1. $EV(A_{BET})$: $\frac{1}{6}*(\frac{1}{2}*(2)+\frac{1}{2}*(1)+1*(1))$ = 0.417
    2. $EV(A_{CHECK})$: $\frac{1}{6}*(\frac{1}{2}*(2)+\frac{1}{2}*(1)+\frac{1}{2}*(2)+\frac{1}{2}*(1))$ = 0.5
    3. $EV(K_{CALL})$: $\frac{1}{6}*(1*(-2)+1*(1))$ = -0.167
    4. $EV(K_{FOLD})$: $\frac{1}{6}*(1*(-1)+ \frac{1}{2}*(1)+\frac{1}{2}*(-1))$ = -0.167
    5. $EV(Q_{BLUFF})$: $\frac{1}{6}*(1*(-2)+\frac{1}{2}*(-2)+\frac{1}{2}*(1))$ = -0.417
    6. $EV(Q_{FOLD})$: $\frac{1}{6}*(1*(-1)+1*(-1))$ = -0.333

- **Player 2 :**
    1. $EV(A_{CALL})$: $\frac{1}{6}*(\frac{1}{2}*(2)+\frac{1}{2}*(1)+\frac{1}{2}*(2)+\frac{1}{2}*(1))$ = 0.5
    2. $EV(A_{CHECK})$: $\frac{1}{6}*(0)$ = 0.0
    3. $EV(K_{CALL})$: $\frac{1}{6}*(\frac{1}{2}*(2)+\frac{1}{2}*(1)+\frac{1}{2}*(-2)+\frac{1}{2}*(-2)$ = -0.083
    4. $EV(K_{FOLD})$: $\frac{1}{6}*(1*(-1)+ \frac{1}{2}*(1)+\frac{1}{2}*(-1))$ = -0.167
    5. $EV(K_{CHECK})$: $\frac{1}{6}*(\frac{1}{2}*(-1)+\frac{1}{2}*(+1))$ = 0.0
    6. $EV(Q_{BLUFF})$: $\frac{1}{6}*(\frac{1}{2}*(-2)+\frac{1}{2}*(-2)+\frac{1}{2}*(+1))$ = -0.25
    7. $EV(Q_{CHECK})$: $\frac{1}{6}*(1*(-1)+1*(-1))$ = -0.333

Nash Equilibrium

- A Nash equilibrium refers to any situation (strategy profile) **where no player can improve** their expected payoff by changing strategies unilaterally, given the other player's behavior.
- In other words, A Nash equilibrium is found when,
    - Player 1 cannot increase their winnings by changing their betting, checking, or folding strategy with any card, given the way Player 2 responds.
    - Player 2 cannot increase their winnings by changing their calling, folding, betting, or checking strategy, given Player 1's behavior.

- However, **No direct Nash Equilibrium** can be achieved through pure strategies alone in the case of AKQ Poker.
- Say, P2 has King and faces a bet from P1,
    - If P2 knows P1 has a Q, he would call the bet.
    - But, If P2 knows P1 has a K, he would fold to avoid losing more.

Mixed Strategies

- We aim to develop an optimal strategy **which cannot be exploited** by any other strategy undertaken by our opponent.
- This can be achieved by **using several strategies** simultaneously with assigned probabilities such that we **equalize the EV** of all possible actions of our opponent.

- As a result, we become **indifferent to our opponent's action** thus reaching an equilibrium as the opponent cannot **positively deviate** from his current strategy.
- Thus, A mixed strategy means assigning probabilities to each possible action and choosing randomly according to those weights.

Equilibrium Strategies

We will be calculating the EV in relative terms, i.e., with respect to normal interactions such as AK will be relative to -1, and QA will be relative to $+1$ after comparing ranks of the drawn cards. This simplifies calculations and does not hinder our calculation as we need to understand how much more or less we can get aside from the drawn interaction.

All EVs are multiplied by $\frac{1}{2} \times \frac{1}{3}$.

Analysing EV of Player 1 to identify Player 2's optimum strategy —

Turn 1

$$\begin{aligned} \text{EV of } A_{1\text{check}} &= 1 \cdot Q_{2\text{bluff}} \\ \text{EV of } A_{1\text{bet}} &= 1 \cdot K_{2\text{call}} \end{aligned}$$

K always checks.

$$\begin{aligned} &\text{EV of } Q_{1\text{bluff}} = -1 \cdot A_{2\text{call}} - 1 \cdot K_{2\text{call}} + 2 \cdot K_{2\text{fold}} \\ \implies &\text{EV of } Q_{1\text{bluff}} = -1 - K_{2\text{call}} + 2 \times (1 - K_{2\text{call}}) \\ \implies &\text{EV of } Q_{1\text{bluff}} = -1 - K_{2\text{call}} + 2 - 2K_{2\text{call}} \\ \implies &\text{EV of } Q_{1\text{bluff}} = 1 - 3K_{2\text{call}} \\ &\text{EV of } Q_{1\text{check}} = 0 \end{aligned}$$

Equating bluffs and checks for indifference:

$$\text{EV of } Q_{1\text{bluff}} = \text{EV of } Q_{1\text{check}} \implies 1 - 3K_{2\text{call}} = 0 \implies K_{2\text{call}} = \frac{1}{3}$$

Turn 3

A always calls.

$$\begin{aligned} &\text{EV of } K_{1\text{call}} = -1 \cdot A_{2\text{bet}} + 1 \cdot Q_{2\text{bluff}} \\ \implies &\text{EV of } K_{1\text{call}} = -1 + Q_{2\text{bluff}} \\ &\text{EV of } K_{1\text{fold}} = -2 \cdot Q_{2\text{bluff}} \end{aligned}$$

Set equal for indifference:

$$-1 + Q_{2\text{bluff}} = -2Q_{2\text{bluff}} \implies Q_{2\text{bluff}} = \frac{1}{3}$$

Thus, P2's optimum actions by the indifference principle are:

$$A_{2\text{call}} = 1,\ K_{2\text{call}} = \frac{1}{3},\ Q_{2\text{bluff}} = \frac{1}{3}$$

which also satisfies P1's A indifference.

Analysing EV of Player 2 to identify Player 1's optimum strategy —

On receiving check:

K checks.

$$\text{EV of } K_{2\text{check}} = 0$$

A bets.

$$\text{EV of } A_{2\text{bet}} = 1 \cdot K_{1\text{call}}$$

$$\begin{aligned}\text{EV of } Q_{2\text{bluff}} &= -1 \cdot A_{1\text{check}} + 2 - 3K_{1\text{call}} \\ \text{EV of } Q_{2\text{fold}} &= 0\end{aligned}$$

On receiving bet:

Q folds.

A calls.

$$\text{EV of } A_{2\text{call}} = Q_{1\text{bluff}}$$

$$\begin{aligned}\text{EV of } K_{2\text{call}} &= -1 \cdot A_{1\text{bet}} + 1 \cdot Q_{1\text{bluff}} \\ \text{EV of } K_{2\text{fold}} &= -2 \cdot Q_{1\text{bluff}}\end{aligned}$$

Solving these equations through intuition and boundary values, the optimum solutions are:

$$A_{1\text{bet}} = 1,\ Q_{1\text{bluff}} = \frac{1}{3},\ K_{1\text{call}} = \frac{2}{3}$$

which gives the following EVs for Player 2:

$$K_{2\text{call/fold}} = -\frac{2}{3},\ Q_{2\text{bluff/check}} = 0,\ A_{2\text{bet}} = \frac{2}{3},\ A_{2\text{call}} = \frac{1}{3}$$

(Note: For $A_{2\text{bet}}$ and $A_{2\text{call}}$, further adjustment may be necessary based on probability of receiving a check or bet.)

Testing and Verification through Python

- Since, it is **not possible** to achieve indifference for A of Player 2, We tested **one more strategy being promoted online for Player 1.**
- All in all, We tested **2 strategies for P1**, against **1 clear best strategy** which achieves indifference for **P2**.
- Player 1 Strategy 1 -
    - A always bets.
    - K always checks.
    - Q bluffs with probability ⅓.
    - K calls with probability ⅔ against bet.
    - Q folds against bet.
- Player 1 Strategy 2 -
    - A,K,Q always checks turn 1.
    - K calls with probability ⅓ against bet.
    - A calls against a bet.
    - Q folds against a bet.
- Player 2 only strategy -
    - A always bets.
    - K checks against a check.
    - K calls with probability ⅓ against a bet.
    - Q folds against a bet.
    - Q bluffs with probability ⅓ against a check.
- Testing Player 1 Strategy 1 - https://www.kaggle.com/code/niceofyou06/akq-poker
    - EV for Player 1 per game : **-0.054**
    - EV for Player 2 per game : **0.054**
- Testing Player 1 Strategy 2 - https://www.kaggle.com/code/vedjani/akq-poker
    - EV for Player 1 per game : **-0.22**
    - EV for Player 2 per game : **0.22**
- We further tested both strategies against **stochastic settings** for P2, and **Strategy 1 clearly outperformed the latter.**
- Thus, **Strategy 1 is clearly better for P1.**

Reflection and Application

The Necessity of Bluffing :

- Bluffing is necessary in AKQ Poker to keep the opponent guessing and to avoid being predictable.
- If a player only bets with strong hands (like Ace), their strategy becomes predictable.
- Bluffing with weak hands (like Queen) introduces doubt in the opponent's mind.
- If the opponent always folds to bets, bluffing allows the bluffer to win pots without having the best hand.
- In equilibrium, players mix their strategies (value bets and bluffs) to keep opponents indifferent between options.
- Bluffing is mathematically necessary to make this equilibrium possible.
- Proper bluffing can increase a player's overall expected winnings across many hands, even if some individual bluffs lose.

Strategic Thinking and Decision Making :

- **EV :** teaches that good decisions prioritize long-term profitability over immediate outcomes. Even if a choice loses once, it's correct if its EV is positive over time, promoting analytical risk-reward assessment.
- **Bluffing :** Bluffing is crucial for unpredictability. If you only bet strong hands, opponents will always fold, preventing profit. Mixing in weak hands balances your strategy, making it harder to exploit.
- **GTO :** AKQ Poker illustrates Game Theory Optimal (GTO) play, where both players use balanced, unexploitable strategies. However, if an opponent deviates from GTO, you can exploit those mistakes to gain more value.
- **Thinking in Ranges** : You learn to think in ranges, not just individual hands, considering what hands your opponent could have based on their actions.
- **Deal with imperfection :** Finally, the game emphasizes decision-making under imperfect information. You often won't know your opponent's cards, so the best move is one that works well over time, not necessarily in

one round.

Real Life Situations like Poker :

- **Penalty Kicks in Football** :
  - The kicker must decide whether to shoot left or right.
  - The goalkeeper must guess the direction without knowing the kicker's choice.
  - Both players use **mixed strategies** to avoid being predictable.
  - A kicker might sometimes shoot to their weaker side (bluffing), just to keep the goalkeeper guessing.
- **Bidding in Auctions** :
  - You don't know how much other bidders are willing to pay.
  - You might place a high bid to scare others off.
  - If everyone bids aggressively, the price goes too high — strategic restraint is sometimes better.
- **Military Strategy** :
  - Commanders, like poker players, must make decisions without knowing the opponent's exact position or plan.
  - Militaries use fake movements or misinformation to mislead the enemy — similar to bluffing with weak hands in poker.
  - Success requires predicting what the enemy might do next and planning accordingly, just like reading an opponent's range in AKQ.
  - Plans must change as new information arrives — just like poker strategy adapts during the hand.