Module - 4

Probability distributions

* Binomial & Poisson distributions - Discrete distributions
* Normal distribution
* Gamma distribution
* Exponential distribution
* weibull distribution

$\left.\right\}$ - Continuous distributions.

Bernoulli Distribution

A random variable x which takes two values 0 & 1 with probabilities q & p respectively.

i.e., $P(x=0)=q$ where $q=1-p$ & $P(x=1)=p$ is called a Bernoulli variate & is said to have a Bernoulli distribution.

Moment Of Bernoulli Distribution

$$\mu_r' = E(x^r) = p \quad \text{for } r=1,2\ldots$$

$$\mu_1' = E(x) = p \,, \quad \mu_2' = E(x^2) = p$$

$$var(x) = \mu_2 = \mu_2' - (\mu_1')^2 = p - p^2 = p(1-p) = pq$$

MGF of Bernoulli Variate

$$M_x(t) = q + pe^t$$

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Binomial distribution

A random variable x is said to follow binomial distribution if its pmf is given by

$$P(X=x) = p(x) = \begin{cases} nC_x\, p^x q^{n-x} & \text{for } x = 0, 1, 2 \cdots n \\ & \& \ q = 1-p \\ 0 & \text{Otherwise} \end{cases}$$

$\& \ 0 \leq q < 1$

Mean $= E(x) = np$

Variance $= V(x) = npq$

Note

$n \,\&\, p$ – Parameter

n – degree of the binomial distribution some times (or) no/. of trials

p – Probability of Success.

$q = 1-p \Rightarrow$ Probability of failure.

Physical conditions for Binomial distribution:

i) Each trial results in two mutually disjoint outcomes

ii) no/. of trials 'n' is finite

iii) Trials are independent to each other

iv) Probability of success $= p$ is constant for each trial.

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Additive Property of Binomial random Variables

If X_1 & X_2 are two independent binomial random variables with parameters (p, n_1) & (p, n_2) then $X_1 + X_2$ is a binomial random variable with parameters (p, n_1+n_2)

Problems:

① Five coins are tossed 3,200 times, find the frequencies of the distribution of heads & tails & tabulate the results. Also calculate the mean no/- of success & SD's.

Sol

$$p = P(\text{getting a head in tossing a coin}) = 1/2$$

$$q = 1-p = 1-1/2 = 1/2$$

$$n = 5 \text{ coins}$$

Let $X \sim B(n, p)$ & it denotes no/- of heads

The values for x are 0, 1, 2, 3, 4 & 5

$$P(X=x) = nC_x \, p^x q^{n-x}$$

$$= 5C_x \left(1/2\right)^x \left(1/2\right)^{5-x} \quad \text{for } x = 0,1,2,3,4,5$$

$$= 5C_x \left(1/2\right)^5 \quad \text{for } 0 \le x \le 5$$

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Frequency distribution (Binomial)

$f(x) = N \cdot P(X=x)$ where N = no. of times an event occurs.

$= N \cdot nC_x \, p^x q^{n-x}$ (or) Event repeats N no. of times

$\therefore N = 3200$

$\therefore f(x) = 3200 \times 5C_x (1/2)^5, \quad 0 \le x \le 5$

$= 100 \times 5C_x, \quad 0 \le x \le 5$

No of heads x	$f(x) = 100 \times 5C_x$
$x=0$	$f(0) = 100 \times 5C_0 = 100 \times 1 = 100$
$x=1$	$f(1) = 100 \times 5C_1 = 100 \times 5 = 500$
$x=2$	$f(2) = 100 \times 5C_2 = 100 \times 10 = 1000$
$x=3$	$f(3) = 100 \times 5C_3 = 100 \times 5C_2 = 1000$
$x=4$	$f(4) = 100 \times 5C_4 = 100 \times 5C_1 = 500$
$x=5$	$f(5) = 100 \times 5C_5 = 100 \times 5C_0 = 100$

Total = 3200

$\therefore$ Mean no. of successes $= np = 5 \times 1/2 = 2.5$

$S.D = \sqrt{npq} = \sqrt{5 \times 1/2 \times 1/2} = 1.118$.

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2) The mean & variance of a binomial Variate are 8 & 6 the find i) $P(X \geq 2)$ ii) $P(X=2)$

Sol

Given $np = 8$ — (1) & $npq = 6$ — (2)

$\frac{(2)}{(1)} \Rightarrow \frac{npq}{np} = 6/8$

$q = 3/4$

$\therefore p = 1-q = 1-3/4 = 1/4$

$np = 8 \Rightarrow n/4 = 8$

$n = 32$

$P(X=x) = nC_x\, p^x q^{n-x} = 32C_x (1/4)^x (3/4)^{32-x}$

for $0 \leq x \leq 32$

$P(X \geq 2) = 1 - \{P(X<2)\}$

$= 1 - \{P(X=0) + P(X=1)\}$

$= 1 - \{(3/4)^{32} + 32C_1 (1/4)(3/4)^{31}\}$

$= 1 - (3/4)^{31}\{3/4 + 32 \times 1/4\}$

$= 1 - 35/4\,(3/4)^{31}$

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ii) $P(X=2) = 32C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^{30}$

$= \frac{\cancel{32}^{2} \times 31}{\cancel{2} \times 1} \left(\frac{1}{\cancel{16}}\right) \left(\frac{3}{4}\right)^{30}$

$= 31 \left(\frac{3}{4}\right)^{30}$

(3) Out of 800 families with 4 children each, how many families would be expected to have i) 2 boys & 2 girls ii) atleast 1 boy iii) atmost 2 girls & iv) children of both sexes. Assume equal probabilities for boys & girls.

Sol

$n = 4$ child

$p = P(\text{birth of a boy}) = 1/2$

$q = 1/2$

X follows $B(n,p)$ & it denotes the no/- of Successes (boys) & $P(X=x) = nC_x \cdot p^x q^{n-x}$

$= 4C_x (1/2)^x (1/2)^{4-x}$

$= 4C_x (1/2)^4$

i) $P(\text{2 boys \& 2 girls}) = P(X=2)$

$= 4C_2 \cdot (1/2)^4$

$= 6 (1/4)^4 = 3/8$

No. of families having 2 boys & 2 girls

$= N_0\, P(x=2) = 800 \times 3/8$

$= 300$

ii) $P(\text{Atleast } 1 \text{ boy}) = P(x \geq 1)$

$= P(x=1) + P(x=2) + P(x=3) + P(x=4)$

(or) $1 - P(x=0)$

$= 1 - 4C_0 (1/2)^0 (1/2)^4$

$= 1 - 1/16 = 15/16$

No. of families having atleast 1 boy

$= 800 \times 15/16 = 750$

iii) $P(\text{atmost } 2 \text{ girls}) = P(x \geq 2)$
(or) atleast 2 boys

$= 1 - P(x < 2)$

$= 1 - \{P(x=0) + P(x=1)\}$

$= 1 - \{4C_0 (1/2)^4 + 4C_1 (1/2)^4\}$

$= 11/16$

∴ No. of families having atmost 2 girls

$= 800 \times 11/16 = 550$

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iv) P(Children of both sexes)

$= 1 - P(\text{children of same sex})$

$= 1 - \{P(\text{all are boys}) + P(\text{all are girls})\}$

$= 1 - \{P(x=4) + P(x=0)\}$

$= 1 - \{4C_4 (1/2)^4 + 4C_0 (1/2)^4\}$

$= 1 - 1/8 = 7/8$

$\therefore$ No. of families having children of both sexes

$= 800 \times 7/8 = 700$

Recurrence formula for calculating theoretical frequencies in Binomial distribution

$$P(x+1) = \frac{n-x}{x+1} \, p/q \, P(x)$$

① An unbiased coin is tossed eight times and the no. of heads noted. The experiment is repeated 256 times & the following frequency distribution is obtained.

No. of heads	0	1	2	3	4	5	6	7	8
Frequency	2	6	30	52	67	56	32	10	1

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Calculte the theoretical frequencies
(Fit a binomial distribution)

Sol

For an unbiased coin

$p = p$(getting head in tossing a coin)

$= 1/2$

$q = 1/2$

$n = 8$

$p/q = \frac{1/2}{1/2} = 1$

Expected frequencies:

$$P(x+1) = \frac{n-x}{x+1}\frac{p}{q}\,p(x) = \frac{8-x}{x+1}\,p(x)$$

$$P(X=x) = p(x) = nC_x\, p^x q^{n-x}$$
$$= 8C_x (1/2)^x (1/2)^{8-x}$$
$$= 8C_x (1/2)^8, \quad 0 \le x \le 8$$

$$P(X=0) = p(0) = 8C_0 (1/2)^8 = (1/2)^8$$

Since coin tossing experiment repeated 256 times

$\therefore N = 256$

(X) Expected frequency $= N.\, P(x)$

For $X = x$

$$N \cdot P(0) = 256 \times (1/2)^8 = 256 \times \frac{1}{256} = 1$$

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For $x=1$

$N.P(x=1) = 256 \cdot P(1) = 256 \times 8C_1 \left(\frac{1}{2}\right)^8 = \cancel{256} \times 8 \times \frac{1}{\cancel{256}} = 8$

For $x=2$

$N \cdot P(x=2) = 256 \cdot P(2) = 256 \times 8C_2 \times \left(\frac{1}{2}\right)^8 = \frac{\overset{4}{\cancel{8}} \times 7}{\cancel{2} \times 1} = 28$

For $x=3$

$N \cdot P(x=3) = 256 \times 8C_3 \times \left(\frac{1}{2}\right)^8 = \frac{8 \times 7 \times \cancel{6}}{\cancel{3} \times \cancel{2} \times 1} = 56$

For $x=4$

$N \cdot P(x=4) = 256 \times 8C_4 \left(\frac{1}{2}\right)^8 = \frac{8 \times 7 \times \overset{2}{\cancel{6}} \times 5}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 70$

For $x=5$

$N \cdot P(x=5) = 256 \times 8C_5 \left(\frac{1}{2}\right)^8 = 8C_3 = 56$

For $x=6$

$N \cdot P(x=6) = 256 \times 8C_6 \left(\frac{1}{2}\right)^8 = 8C_2 = 28$

For $N=7$

$N \cdot P(x=7) = 256 \times 8C_7 \left(\frac{1}{2}\right)^8 = 8C_1 = 8$

For $N=8$

$N \cdot P(x=1) = 256 \times 8C_8 \times \left(\frac{1}{2}\right)^8 = 8C_0 = 1$

No. of heads:	0	1	2	3	4	5	6	7	8
Observed frequencies:	2	6	30	52	67	56	32	10	1
Theoretical frequencies	1	8	28	56	70	56	28	8	1

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② An irregular 6-faced die is such that the probability that it gives 3 even no/s in 5 throws is twice the probability that it gives 2 even no/s in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets?

Sol

Let p – P(getting an even no/- with the unfair die)

X be the R.V & $X \sim B(n, p)$ & it denotes the no/- of even numbers obtained 5 throws.

$\therefore P(X=x) = nC_x p^x q^{n-x}$ & $n=5$

Given: $P(X=3) = 2P(X=2)$

$$5C_3 \, p^3 q^2 = 2 \times 5C_2 \, p^2 q^3$$

$$\cancel{5C_2} \, p^{\cancel{3}} \cancel{q^2} = 2 \times \cancel{5C_2} \, \cancel{p^2} \, q^{\cancel{3}}$$

$$p = 2q$$

$$p = 2(1-p)$$

$$= 2 - 2p$$

$$3p = 2$$

$$p = 2/3 \quad \therefore q = 1/3$$

$$\therefore P(X=x) = 5C_x \left(2/3\right)^x \left(1/3\right)^{5-x}$$

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$P(\text{getting no even no/s}) = P(X=0)$

$= 5C_0 (2/3)^0 (1/3)^5$

$= (1/3)^5 = 1/243$

No/- of sets having no even no/s out of 2500 sets

$= 2500 \times P(X=0)$

$= 2500 \times 1/243 = 10.29$

≈ 10

(3) If X & Y are independent binomial variates & follows $B_1(5, 1/2)$ & $B_2(7, 1/2)$ respectively then Find $P(X+Y=3)$

Sol. Since X & Y are Independent

& $X \sim B_1(5, 1/2)$ & $Y \sim B_2(7, 1/2)$

then $X+Y \sim B(12, 1/2)$

i.e $X+Y \sim B(5+7, 1/2)$
$\sim B(12, 1/2)$

$\therefore P(X+Y=x) = 12C_x (1/2)^x (1/2)^{12-x}$

$= 12C_x (1/2)^{12}$

~~P(X+Y=3)~~ $P(X+Y=3) = 12C_3 (1/2)^{12} = 55/704$

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Moment Generating function of Binomial distribution

Let $X \sim B(n,p)$ then

MGF of Binomial distribution:

$$M_x(t) = E(e^{tx}) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x (pe^t)^x q^{n-x}$$

$$= (q+pe^t)^n$$

Additive Property of a Binomial distribution

Let $X \sim B(n_1,p_1)$ & $Y \sim B(n_2,p_2)$ be independent random variable Then $M_x(t) = (q_1+p_1e^t)^{n_1}$, $M_y(t) = (q_2+p_2e^t)^{n_2}$. what is distribution of $X+Y$?

Sol

$$M_{X+Y}(t) = M_x(t) \cdot M_y(t) = (q+pe^t)^{n_1+n_2}$$

provided ~~where~~ $p_1 = p_2 = p$

~~Bex~~

$\therefore X+Y \sim B(n_1+n_2, p)$

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Poisson Distribution

A random variable X is said to follow Poisson distribution if it assumes only non-negative values & its probability mass function is given by

$$P(X=x) = P(x,\lambda) = \begin{cases} \dfrac{e^{-\lambda}\lambda^{x}}{x!}, & x = 0,1,2\ldots \\ & \& \ \lambda > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Note

① X follows Poisson distribution is denoted by $X \sim P(\lambda)$ where λ is the parameter.

② Cumulative distribution function

$$F(x) = P(X \le x) = \sum_{r=0}^{x} p(r)$$

$$= e^{-\lambda} \sum_{r=0}^{x} \lambda^{r}/r! \quad , \; x = 0,1,2\ldots$$

Some instances where poisson distribution may be successfully employed:

① No/- of deaths from a disease.

② No/- of suicides reported in a particular city

③ No/- of defective items in a packet of some numbers.

etc.

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Remarks

① Poisson distribution is the limiting case of binomial distribution under the following conditions.

i) $n \to \infty$ (Indefinetly large)

ii) $p \to 0$ (Indefinetly small)

iii) $\lambda = np$ (finite)

thus $p = \lambda/n$ & $q = 1 - \lambda/n$ where λ is a +ve real no.

Mean & Variance

$$\text{Mean} = \lambda = \mu_1' \quad , \quad Var = \mu_2 = \lambda$$

MGF of poisson Distribution:

Let $X \sim P(\lambda)$ then

$$M_x(t) = \sum_{x=0}^{\infty} e^{tx} \cdot p(x=x) = \sum_{x=0}^{\infty} e^{t} \cdot \frac{e^{-\lambda} \cdot \lambda^{x}}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda} \cdot (\lambda e^{t})^{x}}{x!}$$

$$= e^{-\lambda}\left\{1 + \lambda e^{t} + \frac{(\lambda e^{t})^2}{2!} + \cdots\right\} = e^{-\lambda} \cdot e^{\lambda e^{t}}$$

$$= e^{\lambda(e^{t}-1)}$$

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Characteristic function of the Poisson distribution

$$\phi_x(t) = \sum_{x=0}^{\infty} e^{itx} p(x,\lambda) = \sum_{x=0}^{\infty} e^{itx} \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= e^{-\lambda} \cdot \sum_{x=0}^{\infty} \frac{(\lambda e^{it})^x}{x!} = e^{-\lambda} \cdot e^{\lambda e^{it}} = e^{\lambda(e^{it}-1)}$$

Additive property of independent Poisson variates:

If x_i for $(i=1,2\ldots n)$ are independent Poisson variates with parameter λ_i for $i=1,2\ldots n$ respectively then $\sum_{i=1}^{n} x_i$ is also a Poisson variate with parameter $\sum_{i=1}^{n} \lambda_i$.

Proof:

$$M_{x_i}(t) = e^{\lambda_i(e^t-1)} \quad , i=1,2\ldots n$$

$$\{\because x_i \sim P(\lambda_i)\}$$

$$M_{x_1+x_2+\cdots+x_n}(t) = M_{x_1}(t) \cdot M_{x_2}(t) \cdots M_{x_n}(t)$$

$$= e^{\lambda_1(e^t-1)} \cdot e^{\lambda_2(e^t-1)} \cdots e^{\lambda_n(e^t-1)}$$

$$= e^{(\lambda_1+\lambda_2+\cdots+\lambda_n)(e^t-1)}$$

is MGF of Poisson variate with parameter $\lambda_1+\lambda_2+\cdots+\lambda_n$.

Note: Converse is not true.

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Note

① If $X_1 \sim P(\lambda_1)$ & $X_2 \sim P(\lambda_2)$ & X_1 & X_2 are independent. then $X_1 + X_2 \sim P(\lambda_1 + \lambda_2)$

but $(X_1 - X_2)$ is not a Poisson variate.

Sums

① A car hire firm has two cars, which it hires out day by day. The no/- of demands for a car on each day is distributed as a Poisson distribution with mean 1·5. Calculate the proportion of days on which i) neither car is used & ii) The proportion of days on which some demant is refused.

Sol

+ Let x be the R.V

+ $x \sim P(\lambda)$ & it denotes the no/- of demands for a car on any day.

$$P(x=x) = \frac{e^{-1.5}(1.5)^x}{x!}, \quad x = 0, 1, 2 \ldots$$

i) Proportion of days on which neither car is used :

$$P(x=0) = e^{-1.5} = 0.2231$$

Note : out of 1000 cars, No/ -of cars not used

$= 1000 \times P(x=0) = 223$ (approximately)

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ii) Proportion of days on which some demand is refused

$$P(X>2) = 1 - P(X \le 2)$$
$$= 1 - \{P(X=0) + P(X=1) + P(X=2)\}$$
$$= 1 - e^{-1.5}\left\{1 + 1.5 + \frac{(1.5)^2}{2}\right\}$$
$$= 1 - 0.2231 \times 3.625$$
$$= 0.19126$$

Note:

Out of 1000 cars, No/- of cars refused

$= 1000 \times 0.19126 = 191.26$

≈ 191 cars.

② In a Poisson frequence distribution, frequency corresponding to 3 successes is 2/3 times the frequency corresponding to 4 successes. Find the mean & standard deviation of the distribution.

Sol:-

Let X be a R-V & $X \sim P(\lambda)$

then the frequency ~~distribution~~ function is

$$f(x) = N \cdot P(X=x) = N \cdot \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \quad x = 0, 1, 2 \ldots$$

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Given,

$$f(3) = \frac{2}{3} f(4)$$

$$\cancel{N}.\frac{e^{-\lambda}.\cancel{\lambda^3}}{3!} = \frac{2}{3}\cancel{N}.\frac{e^{-\lambda}.\lambda^4}{4!}$$

$$\frac{1}{3!} = \frac{2}{3}\,\frac{\lambda}{4!}$$

$$\frac{1}{\cancel{3\times2\times1}} = \frac{2}{3}\times\frac{\lambda}{4\times\cancel{3\times2\times1}}$$

$$\cancel{2}\lambda = 3\times\cancel{4}^{2}$$

$$\lambda = 6$$

$\therefore$ Mean $= 6$ & variaence $= 6$

$$S.D = \sqrt{\lambda} = \sqrt{6}$$

(3) Suppose that the no/- of telephone calls coming into a telephone exchange between 10 A.M & 11 A.M. Say, X_1 is a random variable with poisson distribution with parameter 2. Similarly the no/ of calls arriving between 11 A.M. & 12 noon, say X_2 has a Poisson distribution with parameter 6. If X_1 & X_2 are independent, what is the probability that more than 5 calls come in-between 10 A.M ~~&~~ 12 Noon.?

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Sol

Given $X_1 \sim P(2)$ & $X_2 \sim P(6)$

Let $X = X_1 + X_2$

By additive property $X \sim P(\lambda_1 + \lambda_2)$

$\Rightarrow X \sim P(8)$ where $\lambda = \lambda_1 + \lambda_2 = 6+2 = 8$

P(x calls in between 10 AM & 12 noon)

$$= P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} = \frac{e^{-8}\, 8^x}{x!}, \quad x = 0, 1, 2 \ldots$$

P (More than 5 calls come in between 10 AM & 12 Noon)

$$= P(X>5) = 1 - P(X \leq 5)$$

$$= 1 - \sum_{x=0}^{5} \frac{e^{-8} \cdot 8^x}{x!}$$

$$= 1 - e^{-8} \left\{ \frac{8^0}{0!} + \frac{8^1}{1!} + \frac{8^2}{2!} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!} \right\}$$

$$= 1 - e^{-8} \left\{ 1 + 8 + \frac{\overset{32}{\cancel{64}}}{\cancel{2}} + \frac{8 \times 8 \times \overset{4}{\cancel{8}}}{3 \times \cancel{2} \times 1} + \frac{8 \times 8 \times 8 \times 8}{4 \times 3 \times 2 \times 1} + \frac{8 \times 8 \times 8 \times 8 \times 8}{5 \times 4 \times 3 \times 2 \times 1} \right\}$$

$$= 1 - e^{-8} \left\{ 1 + 8 + 32 + \frac{256}{3} + \frac{512}{3} + \frac{4096}{15} \right\}$$

$$= 1 - 0.1918 = 0.8088$$

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4) If X is a Poisson Variate such that $P(X=2) = 9P(X=4) + 90P(X=6)$ then find i) λ ii) Mean of X

Sol

Let $X \sim P(\lambda)$

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \quad x = 0, 1, 2 \ldots \ \& \ \lambda > 0$$

Given

$$P(X=2) = 9P(X=4) + 90P(X=6)$$

$$\frac{\cancel{e^{-\lambda}} \cdot \lambda^2}{2!} = \cancel{e^{-\lambda}}\left(9\frac{\lambda^4}{4!} + 90\frac{\lambda^6}{6!}\right)$$

$$\frac{\cancel{\lambda^2}}{\cancel{2}} = \frac{\cancel{\lambda^2}}{\cancel{2}}\left(\cancel{9}^{3}\frac{\lambda^2}{\cancel{12}_{4}} + \cancel{90}^{6}\frac{\lambda^4}{\cancel{6}\times\cancel{5}\times4\times\cancel{3}}\right)$$

$$1 = \frac{3}{4}\lambda^2 + \frac{\lambda^4}{4}$$

$$\lambda^4 + 3\lambda^2 = 4$$

$$\lambda^4 + 3\lambda^2 - 4 = 0$$

put $\lambda^2 = t$

$$t^2 + 3t - 4 = 0$$

-4

-1×4

$$(t-1)(t+4) = 0$$

$t = 1, -4.$ ($t = -4$ not possible)

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$\therefore\ t = 1$

$\lambda^2 = 1$

$\lambda = \pm 1$ (Since $\lambda > 0$)

$\therefore\ \lambda = 1$

& Mean $= \lambda = 1$

(5) If X & Y are independent Poisson variates such that

$$P(X=1) = P(X=2) \ \& \ P(Y=2) = P(Y=3).$$

Find the variance of $(X-2Y)$.

Sol

Let $X \sim P(\lambda)$ & $Y \sim P(\mu)$ then we have

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \quad x = 0, 1, 2 \ldots ; \ \lambda > 0$$

$$\& \ P(Y=y) = \frac{e^{-\mu} \cdot \mu^y}{y!}, \quad y = 0, 1, 2 \ldots, \ \mu > 0$$

Given $P(X=1) = P(X=2)$

$$\frac{\cancel{e^{-\lambda}} \cdot \lambda}{1!} = \frac{\cancel{e^{-\lambda}} \cdot \lambda^2}{2!}$$

$2\lambda = \lambda^2$

$\lambda^2 - 2\lambda = 0$

$\lambda(\lambda - 2) = 0$ Since $\lambda > 0$

$\therefore\ \lambda = 2$

$P(Y=2) = P(Y=3)$

$$\frac{\cancel{e^{-\mu}} \cdot \cancel{\mu^2}}{2!} = \frac{\cancel{e^{-\mu}} \cdot \mu^3}{3!}$$

$1 = \frac{\mu}{3}$

$\mu = 3$

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Now, $V(X) = \lambda = 2$, $V(Y) = M = 3$

$$V(X-2Y) = 1^2 V(X) + 4V(Y) = 2 + 4(3)$$
$$= 2+12$$
$$= 14$$

(5) Fit a Poisson distribution to the following data which gives the number of doddens in a sample of clover seeds.

No. of dodders (x)	:	0	1	2	3	4	5	6	7	8 Total
Observed frequency (f)	:	56	156	132	92	37	22	4	0	1 500

Sol

$$\text{Mean} = \frac{\sum fx}{N} = \frac{986}{500} = 1.972 = \lambda$$

& If $X \sim P(\lambda)$ then its pmf is given by

$$P(X=x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

For $X=0$, $P(X=0) = \frac{e^{-1.972} \cdot \lambda^0}{0!} = e^{-1.972} = 0.1392$

For $X=1$, $P(X=1) = \frac{e^{-1.972}(1.972)^1}{1!} = 0.27455$

For $X=2$, $P(X=2) = \frac{e^{-1.972}(1.972)^2}{2!} = 0.27006$

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For $x=3$, $$p(x=3) = \frac{e^{-1.972}(1.972)^3}{3!} = 0.17793$$

For $x=4$, $$P(x=4) = \frac{e^{-1.972}(1.972)^4}{4!} = 0.10964$$

For $x=5$, $$P(x=5) = \frac{e^{-1.972}(1.972)^5}{5!} = 0.03459$$

For $x=6$, $$P(x=6) = \frac{e^{-1.972}(1.972)^6}{6!} = 0.01137$$

For $x=7$, $$P(x=7) = \frac{e^{-1.972}(1.972)^7}{7!} = 0.00320$$

For $x=8$, $$P(x=8) = \frac{e^{-1.972}(1.972)^8}{8!} = 0.00078$$

x:	0	1	2	3	4	5	6	7	8
f: Observed frequency	56	156	132	92	37	22	4	0	1
Expected frequency: $N \cdot P(X=x)$ $= 500 \cdot p(x=x)$	70	137	135	88	44	17	6	2	0

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Normal Distribution:

A random variable X is said to have a normal distribution with parameters μ (called mean) & σ^2 (called variance) if its pdf is given by the

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

(or) $$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

, $-\infty < x < \infty$ &
$-\infty < \mu < \infty$
$\sigma > 0$

Where μ - mean
σ - Standard Deviation

Note

① ~~Def~~ $X \sim$ Normally distributed is denoted by
$X \sim N(\mu, \sigma^2)$

② If $X \sim N(\mu,\sigma^2)$ then $Z = \frac{X-\mu}{\sigma}$ is a Standard Normal Variate with $E(Z)=0$ & $V(Z)=1$ & we write $Z \sim N(0,1)$

④ The pdf of the standard normal variate Z is given by

* ~~$P(-\infty < Z < \infty)$~~

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$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$$

& the corresponding distribution function is denoted by $\Phi(z)$ & it is given by

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \phi(u)\,du = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-u^2/2}\,du.$$

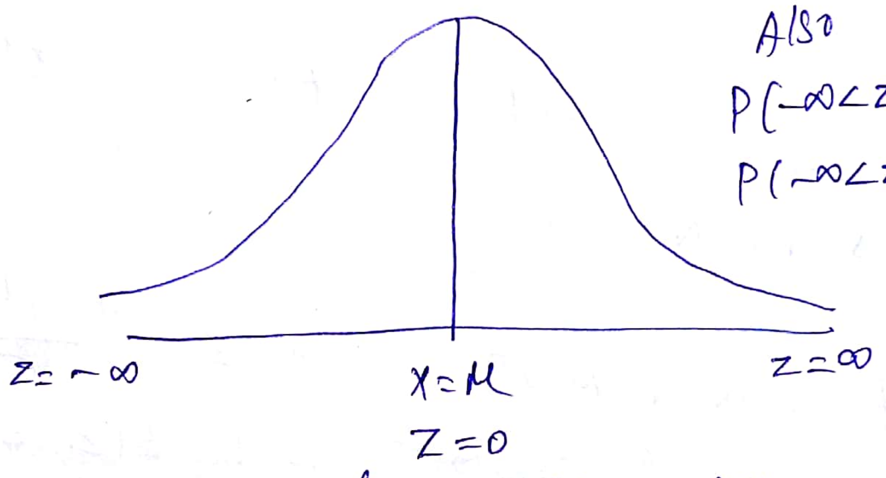
<u>Remarks</u>

① $\Phi(-z) = 1 - \Phi(z)$, $z > 0$

② $P(a \le X \le b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$

where $X \sim N(\mu, \sigma^2)$

③



Also
$P(-\infty < Z < \infty) = 1$
$P(-\infty < Z < 0) = P(0 < Z < \infty) = 0.5$

Fig:- Normal Probability curve

④ The graph of $f(x)$ is famous 'bell-shaped' curve. The top of the bell is directly above the mean μ. For large values of σ, the curve tends to flatten out & for small values of σ it has a sharp peak.

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Binomial distribution :

1. $n \to \infty$
2. neither p nor q is very small.
3.

Note

1. Normal distribution can also be obtained as a limiting case of poisson distribution with parameter $\lambda \to \infty$.

Properties

1. The curve is bell-shaped & symmetrical about the line $x = \mu$.
2. Mean, median & Mode of the distribution coincide.
3. As x increases numerically, $f(x)$ decreases rapidly, the maximum probability occuring at the point $x = \mu$ & it is given by $[p(x)]_{max} = \frac{1}{\sigma\sqrt{2\pi}}$
4. $\beta_1 = 0$ & $\beta_2 = 3$
5. Since $f(x)$ being the probability, can never be negative, no portion of the curve lies below the x axis.
6. x axis is an asymptote to the curve.

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* MGF of Normal distribution:

$$M_x(t) = e^{\mu t + t^2\sigma^2/2}$$

* MGF of a Standard Normal Variate:

$$M_z(t) = e^{t^2/2}$$

* If $x_1, x_2, x_3 \dots x_n$ are independent Normal variates ~~then~~ & $x_i \sim N(\mu_i, \sigma_i^2)$ for any i

then $\sum_{i=1}^{n} x_i \sim N\left(\sum_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \sigma_i^2\right)$

Which is known as additive property of the Normal distribution.

Sums

If x is a normal variate with mean 30 & SD 5. Find the Probabilities that i) $26 \leq x \leq 40$ ii) $x \geq 45$ & iii) $|x-30| > 5$.

Sol

Given $\mu = 30$, $\sigma = 5$

i) $P(26 \leq x \leq 40)$ since $Z = \frac{x-\mu}{\sigma}$

For $x = 26$

$$z = \frac{26-30}{5} = -0.8$$

For $x = 40$,

$$z = \frac{40-30}{5} = 10/5 = 2$$

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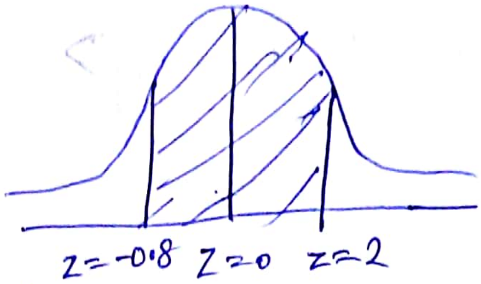
$\therefore P(26 \le X \le 40)$

$= P(-0.8 \le Z \le 2)$

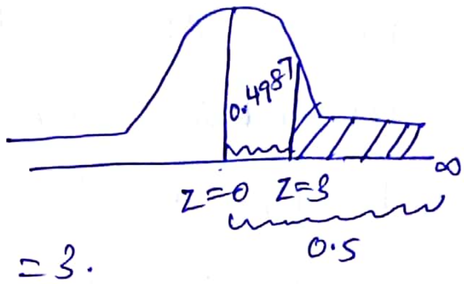


$= P(-0.8 \le Z \le 0) + P(0 \le Z \le 2)$

$= P(0 \le Z \le 0.8) + P(0 \le Z \le 2)$

$= 0.2881 + 0.4772$

$= 0.7653$



ii) $P(X \ge 45)$

For $X = 45$, $\quad Z = \dfrac{45-30}{5} = 15/5 = 3.$

$\therefore P(X \ge 45) = P(Z \ge 3) = 0.5 - P(0 \le Z \le 3)$

$= 0.5 - 0.49865 = 0.00135$

iii) $P(|X-30| \le 5) = P(-5 \le X-30 \le 5)$

$= P(30-5 \le X \le 30+5)$

$= P(25 \le X \le 35)$

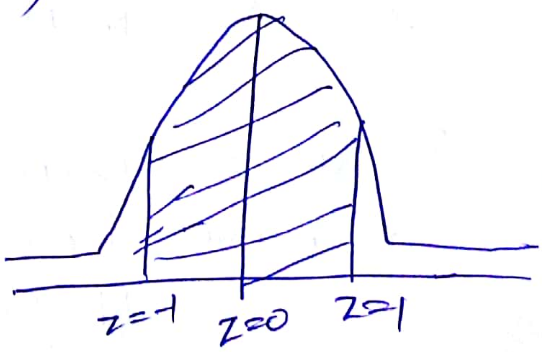
$= P(-1 \le Z \le 1) = 2P(0 \le Z \le 1)$

For $X = 25$

$z = \dfrac{25-30}{5} = -5/5 = -1$

For $X = 35$

$Z = \dfrac{35-30}{35} = 5/5 = 1$



$= 2 \times 0.3413 = 0.6826$

$\therefore P(|x-30| > 5) = 1 - P(|x-30| \leq 5)$

$= 1 - 0.6826 = 0.3174$

② The local Authorities in a certain city instal 10,000 electric lamps in the streets of the city. If these lamps have an average life of 1000 burning hours with a standard deviation of 200 hours, assuming normality, what number of lamps might be expected to fail

i) In the first 800 burning hours?

ii) between 800 & 1200 burning hours?

After what period of burning hours would you expect that

a) 10% of the lamps would fail?

b) 10% of the lamps would be still burning?

Sol

Let X be the R.V

& $X \sim N(\mu, \sigma^2)$ where $\mu = 1000$ & $\sigma = 200$ | $N = 10000$ electric lamps

i) No/- of lamps which fails in the first 800 burning hours

$= N \cdot P(X < 800)$

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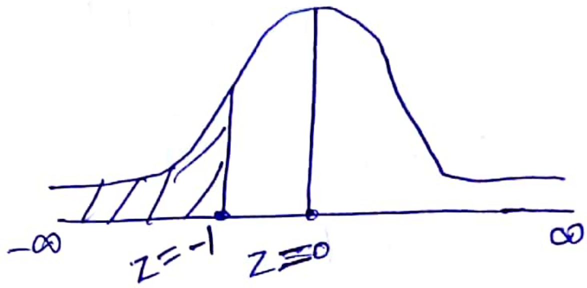
$P(X<800) = P$ (The bulb fails in the first 800 burning hours)

For $X = 800$

$$Z = \frac{800-1000}{200} = \frac{-200}{200}$$
$$= -1$$



$$P(X<800) = P(Z<-1)$$
$$= \cancel{P(Z>1)}$$
$$= P(-\infty < Z < -1)$$
$$= P(-\infty < Z < 0) - P(-1 < Z < 0)$$
$$= 0.5 - P(0<Z<1)$$
$$= 0.5 - 0.3413 = 0.1587$$

$\therefore$ out of 10,000 bulbs, no/- of bulbs which falls in the first 800 hrs $\Big\} = 10{,}000 \times 0.1587$

$= 1587$ bulbs

ii) out of 10,000 bulbs, no/- of bulbs which fails in between 800 & 1200 burning hours $\cancel{N.P}$

$$= N.P(800 < X < 1200)$$

For $X = 800$, $Z = \frac{800-1000}{200} = \frac{-200}{200} = -1$

For $X = 1200$, $Z = \frac{1200-1000}{200} = \frac{200}{200} = 1$

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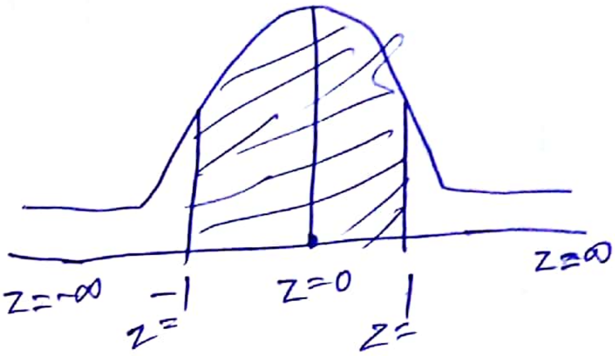
$\therefore P(800 < x < 1200)$

$= P(-1 < z < 1)$

$= 2P(0 < z < 1)$

$= 2(0.3413)$

$= 0.6826$



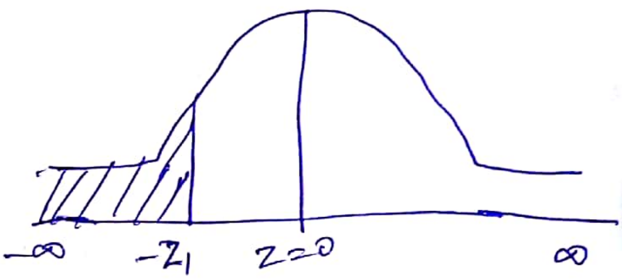
a) Let 10% of the bulbs fail after x_1 hours of burning life.

To find: x_1 such that $P(x < x_1) = 0.10$

When $x = x_1$, $z = \dfrac{x_1 - 1000}{200} = -z_1$ (say)

$\therefore P(z < -z_1) = 0.10$

$\Rightarrow P(-\infty < z < -z_1) = 0.10$



$P(-\infty < z < 0) - P(0 < z < -z_1) = 0.10$

$0.5 - 0.10 = P(z_1 < z < 0) = P(0 < z < z_1)$

$\therefore P(0 < z < z_1) = 0.40$

$P(0 < z < 1.28) = 0.40$

$z_1 = 1.28$

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Since $\frac{x_1 - 1000}{200} = -1.28$

$x_1 - 1000 = -256$

$x_1 = 1000 - 256 = 744$

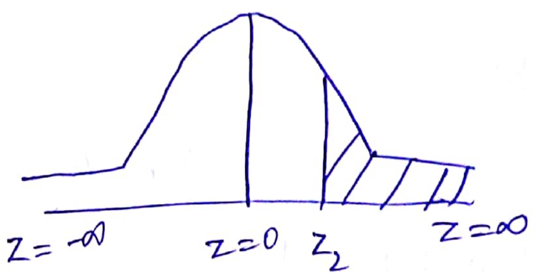
$\therefore$ After 744 hours of burning life, 10% of the bulbs will fail.

b) Let 10% of the bulbs be still burning after (say) x_2 hours of burning life.

Then $P(X > x_2) = 0.10$

$P(Z > z_2) = 0.10$ where $z_2 = \frac{x_2 - 1000}{200}$

$P(z_2 < Z < \infty) = 0.10$



$P(0 < Z < \infty) - P(0 < Z < z_2) = 0.10$

$P(0 < Z < \infty) - 0.10 = P(0 < Z < z_2)$

$P(0 < Z < z_2) = 0.5 - 0.10$
$= 0.40$

$P(0 < Z < 1.28) = 0.40$

$z_2 = 1.28$

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$$\frac{x_2 - 1000}{200} = 1.28$$

$$x_2 - 1000 = 256$$

$$x_2 = 1256$$

$\therefore$ After 1256 hours of burning life, 10% of the bulbs will be still burning.

(2) Two independent random variates X & Y are both normally distributed with means 1 & 2 and S.D's 3 & 4 respectively. If $U = X - Y$, write the probability density function of U. Also state the median, S.D & mean of the distribution of U. Find probability $(U+1 \leq 0)$.

<u>Sol:</u>

Since $X \sim N(1, 9)$ & $Y \sim N(2, 16)$ are independent.

$$U = X - Y \sim N(1-2, 9+16)$$

$$U = X - Y \sim N(-1, 25)$$

$\therefore \mu = -1, \sigma^2 = 25$

$\sigma = 5$

Pdf of U is

$$f(U) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{U+\mu}{\sigma}\right)^2}$$

$$= \frac{1}{5\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{U+1}{5}\right)^2}, \quad -\infty < U < \infty$$

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For U,

$$\text{Median} = \text{Mean} = -1$$

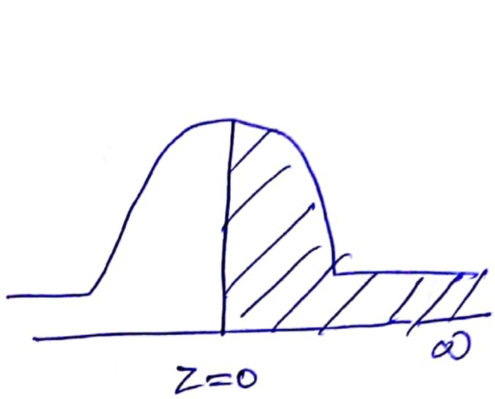
$$\& \ Sd = \sqrt{25} = 5$$

$$P(U+1 \leq 0) = P(U \leq -1)$$

$$Z = \frac{U - M}{\sigma}$$

$$\therefore \text{For } U = -1, \quad Z = \frac{-1+1}{5}$$

$$= 0$$



$$\therefore P(U \leq -1) = P(Z \leq 0)$$

$$= 0.5$$

(3) In an examination it is laid down that a
Student passes if he secures 30% (or) more marks.
He is placed in the first, second or third division
according as he secures 60% or more marks, & marks between 30%.
between 45% & 60% marks
~~And~~ 45% respectively. He gets distinction in
case he secures 80% (or) more marks. It is
noticed from the result that 10% of the Students
failed in the examination, whereas 5% of them
obtained distinction. Calculate the percentage of
Students placed in the second division. (Assume
normal distribution of marks.)

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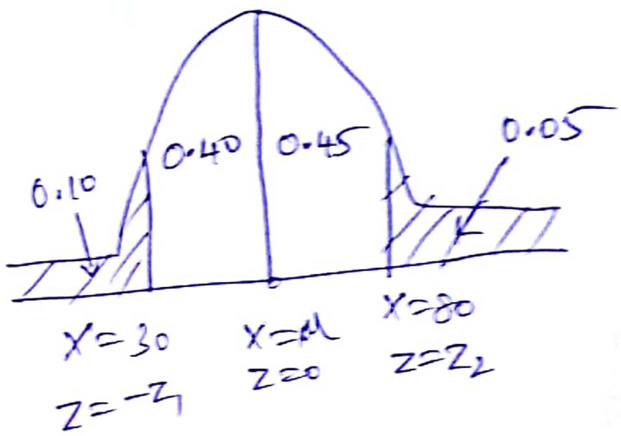
Sol

Let ~~the~~ X be the R-V & $X \sim N(\mu, \sigma^2)$.

It denotes the marks in the examination.

Given

$P(X<30) = 0.10$ & $P(X \geq 80) = 0.05$



When $X=30$

$$Z = \frac{30-\mu}{\sigma} = -Z_1 \text{ (Say)}$$

When $X=80$,

$$Z = \frac{80-\mu}{\sigma} = Z_2 \text{ (Say)}$$

$$\therefore P(0<Z<Z_2) = (0.5-0.05)$$
$$= 0.45$$

$$\& \quad P(0<Z<Z_1) = P(-Z_1<Z<0)$$
$$= 0.5 - 0.10 = 0.40$$

$\therefore$ From Standard Normal table

$$Z_1 = 1.28 \quad \& \quad Z_2 = 1.64$$

$$\therefore \frac{30-\mu}{\sigma} = -1.28$$

$$\frac{\mu-30}{\sigma} = 1.28$$

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$$\mu - 30 = 1.28\sigma \quad \text{— (1)}$$

$$\frac{80-\mu}{\sigma} = 1.64$$

$$80-\mu = 1.64\sigma \quad \text{— (2)}$$

$$-\cancel{\mu} + 80 = 1.64\sigma$$
$$\cancel{\mu} - 30 = 1.28\sigma$$

$$50 = 2.92\sigma$$

$$\sigma = \frac{50}{2.92} = 17.12$$

$$\mu = 30 + 1.28\sigma = 30 + 1.28(17.12)$$

$$= 30 + 21.9136$$

$$= 51.9136$$

$$\simeq 52$$

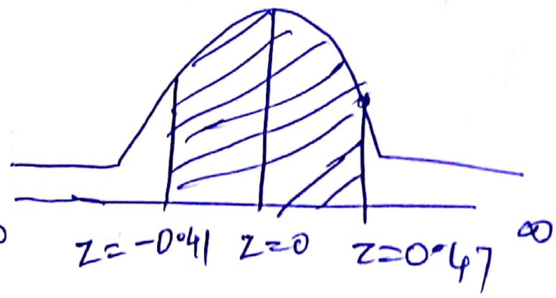
ii) P(Candidate is placed in the Second division)

$$= P(45 < x < 60)$$

$$= P(-0.41 < z < 0.47)$$

Where $x = 45$

$$z = \frac{45-52}{17.12} = -0.41$$



Where $x = 60$

$$z = \frac{60-52}{17.12} = 0.47$$

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$\therefore P(-0.41 < z < 0.47)$

$= P(-0.41 < z < 0) + P(0 < z < 0.47)$

$= P(0 < z < 0.41) + P(0 < z < 0.47)$

$= 0.1591 + 0.1808$

$= 0.3399$

$= 0.34$ (approximately)

$\therefore$ 34% Candidates got second division in the examination.

<u>General Gamma distribution (or) Erlang distribution</u>

<u>Def:</u> A continuous R.V x is said to follow Erlang distribution (or) General Gamma distribution with parameters $\lambda > 0$ & $k > 0$ if its probability density function is given by

$$f(x) = \begin{cases} \dfrac{\lambda^{x} \cdot x^{k-1} \cdot e^{-\lambda x}}{\Gamma k}, & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

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Since $\int_0^\infty f(x)dx = \frac{\lambda^k}{\Gamma k} \int_0^\infty x^{k-1} \cdot e^{-\lambda x} dx$

Let $t = \lambda x$
$dt = \lambda dx$
$x = t/\lambda$

$$= \frac{\cancel{\lambda^k}}{\Gamma k} \int_0^\infty \frac{t^{k-1}}{\cancel{\lambda^{k-1}}} \cdot e^{-t} \frac{dt}{\cancel{\lambda}}$$

$$= \frac{1}{\Gamma k} \int_0^\infty t^{k-1} e^{-t} dt$$

$$= \frac{1}{\cancel{\Gamma k}} \cancel{\Gamma k} = 1$$

Hence $f(x)$ is a legitimate density function.

Case-1

When $\lambda = 1$, The Erlang distribution is Called as Gamma distribution (or) Simple Gamma distribution with parameter k.

Then $f(x) = \begin{cases} \frac{1}{\Gamma k} x^{k-1} \cdot e^{-x}, & x > 0 \\ 0 & \text{otherwise} \end{cases}$

Case - 2 When $k = 1$, The Erlang distribution reduced to exponential distribution with

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parameter $\lambda > 0$ & its pdf

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Some times the erlang distribution itself is called Gamma distribution.

Mean & Variance of Erlang distribution (or) General Gama distribution

* Mean $= E(X) = k/\lambda$
* Variance $= k/\lambda^2$

Mean & Variance of Gamma distribution (Simple) (for $\lambda = 1$)

* Mean $= k$
* Variance $= k$

~~Additio~~ MGF of Gamma distribution

$$M_x(t) = \left(1 - t/a\right)^{-n}$$

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Additive property

If $X_1, X_2, \cdots X_r$ are r independent Gamma variate with parameters n_i then $X_1+X_2+\cdots+X_r$ is also a gamma random variable with parameter $n_1+n_2+\cdots+n_r$

Sums

① In a certain city, the daily consumption of electric power in millions of kilowatt hours can be ~~treated~~ as a R.V having an Erlang distribution with parameters $\lambda = 1/2$ & $k=3$. If the power plant of this city has a daily capacity of 12 millions kilowatt-hours, what is the probability that this power supply will be inadequate on any given day.

Sol

X be the R.V & it follows Erlang distribution.

X denotes the daily consumption of electric power (in millions of kilowatt hours.)

$$f(x) = \begin{cases} \dfrac{\lambda^k \cdot x^{k-1} \cdot e^{-\lambda x}}{\Gamma k}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

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Given $\lambda = 1/2$ & $k = 3$

$$f(x) = \begin{cases} \left(\frac{1}{2}\right)^3 x^2 \cdot e^{-x/2}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

P (The power supply is inadiquate)

$$= P(X > 12) = \int_{12}^{\infty} f(x)\, dx$$

$$= \int_{12}^{\infty} \frac{1}{\Gamma 3} \cdot \frac{1}{2^3} x^2 e^{-x/2}\, dx$$

$$= \cancel{\frac{1}{16}\left\{ x^2 \left(\frac{e^{-x/2}}{-1/2} \right) \right.}$$

$$u = x^2, \quad \int dv = \int e^{-x/2}$$
$$u' = 2x \qquad v = -2e^{-x/2}$$
$$u'' = 2 \qquad v_1 = 4e^{-x/2}$$
$$v_2 = -8e^{-x/2}$$

$$\therefore P(X > 12) = \frac{1}{16} \left\{ x^2 \left(-2e^{-x/2}\right) - 2x\left(4e^{-x/2}\right) + 2\left(-8e^{-x/2}\right) \right\}$$

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$= \frac{1}{16} e^{-6} [288 + 96 + 16]$

$= 25 e^{-6} = 0.0625$

Exponential distribution

A Continuous Random Variable X is said to follow an exponential distribution (or) negative exponential distribution with parameter $\lambda > 0$ if its pdf is

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Since $\int_0^\infty f(x)\, dx = \int_0^\infty \lambda e^{-\lambda x}\, dx = 1$

$\therefore$ $f(x)$ is a legitimate density function.

Mean & Variance of the Exponential distribution

Mean $= \mu_1' = 1/\lambda$	Note: $\mu_r' = r!/\lambda^r$
$Var(x) = 1/\lambda^2$	$\mu_2' = 2!/\lambda^2 = 2/\lambda^2$

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Memoryless property of the exponential distribution:

If x is exponentially distributed then

$$P(X > s+t \mid X > s) = P(X > t) \text{ for any } s, t > 0$$

Note

$$P(X > t) = \int_t^{\infty} \lambda e^{-\lambda x} dx = \left(-e^{-\lambda x}\right)_t^{\infty} = e^{-\lambda t}$$

Sums:

The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$

a) What is the probability that the repair time exceeds 2 hrs?

b) What is the conditional probability that a repair takes atleast 10 hrs given that its duration exceeds 9 hrs?

Sol

Let X be the R.V. $X \sim$ exponential distribution.

& X denotes the time to repair the machine.

$$\therefore f(x) = \lambda e^{-\lambda x} = \frac{1}{2} e^{-x/2}, \; x > 0$$

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a) $P(X>2) = \int_2^\infty \frac{1}{2} e^{-x/2} \, dx$

$$= \left(-e^{-x/2}\right)_2^\infty = -e^{-\infty} + e^{-1}$$

$$= 0 + e^{-1} = 0.3679$$

b) $P(X \geq 10 \mid X>9) = P(X>1)$ (using memory less property)

$$= \int_1^\infty \frac{1}{2} e^{-x/2} \, dx$$

$$= \left(-e^{-x/2}\right)_1^\infty = 0 + e^{-1/2} = 0.6065$$

Note

* Mean $= 1/\lambda = 2$
* Variance $= 2/\lambda^2 - 1/\lambda = 8 - 2 = 6$

Weibull distribution

A continuous r.v X is said to follow weibull distribution with parameters $\alpha, \beta > 0$ then its pdf is $f(x) = \alpha\beta \, x^{\beta-1} e^{-\alpha x^\beta}, \; x > 0$

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Note

When $\beta = 1$ weibull distribution becomes exponential distribution with parameter 'α'.

Mean & Variance of Weibull distribution:

In General $\mu_r' = \alpha^{-r/\beta} \Gamma(r/\beta + 1)$

$\therefore$ Mean $= \alpha^{-1/\beta} \Gamma(1/\beta + 1)$

$$Var(x) = \alpha^{-2/\beta} \left\{ \Gamma(2/\beta + 1) - \left(\Gamma(1/\beta + 1) \right)^2 \right\}$$

Where $E(x^2) = \alpha^{-2/\beta} \Gamma(2/\beta + 1)$

Sums

① Each of the 6 tubes of a radio set has a life length (in years) which may be considered as a R.V that follows a weibull distribution with parameters $\alpha = 25$ & $\beta = 2$. If these tubes function independently of one another, what is the probability that no tube will have to be replaced during the first two months of service?

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Sol

Let X be R.V & $X \sim$ Weibull distribution

i.e, $X \sim WEB(25, 2)$

$\alpha = 25$, $\beta = 2$

& X denotes the life length of each tube

$$\therefore f(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}}, \quad x > 0$$

Given $\alpha = 25$, $\beta = 2$

$$f(x) = 50\,x\,e^{-25x^2}, \quad x > 0$$

P (A tube is not to be replaced during the first 2 months)

$$= P(X > 1/6) = \int_{1/6}^{\infty} 50x\,e^{-25x^2}\,dx$$

$$= \left(-e^{-25x^2}\right)_{1/6}^{\infty} = e^{-25/36}$$

$\therefore$ P(All the 6 tubes are not to be replaced during the first 2 months)

$$= \left(e^{-25/36}\right)^6 \quad \text{(By Independence)}$$

$$= e^{-25/6} = 0.0155$$

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(2) If the life x (in years) of a certain type of car has a weibull distribution with the parameter $\beta = 2$, find the value of the parameter α, given that probability that the life of the car exceeds 5 years is $e^{-0.25}$. For these values of α & β find the mean & variance of x?

Sol

Given $\beta = 2$, $P(X>5) = e^{-0.25}$

$$\text{Pdf} = f(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}}, \quad x>0$$

$$= 2\alpha x e^{-\alpha x^2}, \quad x>0$$

& $P(X>5) = e^{-0.25}$

$$\int_5^{\infty} 2\alpha x e^{-\alpha x^2}\, dx = e^{-0.25}$$

$$\left(-e^{-\alpha x^2}\right)_5^{\infty} = e^{-0.25}$$

$$0 + e^{-\alpha(25)} = e^{-0.25}$$

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$$e^{-25\alpha} = e^{-0.25}$$

$$25\alpha = 0.25$$

$$\alpha = 1/100$$

$$\text{Mean} = E(x) = \alpha^{-1/\beta}\,\Gamma\left(\tfrac{1}{\beta}+1\right)$$

$$= \left(\tfrac{1}{100}\right)^{-1/2}\,\Gamma\tfrac{3}{2}$$

$$= 10 \times \tfrac{1}{2}\,\Gamma\tfrac{1}{2} = 5\sqrt{\pi}$$

$$Var(x) = \alpha^{-2/\beta}\left[\Gamma\left(\tfrac{2}{\beta}+1\right) - \left(\Gamma\left(\tfrac{1}{\beta}+1\right)\right)^2\right]$$

$$= \left(\tfrac{1}{100}\right)^{-1}\left[\Gamma 2 - \left(\Gamma\tfrac{3}{2}\right)^2\right]$$

$$= 100\left[1 - \left(\tfrac{1}{2}\sqrt{\pi}\right)^2\right]$$

$$= 100\left(1 - \pi/4\right) = 25(4-\pi)$$

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