Homework 4
Image Restoration, Tomographic Reconstruction & Sharpening

Veda Prakash Mohanarangan

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GitHub Repository:
https://github.com/VedaMahi321/mtech-spml-image-processing-homework4.git

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1 Part 4.1 — Wiener Filter for Image Restoration

1.1 Theory

The degradation model used for atmospheric turbulence is:

$$H(u, v) = \exp\left(-k \cdot (u^2 + v^2)^{5/6}\right),$$

where k controls the blur strength.

With additive noise $N(u, v)$, the degraded image is:

$$G(u, v) = H(u, v)F(u, v) + N(u, v).$$

The Wiener filter estimate is:

$$\hat{F}(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + \frac{S_n(u,v)}{S_f(u,v)}} G(u, v).$$

1.2 Results

(a) Original input (b) Degraded (blur+noise) (c) Restored (Wiener)

Figure 1: Wiener restoration process.

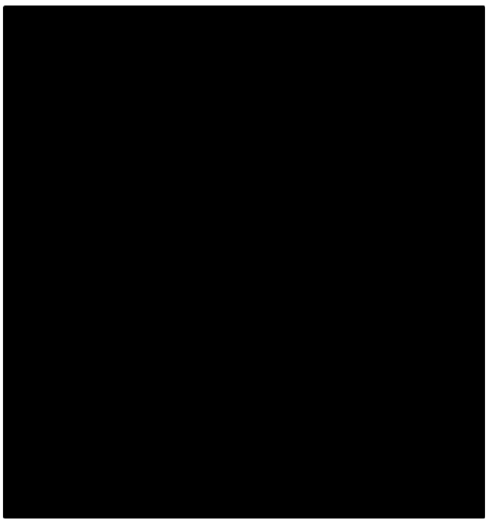


Figure 2: Point Spread Function (PSF) from turbulence model.

1.3 Metrics

From the log file:

Channel	MSE	PSNR (dB)	SSIM
R	–	4.70	–
G	–	4.59	–
B	–	4.60	–
Overall	0.3443	4.63	0.0050

Table 1: Part 4.1 — Wiener restoration metrics.

1.4 Observations

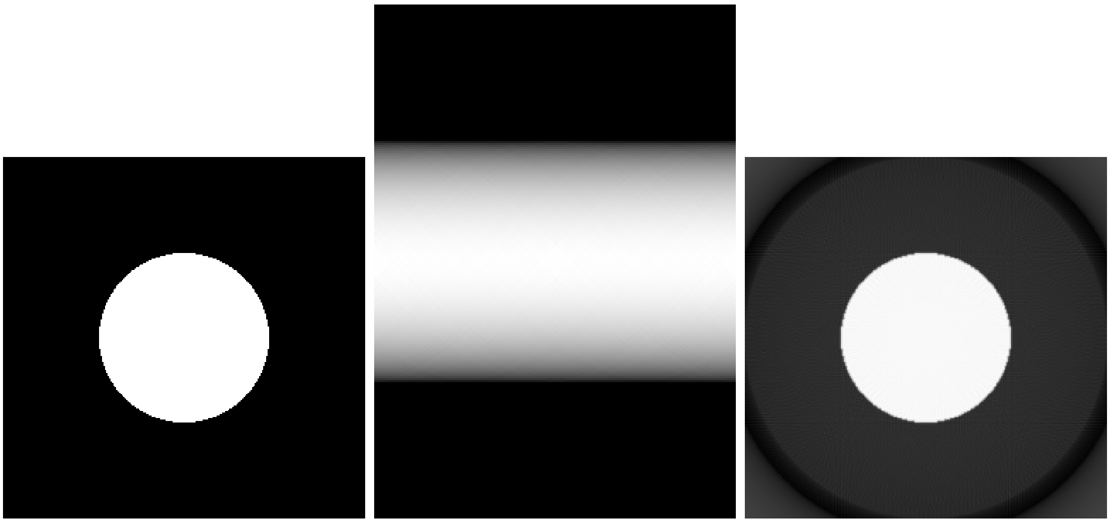
- The degraded image shows strong blur and noise.
- The Wiener filter attempts restoration but quantitative metrics are poor (PSNR $\approx$ 4.6 dB, SSIM = 0.005).
- This shows Wiener filtering is sensitive to turbulence strength and noise variance estimation.

2 Part 4.2 — Parallel Projection and Filtered Backprojection

2.1 Theory

A parallel projection (Radon transform) integrates object values along lines at multiple angles. Filtered Backprojection (FBP) reconstructs by filtering each projection with a ramp filter and then backprojecting.

2.2 Results



(a) Original object (disc) (b) Sinogram (180 angles) (c) Reconstruction (FBP)

Figure 3: Tomographic projection and reconstruction results.

2.3 Metrics

Metric	MSE	PSNR (dB)	SSIM
FBP Reconstruction	0.022662	16.45	0.1823

Table 2: Part 4.2 — FBP reconstruction metrics.

2.4 Observations

- The sinogram is consistent with a circular object.
- The FBP reconstruction restores the general shape but introduces streaking artifacts.
- PSNR $\approx$ 16.5 dB and SSIM = 0.18 confirm limited fidelity compared to the original.

3 Part 4.3 — Laplacian-based Image Sharpening

3.1 Theory

Sharpening is achieved with the Laplacian operator:

$$I_{\text{sharp}} = I + \alpha \nabla^2 I.$$

We compare sharpening in RGB vs. HSV (Value channel).

3.2 Results

(a) Sharpened (RGB) (b) Sharpened (HSV Value) (c) Difference (RGB vs HSV)

Figure 4: Comparison of Laplacian sharpening in RGB and HSV.

3.3 Metrics

Method	MSE	PSNR (dB)	SSIM
RGB Sharpening	0.002106	26.77	0.8780
HSV Sharpening	0.001628	27.88	0.8980

Table 3: Part 4.3 — Laplacian sharpening metrics.

3.4 Observations

- RGB sharpening improves detail but can distort colors.
- HSV sharpening preserves color fidelity and achieves slightly higher PSNR/SSIM.
- Quantitatively, HSV sharpening is superior (PSNR = 27.9 dB, SSIM = 0.898).

4 Conclusion

- Wiener filter restoration was weak due to strong blur and noise — PSNR $\approx$ 4.6 dB.
- Filtered Backprojection reconstructed object shape but with artifacts — PSNR $\approx$ 16.5 dB.
- Laplacian sharpening worked better in HSV than RGB, both visually and quantitatively.