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HW11

EE 16A HW 11

(1) Mechanical Correlation

A IPython → 40

B IPython $[-6, 6, 10, 22, 18, 6, -6, 10, -22, -18]^T$

(2) GPS Receivers

See IPython

(3) Finding Signals in Noise

A The largest inner products are MUCH smaller than the normalized inner product with itself.

B Yes, this moves the spike over

C Some, close to zero, but not zero

D Yes, we can clearly see the shift of the spikes

E As the noise gets higher, it becomes more difficult to differentiate spikes from noise

F We can only find 1 delay. We need an extra reference point (similar concept as in triangulation)

G Not possible to find s_2 accurately with such high noise.

H This strategy accurately cancels out the noise.

I This holds as α_θ is insignificant that it does not factor significantly in the equation

J Estimates are fairly close, . If the noise is insignificant so cancels with the noise, this should detect true

(4) Mechanical least Squares

A $\frac{\langle a, b\rangle - \langle b, a\rangle}{\langle b, b\rangle} = $ $\frac{3+12+7}{9+36+49+64} - \frac{22}{6+24+42+64}$

$\frac{22}{4+36+49+64} = $ $\frac{22}{158} \approx x$

50+50+58

B $\frac{\langle b\ b\rangle - \langle b\ a\rangle}{\langle b\ b\rangle} = \frac{3^2+6^2+7^2+8^2 - 6 - 24 - 42 \cdot 64}{158}$ 22

$\boxed{= \frac{22}{158}}$

C The error vector is orthogonal (near orthogonal) to the columns of A as it is regressed $\|\vec{b} - A\vec{x}\|^2$ with x approaches 0. If the inner product of 2 vectors is 0, they are orthogonal.

Therefore, the error vector is very close to being orthogonal to the columns of A

$\Rightarrow$ orthogonal

5 Linear Analysis

A $a(x^2 + y^2) + dx + ey = 1$

B $ax^2 + bxy + cy^2 + dx + ey = 1$

C

D See Ipython

EE16A Homework 11

Question 1: Mechanical Correlation

In [51]:
```
import numpy as np

def vector_compare(desired_vec, test_vec):
    """This function compares two vectors, returning a number.
    The test vector with the highest return value is regarded as being closest to the
    # Hint: Use transpose for the first argument of np.dot
    # YOUR CODE HERE
    return np.dot(np.transpose(desired_vec), test_vec)

def crossCorrelate(vec1, vec2):
    output = []
    for i in range(len(vec2)):
        output.append(vector_compare(vec1, vec2))
        vec2 = np.roll(vec2, 1)
    return output

def autoCorrelate(vec1):
    return crossCorrelate(vec1, vec1)

array1 = np.array([2, -2, 2, -2, -2, -2, 2, -2, 2, 2])
array2 = np.array([1, 2, 3, 4, 5, 6, 7, 6, 5, 4])

print (autoCorrelate(array1))
print (crossCorrelate(array1, array2))
```

```
[40, -8, 8, -8, 8, -40, 8, -8, 8, -8]
[-6, 6, 10, 22, 18, 6, -6, -10, -22, -18]
```

Question 2: GPS Receivers

In [52]:
```
%pylab inline
import numpy as np
import matplotlib.pyplot as plt
import scipy.io
```

```
Populating the interactive namespace from numpy and matplotlib
```

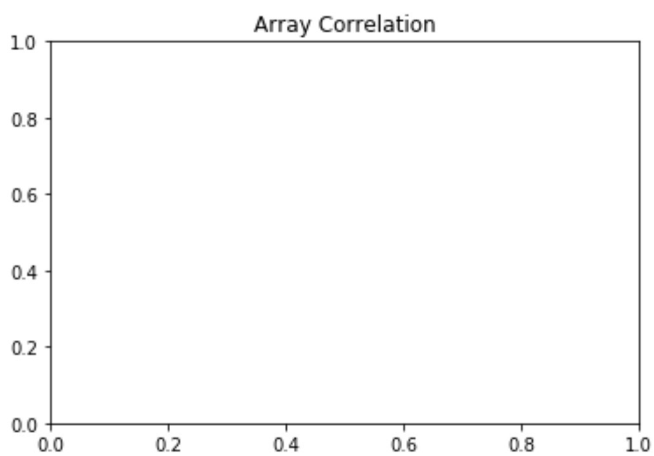
In [53]:
```
## RUN THIS FUNCTION BEFORE YOU START THIS PROBLEM
## This function will generate the gold code associated with the satellite ID using li
## The satellite_ID can be any integer between 1 and 24
def Gold_code_satellite(satellite_ID):
    codelength = 1023
    registerlength = 10

    # Defining the MLS for G1 generator
    register1 = -1*np.ones(registerlength)
    MLS1 = np.zeros(codelength)
    for i in range(codelength):
        MLS1[i] = register1[9]
        modulo = register1[2]*register1[9]
        register1 = np.roll(register1,1)
        register1[0] = modulo

    # Defining the MLS for G2 generator
    register2 = -1*np.ones(registerlength)
    MLS2 = np.zeros(codelength)
    for j in range(codelength):
        MLS2[j] = register2[9]
        modulo = register2[1]*register2[2]*register2[5]*register2[7]*register2[8]*regi
        register2 = np.roll(register2,1)
        register2[0] = modulo

    delay = np.array([5,6,7,8,17,18,139,140,141,251,252,254,255,256,257,258,469,470,47
    G1_out = MLS1;
    shamt = delay[satellite_ID - 1]
    G2_out = np.roll(MLS2,shamt)

    CA_code = G1_out * G2_out

    return CA_code
```

Part (a)

In [58]:

```
# Can see huge amount of noise

def array_correlation(array1, array2):
    """ This function should return two arrays or a matrix with one row corresponding
    the offset and other to the correlation value
    """

    return np.correlate(array1, array2, full)

# Plot the auto-correlation of satellite 10 with itself. Your signal should be centere
# at offset = 0.
# Use plt.plot or plt.stem to plot.

# Plot the autocorrelation:
plt.title("Array Correlation")
corr = array_correlation(Gold_code_satellite(10), Gold_code_satellite(10))
plt.plot(corr)
```

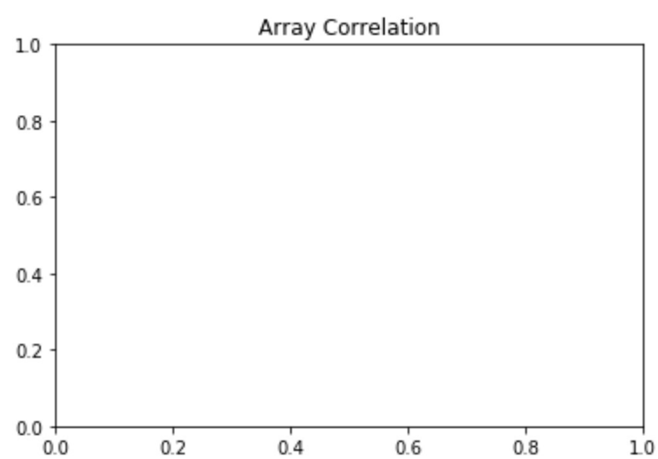
```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
<ipython-input-58-a4b044904941> in <module>()
     14 # Plot the autocorrelation:
     15 plt.title("Array Correlation")
---> 16 corr = array_correlation(Gold_code_satellite(10), Gold_code_satellite(10))
     17 plt.plot(corr)

<ipython-input-58-a4b044904941> in array_correlation(array1, array2)
      6     """
      7
----> 8     return np.correlate(array1, array2, full)
      9
     10 # Plot the auto-correlation of satellite 10 with itself. Your signal shoul
d be centered

C:\Program Files\Anaconda\lib\site-packages\numpy\core\numeric.py in correlate(a,
v, mode)
    973     """
    974     mode = _mode_from_name(mode)
--> 975     return multiarray.correlate2(a, v, mode)
    976
    977

TypeError: an integer is required (got type function)
```



Part (b)

In [59]:
```
plt.title("Array Correlation")
corr = array_correlation(Gold_code_satellite(10), Gold_code_satellite(13))
plt.plot(corr)
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
<ipython-input-59-5bfc934b72c5> in <module>()
      1 plt.title("Array Correlation")
----> 2 corr = array_correlation(Gold_code_satellite(10), Gold_code_satellite(13))
      3 plt.plot(corr)

<ipython-input-58-a4b044904941> in array_correlation(array1, array2)
      6     """
      7
----> 8     return np.correlate(array1, array2, full)
      9
     10 # Plot the auto-correlation of satellite 10 with itself. Your signal shoul
d be centered

C:\Program Files\Anaconda\lib\site-packages\numpy\core\numeric.py in correlate(a,
v, mode)
    973     """
    974     mode = _mode_from_name(mode)
--> 975     return multiarray.correlate2(a, v, mode)
    976
    977

TypeError: an integer is required (got type function)
```



Part (c)

In [ ]:
```
## THIS IS A HELPER FUNCTION FOR PART C
def integernoise_generator(length_of_noise):
    noise_array = np.random.randint(2, size = length_of_noise)
    noise_array = 2 * noise_array - np.ones(size(noise_array))
    return noise_array

plt.title("Array Correlation")
corr = array_correlation(Gold_code_satellite(10), Gold_code_satellite(13))
plt.plot(corr)
```

Part (d)

In [ ]:
```
## THIS IS A HELPER FUNCTION FOR PART D
def gaussiannoise_generator(length_of_noise):
    noise_array = np.random.normal(0, 1, length_of_noise)
    return noise_array

plt.title("Array Correlation")
corr = array_correlation(Gold_code_satellite(10), gaussiannoise_generator(1023))
plt.plot(corr)
```

Part (e)

In [ ]:
```
## USE 'np.load' FUNCTION TO LOAD THE DATA
## USE DATA1.NPY AS THE SIGNAL ARRAY

# YOUR CODE HERE
```

Part (f)

In [ ]:
```
## USE DATA2.NPY AS THE SIGNAL ARRAY

# YOUR CODE HERE
```

Part (g)

In [ ]:
```
## USE DATA3.NPY AS THE SIGNAL ARRAY

# YOUR CODE HERE
```

Question 3: Finding Signals in Noise

In [26]:
```
%matplotlib inline
import numpy as np
import scipy as sp
import scipy.linalg as la
import pylab as plt
```

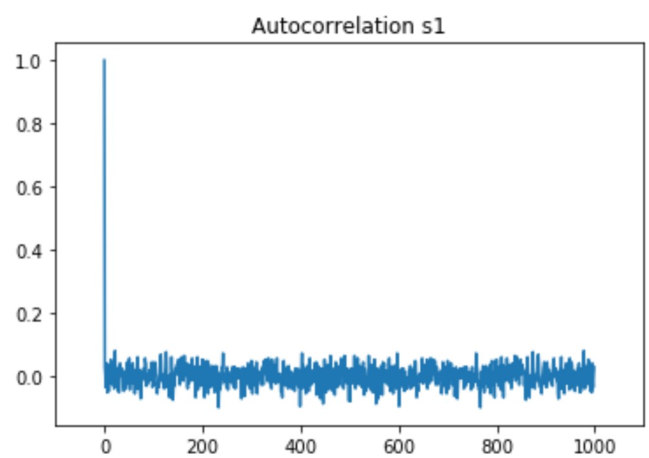
In [27]:
```
N = 1000

def rand_vector(n): # returns a random {+1, -1} vector of length n
    return np.random.randint(2, size=n)*2 - 1.0

def rand_normed_vector(n): # returns a random normalized vector of length n
    x = rand_vector(n)
    return x / la.norm(x)

def cross_corr(f, g):
    # returns the cross-correlation (a vector of all the inner products of 'g' with sh
    C = la.circulant(f)
    corr = C.T.dot(g)
    return corr
```

Part (a)

In [35]:

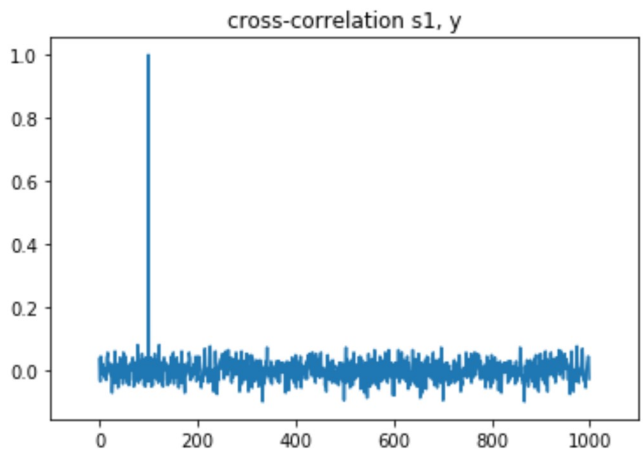
```
# Generate a random normalized vector for s1
# Running this cell again will generate a new random vector
s1 = rand_normed_vector(N)

# Compute all the inner products of s1 with shifted versions of s1
# (i.e., the cross-correlation of s1 with s1)
corr = cross_corr(s1, s1)

# The inner product <s1, s1^(1)> is:
print(corr[1])

# np.roll circularly shifts the signal,
# so the above inner product could be computed as:
print(np.dot(s1, np.roll(s1,1)))

# Plot the autocorrelation:
plt.title("Autocorrelation s1")
plt.plot(corr)

x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
```

```
0.028
0.028
```



Part (b)

In [37]:
```
y = np.roll(s1, 100) # Received y = s1 shifted by 10

# Compute the cross-correlation (all the inner products of y with shifted versions of
corr = cross_corr(s1, y)

# Plot
plt.title("cross-correlation s1, y")
plt.plot(corr)

x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
plt.show()

# Find the index of maximum correlation (inner product)
print(np.argmax(corr))
```




```
100
```

Part (c)

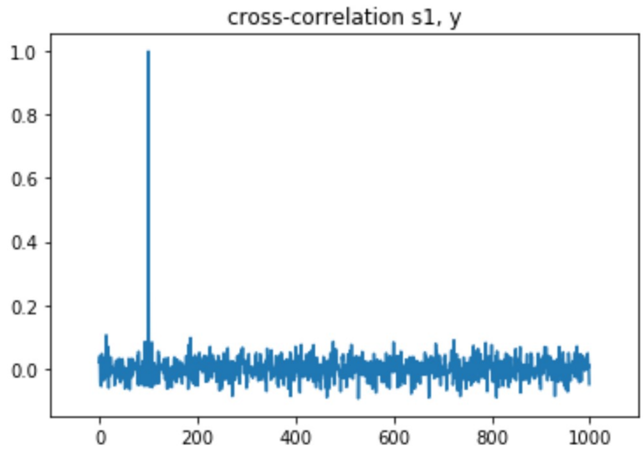
In [40]:
```
# Generate a random normalized vector for s1
# and a random normalized vector for n
# Running this cell again will generate new random vectors
s1 = rand_normed_vector(N)
n = rand_normed_vector(N)
```

```
0.054
```

Part (d)

This is the code from part (b) but with the received signal $\vec{y}$, which is corrupted by noise.

In [42]:
```
s1 = rand_normed_vector(N)
n = rand_normed_vector(N)
y = np.roll(s1, 100) + 0.1*n

corr = cross_corr(s1, y)

plt.title("cross-correlation s1, y")
plt.plot(corr)

x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
plt.show()

# Find the index of maximum correlation (inner product)
```

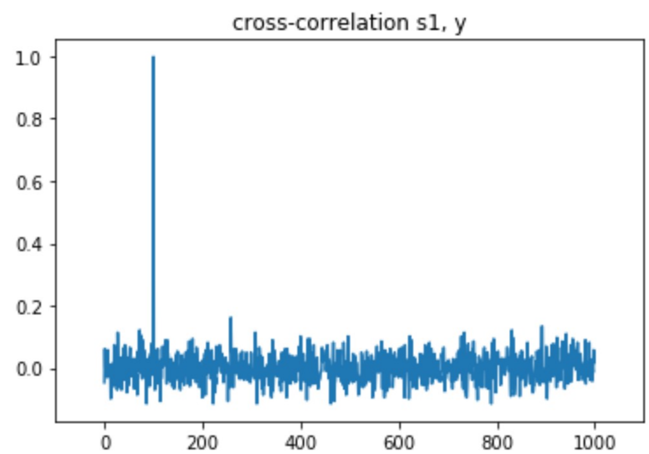


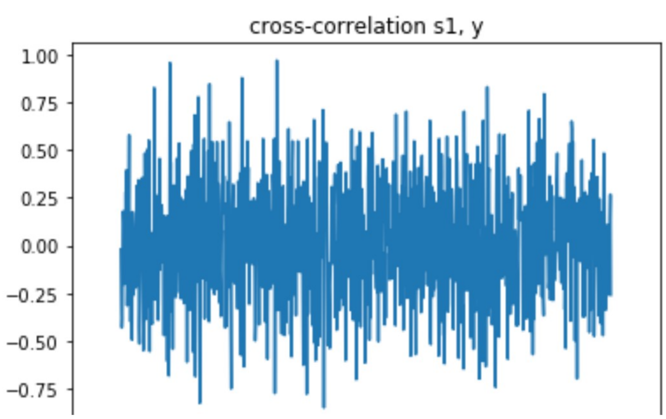
Out[42]: 100

Part (e)

Copy the code provided for part (d), but modify it appropriately, so that the noise is higher. You should generate two cross-correlation plots, one for each noise level in the question. (You can just copy the code from part (d) twice.)

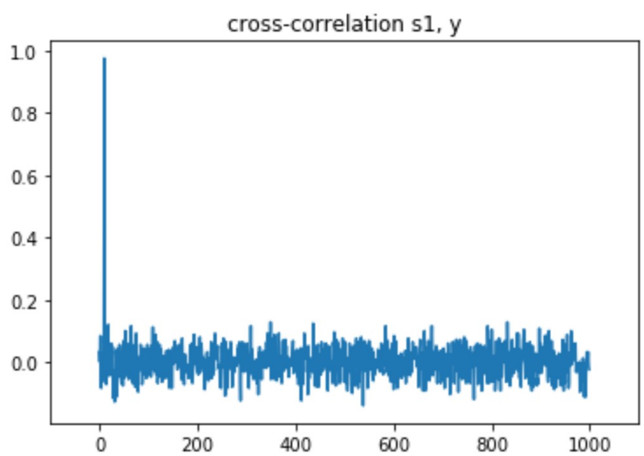
In [43]:
```
s1 = rand_normed_vector(N)
n = rand_normed_vector(N)
y = np.roll(s1, 100) + n

corr = cross_corr(s1, y)

plt.title("cross-correlation s1, y")
plt.plot(corr)

x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
plt.show()

# Find the index of maximum correlation (inner product)
np.argmax(corr)



s1 = rand_normed_vector(N)
n = rand_normed_vector(N)
y = np.roll(s1, 100) + 10*n

corr = cross_corr(s1, y)

plt.title("cross-correlation s1, y")
plt.plot(corr)

x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
plt.show()

# Find the index of maximum correlation (inner product)
```

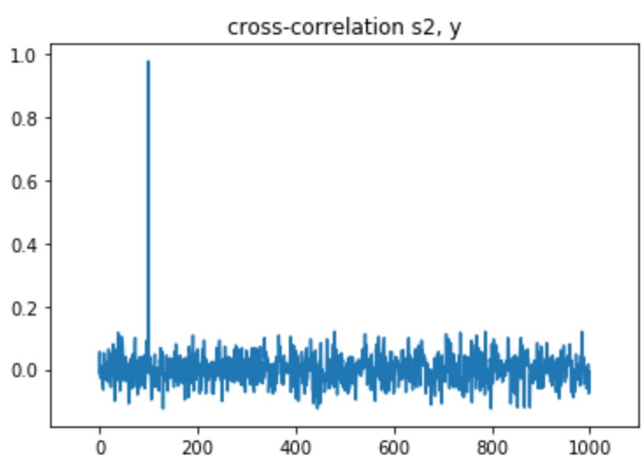




Part (f)

In [44]:
```
s1 = rand_normed_vector(N)
s2 = rand_normed_vector(N)

y = np.roll(s1, 10) + np.roll(s2, 100)

# Compute cross-correlations:
corr_s1_y = cross_corr(s1, y)
corr_s2_y = cross_corr(s2, y)

# Plot cross-correlations:
plt.title("cross-correlation s1, y")
plt.plot(cross_corr(s1, y))
x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
plt.show()

plt.title("cross-correlation s2, y")
plt.plot(cross_corr(s2, y))
x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
plt.show()

j = np.argmax(corr_s1_y) # find the first signal delay (max index of correlation)
k = np.argmax(corr_s2_y) # find the second signal delay
print(j, k)
```



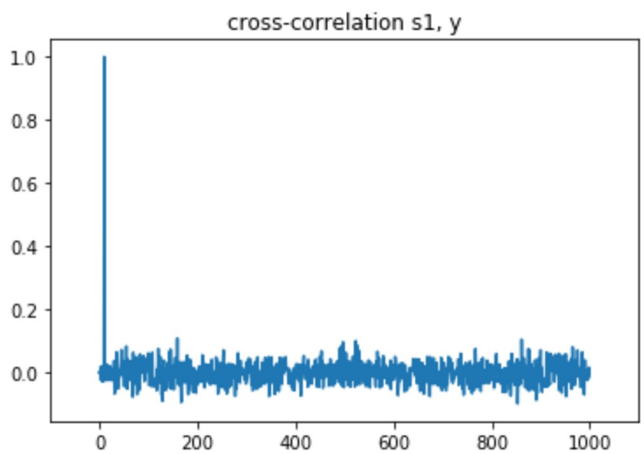


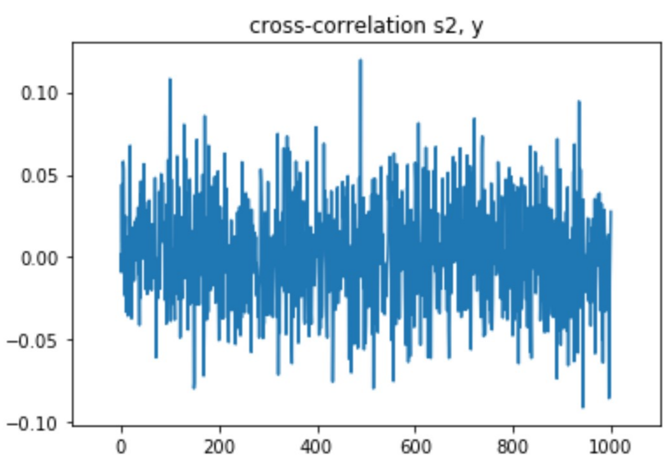

```
10 100
```

Part (g)

This is the same code as in part (f) but with a slight modification to how the received signal y generated. Run the below cell a few times to test for different choices of random signals.

In [45]:
```
s1 = rand_normed_vector(N)
s2 = rand_normed_vector(N)

y = np.roll(s1, 10) + 0.1*np.roll(s2, 100)

# Compute cross-correlations:
corr_s1_y = cross_corr(s1, y)
corr_s2_y = cross_corr(s2, y)

# Plot cross-correlations:
plt.title("cross-correlation s1, y")
plt.plot(cross_corr(s1, y))
x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
plt.show()

plt.title("cross-correlation s2, y")
plt.plot(cross_corr(s2, y))
x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
```

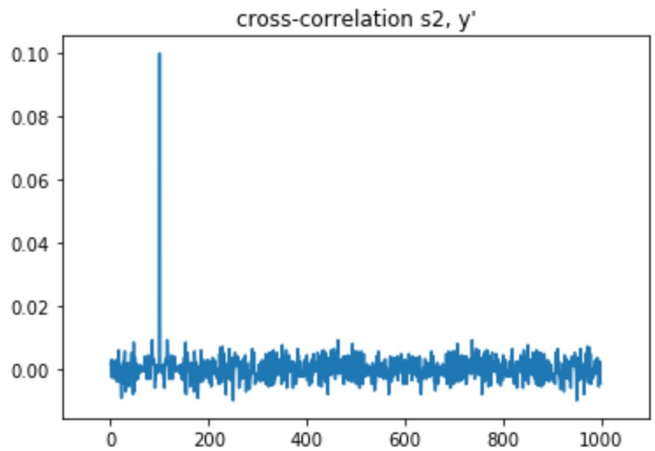




Part (h)

In [46]:
```
corr_s1_y = cross_corr(s1, y)
j = np.argmax(corr_s1_y) # find the first signal delay
print(j)

# Subtract out the contribution of the first signal
y_prime = y - np.roll(s1, j)

# Correlate the residual against the second signal
corr_s2_y = cross_corr(s2, y_prime)

# Plot
plt.title("cross-correlation s2, y'")
plt.plot(corr_s2_y)
x1,x2,y1,y2 = plt.axis()
plt.axis([x1-50,x2+50,y1,y2])
plt.show()

k = np.argmax(corr_s2_y) # find the second signal delay by looking at the index of max
print(k)
```

```
10
```




```
100
```

Part (i)

In [47]:
```
s1 = rand_normed_vector(N)
s2 = rand_normed_vector(N)

y = 0.7*np.roll(s1, 10) + 0.5*np.roll(s2, 100)

corr_s1_y = cross_corr(s1, y)
j = np.argmax(corr_s1_y) # find the first signal delay

corr_s2_y = cross_corr(s2, y)
k = np.argmax(corr_s2_y) # find the second signal delay

print(j, k)

# Once we have found the shifts, estimate the coefficients as inner products:
a1 = np.dot(y, np.roll(s1, j))
a2 = np.dot(y, np.roll(s2, k))

print(a1, a2)
```

```
10 100
0.696 0.4944
```

Part (j)

This is the same code as in part (i) but with noise added to the received signal $\vec{y}$.

In [48]:
```
s1 = rand_normed_vector(N)
s2 = rand_normed_vector(N)
n = rand_normed_vector(N)

y = 0.7*np.roll(s1, 10) + 0.5*np.roll(s2, 100) + 0.1*n

corr_s1_y = cross_corr(s1, y)
j = np.argmax(corr_s1_y) # find the first signal delay

corr_s2_y = cross_corr(s2, y)
k = np.argmax(corr_s2_y) # find the second signal delay

print(j, k)

# Once we have found the shifts, estimate the coefficients as inner products:
a1 = np.dot(y, np.roll(s1, j))
a2 = np.dot(y, np.roll(s2, k))

print(a1, a2)
```

```
10 100
0.7124 0.5188
```

Question 5: Image Analysis

In [ ]:
```
def plot_circle(a, d, e):
    """
    You can use this function to plot circles with parameters a,d,e.
    The parameters are described in the homework pdf.

    You can comment out the line that starts with `plt.title`
    because this makes assumptions regarding the title of your plot.
    """
    is_circle = d**2 + e**2 - 4*a > 0
    assert is_circle, "Not a circle"

    XLIM_LO = -1
    XLIM_HI = 3
    YLIM_LO = -2
    YLIM_HI = 2
    X_COUNT = 400
    Y_COUNT = 400

    x = np.linspace(XLIM_LO, XLIM_HI, X_COUNT)
    y = np.linspace(YLIM_LO, YLIM_HI, Y_COUNT)
    x, y = np.meshgrid(x, y)
    f = lambda x,y: a*(x**2 + y**2) + d*x + e*y

    c1 = plt.contour(x, y, f(x,y), [1], colors='r')
    plt.axis('scaled')
    plt.xlabel('x')
    plt.ylabel('y')
```

In [ ]:
```
def plot_ellipse(a, b, c, d, e):
    """
    You can use this function to plot ellipses with parameters a-e.
    The parameters are described in the homework pdf.

    You can comment out the line that starts with `plt.title`
    because this makes assumptions regarding the title of your plot.
    """
    is_ellipse = b**2 - 4*a*c < 0
    assert is_ellipse, "Not an ellipse"

    XLIM_LO = -1
    XLIM_HI = 3
    YLIM_LO = -2
    YLIM_HI = 2
    X_COUNT = 400
    Y_COUNT = 400

    x = np.linspace(XLIM_LO, XLIM_HI, X_COUNT)
    y = np.linspace(YLIM_LO, YLIM_HI, Y_COUNT)
    x, y = np.meshgrid(x, y)
    f = lambda x,y: a*x**2 + b*x*y + c*y**2 + d*x + e*y

    c1 = plt.contour(x, y, f(x,y), [1], colors='r')
    plt.axis('scaled')
    plt.xlabel('x')
    plt.ylabel('y')
    plt.title(r'${:.2f}x^2 {:+.2f}xy {:+.2f}y^2 {:+.2f}x {:+.2f}y$'.format(a,b,c,d,e))
```

In [ ]:
```
# Here is an example of plot_ellipse.
# This plots (x-1)**2 + (y-1)**2 = 1,
# which is a circle centered at (1,1).

plt.figure(figsize=(6,6))
```

You may find plt.scatter (http://matplotlib.org/api/pyplot_api.html) useful for plotting the points.

Part (c)

In [ ]:
```
def plot_circle(a, d, e):
    """
    You can use this function to plot circles with parameters a,d,e.
    The parameters are described in the homework pdf.

    You can comment out the line that starts with `plt.title`
    because this makes assumptions regarding the title of your plot.
    """
    is_circle = d**2 + e**2 - 4*a > 0
    assert is_circle, "Not a circle"

    XLIM_LO = -1
    XLIM_HI = 3
    YLIM_LO = -2
    YLIM_HI = 2
    X_COUNT = 400
    Y_COUNT = 400

    x = np.linspace(XLIM_LO, XLIM_HI, X_COUNT)
    y = np.linspace(YLIM_LO, YLIM_HI, Y_COUNT)
    x, y = np.meshgrid(x, y)
    f = lambda x,y: a*(x**2 + y**2) + d*x + e*y

    c1 = plt.contour(x, y, f(x,y), [1], colors='r')
    plt.axis('scaled')
    plt.xlabel('x')
    plt.ylabel('y')
```

Part (d)

In [ ]:
```
def plot_ellipse(a, b, c, d, e):
    """
    You can use this function to plot ellipses with parameters a-e.
    The parameters are described in the homework pdf.

    You can comment out the line that starts with `plt.title`
    because this makes assumptions regarding the title of your plot.
    """
    is_ellipse = b**2 - 4*a*c < 0
    assert is_ellipse, "Not an ellipse"

    XLIM_LO = -1
    XLIM_HI = 3
    YLIM_LO = -2
    YLIM_HI = 2
    X_COUNT = 400
    Y_COUNT = 400

    x = np.linspace(XLIM_LO, XLIM_HI, X_COUNT)
    y = np.linspace(YLIM_LO, YLIM_HI, Y_COUNT)
    x, y = np.meshgrid(x, y)
    f = lambda x,y: a*x**2 + b*x*y + c*y**2 + d*x + e*y

    c1 = plt.contour(x, y, f(x,y), [1], colors='r')
    plt.axis('scaled')
    plt.xlabel('x')
    plt.ylabel('y')
    plt.title(r'${:.2f}x^2 {:+.2f}xy {:+.2f}y^2 {:+.2f}x {:+.2f}y$'.format(a,b,c,d,e))
```