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12-6-17
MW 5-6

KEKA HW 12

(1) Method Gram-Schmidt

(1) $v_1 = v_1$ $= [1\ 0\ 0\ 0\ 0]^t$

$u_2 = v_2 - \langle v_2, u_1 \rangle u_1$ $u_2 = [0\ 1/\sqrt{2}\ 1/\sqrt{2}\ 0\ 0]^t$

$u_3 = v_3 - \langle v_3, u_1 \rangle u_1 - \langle v_3, u_2 \rangle u_2$ $u_3 = [0, 0\ 1/\sqrt{2}\ 1/\sqrt{2}]^T$

$$U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$V(V^TV)^{-1}V^tb = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 1 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 5 \\ 1 & 3 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1/2 \\ -1/2 \\ -1/2 \\ -1/2 \end{bmatrix}$$

(B) $u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ -1 \end{bmatrix} - \left(\begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ -1 \end{bmatrix}^T \begin{bmatrix} 1+\sqrt{3} \\ 0 \\ \sqrt{5} \\ 1+\sqrt{3} \\ 0 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ -1 \end{bmatrix}$ $u_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \\ -1/2 \\ -1/2 \end{bmatrix}$

Projection $= \begin{bmatrix} -1/4 \\ 0 \\ -1/4 \\ -1/4 \end{bmatrix}$

C $D = \begin{bmatrix} 1/\sqrt{3} & 0 & 1/\sqrt{3} & 1/\sqrt{3} & 0 \\ 1/2 & 1/2 & -0+ & -1/2 & -1/2 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 2 & -2 \end{bmatrix}$

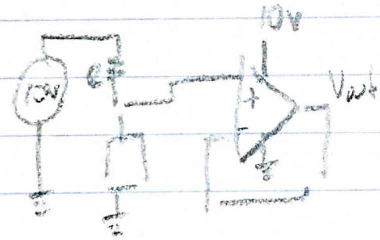
2 How Much is Too much
- See IPython

3 Sparse Imaging
- See IPython

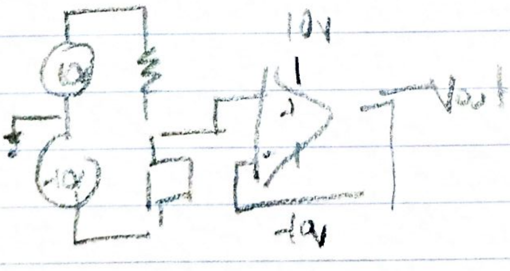
4 Speeding Up OMP
- See Ipython

5 Pet Tol Design

A



B



6 Is EE16A fun
yes, mostly fun, class not so much :)

7 I worked on this homework alone

EE16A Homework 13

Question 2: How Much Is Too Much?

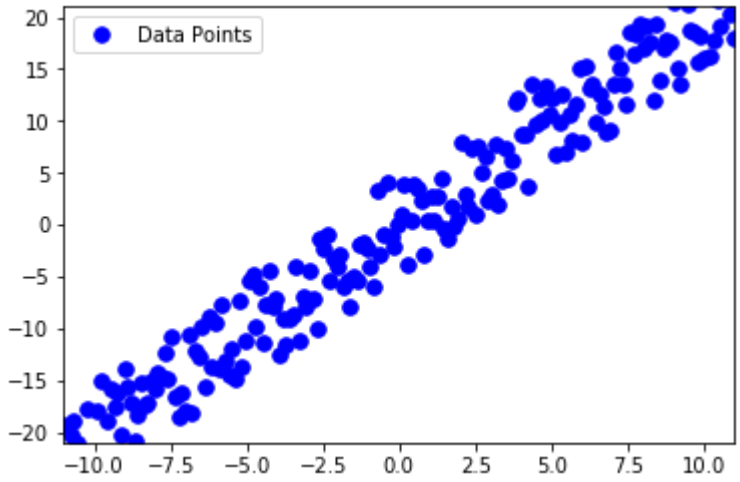
In [1]:
```
import numpy as np
import numpy.matlib
import matplotlib.pyplot as plt

%matplotlib inline

"""Function that defines a data matrix for some input data."""
def data_matrix(input_data,degree):
    # degree is the degree of the polynomial you plan to fit the data with
    Data=np.zeros((len(input_data),degree+1))

    for k in range(0,degree+1):
        Data[:,k]=(list(map(lambda x:x**k ,input_data)))

    return Data

"""Function that computes the Least Squares Approximation"""
def leastSquares(D,y):
    return np.linalg.lstsq(D,y)[0]

"""This function is used for plotting only"""
def poly_curve(params,x_input):
    # params contains the coefficients that multiply the polynomial terms, in de
    degree=len(params)-1
    x_range=[x_input[1], x_input[-1]]
    x=np.linspace(x_range[0],x_range[1],1000)
    y=x*0

    for k in range(0,degree+1):
        coeff=params[k]
        y=y+list(map(lambda z:coeff*z**k,x))
    return x,y

np.random.seed(10)
```

Part (a)

Some setup code to create our resistor test data points and plot them.

In [2]:
```
R = 2
x_a = np.linspace(-11,11,200)
y_a = R*x_a + (np.random.rand(len(x_a))-0.5)*10
fig = plt.figure()
ax=fig.add_subplot(111,xlim=[-11,11],ylim=[-21,21])
ax.plot(x_a,y_a, '.b', markersize=15)
ax.legend(['Data Points'])
```

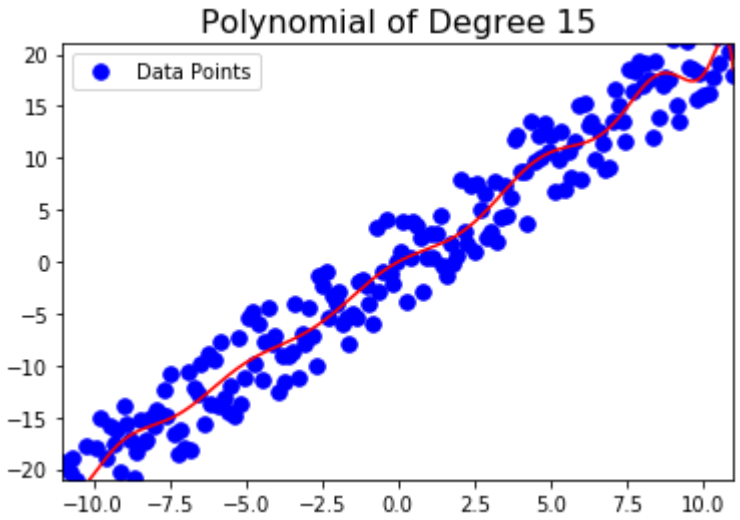
Out[2]: <matplotlib.legend.Legend at 0x112ef8de278>



Let's calculate a polynomial approximation of the above device.

In [12]:
```
# Play around with the degree here to try and fit different degree polynomials
degree=15
D_a = data_matrix(x_a,degree)
p_a = leastSquares(D_a, y_a)

fig=plt.figure()
ax=fig.add_subplot(111,xlim=[-11,11],ylim=[-21,21])
x_a_,y_a_=poly_curve(p_a,x_a)
ax.plot(x_a,y_a,'.b',markersize=15)
ax.plot(x_a_, y_a_, 'r')
ax.legend(['Data Points'])
plt.title('Polynomial of Degree %d' %(len(p_a)-1),fontsize=16)
```

Out[12]: <matplotlib.text.Text at 0x112f171e908>



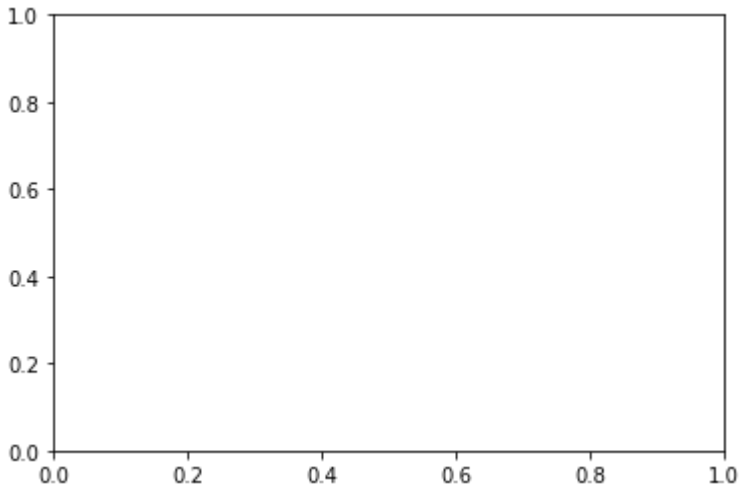
Part (b)

In [14]:
```
def cost(x, y, start, end):
    """Given a set of x and y points, this function
    calculates polynomial approximations of varying
    degrees from start to end and returns the cost
    of each degree in an array. The calculated cost
    should be the mean square error."""
    c = []
    for degree in range(start, end):
        R = 2
        x_a = np.linspace(-11,11,200)
        y_a = R*x_a + (np.random.rand(len(x_a))-0.5)*10
        D_a = data_matrix(x_a,degree)
        p_a = leastSquares(D_a, y_a)
        c.append(p_a*p_a - degree*degree)
    return c
```

In [15]:
```
start = 1
end = 15
fig=plt.figure()
ax=fig.add_subplot(111)
ax.plot(range(start, end), cost(x_a,y_a,start,end))
plt.title('Cost vs. Degree')
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
<ipython-input-15-11266a351dc2> in <module>()
      3 fig=plt.figure()
      4 ax=fig.add_subplot(111)
----> 5 ax.plot(range(start, end), cost(x_a,y_a,start,end))
      6 plt.title('Cost vs. Degree')

C:\Program Files\Anaconda\lib\site-packages\matplotlib\__init__.py in inner(ax,
 *args, **kwargs)
   1895                     warnings.warn(msg % (label_namer, func.__name__),
   1896                                   RuntimeWarning, stacklevel=2)
-> 1897             return func(ax, *args, **kwargs)
   1898         pre_doc = inner.__doc__
   1899         if pre_doc is None:

C:\Program Files\Anaconda\lib\site-packages\matplotlib\axes\_axes.py in plot(se
lf, *args, **kwargs)
   1405
   1406         for line in self._get_lines(*args, **kwargs):
-> 1407             self.add_line(line)
   1408             lines.append(line)
   1409

C:\Program Files\Anaconda\lib\site-packages\matplotlib\axes\_base.py in add_lin
e(self, line)
   1791             line.set_clip_path(self.patch)
   1792
-> 1793         self._update_line_limits(line)
   1794         if not line.get_label():
   1795             line.set_label('_line%d' % len(self.lines))

C:\Program Files\Anaconda\lib\site-packages\matplotlib\axes\_base.py in _update
_line_limits(self, line)
   1813         Figures out the data limit of the given line, updating self.dat
aLim.
   1814         """
-> 1815         path = line.get_path()
   1816         if path.vertices.size == 0:
   1817             return

C:\Program Files\Anaconda\lib\site-packages\matplotlib\lines.py in get_path(sel
f)
    987         """
    988         if self._invalidy or self._invalidx:
--> 989             self.recache()
    990         return self._path
    991
```

```
C:\Program Files\Anaconda\lib\site-packages\matplotlib\lines.py in recache(sel
f, always)
    683                 y = ma.asarray(yconv, np.float_).filled(np.nan)
    684             else:
--> 685                 y = np.asarray(yconv, np.float_)
    686             y = y.ravel()
    687         else:

C:\Program Files\Anaconda\lib\site-packages\numpy\core\numeric.py in asarray(a,
 dtype, order)
    529
    530     """
--> 531     return array(a, dtype, copy=False, order=order)
    532
    533

ValueError: setting an array element with a sequence.
```

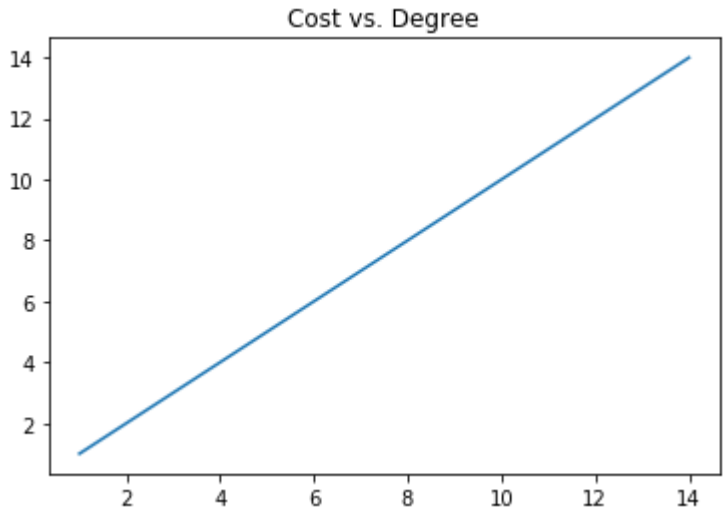


Part (c)

In [27]:
```
def improvedCost(x, y, x_test, y_test, start, end):
    """Given a set of x and y points training points,
    this function calculates polynomial approximations of varying
    degrees from start to end. Then it returns the cost, with
    the polynomial tested on test points of each degree in an array"""
    c = []
    for degree in range(start, end):
        c.append(degree)
    return c
```

In [28]:
```
# Run this to test your new cost function
start = 1
end = 15
x_a_test = x_a[0::2]
x_a_training = x_a[1::2]
y_a_test = y_a[0::2]
y_a_training = y_a[1::2]
c = improvedCost(x_a_training, y_a_training, x_a_test, y_a_test, start, end)
fig=plt.figure()
ax=fig.add_subplot(111)
ax.plot(range(start,end), c)
plt.title('Cost vs. Degree')
```

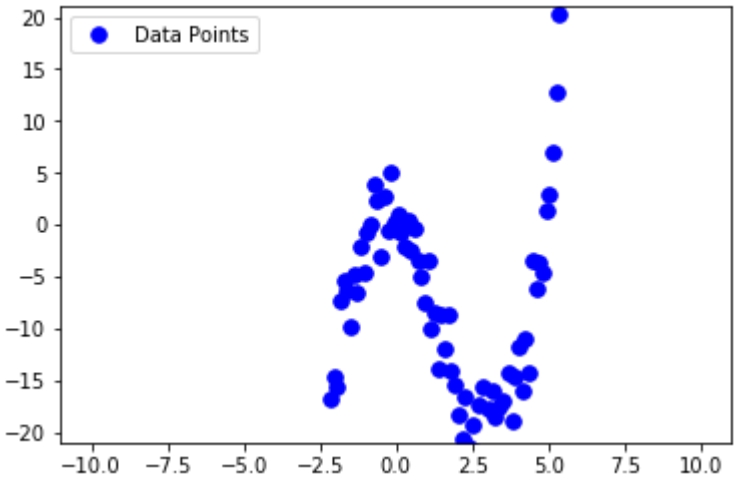
Out[28]: <matplotlib.text.Text at 0x112f412e710>



Part (d)

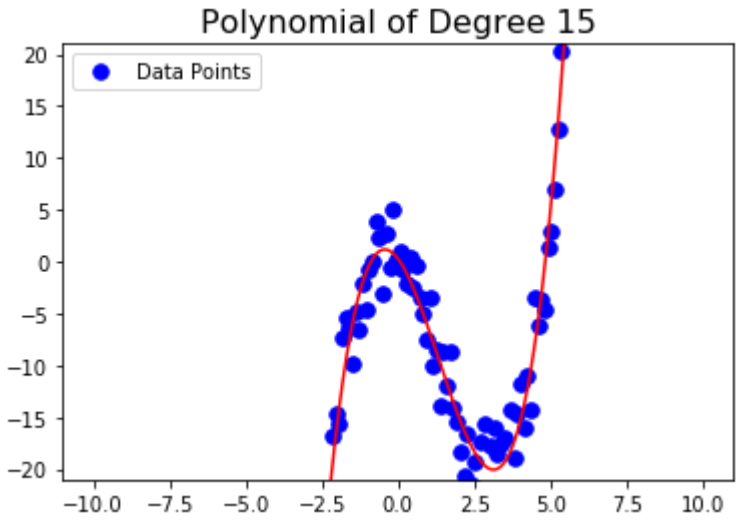
In [29]:
```
x_d_par=np.array([0.1,-4,-4,1])
x_d=np.linspace(-11,11,200)
y_d=np.dot(data_matrix(x_d,3),x_d_par)+(np.random.rand(len(x_d))-0.5)*10

fig=plt.figure()
ax=fig.add_subplot(111,xlim=[-11,11],ylim=[-21,21])
ax.plot(x_d,y_d,'.b',markersize=15)
ax.legend(['Data Points'])
print(len(x_d))
```

```
200
```



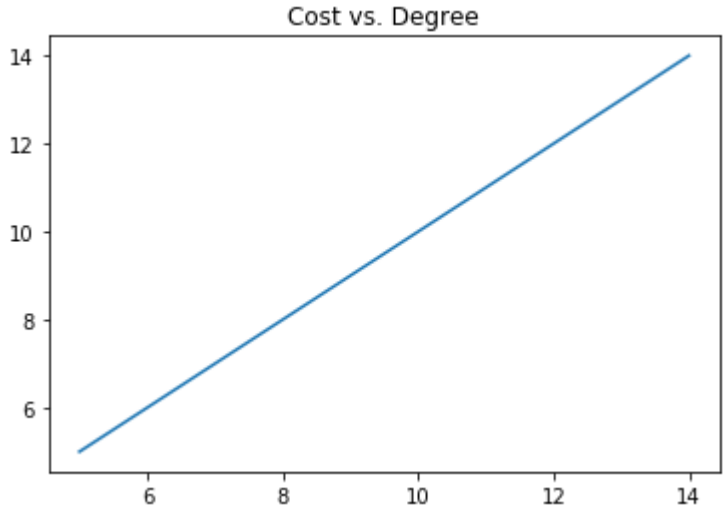
In [30]:
```
# Play With the degree to try to fit different degree polynomials
degree=15
D_d = data_matrix(x_d,degree)
p_d = leastSquares(D_d, y_d)

fig=plt.figure()
ax=fig.add_subplot(111,xlim=[-11,11],ylim=[-21,21])
x_d_,y_d_=poly_curve(p_d,x_d)
ax.plot(x_d,y_d,'.b',markersize=15)
ax.plot(x_d_, y_d_, 'r')
ax.legend(['Data Points'])
plt.title('Polynomial of Degree %d' %(len(p_d)-1),fontsize=16)
```

Out[30]: <matplotlib.text.Text at 0x112f17d8278>



In [31]:
```
start = 5
end = 15
x_d_test = x_d[0::2]
x_d_training = x_d[1::2]
y_d_test = y_d[0::2]
y_d_training = y_d[1::2]
c = improvedCost(x_d_training, y_d_training, x_d_test, y_d_test, start, end)
fig=plt.figure()
ax=fig.add_subplot(111)
ax.plot(range(start,end), c)
plt.title('Cost vs. Degree')
```

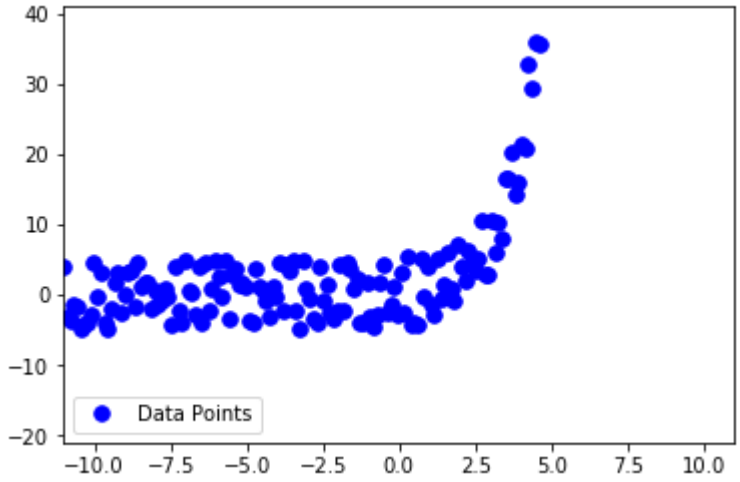
Out[31]: <matplotlib.text.Text at 0x112f407aeb8>



Part (e)

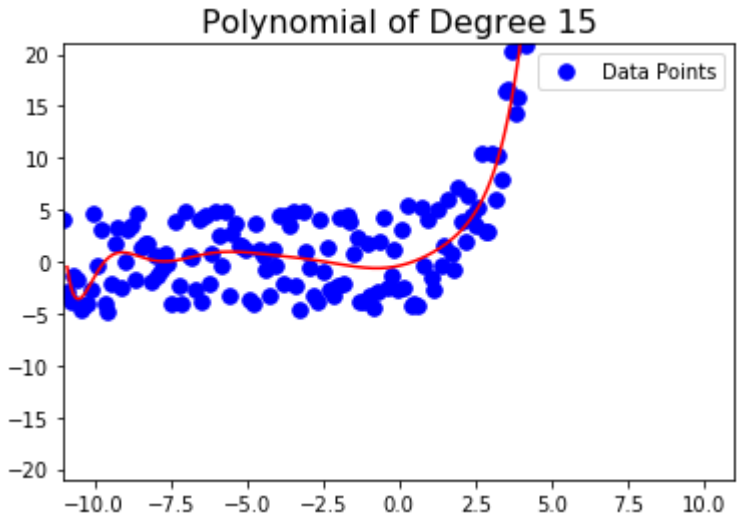
In [32]:
```
x_e=np.linspace(-11,11,200)
y_e=0.4*np.exp(x_e)+(np.random.rand(len(x_e))-0.5)*10

fig=plt.figure()
ax=fig.add_subplot(111,xlim=[-11,11],ylim=[-21,41])
ax.plot(x_e,y_e,'.b',markersize=15)
ax.legend(['Data Points'])
```

Out[32]: <matplotlib.legend.Legend at 0x112f42935c0>



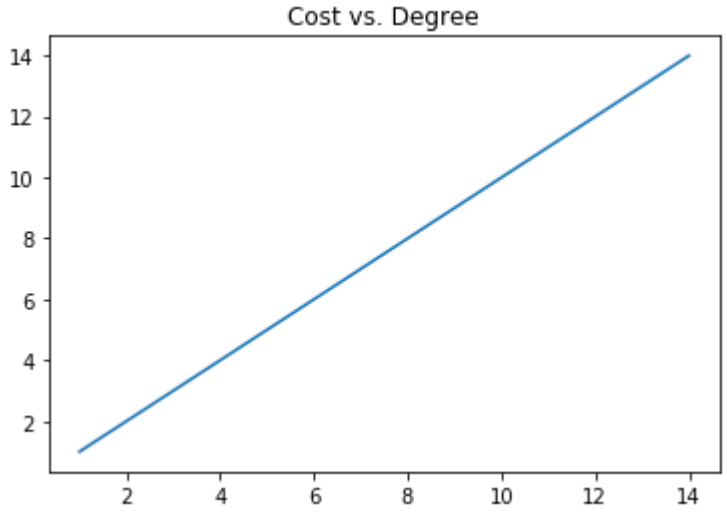
In [33]:
```
# Play With the degree to try to fit different degree polynomials
degree=15
D_e = data_matrix(x_e,degree)
p_e = leastSquares(D_e, y_e)

fig=plt.figure()
ax=fig.add_subplot(111,xlim=[-11,11],ylim=[-21,21])
x_e_,y_e_=poly_curve(p_e,x_e)
ax.plot(x_e,y_e,'.b',markersize=15)
ax.plot(x_e_, y_e_, 'r')
ax.legend(['Data Points'])
plt.title('Polynomial of Degree %d' %(len(p_e)-1),fontsize=16)
```

Out[33]: <matplotlib.text.Text at 0x112f430fa90>



In [34]:
```
start = 1
end = 15
x_e_test = x_e[0::2]
x_e_training = x_e[1::2]
y_e_test = y_e[0::2]
y_e_training = y_e[1::2]
c = improvedCost(x_e_training, y_e_training, x_e_test, y_e_test, start, end)
fig=plt.figure()
ax=fig.add_subplot(111)
ax.plot(range(start,end), c)
plt.title('Cost vs. Degree')
```

Out[34]: <matplotlib.text.Text at 0x112f43d1f28>



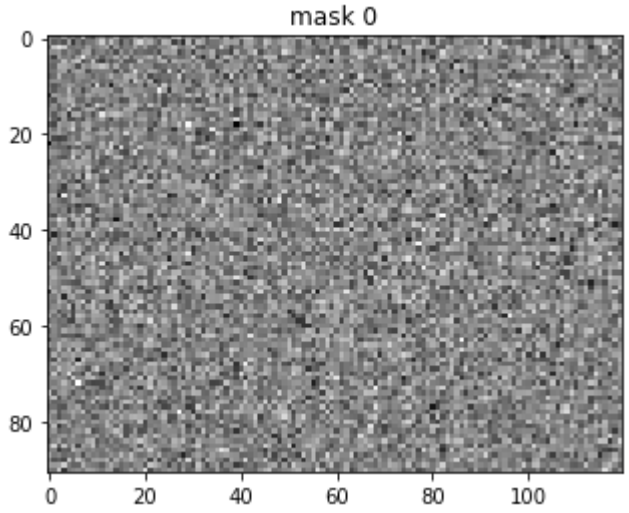
Question 3: Sparse Imaging

This example generates a sparse signal and tries to recover it using the Orthogonal Matching Pursuit algorithm.

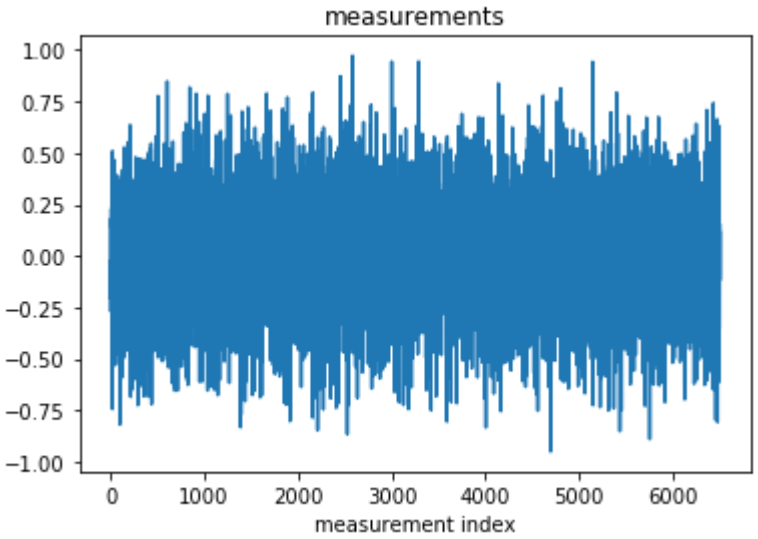
In [35]:
```
# imports
import matplotlib.pyplot as plt
import numpy as np
from scipy import misc
from IPython import display
from simulator import *
%matplotlib inline
```

In [36]:
```
measurements, A = simulate()

# THE SETTINGS FOR THE IMAGE - PLEASE DO NOT CHANGE
height = 91
width = 120
sparsity = 476
numPixels = len(A[0])
```

In [37]:
```
# CHOOSE DIFFERENT MASKS TO PLOT
chosenMaskToDisplay = 0

M0 = A[chosenMaskToDisplay].reshape((height,width))
plt.title('mask %d'%chosenMaskToDisplay)
plt.imshow(M0, cmap=plt.cm.gray, interpolation='nearest');
```



In [38]:
```
# measurements
plt.title('measurements')
plt.plot(measurements)
plt.xlabel('measurement index')
plt.show()
```



In [39]:
```
# OMP algorithm
# THERE ARE MISSING LINES THAT YOU NEED TO FILL
def OMP(imDims, sparsity, measurements, A):
    r = measurements.copy()
    indices = []

    # Threshold to check error. If error is below this value, stop.
    THRESHOLD = 0.1

    # For iterating to recover all signal
    i = 0

    while i < sparsity and np.linalg.norm(r) > THRESHOLD:
        # Calculate the correlations
        print('%d - '%i,end="",flush=True)
        corrs = A.T.dot(r)

        # Choose highest-correlated pixel location and add to collection
        # COMPLETE THE LINE BELOW
        best_index = np.argmax(np.abs(COMPLETE_HERE))
        indices.append(best_index)

        # Build the matrix made up of selected indices so far
        # COMPLETE THE LINE BELOW
        Atrunc = A[:,COMPLETE_HERE]

        # Find orthogonal projection of measurements to subspace
        # spanned by recovered codewords
        b = measurements
        # COMPLETE THE LINE BELOW
        xhat = np.linalg.lstsq(COMPLETE_HERE, COMPLETE_HERE)[0]

        # Find component orthogonal to subspace to use for next measurement
        # COMPLETE THE LINE BELOW
        r = b - Atrunc.dot(COMPLETE_HERE)

        # This is for viewing the recovery process
        if i % 10 == 0 or i == sparsity-1 or np.linalg.norm(r) <= THRESHOLD:
            recovered_signal = np.zeros(numPixels)
            for j, x in zip(indices, xhat):
                recovered_signal[j] = x
            Ihat = recovered_signal.reshape(imDims)
            plt.title('estimated image')
            plt.imshow(Ihat, cmap=plt.cm.gray, interpolation='nearest')
            display.clear_output(wait=True)
            display.display(plt.gcf())

        i = i + 1

    display.clear_output(wait=True)

    # Fill in the recovered signal
    recovered_signal = np.zeros(numPixels)
    for i, x in zip(indices, xhat):
        recovered_signal[i] = x
```

```
    return recovered_signal
```

In [40]:
```
rec = OMP((height,width), sparsity, measurements, A)
```

```
0 -

---------------------------------------------------------------------------
NameError                                 Traceback (most recent call last)
<ipython-input-40-85cdb11237bb> in <module>()
----> 1 rec = OMP((height,width), sparsity, measurements, A)

<ipython-input-39-7e9d2559798c> in OMP(imDims, sparsity, measurements, A)
     18         # Choose highest-correlated pixel location and add to collectio
n
     19         # COMPLETE THE LINE BELOW
---> 20         best_index = np.argmax(np.abs(COMPLETE_HERE))
     21         indices.append(best_index)
     22

NameError: name 'COMPLETE_HERE' is not defined
```

PRACTICE: Part (c)

In [ ]:
```
# the setting

# file name for the sparse image
fname = 'figures/smiley.png'
# number of measurements to be taken from the sparse image
numMeasurements = 6500
# the sparsity of the image
sparsity = 400

# read the image in black and white
I = misc.imread(fname, flatten=1)
# normalize the image to be between 0 and 1
I = I/np.max(I)

# shape of the image
imageShape = I.shape
# number of pixels in the image
numPixels = I.size

plt.title('input image')
plt.imshow(I, cmap=plt.cm.gray, interpolation='nearest');
```

In [ ]:
```
# generate your image masks and the underlying measurement matrix
Mask, A = randMasks(numMeasurements,numPixels)
# vectorize your image
full_signal = I.reshape((numPixels,1))
# get the measurements
measurements = np.dot(Mask,full_signal)
# remove the mean from your measurements
measurements = measurements - np.mean(measurements)
```

In [ ]:
```
# measurements
plt.title('measurements')
plt.plot(measurements)
plt.xlabel('measurement index')
plt.show()
```

In [ ]:
```
rec = OMP(imageShape, sparsity, measurements, A)
```

Question 4: Speeding Up OMP

This example generates a sparse signal and tries to recover it using the Orthogonal Matching Pursuit algorithm.

In [ ]:
```
# imports
import matplotlib.pyplot as plt
import numpy as np
from scipy import misc
from IPython import display
from simulator import *
%matplotlib inline
```

In [ ]:
```
# the setting

# file name for the sparse image
fname = 'figures/pika.png'
# number of measurements to be taken from the sparse image
numMeasurements = 9000
# the sparsity of the image
sparsity = 800

# read the image in black and white
I = misc.imread(fname, flatten=1)
# normalize the image to be between 0 and 1
I = I/np.max(I)

# shape of the image
imageShape = I.shape
# number of pixels in the image
numPixels = I.size

plt.title('input image')
plt.imshow(I, cmap=plt.cm.gray, interpolation='nearest');
```

In [ ]:
```
# generate your image masks and the underlying measurement matrix
Mask, A = randMasks(numMeasurements,numPixels)
# vectorize your image
full_signal = I.reshape((numPixels,1))
# get the measurements
measurements = np.dot(Mask,full_signal)
# remove the mean from your measurements
measurements = measurements - np.mean(measurements)
```

In [ ]:
```
# measurements
plt.title('measurements')
plt.plot(measurements)
plt.xlabel('measurement index')
plt.show()
```

In [ ]:
```
# Write a function that returns a matrix U whose columns form
# an orthonormal basis for the columns of the matrix A.
def gramschmidt(A):
    return U

# A better option is to write a function that takes in a matrix U0
# with orthonormal columns and a single new vector v and adds another
# orthonormal column to U0 creating a new matrix U whose columns are orthonormal
# and span the column space of {U0, v}.
# Note: Using this function will make your code faster.
def gramschmidt_addone(U0, v):
    return U
```

In [ ]:
```
# THERE ARE MISSING LINES THAT YOU NEED TO FILL
def OMP(imDims, sparsity, measurements, A):
    r = measurements.copy()
    indices = []

    # Threshold to check error. If error is below this value, stop.
    THRESHOLD = 0.1

    # For iterating to recover all signal
    i = 0

    ########################################
    ### THIS LINE INITIALIZES THE MATRIX U
    U = np.zeros([np.size(A,0),0])
    ########################################

    while i < sparsity and np.linalg.norm(r) > THRESHOLD:
        # calculate the correlations
        print('%d - '%i,end="",flush=True)
        corrs = A.T.dot(r)

        # Choose highest-correlated pixel location and add to collection
        # COMPLETE THE LINE BELOW
        best_index = np.argmax(np.abs(COMPLETE_HERE))

        ############################
        ### MODIFY THIS SECTION ###
        ## TO USE GRAM-SCHMIDT  ###
        ############################

        indices.append(best_index)

        # Build the matrix made up of selected indices so far
        # COMPLETE THE LINE BELOW
        Atrunc = A[:,COMPLETE_HERE]

        ##############################
        ## CHOOSE ONE OF THESE LINES
        U = gramschmidt(Atrunc)
        ### OR
        v = A[:,best_index]
        U = gramschmidt_addone(U,v)
        ##############################

        # Find orthogonal projection of measurements to subspace
        # spanned by recovered codewords
        b = measurements

        ###################################
        ## COMPLETE THE LINES BELOW AND
        ## REWRITE THESE LINES USING GRAMSCHMIDT TO SPEED UP LEAST SQUARES
        xhat = np.linalg.lstsq(COMPLETE_HERE, COMPLETE_HERE)[0]
        r = b - Atrunc.dot(COMPLETE_HERE)
        ###################################

        # Find component orthogonal to subspace to use for next measurement
```

```
        ## CHANGE THIS LINE

        ############################
        ### ------------------ ###
        ############################

        # This is for viewing the recovery process
        if i % 100 == 0 or i == sparsity-1 or np.linalg.norm(r) <= THRESHOLD:

            # RECOVERING xhat for plotting
            xhat = np.dot(np.linalg.inv(np.dot(Atrunc.T,Atrunc)),np.dot(Atrunc.T

            recovered_signal = np.zeros(numPixels)
            for j, x in zip(indices, xhat):
                recovered_signal[j] = x
            Ihat = recovered_signal.reshape(imDims)
            plt.title('estimated image')
            plt.imshow(Ihat, cmap=plt.cm.gray, interpolation='nearest')
            display.clear_output(wait=True)
            display.display(plt.gcf())

        i = i + 1

    display.clear_output(wait=True)

    # Fill in the recovered signal
    recovered_signal = np.zeros(numPixels)
    for i, x in zip(indices, xhat):
        recovered_signal[i] = x

    return recovered_signal
```

In [ ]:
```
rec = OMP(imageShape, sparsity, measurements, A)
```

(PRACTICE) Question 6: Perceptron Valley

In [21]:
```
%matplotlib inline

import numpy as np
import numpy.random as npr
import time, sys
from IPython.display import clear_output
from IPython.display import display
import matplotlib.pyplot as plt
```

Part (e)

Fill in the missing lines of code in the following function definition to implement the PLA. The update is given below.

if $\text{sign}\left(\langle \vec{w}_j, \vec{x_i} \rangle\right) \neq y_i$
$\quad \vec{w}_{j+1} \leftarrow \vec{w}_j + y_i \vec{x_i}$
else
$\quad \vec{w}_{j+1} \leftarrow \vec{w}_j$
end if

In [22]:
```
def PLA(X,y,w_0,J,ax):
    n = len(y)
    w_j = w_0
    for j in range(J):
        i = npr.randint(0,n-1)
        x_i = X[:,i]
        y_i = y[i]

        # YOUR CODE HERE

        plotHyperplane(ax,X,y,i,j,w_j)
    w_J = w_j
    return w_J
```

In [23]:
```
def plotHyperplane(ax,X,y,i,j,w_j):
    ma = 1.1*max(abs(np.concatenate((X[0,:],X[1,:]))))
    ax.axis('equal')
    ax.axhline(y=0, color='k')
    ax.axvline(x=0, color='k')
    ax.plot(X[0,y==1],X[1,y==1],'r+')
    ax.plot(X[0,y==-1],X[1,y==-1],'bx')
    if y[i] == 1:
        ax.plot(X[0,i],X[1,i],'k+')
    else:
        ax.plot(X[0,i],X[1,i],'kx')
    ax.arrow(0,0,w_j[0],w_j[1],head_width=0.1,head_length=0.1,fc='k',ec='k')
    ax.arrow(0,0,-100*w_j[1],100*w_j[0],head_width=0.1,head_length=0.1,fc='g',ec
    ax.arrow(0,0,100*w_j[1],-100*w_j[0],head_width=0.1,head_length=0.1,fc='g',ec
    ax.axis((-ma,ma,-ma,ma))
    ax.set_title('Iteration {}: training error = {}'.format(j,trainingError(X,y,
    time.sleep(0.1)
    clear_output(True)
    display(fg)
    ax.cla()
```

In [24]:
```
def trainingError(X,y,w_j):
    n = len(y)
    y_p = np.sign(np.dot(X.T,w_j))
    return float(np.sum(y_p!=y)) / float(n)
```

In [25]:
```
# Generate data points
n = 100
d = 2
X = np.concatenate((npr.normal(1,0.5,(d,np.int(n/2))),npr.normal(-1,1,(d,np.int(
y = np.concatenate((np.ones(np.int(n/2)),-np.ones(np.int(n/2))))
```

In [26]:
```
# Run PLA and watch it learn
w_0 = np.array([-5,-5])
J = 25
fg, ax = plt.subplots()
w_J = PLA(X,y,w_0,J,ax)
plt.close()
```

In [ ]:
```
```