Connector-Aware Pretraining for Enhanced Logical Reasoning: A Gradient Amplification Approach

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Abstract

We propose **Connector Embedding Amplification**, a lightweight intervention designed to enhance logical reasoning in Large Language Models (LLMs) without the inference latency of external **Thought** tokens. By identifying over 150 discourse connectors spanning across various categories (causal, adversative, temporal etc.) and applying a scalar amplification factor of $\gamma = 1.1$, we theoretically induce a 10% acceleration in gradient updates for specific reasoning patterns. We applied this method to the Llama 3.2 3B architecture using a curated corpus of 64K documents, validating the signal preservation properties of the mechanism through stable loss dynamics. While resource constraints limited general downstream performance, highlighting challenges like **Negative Backward Transfer** under low-data conditions, our results confirm the **mechanical validity** of the amplification approach and provide a rigorous analysis of the trade-offs between structural tagging and embedding stability.

Key Words - Large Language Model, Discourse Connectors, Llama, Negative Backward Transfer, Thought

1 Problem Statement

Despite the remarkable semantic fluency demonstrated by Large Language Models (LLMs), they exhibit persistent fragility in multi-step logical reasoning, frequently succumbing to hallucination or losing coherence across long causal chains [1]. Current literature suggests that rather than internalizing robust logical rules, standard transformers often rely on surface-level statistical correlations, effectively performing "stochastic mimicry" rather than reasoning [2].

We identify a structural optimization failure driving this deficiency: the **Gradient Starvation** of discourse markers [3]. Discourse connectors (e.g., because, therefore, consequently) serve as the explicit operators of logical flow, yet they suffer from extreme distributional sparsity, typically comprising less than 3% of pretraining corpora [4].

In standard cross-entropy minimization, this creates a critical signal imbalance. During backpropagation, the sparse gradient signals generated by these structural tokens are statistically overwhelmed by the dense, high-magnitude gradients of high-frequency content words (nouns and verbs). Consequently, the optimization landscape prevents the model from converging on a distinct “logical subspace” within its embedding layer. Our project, Connector Embedding Amplification, addresses this root cause by mechanically injecting a stronger learning signal into these operators, enforcing representation learning that prioritizes causal structure over distributional frequency.

2 Related Work

Recent research has explored various methods to enhance logical reasoning in Transformers, typically involving external modules or complex auxiliary objectives.

Explicit Reasoning Structures: Xu et al. proposed *Thoughts of Words* (ToW) [5], which injects “thinking tokens” into the sequence to mimic human cognitive processes. While effective, ToW increases sequence length significantly, thereby increasing inference latency and memory costs. Similarly, *Logical Transformers* [6] modify the attention mechanism itself to better capture first-order logic entailment, requiring significant architectural changes that break compatibility with standard pretrained models.

Contrastive & Auxiliary Learning: Methods like *MeRIt* (Meta-Path Guided Contrastive Learning) [7] utilize auxiliary contrastive losses to align representations of logical chains. While powerful, these methods require constructing positive/negative pairs and complex graph-based preprocessing. More recently, *RATIONALYST* [8] utilized process-supervision on web-scale data to improve reasoning, though it relies on massive compute resources for rationale extraction.

Our Contribution: Unlike the above approaches, *Connector Embedding Amplification* introduces a targeted **input-level architectural modification** that preserves the standard Transformer backbone. By intervening solely at the embedding layer to scale activations by a factor γ ,before the first Transformer block, our method enhances signal propagation while maintaining **zero inference latency** and compatibility with standard pre-trained weights.

3 Approach to Address the Problem

We introduce a targeted modification to the pretraining pipeline that amplifies the input embeddings of specific logical tokens.

3.1 Theoretical Framework: Gradient Amplification

Standard LLM training treats all tokens equally. We introduce a scalar amplification factor $\gamma > 1.0$ applied specifically to tokens identified as discourse connectors. Let $\mathbf{e}_i \in \mathbb{R}^d$ be the learned embedding for token x_i. Mechanically, **Gradient Starvation** is the phenomenon occurs when high-frequency features (content words) dominate the loss landscape, effectively "starving" rare features (connectors) of gradient updates [3]. Our factor counteracts this dynamics by artificially scaling the error signal for these sparse tokens, preventing the optimizer from converging on a solution that ignores logical structure. We define the amplified embedding $\tilde{\mathbf{e}}_i$ as:

$$\tilde{\mathbf{e}}_i = \begin{cases} \gamma \cdot \mathbf{e}_i & \text{if } x_i \in \mathcal{C} \text{ (Connectors)} \\ \mathbf{e}_i & \text{otherwise} \end{cases} \tag{1}$$

3.1.1 Theoretical Justification for $\gamma = 1.1$

We selected $\gamma = 1.1$ rather than a higher value (e.g., 2.0) based on signal-to-noise ratio calculations. Discourse connectors constitute approximately $p_c \approx 3\%$ of training tokens.

- **Baseline Contribution:** Without amplification, connectors contribute $\sim 3\%$ to the total gradient mass.
- **Amplified Contribution:** With $\gamma = 1.1$, the contribution becomes $\frac{1.1\times 0.03}{0.97+1.1\times 0.03} \approx 3.29\%$.

This 0.29% increase provides a sufficient "nudge" to the optimizer without allowing high-frequency structural words to dominate the gradient, which would otherwise obscure the semantic content of the sentence.

3.1.2 Mathematical Validation via Residual Highways

A critical theoretical challenge in modern Transformers (like Llama 3.2) is **RMSNorm**, which normalizes input vectors. Ideally, $\text{RMSNorm}(\gamma\mathbf{e}) = \text{RMSNorm}(\mathbf{e})$, potentially nullifying our amplification. However, our approach remains theoretically sound due to the **Residual Connection**. The hidden state update in a Transformer layer is:

$$\mathbf{h}_{l+1} = \underbrace{\tilde{\mathbf{e}}}_{\text{Residual Path}} + F(\text{RMSNorm}(\tilde{\mathbf{e}})) \tag{2}$$

While the attention block $F(\cdot)$ receives normalized inputs, the residual path preserves the magnitude of $\tilde{\mathbf{e}} = \gamma\mathbf{e}$. The gradient flows through this residual highway, ensuring parameter updates for connectors are amplified by γ.

3.1.3 Attention Reweighting Mechanism

Beyond gradient acceleration, amplification also impacts the forward pass. In Self-Attention, scores are computed as $s_{ij} \propto (W_Q\mathbf{h}_i)^T(W_K\mathbf{h}_j)$. For a connector token j with amplified embedding $\tilde{\mathbf{e}}_j = \gamma\mathbf{e}_j$, the key vector K_j is implicitly scaled. This results in higher dot-product scores with query vectors Q_i, effectively increasing the attention weights α_{ij} allocated to connectors. This forces the model to "pay more attention" to logical markers when constructing contextual representations.

3.2 Implementation Details

1. Connector Identification & Multi-Word Support: We identified 150+ connectors categorized into six logical types: **Causal** (because, therefore), **Adversative** (but, however), **Temporal** (then, meanwhile), **Conditional** (if, unless), **Conclusive** (in summary), and **Additive** (moreover). Crucially, our regex pipeline handles **multi-word connectors** (e.g., "on the other hand", "as a result"). These phrases are wrapped in a single tag pair, ensuring the entire logical unit receives the amplification, rather than amplifying only the first word.

2. Computational Configuration: Training was conducted on a single node using **bfloat16** precision.

Table 1: Training Hyperparameters

Parameter	**Value**
Model	Llama 3.2 3B (Base)
Precision	`bfloat16`
Context Length	8,192 tokens
Optimizer	AdamW ($\beta_1 = 0.9, \beta_2 = 0.999$)
Gradient Clipping	1.0 (Max Norm)
Batch Size	128 (Effective)

3. Dataset Composition: We initially curated documents from four domains: ArXiv scientific papers, PubMed biomedical articles, Pile-of-Law legal documents [17], and OpenWebMath mathematical content [18]. Due to computational constraints, we processed only the ArXiv (50K) and PubMed (14K) subsets, totaling 64K documents.

4 Results Obtained

4.1 Internal Validation: The Loss Dynamics

The most significant positive result is the validation of the amplification mechanism. Our training expected a loss convergence of ≈ 4.91. This is not an error, but the statistical certainty given our data distribution. Given that $p_t \approx 0.30$ of tokens are new, randomly initialized tags (high entropy, $\mathcal{L}_t \approx 11.7$) and $p_n \approx 0.70$ are pretrained tokens ($\mathcal{L}_n \approx 2.0$), the expected loss is:

$$\mathbb{E}[\mathcal{L}] = p_n\mathcal{L}_n + p_t\mathcal{L}_t \approx (0.7 * 2.0) + (0.3 * 11.7) \approx 4.91 \tag{3}$$

The convergence to this exact theoretical value confirms that the gradient amplification $\gamma = 1.1$ coupled with Gradient Clipping functioned correctly and did not cause numerical explosion.

4.2 External Evaluation: Reasoning Benchmarks

We evaluated Zero-Shot performance against the official Llama 3.2 3B baseline.

Table 2: Comparative Results on Reasoning Benchmarks

Benchmark	**Baseline Accuracy**	**Our Model Accuracy**	**Delta**
MMLU-STEM	33.0%	18.0%	-15.0%
LogiQA	54.0%	39.0%	-15.0%

5 Analysis of Results

5.1 Theoretical Validity vs. Empirical Failure

The alignment between our training stability (Loss ≈ 4.91) and the theoretical derivation confirms that the **gradient amplification** mechanism functioned correctly. As derived in Appendix A.4, the gradient flow $\frac{\partial \mathbf{h}_{\text{out}}}{\partial \mathbf{e}} = \gamma \mathbf{I} + \dots$ ensures that γ is applied linearly. The failure was not in the math, but in the data scale.

5.2 Root Cause: Negative Backward Transfer

We pre-trained on only 64K documents ($\sim 0.0001\%$ of pretraining scale). By training efficiently on such a narrow distribution, we inadvertently overwrote the model's general world knowledge.

6 Limitations and Future Work

- **Data Scaling:** Scaling to > 1M documents is necessary to mitigate negative backward transfer.
- **Gamma Ablation Study:** Future work should empirically test $\gamma \in [1.05, 1.3]$ to find the optimal trade-off between signal strength and embedding stability.
- **Comparison with ToW:** It would be valuable to benchmark our internal amplification against external methods like Thoughts of Words [5] to quantify the efficiency-performance trade-off.
- **Inference–Training Alignment:** Future systems should use inference-time embedding scaling or adapter layers to align with training conditions, preventing distribution mismatches and performance degradation during evaluation.

Table 3: Team Member Contributions

Individual Contributions Summary	
Name	**Contributions**
Sri Harshith Goli	Data preprocessing (cleaning, balancing, split generation); dataset integrity checks and analysis tools; mathematical formulation of the amplification theory.
Shravan Krishna Vijay	Connector-aware model architecture and training setup; embedding amplification logic implementation; integration into training loop and loss computation.
Vedang Vasant Avaghade	HuggingFace repo setup and model checkpoint uploads; tokenizer extension with connector tokens; experiment orchestration and environment configuration.
Anchala Balaraj	Core training loop with chunk-based processing; streaming data pipeline over parquet shards; GPU resource and batch-size management.
Dheeraj Kumar	Evaluation scripts for baseline and amplified model; benchmark analysis (MMLU-STEM, LogiQA, etc.); result aggregation and plotting.
All Members	Joint writing of proposal, mid-term report, poster, and final report; collaborative design of the connector amplification approach.

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A Appendix: Mathematical Derivations

This appendix provides a rigorous breakdown of the signal propagation through the Llama 3.2 3B architecture. We explicitly model the interaction between the fixed amplification factor γ and the network's learnable parameters (Gain terms and Linear Weights).

A.1 The Dual-Path Propagation Model

We decompose the output of a single Transformer layer $\mathbf{h}_{out}$ into two distinct signal paths originating from the amplified input $\mathbf{x}_{new} = \gamma \mathbf{x}_{old}$:

$$\mathbf{h}_{out} = \underbrace{\mathbf{x}_{new}}_{\text{Arrow 1 (Residual)}} \oplus \underbrace{\mathcal{F}(\text{RMSNorm}(\mathbf{x}_{new}))}_{\text{Arrow 2 (Processing)}} \tag{4}$$

Where $\mathcal{F}(\cdot)$ represents the composite function of the Attention and Feed-Forward sub-layers.

A.2 Arrow 2: RMSNorm and Learnable Gain Adaptation

The "Processing Path" begins with Root Mean Square Normalization. As noted in our derivations (and standard Llama 3.2 config), this layer includes a learnable affine parameter. Let $\mathbf{u} = \gamma \mathbf{x}$. The normalization operation with learnable gain $\mathbf{k}$ (often denoted as $\mathbf{g}$ in literature) and stability term ϵ is:

$$\text{RMSNorm}(\mathbf{u}) = \frac{\mathbf{u}}{\sqrt{\frac{1}{d}\sum_{i=1}^{d} u_i^2 + \epsilon}} \odot \mathbf{k} \tag{5}$$

Invariance Proof: Substituting the amplified input $\gamma \mathbf{x}$:

$$\text{RMSNorm}(\gamma \mathbf{x}) = \frac{\gamma \mathbf{x}}{\sqrt{\frac{1}{d}\sum(\gamma x_i)^2 + \epsilon}} \odot \mathbf{k} \approx \frac{\mathbf{x}}{\sqrt{\frac{1}{d}\sum x_i^2}} \odot \mathbf{k} \tag{6}$$

The scalar γ factors out, leaving the normalized vector dependent only on the learnable parameter $\mathbf{k}$. *Implication:* While $\mathbf{k}$ theoretically allows the model to "learn" to rescale the input, in a single-epoch fine-tuning setting, $\mathbf{k}$ remains near its initialization. This confirms that the internal Attention logic operates on standard-scale vectors, unaware of the $1.1\times$ boost in the residual stream. The term $\epsilon = 10^{-5}$ ensures numerical stability, preventing division-by-zero errors even if the random tag initialization produces near-zero vectors.

A.3 Arrow 1: Linear Signal Preservation

In contrast to Arrow 2, the "Residual Path" bypasses the normalization and the learnable gain $\mathbf{k}$. It carries the raw amplified vector directly to the addition operator $\oplus$.

$$\mathbf{h}_L = \underbrace{\gamma \mathbf{x}_0}_{\text{Preserved Boost}} + \sum_{l=1}^{L} \Delta_l(\mathbf{k}_l, \mathbf{W}_l) \tag{7}$$

This proves that the amplification factor $\gamma = 1.1$ is **additive and persistent**, independent of the learnable parameters in the processing blocks.

A.4 Feed-Forward Network (SiLU) Dynamics

Inside the "Processing Path" the Llama 3.2 FFN utilizes a SiLU activation with three learnable linear projections $(W_{gate}, W_{up}, W_{down})$. For a normalized input $\hat{\mathbf{x}}$:

$$\begin{aligned} \mathbf{g} &= \mathrm{SiLU}(W_{gate} \cdot \hat{\mathbf{x}}) && (8) \\ \mathbf{y} &= W_{up} \cdot \hat{\mathbf{x}} && (9) \\ \mathrm{FFN}_{out} &= W_{down} \cdot (\mathbf{g} \odot \mathbf{y}) && (10) \end{aligned}$$

Since $\hat{\mathbf{x}}$ is normalized via A.2, the FFN weights operate on standard distributions, preventing saturation of the SiLU activation function.

A.5 Second-Order Effects: Residual Dampening

We identified a subtle interaction when Arrow 1 and Arrow 2 merge. The RMSNorm at Layer $l+1$ normalizes the sum:

$$\mathrm{Denominator}_{l+1} = ||\gamma \mathbf{x}_0 + \Delta_l|| \tag{11}$$

Since $\gamma > 1$, the term $\gamma \mathbf{x}_0$ inflates the norm denominator. Because the learnable weights (W) generating Δ_l have not yet updated to produce proportionally larger outputs, we observe a **dampening effect**:

$$\text{Effective Contribution} \propto \frac{\Delta_l}{||\mathbf{1.1x}_0 + \dots||} < \frac{\Delta_l}{||\mathbf{1.0x}_0 + \dots||} \tag{12}$$

This renders the model "stiff," as the static amplified embedding suppresses the contribution of the learned contextual updates.

A.6 Gradient Amplification (Backward Pass)

Finally, we derive the gradient effect. Using the Jacobian $\mathcal{J}$ of the residual equation:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}} = \frac{\partial \mathcal{L}}{\partial \mathbf{h}_{out}} \cdot \left(\gamma \mathbf{I} + \frac{\partial \mathcal{F}}{\partial \mathbf{x}} \right) \tag{13}$$

The term $\gamma \mathbf{I}$ scales the error signal by 1.1. This validates that the optimizer receives a mathematically amplified signal to update the connector embeddings, specifically targeting the vector $\mathbf{x}$ rather than the downstream parameters $\mathbf{k}$ or $\mathbf{W}$.