

# Pantograph-Catenary System for BRTS in Ahmedabad

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May 2023

# Certificate

This is to certify that the Capstone Project entitled “ Pantograph-Catenary System for BRTS in Ahmedabad”, submitted by the students team to the Capstone Projects Office, Undergraduate College, Ahmedabad University, Ahmedabad-380009, for the partial fulfilment of the requirements for their Undergraduate Degree and is their original work, based on the results of the investigations carried out independently by them during the period 11/22 to 05/23 of the study under the supervision of the Faculty Mentor Professor(s) Sunil Kale, Maryam Kaveshgar, and Saumil Shah at Ahmedabad University.

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# Acknowledgement

We would like to express our sincere gratitude to our professors Dr. Maryam Kavesghar (Assistant Professor, School of Engineering and Applied Science, Ahmedabad University), Dr Sunil Kale (Dean, School of Engineering and Applied Science, Ahmedabad University), and Mr. Saumil Shah (Senior Lecturer, Amrut Mody School of Management, Ahmedabad University) for guiding and supporting us throughout the final year capstone project period. Their expertise and dedication helped us to stay on track and ensured that our project met the highest standards.

Additionally, we would like to thank our team members, Vedanshee Trivedi, Manan Anjaria, Anar Bhagat, Nipun Bhayani, and Rishi Jain for their hard work, creativity, and commitment towards this project. Their eclectic skills and perspectives were crucial in solving our problems and achieving our goals. We would also like to thank Prof. Minal Pathak, and Prof. Shaurya Patel for their guidance regarding the carbon credit and cost calculation in this project.

We are also grateful to Ahmedabad University for providing us with access to resources and facilities, and these contributions were instrumental in the completion of our project.

# Abstract

This capstone project presents a study of the pantograph-catenary system for an electric bus in Ahmedabad. Through the help of the pantograph mechanism, the bus draws power from the overhead wires. This capstone project reviews the existing literature on the different mechanisms that use OHL to draw power, single arm, double arm pantograph and single arm double pantograph and draws a detailed comparison between them to justify the best suitable option to be the single arm pantograph covered with insulators. The lumped mass model of a pantograph, and the effect of contact force between overhead lines and the pantograph have also been reviewed in this project in order to understand the dynamics of the catenary system and kinematics of the pantograph mechanism.

Moreover, the z-axis acceleration, and speed data of a Bus Rapid Transit System (BRTS) of Ahmedabad has been analyzed to determine the contact force between the pantograph-catenary system (PCS) in real-life. A catenary system has been modeled to identify the catenary displacement and nominal stiffness whenever the bus is in motion.

The kinematics of the TATA electric bus was analyzed in order to find the yaw angle of the steering wheel. Furthermore, a kinematic analysis of the pantograph mechanism in accordance with the real-life dimensions of a TATA electric bus and structural parameters of the catenary system was also conducted.

Ultimately, a working model of the pantograph-catenary bus system was made after scaling all the dimensions down by 1/18 times using a remote-control system for the movement of the bus and a servo communication with RFID tags for the upward and downward movement of the pantograph. Apart from this, a cost-benefit analysis has been carried out that would be useful in understanding the feasibility of the system when implemented in real-life.

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# Chapter 1

## Introduction

The pantograph-catenary system is a widely used device for collecting electric power from an overhead line for trains and trolleybuses. The performance and stability of the PCS depend on the contact force between the pantograph and the catenary, which is affected by various factors such as speed, catenary geometry, wind load, and friction.

The major issue with the current BRT system is that the battery-powered electric buses have a limited range and require long charging times. Currently, the Ahmedabad BRT buses can travel 120 km on a single charge which is insufficient for multiple trips. Additionally, the charging infrastructure required to recharge electric batteries can be expensive and time consuming to install.

Another problem encountered is the size and weight of the batteries, which in turn affects the overall weight, and maneuverability of the bus. Consequently, this can affect a bus's performance, and reduce efficiency, especially during harsh weather conditions. It is essential to consider the effects on the environment when a large battery is used. The production of such rechargeable batteries requires the extraction of raw materials including metals and minerals, which can have negative environmental and social impacts.

Apart from this, the idea behind the usage of public transport is that more people commute using that. However, this is not happening in Ahmedabad mainly because the size of the battery is quite large and it reduces the length of the bus. This in turn leads to a reduced number of passengers.

The rationale of this project addresses four main areas that includes sustainability, cost-effectiveness, efficiency, and innovation. Electric buses are more environmentally-friendly as compared to diesel buses. Therefore, by designing a pantograph-catenary system, emissions and pollutants released in the environment would significantly reduce, and thereby making it more sustainable form of transportation.

Although the initial investment in designing and installing a pantograph-catenary system may be higher than other charging solutions, it can ultimately save money in the long run. The reason behind this is that electric buses are cheaper to operate than traditional diesel buses due to lower fuel costs and fewer maintenance requirements. This system will provide a more efficient and reliable charging



solution for electric buses. It can help ensure that the bus remains charged throughout its corridor without needing to go to depots where it stops for extended periods of time for charging.

These types of pantograph-catenary systems are already implemented in western countries such as Germany, Italy, the UK, and the USA. However, designing such a system for an electric bus in India, specifically Ahmedabad is a unique and innovative project that will challenge our engineering skills and creativity. Additionally, it can provide opportunities for research and development in the field of sustainable transportation.

This paper provides information on the kinematics and dynamics of a pantograph-catenary system, and describes a method that will be helpful in calculating the contact force in real-life, that acts when a pantograph comes in contact with the catenary wire. Furthermore, it provides the calculations required to model a catenary system, understand the kinematics of the pantograph, and the kinematics of the TATA electric bus. The above-mentioned method is tested on a 1/18<sup>th</sup> scale prototype of a bus with a single-arm pantograph, along with a catenary system that will continuously power the electric bus.

Additionally, the paper also provides the design and information on the objective of this project, and that is to automate the pantograph attaching and retracting process. It also addresses the behavior of the PAC system whenever a situation where a large turning radius is encountered or speed of the bus increases more than required. It reviews the literature on the existing BRT electric transmission systems, BRT routes, and the existing pantograph designs and mechanisms for trains, and trolley buses that are used in several European countries as well as the use of pantographs on trucks in the USA and Germany. This information has provided a foundation for the project, and it has helped in identifying potential challenges and opportunities.

Furthermore, there are several cost factors that come into consideration when implementing pantograph systems on electric buses. The two major costs involved in this project are capital cost (infrastructure cost) and operational costs (ongoing expenses). The above-mentioned costs were researched and studied for the assessing the economic feasibility and long-term sustainability of adopting pantograph systems. Overviewing through all the costs i.e., infrastructure deployment, the expenses associated with bus modifications, ongoing maintenance and electricity costs, and the overall cost efficiency of the system would help stakeholders in better decision making for the viability of the pantograph charging on the buses. In the initial stage, project has the huge capital expenditure involved and day to day operating expenses but there is a scope that in later date it will cross break-even. It would be convenient to the people once it is implemented in Ahmedabad, India.

# Chapter 2

## Literature Review

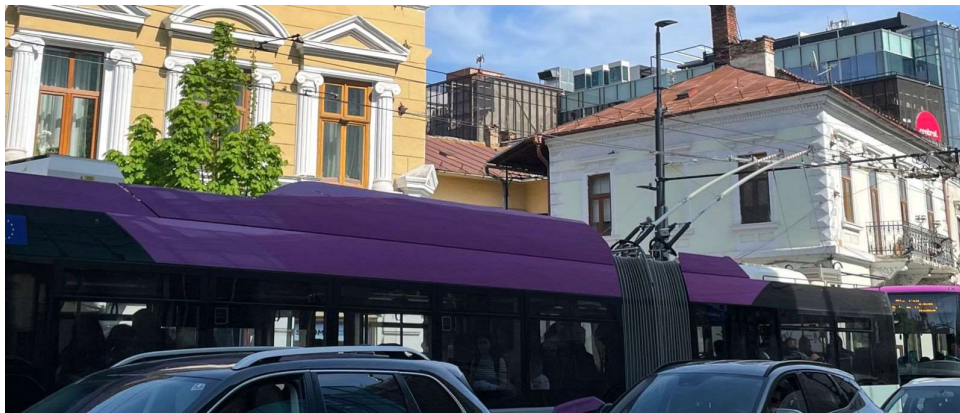
A lot of work is carried out in automating the PAC system that is currently being used in railways. Researchers are working on implementing the electrification technique of a trolleybus catenary system in cities, and how the concept of in-motion charging can increase the efficiency of the current system. For instance, recently, Scania has provided several pantograph-equipped hybrid trucks that operate on an electric highway near Frankfurt, Germany. Due to overhead contact wires, the truck is able to achieve more than 80% efficiency level. [24]

The electrification system is developed by Siemens and it allows the trucks with a pantograph mounted on the roof to travel at a speed of 90 km/hr. Once the truck leaves the electrified highways, it operates on their internal combustion engine with biofuels that reduces CO<sub>2</sub> emissions. The electrified road technology is one such approach that Scania is testing and implementing to shift towards a sustainable transport system. [24] Continental Engineering Services and Siemens Mobility are working on a similar project that is mentioned above [17].

Trolley poles that have a similar functionality are used in local buses and trams in several European countries such as Italy, Switzerland, Poland, etc. However, according to Polom et al, the number of trolleybuses currently operating are around 300, that means the technology is less popular, and more research needs to be performed in this area [25]. Many researchers are exploring the concept of in-motion charging along with the OHL (Overhead Line) infrastructure, which leads to a more efficient public transport system. For example, in Solingen, Germany, the trolleybus system was introduced in 2019 to operate route 695. This line runs under the overhead wire only for 25% of the route, but the combination of in-motion fast charging with additional stationary charging enables service with articulated buses. In Gdynia, Poland, in-motion charging is used, where the disconnection of OHL takes place at a scheduled stop and lasts exactly as long as the replacement of passengers, and the assumed normal stop time. Here, the trolleybuses' OHL infrastructure was designed on the main traffic routes of the city. Many of the sections of routes with high frequency of buses overlap with sections of the trolleybus OHL network. This network can be used to charge electric buses on the move, i.e., for the electrification of bus lines. [25] The trolley poles have springs that keep the trolley wheel or shoe in contact with the wire. Generally, trams and trolleybuses use two trolley poles to control the forward and backward directions of the locomotive. However, the trolley pole is manually changed by

the conductor by pulling the wire either with a rope or a pole, and the poles are usually made up of bamboo because it is lightweight, acts as an insulator, and provides strength.

According to Paternost et al, weight of the bus has a great influence on energy consumption, and a methodology has been developed by them to detect changes in the payload by using pressure sensors on the bus's tires and air suspensions [26]. Thus, weight and other forces acting on the vehicle are necessary when designing a pantograph-catenary system.



**Fig.1** Trolleybus in Cluj-Napoca, Romania

Similarly, bow collectors were used in the past, where the collector head has a bow-shaped cross-section, and it is raised using a spring. Furthermore, there are many companies that manufacture railway pantographs, and some of them are Alstom SA, CRRC Corporation Limited, SCHUNK GmbH & Co.KG, and Wabtec Corporation.

**Table 1:** Comparison of different mechanisms that use OHL to draw power

Parameters	Trolley poles	Bow collector	Pantograph
Working	Has a revolving base and the turning operation is achieved using belts and pulleys	The base of the bow collector is fixed on the roof of the trams	The upward and downward operation is achieved using a pneumatic actuator, and a button is pressed by the driver to facilitate this motion
Degree of Freedom	Limited degree of freedom when taking sharp turns	The degree of freedom is very low	Higher degree of freedom that makes sharp turns easier
Degree of Complexity	Simple to construct	Difficult to construct	Difficult to construct

Weight	Light in weight	Heavy in weight	Heavy in weight
Major issues faced	The conductor needs to manually turn the pole when changing directions, and the revolving mountings can sometimes skip the wire, and are attached to the wrong ones during an intersection	The mountings are fixed so no such issues are encountered here	If it is mounted on an electric bus, the overall weight of the bus will increase
Additional requirements	Wire frogs are required to ensure that the pole goes in correct direction at junction	Overhead frogs and guides are not required	Overhead frogs and guides are not needed
Noise	Less-noisier	More-noisier	More-noisier
Type of overhead lines used	The Overhead wires are not stretched as compared to bow collectors	The Overhead wires are stretched tighter so for short distances the overhead wires have zig-zag lines rather than straight lines	To reduce the wear on the pantograph slide, the overhead wires have zig-zag lines rather than straight lines
Stability	Less stable at high speed	Less stable at high speed	It is more stable at high speed because the pantograph is almost free from dewiring

Additionally, different types of pantograph designs were also studied and analyzed. Table 2 and Table 3 mentioned below shows the major differences between these designs.

**Table 2:** Comparison between single arm and double arm pantograph

Single arm pantograph	Double arm pantograph
Less-heavier	More-heavier

Require less power to raise and lower	Require more power to raise and lower
Connects with overhead single-phase AC system	Connects with an overhead three-phase AC system
Z shape structure	Diamond shape structure
Single point of contact with the OHL	Two points of contact with the OHL
Less energy losses	More energy losses
High efficiency	Low efficiency
The structure is much simpler to build	The structure is complex
If mounted on an electric bus, the overall weight will be less in comparison to the double arm pantograph	If mounted on an electric bus, the overall weight will increase

**Table 3:** Comparison table between Single arm pantograph and single arm double pantograph

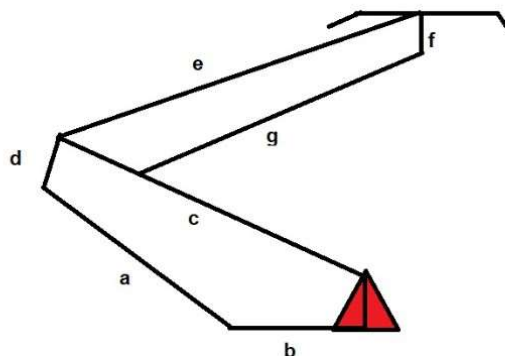
<b>Single arm pantograph</b>	<b>Single arm double pantograph</b>
There is only 1 pantograph	There are 2 pantographs
Structure is much simpler	Structure is complex because two pantographs are used here
One contact head with carbon strip that connects to 2 OHLs	Two contact heads with carbon strip, one each for one pantograph that connects to 2 OHLs
There is an insulator present on the contact head	No such insulator is present here

The TATA electric bus has an Ackermann Steering geometry and a multilink steering mechanism with independent front suspensions. It is designed in such a manner that a 4-bar linkage is located in front of the wheel center. It mainly consists of five components that include an idler arm, relay lever, track rod, two tie rods, and two steering arms. [1] Thus, it is crucial to understand the kinematics of an electric bus before analyzing a pantograph-catenary system for the bus.

A pantograph comprises of two 4-bar mechanisms, and for it to function appropriately, there must be a sufficient contact force between the current collector head and the OHL. Excessive tension on the lower arm can result in the breakage of the OHL because of continuous upward thrust by the pantograph. Therefore, the lower arm of a pantograph is kept at a certain angle by the actuating mechanism.

Consider the four-bar mechanism shown below. When the length  $c$  is rotated in the clockwise direction, a motion is produced in the upper arm, and it results in an angular variation. However, due to the rotation of the upper arm, the angle of the collector's head will change. This will cause improper

contact between the collector head and the OHL. Therefore, a balancing rod is attached in the middle of the existing four-bar mechanism. When the mechanism moves, the distance between the collector head and lower arm reduces, but the length of the balancing rod remains constant. Therefore, the balancing rod pushes the collector head in the opposite direction, and this anticlockwise rotation will reduce the collector head's tilt.



**Fig.2** Mechanism of a single arm pantograph

A catenary system mainly consists of a contact wire, droppers, a messenger wire, a positioner, and an auxiliary messenger wire. The unevenness of the contact wires is one of the factors that influence the dynamic characteristics of current collection systems. There are numerous catenary designs such as a trolley wire that has a single tensioned wire supported by towers, a simple catenary that consists of a messenger wire, droppers, and contact wire, a stitched catenary, and a compound catenary. Higher tension increases the wave speed and makes the catenary stiffness more uniform. When a pantograph moves from maximum to mid position, the catenary will displace more, and thus, the catenary has a parabolic shape. [6]

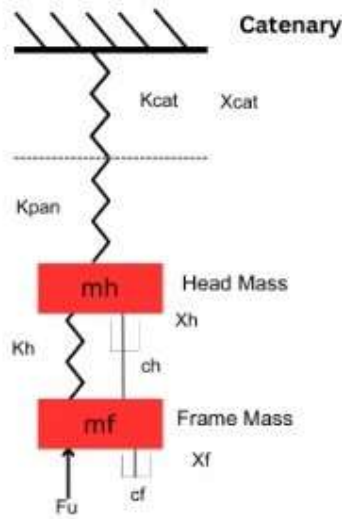
When the vehicle travels, the flow velocity around the frame body and the pantograph rods will generate a lift force that will act on these two 4 bar mechanism arrangements shown in Fig.2 in the vertical direction due to vibration produced during traveling. The force produced when a pantograph pushes the overhead line upwards while traveling is known as the contact force. The current collection of pantograph-catenary systems is directly proportional to the fluctuation of catenary contact force. When the contact force becomes close to zero, the contact between the pantograph head and the

catenary cannot be maintained and arcing is induced which will cause the wear of the pantograph. When the contact force becomes too large then along with the pantograph, the catenary wire is also damaged. As a result, stable current collection is not possible. [6]

According to Ning Zhou et al, the factors that affect a pantograph-catenary system are operating speed, parameter variation, applied force amplitude, weather conditions, and vibrations caused by the locomotive [4]. There are several methods to control the contact force variations and increase the current collection capacity of the system such as using an advanced material for pantograph slide plates, using active control of the pantograph-catenary system, minimizing the acceleration of the collector head, and controlling the contact force based on state estimation using Kalman filter. Mokrani developed an active system with fuzzy logic and sliding mode controller, where PID control law is used to control the contact force and it gives continuous real-time data. The two primary actuation control techniques to reduce contact force fluctuations are PID control and impedance control. The latter one is more efficient due to faster frequency response. Furthermore, several research works have indicated that the usage of multiple sensors on a pantograph reduces the reliability of the control system. [11]

According to Jia-Shiun Chen et al, the electric bus vibrations are caused due to rapid acceleration and deceleration which is produced due to gear shifting. An electric motor is used to suppress driveline torsional vibrations that are transmitted to the driver's seat. This will reduce the vertical vibrations and enhance driver's comfort. [9] Therefore, it is essential to reduce the vibrations, and a damping environment is used, where the total force is equal to the sum of frictional force and the back force provided by the spring. As the mass increases, the spring will move upwards and time  $T$  rises, so SHM (Simple Harmonic Motion) also increases. They provide energy loss by means of fluid friction, absorb mechanical energy and convert that into heat energy. The electric bus vibrations in turn will affect the contact between the collector head and catenary. For example, during the condition of snow accumulation, the mass will increase. As a result, the quality of contact head will deteriorate.[11]

The pantograph is assumed as a lumped-mass model, and is modeled as a set of masses, springs, and shock absorbers on which different forces can act such as pushing force, aerodynamic force, friction, etc. As shown in Fig.3 below,  $F_u$  is the lifting force,  $m_f$  is the mass of the frame,  $m_h$  is the mass of collector head,  $c_f$  is the damping coefficient of frame,  $c_h$  is the damping coefficient of collector head,  $x_f$  is the displacement of frame,  $x_h$  is the displacement of collector head,  $k_{pan}$  is the spring stiffness of pantograph,  $k_{cat}$  is the spring stiffness of catenary, and  $x_{cat}$  is the displacement of catenary. The simulation results after considering the lumped-mass model conducted by Ebru Karakose, and Muhsin Tunay Gencoglu stated that the reference contact force should have a value of 100N. [11]



**Fig.3** Lumped mass model of a single arm pantograph

Another research work by Aleksandrs and Andrejs Matvejevs highlights the combination of a beam model for the contact wire with a mechanical multibody system (MBS) model for the pantograph. Here, the entire system consisting of pantograph, catenary, lower, and upper frames is modeled as a linear system with springs, masses, and dampers. It is expressed as a sum of the natural modes and this technique is known as modal decomposition. [12] Tatsuya Koyama et al has devised a force-equilibrium model to measure the contact force where the pantograph is divided into two subsystems. The displacement measuring method uses an image processing technique, and a line sensor camera is used to mark certain points that will create a new displaced line. This method is useful in measuring the spring deformation and acceleration of the panhead, which will be useful in measuring the contact force in real-life. [2] Thus, the contact force is a summation of aerodynamic force and inertia force. To calculate the aerodynamic force, it is essential to understand the aerodynamic impacts on the locomotive [8]. We analyzed the aerodynamic impacts on a train and how it affects the movement of a pantograph, and based on this we could determine the role of the aerodynamic force acting on an electric bus. High-speed trains have a larger aerodynamic impact because higher the speed of the train, more will be the Reynolds Number. Due to this, a boundary will form, and this boundary layer will produce excessive noise and unnecessary tractive longitudinal resistance. The aforementioned problem can be solved by shrouding which will create a no-slip boundary condition around the pantograph. Therefore, in shrouding we are tilting the pantograph which will change the turbulent flow to laminar flow. According to the boundary layer theory, after the transition point, a boundary layer is formed,



and the flow transits from laminar to turbulent. This will produce a drag force. Thus, the aerodynamic effect is proportional to the square of the velocity. The use of aero foil generates lift above the chord line that will push the pantograph upwards towards the OHL. The inertia force can be measured using an accelerometer that can be placed on the pantograph head. [19]

There are several actuating mechanisms used in a pantograph that will push it upwards or downwards in order to maintain a constant contact force. Generally, in trains, a pneumatic actuating system is used, where the pantograph is operated using an air bellow mechanism for raising and lowering. The compressed air supply in the locomotive varies from 5.5kg/cm<sup>2</sup> to 11kg/cm<sup>2</sup>, depending upon the type of shock, and compressor operation. Pantograph starts lowering when the pressure drops below 4.5 kg/cm<sup>2</sup>. [22]

Some other actuating mechanisms we reviewed were cycloidal drives, planetary gearbox drives, harmonic drives, and servo mechanism. In a cycloidal drive, an eccentric shaft drives the cycloidal disc. The fixed ring pins are arranged outside the eccentric shaft, in which cycloidal disc engages. Due to the eccentric motion, the cycloidal disc will also start rotating around its axis of symmetry. There are holes in cycloidal discs which rotate in clockwise direction whereas the eccentric shaft rotates in the opposite direction. Roller pins are attached to the output shaft in these holes. The cycloidal drive drives the output shaft. [20]

The planet gears are normally mounted on a moving carrier, and the sun gear, which is in the centre, transmits torque to them. The planet gears mesh with an outer ring gear as they orbit the sun gear. Depending on the application, planetary gear systems can range in complexity from extremely simple to complicated compound systems. When weight and space are an issue but a lot of speed reduction and torque are required, planetary gears are frequently used. [20] After researching on the aforementioned gear mechanisms, for the small-scale model the use of servo motor was concluded as the most effective way to actuate the pantograph.

In order to analyse the feasibility of the pantograph-catenary system, we studied several research papers, and news articles that helped us get an approximate estimate and actual cost of the BRTS project, and from where the whole project has been funded. The total capital cost of the project is Rs.12040.75 million (USD 154.37 million) with an annual capital cost of Rs.840 million (USD 10.80 million). Where the bus replacement cost is to be estimated Rs.2000 million and 15 year is the life span taken for e-buses. The one-time cost of e-buses batteries for 5, 10, and 20 years is Rs.1358 million. It will cost Rs.618 million to update the ITMS (integrated traffic management system) infrastructure for 7, 14, and 21 years (USD 7.92 million). This all-combined gives us the total present value of 22,806,21 million of the BRTS project, that includes lanes, building bus shelters, depots,

sliding doors, and terminals, as well as the hardware and software for intelligent transportation systems. The buses are maintained by TATA motors and they charge Rs 59 per km, that includes skilled, unskilled labour required in maintain and smooth functioning of the buses. This above-mentioned cost also includes the electricity cost. Thus, in this paper we have assumed this as an operational cost for e-buses. We have also seen the comparison between the E-BRTS and our proposed system so that we can evaluate the more economically feasible option. The main factors considered during the comparison include the capital costs, operating costs, environmental factors, sustainability, and energy efficiency. [31]

Although the capital or initial cost of an e-bus with a pantograph would be high compared to diesel buses, the operating cost would be lower than diesel buses. To reduce the capital cost, we can use the current BRTS corridors and bus lanes with necessary modifications. In the initial stage, the dimension of the bus can be kept unchanged. In 2009, the cost of BRTS corridor in Ahmedabad was estimated to be 12.5 km long, and had an approximate cost of Rs.103 crores. The current fuel cost for diesel buses, and hybrid buses are Rs. 15 per km & Rs.10-17 per km. On the other hand, the fuel cost of fully electric buses will be Rs.10 per km. The OHL can efficiently provide energy to the bus that would reduce time and money. E- buses that rely on batteries would require charging after a certain range which can be costly and time consuming. Because of the reach and power prerequisites of transports, the electric transport batteries are of high-capacity type, and it has played a significant role in the expense part of the framework. There are numerous battery advancements in the market depending upon the varieties in anode, cathode, and electrolyte blends. For instance, lead corrosive batteries, nickel-based fluid batteries, and lithium-particle batteries are currently being used, where lithium particle is the most suitable, and used for calculating the cost. [32]

When the buses are running, more electricity is required from the OHL and delivered to the vehicle. Thus, it increases the cost of electric power generation, transmission, and distribution. An average electric bus requires about 325 kWh of electricity, and the fleet size has been used to calculate the overall amount of energy needed. For the proposed system we found that the pantograph would cost Rs.1.25 lakh per piece, and the fuel cost would be Rs.10. [32]

The approximate cost of the entire system would be Rs.50 crore per km, which can vary as Ahmedabad BRTS system has its own corridors and ITMS (Integrated Traffic management system). The above-mentioned cost is based on the projects going in Nashik, Maharashtra, where the cost of construction is estimated to be Rs.50 crore, and Kolkata, West Bengal where the cost of construction is estimated between Rs.60-70 crores per kilometre. According to Mr.Bhati, the cost of electrification of the grid can cost between Rs.20-30 crore per kilometre. The cost can vary based on the geographical

condition of the place, and the project will only be practical if it has a stable connection with OHL, and a steady power supply similar to trains. Apart from this, the maintenance cost of the pantograph would take 10-15% of the cost for the whole year. But in our proposed system it can vary as our designed pantograph will be moving and will be detaching itself from the grid. [28], [29]

The 150 BRTS electric buses have reduced the carbon emission and usage of fuel. Ahmedabad Janmarg Ltd has saved 15,127.50 litres of fuel daily, and has reduced its carbon GHG emission by 16,068.58 CO<sub>2</sub>e kg. It can be further be used to finance this project as carbon credit has its separate market. As carbon credit system encourages business and ventures to reduce its carbon emission and give them credits in return for them which can be traded in their market for monetary return. [27]

## Chapter 3

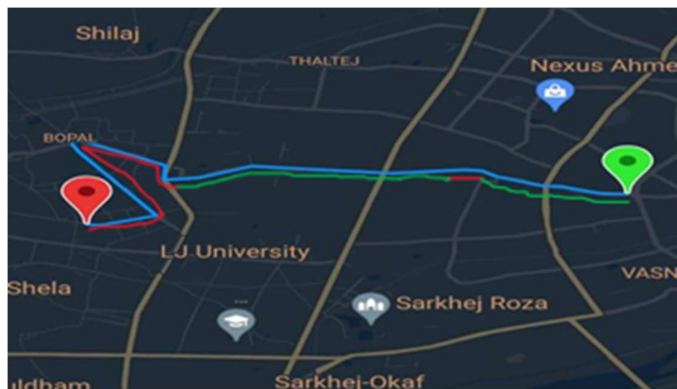
### Research Methodology

The motivation behind choosing a single arm pantograph design among the different types of pantographs can be concluded using Table 1, Table 2, and Table 3 respectively. Some of the main reasons are that the structure is simple, it has a higher efficiency, less power is required to raise and lower it, less energy losses, reduced weight, more stability, and has a higher degree of freedom. It is easier to raise and lower it automatically using a pneumatic actuator or a servo motor. The additional degree of freedom makes sharp turns easier to achieve, and unlike the traditional trolley poles, there is no need to manually turn the arms when changing direction. However, it is important to note that a single arm pantograph's contact head connects with two overhead wires, and to ensure proper grounding of the pantograph-catenary system for an electric bus, an insulator is present on the contact head.

#### *Data collection*

To analyze the kinematics and dynamics of a BRTS bus and its pantograph, we collected data from a trip between Nehrunagar and South Bopal in Ahmedabad. The route from Nehrunagar to South Bopal is shown below.

***Nehrunagar BRTS- Jhansi Ki Rani BRTS- Shivranjani BRTS- Jodhpur Char Rasta- Star Bazaar BRTS- ISRO BRTS- Ramdevnagar BRTS- Iskcon Cross Road BRTS- Iskcon Mandir BRTS- Antariksh Colony BRTS- Ashok Vatika BRTS- Jayantilal Park BRTS- Swagat Bunglow BRTS- Ambli Gam BRTS- Bopal Approach BRTS- Sobo Center- Sukhasan Char rasta- South Bopal***



**Fig.4** BRTS route 17D - Nehrunagar to South Bopal

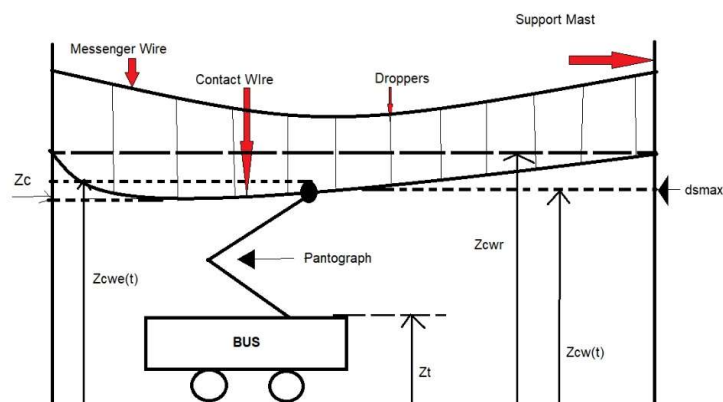
The Fig.4 shown above indicates that an OHL can be installed between the stations marked with a green line, and the red line is where the electric bus should run on a buffer battery without the OHL. Thus, OHL can be installed between Nehrunagar and Ramdevnagar, however from Ramdevnagar to Iskcon Cross Road, it is not economically feasible to install an OHL because there is an overbridge and an intersection here. From Bopal approach onwards, there are no BRTS corridors, and thus, the bus should completely run on a buffer battery. This type of similar approach can be adapted for other BRTS routes.

The data included the bus dimensions, gross vehicle weight, steering wheel diameter, turning radius of road, vertical acceleration, and velocity of the bus. Phyphox app was used to measure the z-axis acceleration and GPS speedometer was used to measure the speed. The steering wheel yaw angle was computed and the actual turning angle of the wheels was measured to study the motion of the bus.

**Table 4:** Specifications of the TATA electric bus [14]

Parameters	Specifications
Length of the bus	9m
Bus width	2.34m
Bus height	3.2m
Turning circle radius (R)	8.75m
Wheel base (L)	4.92m
Acceleration of bus	Greater than or equal to 0.8m/s <sup>2</sup>
Gross vehicle weight	10200 +/- 300 kg
Tire pressure	0.9 MPa
Steering wheel diameter	0.5m
Steering trapezoidal arm	0.2m
Main pin offset distance	0.05m
Wheel tread	2.16 m

To understand the dynamic effects on the entire system, it was crucial to understand the catenary stiffness and displacement that occurs whenever a pantograph comes in contact with the wire. The data related to the catenary system was collected based on the standards defined by a railway catenary system in India, and the trolleybus catenary system in the UK. For simplification, the length



**Fig.5 Simple Catenary System [3]**

**Table 5:** Specifications of a catenary [16, 23]

Parameters	Specifications
Catenary maximum stiffness	3000 N/m
Catenary minimum stiffness	1000 N/m
Average catenary stiffness	2000 N/m
Tensile force of catenary	10000 N
Catenary span	30 m
Distance between dropper	4.8 m
Stiffness variation coefficient	0.5 m
Contact linear mass density	0.95 m
Catenary rest height from ground	5.5 m

## Equations

### *Kinematics of the TATA electric bus*

According to Mehmet Murat Topac et al, to make the steerable wheels turn at different angles that match the bend's center, O, a vehicle steering mechanism should follow a certain rule. This rule is called "Ackermann Geometry" and it means that the wheel axes should meet at O when extended. [1] The equation is expressed as follows:

$$\beta_{aA} = \tan^{-1} \frac{1}{\cot\beta_i + \frac{L}{L_F}} \dots\dots\dots (1)$$

Here,  $\beta_{aA}$  is the ideal turning angle of the outer wheel which is calculated based on the inner turning angle of the wheel  $\beta_i$ . Therefore, it should justify the Ackermann principle by taking into consideration the steering error. The formula to calculate the steering error is as follows:

$$\beta_F = \beta_a - \beta_{aA} \dots\dots\dots (2)$$

According to the vehicle dynamics rule, it is necessary that the Value of the Ackermann error  $\beta_F$  must be less than or equal to 0.5, and the inner turning angle of the wheel  $\beta_i$  must be kept  $\pm 30$  degrees. In this paper,  $\beta_i$  is considered as  $\pm 20$  degrees which is the minimum value needed to fulfill the Ackermann steering conditions.

Using the equations by Qin Zhang in the design of a bus steering system, the steering yaw angle can be computed as follows: Here,  $L_F$  is the wheelbase, R is the turning radius of bus, and B is the wheel tread. [5]

$$\sin\alpha = \frac{L_F}{R} \dots\dots\dots (3)$$

$$\tan\beta = \frac{L_F}{R\cos\alpha} \dots\dots\dots (4)$$

### ***Kinematics of the Pantograph mechanism***

A typical layout and dimensions of a standard pantograph from the company Siemens was considered in the calculation of the kinematic parameters of a pantograph. Later on, after multiple iterations, scaling down of the dimensions was conducted in accordance with the dimensions of the TATA bus. The following are the dimensions of the pantograph:

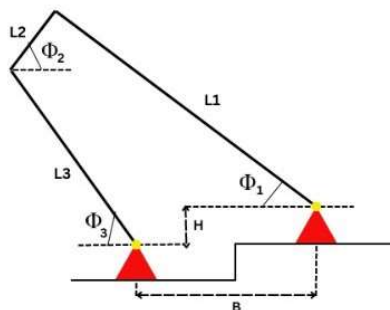
**Table 6:** Dimensions of the pantograph

Parameters	Dimensions

L1	0.96 m
L2	0.1884 m
L3	0.7092 m
L4	1.146 m
B	0.432 m
H	0.09 m

A simple four bar mechanism was considered here to calculate the relative angular formulations in the pantograph.

The equations mentioned below were used to analyze the kinematics of a pantograph. The geometric relations were created by adding the vertical and horizontal lengths of the four-bar mechanism in the beginning.



**Fig.6** Initial four bar mechanism

$$L1 \cos(\phi1) + L2 \cos(\phi2) = c + L3 \cos(\phi3) \dots\dots\dots (5)$$

$$L2 \sin(\phi2) + L3 \sin(\phi3) = d + L1 \sin(\phi1) \dots\dots\dots (6)$$

Then, after some simplifications, one obtains an equation with such a form by rearranging by placing the components with  $L3$  and  $\phi3$  on one side, squaring both equations, and then adding them.

$$k1(\phi1) \sin(\phi2) + k2(\phi1) \cos(\phi2) + k3(\phi1) = 0 \dots\dots\dots (7)$$

Where the independent variable is an arbitrary choice, and  $k_i$  are functions of it. This equation is famously known as the *Freudenstein Equation*.



$$k1(\phi1) = -2 * l2 * (L1 \sin \phi1 + d) \dots\dots\dots (8)$$

$$k2(\phi1) = 2 * l2 * (L1 \cos \phi1 - c) \dots\dots\dots (9)$$

$$k3(\phi1) = L1^2 + L2^2 - L3^2 + c^2 + d^2 + 2 * d * L1 \sin \phi1 - 2 * c * L1 \cos \phi1 \dots\dots (10)$$

The above equations can be analytically solved by assuming:

$$t = \tan\left(\frac{\phi2}{2}\right) \dots\dots\dots (11)$$

From which we also get:

$$\cos(\phi2) = \frac{1-t^2}{1+t^2} \dots\dots\dots (12)$$

$$\sin(\phi2) = \frac{2t}{1+t^2} \dots\dots\dots (13)$$

Moving on, we arrive at the following equation using the *Freudenstein Equation*:

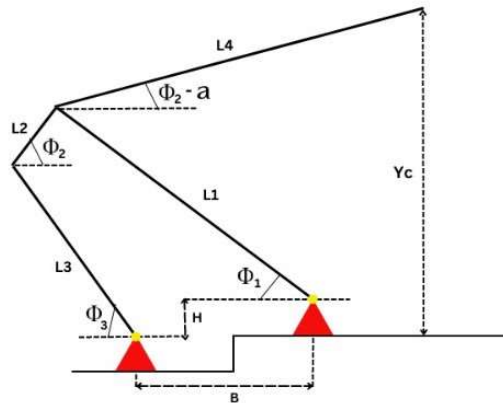
$$t^2 (k3 - k2) + t(2 * k1) + (k2 + k3) = 0 \dots\dots\dots (14)$$

which ultimately provides an answer for  $\phi2(\phi1)$ :

$$\phi2(\phi1) = 2 * \arctan\left(\frac{-k1 \pm \sqrt{k1^2 + k2^2 - k3^2}}{k3 - k2}\right) \dots\dots\dots (15)$$

Hence, by using the above equations, we find:

$$\phi3(\phi1, \phi2) = \arctan\left(\frac{L1 \sin \phi1 - L2 \sin \phi2 + d}{L1 \cos \phi1 + L2 \cos \phi2 + c}\right) \dots\dots\dots (16)$$



**Fig.7** Modified four bar mechanism

If all the bars are thought of as rigid and another bar of length  $L_4$  is attached to the second bar at a fixed angle  $\alpha$ , as indicated in the above picture, the position of the cross bar can be defined by knowledge of just one angle:

$$X_c = -L1 \cos(\phi1) + L4 \cos(\phi2 - \alpha) \dots\dots\dots (17)$$

$$Y_c = L1 \sin(\phi1) + L4 \sin(\phi2 - \alpha) \dots\dots\dots (18)$$

Hence, the component  $Y_c$  gives us the height of the pantograph which has to be 1.7m at its maximum limit to reach and  $\alpha$  be in contact with the contact wires. In order for the pantograph to reach its maximum height, we found to be 32 degrees through an iterative process. [13]

### ***Dynamics of a catenary system***

Considering the pantograph contact wire, and neglecting the messenger wire, the vertical displacement, and nominal stiffness of a catenary can be calculated using the following equations respectively.

$$Z_{cw}(t) = Z_{cwr} - a \sin\left(\frac{vt}{L_{ws}}\right) - b \sin\left(\frac{vt}{L_{ws}/n}\right) \dots\dots\dots (19)$$

Here  $Z_{cw}(t)$  is the catenary displacement,  $Z_{cwr}$  is the catenary height from ground,  $L_{ws}$  is the catenary span, and n is the number of droppers. In our calculations the total number of droppers considered are six. Considering the midspan of the catenary wire, a and b are constants whose values are 0.045 m and 0.01 m respectively. [3]

$$K_c(t) = k_{mean} \left(1 - \alpha \left(\frac{\cos 2\pi \cdot v \cdot t}{L_{ws}}\right)\right) \dots\dots\dots (20)$$

$$k_{mean} = \frac{k_1 + k_2}{2} \dots\dots\dots (21)$$

Here,  $K_c(t)$  is the nominal stiffness of the catenary,  $k_{mean}$  is the average catenary stiffness,  $L_{ws}$  is the catenary span, and v is the velocity of the bus. [16]

### ***System Design and Prototype***

In the trains and trams, there are tracks that usually provide a grounding. However, Ahmedabad BRTS does not have these tracks, and to mitigate this issue, we have designed a small-scale pantograph that is insulated throughout. This design is inspired by a single arm pantograph design that was attached on a 6 m e-bus in Cagliari, Italy as shown in the Fig.8 below [18]. The novel approach we have considered here is developing an entire autonomous mechanism to move the pantograph. To test this entire system, we have designed a 1/18<sup>th</sup> scale remote controlled bus as well as a scaled down version of the catenary system.



**Fig.8** E-bus with a single arm pantograph covered with insulators

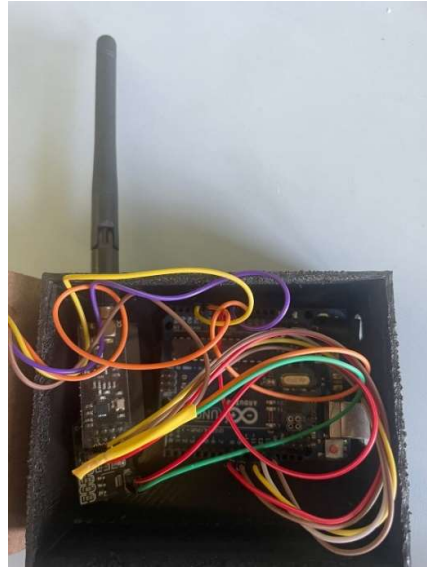
The first part of our prototype mainly includes the design and development of the remote-controlled bus. The components used in building the chassis are 6 rubber tires, screws, nut, bolt, and shaft. The entire body of the bus is made up of wood, the chassis base is made up of steel and the shafts and motor mountings are made up of PLA (Polylactic Acid).



**Fig.9** Remote controlled bus

This bus consists of a transmitter system and a receiver system. The main components used in the transmitter system are Arduino UNO, nRF24L01 radio module, joystick, 9V battery. The joystick has two potentiometers that will transmit values in the x and y direction respectively. The x and y values indicate the motion of the vehicle. A positive y value indicates that the vehicle will move in the forward direction, whereas a negative y value indicates that the vehicle will move in the backward direction.

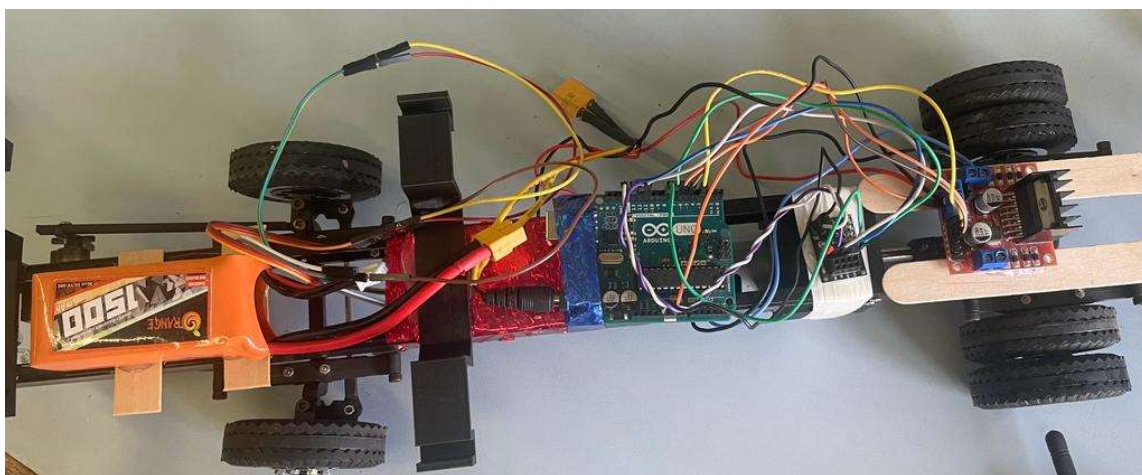
The forward and backward movements of the bus is controlled using a single DC motor that is coupled with a 3D printed mounting. A positive x value and a negative x value indicates that the front wheels of the bus will steer in right and left directions respectively. This steerability is controlled by a servo motor that is attached near the front wheels of the bus with a shaft.



**Fig.10** Transmitter System

The nRF24L01 is a wireless transceiver module, meaning each module can transmit as well as receive data. The nRF module will transmit joystick values and receive 'data received' confirmation. If the confirmation is not received, the same data will be transmitted repeatedly and the connection will not be successfully established. The Arduino UNO is a microcontroller that takes data from the joystick and feeds it to the nRF module.

The receiver system consists of an nRF module, Arduino UNO, L298N motor driver, 12V battery, one DC motor, one servo motor, and a buck converter. The nRF module receives data and transmits a confirmation of the received data. We have used L298N motor driver because Arduino UNO cannot power motors by itself.



**Fig.11 Receiver System**

Additionally, we have designed a scaled down version of the catenary system, where the length of the catenary is 4 m, and the materials used are mild steel, PLA, and Galvanized iron wire. The poles of the catenary are made up of wood, and the two support masts of the catenary are welded.



**Fig.12 Catenary system model**



**Fig.13 Pantograph model**

The Fig.13 shown above is a small-scale model of a 3D printed pantograph, and a servo motor is mounted on along with it. The collector head is insulated with an aluminum foil. The entire pantograph

is placed near the center of gravity of the bus and towards the front side. It is not placed at the rear end because that is more prone to vibrations. In comparison to the trains, the aerodynamic forces acting on the pantograph is less in city buses.

#### *Cost Calculations of the entire PCS*

##### ***Calculation of the Battery required***

Battery size required for EV buses = 325kWh

Cost of 1 lithium battery of 325 kwh = Rs.100,000

For a fleet of 150 buses = Rs.1,50,00,000.

Battery replacement = 5 years.

##### ***Cost of Pantograph***

Cost of 1 Pantograph = Rs.1,25,000 (this price is of freight locomotive pantograph)

Maintenance cost = 10-15% of the total cost per annum.

Rs. 18,750 approx. for one pantograph

Total cost of Pantograph for a whole fleet = Rs.1,87,50,000

Total Maintenance cost per annum = Rs. 28,12,500 approx.

##### ***Infrastructure Cost***

Cost of Electrification of BRTS corridor = Rs.9,255 crores (185.1 km route x Rs. 50 crores per km)

##### ***Cost of Bus Replacement***

Cost of bus replacement = Rs. 330 crores (Rs.2.20 crore x 150 buses)

For the price of bus, we have taken TATA E-buses price.

The bus replacement is 15 years.

##### ***Cost of current Infrastructure***

Cost of Corridors = Rs.1,124.1 crores

Cost of infrastructure maintenance = Rs.13.90 crores per annum.

Cost of replacement of ITMS (every 7 years) = Rs.61.80 crore [31]

##### ***Carbon Credit Calculation***

1 carbon credit = 1 metric ton of CO<sub>2</sub>e

Rate of carbon credit= Rs.22.62 (This rate is taken of 22<sup>nd</sup> May 2023)

Quantity	Monetary value
1 day 16.069 carbon credits (16068.58/1000)	Rs.363.48 (16.069 x 22.62)
1 month 482.07 carbon credits (16.069 x 30)	Rs.10,904.42
1 year 5,865.185 (16.069 x 365)	Rs.1,32,670.48

Ahmedabad Janmarg Ltd has reduced its emissions by 11,037,323.37 CO<sub>2</sub>e kg which can give AJL 11,037.323 carbon credits, and they trade them at a high price. They can get up-to Rs.15,523,111.81 (11,037.323 x Rs1406.46) which can help them cover the cost of operation because the all-time high rate of Carbon credit is Rs.1406.46 per credit.

Since the government has introduced the electric buses in the BRTS, they have started calculating and providing the data related to carbon emissions on the website. The data gets updated on the weekly or fortnightly basis. It was important to understand how the carbon credit provides the finance to companies. According to our analysis it can be seen that companies usually exchange the carbon credits i.e., if one company 'A' has excess carbon credits and company 'B' has less credits as per requirements, then company 'B' would buy remaining credits from them to fill the deficit in exchange of finance. In case of the BRTS, this could be possible between the metro cities like Bangalore, Delhi, Mumbai etc.

### ***BEP Calculation***

Fixed cost	In Rupees
Electrification cost	3,70,20,00,000
Cost of Pantograph	1,87,50,000
Cost of Battery	1,50,00,000
Cost of Replacement of ITMS	8,82,85,714
Bus Replacement Cost	22,00,00,000

<b>Total F.C. (Fixed Cost)</b>	<b>4,04,40,35,714</b>
--------------------------------	-----------------------

### **Replacement of ITMS**

Replacement of ITMS = Rs 61,80,00,000

Life span= 7 years



Per year taken = Rs 8,82,85,714

### Bus Replacement Cost

A= Total cost of 150 buses= Rs 330,00,00,000

B= Life span of buses= 15 years

Annual bus replacement cost= Rs 22,00,00,000(A/B)

**Electrification cost (Catenary system)** = 185.1\*Rs 50,00,00,000/km

A= Total= Rs 9255,00,00,000

B= Life Span= 25 years

Annual Cost (A/B) = ₹ 370,20,00,000

Variable Cost	In Rupees
Pantograph maintenance cost @15%	20,53,12,500
Bus maintenance and operating cost @ Rs 59	83,39,85,945
<b>Total V.C. (Variable Cost)</b>	<b>1,03,92,98,445</b>

Cost of Maintenance is Rs.59 per km which is charged by TATA motors.

38727 km x Rs.59 x 365 days = Rs 83,39,85,945.

1,87,50,500@15%= Rs 28,12,500

A= Life span= 5 years

B= Annual Cost= Rs 102,65,62,500 for 150 EV buses

Cost= (B/A) Rs 20,53,12,500

All the Fixed and Variable cost are taken per annum. And have been divided by their replacement time.  
To get an more accurate data for Annual Fixed Cost and Annual Variable Cost.

Revenue	
Revenue (in Rupees)	<b>58,40,00,000</b>

*In the revenue of the bus, we have only accounted the ticket sales. Which is Rs.16,00,000.*

Rs 16,00,000 x 365 days. = Rs.58,40,00,000.

BEP Formula = (Fixed Cost + Variable cost)/Revenue

<b>BEP (Break-Even Point)</b>	<b>8.70</b>
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The BEP (Break-even point) of the project is high and this indicates that the cost of the project should decrease or revenue of the project should increase.

### *NPV Calculation*

<b>Fixed cost</b>	<b>In Rupees</b>
Electrification cost	3,70,20,00,000
Cost of Pantograph	1,87,50,000
Cost of Battery	1,50,00,000
<b>Total F.C. (Fixed Cost)</b>	<b>3,73,57,50,000</b>

<b>Cash Flow (In Rupees)</b>	<b>58,40,00,000</b>
------------------------------	---------------------

Rs 16,00,000 x 365 days. = Rs.58,40,00,000.

10.25 % is taken as the Discount rate for the calculations.

<b>For 5 Years</b>	
<b>Years</b>	<b>Cash Flow (IN Rupees)</b>
0	-3,73,57,50,000
1	58,40,00,000
2	58,40,00,000
3	58,40,00,000
4	58,40,00,000
5	58,40,00,000
<b>NPV</b>	<b>₹ -1,39,31,94,757.53</b>

<b>For 10 Years</b>	
<b>Years</b>	<b>Cash Flow (In Rupees)</b>
0	-3,73,57,50,000
1	58,40,00,000
2	58,40,00,000
3	58,40,00,000
4	58,40,00,000
5	58,40,00,000
6	58,40,00,000
7	58,40,00,000
8	58,40,00,000
9	58,40,00,000
10	58,40,00,000
<b>NPV</b>	<b>₹ -16,82,90,098.98</b>

For 15 Years	
Years	Cash Flow (In Rupees)
0	-3,73,57,50,000
1	58,40,00,000
2	58,40,00,000
3	58,40,00,000
4	58,40,00,000
5	58,40,00,000
6	58,40,00,000
7	58,40,00,000
8	58,40,00,000
9	58,40,00,000
10	58,40,00,000
11	58,40,00,000
12	58,40,00,000
13	58,40,00,000
14	58,40,00,000
15	58,40,00,000
<b>NPV</b>	<b>₹ 58,36,95,105.23</b>

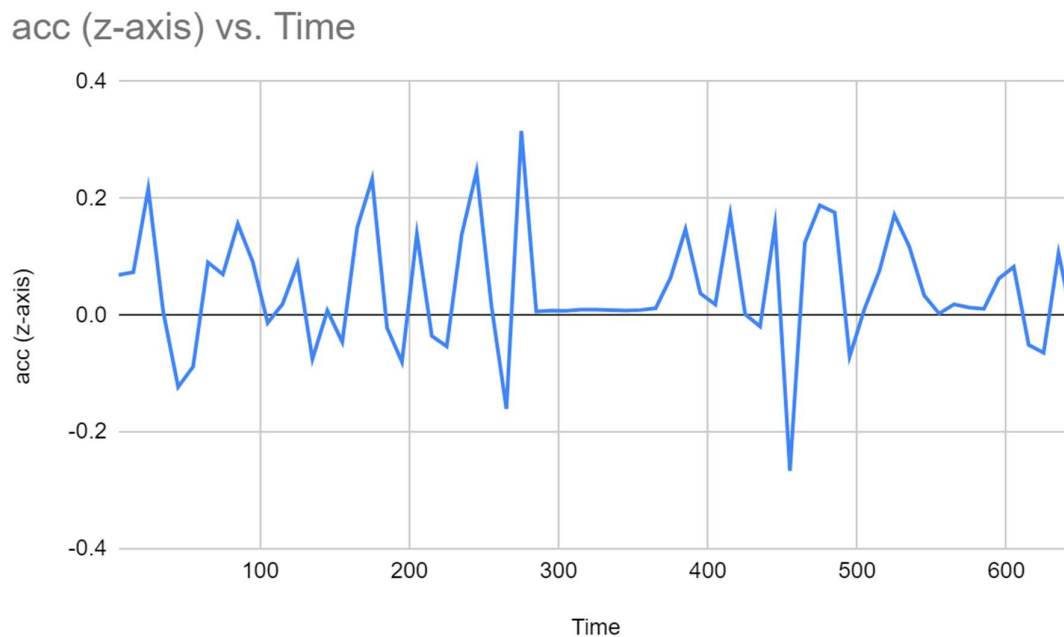
We can look that upon taking different year as base for NPV calculation the NPV become better for the long run as it was negative in 5 years and 10 years and became positive in 15 years. This indicates that this project would be beneficial in the long run.

# Chapter 4

## Results

### *Kinematic Analysis of the TATA electric bus*

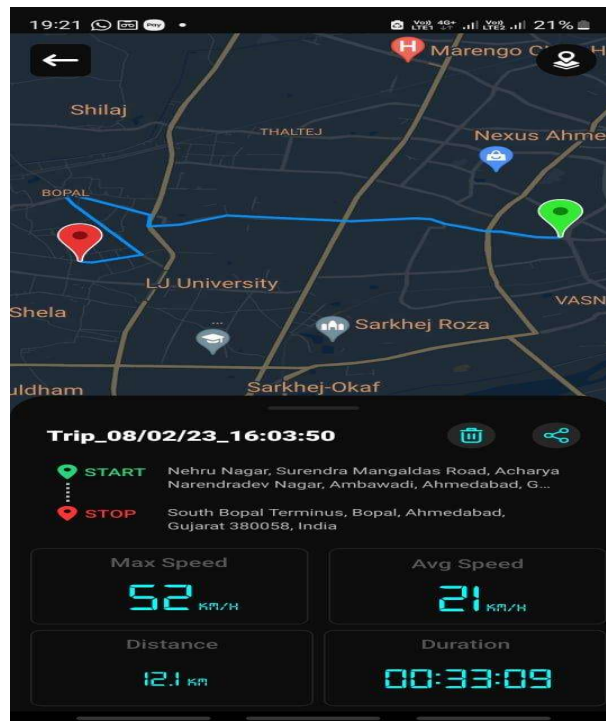
Data sampling was carried out on the vertical acceleration data for the BRTS route and the results are shown below in Fig.14.



**Fig.14** Vertical Acceleration of a BRTS bus on the route Nehrunagar to South Bopal

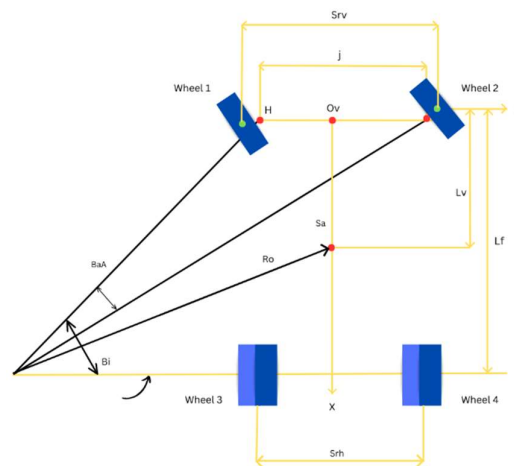
From the figure, it can be seen that the maximum vertical acceleration of the bus without gravity is  $0.3154\text{m/s}^2$ , and the minimum vertical acceleration of the bus without gravity is  $-0.2668\text{m/s}^2$ .

The vertical acceleration data indicates the z-axis motion of the bus, and shows fluctuations in the values whenever a pothole, speed breaker, or a manhole is encountered. These values are crucial in modeling the pantograph and determining the amount of contact force needed to maintain a constant connection between the pantograph and the OHL. The Fig.15 shown below indicates the speed data of the BRT route and, it can be seen that the maximum speed of the bus is 52 km/hr and the average speed is 21 km/hr. Therefore, the maximum speed of the bus will set a boundary condition for the driver, and if the speed is greater than this boundary speed, the pantograph will lose contact with the OHL.



**Fig.15** Speed data of a BRTS bus on the route Nehrunagar to South Bopal

Using the equations (1) and (2), the ideal turning angle of the outer wheel is calculated, where  $L_F$  is the wheelbase,  $j$  is the distance between two pivot points of the wheel as shown in the Ackermann geometry model below. The ideal turning angle  $\beta_{aA}$ , and the actual turning angle  $\beta_a$  of the outer wheel were equal to 17.43 degrees and 17.93 degrees respectively.



**Ackermann Steering Geometry**

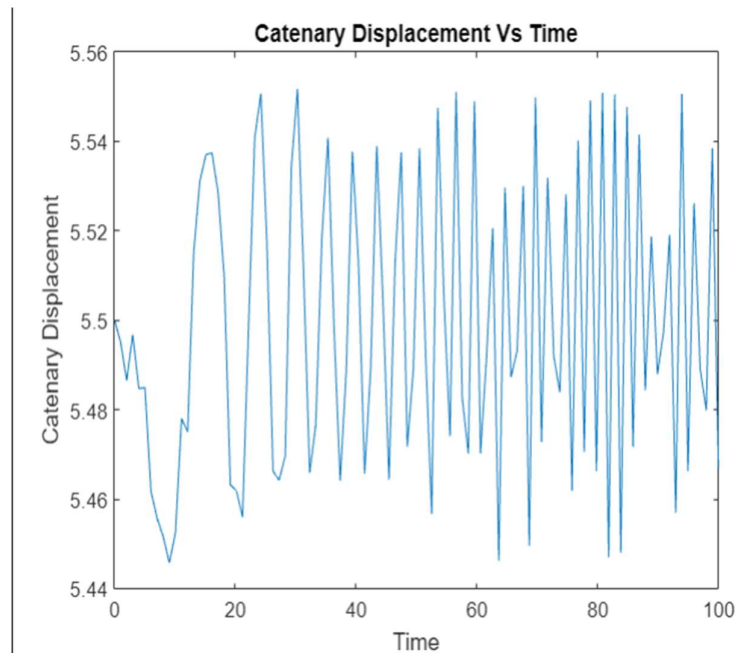
**Fig.16** Ackermann Steering Geometry model

Solving the equations (3) and (4), the input steering yaw angle provided by the driver is calculated and the values are 34.208 degrees, and 44.103 degrees respectively.

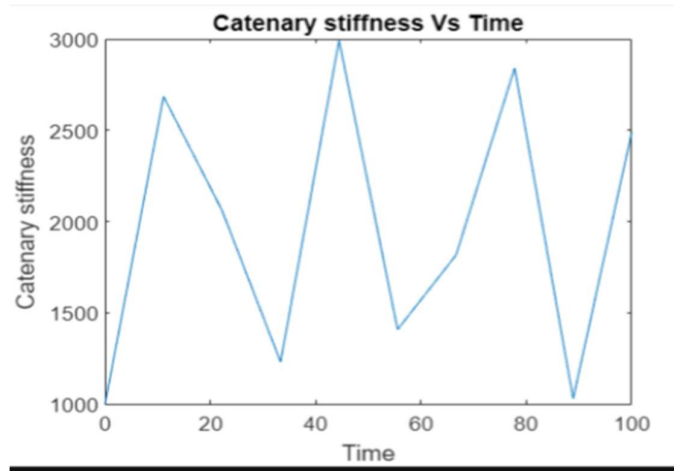
The steering wheel yaw angle refers to the angle of the steering wheel that the driver turns, while the actual turning angle of the front wheels of a bus refers to the angle at which the wheels are turned in response to the driver's input. Factors such as the wheelbase, track width, and caster angle can all have an impact on the way the front wheels respond to the driver's input. Both the angles, will set a boundary condition for the driver, and if the steering yaw angle is greater than the values mentioned above or if the ideal turning angle is more than 17.43 degrees, then the pantograph will lose connection with the OHL.

### ***Dynamic Analysis of the catenary system***

Using the equations (15) and (16), the catenary displacement and catenary stiffness can be calculated, and the results are shown in the graphs below respectively.



**Fig.17** Catenary displacement at  $v = 1\text{m/s}$



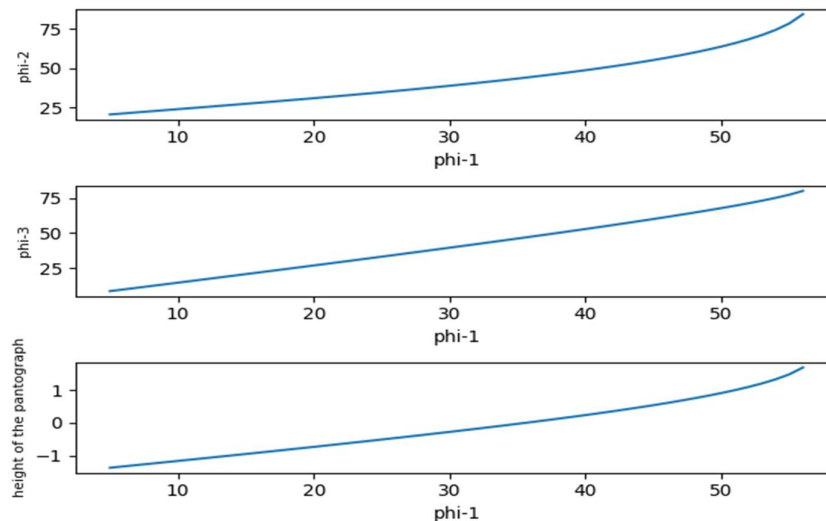
**Fig.18** Catenary nominal stiffness at  $v = 1\text{m/s}$

The Fig.17 shown above indicates the catenary displacement with respect to time when the bus is at the depot, that is the velocity of the bus is  $1\text{m/s}$ , and it can be seen that at different time periods, the catenary gets displaced in a range of  $0.02\text{ m}$  to  $0.06\text{ m}$ .

The Fig.18 shown above indicates the catenary nominal stiffness with respect to time. From the graph, it can be seen that the maximum stiffness is  $3000\text{ N/m}$  during the time interval of 40 seconds to 60 seconds. These values provided a foundation in determining the kinematic relations of the pantograph.

### ***Kinematic Analysis of the pantograph mechanism***

After coding the kinematic equations in Python 3, using NumPy, the real time relations between the input angle  $\phi_1$  with  $\phi_2, \phi_3$  and  $Y_c$  were found by plotting them on a graph using matplotlib.



**Fig.19**  $\phi_1$  v/s  $\phi_2, \phi_3$  and  $Y_c$

The limits of the angles and height being:

$$5 < \phi_1 < 57$$

$$20.4 < \phi_2 < 84.355$$

$$8.75 < \phi_3 < 79.98$$

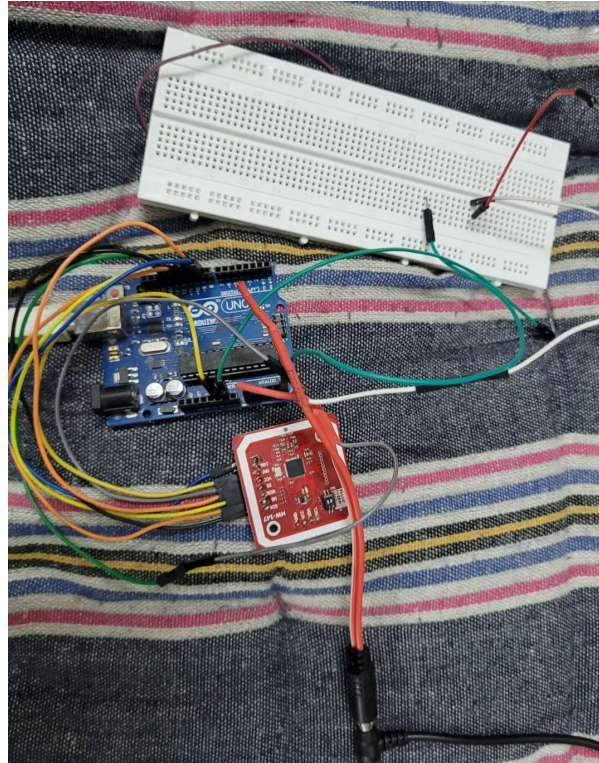
$$-0.146 \text{ m} < Y_c < 1.703 \text{ m}$$

All the three graphs as seen in the above figure are almost linear. As the input angle  $\phi_1$  increases from 5 degrees to 57 degrees, the corresponding angles  $\phi_2$  and  $\phi_3$  also increase along with the height of the pantograph head.

The distance between the roof of the TATA electric bus and the catenary system is precisely 1.7 m and the maximum height of the pantograph head is 1.703 m, which complies with the PCS.

### ***Working Mechanism of the PAC model***

The upward and downward motion of the pantograph is actuated using a servo motor. The servo motor is triggered whenever user scans a RFID sticker using the PN532 NFC reader. The NFC reader is connected with Arduino UNO as shown in the Fig.20 below.



**Fig.20** RFID and Arduino UNO circuit

The links mentioned below shows the working of the entire mechanism and the autonomous movement of the pantograph.

[Panto up movement along with the catenary](#)

[Panto down movement along with the catenary](#)

[panto up and down using servo motor](#)

### **Cost Analysis of Proposed System**

The proposed system has high BEP which indicates that the project will take time to achieve its break even point and will be profitable in long run. The BEP of the proposed system is 8.7. Which is high indicating that the project will take time to generate profits.

The NPV results also show the same results as in 5 years NPV is negative ₹ -1,39,31,94,757.53, it improves in 10 years but is still negative ₹ -16,82,90,098.98. But became positive in 15 years ₹ 58,36,95,105.23. This also indicates us that the project is pursuable for the long run.



# Chapter 5

## Conclusion and Future Works

### *Limitations of the project*

After completing this project, we realized that the catenary design was unstable because the support masts used were quite heavy in comparison to the other components used in the catenary. Secondly, we used a galvanized iron wire, however that was not stable and we should have used a copper wire for better conductivity. The communication protocol used in all the connections was SPI (Serial peripheral interface), and it was quite slow, and instead of this, we should have used a UART communication protocol. Additionally, we believe that we could have integrated the receiver and RFID system, and used only one Arduino UNO instead of two to make the system more streamline and efficient.

The pantograph was not able to maintain a constant contact force when it was moving along with the bus. As a result, it often got disconnected with the catenary wire. The range of the RFID scanner was not suitable for real-life scenarios. However, a long-range RFID scanner, and a GPS based system would increase the reliability of autonomous movement of the pantograph.

The most important limitation according to us is that as the BRTS corridor moves through the city, and it is very important to take safety precaution in count as the power supply to run the buses would be of high voltage. If precautions are not taken, there is a chance of accident and mishap.

### *Future Works*

The next step towards the kinematic and dynamic analysis of pantograph-catenary system is to measure the contact force acting on a large-scale pantograph that will be used for an electric bus in Ahmedabad. The motivation is to calculate the total contact force which comprises of the aerodynamic and inertia forces by attaching load sensors, and accelerometer on the collector head of the pantograph.

This type of similar approach can be used for highways in India, along with the BRTS in the cities. For smooth functioning of such a huge project, we will be requiring a steady flow of power as several buses will be moving on the corridor. And we will also have to develop an integrated system to channel the flow of electricity in different routes. Therefore, more research is required in developing an integrated system to control the supply of power and proper planning is necessary for implementing the high-power electric grid in the city.

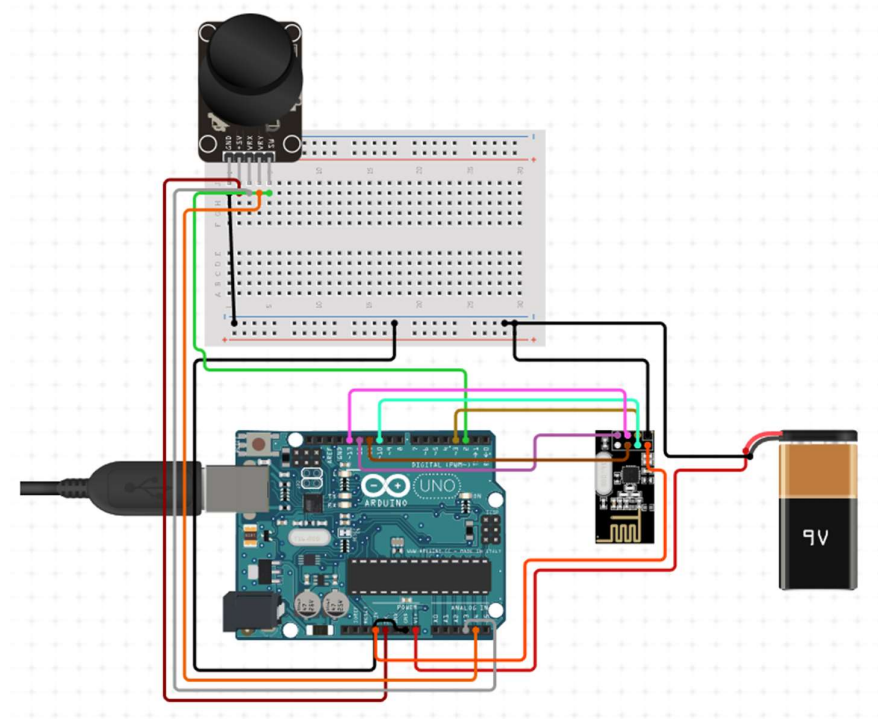
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# Appendix

## *Appendix 1: Transmitter System Circuit for the RC electric bus*



## *Appendix 2: Transmitter System and Receiver System code*

Transmitter System code:

```
// Code when nrf24L01 module transmits the data to Arduino UNO
#include<SPI.h>
#include<nRF24L01.h>
#include<RF24.h>

RF24 radio(9,8); // selecting the Chip enable and chip select not pin
const byte address[6] = "00001"; // byte array that represents address through which
the transmitter and receiver module communicate
char xyData[32] = ""; // converted int datatype into char and initialised as an
empty string
String xAxis, yAxis; // converting and passing x and y values as strings instead of
integers

void setup(){
  radio.begin();
  radio.openWritingPipe(address); // we are writing the address of the receiver to
which we will send the data
```

```

    radio.setPALevel(RF24_PA_MAX); // Set the power amplifier value
    radio.stopListening(); // set as transmitter
    Serial.begin(9600);
}

void loop(){
    xAxis = analogRead(A0); // x and y values are analog values, they indicate the
motion of the bus
    yAxis = analogRead(A1);
    xAxis.toCharArray(xyData, 5);
    radio.write(&xyData, sizeof(xyData)); // passing the x-axis data
    yAxis.toCharArray(xyData, 5);
    radio.write(&xyData, sizeof(xyData)); // passing the y-axis data
    delay(20);
    Serial.print("this is the x axis value");
    Serial.println(xAxis);
    Serial.print("this is the y axis value");
    Serial.println(yAxis);
}

```

Receiver System code:

```

// code to implement Receiver module
#include<SPI.h>
#include<nRF24L01.h>
#include<RF24.h>

#include<Servo.h>
int value1=400;
int value2=600;
int enA = 3;
int in1 = 6;
int in2 = 5;
int x;
int y;
int px;
int py;
Servo servo;
RF24 radio(7,8); // selecting the Chip enable and chip select not pin on the second
arduino UNO
const byte address[6] = "00001";
char receivedData[32] = "";
int servo_pin = 9; // pin 9 of arduino UNO is connected to the servomotor
int xAxis, yAxis;
void setup(){
    pinMode(enA, OUTPUT);
    pinMode(in1, OUTPUT);
    pinMode(in2, OUTPUT);
    radio.begin();
    servo.attach(servo_pin);
}

```

```

    radio.openReadingPipe(0, address);
    radio.setPALevel(RF24_PA_MIN);
    radio.startListening(); // set as receiver
    Serial.begin(9600);
}

void loop(){
if(radio.available()) {
    radio.read(&receivedData, sizeof(receivedData)); // when the signal is detected,
the data will be read by the receiver module
    xAxis = atoi(&receivedData[0]); // converting the string data into integers
    delay(10);
    radio.read(&receivedData, sizeof(receivedData)); // when the signal is detected,
the data will be read by the receiver module
    yAxis = atoi(&receivedData[0]);
    delay(10);
    //Serial.println(xAxis);
}
if (xAxis==0)
{
    xAxis=px;
}
else
{
    px=xAxis;
}
if (yAxis==0)
{
    yAxis=py;
}
else
{
    py=yAxis;
}
//Serial.println(yAxis);
x = map(xAxis, 0, 1023, 50, 130);
y = map(yAxis, 0, 1023, 0, 255);
Serial.println(x);
if(yAxis < value1 || yAxis > value2 )
{
    if(yAxis < value1){
        digitalWrite(in1, HIGH);
        digitalWrite(in2, LOW);
        //Serial.println(y);
        analogWrite(enA,255-y);
        // the motor will turn all the wheels backwards when the joystick is pulled
downwards
    }
}
}

```

```

    }
    else if(yAxis > value2){
        digitalWrite(in1, LOW);
        digitalWrite(in2, HIGH);
        //Serial.println("Backwards");
        //Serial.println(y);
        analogWrite(enA,y);

        // the bus will go forward when joystick is pulled upwards
    }
}
else {
    digitalWrite(in1, LOW);
    digitalWrite(in2, LOW);
    analogWrite(enA,0);
}

if(xAxis < value1 || xAxis > value2){
    servo.write(x);
    //Serial.println(x);          // sets the servo position according to the
scaled value
    delay(15);
}
else {
    servo.write(90);              // sets the servo position according to the
scaled value
    delay(15);
}}

```

### *Appendix 3: MATLAB codes to model the catenary system*

```

% To model the catenary stiffness equation
equation = 'kmean*(1-0.5*cos(0.2093*14*t))';

% Define the range of values for the unknown parameter (time)
t_min = 0;
t_max = 100;
num_of_points = 10;
t_values = linspace(t_min,t_max,num_of_points);
kmean= 2000;
pi = 3.14;

% Using the unknown parameter t, find the vector/matrix values
y_values = kmean*(1-(0.5*cos(0.2093*t_values)));

% Plot the results
plot(t_values, y_values);

```

```

xlabel('Time');
ylabel('Catenary stiffness');
title('Catenary stiffness Vs Time');

```

```

% To calculate the catenary displacement
t_min = 0;
t_max = 100;
num_of_points = 100;
t_values = linspace(t_min,t_max,num_of_points);
g = 9.8;
Tc = 10000;
p = 0.95;

% Using the unknown parameter t, find the vector/matrix values
y_values = 5.5 - (0.045*sin(0.01904116*t_values.^2))-(0.01*sin(0.394384*t_values.^2))

% Plot the results
plot(t_values, y_values);
xlabel('Time');
ylabel('Catenary Displacement');
title('Catenary Displacement Vs Time');

```

#### *Appendix 4: Python code to model the kinematics of a pantograph*

```

import numpy as np
import matplotlib.pyplot as plt
import math

c = 0.720
d = 0.150
l1 = 1.600
l2 = 0.314
l3 = 1.182
l4 = 1.910
phi2_degree_list = []
phi3_degree_list = []
y_crossbar_list = []
for phi1 in range(5, 57, 1):
    radian_phi1 = np.deg2rad(phi1)
    sin_phi1 = np.sin(radian_phi1)
    cos_phi1 = np.cos(radian_phi1)

    k1 = (-2 * l2 * ((l1 * sin_phi1) + d))
    k2 = (2 * l2 * ((l1 * cos_phi1) - c))
    k3 = (l1 * 2 + l22 - l32 + c2 + d * 2 + (2 * l1 * d * sin_phi1) - (2 * c * l1 * cos_phi1))

    phi2 = 2 * np.arctan((-k1 - (math.sqrt(k1 * 2 + k22 - k3 * 2))) / (k3 - k2))
    phi2_degree = np.rad2deg(phi2)
    phi2_degree_list.append(phi2_degree)
print(phi2_degree)

```



```

sin_phi2 = np.sin(phi2)
cos_phi2 = np.cos(phi2)
phi3 = np.arctan(((l1 * sin_phi1) - (l2 * sin_phi2) + d) / ((l1 * cos_phi1) + (l2 * cos_phi2) - c))
phi3_degree = np.rad2deg(phi3)
phi3_degree_list.append(phi3_degree)
print(phi3_degree)
alpha = 73
radian_alpha = np.deg2rad(alpha)

x_crossbar = ((-l1 * cos_phi1) + (l4 * (np.cos(phi2 - radian_alpha))))
y_crossbar = ((l1 * sin_phi1) + (l4 * (np.sin(phi2 - radian_alpha))))
y_crossbar_list.append(y_crossbar)

print(x_crossbar)
print(y_crossbar)

phi1_list = list(range(5, 57, 1))

plt.subplot(311)
plt.xlabel('phi-1', fontsize=10)
plt.ylabel('phi-2', fontsize=8)
plt.plot(phi1_list, phi2_degree_list)

plt.subplot(312)
plt.xlabel('phi-1', fontsize=10)
plt.ylabel('phi-3', fontsize=8)
plt.plot(phi1_list, phi3_degree_list)

plt.subplot(313)
plt.xlabel('phi-1', fontsize=10)
plt.ylabel('height of the pantograph', fontsize=8)
plt.plot(phi1_list, y_crossbar_list)

plt.tight_layout()
plt.show()

```

#### *Appendix 5: RFID code to automate the movement of the pantograph*

```

#include <SPI.h>
#include <PN532_SPI.h>
#include <PN532.h>
#include <NfcAdapter.h>
#include <Servo.h>
PN532_SPI interface(SPI, 10); // create a PN532 SPI interface with the SPI CS
terminal located at digital pin 10
NfcAdapter nfc = NfcAdapter(interface); // create an NFC adapter object
String tagId = "None";
int flag=1;
Servo servo;
int pos = 0;

void setup()
{
  Serial.begin(9600);
  Serial.println("System initialized");
  nfc.begin();

```

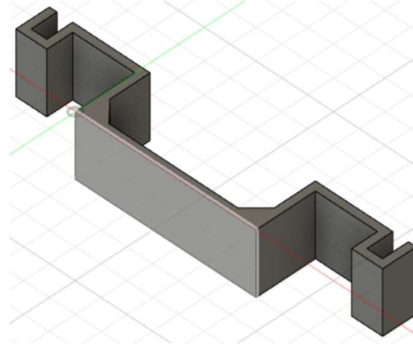
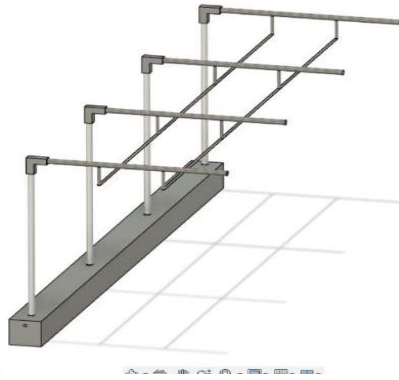
```

servo.attach(A5);
servo.write(0);
}

void loop()
{
  readNFC();
}

void readNFC()
{
  if (nfc.tagPresent())
  {
    NfcTag tag = nfc.read();
    tag.print();
    tagId = tag.getUidString();
    String idu="63 E5 9D 43";
    String idd="73 D0 42 43";
    if (flag==1&&idu==tagId)
    {
      Serial.println("panto upp");
      flag=0;
      for (pos = 0; pos <= 50; pos += 1) {
        servo.write(pos);
        delay(15);
      }
    }
    else if (flag==0&&idd==tagId) {
      Serial.println("panto Down");
      flag=1;
      for(pos = 50; pos >= 0; pos -= 1 ){
        servo.write(pos);
        delay(15);
      }
    }
  }
  delay(5000);
}

```



## Appendix 7: BEP calculations on Excel

N30										
	A	B	C	D	E	F	G	H	I	J
1	Number of EV Buses- 150		Cost considered for the proposed system							
2										
3	Fixed cost	In Rupees	Variable Cost	In Rupees			Pantograph maintenance cost			
4	Electricification cost	3,70,20,00,000					1,87,50,500@15%= Rs 28,12,500			
5	Cost of Pantograph	1,87,50,000	Pantograph maintenance cost @15%	20,53,12,500			A= Life span= 5 years			
6	Cost of Battery	1,50,00,000	Bus maintenance and operating cost @ Rs 59	83,39,85,945			B= Annual Cost= Rs 102,65,62,500 for 150 EV buses			
7	Cost of Replacement of ITMS	8,82,85,714					Cost= (B/A) Rs 20,53,12,500			
8	Bus Replacement Cost	22,00,00,000					Bus Maintenance and Operating cost			
9							Annual cost= 38727 km x Rs.59 x 365 days = Rs			
10	Total F.C. (Fixed Cost)	4,04,40,35,714	Total V.C. (Variable Cost)	1,03,92,98,445			83,39,85,945.			
11							ITMS (Intelligent Transport Management System)			
12	Revenue						Replacement of ITMS = Rs 61,80,00,000			
13	Revenue (in Rupees)	58,40,00,000	BEP (Break-Even Point)	8.70			Life span= 7 years			
14							Per year taken = Rs 8,82,85,714			
15							Bus Replacement Cost			
16							A= Total cost of 150 buses= Rs 330,00,00,000			
17							B= Life span of buses= 15 years			
18							Annual bus replacement cost= Rs 22,00,00,000(A/B)			
19							Revenue (Annual)			
20							Rs 16,00,000 x 365 days. = Rs.58,40,00,000.			
21							Electricification cost (Catenary system) = 185.1*Rs			
22							50,00,00,000/km			
23							A= Total= Rs 9255,00,00,000			
24							B= Life Span= 25 years			
25							Annual Cost (A/B) = ₹ 370,20,00,000			
26										
27										
28										
29										
30										
31										
32										

## Appendix 8: NPV calculations on Excel

B19													
	A	B	C	D	E	F	G	H	I	J	K	L	
1	Cash Flow	16,00,000		NPV (Net Present Value)									
2	Discount rate	10.25%											
3	Initial Investment	4,77,50,48,445		For 5 Years			For 10 Years			For 15 Years			
4				Years	Cash Flow		Years	Cash Flow		Years	Cash Flow		
5	Fixed cost	In Rupees		0	-3,73,57,50,000		0	-3,73,57,50,000		0	-3,73,57,50,000		
6	Electricification cost	3,70,20,00,000		1	58,40,00,000		1	58,40,00,000		1	58,40,00,000		
7	Cost of Pantograph	1,87,50,000		2	58,40,00,000		2	58,40,00,000		2	58,40,00,000		
8	Cost of Battery	1,50,00,000		3	58,40,00,000		3	58,40,00,000		3	58,40,00,000		
9	Total F.C. (Fixed Cost)	3,73,57,50,000		4	58,40,00,000		4	58,40,00,000		4	58,40,00,000		
10				5	58,40,00,000		5	58,40,00,000		5	58,40,00,000		
11				NPV	₹ -1,39,31,94,757.53		6	58,40,00,000		6	58,40,00,000		
12							7	58,40,00,000		7	58,40,00,000		
13							8	58,40,00,000		8	58,40,00,000		
14	Variable Cost	In Rupees					9	58,40,00,000		9	58,40,00,000		
15	Pantograph maintenance cost @15%	20,53,12,500					10	58,40,00,000		10	58,40,00,000		
16	Bus maintenance and operating cost @ Rs 5	83,39,85,945					NPV	₹ -16,82,90,098.98		11	58,40,00,000		
17	Total V.C. (Variable Cost)	1,03,92,98,445								12	58,40,00,000		
18										13	58,40,00,000		
19										14	58,40,00,000		
20	Initial Investment									15	58,40,00,000		
21	3,73,57,50,000+ 1,03,92,98,445 =	4,77,50,48,445								NPV	₹ 58,36,95,105.23		
22													
23													
24													