

## Assignment-2

Vedanshu Goyal  
B19CSE096

### Part1:

$$X_{k+1} = X_k + V_k * (dt) + \frac{1}{2} * a * (dt)^2$$

$$V_{k+1} = V_k + a * (dt)$$

By combining both equations.

### Prediction Step [ Prediction from our previous beliefs]

$$X_{k+1} = A * X_k + B * a + W.$$

$$P_{k+1} = A * P_k * A^T + Q.$$

{ Here I assume W and Q to be 0 because there is no noise in the acceleration. }

$$A \Rightarrow \begin{bmatrix} 1 & 0 & 0 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 & dt & 0 \\ 0 & 0 & 1 & 0 & 0 & dt \\ 0 & 0 & 0 & dt & 0 & 0 \\ 0 & 0 & 0 & 0 & dt & 0 \\ 0 & 0 & 0 & 0 & 0 & dt \end{bmatrix}$$

$$B \Rightarrow \begin{bmatrix} \frac{1}{2}dt^2 & 0 & 0 \\ 0 & \frac{1}{2}dt^2 & 0 \\ 0 & 0 & \frac{1}{2}dt^2 \\ dt & 0 & 0 \\ 0 & dt & 0 \\ 0 & 0 & dt \end{bmatrix}$$

Updation Step [Updating our belief when we have measurement].

$Z_k$  -> measurements ;  $R$  -> Noise in measurements

$$Y = Z_k - H * x$$

$$S = H * P * H^T + R$$

$$K = P * H^T * S^{-1} \text{ {Kalman Gain}}$$

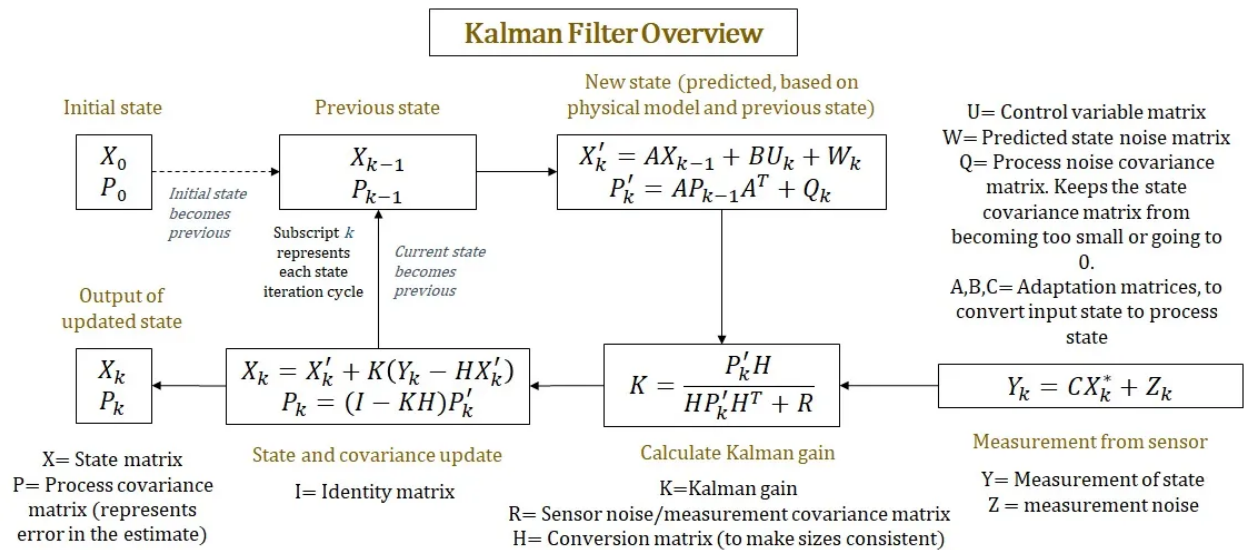
$$x = x' + K * Y$$

$$P = (I - K * H) * P'$$

Here,

$$H = I^6 \text{ { Identity matrix of } 6 * 6 \text{ } }$$

To get a more complete understating of the Algorithm look Flowchart given below...



**Example**

[  $\mathbf{x}_x$  ,  $\mathbf{x}_y$  ,  $\mathbf{x}_z$  ,  $\mathbf{v}_x$  ,  $\mathbf{v}_y$  ,  $\mathbf{v}_z$  ]

```
Init -> [[-0.73977617]
[ 0.36503031]
[10.4960305 ]
[ 5.81195376]
[-0.29592076]
[-7.27067196]]
```

```
-> [[ 5.07217759]
[ 0.06910956]
[ 3.22535854]
[ 5.81195376]
[-0.29592076]
[-7.27067196]]
```

```
-> [[10.88413135]
[-0.2268112 ]
[-4.04531343]
[ 5.81195376]
[-0.29592076]
[-7.27067196]]
```

```
-> [[ 16.69608511]
[ -0.52273196]
[-11.31598539]
[ 5.81195376]
[ -0.29592076]
[ -7.27067196]]
```

```
-> [[ 22.50803888]
[ -0.81865271]
[-18.58665735]
[ 5.81195376]
[ -0.29592076]
[ -7.27067196]]
```

```
-> [[ 28.31999264]
     [-1.11457347]
     [-25.85732931]
     [ 5.81195376]
     [-0.29592076]
     [-7.27067196]]
```

**STATE UPDATE [ GOT MEASUREMENT]**

```
[[ -5.19634191]
 [15.60267359]
 [10.53326648]
 [ 2.08377821]
 [-3.70829697]
 [-3.32472473]]
```

```
[[ -3.75149962]
 [16.21244621]
 [ 7.97960893]
 [-0.22384016]
 [ 2.35102693]
 [-0.72429702]]
```

*Part2:*

*Error after test run: [26.55082577]*

*Error after test run: [36.55834127]*

*Error after test run: [27.99849655]*

*Error after test run: [9.66693333e+106]*

*Error after test run: [3.72375294e+105]*

*[Finished in 244ms]*

### **Part3:**

In the case of a falling projectile, the acceleration of the object is only due to gravity until it reaches its terminal velocity. Once the object reaches its terminal velocity, the net acceleration becomes zero.

Thus, the state equations can be modified accordingly:

$$X_{k+1} = X_k + g * (dt)$$

$$V_{k+1} = V_k = g$$

So, we get the following equations

**Prediction Step [ Prediction from our previous beliefs]**

$$\mathbf{X}_{k+1} = \mathbf{A} * \mathbf{X}_k$$

$$\mathbf{P}_{k+1} = \mathbf{A} * \mathbf{P}_k * \mathbf{A}^T.$$

{ Here I assume W and Q to be 0 because there is no noise in the acceleration. }

$$\mathbf{A} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & dt & 0 & 0 \\ 0 & 1 & 0 & 0 & dt & 0 \\ 0 & 0 & 1 & 0 & 0 & dt \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

And rest of the equations can be used the same as in the first question. Here we can drop matrix B.

The acceleration term is absent from the state equations and measurement equations in Part 3. Also, since the nature of

the measurements and the noise that influences them differs from those in Part 1, the measurement noise for the position measurements may be different in Part 3.