LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Relations & Functions

Lecture: 09

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Today's Goal: :

Examples on Domain of the Functions: } from Telegram.

Types of Mappings:

Recap on Domain of the Functions :

$* \quad f(x) = \sqrt{7-x}\,\big|_{x-3}$

for $f(x)$ is to be real,

$7-x \geq 0 \Rightarrow x \leq 7$

$x-3 \geq 0 \Rightarrow x \geq 3$

& $7-x \geq x-3 \Rightarrow x \leq 5$

$\Rightarrow \boxed{3 \leq x \leq 5}$

But $x \in I$

$\Rightarrow x = 3, 4, 5$

$\Rightarrow D_f = \{3, 4, 5\}$

$R_f = \{f(3), f(4), f(5)\}^*$

$= \{{}^4P_0, {}^3P_1, {}^2P_2\}$

$= \{1, 3, 2\}$

Recap on Domain of the Functions :

$\log_a x \geq 1 \Rightarrow x \geq a^1 \rightarrow (a > 1)$

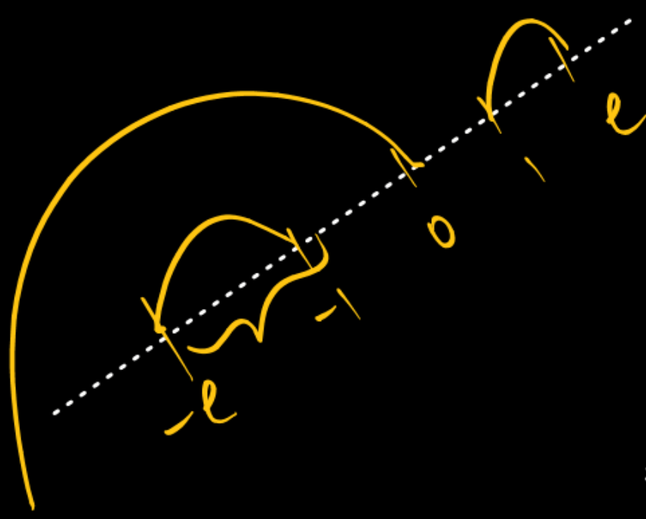
$a < |x| < b \Rightarrow a < x < b$ OR $-b < x < -a$

$1 < x < e$ OR $-e < x < -1$ — (2)

$(1) \cap (2)$: $-e < x < -1$

* $f(x)$ is defined in $(0, 1)$

Domain $\longrightarrow$ $f(e^x) + f(\log|x|)$?



$f(e^x)$ is defined iff $0 < e^x < 1$

$\Rightarrow e^{-\infty} < e^x < e^0$

$\Rightarrow -\infty < x < 0$ — (1)

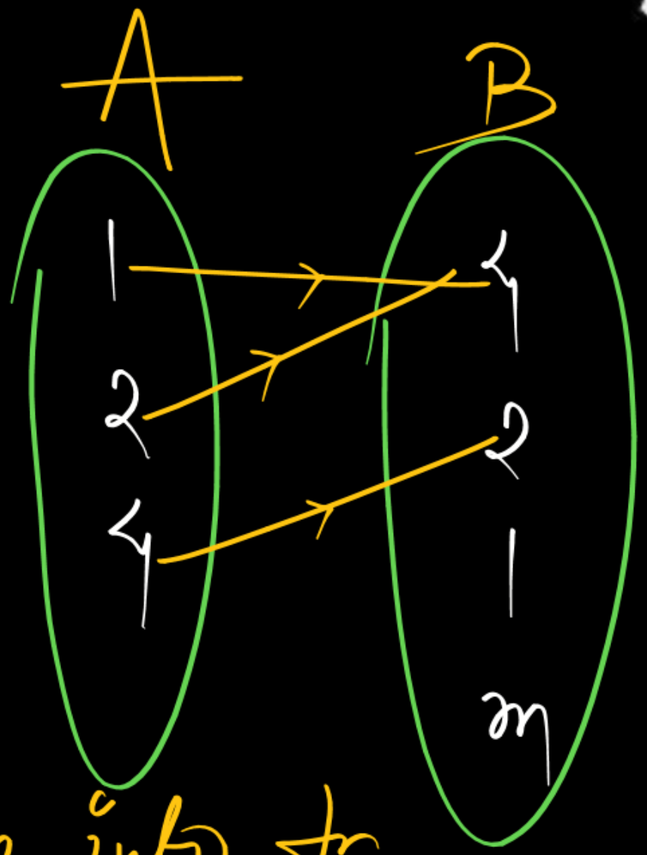
$f(\log|x|)$ is defined iff $0 < \log|x| < 1$

$\Rightarrow e^0 < |x| < e^1$

$\Rightarrow 1 < |x| < e$

Recap of Types of Mappings:

1. One- One Onto Functions (Most Useful)
2. One- One Into Functions
3. Many- One Onto Functions
4. Many- One Into Functions



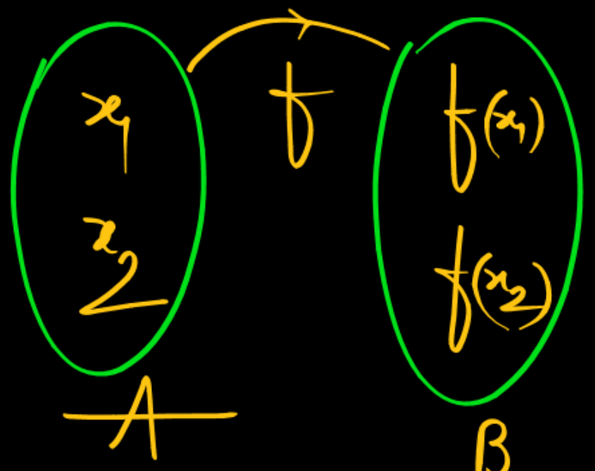
Many One into fr

How to Check One-One/ Many-One?

Yes + NO → NO

$(x_1 = x_2) + (x_1 \neq x_2) \rightarrow (x_1 \neq x_2)$

Method - (1) NCERT method



$f: R \rightarrow R$

$f(x) = 2x + 3$

Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$

$\Rightarrow \boxed{x_1 = x_2}$ One-one fn

$\Rightarrow \boxed{x_1 \neq x_2}$ Many-One

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 + 3 = 2x_2 + 3 \Rightarrow \boxed{x_1 = x_2}$ ∴ One-one

$f: R \to R$: $f(x) = x^2$

let $x_1, x_2 \in R \Rightarrow f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 + x_2)(x_1 - x_2) = 0$

$\Rightarrow x_1 = -x_2 \Rightarrow \boxed{x_1 \neq x_2}$

OR

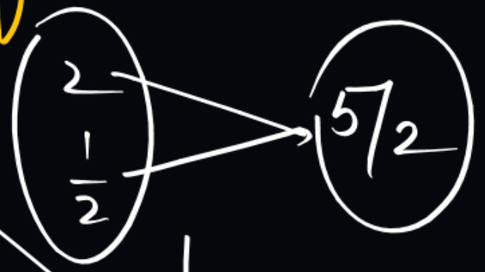
$\boxed{x_1 = x_2}$

$\boxed{x_1 \neq x_2}$ ∴ many-one

$f(x) = x + \frac{1}{x}$ → many-one

$f: R^+ \to R^+$

check f is one-one / many-one



let $x_1, x_2 \in R^+$ show that $f(x_1) = f(x_2)$

$\Rightarrow x_1 + \frac{1}{x_1} = x_2 + \frac{1}{x_2}$

$\Rightarrow (x_1 - x_2) + \left(\frac{1}{x_1} - \frac{1}{x_2}\right) = 0$

$\Rightarrow (x_1 - x_2)\left(1 - \frac{1}{x_1 x_2}\right) = 0$

$\Rightarrow x_1 = x_2$ OR $x_1 x_2 = 1$

$\Rightarrow \boxed{x_1 = \frac{1}{x_2} \neq x_2}$

∴ many-one

Some Basic Examples (From NCERT):

Prove that the greatest integer function f: R to R given by f (x) = [x] , is neither one-one nor onto, where [x] denotes g.i.f less than or equal to x

$f(x) = [x]$

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$

$\Rightarrow [x_1] = [x_2]$

$\Rightarrow x_1 \neq x_2$ OR $x_1 = x_2$

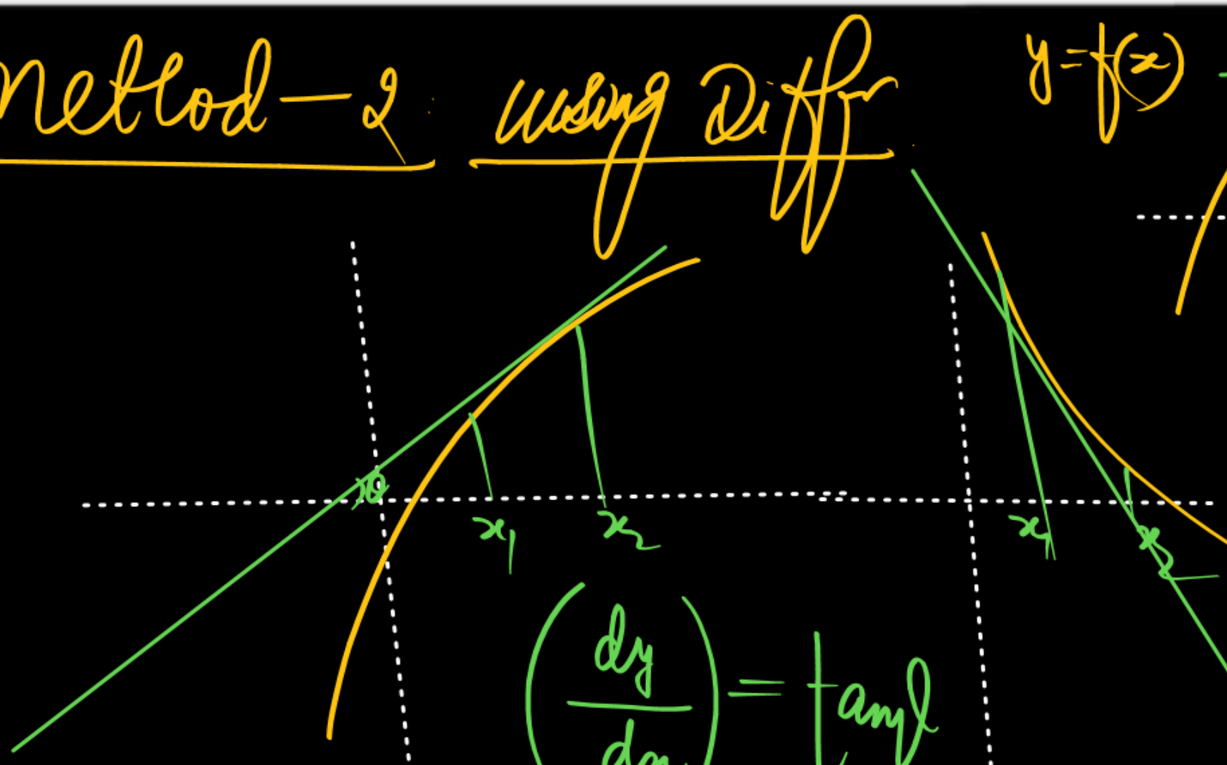
Many One

as $[1.3] = [1.4]$

$\Rightarrow 1.3 \neq 1.4$

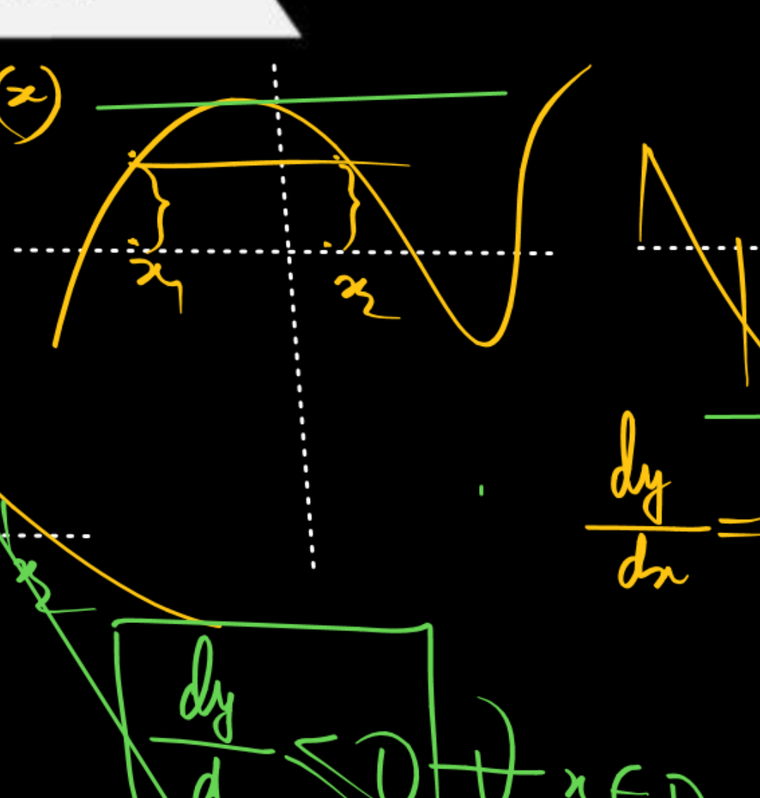
How to Check One-One/ Many-One?

Method – 2 : using Diff

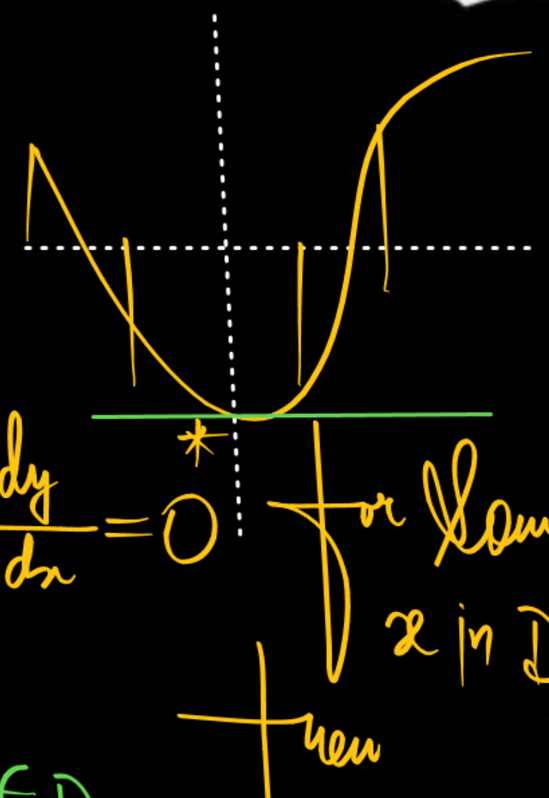


$\left(\frac{dy}{dx}\right)_{(x,y)} = \tan\theta$

$\boxed{\frac{dy}{dx} \geq 0 \ \forall \ x \in D_f}$ then one-one f^n



$\boxed{\frac{dy}{dx} \leq 0 \ \forall \ x \in D_f}$ then one-one f^n



$\frac{dy}{dx} = 0$ for some x in D_f then many-one

$y = f(x)$

Strictly $\frac{dy}{dx} \geq 0$ OR $\frac{dy}{dx} \leq 0$ $\forall x \in D_f$

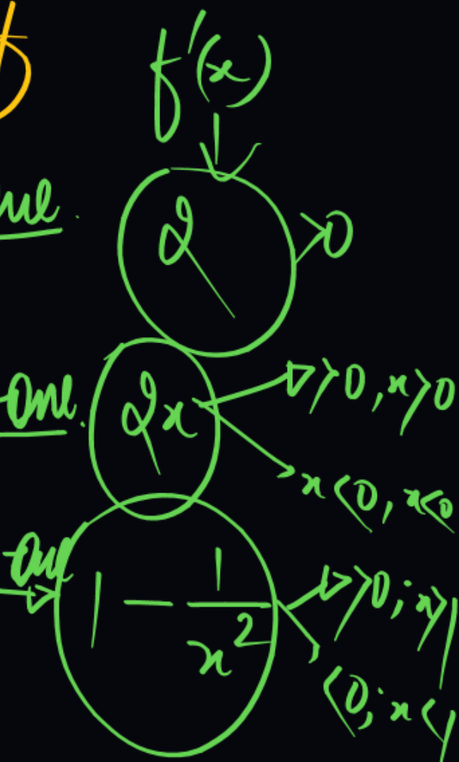
$\Rightarrow$ One-one for elsewhere many-one

Eg① $f(x) = \log x$

$\Rightarrow f'(x) = \frac{1}{x} > 0 \ \forall x > 0$

so one-one

$$f(x) = \begin{cases} 2x+3 & \text{one-one} \\ x^2 & \text{many-one} \\ x + \frac{1}{x} & \text{many-one} \\ 2x + \sin x & \text{one-one} \end{cases}$$



$2 + \cos x > 0 \ \forall x \in R$

Some Standard Examples:

Examples on classification: One-One / Many-One

(1) (a) $f(x) = e^x + e^{-x}$

$\Rightarrow f'(x) = e^x - e^{-x} \begin{cases} > 0 & ; x > 0 \\ < 0 & ; x < 0 \end{cases} \Rightarrow$ Not One-One

(b) $f(x) = x^3$

$\Rightarrow f'(x) = 3x^2 \geq 0 \ \forall \ x \in R$ Not One-One

(c) $f(x) = |x| \operatorname{Sgn} x$

$\Rightarrow f(x) = |x| \times \frac{x}{|x|} = x \Rightarrow f'(x) = 1 \Rightarrow$ One-One

(d) $f: [-1, 1]$ to $[-1, 1]$ $f(x) = \sin 2x$

$\Rightarrow f'(x) = 2\cos 2x$

$f'(x) = 2\cos 2x$

$x \longrightarrow -1$ to 1

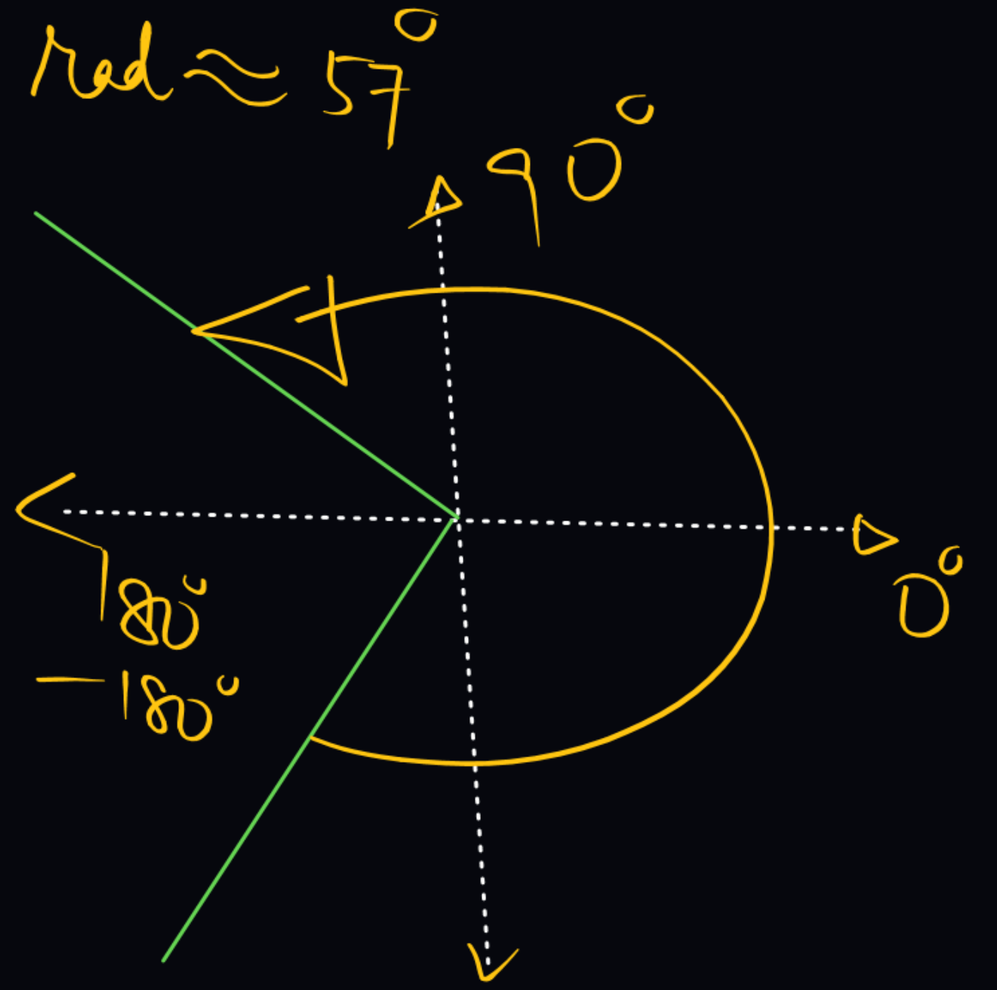
$\longrightarrow -1$ rad to 1 rad

$\longrightarrow -57^\circ$ to 57°

$2x \longrightarrow -114^\circ$ to 114°

$\cos 2x > 0$ or < 0

1 rad $\approx 57^\circ$



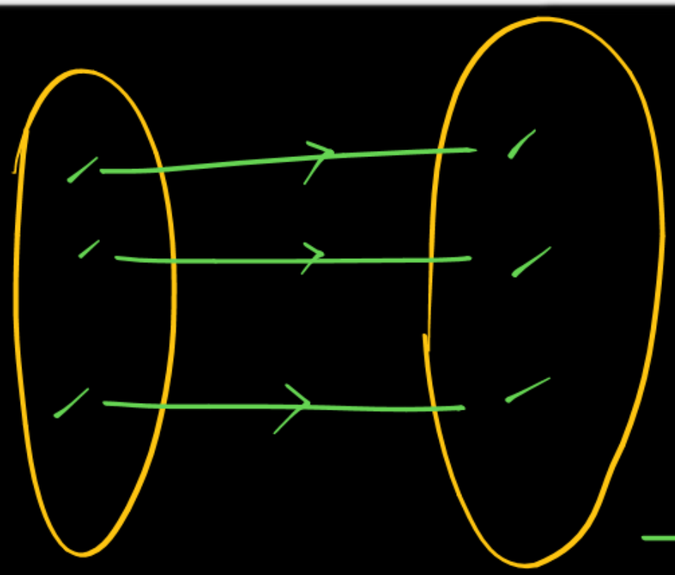
No many one

One-one → Injective

Onto → Surjective

One-one Onto → Bijective

How to Check Onto/Into?



⇒ Given co-domain of f
= obtained range of the f
then Onto f

If Range ≠ co-domain
then into f

Thank You Lakshyians