

# LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI



## Relations & Functions

Lecture: 04



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**Today's Goal:**

**Definition of Functions:**

**Basic Problems of Domain & Range:**



## Definition of Functions:

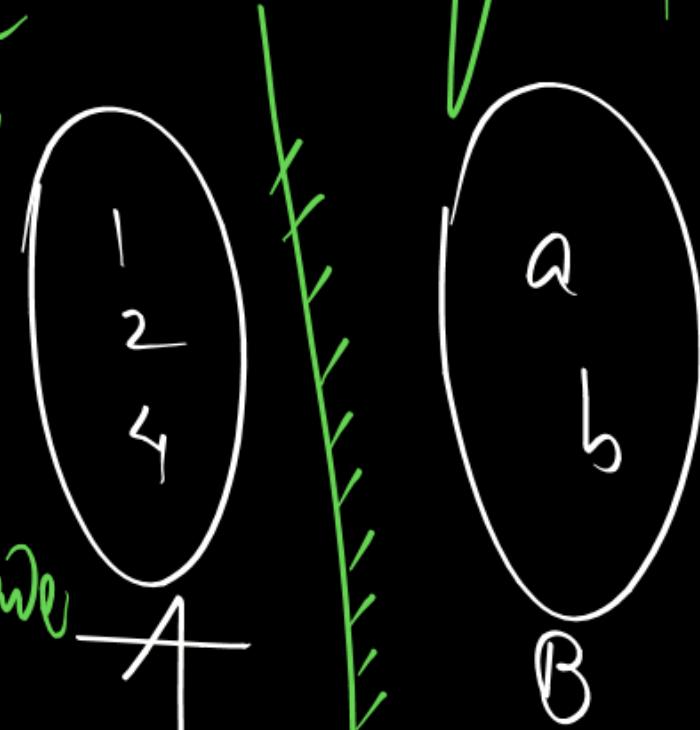
$$A = \{1, 2, 3\}$$

$$B = \{a, b\}$$

Condition for functions:

$R: A \rightarrow B$  is said to be a function

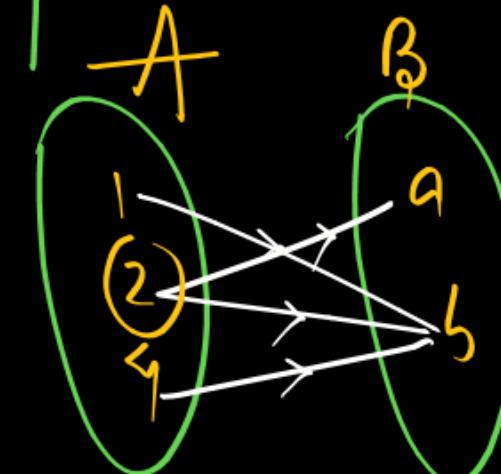
- ① \* One object in Set A will have One image in Set B
- ② \* More than One object in Set A can have One image in Set B
- ③ \* Each & every object in Set A will have a fixed relation with Set B



## Definition of Functions:

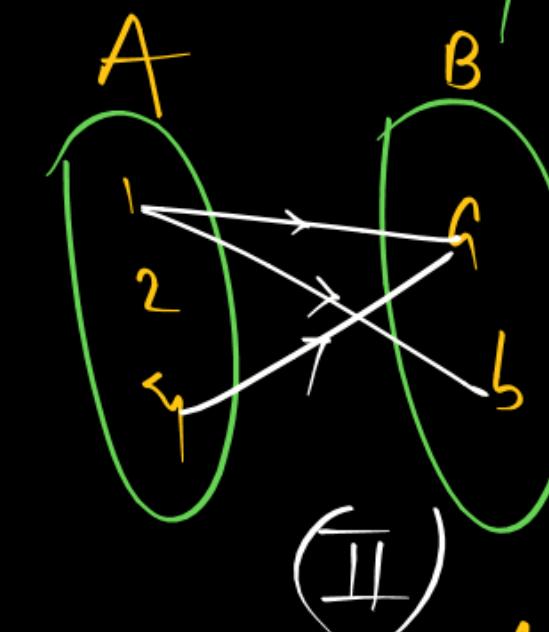
(I) \* There may be a number of elements in set B which have no relation with set A

Eg:



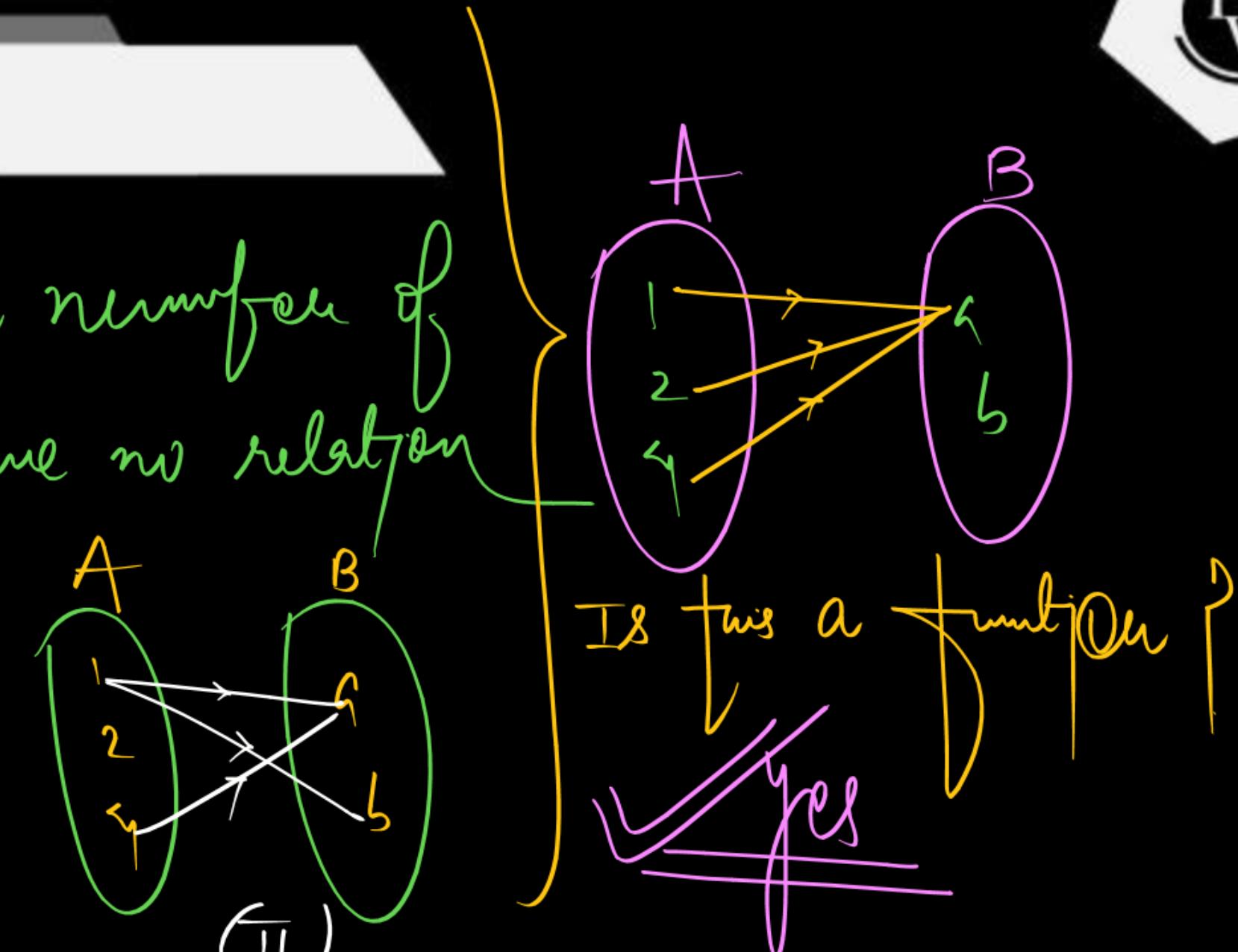
(I)

Not a function



(II)

Not a function



## Examples on Definition of Functions:

**Which of the following is a function?**

$$x \in A, y \in B$$

(a)  $\{(x, y) : y^2 = x, x, y \in R\} \rightleftarrows y^2 = x$

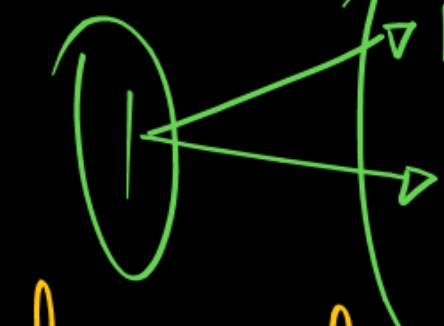
(b)  $\{(x, y) : y = |x|, x, y \in R\}$

(c)  $\{(x, y) : x^2 + y^2 = 1, x, y \in R\}$

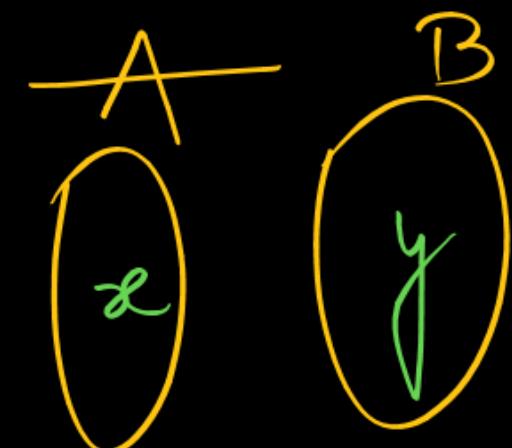
(d)  $\{(x, y) : x^2 - y^2 = 1, x, y \in R\}$



$$\text{at } x=1 \\ \Rightarrow y = \pm 1$$



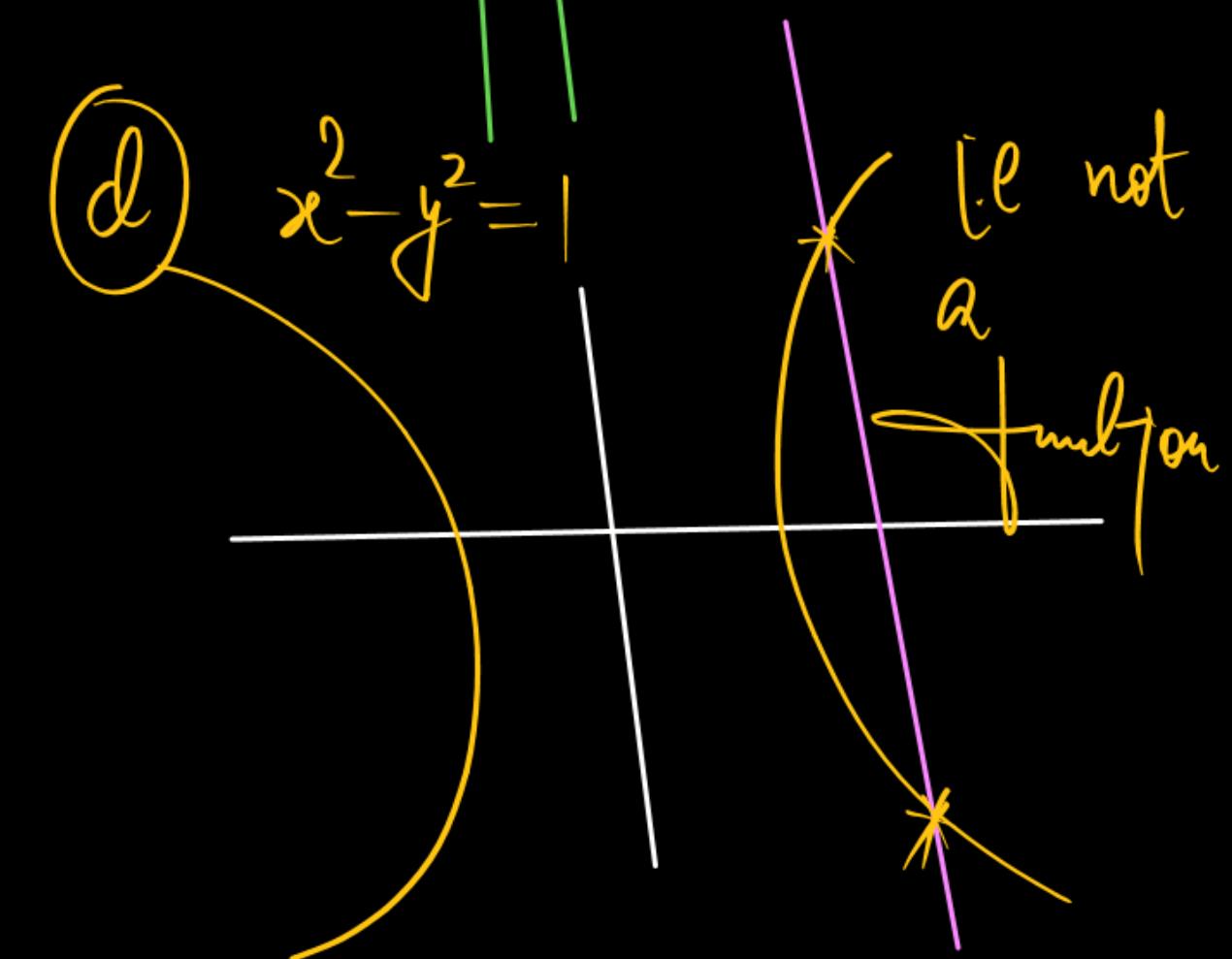
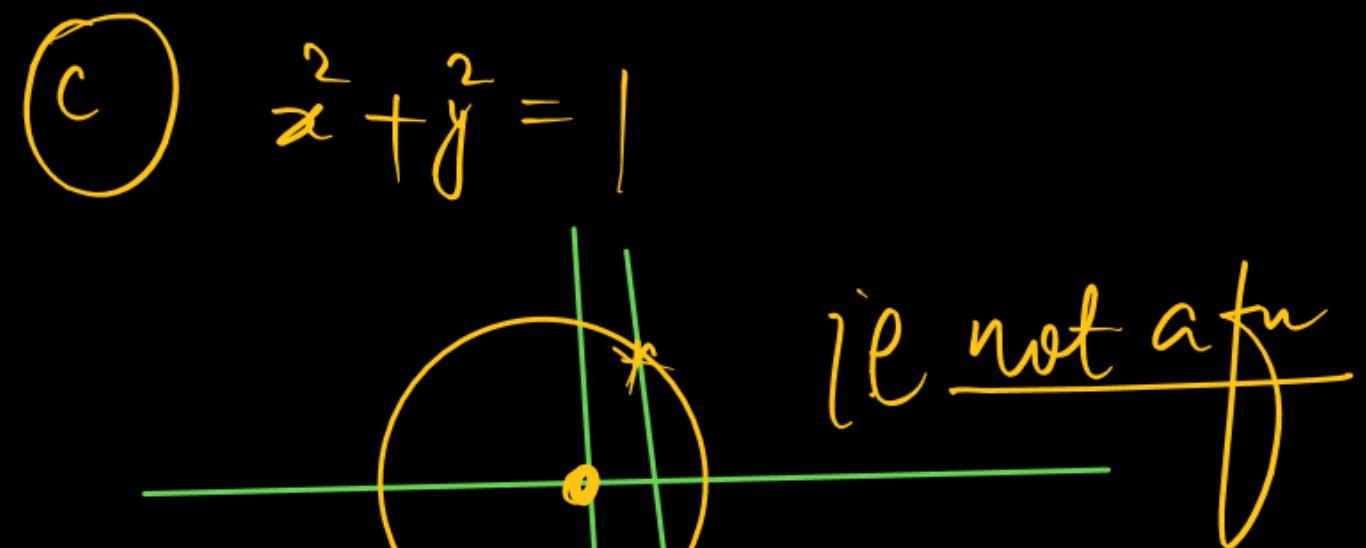
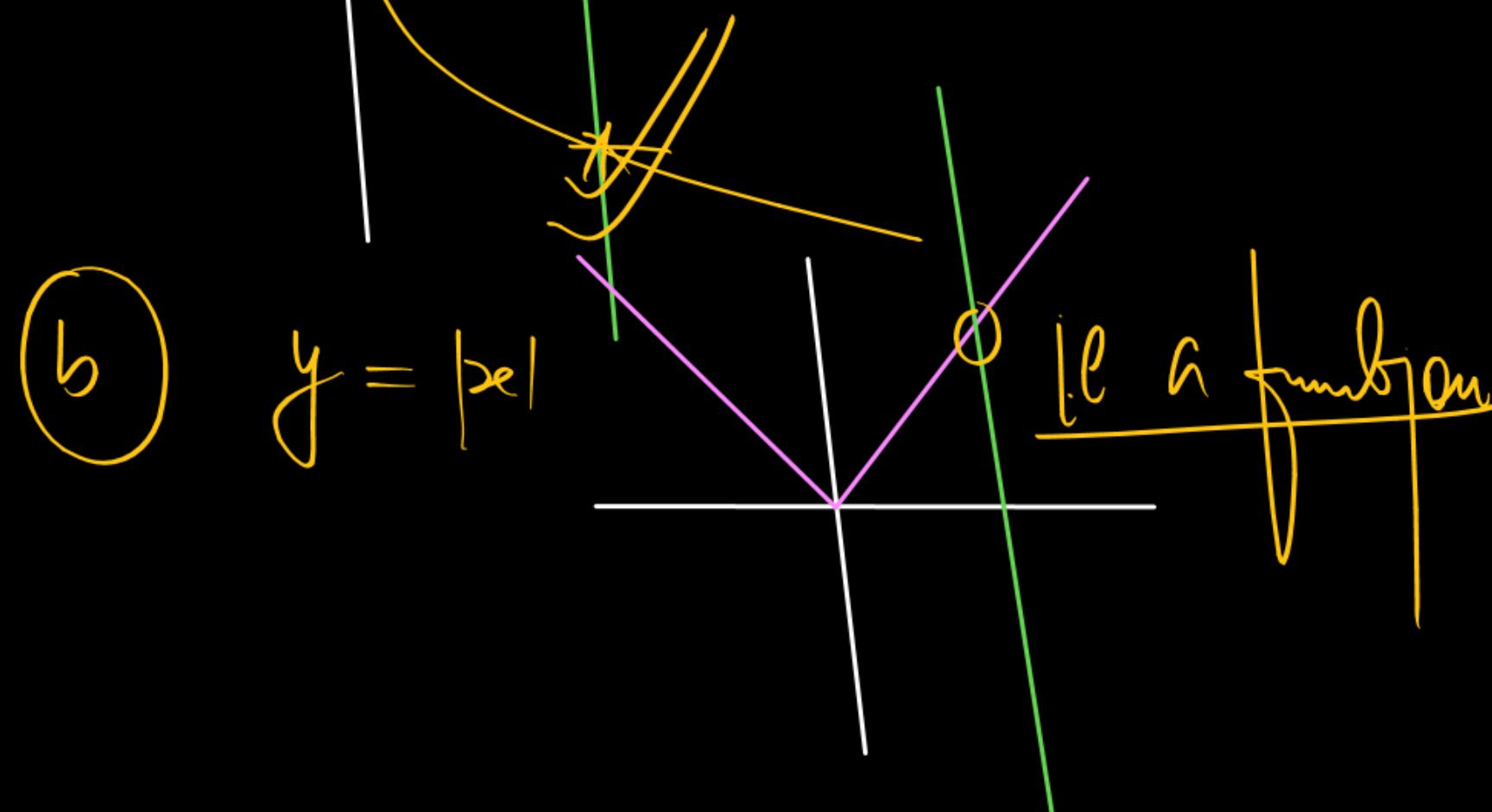
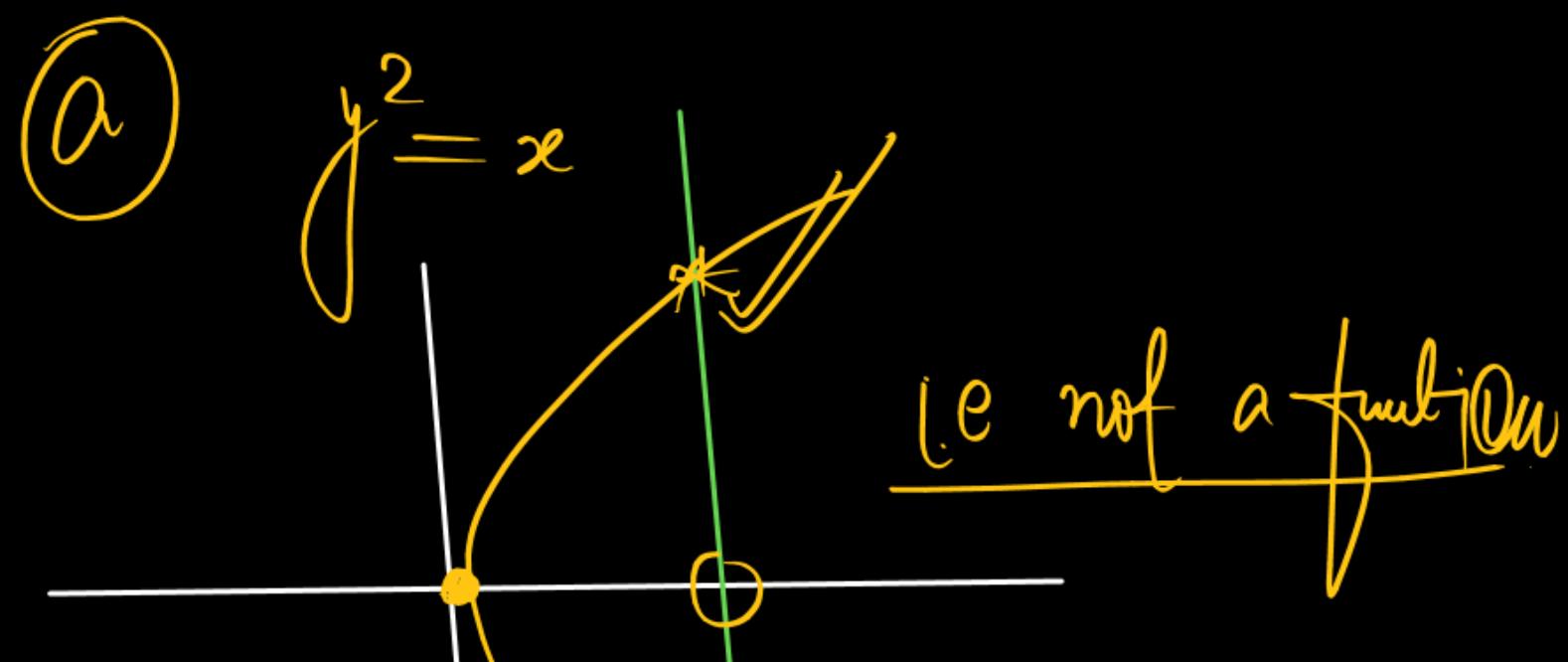
Not a function



$$f: A \rightarrow B$$

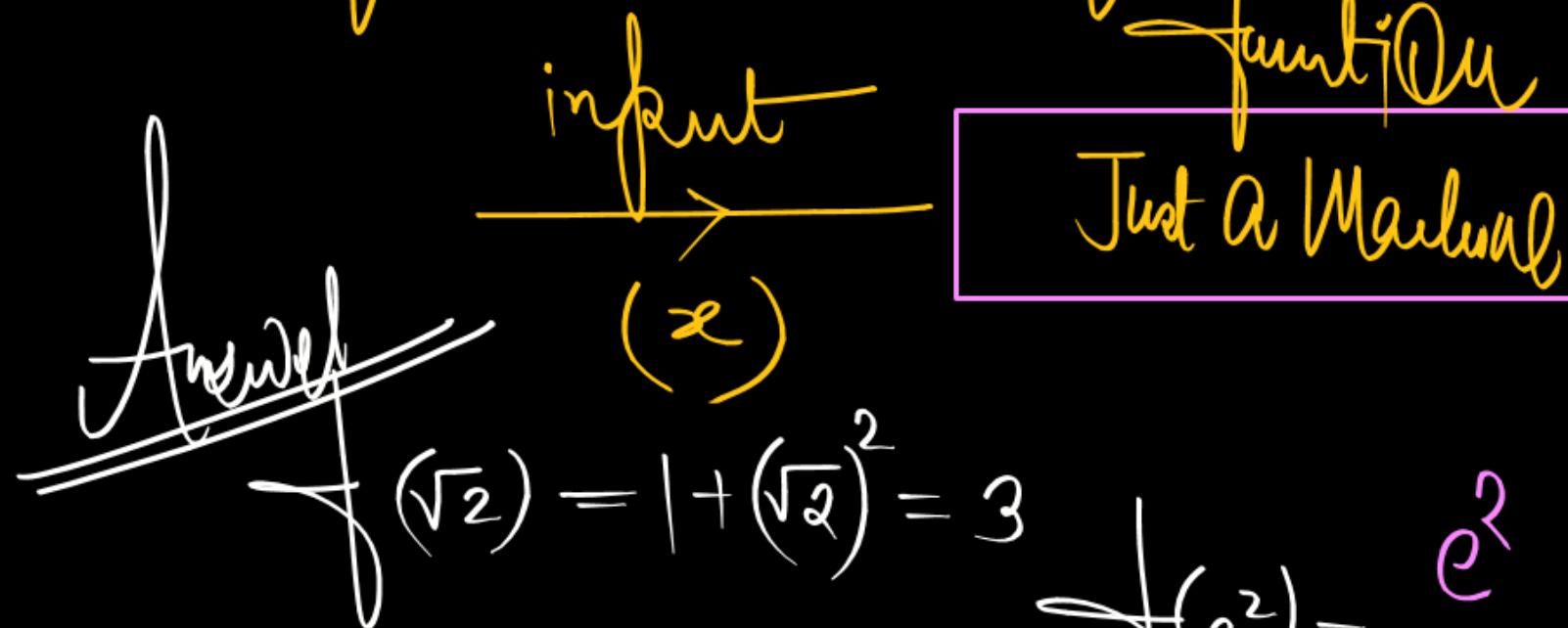
$$\boxed{y = f(x)}$$

Alternative: If a line parallel to  $y$ -axis cuts the given curve, only at a point  
then the given curve is a function.



## Some Standard Problems ( Including PYQs):

$f: \mathbb{R} \rightarrow \mathbb{R}$  or  $f: A \rightarrow B$  i.e.  $y = f(x)$



$$f(\pi) = 3 - \frac{\pi^2}{4}$$

$f(\bar{1}^2) = \text{Not defined}$

$$f(e^2) = \frac{e^2}{1 + (e^2)^2} = \frac{e^2}{1 + e^4}$$

$$\pi \approx \frac{22}{7} \Rightarrow \pi \neq \frac{22}{7}$$

$$\pi \rightarrow 3 \frac{10}{71} \text{ to } 3 \frac{10}{70}$$

Simplify:

3125/1050 ≈

$$(x) = \begin{cases} 1+x^2 & ; 0 < x < 2 \\ 3-x^2 & ; 2 < x < 5 \\ \frac{x}{1+x^2} & ; 5 < x < 9 \end{cases}$$

Find  $f(\sqrt{2}), f(\bar{1}), f(\bar{1}^2), f(e^2) = ?$



## Some Standard Problems ( Including PYQs):

Q: If  $f(x) = \frac{x}{1+x}$

Find  $f(x) + f\left(\frac{1}{x}\right) = ?$

Q: If  $f(x) = \frac{1-x}{1+x}$

Now,  $f\left(\frac{1}{x}\right) = \frac{1/x}{1 + \frac{1}{x}} = \frac{\cancel{(1)}}{\cancel{(x+1)} \cancel{(x)}} = \frac{1}{1+x}$

Find  $f(f(x)) = ?$

So,  $f(x) + f\left(\frac{1}{x}\right) = \frac{x}{1+x} + \frac{1}{1+x} = \frac{x+1}{1+x} = 1 = \frac{1-f(x)}{1+f(x)}$

$$= \frac{1 - \left( \frac{1-x}{1+x} \right)}{1 + \left( \frac{1-x}{1+x} \right)} = \frac{(1+x) - (1-x)}{(1+x) + (1-x)} = \frac{2x}{2} = x$$



## Some Standard Problems ( Including PYQs):

$$\begin{aligned}
 f(x) &= \left\{ 2021 - x^{2020} \right\}^{1/2020} \\
 \text{Find } f(f(x)) &=? \quad \Rightarrow \quad f(f(x)) = \left\{ 2021 - \left( \left\{ 2021 - x^{2020} \right\}^{1/2020} \right)^{2020} \right\}^{1/2020} \\
 \Rightarrow f(f(x)) &= \left\{ 2021 - \left( 2021 - x^{2020} \right) \right\}^{1/2020} = (x^{2020})^{1/2020} = x
 \end{aligned}$$



JEE 2021: \* New

$$\text{If } f(x) = \frac{5^x}{5^x + 5} \Rightarrow \text{find } f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{3}{40}\right) + \dots + f\left(\frac{79}{40}\right)$$

$$\text{find } f(x) + f(2-x) = ?$$
$$\frac{1}{40} + \frac{79}{40} = 2$$

$$\text{Now, } f(2-x) = \frac{5^{2-x}}{5^{2-x} + 5} = \frac{\frac{5^2}{5^x}}{\frac{5^2}{5^x} + 5} = \frac{\frac{5^2}{5^x}}{\frac{5^2 + 5(5^x)}{5^x}} = \frac{5^2}{5^2 + 5(5^x)}$$

$$\therefore f(x) + f(2-x) = \frac{5^x}{5^x + 5} + \frac{5^2}{5^2 + 5(5^x)} = \frac{5^x + 5^2}{5^x + 5(5^x)} = \frac{5^x + 25}{5^x + 5^2} = \boxed{1}$$

$$\int_0^1 f(x) + f(2-x) = 1 \xrightarrow{x=1} f(1) + f(1) = 1 \\ \Rightarrow f(1) = \frac{1}{2}$$

Now put  $x = \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \dots, \frac{39}{40}$

$$\Rightarrow f\left(\frac{1}{40}\right) + f\left(\frac{39}{40}\right) = 1 \quad \checkmark$$

$$f\left(\frac{2}{40}\right) + f\left(\frac{38}{40}\right) = 1 \quad \checkmark$$

$$\vdots \quad \vdots$$

$$\overline{f\left(\frac{39}{40}\right) + f\left(\frac{41}{40}\right)} = 1$$

$$\text{also } f\left(\frac{40}{40}\right) = f(1) = \frac{1}{2}$$

Adding, we get

$$f\left(\frac{1}{40}\right) + \dots + f\left(\frac{39}{40}\right) = 39 + \frac{1}{2} \\ = \boxed{\frac{79}{2}}$$

If 11<sup>th</sup> wasted ...because of....then Revise this..

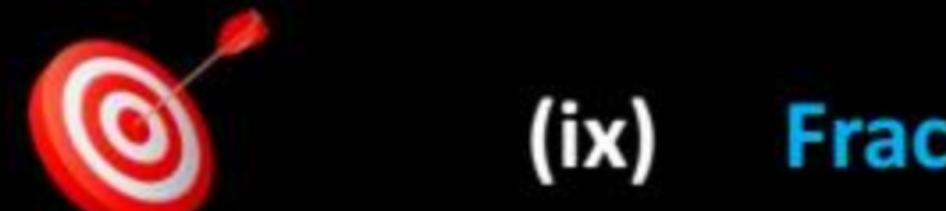
## Important Types Of Functions:

(i) Polynomial Function:

(iii) Fractional Rational Function:

(v) Logarithmic function:

(vii) Signum Function:



(ii) Algebraic Function:

(iv) Exponential Function:

(vi) Absolute Value Function:

(viii) Greatest Integer Or Step Up Function:

(ix) Fractional Part Function:

Homework  
till next class

If 11<sup>th</sup> wasted ...because of.....then Revise this..

# Very Important Terms For Calculus/Real life/Career:

Domain of Definition of Functions: (How to find?)



Range of the Functions: (Used in almost all topics in Mathematics\*)

{ well & defined  
in next class





*Thank You Lakshyians*