LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Relations & Functions

Lecture: 03

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Today's Goal: :

Types of Relations :

Definition of Functions:

Problems based on Types of Relations:

Reflexive, Symmetric, Transitive → equivalence

From NCERT:

Determine whether each of the following relations are reflexive, symmetric and transitive:

1. **Relation R in the set** $A = \{1, 2, 3, \ldots\ldots, 13, 14\}$ **defined as R =** $\{(x, y) : 3x - y = 0\}$

$\Rightarrow y = 3x$

$R : A \to A \Rightarrow R = \{(1,3), (2,6), (3,9), (4,12)\}$

Reflexive: $(a, a) \in R \Rightarrow$ Not $\Rightarrow$ Not reflexive

Symmetric: $(a,b) \in R \Rightarrow (b,a) \in R \Rightarrow$ Not symmetric

Transitive: $(1,3) \in R$ & $(3,9) \in R \Rightarrow (1,9) \in R \Rightarrow$ not transitive

2. **Relation R in the set** $A = \{1, 2, 3, 4, 5, 6\}$ **defined as R = { (x, y): y is divisible by x }**

Reflexive: (x, x) means x is divisible by x i.e. Reflexive

Symmetric

$x R y \Rightarrow y$ is divisible by x

$y R x \Rightarrow x$ is divisible by y

} 4 is divisible by 2 but 2 is not divisible by 4

$\Longrightarrow$ Not Symmetric

for transitive:

$x R y \Rightarrow y$ is divisible by $x \Rightarrow y = n_1 x \; ; \; n_1 \in I$

$y R z \Rightarrow z$ is divisible by $y \Rightarrow z = n_2 y \; ; \; n_2 \in I$

$x R z \Rightarrow z$ is divisible by x

$\Downarrow$

$z = n_2 (n_1 x)$

$\Rightarrow \boxed{z = (\text{integral})\, x}$

$\Rightarrow$ It is transitive

Problems based on Types of Relations:

From NCERT:

Determine whether each of the following relations are reflexive, symmetric and transitive:

3. **Relation R in the set A of human beings in a town at a particular time given by**

(a) R = { (x, y): x is exactly 7 cm taller than y }

$\Rightarrow x = y + 7$

Reflexive ✗ $\Rightarrow x = x + 7$ Not possible

Symmetric ✗ $x = y + 7$, $y = x + 7$ Not possible

Transitive ✗ $x = y + 7$, $y = z + 7$, $x = z + 14$

$xRy : x = y + 7$ ✓

$yRz : y = z + 7$ ✓

$xRz : x = z + 7$ ✗

(b) R = { (x, y): x is wife of y }

R → Not

S → Not

T → Yes ✓

(c) R = { (x, y): x is father of y }

R → Not

S → Not

T → Not

Problems based on Types of Relations:

From JEE MAIN (2010):

Determine whether each of the following relations are reflexive, symmetric and transitive:

R = { (x, y): x, y are real numbers and x = wy for some rational number of w}

R

Reflexive ⇒ Put $y=x \Rightarrow x=wx \Rightarrow w=1 \in Q$ i.e yes

Symmetric ⇒ $x=wy$ & $y=wx$ ⇒ not possible at a time except $w=1$

Transitive ⇒ $\left.\begin{matrix} x=wy \\ y=wz \end{matrix}\right\} \Rightarrow x=w^2z \neq wz$ ⇒ Not transitive

S = { $\left(\frac{m}{n}, \frac{p}{q}\right)$: m, n, p and q are integers such that n, q are non-zero and qm = pn }.

$qm=pn \Rightarrow \frac{m}{n}=\frac{p}{q}$

Reflexive: $\frac{m}{n}=\frac{p}{q}$ ⇒ Yes

Symmetric: $\frac{m}{n}=\frac{p}{q} \Rightarrow \frac{p}{q}=\frac{m}{n}$ ⇒ Yes

Transitive: $\frac{m}{n}=\frac{p}{q}$ & $\frac{p}{q}=\frac{r}{s} \Rightarrow \boxed{\frac{m}{n}=\frac{r}{s}}$ ⇒ Yes

R is not equivalence
S is equivalence

Problems based on Types of Relations:

Some More Standard Problems:

$R = \{(x, y) \mid x - y + \sqrt{2}$ is an irrational no; $x, y \in R\}$

Reflexive: Put $y = x \Rightarrow x - x + \sqrt{2} = \sqrt{2} \notin Q \Rightarrow$ True $\Rightarrow$ yes

Symmetric:

$x R y \Rightarrow x - y + \sqrt{2} \notin Q$

$y R x \Rightarrow y - x + \sqrt{2} \notin Q$

दोनों ही सही

$\Rightarrow$ not Symmetric

$x = \sqrt{2}$, $y = 1$

$\sqrt{2} - 1 + \sqrt{2} \notin Q$

$1 - \sqrt{2} + \sqrt{2} \notin Q$ i.e. wrong

Transitive:

$x R y \Rightarrow x - y + \sqrt{2} \notin Q$

$y R z \Rightarrow y - z + \sqrt{2} \notin Q$

$x R z \Rightarrow x - z + \sqrt{2} \notin Q$

$x = \sqrt{2}$, $y = 1$, $z = 2\sqrt{2}$

$\rightarrow 2\sqrt{2} - 1 \notin Q$ ✓

$\rightarrow 1 - 2\sqrt{2} + \sqrt{2} \notin Q$ ✓

$\rightarrow \sqrt{2} - 2\sqrt{2} + \sqrt{2} \notin Q$ ×

Home assignment :

$a, b \in R$

* Lcm of a & b = 4
* $|a-b| \leq 4$
* l_1 is parallel to l_2
* l_1 is perpendicular to l_2

+ Exercise 1.1 of RD Sharma
+ DPP

Thank You Lakshyians