LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Relations & Functions

Lecture: 01

Topics Covered:

Problems on Set Theorem

Cartesian Product & Its Significance:

Introduction of Relations:

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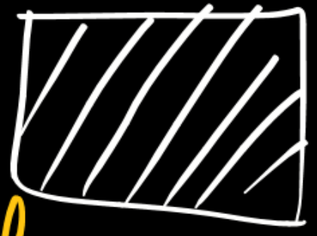
Basic Maths (Useful tool):

Set :⟶ $N \subset W \subset Z \subset Q \subset R \subset C$

Q ↓ rational nos

C ↓ complex no

* 0 is an even integer
* $\frac{1}{0} \neq \infty$ ← $\frac{1}{0}$ is not defined; ∞ is a very ... large but defined } used in LCD
* Lcm of q & $\sqrt{q}$ ⟶ not defined



* $x+2=3 \Rightarrow x=1$

$\Rightarrow x+2 = x+3 \Rightarrow x = ?\ \infty$

Problems on Set Theorem:

1, The number of subsets of a set with 2022 elements having an odd number of elements, is-

$A = \{1, 2, 3, 4, \ldots\ldots n\}$

* No. of total subsets $= {}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n$ } formulae

{ also ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + \ldots = 2^{n-1}$ } याद रखना है

Here, ${}^{2022}C_1 + {}^{2022}C_3 + \ldots + {}^{2022}C_{2021} = 2^{2022-1} = 2^{2021}$

Problems on Set Theorem:

$n(P(B)) = 2^4$ → $n(P(C)) = 2^5 = 32$

2. If A, B, C be three sets such that n (A) = 3, n (B) = 4, n (C) = 5. If P (X) denotes power set of X, $K = \frac{n(P(P(C)))}{n(P(P(A))).n(P(P(B)))}$. Sum of Digits of K is-

Power Set → Set of subset of a given set

$n(A) = 3$

$\Rightarrow n(P(A)) = 2^3 = 8$

$\Rightarrow n(P(P(A))) = 2^8$

$\Rightarrow K = \frac{2^{32}}{2^8 \times 2^{16}} = 2^8 = 256$

⇒ Sum of digits = 2 + 5 + 6 = 13

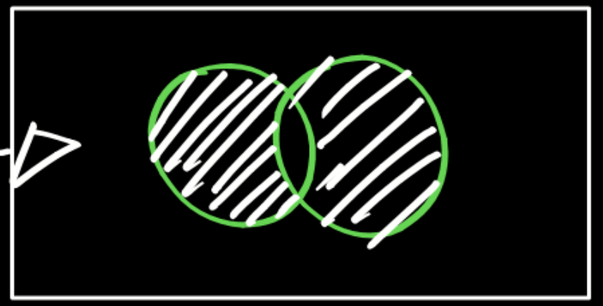
Analysis of Two or More Sets:

all known $\{ A \cup B \quad \cap \quad \sim \quad \Delta \rightarrow$ Symmetric difference

In Recap: $\{ (A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$, $(A')' = A$

$A \sim B$ means elements of set A but not in B.

$A \Delta B = (A \sim B) \cup (B \sim A)$ ⟶



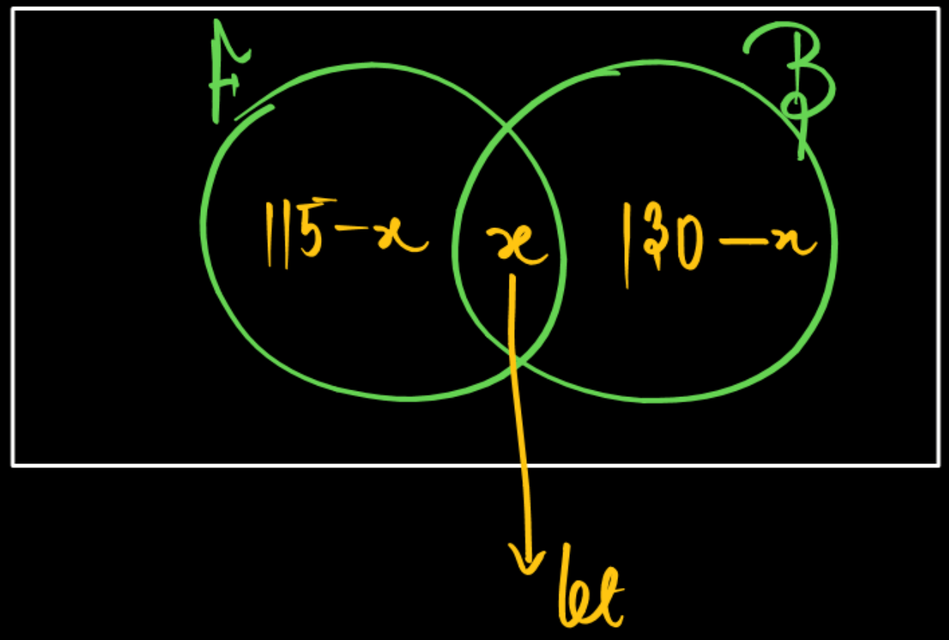
Basic Problems:

In a class of 195 students, 115 like football, 130 like basket ball. If each student likes one of football or basket ball. Then, how many like both football and basketball ?

Given, $(115-x)+x+(130-x) = 195$

$\Rightarrow 245 - x = 195$

$\Rightarrow \boxed{x = 50}$.



Standard Problems:

$12 = n((A \cap B)')$

Out of 14 people, twelve said that it was not the case that they watched television but did not listen to the radio. Also, for nine people it is not the case that they do not watch T.V. and do not listen to the radio. Finally, seven people either watch television or listen to the radio but do not do both. Now let A be number of people that watch T.V. and B be the number of people that listen radio. Then

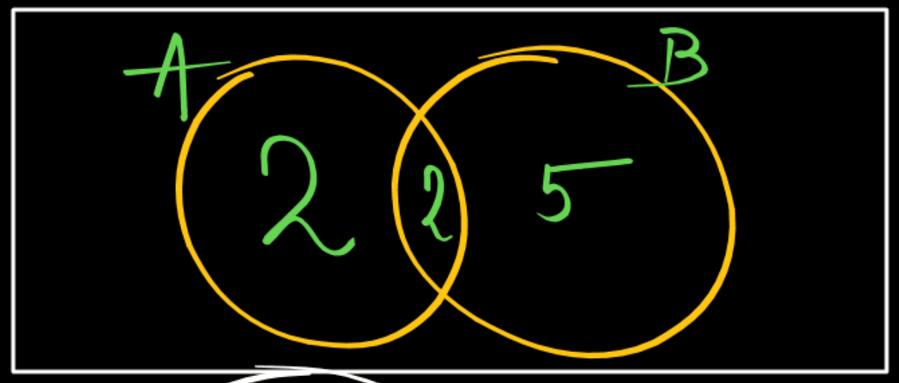
(A) A = 2
(B) B = 5
(C) A = 4
(D) B = 7

$n(U) = 14$

$n(A \cap B') = 2$

$n(A' \cap B') = 5 \Rightarrow n(A \cup B) = 9$

$n(A \Delta B) = 7$



$n(A) = 4, n(B) = 7$

Cartesian Product & Its Significance:

$A = \{1, 2, 3\}$

$B = \{a, b\}$

$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

$(1, a) \longrightarrow$ ordered pair where $1 \in A$, $a \in B$

$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$

$A \times B \neq B \times A$

$\Downarrow$

$(a, b) \neq (b, a)$

$\{a, b\} = \{b, a\}$

$A = \{1, -1\}$

$\Rightarrow A \times A = \{(1, 1), (1, -1), (-1, 1), (-1, -1)\}$

$\Rightarrow A \times A \times A = \{(1, 1, 1), (1, 1, -1), (1, -1, 1), (1, -1, -1), (-1, 1, 1), (-1, 1, -1), (-1, -1, 1), (-1, -1, -1)\}$

$(a_1, b_1) = (a_2, b_2)$

$\Rightarrow a_1 = a_2$ & $b_1 = b_2$

If $\left(2x^2 - 3, \frac{1}{5}\right) = \left(4, \frac{1}{y^2 - 4}\right)$

find x & y

Ans. $2x^2 - 3 = 4$

$\Rightarrow x = \pm\sqrt{\frac{7}{2}}$

& $\frac{1}{5} = \frac{1}{y^2 - 4}$

$\Rightarrow y^2 - 4 = 5$

$\Rightarrow \boxed{y = \pm 3}$

$(x, y) = (a^3 - 3ab^2, 3a^2b - b^3)$

& $\frac{x}{a} + \frac{y}{b} = k(a^2 - b^2)$

find k ?

$x = a^3 - 3ab^2 \Rightarrow \frac{x}{a} = a^2 - 3b^2$

$y = 3a^2b - b^3 \Rightarrow \frac{y}{b} = 3a^2 - b^2$

adding, $\frac{x}{a} + \frac{y}{b} = 4a^2 - 4b^2$

$= 4(a^2 - b^2)$

$\boxed{k = 4}$

Introductions of Relations:

Defn → Relation is defined as subset of cartesian product of two non-empty sets

$A = \{1, 2\}$; $B = \{a, b\} \Rightarrow A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

No. of subset of $A \times B$ are $2^4 = 16$

like $\phi, \{(1, a)\}, \{(1, b)\}, \ldots, \ldots, A \times B$.

Here $\{(1, a)\}$ means 1 is related with a

ie $1 R a$

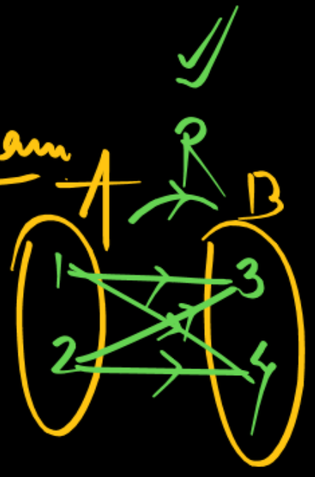
Representations of Relations:

$A = \{1, 2\}$, $B = \{3, 4\}$ $\Rightarrow A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

* Roaster form $\rightarrow$ $R = \{(1,3), (1,4), \ldots\}$
* Set-builder form $\rightarrow$ property of elements are given

$$R = \{(x, y) \mid x + y = 5, \text{ where } x \in A, y \in B\}$$
$$= \{(1,4), (2,3)\}$$

* Arrow diagram ✓

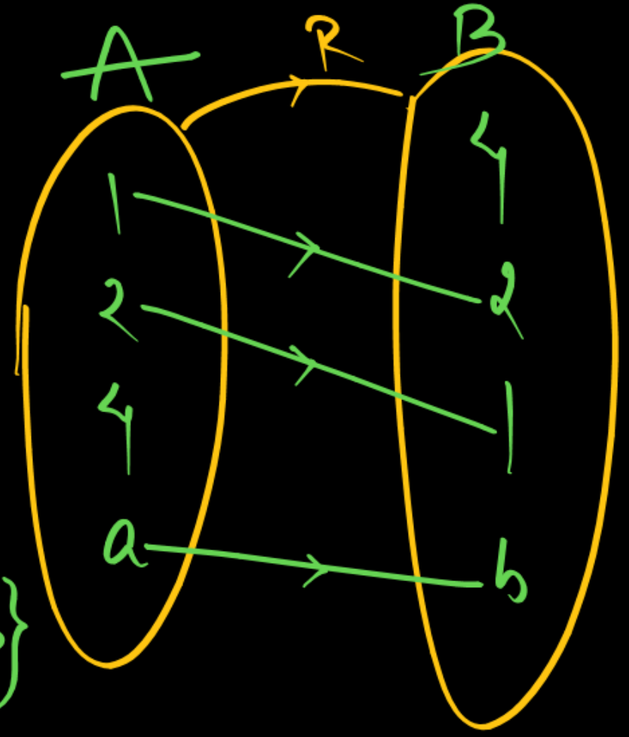


Domain, Codomain & Range of Relations:

Domain : All related elements of Set A $= \{1, 2, a\}$

Co-domain : All elements of set B $= \{4, 2, 1, b\}$

Range : All related elements of set B. $= \{2, 1, b\}$



Problems based on Domain & Range:

The relation R from A to B is given as R = {(5, 3), (2, 7), (8, 5)}. The range of R is

(A) {5, 2, 8} (B) {3, 7, 5} (C) {2, 3, 5, 7, 8} (D) {2, 3, 5, 7}

$A = \{5, 2, 8\}$

$B = \{3, 7, 5\}$

Wrong

But Domain $= \{5, 2, 8\}$

Range $= \{3, 7, 5\}$

Problems based on Domain & Range:

Let A = {4, 5, 7} and B = {2, 4, 6} be two sets and let a relation R be a relation from A to B is defined as $R=\{(x,y):x<y, x\in A, y\in B\}$, then the difference between the sum of elements of domain and range of R is-

(a) 2 **(b)** 3 **(c)** 4 **(d)** 5.

$R=\{(x,y): x<y,\ x\in A,\ y\in B\}$

$=\{(4,6),(5,6)\}$

Domain $=\{4,5\}$

Range $=\{6\}$

Sum of elements in domain $= 4+5=9$

in Range $= 6$

diff $= 9-6=3$

Thank You Lakshyians