LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Relations & Functions

Lecture: 10

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Today's Goal: :

Examples on Domain/Range of the Functions:

Types of Mappings:

Number of Mappings(Functions):

Recap on Domain/Range of the Functions :

Find the range of $f(x) = \log_2\left\{\dfrac{(\sin x - \cos x) + 3\sqrt{2}}{\sqrt{2}}\right\}$

$$-\sqrt{a^2+b^2} \leq a\cos\theta + b\sin\theta \leq \sqrt{a^2+b^2}$$

$$\Downarrow$$

$$\boxed{-\sqrt{2} \leq \sin x - \cos x \leq \sqrt{2}}$$

$$\Downarrow$$

$$2\sqrt{2} \leq (\sin x - \cos x) + 3\sqrt{2} \leq 4\sqrt{2}$$

$$\Downarrow$$

$$2 \leq \frac{(\sin x - \cos x) + 3\sqrt{2}}{\sqrt{2}} \leq 4$$

$$\Rightarrow \log_2 2 \leq f(x) \leq \log_2 4$$

$$\Rightarrow \text{Range} = [1, 2]$$

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Recap on Domain/Range of the Functions :

$$f(x) = \sqrt{\log_e(\cos(\sin x))}$$

Range ?

$D_f = \{n\pi\}$, $n \in I$

For $f(x)$ is to be real:

$$\log_e(\cos(\sin x)) \geq 0$$

$\log_e t \geq 0$

$\Rightarrow t \geq e^0$

$\Rightarrow t \geq 1$

$$\Rightarrow \cos(\sin x) \geq e^0$$

$$\Rightarrow \boxed{\cos(\underbrace{\sin x}_{0}) \geq 1}$$

possible iff $\cos(\sin x) = 1$

$$\text{Range} = \sqrt{\log_e 1} = \sqrt{0} = \{0\}$$

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Recap of Types of Mappings:

1. One- One Onto Functions (Most Useful) Done
2. One- One Into Functions
3. Many- One Onto Functions
4. Many- One Into Functions

How to Check One-One/ Many-One?

$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$ One-One

$\Rightarrow x_1 \neq x_2$ many-one

$y = f(x)$

$\Rightarrow f'(x) \geq 0 \ \forall x \in D_f$

OR

$f'(x) \leq 0 \ \forall x \in D_f$

then One-One fn

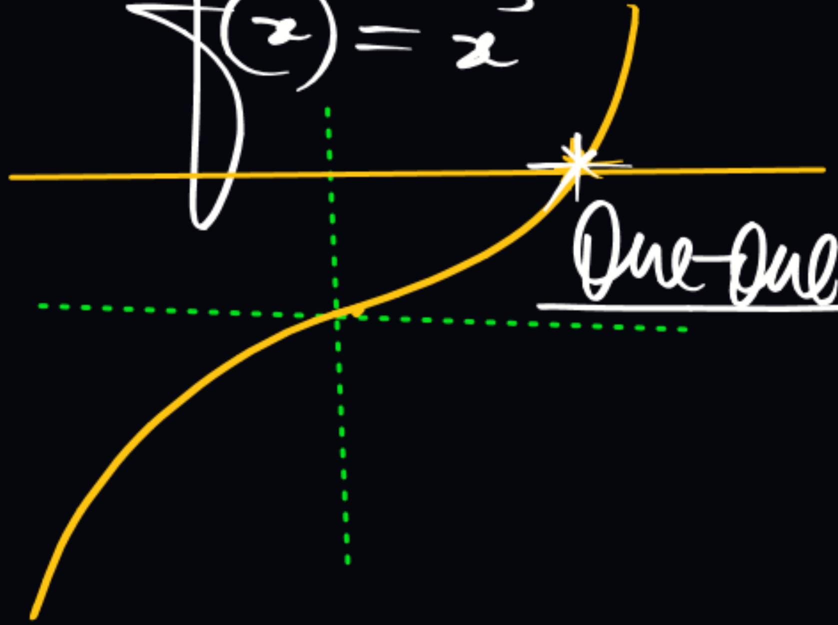
elsewhere many-one

Method-3

If a line Parallel to x-axis cuts the graph of given function Only at One Point, then the function is One-one function* eg. $f(x) = |x|$

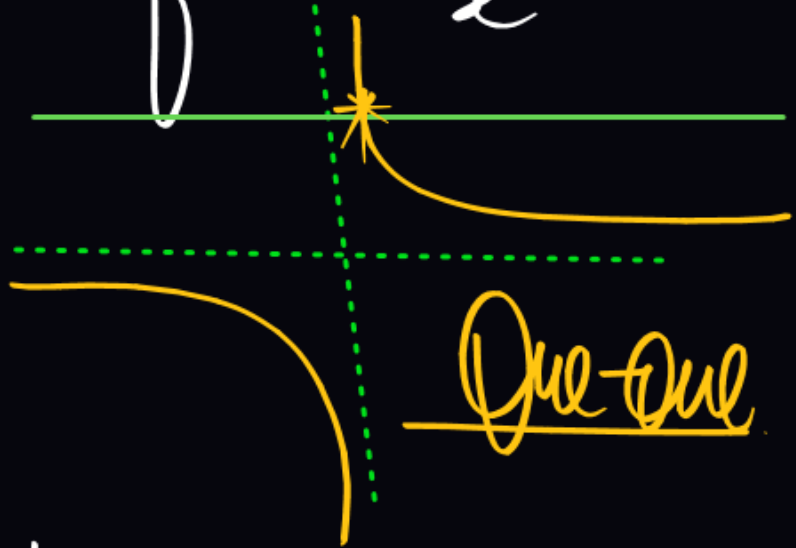
Many-One

$f(x) = x^3$



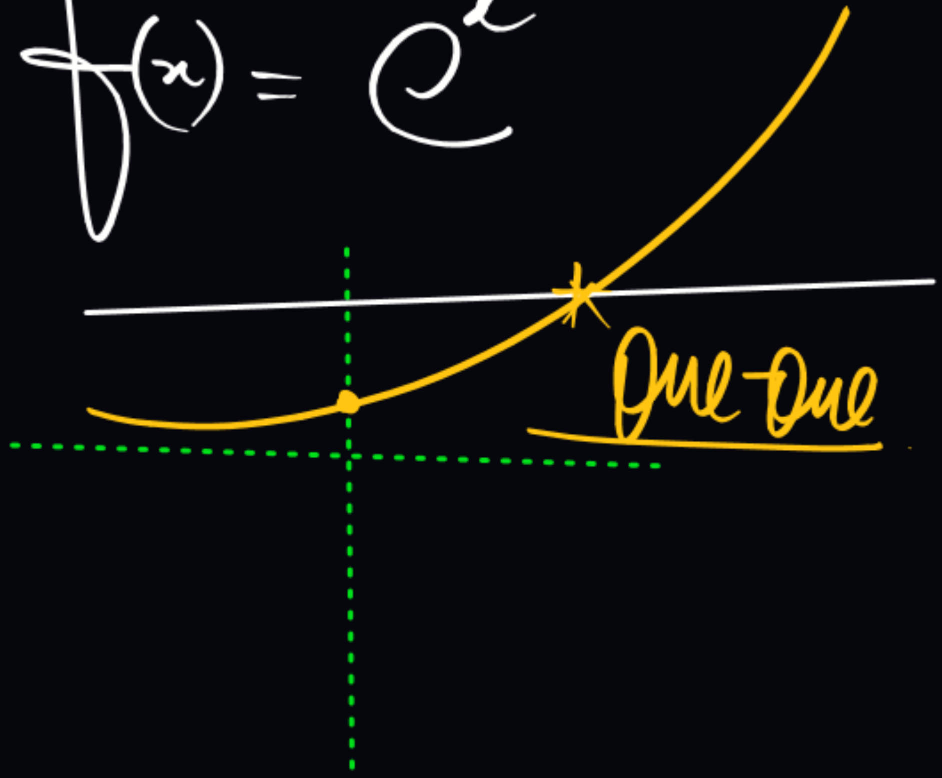
One-One

$f(x) = \frac{1}{x}$



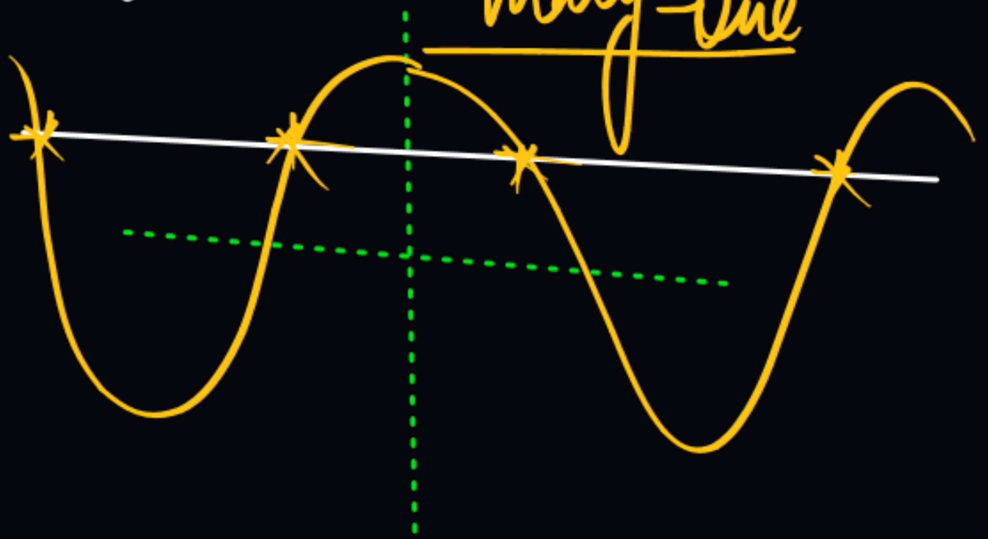
One-One

$f(x) = e^x$



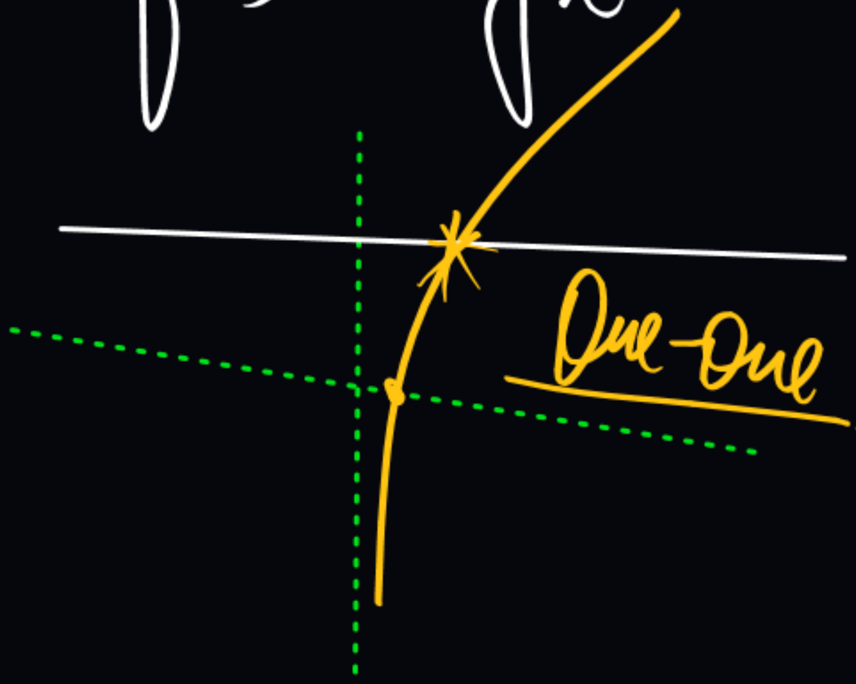
One-One

$f(x) = \cos x$



Many-One

$f(x) = \log x$



One-One

Some Basic Examples (From NCERT):

Prove that the greatest integer function f: R to R given by f (x) = [x] , is neither one-one nor onto, where [x] denotes g.i.f less than or equal to x

Done On last class

$f(x) = \underbrace{[x]}_{\text{integer}}$ is many-one

Co-domain = R

Range = Integer = Z
≠ co-domain

Into fn

How to Check Onto/Into?

$y = f(x)$; $f : A \to B$

Change x in terms of y

let $x = g(y)$

If for every $y \in B$, there exist $x \in A$ then Onto

Otherwise into

A: x → B: y

$f : R \to R$
$f(x) = 2x+3$
Onto/Into?

let $f(x) = y$

$\Rightarrow 2x + 3 = y$

$\Rightarrow \boxed{x = \dfrac{y-3}{2}}$

As for every real y; there exist $x \in R$; so $f(x)$ is Onto

$f : N \longrightarrow N$ $\{N \to \text{natural nos}\}$

$f(x) = 2x+3$ check Onto/into?

let $f(x) = y$

$\Rightarrow 2x+3 = y$

$\Rightarrow \boxed{x = \dfrac{y-3}{2}}$

at $y=1 \Rightarrow x = -1 \times \notin N$

i.e for every natural value of y, we are not getting natural value of x

So, Into f^n.

Range $= \{5, 7, 9, \ldots\}$
$\neq N$

Some Basic Examples (From NCERT):

Let A = R − {3} and B = R − {1}. Consider the function f: A to B defined by $f(x) = \left(\dfrac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

Testing of One-One / Many-one

let $x_1, x_2 \in A$ such that

$f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$\Rightarrow \cancel{x_1 x_2} - 3x_1 - 2x_2 + \cancel{6}$$

$$= \cancel{x_1 x_2} - 2x_1 - 3x_2 + \cancel{6}$$

$$\Rightarrow \boxed{x_1 = x_2} \Rightarrow f(x) \text{ is One-One}$$

$$f(x) = \frac{x-2}{x-3}$$

$f: A \to B$ $\quad A = R - \{3\}$

$B = R - \{1\}$

Testing of Onto | Into

Let $f(x) = y$

$$\Rightarrow \frac{x-2}{x-3} = y$$

$$\Rightarrow x - 2 = yx - 3y$$

$$\Rightarrow x(1-y) = 2 - 3y$$

$$\Rightarrow x = \frac{2-3y}{1-y}$$

as for every $y \in B$, there exist $x \in A$ $\Rightarrow f(x)$ is Onto

$$f(x) = \sin x$$

$f : R \to R \longrightarrow$ Many-one into fn

$f : R \to [-1, 1] \longrightarrow$ Many-one onto fn

$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R \longrightarrow$ One-One into fn

$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1] \longrightarrow$ One-one onto fn

$f : [0, \pi] \to [-1, 1] \longrightarrow$ Many-one into fn

$0 \leq x \leq \pi$

$\Rightarrow \boxed{\sin x \geq 0}$

$\Downarrow$

Range $= [0, 1] \neq [-1, 1]$

Some Standard Examples:

Examples on classification: $\rightarrow f : [2, \infty) \rightarrow Y$

(e) The function f : [2, infinite) to Y defined by $f(x) = x^2 - 4x + 5$ is both one-one and onto if :

$= (x-2)^2 + 1 \geq 1$

(A) Y = R (B) Y = [1, Infinite)

(C) Y = [4, Infinite) (D) Y = [5, Infinite)

$\rightarrow f(x)$ is One-One Onto $\Rightarrow$ Co-domain = Range $\Rightarrow Y =$ Range of $f(x)$

$= [1, \infty)$

(2) (a) $f(x) = x^3 - 2x^2 + 5x + 3$ is one-one-onto

(b) $f : R$ to R $f(x) = 2x^3 - 6x^2 - 18x + 17$ is many one onto function

$$f(x) = \boxed{x^3 - 2x^2 + 5x + 3}$$

i.e $f'(x) > 0 \ \forall x \in R$

$\Rightarrow f(x)$ is One-One f^n

for One-One | Many-one

$$f'(x) = 3x^2 - 4x + 5$$

$$= 3\left\{x^2 - \frac{4}{3}x + \frac{5}{3}\right\}$$

$$= 3\left\{x^2 - 2\cdot x\cdot\frac{2}{3} + \left(\frac{2}{3}\right)^2 - \frac{4}{9} + \frac{5}{3}\right\}$$

$$= 3\left\{\left(x - \frac{2}{3}\right)^2 + \frac{11}{9}\right\} > 0$$

as $f'(x) > 0$

$\Rightarrow f(x)$ is increasing f^n

$f(0) = 3$

i.e Onto

Range = R

Numbers of Mappings (Functions):

will be discussed On next <u>class</u>

Thank You Lakshyians