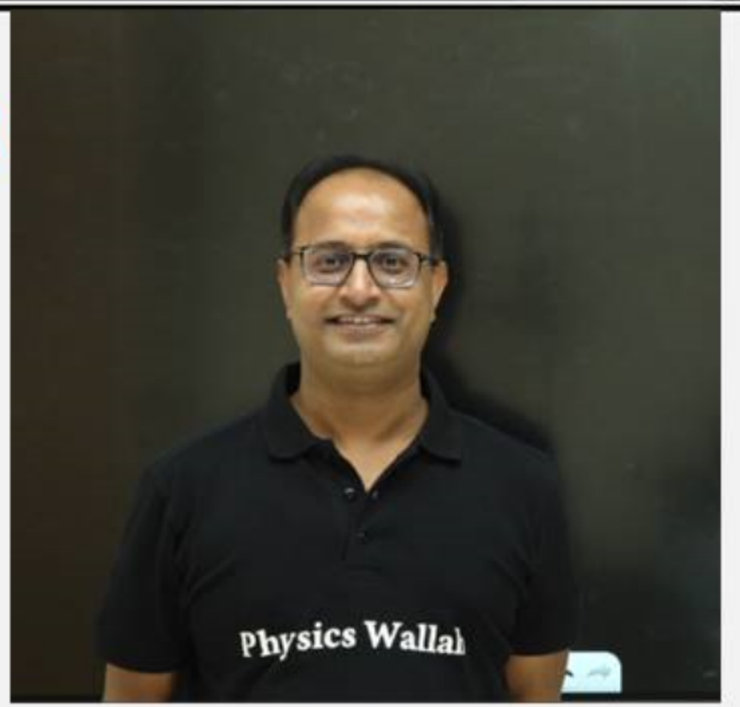
LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Relations & Functions

Lecture: 05



By: KUNDAN KUMAR
(B-Tech, IIT-BHU)
17+ years Teaching Experience

Today's Goal: :

Types of Basic Functions (11th Class):

Basic Problems of Domain of the Functions:

If 11th wasted ...because of.....then Revise this..

Important Types Of Functions:

(i) Polynomial Function:
(ii) Algebraic Function:
(iii) Fractional Rational Function:
(iv) Exponential Function:
(v) Logarithmic function:
(vi) Absolute Value Function:
(vii) Signum Function:
(viii) Greatest Integer Or Step Up Function:
(ix) Fractional Part Function:

Types Of Functions:

(i) Algebraic Function: → $+, -, \times, \div$ between variable & constt
$\Rightarrow +2, x^3 - x + 4, \ldots$

(ii) Polynomial Function: → non-negative degree
eg: $2x^2 + x$, $4x^2 - x^3 + 7x^6$
{$2x - \frac{1}{x}$ is not polyn

(iii) Fractional Rational Function:
$\frac{P(x)}{Q(x)}$ → $P(x)$ → Polynomial
$Q(x)$ → Polynomial $\neq 0$

$x^2 + 1 + \frac{1}{x} \Big\} = \frac{x^3 + x + 1}{x}$ Rational ✓

ILATE
- I → Inverse Trigo.
- L → Logarithm
- A → Algebraic
- T → Trigo.
- E → Exponential

Types Of Functions:

$x^2-3x+2=0$ has roots 1 & 2

$\Rightarrow x^2-3x+2=(x-1)(x-2)$

Basic Problems:

Cubic eqn i.e 3 roots

Let $P(x) = x^3 + ax^2 + bx + c$ be a polynomial such that $P(1) = 1$; $P(2) = 2$; $P(3) = 3$; then find the value of $P(4)$.

$P(1)=1 \Rightarrow P(1)-1=0$

$P(2)=2 \Rightarrow P(2)-2=0$

$P(3)=3 \Rightarrow P(3)-3=0$

$\Rightarrow P(x)-x=0$ has roots 1, 2 & 3

$\Rightarrow P(x)-x=(x-1)(x-2)(x-3)$

Put $x=4$

$\Rightarrow P(4)-4=(4-1)(4-2)(4-3)=6$

$\Rightarrow \boxed{P(4)=10}$

Types Of Functions:

Good Problems:

Let $P(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ be a polynomial such that $P(1) = 1$; $P(2) = 2$; $P(3) = 3$; $P(4) = 4$; $P(5) = 5$ and $P(6) = 6$ then find the value of $P(7)$.

From given data, $P(x) - x = 0$ has 6 roots 1, 2, 3, 4, 5 & 6

$\Rightarrow P(x) - x = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$

$\Rightarrow P(x) = x + (x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$

So, $P(7) = 7 + 6 \times 5 \times 4 \times 3 \times 2 \times 1 \Rightarrow P(7) = 7 + 720 = 727$

Types Of Functions:

(iv) Exponential Function:

eg $2^x = 3^y = 6^z$ find relation in x, y & z

$2^x = 3^y = 2^z \cdot 3^z = k$ (let)

$\Rightarrow 2 = k^{1/x}$

$3 = k^{1/y}$

$6 = k^{1/z} = 2 \times 3 = k^{1/x} \times k^{1/y}$

$\Rightarrow k^{1/z} = k^{\frac{1}{x}+\frac{1}{y}} \Rightarrow \boxed{\frac{1}{z} = \frac{1}{x} + \frac{1}{y}}$

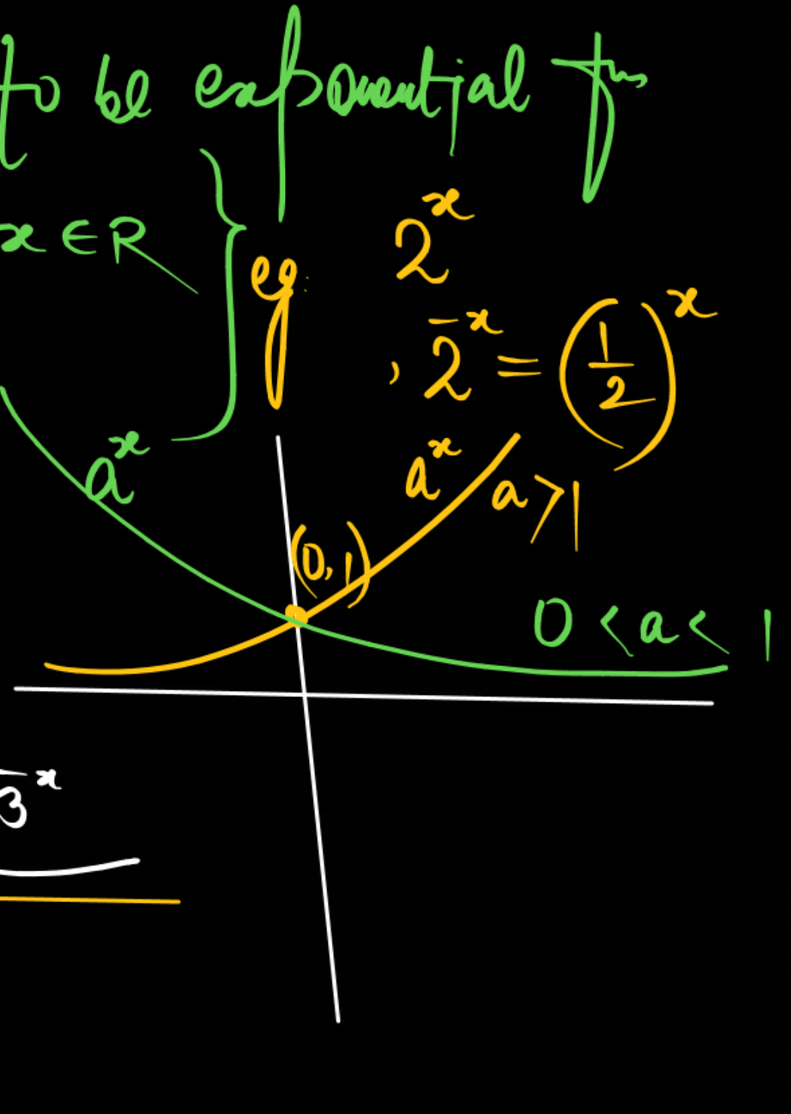
$f(x) = a^x$ is stated to be exponential fn iff $a > 0$ & $a \neq 1$ & $x \in R$

$\Rightarrow f(0) = a^0 = 1$

$f(x) > 0$

eg 2^x, $\bar{2}^x = \left(\frac{1}{2}\right)^x$

$y = \bar{3}^x = \left(\frac{1}{3}\right)^x$



Types Of Functions:

(v) Logarithmic function:

$x^2 = 3 \Rightarrow \boxed{x = \pm\sqrt{3}}$

$x^2 = 5 \Rightarrow \boxed{x = \pm\sqrt{5}}$

If $2^x = 5$ → To solve exponential eqn, we need a new f

$\Rightarrow x = \boxed{\log_2 5}$ read as log of 5 to base 2

$\log_2 4 = 2$

$\log_5 5 = 1$, $\log_{125} 5 = \frac{1}{3}$

OR $\log_{(\frac{1}{4})} 2 = x$ (let)

$\left(\frac{1}{4}\right)^x = 2 = 4^{1/2}$

$= \left(\frac{1}{4}\right)^{-1/2}$

$\Rightarrow \boxed{x = -1/2}$

$2^x = 5 \Rightarrow x = \log_2 5$

In general,

$a^x = b \Rightarrow x = \log_a b$

$a > 0$, $a \neq 1$

(5) $\log_a b = \dfrac{\log_c b}{\log_c a}$

(6) $\log_a b = \dfrac{1}{\log_b a}$

(7) $a^{\log_a x} = x$

(8) $a^{\log_c b} = b^{\log_c a}$

$\log_{10} 1 = 0$; $\log_{10} 2 = 0.3010$; $\log_{10} 3 = 0.4771$ $\Rightarrow$ more useful in chem

To solve problems we use following properties:

(1) * $\log_a(x^n) = n \log_a x$ *

(2) * $\log_{a^m}(x) = \dfrac{1}{m} \log_a x$

(3) $\log_a(xy) = \log_a x + \log_a y$ *

(4) $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$ *

Types Of Functions:

$\log_a x = \log_a y \Rightarrow x = y ;\ x, y > 0$

The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is:

(A) 3 (B) 1

(C) 2 (d) 0

[JEE Adv. 2001]

$x - 1 > 0 \Rightarrow x > 1$

$x - 3 > 0 \Rightarrow x > 3$

$\log_4(x-1) = \log_2(x-3)$

$\Rightarrow \log_{2^2}(x-1) = \log_2(x-3)$

$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\log_2(x-1)^{1/2} = \log_2(x-3)$

$\Rightarrow \sqrt{x-1} = x-3$

Squaring, $x - 1 = x^2 - 6x + 9$

$\Rightarrow x^2 - 7x + 10 = 0$

$x = 2, 5$

as $x > 3$

$\Rightarrow x = 5$

$x > 3$

Thank You Lakshyians