LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Relations & Functions

Lecture: 08

By: **KUNDAN KUMAR**
(B-Tech, IIT-BHU)
17+ years Teaching Experience

Today's Goal: :

Range of the Functions:

Types of Mappings: XII fn

Recap on Domain of the Functions :

Domain of $f(x)$

$y = f(x)$

x ?

$f(x) =$

$x+2 \to R$ (Set of real nos)

$x^2+3 \to R$

$\frac{1}{1+x}$ is real iff $1+x \neq 0 \Rightarrow x \neq -1$

$D_f = R - \{-1\}$

$\frac{1}{\sqrt{1-x}}$ is real iff $1-x > 0 \Rightarrow \boxed{x < 1}$

$\Rightarrow D_f = (-\infty, 1)$

$\frac{1}{\sqrt{x-|x|}} \to$? $D_f = \phi$

$x - |x| > 0$

Case 1 : $x > 0 \Rightarrow |x| = x$

$x - x > 0 \Rightarrow 0 > 0$ ✗

Case 2 : $x < 0 \Rightarrow |x| = -x$

$x - (-x) > 0 \Rightarrow 2x > 0 \Rightarrow x > 0$ ✗

Recap on Domain of the Functions :

$$\log_{g(x)} f(x) \Rightarrow \left\{ \begin{array}{l} f(x) > 0 \\ g(x) > 0 \\ g(x) \neq 1 \end{array} \right\}$$

$$f(x) = \frac{x^2+2x+1}{x^2-6x+8}$$

for $f(x)$ is to be real,

$x^2-6x+8 \neq 0$

$\Rightarrow (x-2)(x-4) \neq 0$

$\Rightarrow \boxed{x \neq 2, 4}$

$\Rightarrow D_f \equiv R - \{2, 4\}$

$$f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$$

$D_f \equiv ?$

for $f(x)$ is to be real; $x+3 > 0$

$\Rightarrow \boxed{x > -3}$

& $x^2+3x+2 \neq 0$

$\Rightarrow (x+1)(x+2) \neq 0 \Rightarrow \boxed{x \neq -1, -2}$

$D_f \equiv (-3, \infty) - \{-1, -2\}$

Range of the Functions :

$\Downarrow$

Mirror image of Domain.

So, for finding Range ? first find Domain.

$y = f(x)$

Case 1: If Domain $= R$, then try to change x as a fn of y

Say $x = g(y)$

Now find y to get real x

& Range is y

eg: $f(x) = \frac{1}{1+x^2}$ $\boxed{D_f \equiv R}$

$\Rightarrow y = \frac{1}{1+x^2}$

$\Rightarrow 1+x^2 = \frac{1}{y}$

$\Rightarrow x^2 = \frac{1}{y} - 1 = \frac{1-y}{y}$

Range of the Functions :

As, $x^2 = \frac{1-y}{y}$

$\Rightarrow x = \pm\sqrt{\frac{1-y}{y}}$

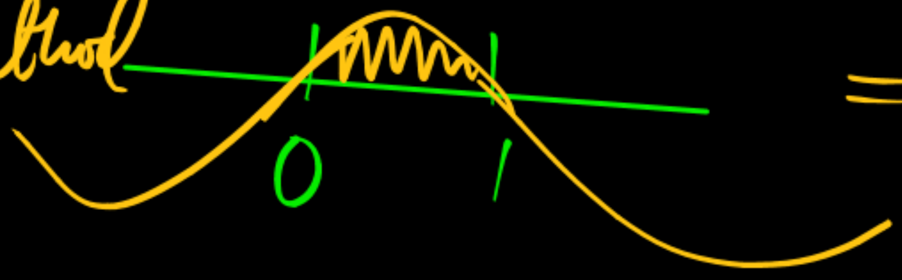
For x is to be real;

$\frac{1-y}{y} \geq 0$ & $y \neq 0$

$1-y \rightarrow y = 1$

$y \rightarrow y = 0$

{Wavy-curve method}



$\Rightarrow y \in (0, 1]$

Range $\equiv (0, 1]$

Alter:

* $f(x) = \frac{1}{1+x^2}$

As, $0 \leq x^2 < \infty$

Range $\equiv (0, 1]$

* $f(x) = \frac{x^2}{1+x^2} = \frac{(x^2+1)-1}{1+x^2}$

$\Rightarrow f(x) = 1 - \frac{1}{1+x^2}$

As $0 \leq x^2 < \infty$

$\Rightarrow R_f = [0, 1)$ ✓

Range of the Functions :

$$f(x)=\frac{x^2+x+2}{x^2+x+1}\,;\ R_f=?$$

Here $D_f=R$ as $x^2+x+1>0\ \forall\ x\in R$

$$\Downarrow$$

$$x^2+2\cdot x\cdot\frac{1}{2}+\frac{1}{4}+\frac{3}{4}$$

$$\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$$

$$\Rightarrow f(x)=\frac{(x^2+x+1)+1}{x^2+x+1}$$

$$\Rightarrow f(x)=1+\frac{1}{x^2+x+1}$$

$$\Rightarrow f(x)=1+\frac{1}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$\Downarrow$$

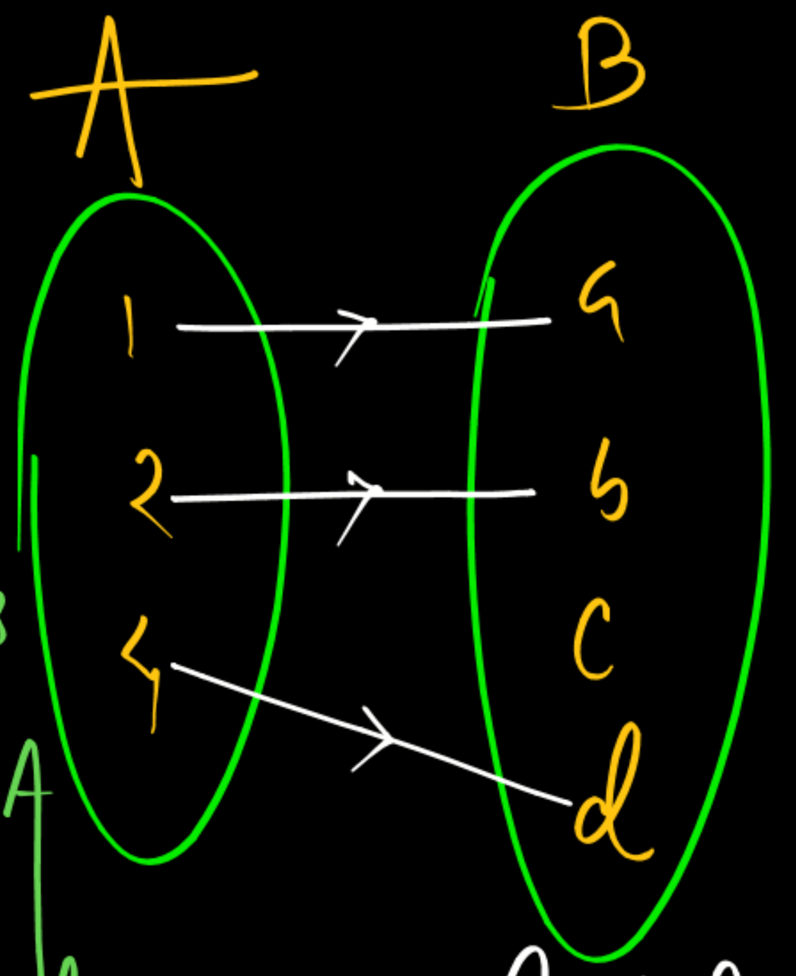
$$[0,\infty)$$

$$\text{Range}=\left(1,\frac{7}{3}\right]$$

Types of Mappings:

1. One- One Onto Functions (Most Useful)
2. One- One Into Functions
3. Many- One Onto Functions
4. Many- One Into Functions

One-One
Many-one } depends on set A



Onto → each of set B
into → at least one in set B having no relation with set A

depends on set B

One-One into fn

Thank You Lakshyians