FTL Analysis results

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Major defendable contributions

From the thorough theoretical analysis of the DFTL method as well as the extensive experimental analysis carried out corresponding to different methods (with the results included for **n**=100 in the subsequent figures), these are the following major observations, and conclusions which I think are defendable (look at all the plot once before reading the further text):

- Experimental evaluations to suggest that DFTL method gives an improper velocity update equation, especially for higher values of Time step (as the steady state velocities are reported non-zero). This error is much more evident for s=1.
- Experimental evaluations to suggest that the DFTL method velocities get closer to the physically accurate velocities for smaller values of time step size (for s=1)
- Theoretical Analysis of the DFTL method to prove that it converges to a physically accurate solution(for s=1)
- Experimental comparisons between DFTL method and BE method to show that DFTL converges to the BE solution for lower values of time step size (for s=1)
- Experimental evaluations to conclude that DFTL will be unstable for values of s lower than 1. Conclusion that the main purpose of a lower s is to make the system less artificially damped by adding more energy to the system in a nonphysical manner.
- Getting a modification to the DFTL method, **FTL Mem**, which would have the following improvements/properties:
  - For the value of s=1, even Mem will converge to the physically accurate solution as DFTL
  - For any value of s, Mem will have a proper velocity update equation (as compared to DFTL)

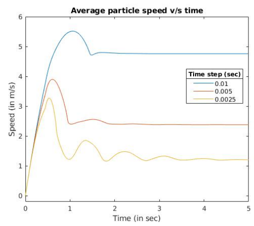
– For the value of s=1, Mem will have better Energy plots than DFTL (because of the improper DFTL velocities). If we take care of the improper DFTL velocities while getting the DFTL energies, then the energies of FTL Mem are very similar to that of DFTL

– For other values of s (for which we would perform the experimentations i.e values which are lesser to one but close to it, like 0.9), FTL Mem has its energies lying above the DFTL energies reducing the artificial damping of the system. The Mem simulations are faster and have higher amplitudes, suggesting less damping

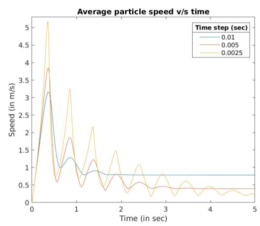
Note that we wouldn't perform simulations for very low values of s, since it will be closer to the traditional FTL update which would get unstable even for very high time step value.

If we look at the equations of the DFTL system and the FTL Mem system then we can see that the only difference between them technically lies in the velocity update equation only, as the extra term which is introduced by the DFTL method to correct the issue of improper mass distribution at the n^{th} time instant will be evident in the RHS of the equation while calculating the predicted positions of the particles at the $(n+1)^{th}$ time instant. It's just that the DFTL method uses an extra term in the velocity equation which decreases monotonously with decreasing time step size, making the DFTL method stable.

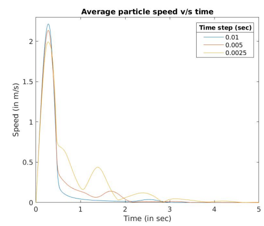
That's why instead of taking an average of the correction terms of i^{th} and $(i+1)^{th}$ particles in the predicted position of i^{th} particle in FTL Mem, if we consider the correction term for just $(i+1)^{th}$ particle (just as done in DFTL $\mathbf{d}_{i+1}$) then the exact same plots are obtained for both the methods.



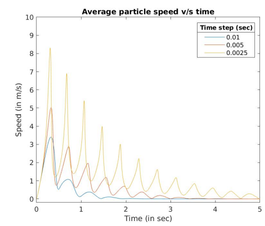
(a) DFTL, s=1



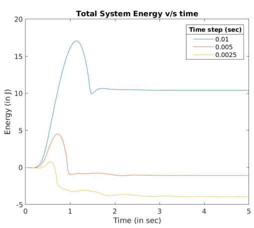
(b) DFTL, s=0.9



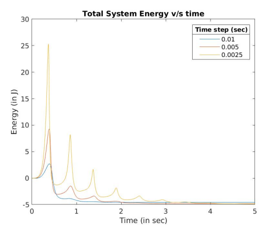
(c) Mem, s=1



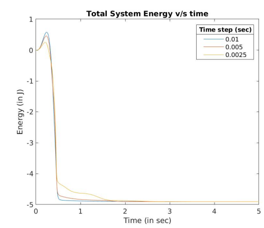
(d) Mem, s=0.9



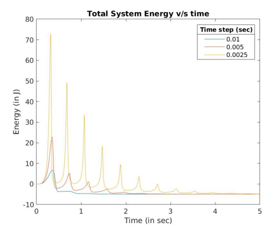
(e) DFTL, s=1



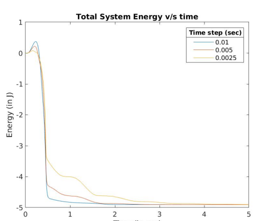
(f) DFTL, s=0.9



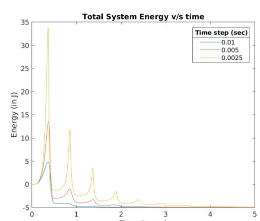
(g) Mem, s=1



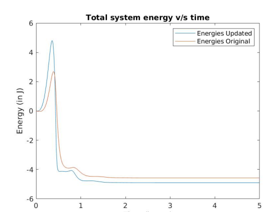
(h) Mem, s=0.9



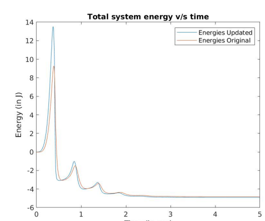
(i) DFTL, s=1,corrected vel



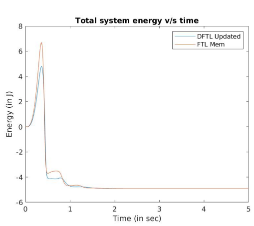
(j) DFTL, s=0.9,corrected vel



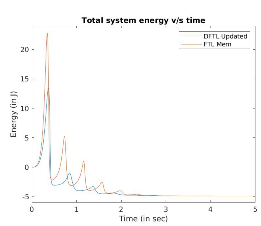
(k) DFTL, s=0.9, TS=0.01s



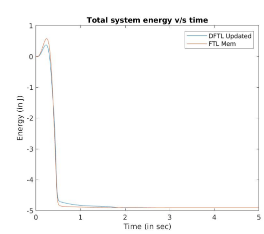
(l) DFTL, s=0.9, TS=0.005s



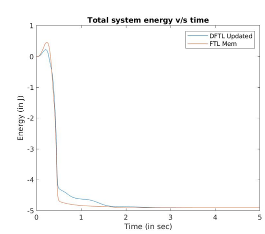
(m) s=0.9, TS=0.01s



(n) s=0.9, TS=0.005s

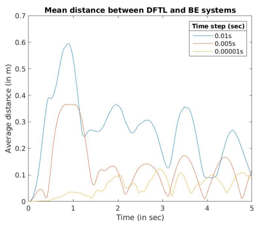


(o) s=1.0, TS=0.01s

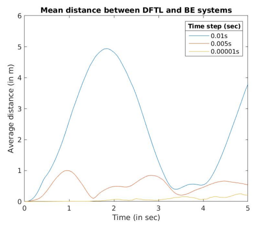


(p) s=1.0, TS=0.005s

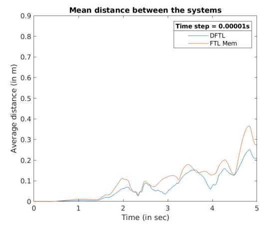
Figure 1: All the Energy and Velocity plots corresponding to **n**=100. TS means the time step involved and s means $s_{damping}$. In the sub-figures (i) and (j), "corrected vel" refers to the velocities of the DFTL system being considered for the Kinetic Energy calculation without the additional term. In the sub-figures (k) and (l): the "Energies Updated" correspond to the DFTL energies with corrected velocity. In the sub-figures (m) etc, "DFTL Updated" corresponds to the Energies of DFTL system with corrected velocity.



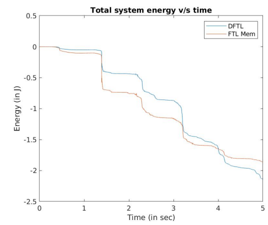
(a) BE with spring & s=1



(b) BE w/o spring & s=1

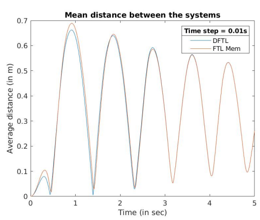


(c) TS = 10^{-5}s, s=1

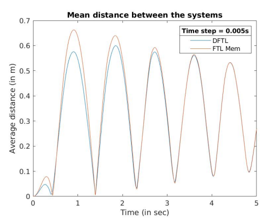


(d) TS = 10^{-5}s, s=1

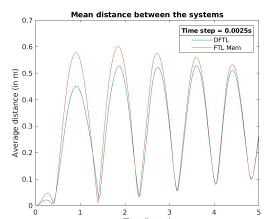
Figure 2: All the Convergence Analysis plots corresponding to **n**=100. The sub-plot (c) has been made w.r.t the Constrained Dynamics BE system (w/o spring-mass assumptions) simulated on 10^{-5}s



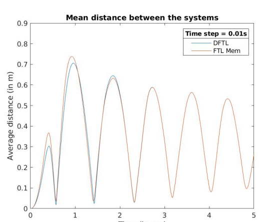
(a) s=1 and TS=0.01s



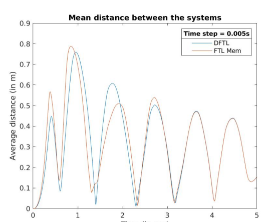
(b) s=1 and TS=0.005s



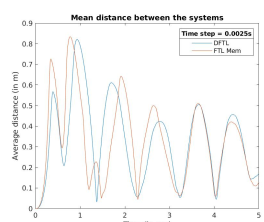
(c) s=1 and TS=0.0025s



(d) s=0.9 and TS=0.01s

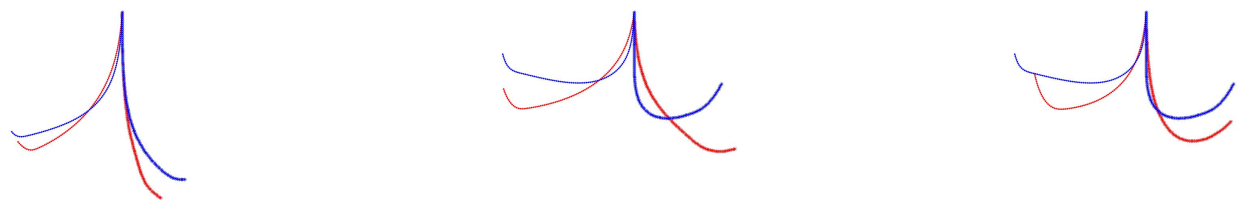


(e) s=0.9 and TS=0.005s



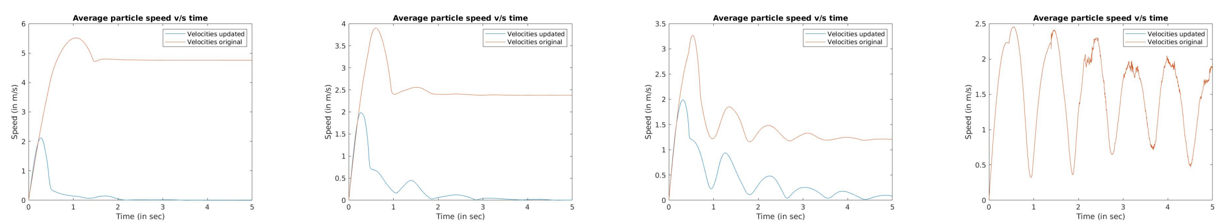
(f) s=0.9 and TS=0.0025s

Figure 3: All the Positional Accuracy plots corresponding to **n**=100. TS means the time step involved and s means $s_{damping}$. We can see for s=1 the distances get lower with decreasing time step size, while for a lower value of s the distances get higher with decreasing time step size. In general, the accuracies for both these methods are approximately the same for all the cases

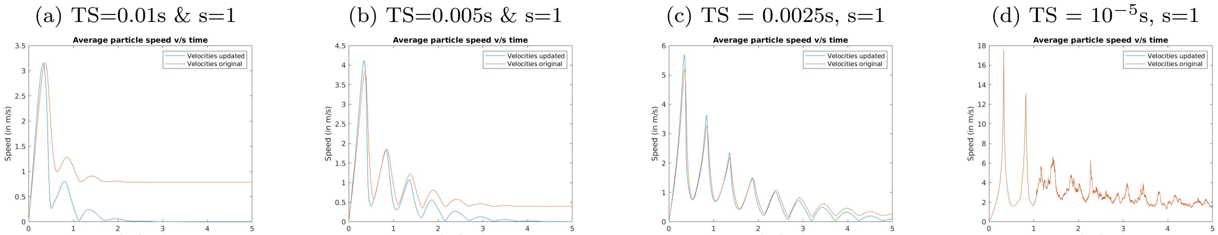


(a) TS=0.01s, t≈0.5s and t≈1s (b) TS=0.005s, t≈0.5s and t≈1s (c) TS=0.005s, 1^{st} & 2^{nd} amplitudes

Figure 4: Simulation comparison plots for these methods for the value of **n** to be 100. The blue particles correspond to FTL Mem and the Red ones are for DFTL. The value of s is 0.9. Here, the dotted markers correspond to the earlier frame, while the crossed markers correspond to a later frame, where **t** denotes the time frame. The total time of simulation for all the cases is 5 seconds. **Amplitude** is defined as the configuration of the system at the instant where the last particle of the system is attaining its highest point i.e a time instant after which all the particles in the system will start moving downwards



(a) TS=0.01s & s=1 (b) TS=0.005s & s=1 (c) TS = 0.0025s, s=1 (d) TS = 10^{-5}s, s=1



(e) TS=0.01s & s=0.9 (f) TS=0.005s & s=0.9 (g) TS = 0.0025s, s=0.9 (h) TS = 10^{-5}s, s=0.9

Figure 5: All the Velocity Convergence Analysis plots corresponding to **n**=100. Here one can see that for any value of s, the predicted DFTL velocities and the corrected DFTL velocities converge towards each other. Though for s=1 only the real velocities are also decreasing in their values; while for s=0.9, the velocities keep blowing up for smaller values of time step, indicating that the system will be unstable as predicted